Information Aggregation and Fuzzy Decision Making Based on Vague Set Theory

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Abstract. Problems that need to accomplish reasoning or decision making tasks in fuzzy environment or fuzzy systems are usually called fuzzy decision making problems, and the fuzzy decision making is widely used in many scenarios such as comprehensive evaluation, assessment and review, and uncertain reasoning. Many extensions of fuzzy set theory, such as intuitionistic fuzzy set theory, Vague set theory and interval valued fuzzy set theory, have been proposed to better adapt to these applications. In the framework of Vague set theory, this paper discusses information aggregation and linear ordering, which are two key problems in the process of fuzzy decision making. Referring to some results of intuitionistic fuzzy set and interval valued fuzzy set theory, this paper proposes a complete fuzzy decision making method based on Vague set theory, and explains the application of this method through the evaluation of teaching quality.

Keywords: Fuzzy set · Vague set · Interval-valued fuzzy set · Fuzzy decision making · Information aggregation · Intuitionistic fuzzy set · Linear ordering

1 Introduction

One of the hot topics in fuzzy mathematics is "fuzzy decision", which is to study the mathematical theory and method of decision making in fuzzy environment or fuzzy system. The goal of fuzzy decision making is to sort the objects in the decision domain in a fuzzy environment, or to select the optimal objects from the decision domain according to some fuzzy constraints [1]. Fuzzy set theory uses membership function to describe fuzzy sets, and its main purpose is to solve fuzzy decision problems [2]. This function maps the elements in the discourse world to the members [0, 1] on the subinterval. The membership level is the membership of the elements to the fuzzy set, and then

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describes the transition state of the boundary of the fuzzy set. The unique membership function defined on a fuzzy set needs to synthesize the support, opposition and neutral evidence of each element belonging to the fuzzy set in the universe. Sometimes it is impossible to objectively and accurately describe the membership relationship between elements. When applied to fuzzy decision making problems, fuzzy set theory cannot accurately and objectively express evidence of support, opposition, and neutral with a single membership function.

In order to show that the elements supporting and opposing evidence belong to a fuzzy set, some scholars put forward a pair of true and false membership functions, so as to give each element a pair of true and false membership, which can more objectively describe the uncertain boundary of the fuzzy set. Intuitionistic fuzzy sets and vague sets are based on this idea. Intuitionistic fuzzy set [3–5] directly use true and false membership pairs $< x, \mu_A(x), v_A(x) >$ is used to represent the membership degree of element x of fuzzy set A, while the vague set uses the interval value $[t_A(x), 1 - f_A(x)]$ as the membership degree of element x with respect to A. The essence of the two theories is the same, and their research results can be used for reference.

Some scholars directly use interval value as membership degree of fuzzy set corresponding to the element, which is called interval value fuzzy set [6, 7]. Scholars have carried out research on this form, including inclusion relation, similarity measure, ranking, and information aggregation. The results are applied to pattern recognition, medical diagnosis, fuzzy decision and image threshold. However, the interval-valued fuzzy set theory system does not delve into the source of element interval value membership, only focusing on processing data for a given interval value. Interval valued fuzzy set theory and vague set theory system are completely consistent in form, so the research results of interval valued fuzzy sets can be directly transplanted to vague set theory. In addition, vague set theory can well explain the source of interval valued fuzzy sets.

In this paper, Vague set theory is used to discuss. Based on the research results of intuitionistic fuzzy sets and interval valued fuzzy sets, the main purpose of this paper is to enrich and improve the fuzzy decision solutions under Vague set theory. This paper mainly has the following three contributions:

- The linear sorting method of interval-valued data is studied;
- The method of information aggregation between interval data and vectors is studied;
- A complete fuzzy decision scheme is proposed, which is based on vague set theory system.

2 Vague Sets

Zadeh put forward the concept of fuzzy set in 1965 to describe the transition state between different sets [2]. Fuzzy set theory describes fuzzy sets through membership function f (*), which maps each element in the discourse world to membership degree $\mu \in [0, 1]$. Membership function in order to obtain accurate membership, we need to synthesize all the evidences that support, oppose and neutral elements belong to fuzzy sets. However, in some application scenarios, such as fuzzy decision problems, a single membership degree function cannot objectively reflect the supporting, opposing, and neutral evidence. Vague set theory is an extension of fuzzy set theory, it uses the true and false membership function t (*) and f (*) to describe the Vague set. For element u, $t(u)$ is called true membership, which is used to measure supporting evidence, $f(u)$ is called false membership, which measures opposing evidence, and subintervals $\left[t(u),\, 1 - f(u)\right]$ can measure neutral evidence. Vague set theory can describe fuzzy concepts objectively and accurately in fuzzy decision making problems.

Let $U = \{u_1, u_2, \ldots, u_n\}$ become the universe of discourse. The vague set A in U is characterized by the true membership function t_A (*) and the false membership function f_A (*), where t_A: $U \to [0, 1]$, f_A: $U \to [0, 1]$ and $t_A(u_i) + f_A(u_i) \leq 1$. The membership degree of element u_i to vague set A is interval value $[t_A(u_i),\, 1 - f_A(u_i)]$. So, Vague set A can be described in the following form:

$$A = \left\{\left[t_A(u_i),\, 1 - f_A(u_i)\right]/u_i\right\},\ \mathrm{i} = 1, 2, 3, \ldots, \mathrm{n} \tag{1}$$

Similarly, a fuzzy set in a continuous universe, A can be written as

$$A = \int_U \left[t_A(u),\, 1 - f_A(u)\right]/u,\ u \in U. \tag{2}$$

The Vague set defined on the universe U is expressed as $\mathrm{VS}(U)$. Since the discrete universe is the basis of most fuzzy decision problems, this paper mainly discusses the discrete case.

For the given Vague sets A and B, we define the following basic operations (intersection, union, and complement):

$$A \cap B = \{[\min(t_A(u_i), t_B(u_i)), \min(1 - f_A(u_i), 1 - f_B(u_i)]/u_i)\}, \tag{3}$$

$$A \cup B = \{[\max(t_A(u_i), t_B(u_i)), \max(1 - f_A(u_i), 1 - f_B(u_i)]/u_i)\}, \tag{4}$$

$$A^c = \{[\max(t_A(u_i)), \max(1 - f_A(u_i), 1 - f_B(u_i)]/u_i)\}, \tag{5}$$

Similar to classical set and fuzzy set theory, there is also an inclusion relationship between Vague sets on the same universe. A set of axioms including interval valued fuzzy sets given by some scholars is also applicable to vague sets. Assume that A, B, and C are Vague sets on the universe of discourse U, and $\underline{A}(u) = t_A(u)$, $\overline{A}(u) = 1 - f_A(u)$, $u \in U$, the following axioms can be obtained:

Axiom 1

$Inc(A, B) = [1, 1]$, if and only if $A \subseteq B$;

Axiom 2

$Inc(A, B) = [1, 1]$, if and only if $\exists u \in U$, $[\underline{A}(u), \overline{A}(u)]=[1, 1]$ and $[\underline{B}(u), \overline{B}(u)]=[0, 0]$;

Axiom 3

$Inc(A, B) \subseteq [0, 1]$;

Axiom 4

$Inc(A, B) = Inc(B^c, A^c)$;

Axiom 5

If $B \subseteq C$, then $Inc(A, B) \leq_2 Inc(A, C)$;

Axiom 6

If $B \subseteq C$, then $Inc(C, A) \leq_2 Inc(B, A)$;

Axiom 7

$Inc(A, B \cup C) \geq_2 \max\{Inc(A, B), Inc(A, C)\}$;

Axiom 8

$Inc(B \cup C, A) \geq_2 \max\{Inc(B, A), Inc(C, A)\}$;

where $Inc(A, B)$ represents the measure of the degree to which fuzzy set A is included in fuzzy set B, and $\leq_2$ represents a partial order relationship, as shown below: $[\underline{a}, \overline{a}] \leq_2 [\underline{b}, \overline{b}] \Leftrightarrow (\underline{a} \leq \underline{b}) \wedge (\overline{a} \leq \overline{b})$.

3 Orders in the Vague Set System

In the process of solving the fuzzy decision problem, we need to determine the candidate solution by linear ranking according to the value of each attribute, and determine the optimal candidate solution. For the fuzzy decision making problem in Vague set system, the first step is to study the linear ordering problem of interval values. In [8–10], some scholars pointed out that the linear order of interval values is called the Admissible Orders. In [8], $L([0, 1])$ is used to represent the set of all sub-intervals of unit closed interval [0, 1], i.e., $L([0, 1]) = \{[a, b] \mid 0 \leq a \leq b \leq 1\}$. Then it gives the definition of admissible orders, and the partial order relationship between interval values is extended to the admissible orders.

Definition 3.1. Suppose any $a, b \in L([0, 1])$, $(L([0, 1], \preccurlyeq)$ is partial order, and $\preccurlyeq$ is called admissible orders. if

(i) $\preccurlyeq$ is linear in $L([0, 1])$
(ii) for all $a, b \in L([0, 1])$, whenever $\left[\underline{a}, \overline{a}\right] \leq_2 \left[\underline{b}, \overline{b}\right]$, $\left[\underline{a}, \overline{a}\right] \preccurlyeq \left[\underline{b}, \overline{b}\right]$.

The membership degree of any element to vague set belongs to $L([0, 1])$ in the universe U which is the concept of admissible order and is applicable to Vague set system. [8] also presents a method for constructing an admissible order using a pair of two aggregate functions defined on $L([0,1])$.

Proposition 3.1. Let Ψ, Υ: $[0,1]^2 \rightarrow [0,1]$ be two continuous aggregation functions, so that for all

$a, b \in L(0, 1)$, the equalities $\Psi(\underline{a}, \overline{a}) = \Psi(\underline{b}, \overline{b})$ and $\Upsilon((\underline{a}, \overline{a})) = \Upsilon((\underline{b}, \overline{b})) \Leftrightarrow A(u) = B(u)$ hold if and only if. If the order $\leq_{\Psi,\Upsilon}$ on $L([0, 1])$ is defined by

$$a \leq_{\Psi,\Upsilon} b \iff \Psi(\underline{a}, \overline{a}) < \Psi(\underline{b}, \overline{b}) \vee \left(\Psi(\underline{a}, \overline{a}) = \Psi(\underline{b}, \overline{b}) \wedge \Upsilon(\underline{a}, \overline{a}) \leq \Upsilon(\underline{b}, \overline{b})\right), \tag{6}$$

Then $\leq_{\Psi,\Upsilon}$ is admissible.

Example 3.1. Here are some admissible orders satisfying proposition 3.1.

(1) the Xu-Yager order

$$[\underline{a}, \overline{a}] \leq_{\mathrm{XY}} [\underline{b}, \overline{b}] \iff (\underline{a} + \overline{a} < \underline{b} + \overline{b} \vee \left(\underline{a} + \overline{a} = \underline{b} + \overline{b} \wedge \overline{a} - \underline{a} \leq \overline{b} - \underline{b}\right), \tag{7}$$

(2) lexicographical orders

$$[\underline{a}, \overline{a}] \leq_{\mathrm{Lex1}} [\underline{b}, \overline{b}] \iff (\underline{a} < \underline{b} \vee \left(\underline{a} = \underline{b} \wedge \overline{a} \leq \overline{b}\right), \tag{8}$$

$$[\underline{a}, \overline{a}] \leq_{\mathrm{Lex2}} [\underline{b}, \overline{b}] \iff (\overline{a} < \overline{b} \vee \left(\overline{a} = \underline{b} \wedge \underline{a} \leq \underline{b}\right), \tag{9}$$

(3) the αβ order

$$[\underline{a}, \overline{a}] \leq_{\alpha\beta} [\underline{b}, \overline{b}] \iff (K_\alpha(\underline{a}, \overline{a}) < K_\alpha(\underline{b}, \overline{b}) \vee \left(K_\alpha(\underline{a}, \overline{a}) = K_\alpha(\underline{b}, \overline{b}) \wedge K_\beta(\underline{a}, \overline{a}) \leq K_\beta(\underline{b}, \overline{b})\right), \tag{10}$$

where $K_\alpha : [0, 1]^2 \to [0, 1]$, $K_\alpha(x, y) = \alpha x + (1 - \alpha)y$, $\alpha, \beta \in [0, 1]$ and $\alpha \neq \beta$, $x, y \in [0, 1]$.

4 Aggregation Function in Vague Set System

Information aggregation is an important research content for fuzzy decision making, which is mainly responsible for the effective integration of evaluation data from multiple sources and comprehensively applying it to the final decision. Fuzzy decision making is carried out under the Vague set theory. When there are multiple decision results from multiple sources, information aggregation of these decision results is needed to get comprehensive decision results. When sorting each candidate object, it is necessary to aggregate the membership degree of interval value corresponding to different attributes to obtain an interval worth score, and then sort them according to some admissible orders to obtain the final candidate object. The interval-valued aggregation function proposed in [11–16] is suitable for information aggregation of decision results of Vague sets.

Definition 4.1. Given operator $A : L([0, 1])^n \to L([0, 1])$ r, where n ≥ 2, A is called an interval-valued aggregation function if it is increasing for the order $\leq$(partial or admissible linear), i.e.,

$$\forall a_i, b_i \in L([0,1]), a_i \leq b_i \Rightarrow A(a_1, a_2, \ldots, a_n) \leq A(b_1, b_2, \ldots, b_n) \text{and}$$
$$A(\underbrace{[0,0], \ldots, [0,0]}_{n\times}) = [0,0],\ A(\underbrace{[1,1], \ldots, [1,1]}_{n\times}) = [1,1].$$

Definition 4.2. An interval-valued aggregation function $A : L([0, 1])^n \to L([0, 1])$ is representable if there exist two aggregation functions $\Psi, \Upsilon : [0, 1]^2 \to [0, 1]$ satisfying:

$$A(a_1,a_2,\dots,a_n) = \left[\Psi\left(\underline{a_1},\underline{a_2},\dots,\underline{a_n}\right), \Upsilon(\overline{a_1},\overline{a_2},\dots,\overline{a_n})\right], \text{ for}$$
all $a_1,a_2,\dots,a_n \in L([0,1])$.

Example 4.1. Given the interval value $a_1 = \left[\underline{a_1}, \overline{a_1}\right], a_2 = \left[\underline{a_2}, \overline{a_2}\right], \dots, a_n = \left[\underline{a_n}, \overline{a_n}\right] \in L([0, 1])$, several common representable aggregation functions with respect to $\leq_2$ are given below.

(i) projections

$$A_L(a_1, a_2, \dots, a_n) = \left[\underline{a_1}, \overline{a_1}\right], A_R(a_1, a_2, \dots, a_n) = \left[\underline{a_n}, \overline{a_n}\right], \tag{11}$$

(ii) the representable arithmetic mean

$$A_{mean}(a_1, a_2, \dots, a_n) = \left[\left(\underline{a_1} + \underline{a_2} + \dots + \underline{a_n}\right)/n, (\overline{a_1} + \overline{a_2} + \dots + \overline{a_n})/n\right], \tag{12}$$

(iii) the representable geometric mean

$$A_{gmean}(a_1, a_2, \dots, a_n) = \left[\sqrt{\underline{a_1}\underline{a_2}\dots\underline{a_n}}, \sqrt{\overline{a_1}\overline{a_2}\dots\overline{a_n}}\right], \tag{13}$$

(iv) the representable power mean

$$A_{power}(a_1, a_2, \dots, a_n) = \left[\sqrt{(\underline{a_1}^2 + \underline{a_2}^2 + \dots + \underline{a_n}^2)/n}, \sqrt{(\overline{a_1}^2 + \overline{a_2}^2 + \dots + \overline{a_n}^2)/n}\right], \tag{14}$$

(v) the representable product

$$A_{prod}(a_1, a_2, \dots, a_n) = \left[\underline{a_1} \wedge \underline{a_2} \wedge \dots \wedge \underline{a_n}, \overline{a_1} \wedge \overline{a_2} \wedge \dots \wedge \overline{a_n}\right], \tag{15}$$

where, A_{mean} function is also an aggregation function referring to admissible orders $\leq_{\alpha\beta}$.

Example 4.2. Given the interval value $a_1 = \left[\underline{a_1}, \overline{a_1}\right], a_2 = \left[\underline{a_2}, \overline{a_2}\right], \dots, a_n = \left[\underline{a_n}, \overline{a_n}\right] \in L([0, 1])$, the following is a description of the aggregate function $\leq_{\mathrm{XY}}, \leq_{\mathrm{Lex1}}, \leq_{\mathrm{Lex2}}$ and $\leq_{\alpha\beta}$

(i) the representable arithmetic mean

$$A_{mean}(a_1, a_2, \dots, a_n) = \left[\left(\underline{a_1} + \underline{a_2} + \dots + \underline{a_n}\right)/n, (\overline{a_1} + \overline{a_2} + \dots + \overline{a_n})/n\right], \tag{16}$$

(ii) the representable weighted arithmetic mean

$$A_w(a_1, a_2, \dots, a_n) = \left[w_1\underline{a_1} + w_2\underline{a_2} + \dots + w_n\underline{a_n}, \omega_1\overline{a_1} + w_2\overline{a_2} + \dots + w_n\overline{a_n}\right], \quad \sum_{i=1}^{n} w_i = 1. \tag{17}$$

5 Fuzzy Decision Making Based on the Vague Set

The task of fuzzy decision making is to comprehensively evaluate and rank the candidate objects in the decision universe of discourse in the fuzzy environment, or select the optimal object according to some fuzzy constraints. It is known from the previous article analysis that the Vague set theory can objectively and accurately describe fuzzy decision making problems. Through the interval-valued aggregation function, decision results from multiple sources can be aggregated, and the interval-valued membership degree of each attribute can be aggregated to obtain the final decision result in the form of interval-valued. Then the admissible orders are used to rank the candidates according to the final decision making result. In this way, a scheme for accomplishing the fuzzy decision making is formed: first, modeling the fuzzy decision problem, then effectively integrating the evaluation results from multiple sources, finally, ranking the candidates according to the integration results. Through the case of teaching level evaluation, the process of fuzzy decision making using the Vague set theory system is described as follows.

Suppose a school evaluates teachers' classroom teaching quality [17–20], and five first-level indicators are selected in the evaluation system, including teaching attitude (A_1), teaching content (A_2), teaching method (A_3), teaching skill (A_4), and teaching effect (A_5). The weight of each indicator is obtained by the demonstration of field experts: $w_1 = 0.1$, $w_2 = 0.2$, $w_3 = 0.2$, $w_4 = 0.1$, $w_5 = 0.4$. The supervision experts, peers, and students are evaluated according to the above evaluation system. The evaluation grades are excellent, good, medium, and poor. Suppose there are n members in an evaluation group, and for the classroom teaching of a teacher's T_i, the evaluation results of students on a certain A_j are as follows: excellent is r_1, good is r_2, medium is r_3, poor is r_4, where $r_1 + r_2 + r_3 + r_4 = n$, and $t_{ij} = (r_1 + r_2) /n, f_{ij} = r_4 /n$ can be obtain. Then the feature of Vague set of the teacher A_j index is [($r_1 + r_2$)/n, 1 - r_4/n].

Table 1. Three kinds of evaluation results of teacher T_1

Index	A_1	A_2	A_3	A_4	A_5
T_{11}	[0.68, 0.89]	[0.75, 0.92]	[0.78, 0.94]	[0.83, 0.96]	[0.80, 0.94]
T_{12}	[0.65, 0.82]	[0.74, 0.85]	[0.68, 0.74]	[0.88, 0.96]	[0.79, 0.85]
T_{13}	[0.61, 0.79]	[0.74, 0.85]	[0.82, 0.90]	[0.91, 0.97]	[0.83, 0.91]

The three evaluation results (T11, T12, T13) for teacher T1 in the Table 1 are interval value vectors of vague set patterns. It is necessary to use the aggregation functions of Example 4.1 and Example 4.2 to aggregate the interval values on the corresponding attributes of the three to obtain the final comprehensive evaluation result. Taking the representable arithmetic mean of Eq. (12) as an example, the aggregation result is:

$$T_1 = ([0.65, 0.83], [0.74, 0.87], [0.76, 0.86], [0.87, 0.96], [0.81, 0.9])$$

In the fuzzy decision making environment, the multi-source evaluation situation is objectively synthesized, and the comprehensive evaluation results in the form of interval-valued vectors for each teacher are obtained. To complete the final evaluation or ranking, it is also necessary to synthesize the interval values of each teacher's evaluation attributes. Since different attributes are given corresponding weights by experts in this field, the weighted arithmetic average method of Eq. (16) can be used for aggregation, i.e., $V_1 = [\sum_1^5 w_i \underline{A_i}, \sum_1^5 w_i \overline{A_i}]$. According to the weights w1 = 0.1, w2 = 0.2, w3 = 0.2, w4 = 0.1, w5 = 0.4 allocated by experts in this field for each attribute of the evaluation system, the aggregation result of T1 teachers is: $V_1 = [0.775, 0.886]$.

Similarly, through the weighted aggregation of each attribute value of each teacher, the final evaluation score in the form of interval value can be obtained. Assume that 5 teachers are participating in the evaluation, and their final evaluation scores are: $V_1 = [0.775, 0.886]$, $V_2 = [0.815, 0.926]$, $V_3 = [0.725, 0.856]$, $V_4 = [0.672, 0.936]$, $V_5 = [0.675, 0.786]$. The admissible orders given in Example 3.1 can be used to sort these interval values. Taking the αβ order given in Eq. (10) as an example, $\alpha = 0.5, \beta = 0.6$, the order of the above interval values can be obtained as: $V_4 \leq_{\alpha\beta} V_5 \leq_{\alpha\beta} V_3 \leq_{\alpha\beta} V_1 \leq_{\alpha\beta} V_2, \alpha = 0.5, \beta = 0.6$. In this way, the linear ranking of participating teachers is realized, and the fuzzy decision making task of the comprehensive evaluation of teaching quality is completed.

6 Summary

The main contribution of this paper is the fuzzy decision problem based on Vague set theory. The true and false membership functions used to describe Vague sets are proposed, which can objectively describe the supporting evidence and opposing evidence in the fuzzy decision making process, while the parts other than the true and false membership can only describe the neutral evidence. Therefore, the theory is very suitable for the application scenarios of fuzzy decision making. This paper improves the information aggregation method and linear ranking method of the decision results of candidate objects under the Vague set theory system, and continues to study on the basis of the research results of intuitionistic fuzzy sets and interval valued fuzzy sets. In addition, this paper improves the decision results of candidate objects under the Vague set theory system, and forms the complete solution of the fuzzy decision problem.

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