Research and Research Culture in Mathematics Education: The Case in Macao, China

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Abstract Macao, a Special Administration Region of China (Macao SAR), is unique in that its mathematics education practice and research have integrated both the eastern and western traditions. Its participation in the Programme for International Student Assessment (PISA) since 2003 inspired the research culture in mathematics education in Macao from international perspectives. In this article, we report on a survey of three kinds of research done in mathematics education in Macao: (1) research related to PISA, in particular PISA 2012 and PISA 2018; (2) research done for master's and Ph.D. theses/dissertations in higher institutions in Macao; and (3) articles published in educational journals, particularly in mathematics education in Macao. The survey reveals emerging research cultures in mathematics education over the past decade—the interests of researchers and practitioners in topics such as comparative studies, lesson studies, mathematical problem-solving, and the use of information technology in mathematics teaching and learning. Lastly, we summarize what Macao has done well and what it needs to do better for further development of Macao's research culture within the global trend of literacy-based mathematics education as modeled by the PISA.

Keywords Research culture · Mathematics education · Macao · PISA

1 Introduction

Macao,[1] a Special Administration Region of China (Macao SAR) since 1999, has been a hub for cultural interflow and mingling between the West and the East for

[1] Macao is the English spelling of the name, and Macau is its Portuguese spelling. Internationally, the two spellings are used interchangeably.

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over 400 years. Related to this long history, the education system in Macao has creatively integrated the western and eastern traditions and has been unique in many respects. Compared with the Chinese mainland, Hong Kong, and Taiwan, Macao is not competitive for high school graduates to succeed in applying for a higher education institution as the transition rate from secondary to tertiary education has been more than 90% in recent years (DSEJ, 2019a). Mathematics education in Macao, on the one hand, has maintained the Chinese tradition focusing on the importance of mathematics for children's future development and the role of practice in their mathematics learning. On the other hand, it is also open to the rapid development of mathematics curriculum, assessment, information and communication technology (ICT), and their implications to the research and classroom practice in mathematics education. Therefore, by studying research culture in mathematics education in Macao, we hope that it can also provide meaningful insight into the global development of mathematics education.

In this chapter, we first briefly introduce the structure of the education system and mathematics education in Macao for readers to understand the contexts of the study. Then, we survey the research in mathematics education in Macao in three areas: (1) research related to the Programme for International Student Assessment (PISA), which is the main research topic that mathematics educators in Macao have done since 2003 and that has helped policy-makers to improve mathematics education practice in schools; (2) research done for the master's and Ph.D. degrees in Macao; and (3) articles published in educational journals in Macao in the past decade, though the majority of these articles were not research-based, rather they were practice-oriented. Including the last collection of articles aims to identify what reflective mathematics educators did at the school level, what they focused on in real mathematics classrooms, and what are the areas that they need to pay attention to. Many of the last set of articles were written jointly by researchers and practitioners, discussing the learning difficulties that students might experience in mathematics classroom and providing better lesson plans to help students overcome their difficulties. This kind of researcher–practitioner partnership for conducting research has been advocated by mathematics educators (Cai et al., 2017). Therefore, we include them in the chapter. Based on the survey, we shall be able to describe the overall characteristics of research culture in mathematics education in Macao and the challenges that it is facing.

2 Characteristics of Macao's Research Culture in Mathematics Education

To cut the story short, education in Macao is characterized by a diverse pluralistic system, allowing the co-existence of various forms of education (Wang, 2009). For example, there are Chinese-medium schools, English-medium schools, and Portuguese-medium schools. The variety of school systems provides alternatives for parents and students. Unlike the Chinese mainland, Hong Kong, and Taiwan where public education is prevailing, private schools are the mainstay of Macao education (Wang, 2009). Based on the education statistics of 2018–2019, there are 74 schools in total in Macao and 77% of them are private schools. About 96.5% of the students were enrolled in private schools (DSEJ, 2019b). For this reason, the Education and Youth Affairs Bureau (DSEJ,[2] abbreviation in Portuguese), which is the administration department of Macao government for education, gives schools great autonomy in determining and selecting instructional medium, teaching materials and contents, and even developing school-based curricula to cater for students' individual and diversified needs. In addition, the DSEJ has initiated a series of educational reforms to promote the development of Macao's basic education since the handover to China in 1999. For instance, in 2006, the Non-tertiary Education Law was promulgated, aiming at protecting the rights of children for a fifteen-year (K1-3 plus 12 years of formal schooling) free compulsory education. Within ten years, the Macao government's investment in non-tertiary education increased from MOP22,819 (about USD2860) per student in 2007 (DSEJ, 2009) to MOP88,555 (about USD11,090) in 2017 (DSEJ, 2019b). With the financial support from the DSEJ, the class size has been reduced from more than 45 to 25–35 (Vong, 2013). The primary characteristics of Macao's research culture in mathematics education are that it is mostly school-based in the private schools and carried out by the autonomous educational practitioners, taking advantage of the Macao Government's investment in basic education and school sponsorship to achieve the ideal of Education-for-All in the first two decades of the new century.

DSEJ invited mathematics educators from local and the Chinese mainland to develop the Basic Academic Attainments (BAA) for major school subjects at the formal education level. Regarding mathematics, the following three documents were released: BAA in Mathematics for Primary Schools (BAAMPS, 小学数学基本学力要求) (Tam, 2016), BAA in Mathematics for Middle Schools (BAAMMS, 初中教育阶段数学基本学力要求) (Tam, 2017b), and BAA in Mathematics for High Schools (BAAMHS, 高中教育阶段数学基本学力要求) (Tam, 2017a). They stipulated the basic proficiency in knowledge, skills, abilities, attitudes, and values upon the completion of primary, middle, and high school mathematics education. The three BAAs in mathematics were implemented since 2016–2018 after being piloted in several schools. They spell out the minimum standards at various stages instead of the "ceilings" which allow schools to develop their own curricula based on their educational visions, missions, and students' abilities (Wong et al., 2015). There are only one series of mathematics textbooks for primary schools and one series for middle schools written by local mathematics educators. However, they are not widely used in local schools. Most Chinese-medium schools actually adopt textbooks used in the Chinese mainland, while most English-medium schools use textbooks from Hong Kong (Tang, 1999). The implementation of different mathematics curricula brings about different teaching practices in schools (Oliveira et al., 2015). However, many Macao teachers tend to teach mathematics in a traditional way emphasizing exercises

[2] Education and Youth Affairs Bureau (DSEJ) and the Higher Education Bureau (DSES) were merged into the Education and Youth Development Bureau (DSEDJ) in February 2021.

with variations and controlling classroom activities although they also encourage students to be engaged in the process of learning (Huang & Leung, 2004). In order to create an atmosphere of research on teaching among school teachers, DSEJ has also initiated the Award Scheme on Instructional Design (ASID) to encourage in-service teachers to develop lesson plans as a teaching unit or a course/program spanning a semester or an academic year, to do action research in their own classrooms and run open classes for peers to observe and for the reviewers to evaluate. The designs that are awarded will be uploaded to the DSEJ website for other teachers to use (DSEJ, 2020). It is hoped that teachers' teaching abilities will be enhanced in the process.

Starting from 2009, the Macao Mathematics Education Research Association has been organizing groups of high school students to attend the International Regions Mathematics League (IRML), which is a part of the American Regions Mathematics League (ARML). Macao teams were the winner of the Onsite Division of IRML in 2009, 2015, and 2018 (ARML, 2019; Macao Daily, 2018). All these programs and efforts have brought about observable achievements in education in Macao, particularly in mathematics education. For instance, the mathematics performance of Macao students in Programme for International Student Assessment (PISA) showed big progress in mathematics literacy, improving its ranking from 12th in 2009 to 3rd in 2018. Table 1 revealed that (a) Macao students' performance (scores) consistently improved from 2003 to 2018, in particular, in the past 10 years; and (b) the performance of the female students improved a lot and the gender differences decreased consistently (Cheung et al., 2020).

Thus, another key characteristic of Macao's research culture is that the government has been increasingly exercising tighter control and monitoring over the quality of academic provision to the students, and this has immense implications on the kind of curriculum and instruction research that the schools and teachers embarked in order to improve student performance in mathematical literacy.

Table 1 Mathematical literacy performance of Macao's 15-year olds in PISA (2003–2018)

PISA cycle	Rank	Score	SD	Males' average	Females' average	Difference (F − M)
2018	3	558	81	560	556	− 4
2015	3	544	80	540	548	8
2012	6	538	95	540	537	− 3
2009	12	525	86	531	520	− 11
2006	8	525	84	530	520	− 10
2003	9	527	87	538	517	− 21

3 Research and Research Culture in Mathematics Education in Macao

Culture is defined differently in different areas of research (Townes, 2018). The one that is closely related to the current study is "*the total range of activities and ideas of a group of people with shared traditions, which are transmitted and reinforced by members of the group*" (Townes, 2018, p. 56). Therefore, it is helpful to examine what mathematics educators including school mathematics teachers, who are the implementers of the intended curriculum and the practitioners of mathematics education, do in the recent years for describing the characteristics of research culture in mathematics education in Macao. In this section, we shall look at the research in mathematics education in Macao in the following three aspects: (1) research related to PISA; (2) research done for master's and Ph.D. degrees in higher institutions in Macao; and (3) articles published in educational journals in Macao. It will help us to know what had been done, which provides a base for us to describe the characteristics of mathematics education and its emerging research culture in Macao.

3.1 Research Related to Macao-PISA

PISA does not only test students' proficiencies in mathematics, science, and reading, but also does collect data through questionnaires from several stakeholders. The data allow us to describe Macao students' mathematics learning characteristics in more details and to identify factors that affect students' performance including quality and equity in mathematics education. In recent cycles, almost all the 15-year olds in Macao are participated in PISA; therefore, the Macao-PISA data are a rather complete set of data and results obtained from PISA are used for DSEJ's policymaking.

3.1.1 Secondary Analysis of the PISA Data

In the past decade, three lines of research are exemplary of the emerging research culture on mathematics education due to the secondary data analysis of the Macao-PISA data. The first line of research is the identification of factors affecting mathematical literacy performance, and after identification of key issues of upmost concern by Macao educational community, mechanisms and processes of the influencing factors cast on the stakeholders need to be clarified. The past two decades also witnessed a shift of assessment from one essentially centered on outcomes of schooling to the processes of classroom instruction effecting student academic performance. In line with this shift, there is a change from print-based to digital assessment, necessitating research into analysis of this kind of process data collected in the assessment platforms.

1. *Identification of factors affecting mathematical literacy performance.* It is important to identify pertinent factors (e.g., mathematics efficacy, mathematics interests, mathematics anxiety, etc.) affecting variations in mathematics performance of the full cohort or a subpopulation of it (e.g., low-achievers) (Cheung, 2015; Cheung et al., 2013, 2014; Sit et al., 2017). For instance, Sit et al. (2017) studied the prediction of digital problem-solving performance of the low-achievers in Macao by perseverance, openness, and use of ICT. Ieong et al., (2016a, 2016b) have investigated the effects of home socioeconomic status on mathematics performance through multilevel mediation analysis of self-regulated learning processes of Macao students in PISA 2012. Their research findings help understand why Macao is rated as a "high-performing high-equity" basic education system among all PISA participating countries/economies.
2. *Clarification of mechanisms and processes of key issues of upmost concern by the Macao educational community.* In the past decade, two thorny issues lingered in the minds of Macao's educational practitioners. Foremost is why Macao is having very high proportion of grade-repeated students and yet its performance in mathematics is very favorable. Sit et al.'s (2015) analysis on studying problems faced by the adolescent grade repeaters in Macao throws lights on this issue uncovering three underlying mechanisms based on evidences from the PISA 2012 data. This research ushered the local government urging schools to curb the rate of grade repetition in schools. Another issue is that the sense of school belonging of Macao students is among the worst in all participating countries/economies. Cheung (2010) identified that factors for remediation and policies were introduced in the subsequent years. Cheung et al. (2018) made another attempt to resolve the attitude-achievement paradox based on anchoring vignettes by evidences from PISA 2012 mathematics data. The conflicting results stemming from data aggregation with regard to attitudinal factors affecting achievement when the secondary data analysis is conducted at the student and system levels are now clarified so as to inform policymaking.
3. *New methodologies in the secondary analysis of the big process data in the information era—Shift of assessment mode from print-based to digital problem-solving in 2012.* The log file data captured by the test delivery system allow researchers to analyze relationships between problem-solving patterns and the cognitive/affective/behavioral learner characteristics (e.g., Ieong et al., 2016a, 2016b; Jin et al., 2016), as well as task- and non-task-specific factors classifying problem-solving experts and novices (e.g., Jin et al., 2017).

3.1.2 Analysis of Macao's Quality and Equity in Mathematics Education from a Comparative Education Perspective

One important aim for Macao to participate in PISA is to assist Macao schools to monitor the quality and equity of the basic education. Figure 1 shows the relationship between mathematical literacy performance of Macao students and their home economic, social, and cultural status (ESCS) in PISA 2003–2018. Compared

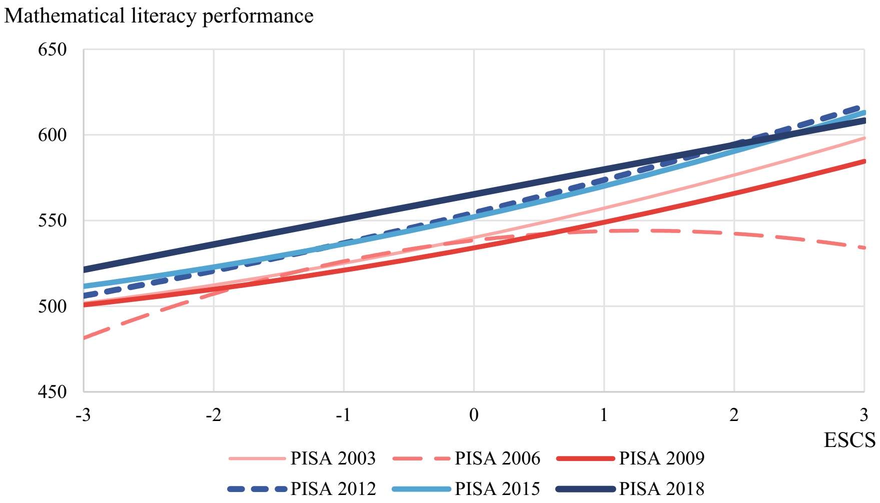

Fig. 1 Relationship between mathematical literacy performance and economic, social, and cultural status (ESCS) in six cycles of Macao-PISA (2003–2018)

with the three earlier ones (i.e., PISA 2003–2009), the intercepts of the regression lines in the latest three cycles (i.e., PISA 2012–2018) are higher indicating a higher level of quality of mathematics education and the slopes are so gentle that performance increase due to one unit increase of ESCS is the lowest in the world. We can see that one unit increase in ESCS is still associated with about 15 score points' increase in mathematical literacy, amounting to approximately to one-third of a grade of schooling among the participating countries of Organization for Economic Co-operation and Development. We still need to pay close attention to it.

Results obtained from PISA reveal that the education system in Macao is the most equitable in the world (IBP, 2016). However, as the insiders, as envisaged in Fig. 1, we know that inequality still exists. Concerning research in mathematics education, overcoming inequity and maintaining quality have becoming two of the most important goals of education in Macao as in other economies in the world (Li & He, 2010; UNESCO, 2008).

3.2 *Research Done for Master's and Ph.D. Degrees*

We have also tried to search for the master's and Ph.D. thesis to see what research the postgraduate students in universities in Macao did for their degrees.

After checking the master and Ph.D. programs offered at the universities in Macao, we found that only the University of Macau (UM) offered mathematics education programs at the postgraduate level; therefore, we shall only need to look at theses and dissertations in the Faculty of Education (FED) of UM. The Ph.D. dissertations on mathematics education in FED/UM were all about PISA under the supervision of the first author of this chapter. Therefore, our focus will be on the master theses in Curriculum and Instruction strand in FED/UM. Similarly, our focus will be on the research done in the past decade, i.e., starting from 2010. The theses after 2017 have not been uploaded to the UM library system; therefore, they were excluded in the current analysis. From 2010 to 2017, in total, 19 students received their master's degrees supervised by six professors in the field of mathematics education, ICT, and educational psychology. We would like to look at these theses from the research methods perspective, then get into the details under each category.

We commonly classify the methods used in educational research into quantitative and qualitative (Mertens, 2015). It is so interesting to find that all the researches done for the master's degrees all used quantitative methods. Educational research is generally classified into three categories, i.e., descriptive, associational, and intervention (Hoy & Adams, 2016). Among the 19 theses, seven used descriptive methods, four used associational methods, and eight used intervention methods.

Descriptive studies. Among the seven descriptive studies, four focused on students' learning, two focused on teachers' teaching, and one involved both teachers and students.

Among the four studies focusing on students, three used PISA data. Two used the Macao data of PISA 2003 (Lam, 2012; Loi, 2011), and one used the Macao and Singapore data of PISA 2012 (Chan, 2017). Lam (2012) used Chi-square tests to compare learning characteristics of students at different problem-solving proficiency levels. Loi (2011) explored the mathematical literacy performance of Macao students in the four content areas, gender differences, and performance differences among students at different grade levels. Chan (2017) compared the performance of Macao and Singapore students and their attributions of failure in mathematics. Chan found that Macao students with low mathematics literacy levels were more likely to attribute their failure in mathematics to external factors like bad luck, wrong guesses, and the difficult nature of mathematics contents.

The study that did not use PISA data was conducted by Leong (2016), who investigated students' geometrical thinking skills, in particular, eighth and ninth grade students' understanding of congruent triangles based on van Hiele model of geometrical thinking. Leong found that about 75% of the participants could reach Level 3; however, only about 30% of them could reach Level 4. About 10% of the participants were at the transit stage. Leong also identified the main difficulties that the students had: low visualization ability, the use of geometric symbols, the use of conditions for determining whether two triangles are congruent, and proof writing skills.

Both Li (2010) and Shi (2012) investigated teachers' teaching strategies. Li (2010) compared the difficulty levels of problems used by three teachers teaching the topic "Pythagorean theorem" in a secondary school in China and those of problems presented in mathematics textbooks. Li found that the number of the problems used in real classrooms is 3–5 times as those presented in mathematics textbooks, and nearly half of the worked examples used in classrooms are from materials other than textbooks. Based on the difficulty model developed by Bao (2002) for textbook analysis, Li found that the problems used in classroom are more difficult than those presented in textbooks in all the five aspects (i.e., investigation, context, computational complexity, reasoning, and topic coverage). Shi (2012) compared the conceptions between five expert and five novice teachers on the use of “One Problem with Multiple Solutions (OPMS)”. Shi found that expert teachers did not only hold a positive attitude toward OPMS, but also realized its positive effect on students’ learning and actively used it in their classroom teaching. Whereas the novice teachers held a negative attitude toward OPMS, they did not think that it might have a positive effect on students’ learning. As a sequence, they thought that it was a waste of time to use OPMS in their classes.

Han (2012) examined what mathematics teachers and their students thought as important for effective mathematics teaching and learning. She found that both teachers and students thought “knowledge and skills”, “mathematical thinking”, “problem solving”, and “emotions and attitudes” important. In terms of classroom activities for effective mathematics teaching and learning, both teachers and the students thought “students’ behavior” (e.g., hands-on activities) followed by “teacher’s behavior” (e.g., teacher’s explanations) and lastly “teacher-student collaborative behavior” (e.g., making a rhyme together) are important.

Associational studies. Among the four associational studies, two investigated the factors that are associated with students’ mathematics achievement (Cheong, 2010; Lao, 2016). Cheong (2010) used Academic Help-Seeking Questionnaire and Mathematical Self-Efficacy Questionnaires to collect data from students at grades 7–9. Cheong found that mathematics achievement is positively associated with students’ mathematics self-efficacy and adaptive help-seeking, but negatively associated with dependent help-seeking and avoidance of help-seeking. Lao (2016) compared the mathematics achievement and attitudes toward mathematics between students of different cognitive styles (i.e., field-independent vs. field-dependent) in an interactive whiteboard instruction environment. It was found that students in a field-independent style performed better in mathematics and had a more positive attitude toward mathematics than those in a field-dependent style. Lao also found that students’ mathematics achievement was positively correlated with their attitudes toward mathematics. Unlike these two studies, Lam (2016) used grade retention as an indicator of students’ performance in schools (not mathematics achievement) and investigated the relationship between parental involvement and their self-regulated learning in mathematics and their experience of grade retention. Lam collected data from students at Grades 8 and 11 and found that students, who experienced grade retention, reported less parent–child communications and lower parents’ school involvement.

Lei (2011) used the Macao data from PISA 2003 and structural equation modeling to investigate how Macao students’ learning characteristics (including motivations in mathematics learning, sense of belonging to the school, mathematics self-efficacy, preference for competitive/co-operative learning situations) affect their learning strategies (including control, elaboration, and memorization strategies). Lei found that all the four characteristics positively are related to students' learning strategies directly or indirectly through intermediate factors.

Interventional studies. All the eight interventional studies used quasi-experimental design. Among them, three had the experimental group working in a Dynamic Geometry Software (DGS)-supported environment (Hu, 2014; Ke, 2014; Zhou, 2017) and one had mathematical games integrated (Ho, 2017). Hu (2014) designed a four-lesson experiment to teach Grade-10 students the translation of quadratic function's graphs based on Action–Process–Object–Scheme (APOS) theory and Relationship–Mapping–Inversion (RMI) principle in a DGS-supported environment. In the post-test, the experimental group outperformed the control group not only in items of point translations and quadratic functions, but also in terms in an extended area including complex functions and circles. Ke (2014) investigated the effects of using a DGS-based instruction on secondary students' achievement in linear functions. The experimental group worked in small groups with DGS to solve inquiry-based tasks, while students in the control group received traditional instruction. Ke found that students in the experimental group showed better improvement in items involving multiple representations of linear functions like $y = kx + b$ but not in items testing students understanding of basic concepts and their application. Zhou (2017) investigated the effect of cooperative learning in a DGS-supported environment on the ninth-grade students' mathematics performance in inverse proportional functions and attitudes. Zhou found that the experimental group had significantly better performance than the control group who received traditional instruction of the same topic, but there were no differences in attitudes toward mathematics between the two groups. Ho (2017) investigated the effect of integrating games into teaching the topic of "Directions" on the fourth-grade students' mathematics performance and attitudes. Ho found that there was no performance or attitude difference between the two groups. However, the students in the experimental group had better improvement in problem-solving abilities and self-confidence dimensions.

Two interventional studies were designed based on Polya's four-stage model (Si, 2014; Wu, 2016). Si (2014) modified the first three steps of Polya's four-stage model for the teaching of geometry and investigated the effect of the modified model on students' performance in geometric proof and their mathematics attitudes. The attitude questionnaire includes a section of self-conceptions on the use of Polya's four-stage approach. Si found that the students in the experimental group did not only perform better in the post-test in geometric proof, but also had an increased self-conception of the use of Polya's four-stage model. Wu (2016) integrated mathematical problem posing into the "Look back" stage of Polya's four-stage model and investigated its effect on students' problem-solving and problem-posing skills. The experiment lasted four lessons on "simultaneous linear equations in two variables". Wu found that the experimental group did not perform better than the control group in mathematical problem-solving, but the experimental group did make better improvement in mathematical problem posing.

Xiao (2010) experimented the dialogic teaching in high school mathematics classes and found that the experimental group could significantly better than the control group for which the instruction was teacher-centered in the post-test though there was no performance difference in the pre-test. Xiao also gave verbal evidence of students' improvement in problem-solving, reasoning and proof, communication, connections, and representation.

Ngai (2012) compared the effect of three kinds of layouts (i.e., dense, moderate, and sparse-layouts) of worked examples on the seventh- and eighth-grade students' mathematics achievements and cognitive load and found that moderate and sparse layouts had lower cognitive load on students than dense layout when the content to learn is at a high difficulty level. The moderate and sparse-layouts made learning easier for the students when the content is simple.

The review of the research done for master's and Ph.D. degrees in mathematics education reveals that the research in mathematics education in Macao is very limited in terms of both numbers and scope. Over eight years, only 19 students finished their master thesis in mathematics education. Research areas covered in these studies are from students' learning characteristics, learning strategies, and their learning difficulties in different topics, to teachers' teaching supported by DGS software and games, based on a modified Polya's four-stage model, dialogic teaching, and to study factors (e.g., parental involvement) that might affect students' mathematics learning and achievement. However, their scope is still very limited. The methods used for data collection and analysis are relatively simple; however, the results obtained in experimental studies are promising because they did help students in some ways.

3.3 Articles Published in Local Educational Journals/Magazines

As in many other countries/economies, there are several educational journals in Macao publishing articles mainly contributed by Macao mathematics educators to share their ideas and experiences. They are: Mathematics Education in Macao (MEM, 澳门数学教育), Teacher Magazine (TM, 教师杂志), and Education in Macao (EM, 澳门教育). Among these three journals/magazines, MEM is a yearly journal with a primary focus on mathematics education. We could not find a full copy of its first ten volumes of MEM; however, we did find two collective volumes with selected articles published in the first ten volumes. Therefore, we use these two volumes and the latest six volumes in the ensuing content analysis of the articles. Of note is that the other two (i.e., TM and EM) publish articles in all areas of education. All the articles related to mathematics education published in 2009–2018 were included in the current analysis. In total, there were 264 articles published, 115 (43.6%) in MEM, 76 (28.8%) in TM, and 73 (27.7%) in EM. After a quick reading of these journals/magazines, we would like to look at the articles from the following aspects:

(1) Authors and their categories (i.e., school teachers, administrators, and professors in mathematics education). The authors are the subjects who are involved in mathematics education research.

(2) Grade levels (kindergarten, lower primary, upper primary, middle school, high school, and higher education institutions).
(3) Content areas (numbers and operations, algebra, geometry, statistics and probability, etc.).
(4) Topic studies (mathematics curriculum, mathematics instruction, mathematics learning, mathematical problem-solving, etc.).

Authors and their categories. About 100 authors contributed articles to the three journals. Among the 264 articles, 211 (79.9%) were contributed by school teachers, 32 (12.1%) were contributed by professors including retired professors, 20 (7.6%) were contributed jointly by school teachers and professors, and 1 (0.4%) was contributed by a research center. Nearly 80% are written by school teachers, and they are about various ways to solve mathematical problems and alternative approaches to teach a particular topic. The one author (including joint writing) whose name appeared most frequently (in 25 articles) is a secondary school teacher. He received his Ph.D. in mathematics education from East China Normal University and now is actively involved in mathematics education research in his school and Macao-wide. He often represented the Macao Association of Research in Mathematics Education to act as one of the ASID reviewers. The one whose name appeared the second most frequently (in 21 articles) is the former head of mathematics department in a secondary school. He has led the mathematics team in his school to write a full serial of middle school mathematics textbooks (Mathematics Division of Hou Kong Middle School, 1995a, 1995b), which were very popular in the 1990s. These two authors also published books individually and jointly (Cheang, 2012, 2015; Cheang & Tang, 2017; Tang, 2011, 2013). In terms of contributions to the mathematics education practice in Macao, they have played a very important leading role.

Less than 10% of the articles are jointly written by school teachers and professors, which indicate that the collaboration between researchers and practitioners is not as common as anticipated. The second author of this chapter often visits schools, does classroom observations, discusses students' learning difficulties with team of mathematics teachers in the school, modifies their original lesson plans, and finally writes articles for these journals to share the instruction ideas with teachers in other schools. The above statistics reveals that more effort is needed, in particular, for lower grade levels as shown below.

Grade levels. We categorize the grade levels that the 264 articles focused on as kindergarten, lower primary level (Grades 1–3), upper primary (Grades 4–6), middle school (Grades 7–9), high school (Grades 10–12), and tertiary education levels. Some of the articles are more general and can be applied to the full primary education level (Grades 1–6) or the full secondary education level (Grades 7–12); some even covered all the 12 years of schooling. Only one grade-level category is determined for one article. The distribution is shown in Table 2. It is interesting to note that about half of the articles focused on mathematics education at the high school levels, the crucial stage before the tertiary education. It is also astonished to see that a bit more than 25% of the articles focused on the primary level, which builds the foundation for

Table 2 Distribution of articles in terms of educational levels

Grade levels	Frequency	Percent
K1–K3	1	0.4
1–6	23	8.8
1–12	20	7.6
7–12	15	5.7
1–3	13	4.9
4–6	17	6.4
7–9	44	16.7
10–12	131	49.6
Total	264	100

children's future development in mathematics. Only one article was for mathematics education at the kindergarten level, which is really very sparse.

Content areas. The content areas involved in the articles were classified into numbers and operations, algebra, geometry, measurement, statistics, and probability as Cai and Jiang (2017) did. Some articles concerned two of the four areas, and some covered all the four areas as PISA. These two categories are listed as a separate category each. The frequencies and percentages are shown in Table 3. About two-fifth of the articles are related to algebra, the main content in the middle and high school mathematics. Comparatively, fewer articles are related to the other four content areas. In particular, only five articles are related to data analysis and probability, which are relatively new to the teachers and students (Cai & Jiang, 2017). It is encouraging to find that about 10% of the articles are related to both algebra and geometry, some of which focused on the teaching of these two main topics. Some focused on the use of combination methods of number and shape, which is an effective problem-solving method that was highly recommended and widely applied in solving mathematical problems.

Topic studies. The topics that the articles focused on were classified into mathematics curriculum, mathematics instruction, mathematics learning, mathematical problem-solving, etc. Only one topic category is determined for one article based on its primary

Table 3 Distribution of articles in terms of content areas

Content areas	Frequency	Percent
Numbers and operations	24	9.1
Algebra	107	40.5
Geometry	38	14.4
Measurement	5	1.9
Data analysis and probability	5	1.9
Algebra and geometry	26	9.8
All	59	22.3
Total	264	100.0

focus. To Chinese mathematics educators, it is a tradition to integrate mathematical problem-solving into mathematics curriculum and instruction (Cai & Nie, 2007). Problems including worked examples and exercises are one of the main components in the mathematics textbooks (Fan & Zhu, 2007). Research on mathematical problem-solving often focuses on the study of multiple solutions of one problem (一题多解), the use of multiple problems to teach one concepts (多题一解), and the discussion of multiple changes of one problem (一题多变) to help students which forms a coherent scheme of an important concept like speed (Cai & Nie, 2007; Jiang et al., 2014). Therefore, we put all the articles on mathematical problem-solving as a single category.

Table 4 shows the frequencies and percentages of articles in different topics. It is not surprising to see that nearly half of the articles focused on mathematical problem-solving. As to the three vertices of the didactic triangle, mathematics instruction caught the most attention from the authors of these articles. School teachers wrote articles to make reflections on their lesson designs and their implementations in real classrooms and to make suggestions for future instruction. Although there is no teaching research office in DSEJ likened that in the Chinese Mainland (Yang & Ricks, 2013), there does exist teaching research offices at the individual school level in Macao. Teachers teaching the same level mathematics have weekly meetings to report on the progress in the current week and discuss their plan for the next week. They also organize open-class activities from time to time to do classroom observations and reflections. The second author of this chapter brings her undergraduate students in secondary mathematics education program to join in local schools' open-class activities twice a year. She also suggested the visited schools to arrange the so-called "Same Content Different Designs" (*Tongke Yigou*) activity (Yuan & Li, 2015) as one of the open-class forms. *Tongke Yigou* helped the prospective mathematics teachers to understand that there are different ways to teach the same content, and it is necessary to select the more appropriate way based on the students' realities and the supporting resources the school can provide. The second author of this chapter also tried to point out the most crucial steps in the different designs. For example, how to show the monotonic property of a function using dynamic geometry software (DGS) is the most important part in teaching the properties of inverse functions (Cheong et al., 2018).

As aforementioned DSEJ released BAAMs for different key stages in 2016–2017, about 16% of the articles discussed what is new in the curriculum standards, what impact the BAAMs will have on classroom teaching. Only 30 articles are concerned about students' mathematics learning, which is the starting point of instructional design. Our serial studies revealed that Macao students' learning is very similar to their peers in the Chinese mainland, Hong Kong, and Taiwan (Jiang et al., 2016, 2019). However, Macao students did not think that One Problem Multiple Solutions (OPMS) are important for their mathematics learning (Jiang et al., 2016, 2019). Our further investigation found that OPMS was not a common practice to mathematics teachers in Macao (Jiang et al., 2017). Furthermore, Macao students thought that memorizing was important to their mathematics learning. In particular, they thought that it is important not only to memorize the definition of mathematical

Table 4 Distribution of articles in terms of topics

Topics	Frequency	Percent
Mathematics curriculum	41	15.5
Mathematics instruction	60	22.7
Mathematics learning	30	11.4
Mathematical problem-solving	128	48.5
Teacher's professional development	2	0.8
Use of ICT in mathematics education	1	0.4
Others	2	0.8
Total	264	100.0

terms, formulae, and mathematical theorems, but also important to memorize exercises finished before the examinations, and the problems where they often make errors (Cheong et al., 2017). Very few articles are concerned about teachers' professional development and use of ICT in mathematics teaching and learning.

In summary, among the 264 articles, nearly 80% were contributed by school teachers, from which two contributed more than 20 articles each. As to the grade levels, about 50% of them are for high school levels, only one for kindergarten level. Concerning the content areas, about two-fifth of the articles are related to algebra, the main content in the middle and high school mathematics. About 10% of the articles are related to both algebra and geometry, and only five articles are related to data analysis and probability. Regarding the topics covered, nearly half focused on mathematical problem-solving, followed by mathematics instruction, then mathematics curriculum, and lastly the mathematics learning.

It seems that the results are conflicting with each other. On one hand, nearly half of the articles are related to mathematical problem-solving, which includes OPMS. On the other hand, OPMS is not considered as important for Macao students' learning. One possible explanation is that OPMS is an important component of teaching practice of very few mathematics teachers to be conducted in a limited number of schools only. Therefore, not too many students could have a lot of experiences in their classroom learning. We need to encourage more mathematics teachers to use OPMS in their classes because OPMS is useful in helping students to make connections among different kinds of problems, different forms of solutions, and different ways/types of approaches/strategies used.

4 Conclusions and Future Directions

In this chapter, we firstly introduce the general information about the education and mathematics education in Macao as a background, secondly describe the two key characteristics of Macao's research culture in mathematics education, that is, it has been mostly school-based in private schools and carried out by autonomous educational practitioners, and meanwhile, the government has been increasingly exercising tighter control over the education, which has shaped the development of mathematics education research at the school level, and finally report the three lines of research done in mathematics education in Macao.

The chapter highlighted several characteristics of mathematics education research in Macao. First, much and important research work has been done around the PISA. This line of research is not only highly policy-oriented and interpretation-oriented, but also an indication of the important influence of outside and international environment on Macao's mathematics education. This is because PISA has been used by Macao government to monitor the schools to be responsible for the academic standards as well as futuristic direction of the basic education they provided to their students.

Second, in terms of the topic areas in the field of mathematics education, the area that has received most attention by Macao's mathematics education researchers is problem-solving, followed by mathematics instruction, mathematics curriculum, and mathematics learning. Some other areas such as teacher education, teacher professional development, ICT, and mathematics education are largely ignored.

Third, in terms of the grade levels, Macao's mathematics education researchers have focused most on senior high school and then junior high school levels, much less on primary level and almost none on kindergarten level. It appears that this characteristic reflects the fact that research culture in junior and especially senior high school levels is much more active than in primary and kindergarten levels.

Fourth, in terms of the authorship, most (about 80%) of the publications in the field of mathematics education were authored by school teachers, which explains the importance of the idea of teachers as researchers in mathematics education research culture in Macao, and in addition, only a small portion (about 20%) was co-authored, which in a sense indicates that mathematics education research in Macao has been done more individually rather than collaboratively. Caveats must be made here to caution the readers that most publications are not research articles as usually stated, but by and large are action research or experience sharing of application of research knowledge to practice for purposes of improvement of curriculum and instruction.

Finally, most publications in mathematics education in Macao are practice-oriented and, to a large extent, based on teachers' own teaching experiences, observations and reflections. While the value of such publications and the related work should never be underestimated for the purpose of sharing for teachers, the weakness in terms of methods for data-collection or evidence gathering from a research perspective should be clearly noted.

To conclude, while the achievement of mathematics education research in Macao should be duly commended, there are also issues and challenges concerning Macao's mathematics education. For the future directions of mathematics education research in Macao, we suggest the following:

First, mathematics educators in Macao should pay more attention to under-researched yet important areas in mathematics education, in particular, teacher education and professional development, and the integration of ICT in mathematics education.

Second, more collaborations between different organizations, between different regions, and between different researchers (especially between school practitioners and university-based researchers) should be encouraged and promoted in order to utilize the knowledge, skills, and experiences of different parties and to make research more concerted.

Third, more research work should also be undertaken with focus on lower grade levels (particularly, junior and primary school levels), given their importance for students' learning of mathematics, and accordingly, related research culture and skills should be developed.

Fourth, there is a need for mathematics educators in Macao, as a research community, to further broaden the scope of research (e.g., go beyond interpretation-oriented, policy-oriented, and practice-oriented types of research) and enhance the capacity of research, in particular, in terms of research methods (e.g., using more qualitative and mixed methods).

Fifth, the educational context in Macao provides a good place to compare the effect of the implementation of different mathematics curricula on teaching practices in schools using different media of instruction. It merits further investigation in this direction.

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