Necessary and Sufficient Delay-Independent Stability Conditions for Commensurate Time Delay Systems

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Abstract This paper presents an algebraic delay-independent stability (DIS) test for the commensurate multiple time delay systems (CMTDSs). We provide necessary and sufficient conditions for stability test. The stability is analysed using stability tests of two univariate polynomials and a generalized eigenvalue problem. The proposed stability approach is advantageous compared to the existing methodologies because it includes less computational complexity. Numerical examples are given to demonstrate the applicability and effectiveness of the proposed approach.

Keywords Delay-independent stability · Algebraic · Linear systems · Commensurate · Multiple time delays

1 Introduction

The motivation to investigate the stability of time delay systems (TDSs) in the context of practical and commercial applications comes from the fact that instability and unexpected changes in system performance [1]. The examples of TDSs are such as gyroscopic system [2] and load frequency control system [3]. Consequently, the stability analysis of TDS has become the most important for qualitative study of TDS [1–7]. The stability of multiple time delay systems (MTDSs) has also gained ample attention amidst TDSs (see [4–7], and references therein). The class of MTDS generally grouped into commensurate [4] and noncommensurate [6] multiple time delay systems where commensurate multiple time delay systems (CMTDS) [4], delay terms rationally depend on a single delay otherwise, it is known as noncommensurate MTDS [6]. The investigation on commensurate multiple time delay system (CMTDS) has become increasingly relevant research because systems having com-

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mensurate type delays can be found in various engineering applications, for instance, N-machine-M-Bus power system [9] and unmanned underwater vehicle [10]. The stability of CMTDSs has been widely investigated over the years [9–14]. Roughly speaking, the stability criteria of TDS can be divided into two broad categories: delay-dependent stability (DDS) [9] and delay-independent stability (DIS) [4], whether or not, the information of delay term is required or not, respectively. In most of the practical systems, either it is not feasible to compute delay terms or delay parameters cannot determine exactly or both; hence, the DIS criterion is preferred [9].

Some of the DIS tests for CMTDS available in the literature are: frequency sweeping tests [4], a polynomial (auxiliary characteristic equation) method [6], a graphical approach using bivariate polynomial [12], zero criteria-based bivariable polynomial approach [13, 15], frequency-dependent one-dimensional (1D) Lyapunov equation [16], and 2D Lyapunov equation [17]. In [14], the stability condition of TDS is given in terms of LMI. The necessary and sufficient stability conditions for DIS of CMTDS were proposed in [8, 13], but later it was established in [15] that these conditions are sufficient. Some sufficient conditions for CMTDS are derived in terms of frequency-dependent 1D Lyapunov equation [16]. In [17], an algebraic procedure for a 2D continuous system was developed using the Lyapunov equation and Kronecker product. These conditions were established to provide Hurwitz stability of polynomial merely. A 2D algebraic technique to check the Routh–Schur stability of multiple TDS was established as sufficient conditions in [18]. The Hurwitz–Schur stability test was derived algebraically in [19] for TDS, but this procedure provided merely sufficient conditions for stability. Thus, Lyapunov approaches, 2D Hurwitz–Schur stability, and LMI techniques result in distinct conservatisms and sufficient conditions. Therefore, the above-discussed stability tests provide merely sufficient conditions. Different from these approaches, an algebraic method for necessary and sufficient conditions for TDSs is required. In [20], an algebraic approach for 2D filters (discrete domain) was established as stability tests of three 1D conditions and one generalized eigenvalue problem (GEVP). Some necessary and sufficient stability conditions to test the stability of 2D linear systems (continuous, discrete, and hybrid) are developed using two polynomials and one GEVP [21].

Motivated by this observation and the stability tests of 2D linear systems in [21], this stability approach can be applied to CMTDSs. In this paper, we investigate the necessary and sufficient conditions for DIS of CMTDS in algebraic manner using this novel stability test. This method has a lower computing complexity and a limited number of 1D tests when compared to previous results [16–19]. The proposed DIS test for CMTDS is based on the bivariate polynomial that is derived using the system's characteristic polynomial. Only two univariate polynomial stability tests and a GEVP are required. To demonstrate the procedure and validity of the proposed approach, numerical examples are given.

Notations. The real and complex number sets are used to represent $\mathbb{R}$ and $\mathbb{C}$ in this study. $\mathbb{R}_+$ states the set of positive real number set. The determinant and transpose of matrix Z are represented by $\det(Z)$ and $(Z)^T$, respectively. $\mathrm{Re}(y)$ stands for real part of y, and conjugate of y is represented by $\bar{y}$. The open left half complex plane and the closed right half complex plane are represented by $\mathbb{C}_-$ and $\mathbb{C}_+$, respectively. The closed unit disc is represented by $\mathbb{D}$, and its complementary set is denoted by $\overline{\mathbb{D}}$. $\delta\mathbb{D}$ states the boundary of unit disc. $*$ denotes the symmetric matrix blocks and the notation iff stands for the if and only if. The $n \times n$ identity matrix is represented by I_n, and 0_n is used for $n \times n$ zero matrix. l is used to represent $\sqrt{-1}$ in this paper.

2 Problem Formulation

In this section, the DIS problem of CMTDS using an algebraic approach is discussed. To illustrate this goal, consider general form of CMTDS represented by delay differential equation as

$$\dot{x}(t) = A_0 x(t) + \sum_{k=1}^{m} B_k x(t - \tau_k), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $A_0, B_1, \ldots, B_m \in \mathbb{R}^{n\times n}$ are known matrices, $\tau_k = k\tau$ $(\tau \in \mathbb{R}_+\{\tau \geq 0\})$ are the commensurate delays, and $k = 1, 2, \ldots, m$. The CMTDS (1) is said to be asymptotically stable if and only if the roots of the characteristic polynomial given by

$$\tilde{P}(s, e^{-\tau s}) \triangleq \det\left(sI - A_0 - \sum_{k=1}^{m} B_k e^{-k\tau s}\right), \tag{2}$$

are located in the left half of the complex plane. Where s is the Laplace operator. By considering $z = e^{-\tau s}$, the bivariate polynomial for characteristic polynomial (2) can be rewritten as follows:

$$\tilde{P}(s, z) \triangleq \det\left(sI - A_0 - \sum_{k=1}^{m} B_k z^k\right). \tag{3}$$

Our objective in this paper is to propose algebraic and simple necessary and sufficient conditions for checking the DIS of CMTDS (1) with its corresponding bivariate polynomial (3). In the next section, we recall some useful definitions for the stability analysis of CMTDS using characteristic polynomial in the next section.

3 Preliminaries

In this paper, we are interested in the DIS analysis of CMTDS (1) using an algebraic procedure. To this end, we interpret here some definitions of stability which are indispensable to derive the main results of this paper. The polynomial (3) of CMTDS (1) can be rewritten as irreducible bivariate polynomial as

$$P(s,z) \triangleq \sum_{i=0}^{n_2}\sum_{j=0}^{n_1} c_{ij}s^i z^j, \tag{4}$$

where n_1 and n_2 are the orders of s and z variables, respectively. For the abovementioned bivariate polynomial (4) of CMTDS (1), we introduce the following definitions.

Definition 1 *A univariate polynomial $d(\lambda)$ is Hurwitz when stability region is $\mathbb{C}_-$ and Schur when stability region is $\mathbb{D}$ with respect to λ iff*

$$d(\lambda) \neq 0, \forall\lambda \in \mathbb{C}_+ \textit{ or } \lambda \in \overline{\mathbb{D}}. \tag{5}$$

This paper deals with DIS of the system (1) with bivariate polynomial $\tilde{P}(s,z)$ (4).

Definition 2 *The CMTDS given in (1) is DIS if*

$$\tilde{P}(s,z) \neq 0, \forall s \in \mathbb{C}_+ \textit{ and } z \in \mathbb{D}. \tag{6}$$

Remark 1 In Definition (2), the discrete TDSs or noncommensurate TDSs consist of $D_1 = D_2 = \mathbb{D}$ (closed unit disc) and stability region for z_1, z_2 variables will be in $D_1 \times D_2$ region.

The stability test for CMTDS (1) is derived using the characteristic bivariate polynomial (4), which will be presented in the next section.

4 Algebraic Necessary and Sufficient Delay-Independent Stability Conditions

In this section, we use the results from Sect. 3 to determine the necessary and sufficient DIS conditions for CMTDS (1).

Lemma 1 *[22] The CMTDS in (1) with its associated characteristic bivariate polynomial (4) is stable if, it holds the following equivalent conditions*

$$P(\hat{s},z) \textit{ is stable in terms of } z, \textit{ for a } \hat{s} \in \mathbb{C}_-, \tag{7}$$

$$P(s,\hat{z}) \textit{ is stable in terms of } s, \ \forall \hat{z} \in \mathbb{D}. \tag{8}$$

The conditions (7) and (8) are equal to each other. Further, we merely discuss here condition (8). The stability test of univariate polynomial in z can be done using (7). The main problem in Lemma 1 is with condition (8), where stability is checked for bivariate polynomial (4). This problem can be solved by transforming some univariate polynomial's stability tests and then solving a generalized eigenvalue problem (GEVP). The bivariate polynomial (4) can be rewritten as follows

$$P(s,z) = P_{a(z)}(s) = \sum_{i=0}^{n_2} c_i(\mathbf{Z})\, s^i = \sum_{i=0}^{n_2} C_i^T \mathbf{Z}\, s^i, \tag{9}$$

where $C_i \triangleq \left[c_{i0},\, c_{i1}\;, c_{i2},\, \ldots,\, c_{in_1}\right]^T$ and $\mathbf{Z} = \left[1,\, z,\, z^2,\, \ldots\, z^{n_1}\right]^T$. Consider that $\overline{P}_{a(z)}(s)$ is equal to its conjugate $P_{a(z)}(s)$ (due to real coefficients) (9) and $\overline{P}_{a(z)}(s)$ can be represented as

$$\overline{P}_{a(z)}(s) = \sum_{i=0}^{n_2} \overline{C}_i(z) s^i = \sum_{i=0}^{n_2} \overline{C}_i^T \bar{Z} s^i. \tag{10}$$

By multiplying (9) and (10), we get new polynomial that is defined as

$$F_{a(z)}(s) \triangleq P_{a(z)}(s)\overline{P}_{a(z)}(s) = \sum_{i'=0}^{n_2}\sum_{i=0}^{n_2} C_i^T \Psi(z)\overline{C}_i s^{i+i'}, \tag{11}$$

where $\Psi(z) \triangleq \frac{1}{2}(\overline{Z}Z^T + Z\overline{Z}^T)$. For stability tests of the (7) and (8) conditions, $F_{a(z)}(s)$ and $\Psi(z)$ are needed to be computed over $z \in \overline{D}$. The matrix $\Psi(z)$ for $z \in \overline{\mathbb{D}}$ can be represented as follows:

$$\Psi(z) = \frac{1}{2} \times \begin{bmatrix} 2 & & & & \\ z+\overline{z} & 2(z\overline{z}) & & * & \\ z^2+\overline{z}^2 & z\overline{z}(z+\overline{z}) & 2(z\overline{z})^2 & & \\ \vdots & \vdots & & \ddots & \\ z^{n_1}+\overline{z}^{n_1} & z\overline{z}(z^{n_1-1}+\overline{z}^{n_1-1}) & \cdots & & 2(z\overline{z})^{n_1} \end{bmatrix}. \tag{12}$$

Lemma 2 *For CMTDS in (1) $z = \overline{\mathbb{D}} = (e^{i\theta} : 0 \le \theta < 2\pi)$ then $\Psi(z) = \gamma(\tilde{z})$, where $\tilde{z} = Re(z)$, $\tilde{z}$ should not belong to $T \triangleq [-1, 1]$, and $\gamma_{i,j}(\tilde{z}) = S_{|i-j|}(\tilde{z})$, $S_0(\tilde{z}) = 1$,*

$$\begin{aligned} &S_1(\tilde{z}) = \tilde{z},\, S_2(\tilde{z}) = 2\tilde{z}^2 - 1, \ldots, S_{n_1+1}(\tilde{z}) = 2\tilde{z}S_{n_1}(\tilde{z}) - S_{n_1-1}(\tilde{z}), \\ &\qquad j = 1, 2, \ldots, n_1 - 1. \end{aligned} \tag{13}$$

Proof From $z = \mathbb{D} = (e^{l\theta} : 0 \le \theta < 2\pi), l = \sqrt{-1}$ it follows that $\tilde{z} \notin T$, and $S_j(x)$ is the first type Tchebyshev polynomial [22], we get,

$$\Psi(z)|_{z\in\overline{\mathbb{D}}} = \gamma(\tilde{z}) = \begin{bmatrix} S_0(\tilde{z}) & & & & \\ S_1(\tilde{z}) & S_0(\tilde{z}) & & * & \\ S_2(\tilde{z}) & S_1(\tilde{z}) & S_0(\tilde{z}) & & \\ \vdots & \vdots & \ddots & & \\ S_{n_1}(\tilde{z}) & S_{n_1-1}(\tilde{z}) & \cdots & S_1(\tilde{z}) & S_0(\tilde{z}) \end{bmatrix}. \tag{14}$$

By using (11) and Lemma 2, we get

$$F_{a(z)}(s)|_{\tilde{z}\in\overline{\mathbb{D}}} = F_{a(\tilde{z})}(s) \triangleq \sum_{i=0}^{n_2}\sum_{i'=0}^{n_2} C_i^T \gamma(\tilde{z})\overline{C}_i s^{i+i'}. \tag{15}$$

Remark 2 From (11), it is noted that $F_{a(z)}(s)$ is of order $2n_2$ and its characteristics will be as same as $P(s, z)$ for stability test. Hence, (8) can be tested by using $F_{a(z)}(s)$, rather than $\tilde{P}(s, z)$. Further, using (12) it is noted that $\Psi(z)$ is real and symmetric matrix. Hence, we observe using (11) and (12), $F_{a(z)}(s)$ is a real polynomial. In our case, conditions (7) and (8) are equal to the stability test of $F_{a(\tilde{z})}(s)$ (15) with respect to $s \forall \tilde{z} \in T$ where $T \triangleq [-1, 1]$. For computation of $F_{a(\tilde{z})}(s)$ (15) and condition (11), we follow the next lemmas and definitions.

Definition 3 *[22] The Jury matrix of dimensions* $(2r-1)\times(2r-1)$ *denoted by* $\Delta[V(\sigma)]$ *for the polynomial* $V(\sigma) = \sum_{h=0}^{2r} V_h \sigma^h$ *with the specified stability region* $\mathbb{C}_- \times \overline{\mathbb{D}}$, *where* $V_h \in \mathbb{R}$ *is as follows:*

$$z = \overline{\mathbb{D}}, \text{ then } \Delta[V(\sigma)] \triangleq K - L, \tag{16}$$

$$\text{where } K = \begin{bmatrix} V_{2r} & V_{2r-1} & V_{2r-2} & \cdots & V_2 \\ & V_{2r} & V_{2r-1} & \cdots & V_3 \\ & & V_{2r} & \cdots & V_4 \\ & 0 & & \ddots & \vdots \\ & & & \cdots & V_{2r} \end{bmatrix}, \text{ and } L = \begin{bmatrix} & & & & V_0 \\ 0 & & & \iddots & \vdots \\ & & V_0 & \cdots & V_{2r-4} \\ & V_0 & V_1 & \cdots & V_{2r-3} \\ V_0 & V_1 & V_2 & \cdots & V_{2r-2} \end{bmatrix}.$$

Lemma 3 *Consider* $F_{a(z)}(s)$ *in (15) is stable in respect to* $s, \forall z \in \overline{\mathbb{D}}$ *iff det*$(\Delta[F_{a(\tilde{z})}(s)])$ *and all elements of* $R(\tilde{z})$ *are positive* $\forall \tilde{z} \in T$, *where*

$$R(\tilde{z}) \triangleq \left\{F_{a(\tilde{z})}(0) \text{ for } s = \mathbb{C}_-\right\}. \tag{17}$$

The condition (8) is checked by finding the roots of univariate polynomials and with the solution of generalized eigenvalue problem (GEVP) in next lemma and theorem.

Lemma 4 *Consider $T \triangleq [-1,\ 1]$ and condition (7) and following statements are equal,*

(a) $P(s,\ \tilde{z})$ is stable in respect to s for a $\tilde{z} \in \delta\mathbb{D}$,
(b) All the elements of $R(\tilde{z})$ (17) are not equal to zero $\forall\ \tilde{z} \in T$,
(c) and the GEVP,

$$\begin{bmatrix} 0 & I & & & \\ & & I & & 0 \\ & 0 & & I & \\ & & & \ddots & \\ \alpha_0 & \alpha_1 & & \cdots & \alpha_{n_1-1} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{n_1-1} \\ w_{n_1} \end{bmatrix} = \tilde{z} \begin{bmatrix} I & & & & \\ & I & & 0 & \\ & & \ddots & & \\ & & & I & 0 \\ 0 & 0 & & \cdots & -\alpha_{n_1-1} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{n_1-1} \\ w_{n_1} \end{bmatrix} \tag{18}$$

of dimensions $n_1(2n_2-1) \times n_1(2n_2-1)$ does not have any roots of GEVP in T, where

$$\alpha_j \triangleq \Delta[C_j(s)], \quad j = 0, 1, \ldots, n_1. \tag{19}$$

When $F_{a(\tilde{z})}(s),\ i = 0, 1, 2, \ldots, n_2$ (15) can be rewritten as follows

$$F_{a(\tilde{z})}(s) = F_{a(s)}(\tilde{z}) \triangleq \sum_{j=0}^{n_1} C_j(s)\tilde{z}^j. \tag{20}$$

Proof The Remark (2) states that stability region for condition (7) in respect to $s, \forall\tilde{z} \in \delta\mathbb{D}$, is equal to the stability test of $F_{a(\tilde{z})}(s)$ in (15) in respect to s can be violated for a $\tilde{z} \in T$ so that one of the elements of $R(\tilde{z})$ is zero. Therefore, $F_{a(\tilde{z})}(s)$ is stable with respect to $s, \forall\tilde{z} \in T$ iff *(a)*. $F_{a(\tilde{z})}(s)$ is stable with respect to s for $\tilde{z} \in T$, (b). All elements of $R(\tilde{z})$ are not equal to zero $\forall\tilde{z} \in T$, and (c). $\det(\Delta[F_{a(\tilde{z})}(s)]) = 0$ does not have any root in T. Using (16) and (20), the Jury matrix for $F_{a(\tilde{z})}(s)$ can be rewritten as

$$\det(\Delta[F_{a(\tilde{z})}(s)]) = \det\left(\sum_{j=0}^{n_1} \Delta[C_j(s)]\tilde{z}^j\right) = 0. \tag{21}$$

We consider the expansion of polynomial (19) as GEVP as follows:

$$(\alpha_0 + \alpha_1\tilde{z} + \alpha_2\tilde{z}^2 + \cdots + \alpha_{n_1}\tilde{z}^{n_1})w = 0, \tag{22}$$

where $w \in \mathbb{R}^{2n_1-1}$ and we define α_j are considered here $j = 0, 1, \ldots, n_1$, where $w_1 \triangleq w,\ w_2 \triangleq \tilde{z}w_1, \ldots, w_{n_1} = \tilde{z}w_{n_1-1}$. Therefore, the DIS condition of CMTDS (1) can derive using the following theorem.

Theorem 1 *Assume that CMTDS (1) with bivariate polynomial (4), and* $T \triangleq [-1, 1]$, *for* $s \in \mathbb{C}_-$ *and* $z \in \overline{\mathbb{D}}$, *respectively. By considering Definitions 1 and 2, CMTDS (1) is delay-independent stable iff, it holds the following statements,*

(a) $P(\hat{s}, z)$ *is stable in respect to* z *for a* $\hat{s} \in \mathbb{C}_-$.
(b) $P(s, \hat{z})$ *is stable in respect to* s, *for a* $\hat{z} \in \delta\mathbb{D}$.
(c) All the members of $R(\tilde{z})$ *in (17) do not have any real roots in* T.
(d) The GEVP in (18) contains no eigenvalue in $T \in [-1, 1]$.

Proof *Using the aforementioned lemmas and definitions, we prove theorem.*

Using above-mentioned Theorem 1 and Lemmas 1–4, the DIS of CMTDS (1) can be algebraically tested using the following algorithm.

Algorithm 1 An algorithm for delay-independent stability for CMTDS (1)

1: Obtain the bivariate polynomial of CMTDS (1) as (4).
2: State the stability region for both variables ($\mathbb{C}_- \times \overline{\mathbb{D}}$), and check DIS condition (a) of theorem 1 and follow the next step. If it is not satisfied, then CMTDS is unstable.
3: Rewrite the bivariate polynomial (4) as (9) and compute C_i.
4: Using Lemma 2 compute $\gamma(\tilde{z})$, $F_{a(\tilde{z})(s)}$, and $R(\tilde{z})$ using (17).
5: Test condition (c) of Theorem 1, If condition is not satisfied, then CMTDS (1) is unstable otherwise follow the next step.
6: Compute $F_{a(\tilde{z})}(s)$ and $C_j(s)$ (20) for $j = 0, 1, \ldots, n_1$.
7: Using Lemma 2 and Definition 3 compute Jury matrix.
8: Obtain the GEVP (18), check the condition (d) of theorem 1, if condition is satisfied then CMTDS (1) is stable, otherwise unstable.

Furthermore, the applicability of the Algorithm **1** can be seen in the next section using numerical example.

5 Numerical Example and Discussion

In this section, numerical example demonstrates the applicability of the proposed algebraic method, and the approach is also compared to existing methods.

Example 1 Consider the following second-order CMTDS (1) (motor-driven pendulum with multiple delays in feedback [25]) with following data,

$$A_0 = \begin{bmatrix} 0 & 1 \\ -59.9568 & -1.4584 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 & 0 \\ -13.5602 & 0 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1.2339 \end{bmatrix}, \quad (23)$$

and $\tau_2 = 2\tau_1$.The CMTDS with given data is delay-free stable, and roots are $-0.1122 \pm \imath 8.5735$. We investigate the necessary and sufficient conditions of DIS of CMTDS using algorithm **1** in the following steps as

(i) Step 1, the characteristic bivariate polynomial for CMTDS is obtained as

$$P(s,\ z) = s^2 - 1.2339sz^2 + 1.4584s + 13.5602z + 59.9568 \tag{24}$$

(ii) Step 2, the stability region $\mathbb{C}_- \times \overline{\mathbb{D}}$ and univariate polynomial $P(0,\ z)$ is stable and its roots are $-0.7292 \pm l7.7088$.
(iii) Step 3, for $P(s, 1)$ polynomial the roots are $-0.1122 + l8.5735$ and $-0.1122 - 8.5735l$. Therefore, $P(s, 1)$ is stable.
(iv) Step 4, $F_{a(\tilde{z})}(s)$ is $-91.9528s^2z - 1.7997s^2 + 27.1205sz^2 + 19.7769sz + 983.3501s + 10141.1348z^2 + 564.7471z + 4280.6336$ and roots of $F_{a(\tilde{z})}(0)$ are $-0.0278 \pm l0.649$. It shows that the $R(\tilde{z})$ does not have any real roots in $T \in [-1, 1]$.
(v) Step 5, by solving GEVP in (18) we find the roots of GEVP are 525.13, $-232.2 \pm l222.3$, $228.3 \pm l236.6$ and 6.20, respectively. Hence, there is no real eigenvalue lie in $T \in [-1, 1]$.

CMTDS (23) is delay-independent stable. For the comparison, the DIS of CMTDS is also investigated using delay Lyapunov matrix in [24], and the results discussed in [24] conclude the sufficient conditions of stability only with more computational time. The algebraic DIS approach developed into single formalism with complex computations and required the computation of α free parameter [23].

6 Concluding Remarks

In this paper, we have proposed necessary and sufficient conditions for testing the delay-independent stability of time delay systems of commensurate type algebraically. The approach we followed here requires merely two univariate polynomials stability tests and a generalized eigenvalue problem. Using this approach, we reduce the mathematical computational burden over existing methods, even if the multiplicity of delay increases. The effectiveness and applicability of the proposed approach to CMTDS are demonstrated using numerical example. The future work will extend the proposed approach to linear systems with noncommensurate, distributed, and time-varying delays.

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