The Evolution of Mathematics Education Research in Singapore

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Abstract Up until 1990, the Institute of Education in Singapore was primarily a teaching institute involved in training teachers for Singapore schools. Since the inception of the National Institute of Education (NIE) in 1990, as an institute of the Nanyang Technological University, the focus of the institute has been enlarged to include research in education. This chapter examines, through a documentary analysis, how a research culture specifically in mathematics education at the National Institute of Education was nurtured, developed and supported from 1990 onwards. Development of the culture for Mathematics Education Research (MER) has been in tandem with all other areas of research at the NIE. Both top-down and bottom-up approaches have been adopted to support research as part of an academic's work at the institute. Policies related to recruitment and promotion of academics were developed to ensure that emphasis was on both teaching and research. Development of research, from individually led bite-sized grains to team-based project with coherent themes, was supported. The setting up of the Centre for Research in Pedagogy and Practice in 2004 and dedicated funding from the Ministry of Education Singapore for research of the Singapore education system heralded an era of MER that has made significant contributions both nationally and internationally. This chapter will also illuminate the four main areas of focus and sources on MER through examples of studies carried out in Singapore since 2000. In addition, it briefly outlines the contribution of MER in ASEAN countries.

Keywords Mathematics education research · Singapore · Evolution · National Institute of Education · Centre for Research in Pedagogy and Practice · Funded research projects · Postgraduate student research

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1 Background

Up until 1990, the Institute of Education (IE) in Singapore was primarily the sole teaching institute involved in training teachers for Singapore schools. Since the inception of the National Institute of Education (NIE) in 1990, as an institute of the Nanyang Technological University, the focus of the institute has been enlarged to include research in education. The transition from an Institute of Education to the National Institute of Education saw the revamping of academic status of its faculty. In the IE, teaching staff were primarily lecturers, senior lecturers and principal lecturers. In the IE, emphasis was on training teachers for schools in Singapore and apparently based on the assumption that tacit knowledge self-perpetuates by way of role-modelling by lecturers at the IE and practising teachers in schools that mentored trainee teachers. The NIE, being an institute of the Nanyang Technological University, applied the same academic ranking used in the university, such as an assistant professor, an associate professor and a professor. As such, the yardsticks for performance of academic staff were also tied to three key aspects: teaching, research and service to the academic/education community. A rubric with a weightage of T (Teaching): R (Research): S (Service) = 5:5:2 for appraisal of staff performance slowly but surely led to developments at the institute that supported research activities of staff. In the following sections, we account chronologically the development of Mathematics Education Research (MER) in NIE which being the sole institute for teacher education in Singapore is also the bedrock for MER in Singapore.

2 Beginnings of Mathematics Education Research in Singapore (Pre-1990)

It may be viewed that in the years leading up to the early 1990s, mathematics education research in Singapore was mainly driven by one of the following needs: (i) postgraduate academic requirements such as a thesis for a master in education (M.Ed.) or doctorate (Ph.D.) in education programme, (ii) individual research interest pursuits or iii) evaluation of the efficacy of teaching approaches and curriculum materials. Two notable documents, a proceeding of a mathematics education conference held in 1987 in Singapore (Institute of Education, 1987) and a review of mathematics education research in Singapore done in 1991 (Chong et al., 1991), provide us with some insights about the state of MER up till the beginning of the 1990s.

The Fourth Southeast Asian Conference on Mathematical Education was held in Singapore in June 1987. This was the first mathematics education conference held in Singapore. It was part of the *South East Asia Conferences on Mathematics Education* (SEACME) series that began in 1978 in Manila (Philippines), and thereafter conferences were held at three-year intervals in Kuala Lumpur (Malaysia) (1981), Haad Yai (Thailand) (1984), Singapore (1987), Brunei (1990), Surabaya (Indonesia) (1993), Hanoi (Vietnam) (1996), and Manila (Philippines) again in 1999. Father, Dr

Bienvenido F. Nebres of the Ateneo de Manila University and Dr Lee Peng Yee of the University of Singapore were instrumental in actualising this series of conferences.

The published proceedings of SEACME 4 shows that there was one keynote paper by Robert Davis from the USA and seven invited papers (two from the UK, two from Australia, one each from the Philippines, Germany and Israel) (Institute of Education, 1987). Another fifty-three papers were contributed, with 18 each from Australia and Singapore, four from India, two each from Japan, Philippines, Papua New Guinea, New Zealand and the USA, and one each from Indonesia, Thailand and the UK. Out of the 18 papers from Singapore, two were from an international institution—the United World College, three were from the Curriculum Development Institute of Singapore (CDIS) involved in innovation and curriculum for schools in Singapore under the purview of the Ministry of Education in Singapore, two were from schools—these were based on the research done by the teachers as graduate students of the IE as part of their Master of Education dissertations and 11 were from the IE showcasing research of the department of mathematics and computer studies at the IE in Singapore.

A close examination of the presentations from Singapore illuminates the nature of MER that was present. The two papers presented by colleagues from the United World College (Binge, 1987; Butler, 1987) were reflections of the Cockcroft Report of the UK (The Cockroft Report, 1982) and implications for mathematics instruction at the United World College in Singapore. The three presentations from the CDIS (Khoo, 1987; Lim, C. L., 1987; Sin, 1987) were reports of ongoing developmental work in school mathematics curriculum at the primary and secondary schools in Singapore. All the reports showcased proposed methods of instruction that were advocated for school mathematics curriculum implementation through textbooks produced by the CDIS. Another two presentations were helmed by school teachers. The first was in partnership with a colleague in the CDIS, and it evaluated the implementation of an individualised computer-assisted remediation package (Gay & Leong, 1987). The second was based on a teacher's survey on the understanding of an aspect of Calculus taught at the junior college and pre-university centres in Singapore (Chen, 1987).

The eleven presentations from the IE were mainly in two areas. The first was empirical, one-off studies on the teaching and learning of mathematics, including the efficacy of teaching materials developed by the CDIS in Singapore schools and pre-service mathematics teacher education at the institute (Chai & Ang, 1987; Fong, 1987a; Gan, 1987; Ong, 1987a; Ong & Lim, 1987; Plant, 1987; Wong, 1987a). The second was on innovative ideas for mathematics instruction in Singapore schools (Fong, 1987b; Lim, S. K., 1987; Ong, 1987b; Wong, 1987b). It is apparent from the presentations that MER was in its infancy and there were no acknowledgements whatsoever for funding of any sort that supported research activities.

In 1991, as part of a state-of-the-art review on mathematics education that was commissioned and funded by the Southeast Asian Research Review and Advisory Group (SEARRAG), mathematics education research in Singapore from 1979 till 1991 was surveyed and documented (Chong et al., 1991). Of the 42 studies documented, 16 (38%) were thesis or dissertation presented for a M.Ed. or Ph.D. degree,

22 (52.4%) were driven by individual research pursuits, and 4 (9.6%) were research projects that evaluated efficacy of teaching approaches and curriculum materials. Of these 22 studies, nine were research reports, seven were journal papers, and ten were conference papers. Six of the seven journal papers were published in the local journals of the IE, two in *Teaching and Learning* and four in the *Singapore Journal of Education*. Only one was published in an international journal—*Studies in Educational Evaluation*. Furthermore, none were published in an international mathematics education research journal.

Almost 62% or 26 out of the 42 studies were on the teaching and learning of mathematics. They were on affective variables and problem solving (Foong, 1985, 1990; Ng-Gan, 1987; Tan, 1989; Wong, 1989), types and levels of understanding (Chu, 1987; Lam, 1985; Purbick et al., 1982; Tan, C. S., 1987; Tay, 1986), analysis of errors (Booth, 1986; Chai & Ang, 1987; Kaur, 1991; Ong & Lim, 1987), low achievers and remediation (Ee, 1991; Fong, 1987c; Yap, 1990), learning strategies (Ng, 1985; Wong, K. Y., 1990; Wong, P., 1990), use of microcomputers (Ho, 1990; Tan, P. K., 1987; Woo-Tan, 1989) and other miscellaneous topics (Ang, 1984; Chai, 1979; Kaur, 1987).

Of the rest of the studies, five were on assessment and examinations in mathematics, five on teacher education in mathematics and six on the efficacy of teaching approached and curriculum materials. Almost all of the studies were one time and exploratory in nature. Chong et al. (1991) noted that:

> the findings even if significant such as the effectiveness of a particular instructional intervention should be considered tentative and perhaps not easily generalizable to other similar situations (p. 49).

Only a few of the studies that had been carried out discussed their findings with a view for possible improvements in mathematics education. The rest merely verified past research findings making no reference to possible improvements in mathematics education. The state-of-the-art review on mathematics education of 1991 concluded that the challenge to raise the level of attention in research on mathematics education towards improvement was best summed up as follows by Sim (1991) cited in Chong et al. (1991):

> [A]n obvious deficiency among studies purportedly to be research in mathematics education is the lack of serious attention to subsequent improvements in mathematics education (p. 59).

3 The Sandwiched Era of Mathematics Education Research in Singapore (1991–2003)

With the inception of the National Institute of Education (NIE) in 1991, as an institute of the Nanyang Technological University, the focus of the institute was enlarged to include research in education. As such, there was an expectation for faculty to partake in research activities. This heralded a top-down push for research activities. Faculty were supported to attend international conferences to present their research, solicit

feedback from international colleagues and network with like-minded researchers. Faculty of the IE, without doctorate degrees, were assessed by an international panel for their long-term fit in NIE. Promising faculty were sent for development at international teacher education institutions. For mathematics education, one such faculty was the first author of this chapter. She went to Monash University in Australia and pursued her PhD in Education. Her three-year study at Monash University was funded by the NIE.

These academics of the NIE, while attending international conferences or doing their studies at international institutions, managed to network with international like-minded colleagues and this facilitated participation in international research. This bottom-up approach, initiated by the academics themselves, was evident in the Kassel Project carried out from 1995 till 1997 by Kaur and Yap (2009), the International Project on Mathematical Attainment (IPMA) carried out from 1999 till 2003 by Kaur et al. (2009a) and a comparative study of primary school pupils' perceptions of their best mathematics teacher in Singapore and Brunei Darussalam from 1997 till 1999 by Wong et al. (2009). Modest funding for the Kassel Project was provided by The Gatsby Charitable Foundation in the UK and the British Council. The IPMA was also modestly funded by the Academic Research Fund of the NIE and the University of Exeter in the UK. The funding was sufficient for data collection purposes but not manpower needs to assist with rigorous data analysis. For the purpose of illustration, Table 1 shows the research outputs of the above three projects that were helmed by the first author of this chapter.

It is apparent from Table 1 that the main goals of the Kassel Project and the IPMA were to document the research as research reports for communication among the communities that were participating in the studies. These were shared at the meetings held by the respective studies. Modest attempts were made to present the research to the wider research community through conferences and other research-related publications. This was partly due to the lack of support for research activity in terms of funding for both "time" and "manpower". Hence, this appears to have impacted both the quantity and quality of the research outputs.

4 Mathematics Education Research in Singapore (Post-2003)

To fuel a research culture at the NIE, it became apparent that research funding was necessary in a structured manner to support educational research. At the beginning of the twenty-first century, the then Dean of the Graduate Programme Office, Professor Lee Sing Kong, initiated the setting up of a Centre for Research in Pedagogy and Practice (CRPP) with a research tranche of about 50 million Singapore Dollars from the Ministry of Education. The CRPP was established at the NIE in 2003 with a focus to improve and sustain student and teacher learning in Singapore schools. The aims of the centre as reflected in the Research Excellence Report (NIE, 2017) are to:

Table 1 Publications from the Kassel project, IPMA and comparative study

Research output	Kassel project	IPMA	Comparative study—Singapore and Brunei Darussalam
Research reports	3 Kaur and Yap (1996, 1997a, 1998)	5 Kaur et al. (2000, 2001a, 2003) and Koay et al. (2003, 2004a)	–
Keynote addresses	–	–	–
Invited presentations	–	–	–
Scholarly book chapters	2 Kaur and Yap (2004, 2009)	2 Kaur et al. (2004, 2009a)	2 Wong et al. (2007, 2009)
Conference papers (published in proceedings)	2 Kaur and Yap (1997b, 1999)	5 Kaur et al. (1999a, 2001b), Koay et al. (2001, 2004b), Thompson et al. (2010)	1 Kaur et al. (1999b)
Journal papers (refereed)	–	1 Thompson et al. (2013)[a]	–

[a] Published in tier 1 MER Journal

- conduct high-quality research and development programmes that are innovative, relevant and responsive locally and internationally;
- understand, design and implement pedagogical innovations in formal and informal contexts, towards more equitable futures for all learners and
- generate rigorous and impactful school-based and system-level educational research that is cognisant of the sociocultural context of Singapore's education landscape (p. 7).

The NIE's Roadmap for the period (2007–2012) (NIE, 2007) outlined three pillars as overarching themes to realise NIE's vision as an institute of distinction. The second pillar (Pillar 2): Achieving international recognition through educational research clearly delineated the following objective for 2012:

> Identifying, developing, implementing and managing a strategically focussed, scientifically rigorous NIE-wide programme of research, development and innovation that seeks to improve the quality of teaching and learning in Singapore schools and consolidate NIE's recent emergence as a leading international research institution (p. 28).

In 2008, the Office of Education Research (OER) was established to forge an institutional-wide programme of research at NIE. A key function of OER is to administer the Education Research Funding Programme, a pool of research funding provided by the Ministry of Education in Singapore. OER aims to:

(i) develop NIE's research capacity in key areas that impact programmatic and pedagogical enhancements both within NIE and in schools and

(ii) deliver evidence-based research-informed pedagogies and programmes to raise the competencies and capabilities of teachers systemically (NIE, 2017, p. 6).

The OER has three research centres. The centres are the CRPP, Education and Cognitive Development Lab (ECDL) and the Learning Sciences Lab (LSL). Over the past two decades, OER has received four tranches of funding from the MOE. In every tranche, MER was funded. It was noted in NIE (2017) that the outstanding research produced by the faculty at NIE makes the Nanyang Technological University (NTU) consistently one of the 20 universities in education and among the top in research performance in the world.

4.1 Institutional Funding for Mathematics Education Research

A meta-analysis of OER funded mathematics-related projects for the period 2008–2017 was carried out by Wong (2017). In his analysis, he noted that a total of 43 projects were funded with a sum of 15.5 million Singapore dollars. The budgets for the projects ranged from $28,000 to $4 million with an overall mean of $350,000. The research grants awarded for the projects included costs of human power, equipment, consumables and other costs such as support for conferences and publications of research outputs.

For the purpose of illustration, Table 2 shows the research outputs of two funded projects, during the period, that were helmed by the first and second authors of this chapter, respectively.

It is apparent from Table 2 that the research outputs of projects that were funded were significant both in quantity and quality when compared to those in Table 1. There were four papers by each of the projects published in tier 1 (Q1) MER journals. In addition, there were also more scholarly contributions in terms of books and chapters. This is certainly a consequence of both the projects:

- Student Perspective on Effective Mathematics Pedagogy: Stimulated Recall Approach Study [The Learner's Perspective Study in Singapore] (CRPP 3/04 BK) and
- Mathematical Problem Solving for Everyone (MPROSE) (OER 32/08 TTL)

being funded appropriately for NIE faculty to engage in research as well as recruitment of research assistants to assist in the research. Capacity building was also apparent as researchers like the first two authors of this chapter worked with teams of colleagues from NIE as well as international peers in the projects. As expected, Wong (2017) noted the academic output of MER projects that were funded during the period 2008–2017 had served the goals of raising the profile of NIE as an influential teacher education institute and to advance the career of NIE scholars.

Table 2 Publications from two MER funded projects at the NIE

Research output	Student perspective on effective mathematics pedagogy: stimulated recall approach study (the learner's perspective study in Singapore) (CRPP 3/04 BK)	Mathematical problem solving for everyone (MPROSE) (OER 32/08 TTL)
Research reports	1 Kaur and Loh (2009)	–
Keynote addresses	3 Kaur (2007a, 2016, 2018)	3 Toh (2013, 2014, 2018)
Invited presentations	1 Kaur (2009a)	–
Professional books	–	2 Toh et al. (2011a, 2011b, 2011c), Leong et al. (2013)
Scholarly books (edited)	2 Shimizu et al. (2010a), Kaur et al. (2013)	–
Scholarly book chapters	11 Kaur and Toh (2019), Leong et al. (2019), Anthony et al. (2013), Mok et al. (2013), Kaur (2010, 2013, 2014), Shimizu et al. (2010b), Kaur et al. (2006a), Mok and Kaur (2006), Seah et al. (2006)	3 Toh et al. (2008, 2011c, 2019)
Conference papers (published in proceedings)	5 Kaur (2007b, 2008a, 2017), Kaur et al. (2005, 2006b)	17 Dindyal et al. (2009, 2010, 2013, 2014); Ho et al. (2013); Leong et al. (2010); Quek et al. (2010, 2012, 2014); Tay et al. (2007); Toh, P. C. et al. (2012, 2014b); Toh (2012); Toh et al. (2009, 2011b, 2012, 2013a)

(continued)

Table 2 (continued)

Research output	Student perspective on effective mathematics pedagogy: stimulated recall approach study (the learner's perspective study in Singapore) (CRPP 3/04 BK)	Mathematical problem solving for everyone (MPROSE) (OER 32/08 TTL)
Journal papers (refereed)	4 Kaur (2008b, 2009b, 2011)[a], Shimizu and Kaur (2013)[a]	13 Deng et al. (2015); Dindyal et al. (2012); Leong et al. (in-press, 2011a, 2011b, 2012)[a]; Liang and Toh (2018); Quek et al. (2011); Tay et al. (2011); Toh, P.C. et al. (2014a)[a]; Toh et al. (2013b, 2014); Yong and Toh (2019)

[a] Published in Tier 1 MER Journal

4.2 Push for Team-Based and Collaborative Research Activities in Mathematics Education Research

The 14th call for research proposals by the OER at NIE, in May 2015, encouraged programmatic research, where programmatic research is defined by an overarching project research theme which focuses on a key educational issue, problem, phenomena or outcome, along with a number of themes, or specific research studies that address important aspects or components of the issue, problem, phenomena or outcome. It therefore has a common strand or focus, supported by a common theoretical framework, and undertakes a coherent, comprehensive, multifaceted approach to understanding and addressing the issue, problem, phenomena or outcome.

The first programmatic research project award at the NIE was granted for a MER project, the Enactment Project, helmed by all the three authors of this chapter and involved a team of nine researchers, ranging from a teaching fellow to a full professor. The project: *A study of the enacted school mathematics curriculum in Singapore secondary schools* had two aims (Kaur et al., 2018). The first was to document how experienced and competent teachers enacted the school mathematics curriculum in secondary schools. It did this by examining: (i) pedagogies adopted by experienced and competent mathematics teachers when enacting the curriculum and (ii) experienced and competent teachers' use of instructional materials for the enactment of the curriculum. The second was to establish how uniform these adopted pedagogies and use of instructional materials by experienced and competent teachers were practised in the mathematics classrooms of Singapore schools.

The four-year-long project also contributed significantly towards the capacity building of younger faculty at NIE in MER. This was through the collaborative research activities of the project, from collecting data using sophisticated methods like the complementary accounts methodology (Clarke, 1998) and disseminating the research through academic papers for conferences, books and journals.

Presently, a group of mathematicians and mathematics educators at the NIE are involved in a study: Big Ideas in School Mathematics. All the three authors of this chapter are also helming the study. The study comprises three sub-studies, as shown in Fig. 1. The team works together but for sub-study 1 the mathematicians take the lead, while for sub-studies 2 and 3 the mathematics educators do the same. The project has been funded for four years (from July 2020 till June 2024) and 13 colleagues are involved with ten from the NIE, two from the Ministry of Education and a lead teacher from a secondary school. Members of the team range from experts to novices in MER.

4.3 The Purpose of Mathematics Education Research

Toh (2020) reviewed MER at the NIE since 2000. He drew on two sources of MER, namely the funded research projects and research carried out by postgraduate master

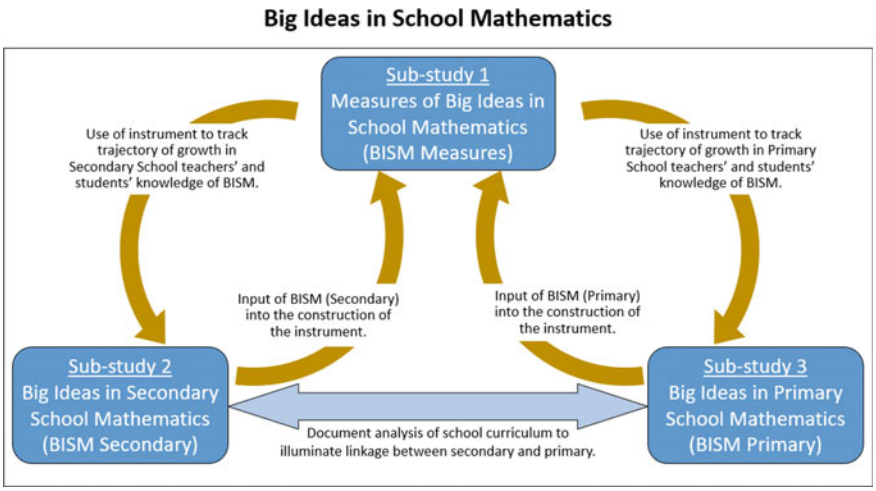

Fig. 1 Conceptual framework of the BISM study

and doctoral students. He classified the MER according to four main categories that depicted the purpose of the research. The categories were as follows:

Policy and curriculum: This category of research involves the study of how policy and curriculum impact the teaching and learning of mathematics. It is exemplified by a research project entitled *Secondary Analyses of Teacher Education and Development Study in Mathematics* which was conducted between 2014 and 2017. This project gathered information about mathematics teacher education in Singapore. It was part of a larger international comparative study, the TEDS-M project. In particular, the findings of this project offered insight into local curriculum and policy matters.

Professional development: This category of research involves the development of teachers through activities practising teachers engage with. It is exemplified by a research project entitled *Enhancing the Pedagogy of Mathematics Teachers to Facilitate the Development of 21st Century Competencies in their Classrooms* that was conducted during the period 2014 to 2017. The project's primarily goal was to build practising teachers' capacity for effective teaching in school classrooms. It proposed long-term collaboration between researchers and the practising teachers. This mode of teacher professional development deviates from the usual workshop mode that is guided by a deficit perspective of professional development.

Teaching and learning practices: This category of research primarily begins with studying the classroom enactment as the primary aim. This category of projects is exemplified by a mathematics education research project entitled *MAthematics is Great: I Can And Like (MAGICAL).* This project developed an alternative approach to teach Lower Secondary Normal (Technical) mathematics using storytelling, comics, and other graphic stimuli in context. It also examined the effect of the

approach on students' mathematical self-concept, motivation to learn mathematics and achievement in mathematics.

Judging from the description, the primary objective of the project was to study the enactment in the mathematics classroom using the researchers' new approach of using comics and storytelling for the low-ability students. The projects which were classified under the category "Teaching and Learning Practices" included those that introduced novel intervention strategies in classroom practice and a study of the impact of these interventions on student learning.

Theory building: Theory building can be explained as "the purposeful process or recurring cycle by which coherent descriptions, explanations, and representations of observed or experienced phenomena are generated, verified and refined" (Lynham, 2000, p. 161). This category of research covers those in which the researchers attempted to develop the "theory" underlying a set of phenomena in education. An exemplar of this category of project was one which was entitled "Portraits of Teacher Noticing during Orchestration of Learning Experiences in the Mathematics Classrooms". The description of the project is shown below:

> This project has two main goals. First, it involves developing a local theory to describe, and prescribe, what and how exemplary teachers notice when they orchestrate Learning Experiences in their classrooms. Second, it is aimed at designing a toolkit that can be used by teachers to promote students' thinking through high-quality Learning Experiences.

When classifying the research projects or research carried out by postgraduate students at times when they fell in more than one category, the classification was based on the original intention of the researchers as provided in the brief description of the research projects, or the abstract of each of the postgraduate doctorate or master thesis (see Toh (2020) for more details). Table 3 shows the spread of MER, since 2000, across the four categories and their source.

It is apparent from Table 3 that under the category "funded mathematics education research projects", the most frequent category of research is "Classroom Teaching and Learning Practices" (37.7%), while the least frequent is "Theory Building"

Table 3 Classification of MER conducted in Singapore since 2000

		Dissertations and thesis	
Source category	Funded projects	Doctoral level	Master level
Policy and curriculum	12 (26.7%)	2 (8.8%)	13 (12.5%)
Professional development	15 (33.3%)	1 (4.4%)	1 (1.0%)
Teaching and learning Practices	17 (37.7%)	5 (21.6%)	38 (36.5%)
Theory building	1 (2.3%)	15 (65.2%)	52 (50.0%)
Total	45	23	104

(2.3%). On the other hand, the most frequent category of research for postgraduate research (including both master and doctorate level studies) level is on "Theory Building" (65.2% for doctorate and 50% for master level studies), while the least common category of education research is "Professional Development" (4.4% for doctorate and 1.0% for postgraduate research).

The above data suggests that different types of mathematics education research serve different functions. Undeniably, the priority of postgraduate studies is to engage candidates to go through the process of research, out of which theory building is an essential component. Thus, significantly many research projects from postgraduate studies have their primary goal as contributing towards the building of theory about specific aspects of mathematics education. This explains the relatively high number of research projects on theory building for the postgraduate research. On the other hand, the funded education research projects conducted in NIE are geared towards improving teachers' effectiveness (Teacher Professional Development, 33.3%) for a more efficient enactment of the curriculum in the authentic classroom (Teaching and Learning Practice, 37.7%). This is in line with the goal of the research funding from the OER at NIE.

5 Singapore's Role in Developing Mathematics Education Research in ASEAN Countries

There are three distinct ways through which MER in Singapore has contributed towards the development of the same in ASEAN countries. The first is similar to how Singapore drew on expertise elsewhere in the period 1991–2003 to develop its own faculty in MER. Since 2005, several students from ASEAN countries have completed their Philosophy of Doctor degrees in Mathematics Education at the NIE. Among them, one from the Philippines is presently a faculty of Ateneo de Manila University in her country and another from Thailand is also a staff of the Institute for the Promotion of Science and Technology in her country. Both students were on scholarships from the respective institutions to do their studies at NIE. Numerous postgraduate students have also come for shorter stints of research attachments with MER scholars at the NIE as part of their PhD studies in their home institutions. Two such students were from the Universiti Pendidikan Indonesia in Indonesia and the Mahidol University in Thailand.

The second and third means of contribution have been through conferences and publications. International and national mathematics education conferences are held periodically at the NIE. The Mathematics and Mathematics Education (MME) Academic Group at the NIE where MER resides has hosted numerous high-profile international conferences since 2012. The first was the second East Asian Regional Conference on Mathematics Education held in May 2002. Next was the Mathematics Education in Research of Australasia (MERGA) 2012 conference held in July 2012 followed by the International Psychology of Mathematics Education Conference in

July 2017 and the MERGA 2021 conference in July 2021. These conferences have in some sense allowed easy excess for MER scholars from the region to engage in rich and current matters in their field of expertise. Since 2005, the Association of Mathematics Educators and the MME Academic Group have jointly organised the Mathematics Teachers Conference. By way of invitations, numerous scholars from the region have also presented their work to teachers from Singapore and elsewhere. These presentations are published in a thematic Yearbook of the Association (see, Kaur et al., 2009b—the first book and Toh and Choy (2021) for the most recent one). This publication has also contributed towards the development of teachers and research scholars in the region and elsewhere.

In 2013, the first author of this chapter together with Catherine Vistro-Yu from the Ateneo de Manila University in the Philippines initiated the Springer Book Series entitled: Mathematics Education—An Asian Perspective. The goal of the series is to facilitate the publication of research in Asia that is often under represented in the international landscape. To date, several volumes have been successfully published by Springer. There are also other ad hoc ways through which MER in Singapore aka NIE continues to contribute in the region and elsewhere. These mainly arise out of the Memorandum of Understanding (MOU) the NIE has with partnering institutions; for example, there is an MOU on Innovations and Teaching and Learning of STEM (Science, Technology, Engineering and Mathematics) with Design Thinking between Singapore and the Philippines (see, https://www.philippine-embassy.org.sg/about-us-2/overview-of-philippines-singapore-relations/).

6 Concluding Remarks

It is apparent from the chronological review of MER in Singapore that from the early 1990s till the present, MER has gradually evolved from one-off small-scale studies on aspects of teaching and learning mathematics to programmatic cum team-based ones that have direct impact on the teaching and learning of mathematics in schools. This evolution has been brought about by firstly research being a necessary aspect of university education at the NIE and also the need for Educational Research and in turn MER by the Ministry of Education in Singapore.

Both top-down and bottom-up initiatives have contributed towards the development of MER. During its infancy, individuals participated in MER either to evaluate some aspects of their teaching, use of resources or simply for interest. They did this alongside their full teaching loads and without any form of research support. But when NIE was established as an institute of the Nanyang Technological University, a significant top-down push was the need to engage in research and create research outputs that were at par with international standards. In reaction to the need to do so, bottom-up initiatives such as networking with international scholars and the creation of centres of research at NIE that funded research in line with the needs of the nation strongly supported MER from then on.

It is apparent from the illustrations in Tables 1 and 2, how research funding impacted the research outputs of mathematics educators at the NIE from 1991 till the present. As would be expected, the trajectory of MER growth has now moved into studies comprising several phases and multiple goals. These studies are engaging in rigorous and purposeful MER that contributes towards local needs and also international MER literature and capacity building of researchers. This is evident in Sect. 4 of the chapter that showcases the Enactment Project and the BISM study. It may be said that MER at the NIE and in turn in Singapore has created a systematic approach of engaging in research with mentors that facilitates rigorous and high yield research outputs. Lastly, it is also apparent that MER in Singapore has contributed towards the development of the same in ASEAN countries and elsewhere. It has done so through their graduate education at the NIE, by hosting international and national conferences and through publications that has not only showcase work at the MIE but also facilitate the publication of works by colleagues in ASEAN countries and Asia.

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