Mathematics Education – An Asian Perspective

Bill Atweh
Lianghuo Fan
Catherine P. Vistro-Yu *Editors*

Asian Research in Mathematics Education

Mapping the Field

Springer

Mathematics Education – An Asian Perspective

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Preface

In 2021, the then newly elected president of the International Commission on Mathematical Instruction (ICMI), Frederick Leung, aspired that "[b]ased on scholarly research, we should facilitate and encourage sharing of best practices and cross-fertilization of ideas, while focusing on capacity building. And in the course of doing so, we should be sensitive to contextual and cultural differences in different countries" (March 1, 2021 ICMI Newsletter). Around two decades previously, Feuer, Towne, and Shavelson (2002) cited the need to raise the quality of educational research worldwide and argued for establishing a strong scientific research culture to help raise the standards of educational research. This collection of chapters examines recent research in mathematics education in various Asian economies.

This is the sixth book in the Springer series *Mathematics Education—An Asian Perspective*, edited by Berinderjeet Kaur and Catherine Vistro-Yu. It is the first book in the series that focuses on recent research in mathematics education in various Asian economies that has developed at different times and rates and in recent decades according to the various economies' history and political priorities. Asia covers a wide region of the globe and is home to over half of the population on earth. During the past two decades, almost all economies have established policies and infrastructure to promote quality research in most areas in education, including mathematics education. For us here, this is an opportune time to examine this research critically in order to highlight its contribution for international audience and to reflect on its strengths and remaining challenges for the benefit of both the international audiences and policy makers, researchers, research managers, and postgraduate students in the continent.

Many economies in Asia are known for their generally high rankings in international large-scale assessments such as in Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA). However, this exemplary achievement is not uniform across the continent. The outstanding performances of Singapore, Japan, Chinese mainland, South Korea, and Taiwan have been consistent, eclipsing other Asian economies unremarkable achievements in these assessments.

In the area of mathematics education research, however, we see a different scenario. Research in the continent is still developing and is establishing its identity at an international scene—albeit at a rapid rate. Currently, almost all economies in Asia are actively engaged in producing mathematics education research and, more importantly, are publishing their own journals with special focus on mathematics education. Below is an alphabetical list of some of these journals that have been publishing mathematics education research, with some having been indexed by reputable international journal databases.

1. *Asian Journal for Mathematics Education* (AJME) published, e-ISSN: 2752-7271 and ISSN: 2752-7263 (Print), by SAGE, founded by East China Normal University. https://uk.sagepub.com/en-gb/asi/asian-journal-for-mathematics-education/journal203738
2. *Brunei International Journal of Science and Mathematics Education* (BIJSME), ISSN 2076-0868, Sultan Hassanal Bolkiah Institute of Education. https://shbieejournal.wordpress.com/journals/bijsme/
3. *Hiroshima Journal of Mathematics Education* (HJME), e-ISSN 2758-5263 and ISSN 0919-1720 (Print), is the official English-language journal of the Japan Academic Society of Mathematics Education (JASME). https://www.jasme.jp/hjme/
4. *Indonesian Journal of Science and Mathematics Education*, e-ISSN: 2615-8639, is a journal that is published by the Research and Scientific Publication Unit. http://ejournal.radenintan.ac.id/index.php/IJSME/index
5. *International Journal of Education in Mathematics, Science and Technology, Turkey* (IJEMST), ISSN 2147-611X, is affiliated with International Society for Technology, Education and Science (ISTES). https://www.ijemst.net/index.php/ijemst
6. *International Journal of Science and Mathematics Education* (IJSME), e-ISSN 1573-1774 and ISSN 1571-0068 (Print) published by Springer, Founded by National Science and Technology Council, Taiwan. https://www.springer.com/journal/10763
7. *Intersection*, ISSN 0118-6876, is the official journal of the Philippine Council of Mathematics Teacher Educators (MATHTED), Inc. http://mathted.weebly.com/intersection.html
8. *Journal of Japan Society of Mathematical Education*, e-ISSN 2434-8619 and ISSN 0021-471X (Print), is published by Japan Society of Mathematical Education. http://www.sme.or.jp/en/
9. *Journal of Science and Mathematics Education in Southeast Asia* (JSMESEA), ISSN 0126-7663, the official Journal of SEAMEO Regional Centre for Education in Science and Mathematics (RECSAM). http://www.recsam.edu.my
10. *Journal on Mathematics Education*, e-ISSN 2407-0610 and ISSN 2087-8885 (Print), published by Universitas Sriwijaya in collaboration with Indonesian Mathematical Society (IndoMS). http://jme.ejournal.unsri.ac.id/index.php/jme/

11. *Southeast Asian Mathematics Education Journal* (SEAMEJ), e-ISSN 2721-8546 and 2721-8546 (Print), published by SEAMEO Regional Centre for QITEP in Mathematics, Indonesia. https://doaj.org/toc/2721-8546
12. *Journal of Educational Research in Mathematics* (JERM), e-ISSN 2288-8357 and ISSN 2288-7733 (Print) is the official journal of the Korean Society of Educational Studies in Mathematics. https://www.jerm.or.kr/main.html
13. *The Mathematician Educator* (TME) (ISSN: 2717-5634; formerly The Mathematics Educator from 1996 to 2019) is an official publication of the Association of Mathematics Educators, Singapore. https://ame.org.sg/tme/
14. *Turkish Journal of Computer and Mathematics Education*, e-ISSN 1309-4653, published by Science Research Society, Turkey. https://www.turcomat.org/index.php/turkbilmat

At least a few of these have received international circulation and reputation beyond Asia, attesting to the developing research activity in the continent. Asian mathematics education research, as a theme of international academic discourse, has received some attention, as shown in the two volumes edited by Sriraman, Cai, Lee, Fan, Shimizu, Lim, and Subramaniam (2015). It is worthy to note that that collection highlights research developed only in five high income economies in the continent: China, Korea, Singapore, Japan, Malaysia, and India. Our current book provides an update of contemporary research in the continent and adds contributions from other economies that are not often published internationally.

For this book, attempts were made by the editors to solicit contributions from a wide range of economies, especially those less familiar in international publications. However, this call for contributions has coincided with the outbreak of the COVID-19 pandemic, preventing the participations of certain groups of educators to develop their own chapters. In this collection, we attempted to encourage collaborative authorship where more experienced researchers support novice ones in compiling their chapters. Similarly, we were interested in critical reflections on the accounts and evidence-based learning from them in identifying arising or persisting problems in developing research in the respective economies.

The book consists of two parts. The first part consists of three chapters addressing issues in the *Developing Research Culture in Mathematics Education* in the continent. The first chapter entitled "Building a Research Culture in Philippine Graduate Education: Reflections on Experiences in Mathematics Education", contributed by Enriqueta D. Reston and Richard R. Jugar from the Philippines, discusses research culture as a construct understood in five dimensions: individual identities, institutional attributes, community of practice, research environment, and research artifacts. Based on the experiences of one university in the Philippines, the chapter identifies two factors that contribute toward building a research culture in graduate research: graduate student identity and research learning environment. Finally, the chapter describes four concrete models for graduate research mentoring. The chapter concludes by arguing that the synergy of the three aspects, that is, the multi-dimensional conception of research culture in higher education, the factors that contribute to the development of research culture in graduate education, and the mentoring models, can serve as both a guide and a tool for the development of the research culture in graduate education.

The second chapter "Development of Chinese Mathematics Education Research Culture: A Case Study", by Jian Liu, Yaoyao Dong, Qimeng Liu, and Jiaxin Yan, discusses the Beijing Mathematics Education Seminar (BMES) founded in 1995, which brought together teachers, postgraduate students, and scholars working in the field of mathematics education working together to advance the research and teaching practices in mathematics education. Based on the core elements of Cultural–Historical Activity Theory, this diachronic case study explores the development strategies used and the challenges faced from three aspects: the *subject*, the *object*, and the *instruments*. The authors have identified some challenges faced in maintaining the group activities for over 25 years including some contradictions: maintaining the balance between theoretical and practical research perspectives, the orientation toward focused or more diverse agendas, and the relative effectiveness of both the "lecture" and "discussion" formats. The authors remain hopeful that their experience may provide an inspiration for the construction of mathematics education research culture in other contexts.

The third chapter in this part, "The Evolution of Mathematics Education Research in Singapore" by Berinderjeet Kaur, Tin Lam Toh, and Eng Guan Tay, examines how a research culture specifically in mathematics education at the National Institute of Education of Singapore was nurtured, developed, and supported from 1990 onward. The authors discuss both top-down and bottom-up approaches at play. First, institutional policies related to recruitment and promotion of academics were developed to ensure that emphasis was on both teaching and research. Second, the development of research progressed from individually led bite-sized grains that eventually developed into team-based projects with coherent themes. Significantly, through hosting of postgraduate students and holding of annual research conferences, Singapore has also assisted in the development of a research culture and skills in other Asian countries such as Thailand, Indonesia, and the Philippines.

The second part of the book focuses on *Reviews of Research in Mathematics Education in Different Economies*. The fourth chapter "Critical Analysis of Mathematics Education Doctoral Dissertations in the Philippines: 2009–2021" by Bill Atweh, Minie Rose C. Lapinid, Auxencia A. Limjap, Levi E. Elipane, Michel Basister, and Rosie Conde, presents a critical analysis of doctoral dissertations in mathematics education in the Philippines during the past ten years. It analyzes the dissertations with respect to the topics in the discipline they address, the targeted participants in terms of educational level, and roles of stakeholders and theoretical frameworks used to construct this research. In particular, the authors noted a diversification in this research. However, they also identified a few less traversed areas of research that focus on social goals of education, the relationship of mathematics teaching and learning to other disciplines, equity in terms of language and access to quality mathematics education due to poverty, elementary and kindergarten mathematics, assessment, technology use in informal settings, and research areas that are informed by critical and sociopolitical perspectives.

The fifth chapter "A Critical Review of Mathematics Education Research in Korea: Trends, Challenges, and Future Directions", in this collection by JeongSuk Pang and Minsung Kwon, commences by a summary of two recent studies on the trends in mathematics education research using different approaches: content analysis and topic modeling. This chapter then provides critical reviews on the research trends, including an increase in research articles, diversification of research topics, and balance of research methods, while comparing and contrasting them with the international trends in mathematics education research. The authors concluded that, while it may be too early to characterize Korea's research trends in one specific research topic or approach, two popular research topics (research on curricula/textbooks and research on teacher education) illustrate the particular issues, values, and contexts of mathematics education in Korea. The authors argued that mathematics education researchers need to activate international comparative or collaborative studies to better understand the research topics of a country, to better notice what gaps exist in the research trends, and to search for alternative approaches.

The sixth chapter is "Mathematics Education Research Trends in Turkey: International Research Context" by Yüksel Dede and Veysel Akçakın. Similar to the previous chapter, the authors investigated patterns in Turkish research published in three local journals in comparison with research published in two leading international research journals in mathematics education. The chapter demonstrates the dominance of quantitative techniques in Turkish research in comparison with international publications. Similarly, there is a tendency in Turkish research to be conducted on teacher education context rather than on school teachers and students. The authors argue that such analysis is helpful to identify gaps in research in the country toward widening of the research agenda.

Kwok Cheung Cheung, Chunlian Jiang, and Lianghuo Fan are the authors of the seventh chapter in this collection and discuss the "Research and Research Culture in Mathematics Education: The Case in Macao, China". The authors argue for the unique characteristic of research published in Macao in that its mathematics education practice and research have integrated both the eastern and western traditions. The research reported here is dominated by those related to PISA assessments. Also, the survey revealed emerging research cultures in mathematics education over the past decade—the interests of researchers and practitioners in topics such as comparative studies, lesson studies, mathematical problem solving, and the use of information technology in mathematics teaching and learning. Some other areas such as teacher education, teacher professional development, ICT, and mathematics education are largely ignored. An interesting finding for this chapter is that the vast majority of mathematics education articles reviewed were written by school teachers. The authors conclude that there is a need for mathematics educators in Macao to broaden the scope of research (e.g., go beyond interpretation-oriented, policy-oriented, and practice-oriented types of research) and to enhance the capacity of research, in particular, in terms of research methods (e.g., diversification of research methods).

Finally, in the eighth chapter "Trends in Mathematics Education Research in Indonesia", the authors, Nurwati Djam'an, Neni Mariana, and Mangaratua M. Simanjorang, survey mathematics education research in the country with respect to the trending types of research conducted, research topics investigated, the focus of research, targeted educational levels, and the outputs of the research. In terms of type of research, the largest group of studies were found to be experimental research. In terms of topics that the researchers targeted, learning media and ICT constituted 66% of the studies reviewed. The authors noted that academic achievement in mathematics remains the variable most focused on in such research. The authors also demonstrated the underrepresentation of studies aiming at primary years and early childhood. The most common output of such research consisted of instructional materials and learning models. The authors go on to argue that the new challenge of mathematics education research in Indonesia is investigating the social, cultural, and political views about mathematics and mathematics education including social justice issues.

We conclude this Preface by outlining some of the editors' own reflections on themes arising from this review.

First, we note that, based on the chapters here, the research culture in mathematics education is highly active and rapidly evolving. In addition to five advanced countries which were the subject of the Sriraman et al. review, in 1995, research policy in the different economies was successful in promoting research and publication activities in more economies represented here. Six of the eight chapters in this book report on research in economies not covered by the 1995 review, including Indonesia, Korea, Macao, the Philippines, and Turkey.

Second, the scope of research reported in this book has grown to cover most of the familiar areas of research internationally. Naturally, there are some variations in the emphasis of research topics between countries. For example, teacher education received more attention in Korea (Chapter "A Critical Review of Mathematics Education Research in Korea: Trends, Challenges, and Future Directions") and Turkey (Chapter "Mathematics Education Research Trends in Turkey: International Research Context"), while the integration of ICT into mathematics classrooms, with more from a practical perspective, received more attention in Macao (Chapter "Research and Research Culture in Mathematics Education: The Case in Macao, China") and Indonesia (Chapter "Trends in Mathematics Education Research in Indonesia"). However, there are some areas of research that remain underrepresented in the choice of research topics. Perhaps research on social dimensions of mathematics education and social justice aspects may be not as prominent as didactic or pedagogical research (Chapter "Critical Analysis of Mathematics Education Doctoral Dissertations in the Philippines: 2009–2021" from the Philippines, Chapter "Mathematics Education Research Trends in Turkey: International Research Context" from Turkey, and Chapter "Trends in Mathematics Education Research in Indonesia" from Indonesia).

Third, research in mathematics education in the continent has employed a variety of designs and research methods: for example, experimental research in Indonesia

(Chapter "Trends in Mathematics Education Research in Indonesia"), content analysis in Korea (Chapter "A Critical Review of Mathematics Education Research in Korea: Trends, Challenges, and Future Directions"), and quantitative techniques in Turkey and Macao (Chapters "Mathematics Education Research Trends in Turkey: International Research Context" and "Research and Research Culture in Mathematics Education: The Case in Macao, China"). Several of the chapters discuss a range of methodologies used by researchers within the respective researchers. We believe that the development in this aspect is a highly positive indicator of Asian mathematics education research, given the strengths of different research methods and what can they inform about the complex nature of educational practice. However, several chapters have identified the yet evolving tradition of qualitative research methods (Chapter "Trends in Mathematics Education Research in Indonesia" from Indonesia, Chapter "Mathematics Education Research Trends in Turkey: International Research Context" from Turkey) and some of the challenges to researchers in utilizing them (Chapters "Building a Research Culture in Philippine Graduate Education: Reflections on Experiences in Mathematics Education" and "Critical Analysis of Mathematics Education Doctoral Dissertations in the Philippines: 2009–2021" from the Philippines).

Fourth, there is noted evidence from these chapters that the pool of researchers in many Asian countries and economies is expanding. As more universities in the continent are guided by the focus of their governments on research, more faculty members are engaged in research as a normal requirement for their employment. Further, many universities have developed postgraduate degree programs that demand research projects. Several chapters in this collection discuss master's and doctoral research projects, some of which lead into local publications, and in some cases at least, in international venues (Chapters "Building a Research Culture in Philippine Graduate Education: Reflections on Experiences in Mathematics Education" and "Critical Analysis of Mathematics Education Doctoral Dissertations in the Philippines: 2009–2021" from the Philippines, Chapter "Development of Chinese Mathematics Education Research Culture: A Case Study" from the Chinese mainland, and Macao, Chapter "Research and Research Culture in Mathematics Education: The Case in Macao, China" from Macao). In particular, Chapter "Development of Chinese Mathematics Education Research Culture: A Case Study" discusses the infrastructure that allows the inclusion of school teachers, along with postgraduate students and university researchers, in discussing research matters on a regular basis. With this widening pool of researchers, research productivity would continue to increase in the future.

Finally, we note that Asian research in mathematics education exists in a globalized world. This is evidenced in the similarities in research topics and methodologies adopted in the different regions of the globe. Here, we do not take the stance that these topics and methodologies are to be developed in one region and exported to the rest of the world. Topics and methodologies are always contextualized and modified according to the locality's needs, traditions, and values. While it is beyond the aim of this collection to propose an identity for Asian research in mathematics education in contrast to other regions of the globe, it is our belief that with continued collaborations in research and sharing of research outputs and internationalization of research publications, strong research in Asia can contribute to strengthening of research internationally. We hope and trust this book may increase the international recognition of Asian research as well as encourage continued dialogue about research in our field.

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Contents

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Developing Research Culture in Mathematics Education

Building a Research Culture in Philippine Graduate Education: Reflections on Experiences in Mathematics Education

Enriqueta D. Reston and Richard R. Jugar

Abstract With the recognition of the lead role of graduate education in enhancing the quality of teaching and research in higher education, this chapter draws upon the literature and practice on how the development of a research culture in graduate education may provide the means toward attainment of the goals and desired outcomes of graduate education and research productivity in Philippine higher education. The construct of research culture in higher education is examined in light of its evolving meaning, and five dimensions were identified namely: individual identities, institutional attributes, community of practice, research environment, and research artifacts. With reference to regional and national qualification frameworks that define the nature and outcomes of graduate education, the factors that contribute toward building a research culture in graduate education were identified based on two dimensions: the graduate student identity and the research learning environment. Four concrete models for graduate research mentoring are described, and reflections from experiences in graduate mathematics education are presented. The chapter winds up with recommendations for future directions.

Keywords Research culture · Graduate education · Higher education research

We regard the development of a research culture in higher education as a means toward attainment of research quality, productivity and impact in higher education institutions (HEIs). With research and publications as a primary criterion in rankings among colleges and universities, the past two decades have witnessed growing attention to research activities such as research capacity building programs of faculty, increased funding, and emerging policy and infrastructure support for research in higher education. With graduate education considered the "apex of the educational system," graduate education is expected "to take the lead role in

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enhancing the quality of Philippine higher education towards global competitiveness and world-class scholarship" (Commission on Higher Education, 1998, p. 1).

Qualification frameworks in higher education such as the European Qualifications Framework (EQF, 2017), ASEAN Reference Qualifications Framework (AQRF) (ASEAN, 2014), and the Philippine Qualifications Framework (PQF, 2018) are in general agreement on the role of graduate education in developing graduates who are able to apply their advanced specialized knowledge and skills through systematic inquiry and creation of new knowledge and innovations through research. Further, the role of graduate education in research systems is recognized as an investment of human capital, both for personal and national development (Kearney, 2008). As graduate programs lead to advanced degrees in chosen disciplines, the development of essential research skills toward specialized fields of expertise is expected to advance knowledge and innovation systems which are key drivers to national development. The critical role of graduate education in university research output is also recognized in a large-scale study on the impact of graduate students on research productivity of universities in Korea which reveals that the number of graduate students correlates significantly with researchers' productivity (Kwon et al., 2015).

1 Graduate Education in the Philippine Context

Over the past two decades, graduate education in the Philippines has witnessed various national reform initiatives for upgrading the quality and outcomes of graduate programs. In terms of policy support mechanisms, the Commission on Higher Education (CHED) issued CHED Memorandum Order (CMO) No. 9 on Policies and Standards in Graduate Education in addendum to CMO 36 (s. 1998) which emphasized the requirement of "a strong research program" in the graduate areas of study (Commission on Higher Education, 2003).

In 2013, the Task Force on Graduate Education Reform was created by CHED which reviewed the state of graduate education in the country, and "the cultivation of a culture of research and innovations in graduate programs" was among the recommendations (Commission on Higher Education, 2019, p. 1). The recommendations of this task force also provided the rationalization for CMO 15, series of 2019 on upgrading the policy, standards, and guidelines for graduate programs. In Sect. 2 of this CMO, the defining nature and philosophical underpinnings of graduate programs include the production of original research and creative work as among the general outcomes of graduate programs (Commission on Higher Education, 2019).

Further, in a report on the profile of graduate programs in the Philippines based on data generated by the CHED Task Force on Graduate Education Reform, the data for AY 2011–2012 revealed a total of 960 HEIs offering master's programs and doctoral programs with around 12,000 graduates from master's program and approximately 1,300 graduates from doctoral programs for that academic year (Ofreneo, 2014). These figures must have increased and accumulated over the years. Moreover, there is rarely any report on the extent to which graduate thesis and dissertation research outputs have contributed to the overall research and publication productivity of HEIs in the Philippines. With these policy mechanisms and support systems that have been placed for graduate education, there is much potential for graduate education to lead in Philippine higher education research productivity, knowledge generation, and innovations for national development. In this chapter, we argue that the development of a strong research culture in graduate education would provide the mechanism for these desired goals of graduate education.

The need to enhance research culture in Philippine higher education is explicit in the study of Quimbo and Sulabo (2014) which analyzed research productivity of five selected HEIs in the Philippines with 377 randomly selected faculty members as research participants. Results of path analysis showed that educational attainment, research benefits, and incentive systems are important predictors of both research self-efficacy and research productivity. From the findings, recommendations toward enhancement of the research culture in higher education were made through a faculty development program, enhanced research collaboration, and good incentive system. However, the role of developing a research culture in graduate education to contribute toward research productivity in higher education was not explicitly identified in this study.

The pursuit of research quality and productivity in Philippine graduate education may be also considered a "sustainable goal" for collective action as HEIs respond to the growing challenges and increasing pressure and accountability to advance knowledge through research. In this chapter, we draw from models in the literature and reflections from our own practice to argue on how the development of a research culture, particularly in graduate education, may provide the impetus toward a collective action for sustained growth in research quality, productivity and impact for research productivity in HEIs. In particular, this chapter aims to expand understanding on how building a research culture in graduate education may lead to sustained research quality, productivity, and impact in higher education. Toward this end, we seek to address the following questions:

1. What constitutes research culture in higher education? What are the foundations and dimensions upon which it is built?
2. What factors contribute toward building a research culture in graduate education?
3. What models of graduate research mentoring may contribute toward building a strong research culture?
4. What experiences in mathematics education research at a university may guide future directions in the development of research culture?

2 Research Culture in Higher Education

In the literature, discourses and documents on research in higher education, the concept of a *research culture* emerges as a construct that is used with increasing popularity despite its multiple meanings and contexts. In 2001, the first author did for her dissertation research on the research culture among teacher-researchers in the universities of Cebu City, Philippines (Reston, 2001). Following Shore's (1996) definition of culture, research culture was defined in terms of the "mind of a community" comprising faculty researchers in HEIs and the public artifacts of research. A characterization of teacher-researchers called faculty vitae, and the perspectives of teacher-researchers and administrators on research were explored in terms of the following areas: personal research capabilities, institutional support system, research productivity, and research linkages. On the public artifacts of research, these comprise the research and publication outputs of faculty which were classified by discipline and methods used. Moving forward, this conceptualization of a research culture may now be inadequate to cope with the complexities of higher education; hence, there is the need to take a more holistic approach to understanding the foundations and dimensions of this construct.

Research is often viewed as inquiry, a study or investigation, and an agency for change, development or improvement. Vogt (2007, p. 5) defines research as "the systematic collection and/or study of evidence in order to answer a question, solve a problem, or create knowledge." Simon (2004) further contends that research is an authentic inquiry that attempts to answer important questions that represent the interest of a field and thus advances knowledge in that field in a significant way.

The nature of research as a form of scholarship is also linked in Boyer's (1990) *Scholarship Reconsidered*, with the four domains of scholarship, namely the scholarship of discovery, the scholarship of integration, the scholarship of application, and the scholarship of teaching. The scholarship of discovery has been considered as what comes closest to research in terms of "the commitment to knowledge for its own sake, to freedom of inquiry and to following, in a disciplined fashion, an investigation wherever it may lead." Research as a scholarship of discovery is built upon the intrinsic value of all knowledge, its disciplined methods of inquiry, and its communal nature as it draws upon and contributes to the expertise and experience of a learned community in a specific field of study (Hill, 2010). In graduate education, research is considered a field of study as embedded in Research Method courses and as a process and demonstration of outcomes of graduate education as evident in the production of graduate theses and dissertations. In this chapter, we adopt the view of research as authentic inquiry and praxis toward advancing knowledge in a field for its intrinsic value and for practical applications in solutions of problems and improvement of practices in a community of practice.

On the other hand, the foundations of the concept of **culture** apart from research find its convergence in the fields of anthropology, psychology, philosophy, social, and cognitive sciences. Shore (1996) in his book *Culture in Mind: Cognition, Culture and the Problem of Meaning* provided a more extensive synthesis of literature on the concept of culture from these fields through a cognitive approach where culture is depicted as a cognitive system comprising an extensive and heterogeneous collection of models defined by a number of parameters. These models are representations of reality that exist both in the mind of a community as cognitive constructs and as empirically observable artifacts in human behavior and social institutions. Shore's (1996) conception of culture is anchored on two opposing principles: relativism and psychic unity, and he further argued on the complex relationship between culture in mental representations and in social institutions. In a book review on Shore's (1996) conception of culture, Nuckolls (1998) asserted that although the concept of culture has suffered from conceptual vagueness, yet it has permeated into popular usage of people from various fields or disciplines.

Research culture in higher education has multiple meanings, yet there is a consensus in literature that it is one key ingredient toward research productivity. In an address to the academic senate of the University of Western Sydney, Cheetham (2007) contended that the growing concern on building a research culture in higher education may be due to the increasing recognition that research is the basis of how a university education works; it is the intellectual life blood of academics, the fundamental support for teaching, and the basis for community extension. While there may be no consensus definition of research culture, it is easy to recognize the importance of having one. Developing a culture of research is viewed as essential among HEIs with higher levels of research productivity and “a culture of research provides a supportive context in which research is uniformly expected, discussed, produced, and valued” (Hanover Research, 2014, p. 5). This description of research culture draws its basis from the definition of culture as “a system of widely shared and strongly held value” (Marchant, cited in Hanover Research, 2007, p. 5). Further, this shared view and value of research held by a community that comprise research culture may be concretely reflected in practice such as in the management of research at the university including decisions on faculty support, funding, and recognition of its importance in promotions and tenure of faculty.

The perspectives of selected higher education faculty on the prevailing research culture in Philippine HEIs were investigated by Clemeña and Acosta (2007), and they came up with a conceptual framework of research culture upon which to analyze the dynamics of the interaction of the policies and mandates on research of CHED and the practices of HEIs relative to the research orientation in their institutions and the perspectives of active faculty researchers. Their findings provided the basis for understanding research culture as an operational construct concerned with the dynamics of interrelationships of three domains, namely (1) the trifocal function of teaching, research, and community service/extension in a university; (2) the individual attributes and output of faculty engaged in research; that is, reflecting their knowledge, skills, values, and attitudes relative to the conduct of research, as well as their readiness, capacity, and experience as regards research; and (3) the institutional attributes and policies set by the institution toward developing a research orientation. This conceptual framework for understanding research culture in Philippine HEI’s comprises an interaction of functions, people, institutional policies, and support system toward research.

In the pursuit of research quality and productivity in higher education, Ridley (2011) also argued on the importance of developing a research culture in research capacity building in her documentation of the case of Addis Ababa University in Ethiopia. She contended that research capacity building is not just a body of knowledge or set of techniques to be delivered through workshops, but essentially the development of a research culture. She contended that “research culture refers not so much to systems (reflected in the notion of research infrastructure) as to values, dispositions and habitual behaviors” of a community (Ridley, 2011, p. 288). A descriptive characterization of an active research culture is also provided by Nagy (2011) in terms of human resources and a working environment of people who are actively engaged successfully in research, with a steady stream of research output in the form of discoveries and inventions, international publications, invited presentations, conferences and exhibitions, and other forms.

With the evolution of the definition of research culture over the past two decades, we draw out its dimensions grounded on the foundation of the nature of research as a process of authentic inquiry and as praxis in the form of informed commitment of action and reflection toward a goal of contributing new knowledge or providing solutions to problems within the context of a community or a field of practitioners in specific disciplines. Further, it takes into account the conceptions of culture as a multi-dimensional construct. By examining these various conceptions of research culture that has evolved over the past two decades, we built our conceptualization of research culture on the premise that it is a dynamic entity and shared praxis of a group of individuals as both agents or actuators of research with collective identity within a community of practice. A summary table on the multi-dimensional conception of research culture in higher education is given in Table 1.

These various conceptions of research culture have provided a broader conceptualization or research culture and a deeper understanding on how it may be developed and sustained in higher education. In particular, these conceptions will be used to describe the cases of developing graduate mathematics and science education at a university.

3 Factors Contributing to Building Research Culture in Graduate Education

The definition of graduate education may vary depending on both level and context. However, what is common is the requirement of an accomplished bachelor's or undergraduate degree as a minimum prerequisite. Generally, graduate education may be viewed as a continuum that proceeds from the completion of a bachelor's degree moving toward the highest level which is a doctoral degree. In terms of pathing, graduate education may be done by coursework and research, or by research alone. Graduate education by coursework requires students to enroll and accomplish a set number of courses that culminates in the submission of a thesis or a dissertation. Graduate education by research or higher degree by research does not require enrollment to a set of specific courses but usually demands a set number of interrelated scholarly publications, the totality of which becomes the equivalent of a complete dissertation (Kehm, 2006). The development of a thesis or a dissertation is central and fundamental in defining graduate education as a distinct higher level of formal education. In the Philippines, graduate education in the master's and doctoral levels is predominantly acquired through coursework and a thesis or dissertation. While there are some initiatives at the level of policy development for higher education

Table 1 Multi-dimensional conception of research culture in Philippine higher education research

Dimension	Description and indicator	Source
Individual identities	This refers to the identities of individual persons (faculty, staff, administrators, students) engaged in research or supporting the mechanism for doing research in HEIs. These individual identities are reflected in their • knowledge, skills, values, and attitudes relative to the conduct and goals of research, • research readiness, capacity, and experience • faculty Vitae: educational attainment and personal research capabilities and productivity	Clemeña and Acosta (2007) Reston (2001)
Institutional attributes	This comprises the characterization of the institution toward developing a research orientation in terms of • policies and institutional support system • research infrastructure and systems • research productivity incentives • research linkages and collaborations	Clemeña and Acosta (2007) Reston (2001) Quimbo and Sulabo (2014)
Community of practice with shared praxis	This refers to the communal nature of research practice as it draws upon and contributes to the expertise and experience of a learned community with • shared values, dispositions, and habitual behaviors • commitment to the pursuit of inquiry to advance knowledge and contribute to both theory and practice in one's field • a supportive context in which research is uniformly expected, discussed, produced, and valued	Ridley (2011) Hanover Research (2014) Hill (2010)
Research environment	This refers to the working and learning environment of people who are actively engaged in the teaching and conduct of research, with a steady stream of research outputs, including • ongoing faculty development programs • enhanced research collaboration • embedding research in teaching and learning	Ridley (2011) Quimbo and Sulabo (2014)

(continued)

Table 1 (continued)

Dimension	Description and indicator	Source
Empirical artifacts	This refers to the material or public artifacts of research outputs in the form of • publications in peer-reviewed journals • proceedings from invited presentations conferences, discoveries, patents, and inventions • research exhibitions and community engagement for utilization of research outputs	Reston (2001) Ridley (2011)

institutions to offer higher degrees by research, the extent of its implementation is still limited as it is in its early stages of adoption (Commission on Higher Education, 2019).

The thesis or dissertation as a focal and final requirement of graduate programs where the candidate forwards a scholarly contribution as validated by the experts in the field may be considered as an authentic form of assessment. In the master's level, the thesis should sufficiently demonstrate that the student has "mastered" the rudiments of research. In the doctoral level, the dissertation should not only demonstrate sound research conceptions and practices but has to have concrete and sufficient evidence of contributing to and expanding the existing body of knowledge. The quality of graduate publications in general, or theses and dissertations in particular, is largely reflective of the overall quality of graduate education of a particular higher education institution (Kyvik & Thune, 2015). This is coming from the premise that research as a systematic form of inquiry is both a central undertaking and an indispensable tool in graduate education.

In examining what could possibly contribute to developing an ideal research culture in graduate education, we focus on a two-dimensional analysis framework on the context of research in graduate education. These dimensions are (a) *the graduate student identity* and (b) *the research learning environment.* These dimensions focus on the graduate student as the central figure to which culture is created and manifested. To provide consistency in the usage of the identified dimensions, the following premises are considered:

- Research culture as a shared praxis.
- Research culture as a dynamic entity.
- The individual as both agent and actuator of research culture.
- The collective actions of individuals constitute the research culture identity.

The premises encompass both the individual and social aspects of research culture as reflected in Table 1. Moreover, these premises focus on the individual as the main actor to which research culture is established and practiced, as well as on the environment to which research culture is both developed and manifested.

While graduate education deals with relatively experienced and seasoned learners, the program and approach of graduate education should still provide learning opportunities for graduate students to be mentored in the learning and conduct of research. Moreover, enabling mechanisms and supportive structures should also be in place that complement the mentoring approach to ensure that the research culture in graduate education translates to the attainment of the desired educational outcomes for both masters and doctoral levels.

The first dimension focusing on the graduate student identity highlights the need to clearly set what is a graduate student and to what ends graduate education seek to accomplish. There are a number of documents in the form of qualification frameworks that may be used to define the nature of graduate education outcomes. Table 2 shows two examples from the ASEAN Reference Qualifications Framework (AQRF) (ASEAN, 2014) and the European Qualifications Framework (EQF, 2017).

Table 2 Descriptions of graduate education from AQRF and EQF

Framework	Level	Academic qualification	Description
AQRF	7	Masters	• Is at the forefront of a field and show mastery of a body of knowledge • Involves critical and independent thinking as the basis for research to extend or redefine knowledge or practice
	8	Doctoral	• Is at the most advanced and specialized level and at the frontier of a field • Involves independent and original thinking and research, resulting in the creation of new knowledge or practice
EQF	7	Masters	• Highly specialized knowledge, some of which is at the forefront of knowledge, in a field of work or study, as the basis for original thinking and/or research • Critical awareness of knowledge issues in a field and at the interface between different fields • Specialized problem-solving skills required in research and/or innovation in order to develop new knowledge and procedures and to integrate knowledge from different fields
	8	Doctoral	• Knowledge at the most advanced frontier of a field of work or study and at the interface between fields • The most advanced and specialized skills and techniques, including synthesis and evaluation, required to solve critical problems in research and/or innovation, and to extend and redefine existing knowledge or professional practice

While the level assignment might vary depending on which qualification framework is used, i.e., the Arab Qualifications Framework assigns Levels 9 and 10 for masters and doctoral levels, respectively (AQF, 2012), there is a general agreement on the educational outcomes of graduate education which are to (1) expand the field of knowledge and (2) for the graduates to demonstrate mastery and ability to conduct independent research. The descriptions in Table 2, while inherently broad, are used to identify what enabling mechanisms may be developed for their attainment. By dissecting these generally agreed descriptors, graduate students have therefore two distinct identities: (a) as creators of knowledge and (b) as systematic enquirers. Considering the graduate student as both the agent and actuator of research culture by creating knowledge through a systematic form of inquiry, there are *essential attributes* that need to be developed to attain this desired identity. Drawing from our experiences in mentoring graduate students and implementing graduate programs, these attributes are identified as follows:

A. **Exhibits Self-awareness**. The starting point of research is usually thought to be the conceptualization of the research problem. While it is true that the research problem is central in the development of the research process, the act of conceptualizing the research problem is highly influenced by the research paradigm of the researcher (Mittwede, 2012). The graduate student should therefore be made aware of the differing paradigms in research to have an understanding of these differences and to reflect on his or her own. In doing so, the graduate student will develop a better sense of self-awareness in terms of his or her own ontological and epistemological beliefs—factors that will consequently influence his or her choices in developing his or her identity as a researcher.

B. **Knowledgeable and Informed**. As creators of knowledge, the graduate student should have both foundational and advanced knowledge of the field. One can only advance or extend the knowledge in the field if one knows where the extant boundary is (Zhang & Sternberg, 2011). In effect, a graduate student needs to possess both *relevance* and *recency* of the knowledge in the field. The coursework-based approach in graduate education usually utilizes the course component to ensure that the graduate students have the sufficient background prior to conducting the thesis or dissertation. Higher degree by research model on the other hand incorporates this aspect in the process of creating a series of publications. In most cases, relevance is readily established as dictated by the focus or specialization of research (Kehm, 2006). Recency on the other hand proves to be challenging to most graduate students as it requires constant evolution of curricular content—a process that necessitates both access to the latest research outputs and the drive to keep oneself updated through constant review of literature.

C. **Open to Constructive Criticism**. Both knowledge creation and the methodologies associated with the conduct of research are largely based on the consensus of the experts in the field. Peer review or expert refereeing is considered as a practice that ensures the quality of the new set of information or knowledge generated as a product of research (Horbach & Halffman, 2018). In practice, theses and dissertations are generally presented to a group of experts in the field to be scrutinized and critiqued. This practice provides the graduate students the experience to defend their work and consequently improve the overall quality of the study. The practice of asking and providing constructive criticism should form part of the graduate students' experience. This may be integrated in the coursework activities as a series of steps in developing this attribute with the final defense as the highlight. The practice may take both oral (presentation) and written (publication) forms to simulate the real-life context as future researchers and experts in the field.

D. **Adheres to Ethical Practice**. The pursuit of knowledge and the conduct of research necessitate consideration and adherence to the ethics governing the practice of the profession (Reijers et al., 2018). Graduate students need to be aware of multiple ethics-related policies and practices including those beyond the usual regulatory frameworks (Cascio & Racine, 2018). Ethics approval is one of the key documents required in the process of evaluating the proposed thesis or dissertation. Higher education institutions normally have an established research ethics policy consistent with national and international accords that stipulates the guidelines on how to ensure the conduct of ethically acceptable research and projects. Adherence to ethical practice as an attribute encompasses all facets of graduate education and should therefore be explicitly incorporated in all forms of activities and outputs.

The second dimension focuses on the research learning environment of the graduate students. The attainment of outcomes for graduate education is a delicate balance of both the learners' identity and the learning environment. In this context, the learning environment refers to the collective factors external to the learner or graduate student that either directly or indirectly affect the learning process and consequently the attainment of the intended educational outcomes. Learning environment therefore includes physical facilities, instructional practices, and institutional policies among others in the context of graduate education taking place in a relatively structured environment (UNESCO-IBE, 2021). With respect to the learning environment, we forward these *essential elements* necessary in building the ideal research culture in graduate education as complementary to the essential attributes that constitute the graduate students' desirable identity.

A. **Research Agenda.** Research institutions in general, and higher education institutions in particular, follow a specific mandate or niche when it comes to the services or specializations they offer. This niche is basically based on the area of strength or interest of the institution. The defining programs or the flagship projects and initiatives are likewise anchored to this niche that include the research agenda. The formulation of a clear research agenda is an essential component in developing an ideal research culture in graduate education as it serves as the anchor of the contextualization of the graduate education curriculum and the consequent research thrusts. Research agenda is usually encompassing and broad to accommodate the varied contexts and disciplines of the graduate education programs being offered (Ertmer & Glazewski, 2014). Units in the institution offering different graduate programs are expected to align their respective research thrusts to the institutional research agenda, thereby providing a relatively specific area of interest for the graduate students to focus on.

B. **Research Groups.** The existence of an institutional agenda is a good starting point in the formulation of the research thrust of the different units in the institution such as a school or college, a department, or even a program. Based on these identified thrusts, a specific clustering of the faculty composition along with the graduate students may be formed. These clusters constitute the different research groups with each having a specific area of focus or interest. Participation in research groups generally enhances the graduate students' experience as it allows interaction between and among other graduate students and mentors working on the same area of research. Research groups also facilitate sharing of resources among members. Further, membership to research groups affords graduate students an accessible venue for support in the conduct of their respective thesis or dissertations that may include but not limited to peer critiquing and advising among others. Lastly, inter-group collaboration can readily facilitate translational and interdisciplinary research and projects that are deemed necessary for progress (Begg et al., 2015).

C. **Faculty Exemplars**. Mentors or faculty members of the graduate program play a vital role in the development of both the identity and practice of graduate students (Wright-Harp & Cole, 2008). The usual method of assigning or appointing graduate faculty members is largely based on the level of education, that is, faculty at the masters and doctoral levels should have completed a masters and doctoral degrees, respectively. The highest educational attainment, however, should not be used as the sole basis to qualify as a mentor in the graduate program. Consistent with the identity of the graduate student that the graduate program is trying to form, faculty members of the graduate program should consistently exhibit the four previously identified attributes of exhibiting self-awareness, being knowledgeable and informed, being open to constructive criticism, and adhering to ethical practice. In effect, graduate faculty members or mentors should demonstrate with constancy and excellence the graduate identity of expanding the field of knowledge and employment of sound research practices including dissemination and application of results through publications and community extensions.

D. **Functional Facilities**. The conduct of research in general requires the use of instruments and materials especially in the collection and analysis of data. Different areas of research will require different facilities such as physical laboratories, raw materials, analytical instruments and software, reproduction lines, and virtual spaces among others. The presence or absence of a certain material, instrument, or facility can greatly affect the quality and even the capacity of conducting research. Institutions with excellent graduate programs are always complemented by the presence of functional facilities (Vidalakis, Sun, & Papa, 2013). However, investing in functional facilities should proceed from having a comprehensive and sound research program and not the other way around.

This is specifically true for institutions of higher education in countries where resources are relatively scarce.

E. **Institutional Support**. The first four elements identify specific aspects of the graduate students' learning environment with respect to developing an ideal research culture for graduate education. While these aspects may be addressed or are already extant in an institution, a concrete and consistent institutional support is necessary for the optimal functioning and upkeep of these elements. Institutional support should come in the form of *enabling* and *corrective* policies as well as allocation of a reasonable number of resources (Kramer & Libhaber, 2016) that may include provision of project and scholarship supports, reasonable budget allocations, physical structures to support higher education studies, adequate human resources, and functional student services. The presence of enabling policies generally promotes growth and development, while corrective policies ensure responsibility and accountability in the process of growing. Conversely, the absence of a clear set of policies basically devoid the other elements with a solid anchor that may result in inconsistencies, misinterpretations, and consequent dissolution or dilution of the intended ideal research culture for graduate education.

In summary, this section identifies the different factors that may contribute to the development of an ideal research culture in graduate education. The graduate student identity and the learning environment in graduate education served as the dimensions of focus as these encompass both the person and the context to which this person is formed. The four essential attributes that form the essence of the graduate student identity and the five essential elements that constitute the learning environment may be viewed as the aspects that have to be focused on and sufficiently developed to attain the desired research culture in graduate education.

4 Mentoring in Graduate Education

Graduate education, in the context of higher education, is generally described as the level of education where students are expected to practice substantial independent learning compared to their undergraduate counterparts. Independent learning in this context may not be defined traditionally as the absence of an actual structured face-to-face instruction. Rather, it is more on the idea that graduate students are expected to make a series of informed choices and decisions on what area and methods of inquiry they intend to pursue. This understanding of independent learning complements the process of developing the graduate student identity as creators of knowledge and systematic enquirers. The level of student independence in the continuum of higher education from undergraduate to doctoral levels may be viewed through the lens of the Gradual Release of Responsibility Instructional Framework (GRRIF) (Fisher & Frey, 2013). Following the levels of GRRIF, there is a distinctive increase in the responsibility of students to be accountable and responsible for their own learning as the level of education increases. In effect, the undergraduate level may be characterized as being led, the master's level as walking together, and the doctoral level as being guided from a step behind.

Graduate students are still in the process of establishing their identity as creators of knowledge and researchers. It is therefore imperative to provide both guiding and enabling support to ensure their development. In practice, the main support comes in the form of a mentor—an individual responsible for the development of the graduate student's potential as evidenced by the successful completion of the degree (Simpson, 2012; Wright-Harp & Cole, 2008). In this chapter, we may refer to the mentor as a research adviser or research supervisor. The responsibility of the mentor constitutes all academic-related advising as well as helping the advisee transition from being a student to being a professional. The role of a mentor in graduate education is one of the key determinants for both success and attrition. In particular, mentor–mentee relationship in graduate education is found to be one of the key factors that could either facilitate or impede the graduate students' progress and consequent completion (Golde & Dore, 2001).

The qualifications to become a mentor are defined differently depending on the institution. Some institutions offering graduate education classify the level of a mentor based on experience. Despite the differences, common qualifying requirements to become a mentor in graduate education are generally academically driven such as the (a) the possession of a graduate degree and (b) research productivity (Curtin University, 2019). Additional requirements have been identified to complement these baseline requisites to specifically address dispositional targets that include the prospective mentor's attitude toward mentoring.

Mentoring Models in Graduate Education. As graduate education essentially takes place within the context of a mentor–mentee relationship, there are a number of models that have been developed in an attempt to decrease attrition rate among graduate students, ensure completion of the program, improve the graduate schooling experience, and increase the quantity and quality of research outputs. This section presents some of these developed models with emphasis on each model's implication toward the development of research culture in graduate education and consequent production of research outputs. Further, we reflect on our experiences in graduate mathematics education in which these models apply in mentoring students in graduate mathematics education.

A. **The Classic Apprentice-Master Model**. The Apprentice-Master Model (AMM) is considered as the oldest and the most common model of graduate student mentorship. As the name suggests, it is a one-to-one arrangement where an experienced mentor is assigned to a graduate student. The mentor essentially guides the student in all aspects of the graduate program experience with particular emphasis on the choice and conduct of the student's thesis or dissertation (Yeatman, 1995). The merits and limitations of AMM have been widely studied and documented in literature (Carter-Veale et al., 2016). Even today, AMM is still widely adopted in the ASEAN context of graduate education. As there is only a single mentor, the student mentoring experience will relatively be limited, and the resulting research culture of the student will be an extension of that of the mentor. It was also documented that the mentoring style of a mentor usually mirrors the mentor's own experience as a graduate student. In terms of the production research output, the quantity and quality will be mentor dependent as the mentor has the final say on both the direction and modalities of dissemination.

B. **Collaborative Cohort Model**. The Collaborative Cohort Model (CCM) is characterized by multiple graduate students or cohorts under the supervision of a mentor in a single academic department. This model is essentially one-to-many as the graduate students take the majority of their courses together (Burnette, 1999). The CCM has both the advantages and limitations of the AMM with respect to the development of research culture and quality of research output as described above. The CCM however provides an additional affordance for the graduate students as it addresses the identified negative influence of the feeling of isolation as one of the causes of attrition and non-completion of theses and dissertations. It was also argued that the CCM promotes the formation of a community of practice albeit tempered by the presence of only a single mentor. CCM also facilitates sharing of resources among the cohort and diffusion of ideas. In practice, the presence of multiple advisees under a single mentor does not automatically mean that the model being followed is CCM. The defining characteristic of CCM is the collaboration in the cohort. A number of graduate students working independently on separate and stand-alone research even if under the tutelage of the same mentor still fall under the AMM.

C. **Dissertation House Model**. The Dissertation House Model (DHM) proceeds from the expansion of the AMM and the CCM. It addresses the limitation of AMM and CCM in terms of the presence of a single mentor and enhances the affordances of CCM by allowing increased contact and collaboration among graduate students from different disciplines. The DHM may be described as a many-to-many type of mentoring model (Carter-Veale et al., 2016). By extension, the DHM also facilitates translational and interdisciplinary research as the collaboration between and among students and mentors organically diffuses learnings and ideas across disciplines. The development of the research culture in graduate education in this setting will most like be positive as it exposes students to a number of research paradigms and approaches while focusing on their area of interest. It also allows viewing of the research problem through multiple lenses that consequently translates into increased research outputs. The caveat of DHM is the requirement to have multiple established mentors that are open to translational and interdisciplinary collaborations as well as a strong institutional support as it necessitates resources in congregating graduate students and mentors in a shared physical space.

D. **Partnership for Development Model**. The Partnership Development Model (PDM) is a complementary mentoring model as it does not offer an academically focused mentoring approach that AMM, CCM, and DHM provide. The first three identified models essentially assign a specific mentor to a graduate student who is responsible from admission to graduation. PDM was formulated to augment the existing mentoring models by providing a support group composed of a graduate school faculty member, fellow graduate students, and a postdoctoral student. The group generally is kept small typically composed of five to six individuals. The focus of PDM as a supplemental and complementary model may include both academic and non-academic-related topics such as work-life balance and professional development among others (Lewinski et al., 2017). While PDM may not directly influence the development of an ideal research culture and research output in graduate education, the support it provides to graduate students to continue and eventually finish the program indirectly reinforces the quality of graduate programs as it decreases attrition rate and helps build a positive graduate student experience.

The different mentoring models provide us with the overview in terms of the extent of interactions between and among graduate students and mentors. Based on these available interactions, the implications toward the development of an ideal research culture for graduate education may be deduced including the relative quantity and quality of research outputs. The adoption of a model per se may not necessarily translate to an improvement in quality of graduate education. These models, while possessing inherent strengths depending on context, are still situated within an institution of higher learning. AMM may work best for most institutions of higher learning that are still in the process of improving their graduate education faculty profile. CCM on the other hand works well for established mentors with a defined research agenda or are involved in research projects. DHM may be useful and applicable for institutions with a strong faculty profile, established graduate programs, and available disposable resources.

The choice of a specific mentoring model in graduate education may be driven by institutional aspirations but is ultimately dependent on the available academic assets and resources. In today's globalized educational landscape, translational, transdisciplinary, and interdisciplinary approaches to knowledge generation and utilization are highly encouraged and prioritized over silo-based approaches (Neuhauser et al., 2007). Model-wise, DHM is the best choice to this end. Given the differing state of higher education institutions all over the world in general and in Asia in particular, we are advocating a paced and transitional approach of model adoption. The institution currently using the classical AMM model may have to invest on mentor profile development prior to moving toward the CCM specifically in developing a specific research niche. For institutions currently adopting the CCM, additional mentor development focusing on translational and interdisciplinary approaches will need to be developed as well as the allocation of adequate resources as prerequisite to the adoption of the DHM. With regard to the adoption of PDM, it may be relatively challenging for institutions currently adopting the AMM. PDM as a complementary and supplemental mentoring model is easier to incorporate for CCM and is an ideal pair for DHM as both CMM and DHM have embedded the idea of collaborative learning which is an essential component of PDM.

5 Reflections from Experiences in Graduate Mathematics Education

The University of San Carlos (USC) offers graduate mathematics and science education programs through the School of Education. Since 2008, it has been a delivering institution of graduate science and mathematics education scholarships of the Department of Science and Technology-Science Education Institute (DOST-SEI). The Classic Apprentice-Master Model where one-to-one arrangement between thesis/dissertation mentors (generally called advisers) and graduate students has been the dominant model over the past several years. Moreover, there are meetings and capacity building programs for thesis/dissertation mentors organized by the school administration and from DOST-SEI to provide the venue for shared understanding on the policies, guidelines, and practice of thesis/dissertation advising.

Graduate Mathematics Education Research. The Master of Arts in Mathematics Education (MAMEd) program comprises 33 units of coursework and six units of thesis work. The coursework includes courses such as Research Methods in Education and Thesis Proposal Preparation aside from the basic, major, and elective courses in Mathematics and Mathematics teaching. As early as in the Research Methods course, students are exposed to different research areas in mathematics education to facilitate their own formulation of a research problem for their own master's thesis. Moreover, defended MAMEd theses show that most of the research problems revolve around investigating effects of certain teacher-based interventions on student learning outcomes within some aspects of the secondary mathematics curriculum, as reflected in the following titles:

- *Effect of Implementing Inquiry-based Teaching and Learning on Grade 7 Students' Level of Geometric Understanding on Polygons Using the Van Hiele's Model*
- *Using Concrete-Representational-Abstract Instruction (CRA-I) In Developing Students' Conceptual Understanding of Measures of Position*
- *The Effect of Computer-Supported Collaborative Learning on Students' Motivation and Problem Solving Skills When Mediated by Engagement.*

These sample titles of mathematics education thesis research indicate a focus on mathematics teaching and learning which are connected to the curriculum. The introduction of teacher-based interventions and their effect on student learning outcomes lead to the popularization of quantitative experimental designs and to mixed-methods research but to a limited extent. Moreover, this also reflects the need to broaden the sphere of mathematics education research in areas beyond the curriculum and methods beyond quantitative experimental designs. As Simon (2004) claimed, the evolution of mathematics education research over the past two decades has witnessed the acceptance and subsequent predominance of qualitative research methodologies. It is also observed that the faculty are developing research capacity in mathematics education research while serving as thesis mentors. This calls for the need of faculty with mathematics education degrees and research experience in qualitative methods, as well as further capacity building among the current thesis mentors in the field.

While the dominant mentoring model is the classical AMM, the practice is currently transitioning toward CCM. This transition appears to come naturally as mentors or advisers accept up to five graduate students as mentees or advisees. In the process, these students are treated as cohorts under a single mentor and are provided with relatively similar support. In practice, these groups of graduate students do not have a defined schedule to meet as a group but are interacting informally resulting in the diffusion of ideas and a sense of belonging consistent with the affordances of AMM. Furthermore, mentors with a relatively well-defined research agenda tend to gravitate toward CCM since the focus of the graduate students can be aligned or directed toward that of the research adviser while providing students the freedom to formulate their own specific research questions.

6 Recommendations for Future Directions

The identification and reflection on the current research culture in the graduate program offerings of an institution are an excellent starting point for its consequent upkeep or improvement. In this chapter, we presented a multi-dimensional conception of research culture that may be used as a lens in defining the current state of institutional research culture relative to the institution's aspirations. The dimensions along with the posited descriptions may be useful in identifying relevant artifacts to empirically assess the quality of research culture as understood. Higher education institutions can use the understanding of research culture as presented in the chapter to critically reflect on their explicit and implicit conceptualization of research culture. Clarifying the institution's conceptions of research culture will help in improving and establishing the institution's identity and niche in terms of research in graduate education. These clarified understanding and conceptions will consequently translate to changes in the institution's policies, practices, and processes.

With regard to the factors affecting the research culture in graduate education, the essential attributes comprising the graduate student identity and the essential elements constituting the graduate student learning environment may be used as a functional framework in evaluating current degree programs. The identified attributes and elements can be used as domains or areas with which current policies, practices, and policies in place may be assessed. Further, it may also be used as a planning tool for the development and deployment of new graduate programs. The four essential attributes and the five essential elements were provided with descriptions and roles. These may be expanded and translated to actual research instruments to assess the presence or absence of these constructs as well as the extent or quality if identified as present. Understanding what specific aspect of the graduate student identity or learning environment that a specific institution or graduate program is doing well will help in ensuring that the desired quality is kept consistent. Conversely, aspects identified as absent or underperforming will allow the institution to proactively respond to attain the desired improvement.

Finally, the different mentoring models as described are useful in understanding the current state mentoring praxis in HEIs. Different institutions may identify which specific mentoring model is currently at work. The PDM may be also explored in cases where the academic unit has limited faculty resources and expertise in the field. The descriptions of these mentoring models along with their respective affordances, challenges, and requirements may be used by HEIs in formulating policies that will set the direction of their current mentoring model toward one which is holistic, collaborative, translational, and interdisciplinary as desired.

The development of the research culture in graduate education is a process that takes time and resources. The synergy of the three main points in this chapter, namely (1) the multi-dimensional conception of research culture in higher education, (2) the factors that contribute to the development of research culture in graduate education, and (3) the mentoring models, can serve as both a guide and a tool to this end.

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Development of Chinese Mathematics Education Research Culture: A Case Study

Jian Liu, Yaoyao Dong, Qimeng Liu, and Jiaxin Yang

Abstract Inspired by the Bourbaki Seminar, the Chinese Professor Yan's Seminar, and the needs of domestic mathematics curriculum reformation, Beijing Mathematics Education Seminar (BMES) was founded in 1995, bringing together teachers, postgraduates, and scholars working on mathematics education. It has been recognized as a typical case of local mathematics education research culture with Chinese characteristics. Based on the core elements of cultural–historical Activity Theory, the current study, as a diachronic case study, explores the development strategies and challenges that BMES has been confronting from three aspects, that is, the subject, the object, and the instruments. Qualitative research methodology of the text analysis, informal conversations, and semi-structured interviews has been applied. The results demonstrate that the development strategies of BMES mainly include the following three points: the relay of leaders and their followers, the leading and contemporary research agenda, and inclusive and cooperative discussion format. For the current and future development, BMES is also facing a number of challenges, including the following tensions: maintaining the balance between theoretical and practical research perspectives, the orientation for the research agenda, and the relative effectiveness of "lecture" and "discussion" formats. Furthermore, the case of BMES reflects the development of Chinese mathematics education research culture in the past 25 years and to a certain extent may provide an inspiration for the construction of mathematics education research culture in other countries.

Keywords Seminar · Mathematics education · Research culture · Chinese characteristics · Beijing · Development strategy

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1 Background

1.1 A Seminar as a Scholarly Communication Activity

In the process of advancing research on mathematics education, informal scholarly communication activities between academics play a vital role in the creation of knowledge and information (Swigon, 2015). A seminar is a common informal scholarly communication activity. In English, "seminar" refers to a class or group, usually at a university or a summer school, where certain issues are discussed, and it focuses on discussions among learners (Ma, 2003). Discussion as a means to generate knowledge is an activity practiced by philosophers like Plato as well as contemporary researchers (Swigon, 2017).

In the mid-1930s, the establishment of the Bourbaki group in Paris promoted such informal scholarly communication organized in the form of seminars. While studying at the École Normale Supérieure in Paris, the first generation of the Bourbakists participated in a seminar organized by the famous French mathematician Jacques Hadamard. At that time, the seminar represented a novelty for mathematicians in Paris (Remmert et al., 2016). In 1933, inspired by the organization of the Hadamard' seminar, and under the academic guidance of Gaston Julia who was a professor at the Sorbonne, the young French mathematicians of the early Bourbakists founded the Bourbaki Mathematics Seminar. The seminar was organized for the writing of Elements of Mathematics, during which heated questions, corrections, debates, and refutations had been launched. The seminar is "the laboratory of a restricted team" of mathematicians working together on definite topics (Beaulieu, 1989), and it became a research team with the characteristics of joint participation, unification, and collaboration. In the 1960s, this kind of collective organization for mathematical research quickly exploded. These collective practices gradually came to characterize the mathematical life of the period (Paumier, 2014). Affected by the Bourbaki Seminar, this collective academic activity for mathematics research quickly spread in France and soon became an important international scholarly communication activity (Remmert et al., 2016). At the same time, Chinese mathematician Professor Shijian Yan also organized a seminar on probability theory in China, which impressed a group of scholars to devote themselves to mathematics research at that time.

From the 1980s to the 1990s, "mass education", "activity-orientated", "personality-orientated", and "life-orientated" have become a universal value of mathematics curriculum reformation in various countries (Zheng, 2003). However, in China, restrained by the traditional "subject-orientated" curriculum mode, the mathematics curriculum goal has deviated from the demand of social development, while the mathematics curriculum has ignored the actual applications of mathematics, and it resulted in the serious lag of mathematics education behind social development (Liu, 1997). In this regard, at the end of the twentieth century, the mathematics education group represented by the Beijing Mathematics Education Seminar (BMES) was established in China to meet the demand for domestic mathematics curriculum reformation. BMES was centered in Beijing, with domestic university teachers in the field of mathematics education as the core members. Young scholars, postgraduates, and mathematics teachers are the main participants of BMES. The group deliberations were conducted through a combination of online and offline methods. The academic exchange platform has become a cultural center of Chinese mathematics education and research with the main focus on mathematics education reformation.

1.2 Analysis of Research Culture Under Activity Theory

A research culture is the product of the life style, spirit, and system of the academic research group of scholars (Liang, 2006). As a scholarly communication activity, BMSE reflects the development of mathematics education research culture in China. Among the existing theories, from the perspective of social culture and history, cultural–historical Activity Theory (or Activity Theory, for short) is considered to be a classic theory for studying human activities, which helps to analyze the cultural factors behind human activities. From this theoretical perspective, the activity of the subject is not an individual behavior, but a collective social practice activity, which exists in a social, cultural, and historical environment composed of rules, communities, and division of labor (Cole, 1996). At present, Activity Theory has been applied in many fields such as psychology, pedagogy, and linguistics. Some current studies have used this theory to analyze the teacher education research culture (Wang, 2017a, 2017b; Wei, 2019; Yan & Yang, 2017). In this context, Activity Theory provides a foundation for this case study to explore the mathematics education research culture in China.

The first generation of Activity Theory was embodied in Vygotsky's work. Activity Theory redirects our gaze from what is going on inside the individual to what happens between human beings, their objects, and their instruments when they pursue and change their purposeful collective activities (Sannino & Engeström, 2018). The second-generation Activity Theory expands the model into six elements: subject, object, instruments, community, rules, and division of labor. The process of the subject's achievement of the goal cannot be separated from historical conditions and social circumstances (Yan & Yang, 2017). Specifically, the subject, object, and instruments are located at the top of the model and are the basic units that make up the activity (Engestrom, 1987). On this basis, the third-generation Activity Theory also proposed by Engestrom (1987) further considers the cultural complexity of activities and proposes the interaction between systems.

In general, Activity Theory emphasizes the two-way interaction process between human beings and their environment. Its core elements include subject, object, and instruments. Among them, the subject is considered the individual or subgroup whose position and point of view are chosen as the perspective of the analysis. Object refers to the raw material or problem space at which the activity is directed, which is turned into outcomes with the help of instruments, that is, tools and signs (Sannino & Engeström, 2018). Furthermore, instruments are the means or intermediaries that the subject acts on the object, including language, discussion environment, and context, cooperation and communication, etc. (Wang, 2017a, 2017b). Therefore, Activity Theory can be used to explore the question of "WHO" does "WHAT" through "WHAT MEANS" in activities under a certain social and cultural background and then to explain the cultural formation and development of a group in its activities.

In summary, based on the Activity Theory, this case study reflects the culture of Chinese mathematics education research by explaining the development strategy and analyzing the challenges of BMES. It is hoped that this study may enlighten mathematics educators in their scholarly communications and reflect on the development of the research culture.

2 Methodology

2.1 Participants

This investigation selects Beijing Mathematics Education Seminar as a case study. BMES is an informal scholarly communication activity. During the past 25 years between 1995 and 2020, BMES aimed to advance the reformation and academic prosperity of mathematics education, gathering Chinese mathematics education researchers from universities, primary schools, and middle schools. Under the core leadership from scholars including Xiaoda Zhang, Shijian Yan, Jian Liu, Xiaotian Sun, Xiaomei Liu, Fu Ma, Yiming Cao, Chunxia Qi, and Dan Zhang, the seminar was jointly hosted by the Mathematics New Curriculum Advancement Committee, the New Century Mathematics Curriculum Development Fund, and the School of Mathematical Sciences of Beijing Normal University. The participants of BMES include both researchers and practitioners in mathematics education.

As of June 2021, BMES has organized more than 180 sessions, both online and on-site. During that period, more than 100 noted mathematics scholars and educators from Cambridge University, Vanderbilt University, Beijing Normal University, and East China Normal University, together with young scholars and mathematics teachers, presented excellent speeches and talks. Postgraduates majoring in mathematics education and mathematics teachers participated in relevant discussions.

2.2 Analytical Framework

Based on the core elements of Activity Theory, the current study explores the development strategies and challenges faced by BMES from the three aspects, i.e., the subject, object, and tools of activities. From the sociocultural perspective, we analyzed the mathematics education research culture embodied in seminar activities (see Fig. 1). In the diagram, the subject specifically refers to the participants of BMES; the object

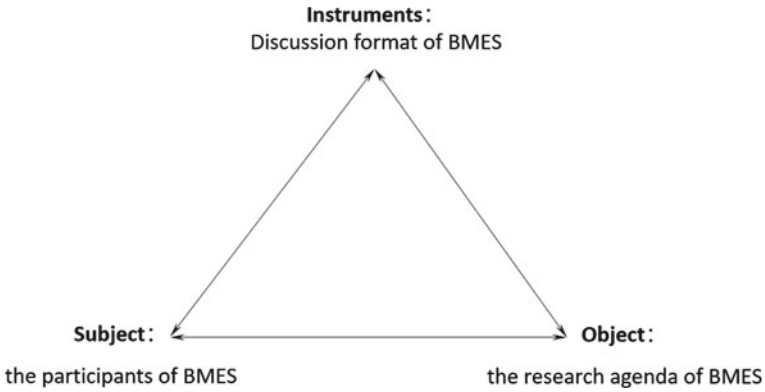

Fig. 1 Analytical framework of current study

is considered the target task, that is, the research agenda of BMES; the instruments are the means or forms to complete the objects, which are manifested in discussion format. Our framework analyzed the culture of Chinese mathematics education research by explaining "WHO" does "WHAT" through "WHAT MEANS".

2.3 Data Collection

This study is a diachronic case study (Merriam, 2001). We collected the materials through various methods such as text analysis, informal conversation, and semi-structured interview in 2 months. In order to achieve the purpose of "triangular verification", materials were collected from multiple perspectives such as university teachers, postgraduates, and mathematics teachers. Table 1 presents the outline of research materials.

Table 1 Outline of research materials

Purpose	Method	Materials collected
Development strategy	Text analysis	Minutes, annual reports, briefings, and other text materials of BMES
	Informal conversation	Face-to-face chats, phone calls, and emails with the core organizers of BMES are recorded in the research log
Challenges	Semi-structured interview	Online interviews with four postgraduates, four mathematics teachers, and four professors, and each interviewee was interviewed for about 30 min. The interview data was collected by real-time recording and audio recording

In view of the development strategy of BMES, research was carried out through the collection of minutes, annual reports, briefings, and other text materials, as well as the informal conversation materials of the core guidance. In response to the challenges faced by BMES, based on the interview outlines, one-to-one formal interviews were conducted through WeChat conference call, and each interviewee was interviewed one-to-one for about 30 min. The interview data was collected by real-time recording and audio recording.

For the selection of interviewees, first, a stratified sampling method has been applied to conduct sampling according to the group category and personal occupations of those who are participating in the seminar. Secondly, purposive sampling was applied to identify specific interviewees. According to the purpose of the research, purposive sampling is likely to provide the largest amount of information for the research questions (Chen, 1996). Therefore, the selected interviewees are those who have participated in BMES for a number of times and who have the capacity to evaluate and predict the challenges of BMES. Overall, we mainly selected the leaders and their followers for interviews, including four postgraduates from mathematics education majors, four mathematics teachers from primary and middle schools, and four professors of mathematics department in key universities.

2.4 Data Analysis

The sorting process included the transcription, coding, and analysis of the materials. The collected interview records were processed and coded independently by two master's students in mathematics education. After transcribing the interview records to form the interview text for coding analysis, different methods such as word-by-word coding, line-by-line coding (sentence-by-sentence coding), and event-by-event coding were used to segment the text into data units (Charmaz, 2009). Based on grounded theory, Strauss' three-level coding format was used to code and analyze data (Chen, 2015; Strauss & Corbin, 1990). The three-level coding included three parts: open coding, axial coding, and selective coding. Firstly, the original data was sorted in the form of open coding, and the pre-settings were suspended as much as possible. The research data was conceptualized and classified according to theoretical sensitivity; secondly, the axial coding was carried out, that is, the categories of the study were summarized to form tree nodes; finally, based on the axial coding, the results of core category analysis were formed by selective coding. The percentage of the number of identical codes between two researchers to the total number of codes was set as the inspection index. It turned out that the code consistency between the two researchers was as high as 91%, indicating the satisfactory result of dual-independent coding.

3 Development Strategies

3.1 The Leaders and Their Followers Are the Basis

From the perspective of Activity Theory, the subjects of BMES are the leaders and their followers, who are also the basis for the formation and development of mathematics education research culture. BMES is centered in Beijing to promote the development of mathematics education research culture in Northern China and the whole country. Specifically, the leaders and their followers are the source of the long-lasting vitality of BMES. Among them, the leaders are also the soul of the cultural development. The followers practice the mathematics education thought, and they are the backbones of mathematics education research culture to be disseminated and passed on.

As the first generation of leaders, the famous mathematician Professor Shijian Yan and mathematics educator Professor Xiaoda Zhang played a pioneering role at the early construction of BMES. The year of 1989 witnessed the opening of the annual conference held by the National Committee on Secondary Mathematics Teaching of the Chinese Society of Education. At the conference, Professor Zhang, Chairman of the National Committee on Secondary Mathematics Teaching of the Chinese Society of Education, encouraged young scholars to study mathematics education to meet the challenges of the twenty-first century. In addition, a celebrated mathematician who has strong passion for mathematics education, Professor Yan is fully devoted to the mathematics education research. Relying on the National Natural Science Fund research projects, and with his extensive academic influence, Professor Yan has called on a group of academic leaders in mathematics to deliberate on the most crucial issues on mathematics curriculum reformation, so that more mathematicians participated in mathematics education research. In 1995, under the guidance of Professor Yan and Professor Zhang, BMES was established.

As followers of the first generation of leaders, the second-generation leaders are composed of researchers in the field of mathematics education in Beijing. They are mainly professors Jian Liu, Xiaotian Sun, and Xiaomei Liu and faculty members from the Department of Mathematics at Beijing Normal University, Minzu University of China, and Capital Normal University, respectively. Beijing Normal University (BNU) is recognized as one of the leading normal universities in China that conducts educational research. As a young faculty member in the field of mathematics education at BNU, Mr. Jian Liu, with the recommendation from Professor Zhang, took the lead in the formation of the research project entitled "Prospects of Chinese Mathematics Education in the 21st Century: The Theory and Practice of Mathematics for All". And this research project planted the seed for the development of BMES. In the past three years, the next generation of leaders is rising. Most of them are academic youths from the field of mathematics education in Beijing, conveying the mathematics education thought and research spirit of their predecessors. They are also where the vitality of BMES's mathematics education research culture continues.

The leaders' interest and direction in mathematics education research make the theme of BMES using the minority to bring along the majority. With the university faculty team as the core, it continues to spread out. In this process, the followers include postgraduates majored in mathematics education and mathematics teachers.

On the one hand, in their capacity as the professors in various universities, the leaders have not only provided their master's and doctoral students with exchange and presentation opportunities at the seminar, but also encouraged postgraduates from different universities to share topics around mathematics education research, making the seminar a professional development platform for a large number of mathematics education postgraduates around China. As a result, the seminar has attracted groups of graduate students who love mathematics education. For instance, those from Capital Normal University, Nanjing Normal University, and Wenzhou University have all shared and discussed their research results and viewpoints at the seminar. On the other hand, BMES encourages and invites the participation of primary and secondary school principals, and they also drive the participation and enthusiasm of mathematics teachers. For example, Li Xin, Principal of Beijing No. 5 Middle School, Wei Zhang, Principal of Beijing No. 11 School, Yi Qiu, Vice Principal of Sichuan Emeishan Second Middle School, and other principals and teachers have actively participated in the themed seminars. They shared their first-hand classroom instruction experiences, thus opening up a communication bridge between educational theory and practice. The following is part of the minutes where Principal Wei Zhang from Beijing No. 11 School shared his case study at BMES:

> A doctoral student from Northeast Normal University participated in the discussion through a conference call where she raised a question: 'In the process of instruction design, there will be a series of content restructure, what method should be applied?' Principal Wei Zhang responded by taking the No. 11 School as an example. He said, since courses at middle school are offered hierarchically, each chapter in the book 'How to Create and Use Rubrics: for formative assessment and grading' will be processed based on the national textbooks. however, the overall structure remains unchanged. We will use an integrated approach when instructing, and the content restructure focused more on the nature of mathematics. (Issue 4, 2018)

As the political and cultural center of China, Beijing has the largest number of universities and very rich educational and academic resources, which drives the development of basic education in Beijing and the level of teachers' teaching research in the form of "top-down". At this background, teachers in Beijing have higher education research enthusiasm and self-development beliefs. Accordingly, it is also an important factor for BMES to attract many mathematics teachers. Influenced by the regional culture of Beijing, BMES has university faculty members in the field of mathematics education as the leaders and postgraduates in mathematics education and mathematics teachers as the backbone, forming an environment for stable and rigorous spirit in research. It not only gathers researchers in mathematics education in Beijing, but also calls on scholars and teachers from Taiwan, Hong Kong, and other regions to ignite the torch of mathematics education research.

3.2 *The Leading and Contemporary Research Agenda Is the Core*

Based on the Activity Theory, the object is the goal of subject's activity (Wang, 2017a, 2017b). The object of BMES refers to the specific research agenda. Identifying the themes for discussions and research is central for the development of mathematics education research culture. Specifically, the research agenda that points to mathematics education reformation is the goal of BMES. "Reformation and innovation" is the academic philosophy upheld by generations of seminar members. The research agenda of BMES can be divided into three stages.

The first stage is between 1995 and 2001. During this stage, BMES conducted themed discussion mainly serving the formulation of mathematics curriculum standards for schools. Entrusted by the Ministry of Education, China, Jian Liu and other leaders participated in the development of the *Compulsory Education Mathematics Curriculum Standard (Experimental Draft)* and the compilation of a new round of experimental textbooks at the end of the twentieth century. Thus, BMES provided a communication platform to promote the reformation of Chinese mathematics curriculum. During this period, BMES gathered a group of researchers dedicated to mathematics curriculum reformation. They advocated the proposition that "mathematics education is for everyone in schools". In the sense of "Maths for all", they also built a mathematical education theory correspondingly. Such academic proposition formed the unique cultural cohesion of BMES and built a common vision of "Maths for all", which profoundly influenced the field of Chinese mathematics education research at that time.

The second stage is from 2002 to 2010. BMES took mathematics curriculum and textbooks as the research agenda and openly discussed various debates about mathematics curriculum reformation including debates on "mathematics for all" versus "mathematics for elites", mathematical "two basics (include basic mathematics knowledge and basic skills)", receptive learning versus inquiry learning, and textbook designs (Bao & Xu, 2013; Hu & Liu, 2015; Luo et al., 2008; Zhao & Song, 2010). In this regard, BMES invited and gathered many renowned scholars and senior teachers to discuss the above debates. For example, in 2009, BMES invited Jinfa Cai to share the foreign curriculum reformation experience and the "California Mathematical War" on the "International Mathematics Curriculum Reformation".

The third stage is from 2011 until the present time. At this stage, the research agenda of BMES has shifted to diversify and is informed by international research trends, reflecting the cutting-edge concepts of Chinese mathematics education research. Since the "Compulsory Education Mathematics Curriculum Standards (2011 Edition)" was formally released in 2011, the research agenda has shifted from responding to the debates on curriculum reformation to focusing on specific issues and paying more attention to the integration of mathematics education with psychology, statistics, etc. There are themed discussions around the recent advances in mathematics education such as mathematics competencies, mathematics activity curriculum development, mathematics education assessment, mathematics cognitive mechanism, and mathematics textbook compilation. Excerpt of the minutes from a seminar entitled "Why emphasis on the sense of measurement" is displayed as below:

> Why do we need to talk about 'the sense of quantity'? From the foreign curriculum standards, it can be seen that many countries attach great importance to 'measurement'. There are four areas of primary school mathematics in our country: number and algebra, graphics and geometry, statistics and probability, and synthesis and practice. There is no setting for 'measurement' in the content of the field. Also, it is more difficult to add the field of 'measurement' in it, and teachers may not accept it. In this regard, we still do not pay enough attention to 'measurement'. Therefore, in order to emphasize the matter of 'measurement', 'the sense of quantity' is added to the performance of literacy in the new round of curriculum standard revision. (Issue 2, 2021)

In summary, the research agenda of the task-driven BMES is both leading and contemporary. This is the core of the BMES's development and continuation, which reflects the value pursuit of the seminar members for continuous breakthroughs and innovations in traditional mathematics education research. On the one hand, mathematics education researchers in Beijing have greater policy acumen and more opportunities to participate in policy making. With the unique policy appeal, BMES has gathered the essence of the mathematics curriculum reformation thoughts of Chinese researchers since the new round of curriculum reformation. It is an important vane of Chinese mathematics curriculum and textbook research. On the other hand, BMES keeps up with the times and explores the hotspots problems of mathematics education research. The research perspective of BMES starts from the research of mathematics curriculum and textbooks and continues to strive for diversification and internationalization. Furthermore, it reflects the trend of Chinese mathematics education research gradually moving from practical problems to diversified development.

3.3 Inclusive and Cooperative Discussion Format Is a Productive Means

According to the language of the Activity Theory, the subject can act on the object through certain instruments. For BMES, inclusive and cooperative discussions are important instruments for the seminar members to achieve their goals. Through the in-depth discussions of multiple issues and the generation of creative views, the inclusive and collaborative atmosphere promotes the deepening of the discussion.

Based on the respect of individual differences, BMES encourages free sharing of opinions and the presentation of various viewpoints, which has contributed to the forming of typical inclusive culture. For example, at the themed seminar of "The prospects of International Conference on Mathematics Textbook Research and Development (ICMT)" in December 2019, researchers, mathematics teachers, and textbook editors all put forward different opinions upon the research work on mathematics textbooks conducted by the teachers. Professors Chunxia Qi and Xiaomei Liu suggested that teachers may try to design textbooks by themselves and investigate the use of textbooks by students. Master's student Pingting Feng proposed to study the degree to which primary school mathematics teachers pay attention to textbooks. Principal Wang recommended that comparative research on Chinese and foreign textbooks be conducted. The excerpt of the minutes in that discussion states.

> Professor Chunxia Qi suggested that teachers should focus on the trend of research, emphasizing that no matter what research is being done, it is important to generate the results based on evidence. School teachers can pilot on the development of teaching materials.
>
> Principal Wang introduced the research on teaching materials within Fangcaodi School. At this school, comparison study on Chinese and foreign teaching materials has been conducted for more than one year.
>
> Professor Jian Liu's students presented a research project on primary school mathematics teachers' attention to textbooks. This project focuses on how teachers pay attention to the use of textbooks and divide them into different dimensions. (Issue 4, 2019)

By encouraging cooperation and especially collaborative discussion, BMES forms a typical cooperative culture. On the one hand, BMES calls on mathematics education researchers from various universities. The formation of the academic circle of local mathematics educators relies on local universities, which are mostly operated as isolated silos. Including the regions of Hong Kong and Macau, mathematics education research often tends to reflect the phenomenon of "mind your own business". There is hardly any cooperation between different universities or regions. Different from the traditional "tribal" academic circles, BMES, relying on the academic influence and platform of Beijing Normal University, which was a top university under the Ministry of Education, has opened its communication to the public, breaking the previous regional boundary centered on universities. A cutting-edge mathematics education academic ecosystem has been created through the construction of a scholar community. To a certain degree, it has realized the exchange, communication, and cooperation of the circle of mathematics education research in Beijing including researchers from other universities, which has helped forge a scholarly community with Beijing Normal University as the hinterland.

On the other hand, with universities as the core, and driving teachers to work together in mathematics education research, BMES has built a bridge between educational theory and practice. By creating an atmosphere of collaborative culture, BMES has stimulated the shared belief and vision of all participants. The shared belief has contributed to the mutual vision among BMES' members, that is, to promote the prosperity and development of mathematics education in China. A shared identity and mission have formed among the participants of BMES. Taking the themed seminar of "Possibilities for the Integration of Science and Technology Museum Education and School Mathematics Education" in 2018 as an example, researchers and practitioners in Beijing have discussed the mathematical issues in science and technology museums from different research perspectives. Participants expressed their willingness to cooperate in teaching research during the process of sharing and learning, e.g., joining the practical research project on the development and utilization of popular science museum resources in the Teachers Research and Training Center of Dongcheng District. The following presents some of the minutes of a discussion session:

> In addition to 'encouraging students to go outside', 'bringing in the science and technology museums to the school' is also another form of educational activities that teachers have heatedly discussed. Ms Gao from Zhongguancun No.2 Elementary School shared her experience of setting up a maths activity room in the school and attempt to bring the science museum to school, which has to some extent increased the frequency of students hands-on learning. It is hoped that the children will experience the fun part of mathematics and fall in love with mathematics. (Issue 3, 2018)

In order to promote mathematics education research, encouraging inclusive and cooperative discussions is an important means for BMES. The culture of inclusiveness and cooperation of BMES is rooted in the excellent traditional Chinese culture. With Chinese characteristics, it forms a mathematics education research culture. As for the inclusive culture, the overall academic atmosphere of the capital's "atmospheric and open" regional culture has contributed to the formation of the inclusive culture. At the same time, the ideas of "harmony without difference", "a hundred schools of thought", and "inclusiveness" in Chinese culture also provide cultural fertile ground for the inclusive culture. For cooperative culture, influenced by Confucian culture, the spirit of cooperation has been emphasized since ancient times. For example, Confucius once said, "When I walk along with two others, one of them may surely serve me as my teacher. I will select their good qualities and follow them, and I will also get rid of their bad ones", Chinese ancient books *Xue Ji* pointed out that "study alone without friends, then lonely and ignorant". Moreover, in the context of Chinese culture, collectivism is deeply ingrained and invisibly promotes mutual learning and cooperation between people.

All in all, an inclusive and cooperative discussion is an important factor for BMES to attract many mathematics teachers, postgraduates, and university scholars, which is also a vital support to gather researchers to form a joint force in mathematics education research.

4 Challenges

4.1 The Balance Between Theory and Practice

When educational academics and practitioners jointly study mathematics education, they need to face the problem of balance between theory and practice. The essence of this problem is the tension between the complex subjects of mathematics education research.

In the semi-structured interview, a leader of BMES pointed out the "separation" between theory and practice at current agenda: "*When deliberating on academic issues, the educational practitioners often put forward many practical questions, which makes it difficult to integrate with theory. Meanwhile, what the practitioners shared normally involve topics related to classroom applications and teaching activities, which in turn makes researchers confused*".

On the one hand, educational practitioners, represented by mathematics teachers, prefer to engage in theoretical issues in BMES. Just like a math teacher in a primary school said: "*Participating the BMES gives me more confidence in theory-related issues, which makes me dare to speak and write*". The discussion of teachers' teaching experience is more likely to be analyzed from a theoretical perspective, which can help teachers refine and sublimate their practical experience and also provide more theoretical support for their mathematics education research. However, mathematics teachers are better at solving practical problems in education, so that their research on mathematics education is always "bottom-up" and more inclined to speculative experience research.

On the other hand, educational academics, represented by university scholars and postgraduates, prefer to learn educational practice content from BMES. A third-year master's student of mathematics education pointed out, "*BMES allowed me to understand the students better and know exactly where their confusion and perplexity are. This is something I can't learn from books*". However, educational academics have more theoretical and methodological perspectives, and their research on mathematics education may be "top-down".

According to the Activity Theory, the subjects' personal experience, as well as social and cultural environment, plays an important role for their development (Qin & Dai, 2003). Just as said above, the subject of BMES is the participants. They are mainly from universities and primary and secondary schools, who have different personal experiences and are in different social and cultural environments as well. Accordingly, the heterogeneity of the environment and the diversity of the subjects lead to the difference of participants' research preferences, perspectives, and paradigms. In order to build a community of mathematics education research, how to balance the theoretical and practical issues to give full play to the wisdom of both educational academics and practitioners is a key point. This remains a challenge in the development of mathematics education research culture.

4.2 The Lack of Orientation for the Research Agenda

Under the influence of social demand and discipline development, the research agenda of BMES presents a trend from "focus" to "dispersion", whose essence is the goal-oriented problem of the object.

The generation of BMES is the product of Chinese social development and education reformation, which also reflects the application orientation of mathematics education research. Driven by the formulation of *Compulsory Education Mathematics Curriculum Standard*, BMES carried out systematic and in-depth research on the curriculum and textbooks in its early stages. However, with the end of curriculum standard formulation and the diversified development of mathematics education research, BMES lacked a clear task driven for a long time, which allowed for the research agenda to change from "focus" to "dispersion". The specific performance is that the research agenda of each period presents a non-continuity. In the interviews, a leader pointed out: "*The current agenda is rather divergent without focus. This is both the characteristics and challenge of BMES*". The gradual "dispersion" of the research agenda also makes it difficult to build the research foundation and to accumulate the research results of BMES. In that case, every discussion often just scratches the surface and is really hard to carry on the deeper exploration to mathematics education research among people with alternatives interests and expertise.

Based on the Activity Theory, the subject's activities point to the object that is the target task of the subject. Furthermore, the change of these tasks drives and influences the subject's activities. Influenced by the thought of "study for the purpose of application" in Confucian culture, Chinese scholars have academic aspirations to "maintain the state rightly and make all peaceful" and stronger practical consciousness (Chu & Li, 2014). Meanwhile, the social demand is often greater than the internal logic of the discipline in the motivation of Chinese mathematics education researchers. Therefore, external factors such as history, culture, and social environment affect the culture of mathematics education research in China. This is also the deep-rooted reason why BMES' agenda lacks a stable orientation after losing its external task drive.

4.3 The Relative Effectiveness of "Lectures" and "Discussions" Formats

BMES takes special communication as the main instrument to promote subject's mathematics education research. However, on several occasions invited guests have delivered lectures to the participants. Hence, the effectiveness of the instrument is the essence of how to properly deal with the problem of balance and the relative efficacy of "lecture" and "discussion".

The communication format of BMES is dominated by traditional lectures, where the time for actual discussion is insufficient. In the interview, a math education expert pointed out: *"Due to limited access to information in the past, BMES used to focus on "lecturing", as a result, the presentations delivered were more of value*". However, the development of the times has promoted the diversification of information acquisition channels, especially in the post-epidemic era. The opening of various academic websites, international databases, and other network resources had helped researchers and teachers acquire massive amounts of educational information. Nowadays, online discussion is popular all around the world, and people have a lot of opportunities to "listen to lectures". In addition, the time left for participants to discuss and communicate with each other is limited in BMES. In each seminar, a large part of the time is taught by the sharer, and the real time for discussion is not enough. It is often difficult to carry out in-depth discussion, leading to the low quality of the discussion.

The academic discourse power of BMES is in the hands of a few people, and the opportunity of discussion is not equal. As for the members of BMES, they rarely become the presenter. For most members, the discussion session is what they can participate in. Then, in the discussion session, the discourse power of university teachers is greater than that of school practitioners, and the discourse power of practitioners is greater than that of graduates. It indicates the inequality of discussion opportunities. The remark from a third-year doctoral student in mathematics education is typical, *"We have limited opportunities to present, those who raise their hands to share are often university lecturers and school teachers"*.

According to the Activity Theory, in either way of knowledge sharing and building, i.e., of lecture or discussion, the intermediary is the interaction between the subject's activities and the external social, cultural, and historical environment (Engestrom, 2001). The unbalanced allocation of time in lecture and discussion essentially ignores the change of the subject's environment and its influence on the subject's cognition. Due to the personal "qualifications" and the influence of Chinese history and culture, participants have different opportunities for discussion. In other words, senior university scholars and frontline practitioners tend to have more discourse power than younger graduates, which reflects the "authority of elders" in Chinese academic discourse system.

Based on the Activity Theory, the above analyzes the challenges faced by BMES from the subject, object, and tools of the activity, which can provide revelations for the development of other seminars. First, the seminar promotes communications and cooperation between educational academics and practitioners. From the perspective of the participants and the research agenda they are concerned about, the seminar needs to balance the relationship between theory and practice. Specifically, the seminar should pay attention to the topics of common concerns, and the problems that can be better solved through the combination of theory and practice. However, it is not a good idea to make the research agenda too "academic" or completely based on "practical experience". Second, whether the research agenda of the seminar is focused or divergent is a question that needs to be considered. Continuous research agenda driven is helpful for continuous focus and deepening of content, but whether the content must be "focused" is worth thinking about. Third, from the perspective of discussion format, online seminars are emerging in the context of the epidemic. How to deal with the problems of "lectures" and "discussions" is related to the effectiveness of the seminar. The seminar needs to ensure the "quantity" of discussion, which means that we need enough time to discuss with each other. Also, equal discussion opportunities are related to the "quality" of discussion.

5 Conclusions and Limitations

Based on the culture-historical Activity Theory, this study takes BMES as a case for qualitative research and analyzes the development strategies and challenges from three aspects: subject, object, and instruments (see Fig. 2). Our study reflects the formation and development of Chinese mathematics education research culture and mainly draws the following conclusions:

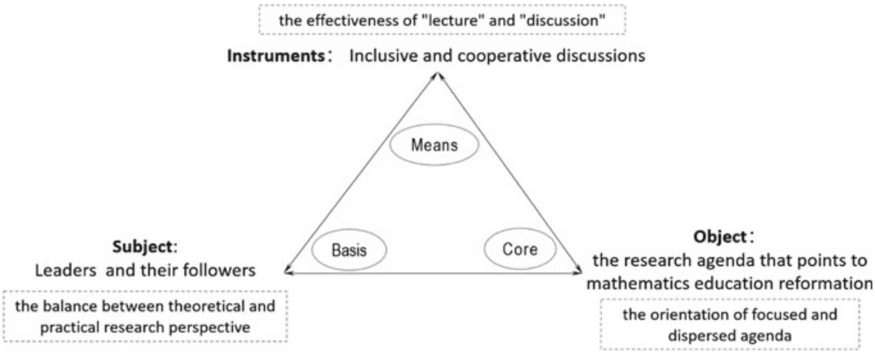

Fig. 2 Development strategies and challenges of BMES

First, the development strategy of BMES mainly includes three aspects: the relay of leaders and their followers, the leading and contemporary research agenda, and the inclusiveness and cooperative discussion format. Among them, the leaders and their followers, as the subject, are the foundation and source of the sustainable development of BMES. As the object that the subject needs to be reformed, the task-driven research agenda points to mathematics education reformation, which is both leading and contemporary. And it is the core of the continuation of BMES. The inclusive and cooperative discussion is the instrument for the subject to act indirectly on the object, as well as an important means for the development of BMES. Secondly, the characteristics and problems of BMES coexist. BMES is faced with the following challenges: the balance between theoretical and practical research perspective, the lack of orientation for the research agenda, and the effectiveness of "lecture" and "discussion". It is also the problem that needs to be concerned in the development of Chinese mathematics education research culture.

In addition, this case study has two main limitations. On the one hand, we select Beijing Mathematics Education Seminar as the case to analyze, which is actually limited in representativeness and can only reflect the development of Chinese mathematics education research culture to a certain extent. On the other hand, although the study aims for authenticity, the formats of online audio interviews, to some extent, have led to the loss of information in terms of the expressions and actions of the interviewees. Meanwhile, the interviewees may "make up" in some circumstance, as a result of which certain sensitive and tacit issues might not be fully raised.

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The Evolution of Mathematics Education Research in Singapore

Berinderjeet Kaur, Tin Lam Toh, and Eng Guan Tay

Abstract Up until 1990, the Institute of Education in Singapore was primarily a teaching institute involved in training teachers for Singapore schools. Since the inception of the National Institute of Education (NIE) in 1990, as an institute of the Nanyang Technological University, the focus of the institute has been enlarged to include research in education. This chapter examines, through a documentary analysis, how a research culture specifically in mathematics education at the National Institute of Education was nurtured, developed and supported from 1990 onwards. Development of the culture for Mathematics Education Research (MER) has been in tandem with all other areas of research at the NIE. Both top-down and bottom-up approaches have been adopted to support research as part of an academic's work at the institute. Policies related to recruitment and promotion of academics were developed to ensure that emphasis was on both teaching and research. Development of research, from individually led bite-sized grains to team-based project with coherent themes, was supported. The setting up of the Centre for Research in Pedagogy and Practice in 2004 and dedicated funding from the Ministry of Education Singapore for research of the Singapore education system heralded an era of MER that has made significant contributions both nationally and internationally. This chapter will also illuminate the four main areas of focus and sources on MER through examples of studies carried out in Singapore since 2000. In addition, it briefly outlines the contribution of MER in ASEAN countries.

Keywords Mathematics education research · Singapore · Evolution · National Institute of Education · Centre for Research in Pedagogy and Practice · Funded research projects · Postgraduate student research

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1 Background

Up until 1990, the Institute of Education (IE) in Singapore was primarily the sole teaching institute involved in training teachers for Singapore schools. Since the inception of the National Institute of Education (NIE) in 1990, as an institute of the Nanyang Technological University, the focus of the institute has been enlarged to include research in education. The transition from an Institute of Education to the National Institute of Education saw the revamping of academic status of its faculty. In the IE, teaching staff were primarily lecturers, senior lecturers and principal lecturers. In the IE, emphasis was on training teachers for schools in Singapore and apparently based on the assumption that tacit knowledge self-perpetuates by way of role-modelling by lecturers at the IE and practising teachers in schools that mentored trainee teachers. The NIE, being an institute of the Nanyang Technological University, applied the same academic ranking used in the university, such as an assistant professor, an associate professor and a professor. As such, the yardsticks for performance of academic staff were also tied to three key aspects: teaching, research and service to the academic/education community. A rubric with a weightage of T (Teaching): R (Research): S (Service) = 5:5:2 for appraisal of staff performance slowly but surely led to developments at the institute that supported research activities of staff. In the following sections, we account chronologically the development of Mathematics Education Research (MER) in NIE which being the sole institute for teacher education in Singapore is also the bedrock for MER in Singapore.

2 Beginnings of Mathematics Education Research in Singapore (Pre-1990)

It may be viewed that in the years leading up to the early 1990s, mathematics education research in Singapore was mainly driven by one of the following needs: (i) post-graduate academic requirements such as a thesis for a master in education (M.Ed.) or doctorate (Ph.D.) in education programme, (ii) individual research interest pursuits or iii) evaluation of the efficacy of teaching approaches and curriculum materials. Two notable documents, a proceeding of a mathematics education conference held in 1987 in Singapore (Institute of Education, 1987) and a review of mathematics education research in Singapore done in 1991 (Chong et al., 1991), provide us with some insights about the state of MER up till the beginning of the 1990s.

The Fourth Southeast Asian Conference on Mathematical Education was held in Singapore in June 1987. This was the first mathematics education conference held in Singapore. It was part of the *South East Asia Conferences on Mathematics Education* (SEACME) series that began in 1978 in Manila (Philippines), and thereafter conferences were held at three-year intervals in Kuala Lumpur (Malaysia) (1981), Haad Yai (Thailand) (1984), Singapore (1987), Brunei (1990), Surabaya (Indonesia) (1993), Hanoi (Vietnam) (1996), and Manila (Philippines) again in 1999. Father, Dr

Bienvenido F. Nebres of the Ateneo de Manila University and Dr Lee Peng Yee of the University of Singapore were instrumental in actualising this series of conferences.

The published proceedings of SEACME 4 shows that there was one keynote paper by Robert Davis from the USA and seven invited papers (two from the UK, two from Australia, one each from the Philippines, Germany and Israel) (Institute of Education, 1987). Another fifty-three papers were contributed, with 18 each from Australia and Singapore, four from India, two each from Japan, Philippines, Papua New Guinea, New Zealand and the USA, and one each from Indonesia, Thailand and the UK. Out of the 18 papers from Singapore, two were from an international institution—the United World College, three were from the Curriculum Development Institute of Singapore (CDIS) involved in innovation and curriculum for schools in Singapore under the purview of the Ministry of Education in Singapore, two were from schools—these were based on the research done by the teachers as graduate students of the IE as part of their Master of Education dissertations and 11 were from the IE showcasing research of the department of mathematics and computer studies at the IE in Singapore.

A close examination of the presentations from Singapore illuminates the nature of MER that was present. The two papers presented by colleagues from the United World College (Binge, 1987; Butler, 1987) were reflections of the Cockcroft Report of the UK (The Cockroft Report, 1982) and implications for mathematics instruction at the United World College in Singapore. The three presentations from the CDIS (Khoo, 1987; Lim, C. L., 1987; Sin, 1987) were reports of ongoing developmental work in school mathematics curriculum at the primary and secondary schools in Singapore. All the reports showcased proposed methods of instruction that were advocated for school mathematics curriculum implementation through textbooks produced by the CDIS. Another two presentations were helmed by school teachers. The first was in partnership with a colleague in the CDIS, and it evaluated the implementation of an individualised computer-assisted remediation package (Gay & Leong, 1987). The second was based on a teacher's survey on the understanding of an aspect of Calculus taught at the junior college and pre-university centres in Singapore (Chen, 1987).

The eleven presentations from the IE were mainly in two areas. The first was empirical, one-off studies on the teaching and learning of mathematics, including the efficacy of teaching materials developed by the CDIS in Singapore schools and pre-service mathematics teacher education at the institute (Chai & Ang, 1987; Fong, 1987a; Gan, 1987; Ong, 1987a; Ong & Lim, 1987; Plant, 1987; Wong, 1987a). The second was on innovative ideas for mathematics instruction in Singapore schools (Fong, 1987b; Lim, S. K., 1987; Ong, 1987b; Wong, 1987b). It is apparent from the presentations that MER was in its infancy and there were no acknowledgements whatsoever for funding of any sort that supported research activities.

In 1991, as part of a state-of-the-art review on mathematics education that was commissioned and funded by the Southeast Asian Research Review and Advisory Group (SEARRAG), mathematics education research in Singapore from 1979 till 1991 was surveyed and documented (Chong et al., 1991). Of the 42 studies documented, 16 (38%) were thesis or dissertation presented for a M.Ed. or Ph.D. degree, 22 (52.4%) were driven by individual research pursuits, and 4 (9.6%) were research projects that evaluated efficacy of teaching approaches and curriculum materials. Of these 22 studies, nine were research reports, seven were journal papers, and ten were conference papers. Six of the seven journal papers were published in the local journals of the IE, two in *Teaching and Learning* and four in the *Singapore Journal of Education*. Only one was published in an international journal—*Studies in Educational Evaluation*. Furthermore, none were published in an international mathematics education research journal.

Almost 62% or 26 out of the 42 studies were on the teaching and learning of mathematics. They were on affective variables and problem solving (Foong, 1985, 1990; Ng-Gan, 1987; Tan, 1989; Wong, 1989), types and levels of understanding (Chu, 1987; Lam, 1985; Purbick et al., 1982; Tan, C. S., 1987; Tay, 1986), analysis of errors (Booth, 1986; Chai & Ang, 1987; Kaur, 1991; Ong & Lim, 1987), low achievers and remediation (Ee, 1991; Fong, 1987c; Yap, 1990), learning strategies (Ng, 1985; Wong, K. Y., 1990; Wong, P., 1990), use of microcomputers (Ho, 1990; Tan, P. K., 1987; Woo-Tan, 1989) and other miscellaneous topics (Ang, 1984; Chai, 1979; Kaur, 1987).

Of the rest of the studies, five were on assessment and examinations in mathematics, five on teacher education in mathematics and six on the efficacy of teaching approached and curriculum materials. Almost all of the studies were one time and exploratory in nature. Chong et al. (1991) noted that:

> the findings even if significant such as the effectiveness of a particular instructional intervention should be considered tentative and perhaps not easily generalizable to other similar situations (p. 49).

Only a few of the studies that had been carried out discussed their findings with a view for possible improvements in mathematics education. The rest merely verified past research findings making no reference to possible improvements in mathematics education. The state-of-the-art review on mathematics education of 1991 concluded that the challenge to raise the level of attention in research on mathematics education towards improvement was best summed up as follows by Sim (1991) cited in Chong et al. (1991):

> [A]n obvious deficiency among studies purportedly to be research in mathematics education is the lack of serious attention to subsequent improvements in mathematics education (p. 59).

3 The Sandwiched Era of Mathematics Education Research in Singapore (1991–2003)

With the inception of the National Institute of Education (NIE) in 1991, as an institute of the Nanyang Technological University, the focus of the institute was enlarged to include research in education. As such, there was an expectation for faculty to partake in research activities. This heralded a top-down push for research activities. Faculty were supported to attend international conferences to present their research, solicit feedback from international colleagues and network with like-minded researchers. Faculty of the IE, without doctorate degrees, were assessed by an international panel for their long-term fit in NIE. Promising faculty were sent for development at international teacher education institutions. For mathematics education, one such faculty was the first author of this chapter. She went to Monash University in Australia and pursued her PhD in Education. Her three-year study at Monash University was funded by the NIE.

These academics of the NIE, while attending international conferences or doing their studies at international institutions, managed to network with international like-minded colleagues and this facilitated participation in international research. This bottom-up approach, initiated by the academics themselves, was evident in the Kassel Project carried out from 1995 till 1997 by Kaur and Yap (2009), the International Project on Mathematical Attainment (IPMA) carried out from 1999 till 2003 by Kaur et al. (2009a) and a comparative study of primary school pupils' perceptions of their best mathematics teacher in Singapore and Brunei Darussalam from 1997 till 1999 by Wong et al. (2009). Modest funding for the Kassel Project was provided by The Gatsby Charitable Foundation in the UK and the British Council. The IPMA was also modestly funded by the Academic Research Fund of the NIE and the University of Exeter in the UK. The funding was sufficient for data collection purposes but not manpower needs to assist with rigorous data analysis. For the purpose of illustration, Table 1 shows the research outputs of the above three projects that were helmed by the first author of this chapter.

It is apparent from Table 1 that the main goals of the Kassel Project and the IPMA were to document the research as research reports for communication among the communities that were participating in the studies. These were shared at the meetings held by the respective studies. Modest attempts were made to present the research to the wider research community through conferences and other research-related publications. This was partly due to the lack of support for research activity in terms of funding for both "time" and "manpower". Hence, this appears to have impacted both the quantity and quality of the research outputs.

4 Mathematics Education Research in Singapore (Post-2003)

To fuel a research culture at the NIE, it became apparent that research funding was necessary in a structured manner to support educational research. At the beginning of the twenty-first century, the then Dean of the Graduate Programme Office, Professor Lee Sing Kong, initiated the setting up of a Centre for Research in Pedagogy and Practice (CRPP) with a research tranche of about 50 million Singapore Dollars from the Ministry of Education. The CRPP was established at the NIE in 2003 with a focus to improve and sustain student and teacher learning in Singapore schools. The aims of the centre as reflected in the Research Excellence Report (NIE, 2017) are to:

Table 1 Publications from the Kassel project, IPMA and comparative study

Research output	Kassel project	IPMA	Comparative study—Singapore and Brunei Darussalam
Research reports	3 Kaur and Yap (1996, 1997a, 1998)	5 Kaur et al. (2000, 2001a, 2003) and Koay et al. (2003, 2004a)	–
Keynote addresses	–	–	–
Invited presentations	–	–	–
Scholarly book chapters	2 Kaur and Yap (2004, 2009)	2 Kaur et al. (2004, 2009a)	2 Wong et al. (2007, 2009)
Conference papers (published in proceedings)	2 Kaur and Yap (1997b, 1999)	5 Kaur et al. (1999a, 2001b), Koay et al. (2001, 2004b), Thompson et al. (2010)	1 Kaur et al. (1999b)
Journal papers (refereed)	–	1 Thompson et al. (2013)[a]	–

[a] Published in tier 1 MER Journal

- conduct high-quality research and development programmes that are innovative, relevant and responsive locally and internationally;
- understand, design and implement pedagogical innovations in formal and informal contexts, towards more equitable futures for all learners and
- generate rigorous and impactful school-based and system-level educational research that is cognisant of the sociocultural context of Singapore's education landscape (p. 7).

The NIE's Roadmap for the period (2007–2012) (NIE, 2007) outlined three pillars as overarching themes to realise NIE's vision as an institute of distinction. The second pillar (Pillar 2): Achieving international recognition through educational research clearly delineated the following objective for 2012:

> Identifying, developing, implementing and managing a strategically focussed, scientifically rigorous NIE-wide programme of research, development and innovation that seeks to improve the quality of teaching and learning in Singapore schools and consolidate NIE's recent emergence as a leading international research institution (p. 28).

In 2008, the Office of Education Research (OER) was established to forge an institutional-wide programme of research at NIE. A key function of OER is to administer the Education Research Funding Programme, a pool of research funding provided by the Ministry of Education in Singapore. OER aims to:

> (i) develop NIE's research capacity in key areas that impact programmatic and pedagogical enhancements both within NIE and in schools and
>
> (ii) deliver evidence-based research-informed pedagogies and programmes to raise the competencies and capabilities of teachers systemically (NIE, 2017, p. 6).

The OER has three research centres. The centres are the CRPP, Education and Cognitive Development Lab (ECDL) and the Learning Sciences Lab (LSL). Over the past two decades, OER has received four tranches of funding from the MOE. In every tranche, MER was funded. It was noted in NIE (2017) that the outstanding research produced by the faculty at NIE makes the Nanyang Technological University (NTU) consistently one of the 20 universities in education and among the top in research performance in the world.

4.1 Institutional Funding for Mathematics Education Research

A meta-analysis of OER funded mathematics-related projects for the period 2008–2017 was carried out by Wong (2017). In his analysis, he noted that a total of 43 projects were funded with a sum of 15.5 million Singapore dollars. The budgets for the projects ranged from $28,000 to $4 million with an overall mean of $350,000. The research grants awarded for the projects included costs of human power, equipment, consumables and other costs such as support for conferences and publications of research outputs.

For the purpose of illustration, Table 2 shows the research outputs of two funded projects, during the period, that were helmed by the first and second authors of this chapter, respectively.

It is apparent from Table 2 that the research outputs of projects that were funded were significant both in quantity and quality when compared to those in Table 1. There were four papers by each of the projects published in tier 1 (Q1) MER journals. In addition, there were also more scholarly contributions in terms of books and chapters. This is certainly a consequence of both the projects:

- Student Perspective on Effective Mathematics Pedagogy: Stimulated Recall Approach Study [The Learner's Perspective Study in Singapore] (CRPP 3/04 BK) and
- Mathematical Problem Solving for Everyone (MPROSE) (OER 32/08 TTL)

being funded appropriately for NIE faculty to engage in research as well as recruitment of research assistants to assist in the research. Capacity building was also apparent as researchers like the first two authors of this chapter worked with teams of colleagues from NIE as well as international peers in the projects. As expected, Wong (2017) noted the academic output of MER projects that were funded during the period 2008–2017 had served the goals of raising the profile of NIE as an influential teacher education institute and to advance the career of NIE scholars.

Table 2 Publications from two MER funded projects at the NIE

Research output	Student perspective on effective mathematics pedagogy: stimulated recall approach study (the learner's perspective study in Singapore) (CRPP 3/04 BK)	Mathematical problem solving for everyone (MPROSE) (OER 32/08 TTL)
Research reports	1 Kaur and Loh (2009)	–
Keynote addresses	3 Kaur (2007a, 2016, 2018)	3 Toh (2013, 2014, 2018)
Invited presentations	1 Kaur (2009a)	–
Professional books	–	2 Toh et al. (2011a, 2011b, 2011c), Leong et al. (2013)
Scholarly books (edited)	2 Shimizu et al. (2010a), Kaur et al. (2013)	–
Scholarly book chapters	11 Kaur and Toh (2019), Leong et al. (2019), Anthony et al. (2013), Mok et al. (2013), Kaur (2010, 2013, 2014), Shimizu et al. (2010b), Kaur et al. (2006a), Mok and Kaur (2006), Seah et al. (2006)	3 Toh et al. (2008, 2011c, 2019)
Conference papers (published in proceedings)	5 Kaur (2007b, 2008a, 2017), Kaur et al. (2005, 2006b)	17 Dindyal et al. (2009, 2010, 2013, 2014); Ho et al. (2013); Leong et al. (2010); Quek et al. (2010, 2012, 2014); Tay et al. (2007); Toh, P. C. et al. (2012, 2014b); Toh (2012); Toh et al. (2009, 2011b, 2012, 2013a)

(continued)

Table 2 (continued)

Research output	Student perspective on effective mathematics pedagogy: stimulated recall approach study (the learner's perspective study in Singapore) (CRPP 3/04 BK)	Mathematical problem solving for everyone (MPROSE) (OER 32/08 TTL)
Journal papers (refereed)	4 Kaur (2008b, 2009b, 2011)[a], Shimizu and Kaur (2013)[a]	13 Deng et al. (2015); Dindyal et al. (2012); Leong et al. (in-press, 2011a, 2011b, 2012)[a]; Liang and Toh (2018); Quek et al. (2011); Tay et al. (2011); Toh, P.C. et al. (2014a)[a]; Toh et al. (2013b, 2014); Yong and Toh (2019)

[a] Published in Tier 1 MER Journal

4.2 Push for Team-Based and Collaborative Research Activities in Mathematics Education Research

The 14th call for research proposals by the OER at NIE, in May 2015, encouraged programmatic research, where programmatic research is defined by an overarching project research theme which focuses on a key educational issue, problem, phenomena or outcome, along with a number of themes, or specific research studies that address important aspects or components of the issue, problem, phenomena or outcome. It therefore has a common strand or focus, supported by a common theoretical framework, and undertakes a coherent, comprehensive, multifaceted approach to understanding and addressing the issue, problem, phenomena or outcome.

The first programmatic research project award at the NIE was granted for a MER project, the Enactment Project, helmed by all the three authors of this chapter and involved a team of nine researchers, ranging from a teaching fellow to a full professor. The project: *A study of the enacted school mathematics curriculum in Singapore secondary schools* had two aims (Kaur et al., 2018). The first was to document how experienced and competent teachers enacted the school mathematics curriculum in secondary schools. It did this by examining: (i) pedagogies adopted by experienced and competent mathematics teachers when enacting the curriculum and (ii) experienced and competent teachers' use of instructional materials for the enactment of the curriculum. The second was to establish how uniform these adopted pedagogies and use of instructional materials by experienced and competent teachers were practised in the mathematics classrooms of Singapore schools.

The four-year-long project also contributed significantly towards the capacity building of younger faculty at NIE in MER. This was through the collaborative research activities of the project, from collecting data using sophisticated methods like the complementary accounts methodology (Clarke, 1998) and disseminating the research through academic papers for conferences, books and journals.

Presently, a group of mathematicians and mathematics educators at the NIE are involved in a study: Big Ideas in School Mathematics. All the three authors of this chapter are also helming the study. The study comprises three sub-studies, as shown in Fig. 1. The team works together but for sub-study 1 the mathematicians take the lead, while for sub-studies 2 and 3 the mathematics educators do the same. The project has been funded for four years (from July 2020 till June 2024) and 13 colleagues are involved with ten from the NIE, two from the Ministry of Education and a lead teacher from a secondary school. Members of the team range from experts to novices in MER.

4.3 The Purpose of Mathematics Education Research

Toh (2020) reviewed MER at the NIE since 2000. He drew on two sources of MER, namely the funded research projects and research carried out by postgraduate master

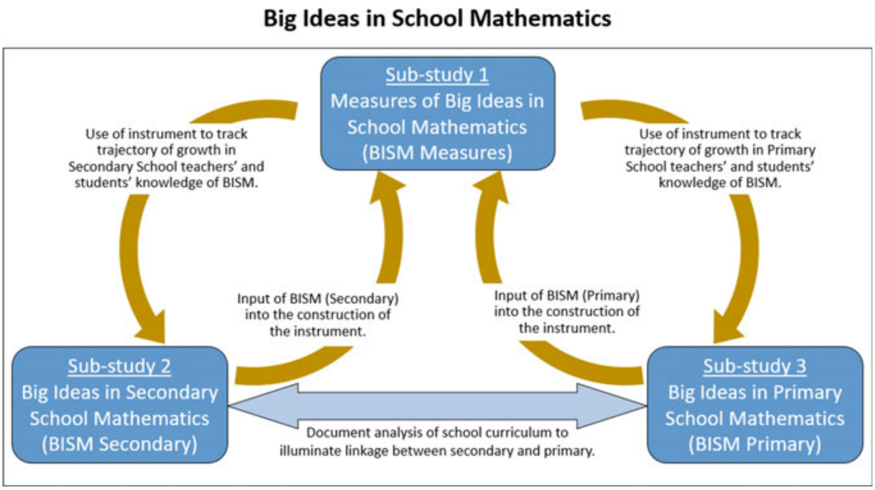

Fig. 1 Conceptual framework of the BISM study

and doctoral students. He classified the MER according to four main categories that depicted the purpose of the research. The categories were as follows:

Policy and curriculum: This category of research involves the study of how policy and curriculum impact the teaching and learning of mathematics. It is exemplified by a research project entitled *Secondary Analyses of Teacher Education and Development Study in Mathematics* which was conducted between 2014 and 2017. This project gathered information about mathematics teacher education in Singapore. It was part of a larger international comparative study, the TEDS-M project. In particular, the findings of this project offered insight into local curriculum and policy matters.

Professional development: This category of research involves the development of teachers through activities practising teachers engage with. It is exemplified by a research project entitled *Enhancing the Pedagogy of Mathematics Teachers to Facilitate the Development of 21st Century Competencies in their Classrooms* that was conducted during the period 2014 to 2017. The project's primarily goal was to build practising teachers' capacity for effective teaching in school classrooms. It proposed long-term collaboration between researchers and the practising teachers. This mode of teacher professional development deviates from the usual workshop mode that is guided by a deficit perspective of professional development.

Teaching and learning practices: This category of research primarily begins with studying the classroom enactment as the primary aim. This category of projects is exemplified by a mathematics education research project entitled *MAthematics is Great: I Can And Like (MAGICAL)*. This project developed an alternative approach to teach Lower Secondary Normal (Technical) mathematics using storytelling, comics, and other graphic stimuli in context. It also examined the effect of the approach on students' mathematical self-concept, motivation to learn mathematics and achievement in mathematics.

Judging from the description, the primary objective of the project was to study the enactment in the mathematics classroom using the researchers' new approach of using comics and storytelling for the low-ability students. The projects which were classified under the category "Teaching and Learning Practices" included those that introduced novel intervention strategies in classroom practice and a study of the impact of these interventions on student learning.

Theory building: Theory building can be explained as "the purposeful process or recurring cycle by which coherent descriptions, explanations, and representations of observed or experienced phenomena are generated, verified and refined" (Lynham, 2000, p. 161). This category of research covers those in which the researchers attempted to develop the "theory" underlying a set of phenomena in education. An exemplar of this category of project was one which was entitled "Portraits of Teacher Noticing during Orchestration of Learning Experiences in the Mathematics Classrooms". The description of the project is shown below:

> This project has two main goals. First, it involves developing a local theory to describe, and prescribe, what and how exemplary teachers notice when they orchestrate Learning Experiences in their classrooms. Second, it is aimed at designing a toolkit that can be used by teachers to promote students' thinking through high-quality Learning Experiences.

When classifying the research projects or research carried out by postgraduate students at times when they fell in more than one category, the classification was based on the original intention of the researchers as provided in the brief description of the research projects, or the abstract of each of the postgraduate doctorate or master thesis (see Toh (2020) for more details). Table 3 shows the spread of MER, since 2000, across the four categories and their source.

It is apparent from Table 3 that under the category "funded mathematics education research projects", the most frequent category of research is "Classroom Teaching and Learning Practices" (37.7%), while the least frequent is "Theory Building"

Table 3 Classification of MER conducted in Singapore since 2000

		Dissertations and thesis	
Source category	Funded projects	Doctoral level	Master level
Policy and curriculum	12 (26.7%)	2 (8.8%)	13 (12.5%)
Professional development	15 (33.3%)	1 (4.4%)	1 (1.0%)
Teaching and learning Practices	17 (37.7%)	5 (21.6%)	38 (36.5%)
Theory building	1 (2.3%)	15 (65.2%)	52 (50.0%)
Total	45	23	104

(2.3%). On the other hand, the most frequent category of research for postgraduate research (including both master and doctorate level studies) level is on "Theory Building" (65.2% for doctorate and 50% for master level studies), while the least common category of education research is "Professional Development" (4.4% for doctorate and 1.0% for postgraduate research).

The above data suggests that different types of mathematics education research serve different functions. Undeniably, the priority of postgraduate studies is to engage candidates to go through the process of research, out of which theory building is an essential component. Thus, significantly many research projects from postgraduate studies have their primary goal as contributing towards the building of theory about specific aspects of mathematics education. This explains the relatively high number of research projects on theory building for the postgraduate research. On the other hand, the funded education research projects conducted in NIE are geared towards improving teachers' effectiveness (Teacher Professional Development, 33.3%) for a more efficient enactment of the curriculum in the authentic classroom (Teaching and Learning Practice, 37.7%). This is in line with the goal of the research funding from the OER at NIE.

5 Singapore's Role in Developing Mathematics Education Research in ASEAN Countries

There are three distinct ways through which MER in Singapore has contributed towards the development of the same in ASEAN countries. The first is similar to how Singapore drew on expertise elsewhere in the period 1991–2003 to develop its own faculty in MER. Since 2005, several students from ASEAN countries have completed their Philosophy of Doctor degrees in Mathematics Education at the NIE. Among them, one from the Philippines is presently a faculty of Ateneo de Manila University in her country and another from Thailand is also a staff of the Institute for the Promotion of Science and Technology in her country. Both students were on scholarships from the respective institutions to do their studies at NIE. Numerous postgraduate students have also come for shorter stints of research attachments with MER scholars at the NIE as part of their PhD studies in their home institutions. Two such students were from the Universiti Pendidikan Indonesia in Indonesia and the Mahidol University in Thailand.

The second and third means of contribution have been through conferences and publications. International and national mathematics education conferences are held periodically at the NIE. The Mathematics and Mathematics Education (MME) Academic Group at the NIE where MER resides has hosted numerous high-profile international conferences since 2012. The first was the second East Asian Regional Conference on Mathematics Education held in May 2002. Next was the Mathematics Education in Research of Australasia (MERGA) 2012 conference held in July 2012 followed by the International Psychology of Mathematics Education Conference in

July 2017 and the MERGA 2021 conference in July 2021. These conferences have in some sense allowed easy excess for MER scholars from the region to engage in rich and current matters in their field of expertise. Since 2005, the Association of Mathematics Educators and the MME Academic Group have jointly organised the Mathematics Teachers Conference. By way of invitations, numerous scholars from the region have also presented their work to teachers from Singapore and elsewhere. These presentations are published in a thematic Yearbook of the Association (see, Kaur et al., 2009b—the first book and Toh and Choy (2021) for the most recent one). This publication has also contributed towards the development of teachers and research scholars in the region and elsewhere.

In 2013, the first author of this chapter together with Catherine Vistro-Yu from the Ateneo de Manila University in the Philippines initiated the Springer Book Series entitled: Mathematics Education—An Asian Perspective. The goal of the series is to facilitate the publication of research in Asia that is often under represented in the international landscape. To date, several volumes have been successfully published by Springer. There are also other ad hoc ways through which MER in Singapore aka NIE continues to contribute in the region and elsewhere. These mainly arise out of the Memorandum of Understanding (MOU) the NIE has with partnering institutions; for example, there is an MOU on Innovations and Teaching and Learning of STEM (Science, Technology, Engineering and Mathematics) with Design Thinking between Singapore and the Philippines (see, https://www.philippine-embassy.org.sg/about-us-2/overview-of-philippines-singapore-relations/).

6 Concluding Remarks

It is apparent from the chronological review of MER in Singapore that from the early 1990s till the present, MER has gradually evolved from one-off small-scale studies on aspects of teaching and learning mathematics to programmatic cum team-based ones that have direct impact on the teaching and learning of mathematics in schools. This evolution has been brought about by firstly research being a necessary aspect of university education at the NIE and also the need for Educational Research and in turn MER by the Ministry of Education in Singapore.

Both top-down and bottom-up initiatives have contributed towards the development of MER. During its infancy, individuals participated in MER either to evaluate some aspects of their teaching, use of resources or simply for interest. They did this alongside their full teaching loads and without any form of research support. But when NIE was established as an institute of the Nanyang Technological University, a significant top-down push was the need to engage in research and create research outputs that were at par with international standards. In reaction to the need to do so, bottom-up initiatives such as networking with international scholars and the creation of centres of research at NIE that funded research in line with the needs of the nation strongly supported MER from then on.

It is apparent from the illustrations in Tables 1 and 2, how research funding impacted the research outputs of mathematics educators at the NIE from 1991 till the present. As would be expected, the trajectory of MER growth has now moved into studies comprising several phases and multiple goals. These studies are engaging in rigorous and purposeful MER that contributes towards local needs and also international MER literature and capacity building of researchers. This is evident in Sect. 4 of the chapter that showcases the Enactment Project and the BISM study. It may be said that MER at the NIE and in turn in Singapore has created a systematic approach of engaging in research with mentors that facilitates rigorous and high yield research outputs. Lastly, it is also apparent that MER in Singapore has contributed towards the development of the same in ASEAN countries and elsewhere. It has done so through their graduate education at the NIE, by hosting international and national conferences and through publications that has not only showcase work at the MIE but also facilitate the publication of works by colleagues in ASEAN countries and Asia.

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Reviews of Research in Mathematics Education in Different Economies

Critical Analysis of Mathematics Education Doctoral Dissertations in the Philippines: 2009–2021

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Abstract This chapter presents a critical analysis of educational research as reflected in doctoral dissertations completed during the past decade in mathematics education in the Philippines. It analyzes the published dissertations with respect to the topics in the discipline they address, the targeted participants in terms of educational level and roles of stakeholders and theoretical frameworks used to construct the research. The analysis revealed a strong emphasis on research on teaching educational goals of mathematics, teacher development, and the use of technology in teaching. The less traversed areas were the research on social goals of education, the relationship of mathematics teaching and learning in other disciplines, the equity in terms of language and access to quality mathematics education due to poverty, elementary and kindergarten mathematics, assessment, technology used in informal settings, and research areas that are informed by critical and sociopolitical perspectives. The analysis calls for the need for better articulation of the role of the theory in a dissertation and a greater diversification of theoretical stances, in particular the critical and sociopolitical perspectives that remain underutilized.

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During the past fifteen years, in several developing countries, some included in this book, government authorities responsible for higher education, and the higher education institutions themselves embarked on developing policies and agendas promoting their countries' research productivity. In many cases, such policies included increasing the numbers of faculty with doctoral qualifications and/or expanding the doctoral programs in general.

This is not to say that research activities were totally new in many of the developing countries. Some faculty members within the more prestigious universities in each country may have had the opportunity to conduct research as part of their normal duties. Further, selected faculty members from these countries were the beneficiary of international scholarships to pursue higher degrees abroad. However, only a few of those returning to their institutions had the opportunities to conduct research due to limited opportunities (e.g., funding and heavy teaching and administration loads) and expectations of their roles.

Perhaps expectedly, the early attempts for research in developing countries mirrored research conducted in developed countries (Atweh et al. 2003). However, less common are research studies that critically reflect on the research conducted in these countries in terms of their characteristics, scope, and quality. We hope that such reflections might be conducted in the future once the culture of research is well established in the respective countries. This chapter is such an attempt for a critical reflection on educational research at doctoral level in the Philippines conducted during the past ten years, and in one specific area of mathematics education. It aims at analyzing the published dissertations with respect to the topics in the discipline they address, the targeted participants in terms of educational level and roles of stakeholders and theoretical frameworks used to construct this research. The expected benefit from such an analysis is to identify patterns that are common across many of the universities; identify possible quality issues with this research, particularly, with respect to their theoretical formulation, and, importantly, to identify possible silences in that research toward increasing its diversification.

First, we discuss the context of research development in the Philippines during the last decade and the methods we used in conducting this analysis.

1 Educational Research Development in the Philippines

In the Philippine Development Plan (National Economic & Development Authority, 2017), the national government identified the vital role of the human resources in achieving the goal of economic development as reflected in its commitment to improving education at all levels. To empower the citizens to contribute in a globally competitive knowledge economy, the Government realized the need to sustain its effort to institutionalize reforms in education nationwide, including reforms in teacher education (Atweh et al. 2007). The broad agenda for reforms in the country's educational system dates back to 1992 when three governing bodies were created to manage the different educational sectors: (1) The Department of Education managing basic education, (2) Technical Education and Skills Development Authority managing technical and vocational education, and (3) the Commission on Higher Education (CHED) managing the tertiary and graduate education. Since then, the policies, standards, and guidelines on implementations of programs and policies in higher education are disseminated through the issuance of CHED memoranda.

The earliest known CHED Memorandum on the Doctor of Philosophy (PhD) programs was released in 1998 to provide the policies and standards on graduate education. It affirmed the lead role the graduate education should take to stir the Philippine higher education "toward globalization and world-class scholarship." (Commission on Higher Education, 1997, Sect. 1, no. 3, p. 1). It stipulated the standards in graduate education on research requiring the writing of dissertations. It was only in 2007 that CHED spelled out in detail the expectations in research undertakings for Doctor of Philosophy (PhD) courses in contrast to Doctor of Education (EdD) courses. The CHED memorandum specified that while expertise in theoretical knowledge in an area of specialization is expected in both programs, PhD dissertations are expected to "pose theoretically meaningful programs and hypotheses, gather and analyze data, and communicate the results and theoretical and practical implications of the research to diverse audience." (Commission on Higher Education, 2007). On the other hand, the EdD dissertations should "develop specific complex educational programs and systems to evaluate such complex programs and systems" (p. 4) and disseminate the theoretical and practical implications to various sectors of the academic community. All the dissertations reviewed in this chapter are for Doctor of Philosophy in mathematics education.

The doctorate program in mathematics education has been offered by some private universities since the early 1980s. Among these are the Centro Escolar University and the De La Salle University in Manila. Currently, there are a total of 22 universities across the country that confer different doctorate degrees in mathematics education under different designations: PhD in Mathematics Education, PhD in Science Education with major in Mathematics, Doctor of Education (EdD) in Mathematics, PhD in Education (Mathematics), PhD in Education with specialization in Mathematics Education, and PhD in Mathematical Science with major in Mathematics Education. These programs in essence have similar structures, courses, and types of research but may vary in terms of rigor of requirements. The offering of doctoral programs in the Philippines requires CHED approval which opens the opportunity for scholarships awarded to students and faculty for those programs.

Doctorate programs in mathematics education follow the curriculum prescribed in the CHED Memorandum No. 53 s. 2007 on "Policies and Standards for Graduate Programs in Education for Teachers and Other Education Professionals." The memorandum identifies the following:

1. Demonstration of highly advanced systematic knowledge and skills in highly specialized and/or complex interdisciplinary or multidisciplinary field of learning;

2. Utilization of complex research/creative work and/or professional practice and/or the advancement of learning with full independence in individual work and/or in teams of interdisciplinary or multidisciplinary setting and,
3. Application of significant level of expertise-based autonomy and accountability to professional leadership for innovation, research and/or development management in highly specialized or interdisciplinary or multidisciplinary field.

These programs consist of a total of 60 units; 18 units are dedicated to *core* or *foundation* courses, 21–24 units to *major courses*, 6–9 units to *cognates or electives,* and 12 units to *dissertation.* It should be noted that coursework constitute a large part of doctoral degrees in the country, devoting only 20% of the curriculum for conducting the research and writing the dissertation. Typically, students have one to two years to finish their research and dissertation writing, with the provision that drafts of the proposals may have been developed in some of the coursework. Many of these candidates are non-scholarship holders and, hence, have full-time employment. However, there is another point that characterizes doctoral programs in the Philippines different from some other countries. Major courses specified at a doctoral level include mathematics content as well as mathematics education courses, under *major courses* mentioned above. In many cases, the number of mathematics content courses exceeds those in mathematics education. Further, they may include research methods courses. This raises a concern that the graduates' knowledge of the wide range of research areas and theories in mathematics education may not be sufficiently wide to cover many of the research areas and theories utilized internationally in the field. There is a wide concern among teacher educators, and graduate students themselves, about the need for longer time to prepare and conduct quality research in the programs offered. The relatively short time available for conducting or writing of research and dissertation may be a challenge in light of the CHED expectation to promote research or creative work that demonstrate originality, critical thinking, problem-solving skills and leadership in research and practice of profession and for that research to be globally competitive and locally relevant (Commission on Higher Education, 2019).

It is worthwhile to mention in this context that recent developments in doctoral programs in the Philippines include offering of a three-year Doctor of Philosophy (PhD) by research, in which seventy five percent (75%) of the total units of the program is focused on research-dissertation work. The development and writing of the dissertation should be undertaken in the remaining twenty five percent (25%) of the program. Another novice feature of these PhD programs is that graduates should have a "publication or evidence of acceptance to an internationally or nationally indexed journal or juried creative work outlet" as a requirement for graduation (Commission on Higher Education, 2019). Due to their recency, no thesis reviewed here is from these programs.

The last relevant changes in the education system in the Philippines that impacted the expansion of doctoral degrees in mathematics education are the recent changes to the basic education curriculum. In year 2012, the Philippine government embarked on a new K-12 curriculum that added two years to its basic education which implied reduction in numbers of enrolling students at higher education for a few years starting at 2016. As part of the effort to manage the oversupply of teaching faculty in higher education, the governments, and the institutions themselves, encouraged faculty members who do not hold doctoral degrees, to undertake doctoral degrees through scholarships Commission on Higher Education (2016d).

1.1 Processes Adopted in this Analysis

To gather the relevant dissertations for the conduct of the analysis in this chapter, e-mail messages were sent to the identified officials of higher education institutions in the Philippines offering doctoral programs in mathematics education, requesting electronic copies of the completed doctoral dissertations during the past decade. Additionally, the social media including personal accounts of educators known to the authors and professional lists was used to request such dissertations. Personal and public requests were necessary since not all dissertations held by some institutions were electronically available—neither as abstracts nor as full dissertations. A total of fifty dissertations were submitted for analysis. However, an examination of the titles and the abstracts yielded only 36 dissertations in mathematics education completed for the last twelve years. While it is not possible to determine if this is the total collection of doctoral dissertations in mathematics education offered in the period of study, we are confident that it is a highly representative sample of those that were successfully defended in the set period. The dissertations came from a total of eight universities, five public and three private universities. Twelve theses (33%) were offered by public universities and twenty-four (66%) came by public universities. Similarly, twenty-eight (78%) of these dissertations came from institutions from national capital region and only eight (22%) were completed by graduates of provincial universities. It should be noted, however, that metropolitan universities attract a significant number of doctoral candidates from the provinces.

In terms of the process used in the analysis, the authors as a group identified the main dimensions for the analysis based on similar reviews from international literature, and also based on their own experiences and interests as supervisors of postgraduate students. Several electronic meetings were held among the authors to discuss their preliminary analysis and to re-structure the analysis when needed. While, more categories for the analysis were used than represented in this chapter, in this context we will only discuss three of the main ones: the topic area of the dissertations, the targeted participants and sites for the research, and the theoretical framing of the research.

2 Findings

2.1 Research Areas Investigated

The first criterion for the analysis adopted here is looking for patterns and absences in the research areas considered by the various doctoral students. While there are different ways to classify research in terms of topic/areas of research, here we adopted the eight themes utilized by Bakker et al. (2021). We note that any classification of research is somewhat problematic since it is always possible to find research that covers more than one area. In other words, these themes are not to be taken as mutually exclusive. For example, one particular dissertation in our collection (#31) dealt with realistic mathematics approach, mathematical communication, problem-solving skills, and high-functioning autistic children, which was classified under approaches to teaching, educational goals, and relation of mathematics to other disciplines, in this case, special education. Nevertheless, it is useful to investigate overall patterns of research areas undertaken/neglected in a particular time by a particular group of researchers for informing future research agendas.

In the first stage of analysis, we used the themes by Bakker and colleagues to tally the number of dissertations addressing each theme. The results are given in Table 1. Further sections below discuss the dissertations under each specific theme in some detail.

A few comments are needed to further clarify our classification. First, subsumed in Approaches to Teaching are two subclassifications: Teaching Strategies and Curriculum. As explained by Bakker et al., (2021, p. 6), "any challenges around developing a coherent mathematics curriculum, smoothing transitions to higher

Table 1 Dissertations' research area themes

Themes	Frequency
1. Approaches to teaching	19 (53%)
a. Teaching strategies (only)	10 (28%)
b. Curriculum (only)	4 (11%)
c. Both	5 (14%)
2. Goals of mathematics education	14 (39%)
a. Educational goals	14 (39%)
b. Societal goals	0 (0%)
3. Affect	11 (31%)
4. Assessment	3 (8%)
5. Relation of math to other disciplines	1 (3%)
6. Teacher professional development	13 (36%)
7. Technology	10 (28%)
8. Equity, diversity, and inclusivity	6 (17%)

school levels, balancing topics, typical overload of topics, the influence of assessment on what is taught, and what teachers can teach" are considered research areas in curriculum. This includes research focus covering instructional materials and resources. To avoid counting a dissertation more than once in the subcategories under approaches to teaching, we delineate each into only one of the following: those that focus only on strategies, those that focus on curriculum only, and those that focus on both. Second, the category of Goals of Mathematics Education is further classified as educational and societal as indicators of which educational outcomes seem a priority focus of the research. Societal goals pertaining to learning "to function in the economy and in society more broadly" such as "goals related to students developing as a human being" and "learning to see mathematics in the world and develop a relation with the world," whereas educational goals include mathematical literacy, numeracy, critical, and creative thinking (Bakker et al., 2021). Examples of mathematical educational goals frequently mentioned in Bakker et al. were achievement, statistical literacy, computational and algorithmic thinking, reasoning, argumentation, and proof, artificial intelligence, modeling, and data science.

It is possible to make a few overall observations about the patterns represented in Table 1. The research areas of the dissertations reviewed tend to lean toward the application of teaching approaches and looking at their effects on students' academic achievement and attitudes. The high percentage of dissertations in this area may be explained by several possibilities. Research agendas from the Department of Education, and some of the universities themselves, often highlight research on teaching and learning, and related problems that students face and how to improve student outcomes, perhaps at the expense of research that may focus on other aspects of education such as social factors and issues of equity. Further, the relatively low number of studies in the area of technology and its application may be due to the relative low number of schools with access to sufficient technology for educational use. We expect that as one of the effects of the current pandemic, this pattern would possibly be reversed since the pandemic necessitated the use of telecommunication in teaching and learning. Here, we note that a high percentage of research proposals in development by current doctoral candidates relate to distant teaching and in particular online teaching. Lastly, of interest is the low number of dissertations on the relationships of mathematics to other disciplines.

2.1.1 Approaches to Teaching

Teaching Strategies

There is a variety of theories and models used by the different dissertations as approaches to teaching, including van Hiele levels of geometric thinking (#7), Local Instruction Theory (#13), explicit mathematics instruction with a focus on mathematical thinking, 5Es instructional model (#25), collaborative learning (#12, #24), and repetition with complex variations (#26). Likewise, there are dissertations that aimed at developing approaches to teaching problem solving and problem posing. These include teaching students to use keywords and visual representations in solving probability problems (#32), gradual release of assistance instruction (#3a), problem-based learning (#24), and problem posing activities (#12, #24).

Several of the dissertations examined, employed, and tested the effectiveness of a variety of teaching strategies. Doctoral research is necessarily limited in time and often done in semi-controlled environments. Considering the short time provided in Filipino doctoral programs for research and dissertation writing, interventions of a few weeks up to a maximum of three months are common in the dissertation reviewed. Further, research interventions are always subject to the Hawthorn effect (McCambridge et al., 2014). It may be interesting to see how much of these interventions are actually effective in long-term implementations, if they remain being practiced, maintained, sustained, and better yet, improved in the settings for which they were developed. Similarly, a study may need to be conducted to document the various strategies Filipino mathematics teachers employ in their teaching in a non-obtrusive natural setting to see which of these practices were deliberately adopted from research studies and which ones were used to address a pressing need. Further, there is no dissertation that we could identify that looked in-depth at the process of student thinking and interactions while being involved in these different teaching strategies. More common were studies that looked at the effectiveness of such methods in achieving certain outcomes. We will return to this observation in the final section of this chapter.

Curriculum

To address the dearth of quality instructional resources in many Filipino schools, a number of dissertations focused on the design, development, and implementation of relevant materials for classroom use. Sources for such materials came from a variety of theories and models of teaching suggested in the literature. These include ethnomathematically enriched learning materials (#5), reflective teaching strategy in a flipped classroom using the QE7Cs learning model (#3b), culturally sensitive lessons in modular approach to teaching (#6), reflection embedded instructional materials (#14), and metacognitive activities (#16). One of these dissertations developed a course material in algebra in an online environment: Blackboard Mobile Learn (#1). A single dissertation targeted one characteristic of the K-12 curriculum adopted in the Philippines, namely the spiral curriculum. This dissertation examined the fidelity of the actual implementation by teachers of this principle in mathematics teaching (#35). On the basis of the results, recommendations and policies were proposed.

While the majority of dissertations dealt with curriculum materials for formal education settings, it is noteworthy to mention a single dissertation conducted on the Alternative Learning System (ALS) curriculum (#34) for adult learners who failed to attend and finish their formal basic education due to various reasons such as non-existence of schools in the community, dropping out of school due to poor performance disciplinary actions, or financial difficulties.

2.1.2 Goals of Mathematics Education

The previous section considered the analysis of the dissertations in terms of their teaching or curriculum focus in designing their study. In the next two sections, we will present the analysis based on the intended learning outcomes targeted by the different dissertations. Roughly speaking, the above section looked at the independent variables used, and these two sections will focus on the dependent variables. In this context, we use these terms, i.e., dependent and independent variables, in a general sense without implying that quantitative methods were used to investigate them.

Educational Goals

Some of the reviewed studies evaluated the effectiveness of a certain teaching approach intervention on measures of learning such as students' conceptual understanding (#3b), mathematics achievement (#16, #24, #27, #29) and performance (#25). Others are more targeted toward the twin goals of the K to 12 Philippine Mathematics Education which are critical thinking (#13, #25, #16, #24, #27) and problem solving (#3a, #32) (Department of Education, 2016). Some studies are more specific on developing certain aspects of these skills such as students' commognition and creativity (#24, #26), and proving and reasoning skills (#30). All the studies mentioned here have predominantly targeted students' academic goals in the cognitive learning domain. Perhaps there are two exceptions, where one looked at students' patience and persistence in solving problems (#26), and the other discussed student engagement (#27).

Although tacitly, some dissertations used certain approaches targeting students' appreciation of the usefulness, nature, power, and beauty of mathematics—e.g., using the realistic mathematics approach (#31) and culturally sensitive lessons (#6). Some of the constructs not considered by the reviewed dissertations include the development of students' mathematical thinking skills vis-a-vis transversal skills (at times called soft skills), students' logical thinking, reasoning, motivation to study mathematics, creativity, attitudes toward pursuing a mathematics related career, among others. Likewise, contemporary constructs such as agency and identity are not yet common in mathematics education dissertations in the Philippines.

Societal Goals

When students realize the various applications of mathematics in sciences, business, and engineering, they get to see the importance of learning the subject and of equipping themselves with the necessary skills they could acquire in a mathematics classroom to function in the economy and the society. Societal goals pertain to social empowerment through mathematics contributing to critical citizenship. Many if not most of the policies in the government and industry are data driven and involve the use of mathematics in its analysis. Ernest (2015) explained it best by saying,

> Mathematics should be taught so as to socially and politically empower students as citizens in society. It should enable learners to function as numerate critical citizens, able to use their knowledge in social and political realms of activity, for the betterment of both their own selves and for democratic society as a whole. This involves critically understanding the uses of mathematics in society: to identify, interpret, evaluate and critique the mathematics embedded in social, commercial and political systems and claims, from advertisements, such as in the financial sector, to government and interest-group pronouncements. Every citizen needs to understand the limits of validity of such uses of mathematics, what decisions it may conceal, and where necessary reject spurious or misleading claims. (p. 191)

None of the dissertations reviewed focused on such identification of social goals in mathematics education. This is a big gap in research in the Philippines. Future research studies are enjoined to aim at developing societal goals in mathematics education addressing social problems, understanding of society, encourage active, responsible, and morally upright national and global citizenship. Mathematics can be one of the disciplines that can help address such issues as justice, equity, and environment stewardship, and our students are envisioned to be productive members of the society and future leaders who can contribute to community development and the society at large.

2.1.3 Affective Factors in Mathematics Education

Arguably, next to academic achievement, affective factors were a very common dependent variable considered by many of the dissertations reviewed. As argued by McLeod (1992), integrating affective issues in studies on cognition and instruction help strengthen and refine some of the theories in teaching and learning mathematics. Some of these affects are attitude in problem solving (#3a), students' (#7) and teachers' (#19) epistemological beliefs in mathematics, self-efficacy (#16), motivation (#18), affective instructional strategies and burnout model (#22), non-cognitive factors of success such as attitude toward research in mathematics education (#23), persistence in learning mathematics (#25, #26), and perception toward the 5E instructional model (#25). The approaches in the mentioned dissertations were characterized as traditional in terms of research paradigm as these were predominantly measured quantitatively using questionnaires with self-report responses. Some affects such as outstanding teachers' personal characteristics and beliefs (#8) and students' confidence in problem solving (#3a) were appropriately studied using the interpretative research paradigm through observations and interviews. In this regard, we note that the traditional constructs of attitude and motivation permeated the dissertations under considerations—in contrast to the construct of engagement which is underutilized. While the two constructs are related, the focus of attitudes is on the emotional reaction to the learning experience, while engagement is the actual level of effort and activity that students demonstrate in their study (Finn & Zimmer, 2012).

2.1.4 Assessment

Several studies have made use of assessment tools to show the effects of proposed instructional intervention strategies on students' performance (#17, #25, #33), mathematics proficiency (#18), problem-solving skills (#34), problem posing skills (#12) using paper and pen tests, performance tasks and rubrics. However, the focus of such studies was not on the assessment, and assessment was just a means to determine the effectiveness of the teaching strategy. Nonetheless, there are three dissertations that used assessment as their focal point. These are on teachers' assessment of twenty-first century skills in mathematics (#2), online homework (#15), and structured note-taking (#29) as formative assessments. Future dissertations may venture in this direction especially that assessments may be problematic with the distance learning currently being held through flexible modes of delivery using printed self-learning modules (SLM), online learning, and DepEd TV (Department of Education, 2020). Perhaps a naturalistic study may be conducted to elucidate how assessments are being conducted in the Philippines during the remote teaching and flexible modes of learning in the pandemic period and how authentic these assessments are. Arguably, there is a common belief that teacher assessment techniques used in the country tend to be traditional with a heavy focus on exams. However, educational policies attempt to diversify classroom assessment by using performance tasks (OneNews, 2021). Here we note that very few of the dissertations reviewed focus on these non-exam assessment methods. Arguably, as the result of the migration of courses online during the pandemic, non-exam assessment may have become more common. Perhaps this may be reflected in the upcoming research in mathematics education in the next few years.

2.1.5 Relation of Mathematics to Other Disciplines in Education

The review by Williams et al. (2016) showed interdisciplinarity in mathematics education a relatively underdeveloped research subfield. They further explained that progress in this field of research is hampered due to the vagueness and lack of consensus about disciplinarity concepts, the lack of interdisciplinarity interventions and programs, and the lack of depth and breadth of research in this field.

The only attempt to relate Mathematics to other disciplines dealt with special education. This study had high-functioning autistic children (#31) as participants. The study implemented the Real Mathematics Education (RME) approach in 2 months using the discrete trial training to trace the effects on students' progress in their communication and problem-solving skills. Conducting dissertation relating mathematics to other disciplines may be considered as less traversed area in mathematics education research as regards dissertations and thus, may be a good research area to explore in the future.

Future studies may also include the teaching of mathematics to students who are more into the arts and design—e.g., origami, tiling, textile designs, Islamic art, and even sports. Since mathematics is a tool in many disciplines such as the sciences

(General Science, Biology, Chemistry, and Physics) and Business and Economics, it may be interesting to look at how aptly we have prepared our students for them to use mathematics and how we can train teachers to teach mathematics with the aim in mind of preparing students in these courses of their study.

2.1.6 Teacher's Knowledge and Development

While the studies above focused on student learning, there were a few studies that are related to teachers' learning and practice. One dissertation examined teachers' assessment practices of twenty-first century skills (#2); another dissertation examined readiness of pre-service teachers in teaching mathematics (#11); while two dissertations utilized Schulman's mathematics pedagogical-content-technological knowledge (#20, #28). Dissertation #20 particularly looked into Senior High School mathematics teachers' mathematics content and pedagogical knowledge through a test, analysis of lesson plans, classroom discourse observations, and interviews, while dissertation #28 provided a didactics of mathematics course prototype integrated in the teacher education curriculum in view of improving the mathematics pedagogical content knowledge (MPCK) of pre-service teachers.

Other teacher-related constructs studied were reflective thinking skills (#14, #17), epistemological beliefs (#19), heuristics, confidence, and attitude toward problem solving (#3a). In order to address the necessity of equipping students of the twenty-first century skills to meet the demand of the changing workplace and society, the dissertations "assessing teachers' knowledge and practices in assessing students' twenty-first century skills in mathematics" (#2) and "twenty-first century attributes of mathematics teachers" (#10) worked along this line.

A few recent dissertations conducted studies on community of inquiry among teachers to serve as support groups through lesson study (#9, #36) and professional learning communities called learning action cells by the Department of Education (#6, #36). Notwithstanding, in order to celebrate and recognize teachers' value and their contribution to education, a case study (#8) described the personal and professional characteristics, beliefs, and teaching practices of outstanding teachers.

Arguably, the focus on teachers' knowledge and development, to a certain degree, paralleled the focus on students' learning, reviewed above. In both cases, the traditional models of knowledge and its development permeated such research. This is not to say that such models are outdated, but to point out that contemporary theories of learning and society may still be unfamiliar to many doctoral students in mathematics education. These will be discussed further in the section of Theoretical Framing of the Research.

2.1.7 Technology

The dissertations reviewed were conducted prior to the pandemic. None of these have studied a fully online instruction delivery. Studies on blended learning using Pearson learning solutions software in a full assessment-driven environment (#4) and flipped classroom (#3b) indicated how online and face-to-face instruction delivery could be combined. Other studies suggested putting up a course site using Blackboard mobile learn (#1), giving online homework (#15), and using GeoGebra (#33) for concept building.

The unprecedented pandemic has changed the landscape of mathematics education. Remote or distance learning has become a recurring vocabulary. Classes have been shifted to the online mode, but this has posed challenges to a third world country such as the Philippines in terms of equal access to education as most are confronted with problems in infrastructures such as Internet connectivity and availability of resources. A lot of these teaching strategies had to be tweaked and tested for its viability in the remote learning context. As pointed out by Borba (2021),

> In normal times, such papers become old because digital technology changes so fast, and we rarely even have the time to implement a given technology in the classroom before a new one comes up. However, at this point, everything may become outdated, because we cannot predict the evolution of the COVID-19 crisis, nor whether a new crisis will follow it. (p. 1)

With the proliferation of computer applications, future directions may look into technology-related skills in mathematics teaching and learning. With regard to teacher educational goals, how could we prepare our future teachers and upskill practicing teachers to use technology more in teaching and in their preparation of instructional materials such as creating, editing and managing digital images, audio and video, social bookmarking, organizing and sharing web content, creating and editing photo caricature and cartoons, infographics and posters, conducting quick polls, and using technology in assessments (Bigari, 2019). With regard to basic educational goals, with the already jam-packed curriculum and the lack of infrastructures, could computational skills, robotics, ICT literacy, among others be integrated? Since this generation of students are digital natives, embedding digital technologies in mathematics education is preferable; future dissertations may consider including "mathematical digital competency" (MDC), a construct introduced by Geraniou and Jankvist (2019). We envisioned that in the near future, dissertations may be a collaborative work between mathematics education and computer studies which may include but not limited to game-based learning, computational thinking skills, logic and programming, and robotics. However, keeping in mind the challenges pointed by Borba, further research should also look at empowering teachers, and in turn students, to challenge their traditional views about teaching and learning to deal with the deep implications of the effective use of technology in the future of education.

2.1.8 Equity, Diversity, and Inclusivity

Dissertations under this theme tended to focus on how culture could be integrated into the instruction for a more relevant learning to all students. Only three such dissertations were identified: ethnomathematically enriched learning materials (#5), culturally sensitive lessons in a multicultural locale (#6), and the use of cultural historical activity theory in mathematics (#27).

One dissertation made an effort to establish a critical understanding of the role of social factors in influencing mathematics proficiency and motivation (#18). The social factors in the study included classroom cultural climate that touched on gender equity. The studies on using realistic mathematics approach, communication skills, and problem-solving skills among high-functioning autistic children (#31), and the analysis of mathematical problem-solving skills among adult learners in alternative learning system (#34) are the initial attempts to address diversity and inclusivity in mathematics education. Future studies may seek to answer questions such as “How can teaching in remote areas where a community school of very few teachers and each teacher handles different grade levels be able to optimize learning” and “How are issues of equity, diversity, and inclusivity being addressed in online teaching especially to those in far flung rural areas where internet access is not possible.”

Since instructional materials in mathematics in the Philippines are in English, the issue of language diversity in education in the Philippines may not be as apparent as compared to its neighboring melting pot countries such as Singapore. Perhaps surprising that none of the dissertations reviewed here have dealt with issues related to language and mathematics education. Hence, future studies may examine the role of language in mathematics teaching and learning. For example, was the use of the mother tongue in the early years of education been helpful in pupils’ numeral literacy and skills? Could a bilingual instruction be considered as a transition from the use of the mother tongue to the English only medium of instruction in teaching and learning mathematics? What are the effective strategies to teach students comprehend worded problems? How proficient are our students in understanding and using mathematical terminologies, symbols, and notations in communicating mathematical ideas? How do teachers use code switching in their teaching and to what effect? With the use of the content and language integrated learning (CLIL) methodology, subject content teachers are no longer viewed as teachers teaching their subject content but may also be teachers teaching students the English language through introducing relevant vocabulary and functional language related to mathematics for better comprehension and critical thinking (Darn, n.d.). This may entail teacher training on CLIL and collaborations between English language teachers and mathematics teachers. Dissertations along this line are strongly encouraged.

Other important areas related to equity research that is important in the context of the Philippines are poverty and socioeconomic inequality. Once again, there were no dissertations that considered mathematics education in poorer schools and communities.

2.2 Targeted Participants and Levels

The second dimension of the analysis conducted here is the focus of the different dissertation on the targeted participants of the research. Here we consider the research sites as an indication of the levels of education targeted, the type of institutions, and the type of participants as stakeholders in education.

2.2.1 Research Sites

The first analysis reported here relates to the location of research sites in terms of level of educational ladder of the participants' subjects and the type of institutions that they come from. These are summarized in Table 2.

Two overall observations are possible from Table 2. There are more dissertations that targeted teaching and learning at higher education (16) than basic education (20). Secondly, there are more dissertations targeting public institutions (15) than private institutions (7). Considering higher educational institutions alone, records from Commission on Higher Education (2017) of the Philippines shows that in 2016–2017, there were 1710 private higher education institutions (HEIs) and 233 public HEIs. Despite having more private than public higher education institutes, it can be noticed that there are more studies conducted in public (6) than in private ones (4), and 10 dissertations were conducted in both types of universities. Arguably, the specific site chosen for research may be reflection of the local of their employment as much as students' interest in particular research questions. As discussed above in the section on the context of research development in the Philippines, many of the doctoral students in the last five years were higher education faculty beneficiaries of special programs designed to manage the effect of the K-12 reforms in basic education.

Interestingly, there are 14 studies conducted in both private and public institutions. These studies either aimed at coming up with a generalization (e.g., #s 11, 17, 22, 35) or delved on topics that require convening a community of teacher practitioners to discuss best practices toward attaining a particular goal (#5). For example, in order to determine the readiness of secondary pre-service teachers in his locale, the PhD student in his dissertation (#11) involved students of 4 public and 6 private teacher education institutes (TEIs) in 4 provinces in the Cordillera Administrative Region (CAR). On the other hand, a study (#5) looked into how the local culture could be integrated into the teaching of Grade 7 mathematics. This necessitates the

Table 2 Institutional classification of dissertations' research site

	Private	Public	Both private and public	Total
Basic education	3 (8%)	9 (25%)	4 (11%)	16 (44%)
Higher education	4 (11%)	6 (17%)	10 (28%)	20 (56%)
Total	7 (19%)	15 (42%)	14 (39%)	36 (100%)

involvement of both the public and private, elementary and high school mathematics teachers since transitioning from Grade 6 to Grade 7 was identified as a crucial period in learning as the learner needs to make adjustment in a new setting, curriculum standards, rules, and processes.

Aside from these, there is one study (#4) that was conducted outside the Philippines, specifically in the middle east. While the PhD student has yet to complete his degree program, he had to work overseas. Given his circumstances, he conducted his dissertation in the institution where he was working. This dissertation was the first attempt of such kind where consultations were remotely done through email correspondence with his adviser in the Philippines. The dissertation defense was held in person in the campus when he had a term break from his work duties to come home in the Philippines for a vacation. In the following recent years before the pandemic, unchartered avenues were explored such as inviting external panelists from foreign countries in a synchronous online defense. Eventually, during the pandemic, all defenses are being held online as campuses are closed and only a skeletal workforce are allowed campus entry.

2.2.2 Research Participant Stakeholders

As can be gleaned from Table 3, there is only one study involving elementary pupils as participants. Likewise, only one study had adult learners in the alternative learning system as participants. There are relatively few studies that involved secondary students. Of the 17 studies with tertiary level student participants, 10 had pre-service teachers or Education students, 3 studies had non-education students, and 4 involved both education and non-education students. It could be deduced that the pre-service teachers are more readily accessible to the PhD students as most of them are teaching in TEIs than the basic education students and non-education tertiary level students. Moreover, there are considerably many studies (12) on in-service teachers. These studies underscored the importance of the teachers in the teaching and learning process. This is evident in the several award-giving bodies such as the Global Teacher Prize (GTP) and the Metrobank Foundation that have been giving recognition to the efforts, accomplishments, and contribution of Filipino teachers in developing the nation's future work force and professionals for its economy.

Very few of the dissertations included participants other than students and teachers such as administrators (#s 21, 9, 6), alumni (#21), and community members (#6). Immediate supervisors, administrators, and community members knowledgeable of the local culture served as resources in the crafting of culturally sensitive lessons in mathematics lesson (#6). Alumni, non-teaching staff and administrators worked together to craft a 5-year strategic plan toward quality mathematics instruction (#21). The coordinating teachers, their subject coordinators and administrators worked together in laying down the ground works of pre-service teachers during the latter's internship using the lesson study approach (#9). An area we find worthy of consideration in future studies is on collaboration with the industry. There had been a number

Table 3 Distribution of dissertations' participants

Participant types	Frequency
Elementary students	1 (3%)
Secondary students	8 (22%)
Tertiary students	17 (47%)
Non-education	3 (8%)
Education or pre-service teachers	10 (28%)
Both education and non-education	4 (11%)
In-service teachers	12 (33%)
Administrators and non-teaching staff	3 (8%)
Community members	1 (3%)
Alumni	1 (3%)
Adult learners	1 (3%)

of dissertations in science education along this line (e.g., Cajimat, 2019), but none so far with regard to mathematics education.

In these dissertations analyzed in terms of their participants and their research sites, it may be safe to say that most of the participants are the doctoral students' own students in the basic education, in the collegiate level, and education students in teaching internship, their community or other stakeholders within their circle of influence to take part of the study. The PhD students do research in the context that is most relevant and at the most accessible research site. The choice may perhaps be perceived as opportunistic but it also depicts their interest and their concern to address issues and problems that are within their proximity. While action research may contribute to the design of many of these studies, it remains underutilized as a main methodology informing many of these doctoral dissertations.

2.3 Theoretical Framing of the Research

The third dimension considered in our analysis is the theoretical underpinnings of doctoral research in mathematics education in the Philippines during the period of the study. Traditionally, theoretical discussion is accomplished in two possible sections in the dissertation: the conceptual and the theoretical framework. Perhaps, prior to presenting the findings, two comments are pertinent. First, in the Philippines research proposals normally require the discussion of a conceptual framework, but make a theoretical framework optional or "if applicable." Secondly, in the international research, these two constructs are not universally understood in the same way. For example, on one hand, Merriam and Tisdell (2016) noted that the two constructs are often used interchangeably by some authors. On the other hand, Creswell (2002), a leading author of widely used books on research methodologies, does not even mention theoretical framework in his book. However, some other authors such as

Schwandt (1993) argues that "[a]theoretical research is impossible" (p. 7). Hence, from this stance, even if the identified theoretical framework is not directly identified, it can be deduced from the literature review and the design.

For an understanding of a theoretical framework used in our analysis, we adopt Merriam's understanding that it makes explicit the underlying approach to the research structure consisting of concepts, variables or theories that inform the study research questions and to a certain degree the analysis of the data. However, we also note that for some authors, (e.g., Miles & Huberman, 1994), this is precisely the definition of a conceptual framework. For us here, theoretical frameworks refer to more general and well-established theories of education or social life that inform the way the phenomenon under investigation is understood and is reflected in the conceptualization of the study including the construction of the research questions and data analysis. Hence, this understanding locates the conceptual framework within a larger theoretical framing.

Similarly, there is no standard way in research in mathematics education as to the identification and the classification of the different available theories informing research. Here we mention three categories of theories that developed historically in mathematics education. Using the concept of recontextualization borrowed from Bernstein (1996), Lerman (2000) noted that most of the theories in mathematics education were adapted from other theories of the day and re-interpreted within mathematics education. We may add, as Merriam rightly noted, such an application to another field or specific problem of investigation is not pre-determined by the parent theory. Traditionally, the influence of psychological theories and constructs have had a strong influence on early mathematics education research (Kilpatrick, 1992; Lerman, 2000). Lerman identified a general characteristic of psychological models of learning arguing that "[s]tudies in epistemology, ontology, knowledge, and knowledge acquisition tend to focus on how the individual acquires knowledge and on the status of that knowledge in relation to reality" (p. 8). Lerman goes on to discuss how the early constructs of learning developed under behaviorism have influenced much of the early research in mathematics education, and hence its practice, we may add. He then discussed the shift in psychology initiated by Piaget that led to all but replacement of behaviorism by the various constructivist theories of learning. Finally, Lerman acknowledged the strong relationship of the psychological theories and the discipline of mathematics itself. In this chapter, we will refer to such theories as psycho-mathematical theories to highlight their origin and focus.

In his seminal paper in the early 2000s, Lerman (2000) goes on to discuss the "social turn" in research in mathematics education. In turn, Atweh (2007) highlighted three aspects of this social turn. First, its theories informing this research came from outside the field of psychology and education themselves and are rooted in anthropology, cultural psychology, and sociology. Secondly, these social theories informed further research focusing on students' background as intrinsically related to their achievement, participation, and opportunity to learn in schools. Thirdly, these theories implied a view of learning as not an individual effort and achievement by a student, but as a social activity between different social players. In this context, it is relevant to briefly discuss a third major turn (branching) in mathematics education discussed by Gutierrez (2013) informed by theories recontextualized from postmodern thought. Such sociopolitical theories that problematize knowledge, its development and the roles it plays in society, have emerged as significant trend in mathematics education. Constructs of power and identity are familiar components of many such theories. In the context of this particular analysis, we include research from a critical theory perspective that question educational discourse, constructs and practices, rather than accept them as given and unproblematic constructs. Questions of equity and social justice usually form an integral part of such frameworks.

2.3.1 Author Identified Theories in the Dissertations

The first level analysis of the dissertations in mathematics education in terms of their theoretical framing was carried out on the theories as the researchers have identified themselves. Consistent with the writing of Lerman above, several of the theories elaborated by the various authors in the respective dissertations come from psycho-mathematical approaches to studying, teaching, and learning. It is worth to point out in this context that some of those theories, for example, Vygotsky's theory and Realistic Mathematics Education, bridge the divide between the social and the psychological. However, it seems to us the way these theories are used in these dissertations focus more on the individual learning than on the social interactions and social outcomes of learning, thus they are classified, for our purposes here as psycho-mathematical theories. Table 4 exemplifies dissertation identified theories based on both psychological and social background identified by Lerman.

Having said that, there are a number of dissertations that do not seem to adhere to a particular theoretical theory. Examples of these are studies that employed methodologies of professional development such as lesson studies, twenty-first century learning and certain accreditation models adopted by educational authorities.

Based on this analysis, it appears that most of the theoretical frames identified by the various dissertations came from what Lerman (2000) calls the psychological theories recontextualized in mathematics education. On the other hand, there is some evidence that sociocultural theories are finding their way into dissertation research in mathematics education in the Philippines.

However, it is also very clear that none of the theoretical frameworks identified in these reviewed dissertations came from the sociopolitical theories identified by Gutierrez (2013). In particular, there are no studies that use the constructs of identity, agency, or power. There is no research reported here that uses methodologies and theories such as developed by Michael Apple, Peter McLaren, Michel Foucault, Jürgen Habermas and Pierre Bourdieu, or Paulo Freire, just to mention a few that have informed a number of investigations internationally.

Perhaps more importantly is to consider in a bit more detail how these theories were actually used in the respective dissertations.

Table 4 Examples of Identified psycho-mathematical and sociocultural theories

Psycho-mathematical-based theories	
Constructivism	Unified theory of acceptance and use of technology
Vygotsky's zone of proximal development	Bigg's model of constructive alignment
Gardner's multiple intelligences	Gradual release of responsibility
Mathematical pedagogical content knowledge	Flipped classroom
Realistic mathematics education	7C framework
Levels of epistemological beliefs in geometry	Mathematical problem solving
Teacher's reflective thinking and practices	Self-efficacy
van Hiele phases of geometric learning	
Sociocultural-based theories	
Social learning theory	Transformative learning theory
Ethnomathematics	Culture-sensitive lessons
Anthropological theory of didactics	Diffusion of innovations theory
	Cultural historical activity theory

2.3.2 Utilizing Theoretical Frames in the Dissertations

The way a theoretical framework is actually used by the various authors is not consistent across the different dissertations. In some dissertations, the role of the theory was quite explicit and consistently used all through the dissertation. For example, dissertation #27 adopted the Cultural Historical Activity Theory in Mathematics which is directly reflected in the research questions, in identifying the constructs to focus upon, and in the development of the instruments it used to collect the data. In a number of dissertations, there was a clear association between the theoretical framing of the research and the overall conceptualization of the dissertation. Dissertation (#28) used Anthropological Theory of Didactics (ATD) and Mathematical Pedagogical Knowledge in developing a Didactics in Mathematics Course for Pre-service Teachers in the Philippines. Another dissertation that used the Gradual Release of Responsibility (#3A) was able to evidently use its theoretical frame in designing an intervention to measure its effect on cognitive domain in problem solving and affective skills such as confidence and attitudes.

However, this was not the case in all dissertations. For example, as dissertation #16 identified the Social Cognitive Theory as one of its theoretical frames, it also discussed other constructs such as self-efficacy and metacognition whose understandings arise from other theoretical backgrounds and take a primary role in the construction of the research questions. However, other constructs such as social interactions that are related to the Social Cognitive Theory, although discussed in the literature review, was not utilized in the research questions, and hence, the analysis.

This lack of clarity in how the theoretical lens identified in the dissertation is actually used in the research is quite common in the dissertations reviewed. The dissertation (#2) that discussed Vygotsky's Theory of Cognitive Development and the theoretical frame Bigg's Model of Constructive Alignment in the discussion of the theoretical framework of the research did not actually use these in the research questions, instruments, and analysis. Instead, the researcher seemed to have used a much more domain-specific frame of knowledge and practices of teachers in assessment of twenty-first century skills. Several other dissertations discussed a number of theoretical underpinnings in the beginning parts (Review of Related Literature and Conceptual Framework) of the dissertation, but only one of these frameworks were actually used in the other parts of the research, if at all (i.e., #11, #18, #22, #33, #34).

A number of studies seem to have identified a theoretical framework that is only used to develop the teaching material used in the study—usually without demonstrating in depth how this was actualized. This relates in particular in those studies that have identified constructivism (including theories of Piaget and Vygotsky) as one principal theoretical framing of their research. For example, dissertation #5 utilized both ethnomathematics and constructivism as its theoretical framing and the dissertation #36 utilized both Organizational Learning Theory and Social Constructivism as their theoretical frameworks. In both cases, the research questions and the analysis did not reflect constructs and principles of constructivism. Presumably, constructivism was used to develop the teaching material employed. This raises questions how theoretical frameworks are interpreted by the various dissertations. In particular, in both cases, the descriptions of the teaching materials were not provided in sufficient detail to demonstrate the use of such theories.

2.3.3 Prevalence of Process–Product Thinking

Based on the seminal work of Brophy in the 1980s, the area of research that has received significant following in research conducted in general education, including mathematics education is process–product research. Traditionally, this research focused on specific effective classroom teachers' behaviors on student outcomes (Creemers & Kyriakides, 2016). It is relevant to point out that none of the dissertations reviewed here has specifically identified process–product research as their theoretical framework. However, taken in a more general understanding of this research as research that identifies specific variables and investigates their "effect" relationships on other variables, it seems to us that this type of research is demonstrated in the majority of studies reviewed. Creemers and Kyriakides point out one characteristic of this research.

> Because of the definition of process and product variables, the process-product paradigm preferred empirical research of a quantitative nature ... Critical advocates of the qualitative approach stated that the quantitative approach did not do enough justice to the "richness" of education at classroom level. Ethnographic, detailed descriptions of education processes at classroom level were presented as an alternative to paying very little attention to the outputs of education". (p. 112)

In our analysis, it appeared that the vast majority of dissertations reviewed here remain focused on the relations of specific variables and their effect on specified outcomes. To illustrate the trend, we provide three examples. The dissertation #24 aimed to study creativity in problem solving and posing. Arguably, several research questions may have led to an in-depth description of creativity and problem solving and posing used by students. Instead, a quasi-experimental design was chosen to investigate the effects in creativity and academic performance of the students. Similarly, dissertation #25 made use of Rigorous Mathematical Thinking Approach and 5E's Instructional Model. Instead of an in-depth investigation about how the intervention was experienced by the students and demonstrate its effectiveness with actual problems solved by the students, the dissertation developed questionnaires to determine students' achievement, conceptual understanding, and persistence scores. Finally, dissertation #26 makes use of researcher-made instruments to measure commognition via a 5-item open ended questionnaire, and a 16-item self-test for persistence level. These were used to implement a quasi-experimental study on developing students' commognition, creativity, and persistence through repetition with complex variation.

Indeed, even in the case of studies that are based on sociocultural theories and in cases that use qualitative analysis, the research questions remain focused on the effect of certain variables and the development of variables. Holistic ethnographic types of questions are all but absent. One example is Dissertation #5, which was about designing, developing, and implementing an ethnomathematically enriched learning material in high school geometry. Though there were some classroom observations done, most of the data analysis heavily gleaned on the quantitative measurement of the ethnomathematical perspective being espoused. Again, the analysis was more on the "effects", rather than deeper rationalizations on the processes of students' thinking vis-à-vis the materials/interventions being used.

The intention here is not to argue that such research is not useful, but to demonstrate a possible general assumption about what research can look like and that it can only be in the form of examining relationships between variables. In such cases, the depth of students' and teachers' experiences of such intervention may all but be lost. In this context, we are in agreement with Creemers and Kyriakides that such research fails to examine the day-to-day richness of students and teachers experiences of the educative process in sufficient depth.

3 Concluding Remarks

Doctoral level research in mathematics education is rather recent in many universities in the Philippines. In a relatively short period of time, around 22 universities in the country offer doctoral degrees for an increasing number of candidates. One of the products of such research is already being translated to conference presentations and publications encouraged and supported to various degrees by the institutions themselves. Undoubtedly, the rapid evolution of doctoral research in the Philippines is a credit to the hundreds of graduate school faculty members and active researchers. Research such as represented by this chapter is both opportune and useful toward assessing the overall commitment to research in terms of coverage and quality.

This review in this chapter has identified a few silences of research in mathematics education—at least in doctoral research. In particular we point to the limited research in terms of

1. Its focus on the social goals of education in terms of development of the subjectivity of students as users of mathematics and as general citizens.
2. The relationship of mathematics teaching and learning with the other disciplines in the school curriculum.
3. The focus of research on equity that is of particular relevance to the Philippines such as language and poverty.
4. Its limitation to focus on the foundation years of the elementary and kindergarten that form the foundation for later learning of mathematics.
5. Limited focus on topics such as assessment, the use of technology, and informal settings.
6. Research that is informed by critical and sociopolitical perspectives.

In addition, this review has pointed out inconsistencies in terms of theoretical framing of the dissertations. On one hand, there is a need for better articulation of the role of the theory in a dissertation in light of its consistency in the different sections of the dissertation. On the other hand, there is a need for greater diversification of theoretical stances, in particular the critical and sociopolitical perspectives that remain underutilized.

As the field of research in higher education is expanding in the Philippines, perhaps many of these silences may be addressed by future doctoral candidates. In particular, perhaps one of the effects of the pandemic might be an increase in number of researchers investigating topics such as assessment and technology in mathematics education. However, other silences may be a bigger challenge to meet. Programs for doctoral degrees need to be re-examined with respect to the relative limited opportunity in them to deal with the wide range of theories and perspectives in mathematics education research and the relative short time for candidates to complete their research design and implementation that provide limitations to the type of research that students may attempt.

In this context, we do not make general statement as to the relevant benefit of one particular perspective of research over the others. However, there are two reasons why diversification of research may be important. First, new insights in understanding mathematics education arise from the different perspectives. Different research perspectives imply different research questions and yield different answers. Second, each one of these perspectives has extensive presence in international literature. For a developing country to participate in international dialogue (in contrast to the discourse of comparativeness), diversification of research contributes to widening of avenues of dialogue.

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A Critical Review of Mathematics Education Research in Korea: Trends, Challenges, and Future Directions

JeongSuk Pang and Minsung Kwon

Abstract The purpose of this chapter is to critically review the overall trends of mathematics education research in Korea. For this purpose, we summarize two recent studies on the trends in mathematics education research using different approaches: content analysis and topic modeling. This chapter then provides critical reviews on the research trends, including an increase in research articles, diversification of research topics, and balance of research methods, while comparing and contrasting them with the international trends in mathematics education research. This chapter further elaborates on two popular research topics in Korea, curriculum and textbooks and teacher education, including the significances, challenges, and future directions.

Keywords Mathematics education research trends in Korea · Research topics · Research methods · Research on curriculum and textbook · Research on teacher education

1 Introduction

Over the last 20 years, Korean students have demonstrated their outstanding performance in a series of international mathematics assessments, in particular in the Trends in International Mathematics and Science Study (e.g., Mullis et al., 2020) and the Program for International Student Assessment (e.g., OECD, 2019). Korean students' excellent accomplishments have attracted considerable attention from international educators, researchers, and policy makers. In particular, researchers have investigated several aspects of Korean mathematics education, including curricular changes and its challenges (Pang, 2014; Wong et al., 2014), textbooks development and comparative textbook analysis (Hong & Choi, 2014; Pang, 2008), prospective teachers' profound

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mathematics conceptual knowledge for teaching (Li et al., 2020), and key characteristics of effective mathematics instruction (Grow-Maienza et al., 1999; Pang, 2009; Park & Leung, 2005). These studies illustrate the detailed accounts of the corresponding research topics, but they do not depict a holistic overview of mathematics education research in Korea.

In contrast, some efforts have been made to address Korean mathematics education from a comprehensive perspective. For instance, a special issue of the ZDM Mathematics Education Journal introduced the main features of Korean mathematics education (Kwon & Cho, 2012), such as addressing challenges with the national mathematics curriculum, comparative analyses of reform curricula or textbooks in Korea and the US, characteristics of effective mathematics instruction and teaching practices, the expertise of mathematics teachers, and changes in assessment. Similarly, the first sourcebook on Asian research in mathematics education (Sriraman et al., 2015) included various aspects of Korean research, such as a review of studies on philosophical aspects of mathematics education, issues of curricula and textbooks, the use of history of mathematics in teaching mathematics, mathematical reasoning, mathematical modeling, gender, assessment, and teacher education. In addition, two books on Korean mathematics education were published. The first volume included the historical developments and future directions of the national mathematics curriculum and textbooks, various instructional practices by different content or process strands, and assessment (Kim et al., 2012). The second volume addressed mathematics teacher education, special programs of mathematics education (e.g., gifted education, mathematics camp), development of mathematics education, and implications for future mathematics education (Kim et al., 2015).

The aforementioned studies provide important features of Korean mathematics education in the international context, but they do not include quantitative analyses on the overall trends of mathematics education research in Korea. In particular, mathematics education research in Korea showed a rapid quantitative growth in the past two decades (Pang, 2020). Given these, the purpose of this chapter is to survey the overall trends of mathematics education research in Korea and to critically review such trends. This chapter first summarizes two recent studies that analyzed research articles published in the Korean journals to identify the trends of mathematics education research. This chapter then provides critical reviews on such research trends while comparing and contrasting them with the international trends in mathematics education research. It further elaborates on two popular research topics in Korea, curriculum and textbooks and teacher education, including the significances, challenges, and future directions. As such, this chapter aims to provoke subsequent discussions concerning mathematics education research in the international context as well as to inform readers of the overall trends of mathematics education research in Korea.

2 Mathematics Education Research Trends in Korea

This section begins with an explanation of why two specific studies, among the studies that analyzed trends of mathematics education research in Korea, were selected for a detailed review in this chapter. It then provides a brief description of seven peer-reviewed mathematics education journals in Korea, partly because the two studies analyzed the articles published in these journals. This section ends with an overview, methods, and main results of the two studies, which serve for the subsequent reflections on mathematics education research in Korea.

2.1 Two Studies Selected to Illustrate Mathematics Education Research Trends in Korea

With the rapid quantitative increase of research articles in Korea, various efforts have been made to analyze the trends in mathematics education research. Some studies focused on the research trends concerning specific school levels, such as elementary mathematics education (e.g., Kim & Pang, 2017), secondary mathematics education (e.g., Park, 2003), or university mathematics education (e.g., Kwon & Ju, 2003). Other studies focused on the research trends on specific topics, such as mathematics gifted education (e.g., Min et al., 2011), mathematics instruction (e.g., Kim, 2010), research methods in mathematics education (Kim et al., 2014), or mathematics teacher education (e.g., Sunwoo & Pang, 2019). These studies have benefits of examining research trends in detail by the selected school levels or research topics. However, they may be limited in identifying the overall research trends across different school levels or connections across research topics investigated.

In contrast, recent efforts have been made to identify the overall trends of mathematics education research in Korea. Among them, two studies were selected for a detailed review in this chapter. Study 1, by Pang et al. (2019), analyzed 4559 articles published from 1963 to June of 2019 in Korea using content analysis. Study 2, by Shin (2020), compared 3114 articles in Korea with 1636 international articles published from 2000 to 2019 using a topic modeling method. These two studies were chosen for the review because they provide a comprehensive overview of the current trends in mathematics education research in Korea across different school levels, research methods, and research topics. Although these two studies share similarities in the scope of articles covered and the number of articles analyzed, they differ in terms of the period of publications (i.e., 50 years vs. 20 years), the method employed (i.e., content analysis vs. topic modeling), and international comparison (i.e., domestic only vs. comparison between domestic and international). As such, the two studies can complement each other in indicating the trends in mathematics education research in Korea.

Table 1 Seven domestic professional journals

Journal	Published since	Listed on the KCI since
The Mathematical Education	1963	1999
Journal of Educational Research in Mathematics	1991	2002
School Mathematics	1999	2002
Journal of the Korean School Mathematics Society	1998	2004
Communications of Mathematical Education	1997	2007
Journal of Elementary Mathematics Education in Korea	1997	2008
Education of Primary School Mathematics	1997	2010

2.2 Professional Mathematics Education Journals in Korea

The two studies selected for this chapter, Pang et al. (2019) and Shin (2020), analyzed peer-reviewed articles published in the seven domestic professional journals listed in the Korea Citation Index (KCI) of the National Research Foundation.[1] Table 1 shows a list of seven professional journals in the order indexed in the KCI, along with the year first published.

Three out of the seven journals are published by the Korean Society of Mathematical Education, which is the oldest professional mathematics education society in Korea. *The Mathematical Education* is the oldest mathematics education research journal in Korea. This journal, published since 1963, deals with all aspects of mathematics education. *Communications of Mathematical Education* and *Education of Primary School Mathematics* have been published since 1997. As the journal title indicates, the latter is specifically intended to deal with research related to primary schools.

Two journals are published by the Korea Society of Educational Studies in Mathematics. *Journal of Educational Research in Mathematics* has been published since 1991 and *School Mathematics* has been published since 1999. Initially, the former aimed to focus on research based on the review of literature relating to mathematics education, while the latter aimed to focus on practical issues directly related to teaching and learning mathematics. These different foci between the two journals have become blurred in recent years.

The remaining two journals are published by other mathematics education organizations. *Journal of the Korean School Mathematics Society* was launched in 1998, aiming to publish papers connecting theories of mathematics education with actual teaching practices to improve the quality of school mathematics. *Journal of Elementary Mathematics Education in Korea* was launched in 1997 by the Korea Society of

[1] The journals listed in the Korea Citation Index ensure that the research articles published in the journals are of high quality in the Korean context. The journals have to pass a regular and rigorous evaluation to remain listed on the Korea Citation Index.

Elementary Mathematics Education, aiming to promote research and practice specifically related to elementary mathematics education. All the aforementioned journals are currently published four times a year. These journals play a significant role among Korean researchers sharing their various studies on mathematics education.

2.3 *Study 1: A Comprehensive Analysis of Mathematics Education Research Trends in KCI Journals Over 50 Years*

2.3.1 Overview

This section reports on a study that comprehensively analyzed the mathematics education research trends in Korea by reviewing the articles published in the seven mathematics education professional journals over the last 50 years (Pang et al., 2019). As the study analyzed almost all the articles[2] published in the journals according to publication periods, research topics, research methods, and target research population (e.g., teachers, students, textbooks), it provides a comprehensive reflection of mathematics education research trends in Korea.

2.3.2 Method

A total of 4559 research articles were analyzed by the following four analytic elements: publication periods, research topics, research methods, and target research population. For publication periods, the initial approach analyzed how many articles were published each year and then the years were grouped into either a five-year or a ten-year time period to analyze how research topics, research methods, or target research population had changed across the specific periods. For research topics, the following seven major topics were used: (a) general research; (b) curricula or textbooks; (c) students' abilities or characteristics; (d) instruction or teaching methods; (e) assessment; (f) technology or manipulatives; and (g) teacher education. Each major topic was then classified into four to six subtopics, resulting in a total of 36 subtopics.

Regarding research methods, the following four main categories were used: document analysis, quantitative, qualitative, and mixed methods. Each main research method was then classified into two to four sub-methods, resulting in a total of 11 subtopics. Analyses were conducted to examine which research methods were used most often in relation to each research topic, beyond the frequency of each research method. Finally, regarding the target research population, an initial analysis identified whether the paper targeted elementary school, secondary school, or university

[2] The only excluded papers were related to pure or applied mathematics, which were published in the oldest mathematics education journal from 1963 to 1980s.

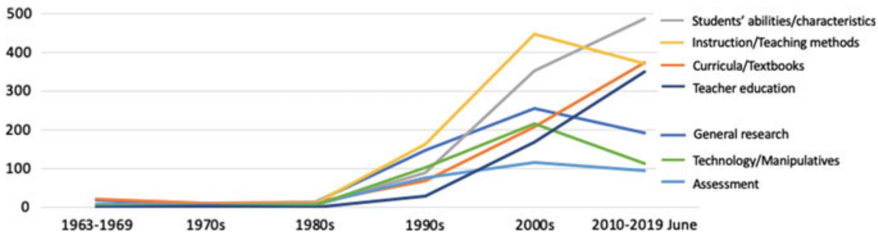

Fig. 1 Frequency of seven research topics over a ten-year period

level education. Then, additional analyses were conducted at each school level to identify who (e.g., teachers or students) or what (e.g., curricula or textbooks) was being studied.

2.3.3 Main Results

Regarding publication periods, the number of research papers on mathematics education has increased in the 1990s, with about 200 papers published each year since the late 2000s. The quantitative increase of research papers related to the publications of new mathematics education journals in the 1990s, along with their subsequent lists in the Korea Citation Index in 2000s, is given in Table 1.

Figure 1 shows the trend in research topics divided into approximately ten-year publication periods. The most popular research topic is instruction or teaching methods (21.96%), which has been popular since the 1990s. The second most popular research topic is students' abilities or characteristics (20.60%), which has been popular since 2000. The third most popular topic is curricula or textbooks (15.17%), receiving much attention from researchers specifically in the 2010s. In contrast, general research[3] was popular between 1963 and 1999 but did not continue in popularity after that. Assessment has not received much attention from researchers over the last 50 years, accounting for only 6.71% of the total research. Note that the topic of technology or manipulatives was popular in the 2000s, and teacher education was popular in the 2010s.

Table 2 shows the top 10 out of the 36 subtopics used in the study. The two most popular subtopics were the development or application of mathematical tasks or programs (9.90% of the total research) and student's mathematical knowledge, concepts, or understanding (9.40%). The high frequency of these two subtopics explained why *instruction or teaching methods* and *students' abilities or characteristics* were the top two major topics most frequently studied over the last 50 years. The next popular subtopics were general analysis of mathematical concepts, terms, or symbols (6.39%) and the development and use of educational software or programs (6.17%). The remaining six popular subtopics showed similar frequencies. Note that

[3] General research includes theory of mathematics education, history of mathematics, or general analysis of mathematical concepts.

the top ten subtopics were distributed across different major topics (i.e., from general research to teacher education) except assessment.

Regarding research methods, document analysis was dominant between 1963 and 1999, accounting for more than 50% of the total research. Since 2000, document analysis has decreased, and other research methods such as qualitative and quantitative research methods have increased. Figure 2 displays the distribution of seven research topics by four main research methods, and Table 3 shows the frequency of seven research topics by detailed research methods.

Table 2 Frequency of the top ten research subtopics

Subtopic	Frequency	(%[a])
Development or application of mathematical tasks or programs	451.5[b]	(9.90)
Students' mathematical knowledge, concepts, or understanding	428.5	(9.40)
General analysis of mathematical concepts, terms, or symbols	291.5	(6.39)
Development and use of educational software or programs	281.5	(6.17)
Teaching methods in analyzing curricula or textbooks	217.5	(4.77)
Students' mathematical competencies (e.g., problem solving, reasoning, communication, creativity, or convergence)	193	(4.23)
Instruction or teaching methods to foster students' mathematical knowledge or skills	191.5	(4.20)
General research on curricula or textbooks (e.g., trend, changes, theory)	186	(4.08)
Teacher preparation or professional development programs	164.5	(3.61)
Theory of mathematics education (e.g., epistemology, learning theory, psychology, philosophy)	162	(3.55)

[a] The percent (%) was based on the total number (4559) of research papers analyzed in the study
[b] In cases where a research paper covered two topics evenly, the frequency of each topic was calculated as 0.5

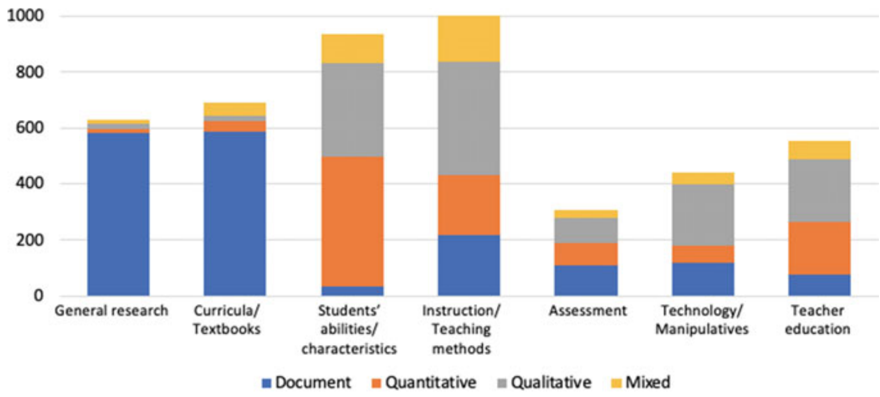

Fig. 2 Distribution of seven research topics by four main research methods

Table 3 Frequency of seven research topics by detailed research methods

Research methods		GR[a]	C/T	SA/C	I/T	AS	T/M	TE	Frequency (%)	
Document analysis	Critique/Review/Summary	**196**[b]	35	9	**63**	20	33	17	373 (8.18)	1727 (37.88)
	Pedagogical analysis	**384.5**	**553**	26.5	155	88	86.5	60.5	1354 (29.70)	
Quantitative research methods	Survey	11	29	**196**	28	37	9	**114**	424 (9.30)	1055 (23.14)
	Experimental	2	2	**101.5**	**165**	9	40.5	26	346 (7.59)	
	Others	3	4	**165.5**	20	35.5	9.5	**47.5**	285 (6.25)	
Qualitative research methods	Case study	5	6	**285**	137	17	73.5	**172.5**	696 (15.27)	1308 (28.69)
	Development	3.5	7	15	**194**	51.5	**101**	21	393 (8.62)	
	Action research	5	3	**18.5**	**36**	5.5	15.5	12.5	96 (2.11)	
	Others	7	4	13.5	**37.5**	16	**30**	15	123 (2.70)	
Mixed research methods	Quantitative and Qualitative	0	12	**84.5**	**103**	18.5	20.5	45.5	284 (6.23)	469 (10.29)
	Others[4]	12	**36.5**	24	**62.5**	8	21.5	20.5	185 (4.06)	
Total (%)									4559 (100)	

[a] GR: general research; C/T: curricula or textbooks; SA/C: students' abilities or characteristics; I/T: instruction or teaching methods; AS: assessment; T/M: technology or manipulatives; TE: teacher education
[b] The two most popular topics for each research method are in bold

[4] This includes other mixed methods such as document analysis and qualitative research methods.

The most popular research method was document analysis (37.88%), specifically pedagogical analysis (29.70%) used for studying curricula or textbooks and general research. Under document analysis, note that critique, review, or summary was mainly used for studies on general research and instruction or teaching methods. The second most popular research method was qualitative research methods (28.69%). Of the qualitative research methods, the case study method was the most frequently used (15.27%), mainly for studies on students' abilities or characteristics, followed by studies on teacher education. It is also noticeable that development research was mainly used for studies on instruction or teaching methods, followed by studies on technology or manipulatives.

Quantitative research methods were used for 23.14% of the total research. Of the various quantitative research methods, the survey method was the most frequently used (9.30%), followed by the experimental research method (7.59%). The former was mainly used for studies on students' abilities or characteristics, and the latter was used mainly for studies on instruction or teaching methods. Mixed research methods were the least frequently used, accounting for 10.29% of the total research. Of the various mixed research methods among document analysis, quantitative methods, and qualitative methods, a mixture of quantitative and qualitative methods was the most popular (6.23%), specifically for studying instruction or teaching methods.

Finally, regarding the target research population, the most prevalent participants have been from either elementary or secondary school level since 1963. In contrast, preservice teachers or participants from at least two different school levels have been studied since 1990s. Figure 3 shows the number of research papers based on the target research population. Both at the elementary and at the secondary school level, the most popular research participants were students (15.40% and 18.53%, respectively), followed by curricula or textbooks (8.60% and 7.87%), teachers (2.83% and 3.07%), and groups of both teachers and students (1.25% and 1.38%). At the university level, preservice secondary school teachers (5.68%) were more popular research subjects than preservice elementary school teachers (2.11%). The percentage of the mixed groups from at least two different school levels was relatively low compared with papers using only elementary school levels or secondary school levels.

2.4 Study 2: A Comparative Analysis of Mathematics Education Research Trends in KCI and SSCI Journals Over 20 Years

2.4.1 Overview

This section reports a study that compared mathematics education research trends between KCI and Social Science Citation Index (SSCI) journals over the last 20 years (Shin, 2020).[5] Using a topic modeling method, this study identified 16 similar

[5] The summary reported here was reviewed and approved by the author.

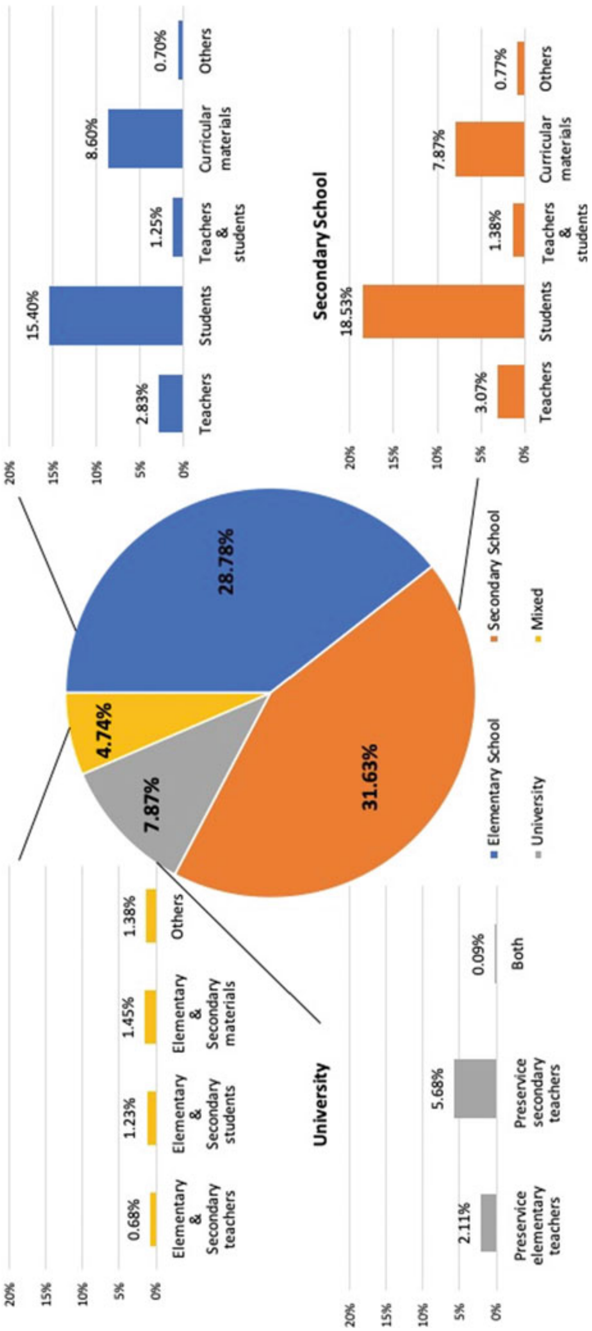

Fig. 3 Frequency of target research population

research topics and seven distinct research topics. This comparative analysis helps us better understand culturally specific features that may be overlooked through an analysis of research articles in Korea.

2.4.2 Method

At the initial stage of analysis, a total of 3125 articles published in seven KCI journals and 1652 articles published in five SSCI journals[6] from 2000 to 2019 were retrieved. Because it is important to perform the same pre-processing steps[7] for both KCI and SSCI journals, only English abstracts were used. As a result, excluding articles without English abstracts resulted in 3114 KCI articles and 1636 SSCI articles for the analysis. To analyze a large number of articles more efficiently and effectively, this study employed a topic modeling method. Topic modeling classifies topics based on the frequency of a simultaneous appearance of words in the abstract.

To identify the topics that best fit the articles, the Latent Dirichlet Allocation (LDA)-based topic modeling method was employed. Using the perplexity K-curve, the optimal number of topics was determined ($K = 23$ for both KCI and SSCI). After extracting 23 topics, the topic names were determined based on a set of the top ten words and several representative articles. As a result, this study identified 16 similar research topics and seven distinct research topics in KCI and SSCI journals. Unlike the traditional coding method in analyzing the research trends (i.e., calculating frequency evenly across multiple topics), this study used the topic distributions across multiple topics.

2.4.3 Main Results

Using a LDA-based topic modeling method, 23 topics were extracted. Figure 4 illustrates the distribution of 23 topics from KCI journals and the distribution of 23 topics from SSCI journals. The most popular research topic, both in KCI and SSCI journals, is preservice teacher.

After comparing 23 research topics between KCI and SSCI journals, this study identified 16 similar topics which had a similar probability distribution of words: algebra/algebraic thinking, fraction, functison/representation, statistics, geometry,

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[6] Five SSCI journals are Journal for Research in Mathematics Education, Educational Studies in Mathematics, Mathematical Thinking and Learning, Journal of Mathematics Teacher Education, and ZDM.

[7] The following pre-processing steps were performed: (a) removing pronouns, conjunctions, prepositions, adverbs, (auxiliary) verbs, and articles except nouns and adjectives; (b) checking words appearing less than five times in the English abstracts and removing meaningless words (e.g., enough, thing); (c) changing plural nouns to singular nouns; (d) removing non-topic words appearing frequently in the abstract (e.g., study, finding); (e) replacing synonyms into a single word (e.g., replacing pupil and learner to student); and (f) removing the two most common words in the articles (i.e., mathematics and education).

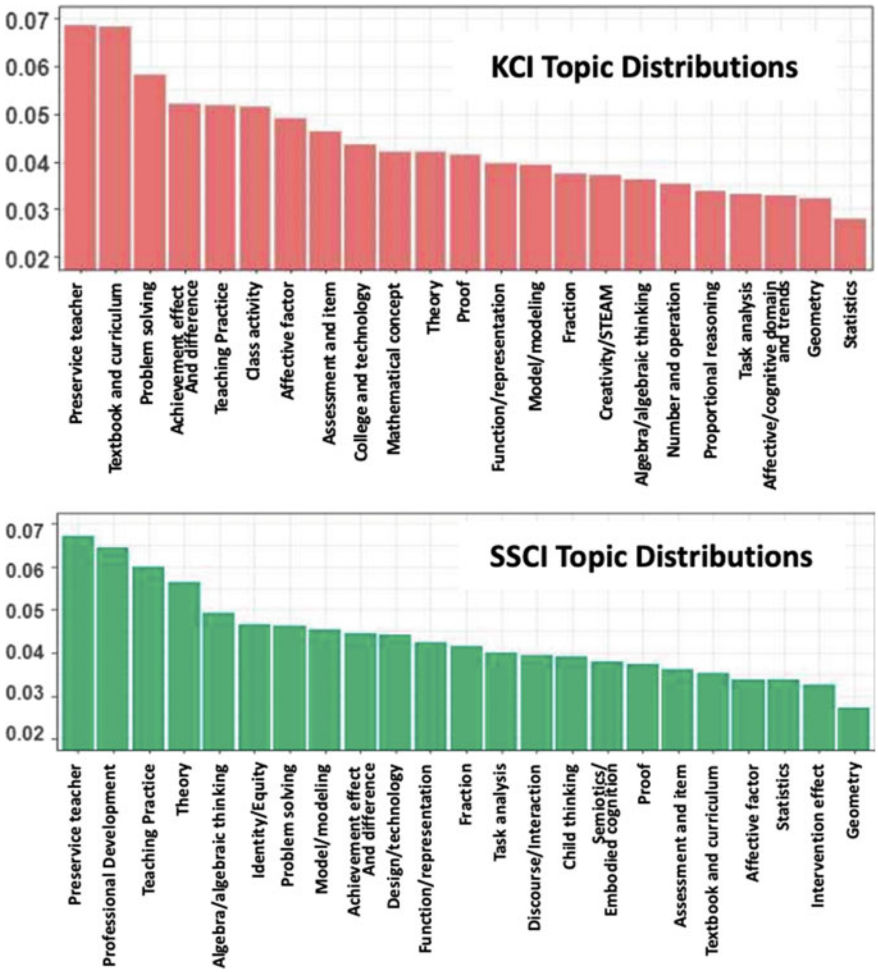

Fig. 4 Topic distributions from KCI and SSCI journals (Shin, 2020, p. 70)

problem solving, model/modeling, proof, achievement effect/difference, affective factor, preservice teacher, teaching practice, textbook/curriculum, task analysis, assessment, and theory.

Figure 5 illustrates the top ten words that characterize four selected similar research topics in KCI and SSCI journals. For example, a set of common words for the most popular research topic, preservice teacher, includes teacher, preservice, knowledge, teaching, elementary, program, school, secondary, and pedagogical. The research on preservice teachers includes research on elementary and secondary preservice teacher education program and preservice teachers' knowledge. A set of common words for research on textbook and curriculum includes textbook, curriculum, school, and difference. This research topic addressed the analysis of mathematical content, terms, and other factors of textbooks, comparison of revised curricula, or comparison of textbooks across countries. Another similar research topic, affective factor, includes a set of common words: student, factor, anxiety, efficacy, and belief. This research topic addressed both affective domains (e.g., belief, attitude, interest, and self-efficacy) and factors impacting affective domains. In the KCI journals, the word achievement was included for this research topic, which examined the relationship between mathematics achievement and the affective domain. A set of common words for research on assessment items include assessment, item, test, and response. KCI articles focused on assessment items and the development of assessment standards, whereas SSCI articles focused on developing and using assessment items for instruction, instructional quality, and validity.

This study also identified seven distinct topics which had different probability distributions of words between KCI articles and SSCI articles. The seven distinct topics in the KCI articles are affective/cognitive domain and research trends, mathematical concept, class activity, number and operation, creativity/STEAM, proportional reasoning, and college/technology, whereas the seven distinct topics in the SSCI articles are discourse/interaction, professional development, identity/equity, child thinking, semiotics/embodied cognition, intervention effect, and design/technology. In this study, the research on number and operation in the KCI articles and child thinking in the SSCI articles might be considered similar topics but was differentiated by grade-level and research focus. Also, research on college/technology in the KCI articles and research on design/technology in the SSCI articles are connected in some sense. However, the KCI articles focus on college engineering students and the SSCI articles focus on using technology in instructional design.

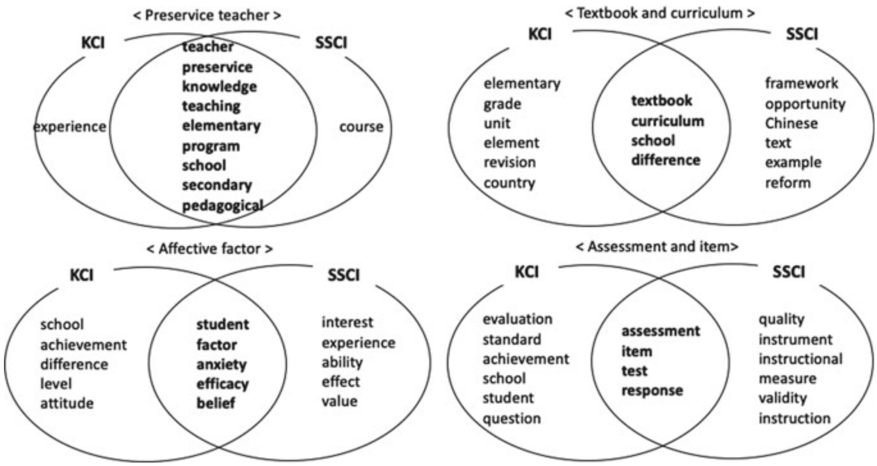

Fig. 5 Top ten words that characterize four selected similar research topics in KCI and SSCI journals

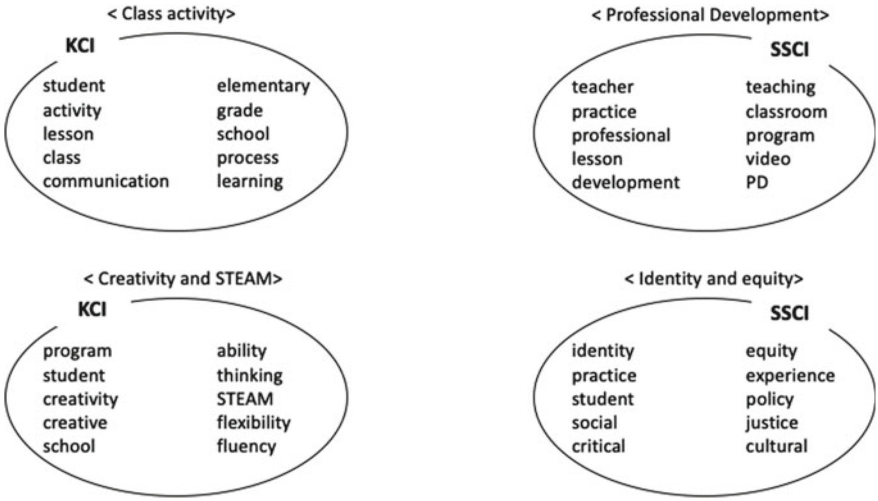

Fig. 6 Top ten words that characterize two selected distinct research topics in KCI and SSCI journals

Figure 6 illustrates the top ten words that characterize the selected two distinct research topics in KCI journals and two distinct research topics in SSCI journals. Although class activity in KCI journals includes communication, it includes various activities such as reading, writing, project-based learning, discussion, and flipped learning. One distinct research topic in KCI journals is creativity/STEAM. One of the differences was that the professional development of in-service teachers appeared as one independent topic only in SSCI journals. Identity and equity have often attracted attention from multicultural countries, using critical theory to explore equity, social justice, and minorities' identities.

3 Reflections on Mathematics Education Research Trends in Korea

Building on the two studies summarized above, this section reflects on mathematics education research trends in Korea. Specifically, three distinctive features of mathematics education research trends were identified: the quantitative increase of research articles, the diversification of research topics, and the balance of research methods. In this section, we provide specific examples for each feature and explain potential factors contributing to each feature.

3.1 Quantitative Increase of Research Articles

There is a rapid increase of research articles published in Korea, especially between 2000 and 2019. For instance, the number of articles published in these two decades makes up 82% of the total articles published in the past 50 years (38.6% in 2000s and 43.4% in 2010s, respectively). The rapid increase of research articles can be explained by the emergence of new professional organizations and their journals. As given in Table 1, five professional mathematics education journals were launched in the last 1990s, and they were listed on the KCI in the 2000s. In fact, the years in which the rapid increases of research articles were observed roughly correspond to the years in which a new mathematics education journal was first listed on the KCI (Pang, 2020). Besides this general factor contributing to the quantitative increase of research articles in the past two decades, doctoral programs and faculty hiring process are also contributing to the quantitative increase of research articles in Korea. A number of doctoral programs tend to encourage graduate students to present their research ideas in the annual meetings of the professional organizations and to publish journal articles to receive a doctoral degree. As part of the hiring process, universities require publication records for faculty candidates and evaluate the quantitative and qualitative quality of their publications. For instance, the quantitative measures of publication records vary by universities, but many universities require at least 300%[8] of publication records. However, as the hiring process in Korea is very competitive, some faculty candidates often far exceed the requirement. Lastly, the KCI journals mentioned above have a short turnaround time. In many cases, the first decision has been made within one month after the initial submission in Korea, which also contributes to the rapid increase of research articles. With these social, cultural, and contextual factors, the increase of research articles in Korea will probably continue, at least in the near feature.

3.2 Diversification of Research Topics

Another distinctive feature includes diverse research topics in Korea, demonstrated by seven main topics and 36 subtopics. As summarized in Study 1, the top ten subtopics were distributed across different topics except assessment. The popular research topics are often influenced by periodic curriculum revisions or new educational policies in Korea. The most popular research subtopics include mathematics task development, educational software development, and students' mathematical competencies, which reflect the main focus of revised curriculum. For instance, because new mathematical constructs (e.g., possibility instead of probability for

[8] Although the criteria of quantitative evaluations may differ by universities, a general measure is as follows: 100% for publishing an article as a sole author, 70% for publishing an article by two authors, and 50% for publishing an article by three or more than three authors.

elementary school students) or mathematical competencies (e.g., creativity or convergence, data processing) were introduced to the national curriculum, new units or alternative approaches were developed and implemented in mathematics classrooms to assess their suitability. Similarly, instructional programs reflecting new educational policies or various social expectations, such as STEM-based lessons, mathematics lessons using educational technology, gender equity, and character-building through mathematics lessons, have been developed and implemented (Pang, 2020). Other popular research topics also include students' cognitive and affective aspects, teaching methods, and teacher education.

In fact, the diversification of mathematics education research topics in Korea is similar to the international research trends in mathematics education. Although the increase or decrease of certain research topics are observed over time, new research topics do not replace the old ones (Hannula, 2009; Inglis & Foster, 2018; Pang et al., 2019; Shin, 2020). Furthermore, research on mathematics education makes the spectrum richer, increases the complexity and diversity of research theoretically and methodologically, and increases the connectivity across research frameworks (Hannula, 2009; Inglis & Foster, 2018).

It is noticeable that the similar research topics do not always appear with the same frequency in KCI and SSCI articles. For instance, as shown in Fig. 4 in Study 2, the research topic of textbook and curriculum appeared as the second most popular research topic in the KCI articles, but appeared as the 19th popular research topic in the SSCI articles. Research on affective factors or assessment/item appeared more frequently in the KCI articles than in the SSCI articles, whereas research on algebra/algebraic thinking appeared more frequently in the SSCI articles than in the KCI articles. Given these, further research is needed not only to investigate the overall research trends in mathematics education within a country but also to compare or contrast popular research topics across different countries to better understand social, cultural, and contextual significance, needs, and factors.

3.3 Balance of Research Methods

According to Study 1, document analysis was dominant up to 1999, accounting for more than 50% of the articles but other research methods such as quantitative, qualitative, and mixed methods have been increasing since 2000. As a result, over the past 50 years in Korea, 38% of the articles employed document analysis, 23% of them employed quantitative methods, 29% of them employed qualitative methods, and 10% of them employed mixed methods. Unlike this balanced approach to research methods employed in Korea, international journal articles employed more qualitative methods than quantitative methods. For instance, in the review of 710 research articles in six dominant international journals, Hart et al. (2009) found that 21% of journal articles were quantitative, 50% of them were qualitative, and 29% of them were mixed. Similarly, Hannula (2009) found that 23% of the submissions for the annual conference of International Group for the Psychology of Mathematics Education were quantitative and 66% were qualitative. On the other hand, quantitative methods (60%) were more dominant than qualitative methods (35%) in Turkey (Çiltaş et al., 2012), whereas qualitative methods were more dominant than quantitative methods in Canada (Hannula, 2009).

It is interesting to observe the uniqueness of research methods employed in each country, either balanced, dominant, or integrated approaches of various research methods. The choice of research methods can be made by individual researchers' paradigmatic perspectives and research problems but might reflect dominant research paradigms of each country or be influenced by research topics. As shown in Fig. 2 and Table 3 which illustrate the breakdown of the research methods by research topics, the most frequently employed research methods differ by research topics. For instance, document analysis, specifically pedagogical analysis, was used for the studies on general research and curricula/textbooks. Quantitative research methods, specifically survey research, were the most frequently used for the studies on students' abilities/characteristics. Among qualitative research methods, the case study was the most popular for studies on students' abilities/characteristics, while development research was employed most frequently for studies on instruction/teaching methods. Some of these findings are quite obvious, while others raise the question why this could be the case. In the future studies, it would be worthwhile to analyze how the same research topics employ different research methods across different countries and to investigate whether the selection of different research methods has implications for the findings of the research topics.

4 Elaborations on Two Popular Research Topics in Korea

In critically reviewing and reflecting on Study 1 and Study 2, we found two popular research topics in Korea that need further elaboration: research on curricula or textbooks and research on teacher education. This section discusses the significance, challenges, and future directions of these two research topics.

4.1 Research on Curricula or Textbooks

Research on curricula or textbooks needs to be further elaborated in the Korean context. According to Study 1, research on curricula or textbooks was one of the most popular research topics and, more specifically, the top ten popular subtopics included both an analysis of teaching methods described in textbooks and general research on curricula or textbooks. According to Study 2, the research topic of curricula/textbooks placed second in the 23 topics in KCI articles, whereas it placed only 19th in SSCI articles. In fact, mathematics textbook research as a research topic is relatively new but has received growing interest by including various international comparisons, new or alternative forms such as interactive or electronic textbooks, textbook assessment, historical reflections, or cultural influence on textbook development (Schubring & Fan, 2018).

Korea has a national mathematics curriculum and employs mathematics textbooks aligned with the curriculum. Textbooks are the main resources for teachers to teach mathematics, and Korean teachers tend to faithfully cover mathematical tasks in the textbooks (Pang, 2008). Given these contexts, every effort is made to develop high-quality mathematics textbooks whenever the curriculum is revised. Various textbook-related studies have rapidly increased in recent years, specifically since the most recent revision of the mathematics curriculum in 2015. Such studies include an analysis of mathematical constructs or teaching methods depicted in the previous series of textbooks or teacher manuals. Curricula or textbooks from other countries are often compared and contrasted with Korean documents to search for alternative approaches. For instance, Lee and Pang (2019) compared and contrasted the teaching methods of fraction multiplication in Korean and Japanese elementary mathematics textbook series in terms of quantities with referent units, the meanings of fraction multiplication, and visual representations. This is why the topic of curricula or textbooks was the most frequent topic in the international articles published in the KCI journals that dealt with foreign documents or participants or were co-authored by scholars from other countries.

However, research on teachers' use or perception of textbooks has been scarce, compared to other curricula or textbook-related studies (Pang et al., 2019). Just as important as making good mathematics curriculum materials, teachers need to properly understand such materials and implement them in actual mathematics instruction. Specific attention is required in Korea to how teachers understand the intentions of tasks in the textbooks, to what extent they modify mathematical tasks in what contexts, and how such a modification leads to different opportunities for students to learn mathematics. Considering the popularity of the research on teachers' curriculum use in the international context (e.g., Remillard, 2005; Remillard & Heck, 2014), the lack of such research in Korea requires further attention. In a similar vein, students' use of textbooks and diverse types of curricular resources including digital resources are the research areas to be further studied.

4.2 Research on Teacher Education

Similar to international research trends (Hannula, 2009; Lo et al., 2014; Shin, 2020) and research trends in other individual countries (e.g., Çiltaş et al., 2012), research on teacher education has been rapidly increasing in Korea. Study 1 shows that 552 out of 4559 articles dealt with teacher education research, accounting for 12% of the total research. More specifically, only about 30 articles on teacher education were published in the 1990s, whereas about 170 articles were published in the 2000s and about 350 articles were published in the 2010s. According to Study 2, research on preservice teachers was the most popular among the 23 topics in KCI journals, approximately accounting for 7% of topic distributions. The rapidly growing attention on teacher education in Korea is noticeable but its associated contexts and challenges need further elaboration.

Unlike some other countries, teacher preparation programs in Korea do not face challenges of recruiting high-quality teacher candidates, challenges of rapid expansion of programs due to teacher shortage, or challenges of high teacher turnover rates (Akiba et al., 2007; Ingersoll, 2001; Kang & Hong, 2008). Due to job security and high respect for the teaching profession in Korea, only outstanding high school graduates are admitted to teacher preparation programs. Korea has a somewhat uniform curriculum across teacher preparation programs and similar teaching credential requirements across different geographical regions. Upon the successful completion of the four-year coursework, preservice teachers have to pass a competitive national teacher employment test to be hired as public school teachers. On the one hand, these sociocultural and institutional contexts of teacher preparation programs contribute to highly qualified beginning teachers. On the other hand, there has been a lack of research on critical reflections on teacher preparation programs along with their effects on teacher expertise. Specifically, further research is needed, beyond studies on preservice teachers' knowledge or beliefs, regarding alternative approaches to foster preservice teachers' learning and also effective methods to connect between university coursework and teaching practicum. As an example of such studies, Pang and Sunwoo (2021) analyzed the changes in preservice teachers' noticing through an elementary mathematics methods course along with a practicum, reporting that substantial changes in their noticing occurred after a practicum and subsequent discussions on their own lesson planning, implementation, and reflections.

Professional development programs for in-service teachers are also worthwhile to mention. According to Study 2, the research topic of professional development appeared as a distinct topic not in KCI journals but in SSCI journals. Moreover, this research topic was the second most popular research topic among 23 topics in SSCI journals. According to Study 1, among teacher groups, preservice teachers were more popular research participants than in-service teachers. Interestingly, the dominant research topic of preservice teachers with little research on in-service teachers was also observed in Turkey (Çiltaş et al., 2012). Owing to affordable teacher recruitment, selection, and employment in Korea as mentioned above, the effects of professional development programs for in-service teachers are not well explored. One reason for this may be that professional development opportunities for in-service teachers in Korea often emerge from their professional learning communities instead of from a large-grant research program developed by researchers at universities. Yearly training throughout a teaching career and professional development to enhance teacher expertise (e.g., professional learning community among teachers) often rely on teachers' voluntary willingness. Against this trend, a long-term sustained system of professional development has been called for, beyond supervision and observation in local schools as well as district-level training courses (Pang, 2018). Teacher professional development in China, Israel, Japan, Singapore, and the US were reviewed to explore implications for Korean mathematics teachers (Kwon et al., 2012). Weighing teachers' long careers in Korea, various and career-appropriate professional development programs need to be designed and implemented, followed by critical evaluations of such programs on whether they stimulate teacher learning and ultimately elicit high-quality teaching practices.

5 Closing Remarks

In closing this chapter, we would like to highlight that both commonalities and differences exist between Korean and international research trends in mathematics education. In particular, the increase of research articles and the diversification of research topics in Korea reflect the international research trends, but the underlying factors clearly indicate unique features in social, cultural, political, institutional, and educational contexts.

Despite sharing many commonalities, each country investigates specific topics and establishes its own research trends. For example, France has an emphasis on theoretical framework (e.g., didactical contract), Netherlands has a strong foundation on Realistic Mathematics Education, Russia focuses on talented education (e.g., mathematics Olympiads), Japan has a tradition on lesson studies and problem solving, and South Africa has an emphasis on equity and language (Hannula, 2009). It might be too early to characterize Korea's research trends in one specific research topic or approach, but two popular research topics discussed above (research on curricula/textbooks and research on teacher education) illustrate the particular issues, values, and contexts of mathematics education in Korea.

As a final remark, we would like to argue the usefulness of locating mathematics education research trends in the international context. We elaborated that there are considerable challenges and future directions for improvement in research on curricula or textbooks and research on teacher education in Korea. It is interesting that such challenges and directions emerge when we compare and contrast research topics in the KCI articles with those in the SSCI articles. We, as mathematics education researchers, need to activate international comparative or collaborative studies to better understand the research topics of a country, to better notice what gaps exist in the research trends, and to search for alternative approaches.

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Mathematics Education Research Trends in Turkey: International Research Context

Yüksel Dede, Gürcan Kaya, and Veysel Akçakın

Abstract The purpose of this chapter is to reveal trends in mathematics education research in Turkey and to discuss its similarities and differences in comparison with international research. This chapter starts with a section focusing on several challenges facing the education system, and mathematics education research in particular, in Turkey. In the following section, in order to better understand the trends of mathematics education research in Turkey, the trends of Turkish journals are compared with those in the two international journals. Articles published in three journals in Turkey (Educational Sciences-Theory and Practice [ESTP], Education and Science [E&S], and Hacettepe University Journal of Education [HJE]) and two journals that are popular in Europe and Asia (Educational Studies in Mathematics [ESM] and International Journal of Science and Mathematics Education [IJSME]) in terms of mathematics education and indexed in Social Sciences Citation Index (SSCI) were subjected to semantic content analysis in two sub-dimensions: their research content and their methods and techniques of research. Finally, implications for the future of mathematics education research in Turkey are discussed as conclusions.

1 Introduction

Until the second half of the twentieth century, education and culture were thought to be independent of values, as the positivist belief that scientific discovery and technological developments were based on rational, experimental, and objective criteria that

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were not tied to any particular social value system (Lee, 2001). On the contrary, nowadays, the view is that education, and mathematics education in particular, reflects the values of the culture it is in and is influenced by these values (see Bishop, 2002; Ernest, 2007). According to this new perspective, it is necessary to interpret the relations between people and the world with scientific knowledge and methods, rather than placing scientific knowledge, and mathematical knowledge in particular, in a field of cognition independent of the world (Tan & Kim, 2012). This new point of view toward mathematics naturally has an impact on the classroom applications of mathematics (Ernest, 1991) and also prompts teachers to review their teaching approaches (see Seah, 2003). It also provides a good basis for students to better appreciate how important mathematics is to understand and interpret the situations in today's complex economies and constantly changing conditions (Van de Walle et al., 2019). Similarly, mathematics education research is necessary to understand what mathematics is for students' lives and the whole society and how to teach it more effectively and meaningfully. In addition, mathematics education research needs to pay more attention to real-life contexts to uncover interdependent relationships between mathematics, people, and society (see Tan & Kim, 2012). In this context, firstly a brief information about mathematics education research in Turkey is given. Then, findings of the content analysis of the articles of International and Turkish journals, indexed in the SSCI in the last five years, are presented. With these findings, the similarities and differences between International and Turkish journals in terms of research contents, methods, and techniques in mathematics education, and thus the research trends of these journals, are revealed. Finally, implications for the future of mathematics education research are discussed in this chapter.

1.1 Mathematics Education Research in Turkey

As of April 2020, there are 203 universities and academies in Turkey, 129 of which are state institutions and 74 private universities (see Higher Education Council of Turkey [in Turkish: YÖK], 2020). These universities have 92 faculties of education and 90 faculties of Science and Literature. In the faculties of science and literature, only pure mathematics content courses are given, and no courses for educational sciences are included in the curricula of these faculties. In education faculties, on the other hand, pedagogical courses are given in addition to pure content courses, but pure content courses are not handled at a deep level as in science and literature faculties. In order for the graduates of the Faculty of Science and Literature to become teachers, they must also take initial teacher training courses (e.g., special teaching methods, introduction to education, assessment, and evaluation). However, the main responsibility for teacher training rests heavily on the Faculties of Education. The student-centered education-based accreditation of Education Faculties, started in 1997 (see Education Information Network in Europe [EURYDICE], 2010; YÖK, 2018). Within the scope of this accreditation process, two different mathematics teacher training programs were implemented: elementary mathematics teaching program (in Turkish: İMÖ) and secondary mathematics teaching program (in Turkish: MÖ). As of April 2020, there are 107 İMÖ programs in Turkey (at 95 state and 12 private universities) and 13 MÖ programs, all at state universities. Compared to overseas universities, academic research in mathematics education in Turkey started only recently. For example, while elsewhere doctoral studies on mathematics education and the establishment of mathematics education research societies date back to the 1900s, studies on mathematics education in Turkey only started in the early 2000s (see Education Information Network in Europe [EURYDICE], 2010; YÖK, 2018).

With the aforementioned accreditation process, since the beginning of 2000s, Educational Sciences Institutes were established for the first time in addition to the Institutes of Science (mathematics, physics, chemistry, engineering, etc.) and Social Sciences (history, psychology, law, etc.). Educational sciences institutes, unlike education faculties, focus on graduate education rather than undergraduate education. During this period, the number of graduates and doctorates in educational sciences in the educational sciences institutes increased, and educational sciences began to be seen as a separate branch of knowledge from the content areas. Parallel to the establishment of mathematics education as a separate discipline was the rapid increase in the number of educational science journals (e.g., Education and Science, Gazi University Journal of Gazi Educational Faculty, Hacettepe University Journal of Education, Eurasian Journal of Educational Research, Educational Sciences: Theory and Practice), and the inclusion of articles about mathematics education in these journals. In addition, journals that only publish research on mathematics education began to emerge during this period (e.g., Turkish Journal of Computer and Mathematics Education, Necatibey Faculty of Education, and Electronic Journal of Science and Mathematics). Similarly, the projects supported by the Scientific and Technological Research Council of Turkey (in Turkish TÜBİTAK) started to take place in the field of mathematics education for the first time (e.g., Akkoç et al., 2011; Kılıç & Doğan, 2018; Öksüz et al., 2011; Tanışlı et al., 2019). During this period, students were also sent to universities abroad (especially the USA, England, Germany, and France) for master's and doctorate degrees in mathematics education with the cooperation of Ministry of National Education of Turkey (in Turkish MEB) and YÖK. With the return of these students to Turkey at the end of their education, research on mathematics education in the country gained even more traction. In addition, Turkish mathematics education researchers started publishing articles in reputable international journals in the 2000s and continue to do so today.

2 Aims and Importance of the Study

The first of the important tasks of educational research is to conceptualize, observe, and systematically record the events and processes related to learning; the second is to analyze the data recorded in order to accurately determine the conditions and results of these observations, and the third is to contribute to the related subject using various theories. Furthermore, finding solutions to problems that do not require specialization by their nature, systematically and productively in social and human science research, can be assumed another important task (Yates, 2012). In addition, educational research helps to review and develop educational practices; in other words, they form the basis of educational decision-making (Kida, 1984). For example, conducting research on relevant policies and practices to reveal and prevent the causes of educational inequality will contribute to evaluations by providing new information and insights into these policies and practices (Reid, 2013). In this context, investigating the trends of these research activities will contribute to the improving practice. In the same way, international comparison studies are a prominent phenomenon in policymaking (Adamson, 2012). For these reasons, it is important to examine the researches according to the purpose and content and the methods used, and thus determine the possible trends and direction of these researches over time. In this respect, many studies have been conducted to determine the trends of research in the Turkish context both nationally and in comparison with international mathematics education research. For example, in the context of Turkey, Ulutaş and Ubuz (2008) examined the articles published in four Turkish educational journals between 2000 and 2006 and they found that most of the studies in the articles they examined were quantitative studies and were mainly conducted with preservice teachers that were about cognitive and affective dimensions of teaching and learning. Baki et al. (2011) examined doctorate and master's theses published in Turkey and they found that most of the theses about mathematics education between 1998 and 2007 were conducted using quantitative experimental design, usually in 6th, 7th, and 8th grades. Doğan and Tok (2018) examined articles in the field of educational sciences published in SSCI indexed Turkish journal Education and Science between 2007–2014 and they found that the examined articles were mostly quantitatively conducted with mostly preservice teachers chosen as participants. Selçuk et al. (2014) also examined articles published between 2007 and 2013 in terms of contents in Education and Science and found that the studies were mostly conducted using quantitative methods, especially descriptive survey method and preservice teachers were mostly chosen as participants.

Few studies in mathematics education have published internationally. For example, Inglis and Foster (2018) examined the articles in ESM and JRME journals since the date they first started publishing research. Foster and Inglis (2019) reviewed two of the UK's leading mathematics education journals (Mathematics Teaching and Mathematics in School) since their first publication. However, these studies can be seen as reviews of researches that remain at the national level. There has not been any comparison study between internationally published journals with Turkish journals. In this context, unlike the studies mentioned above, in this chapter we consider trends of articles published between 2009 and 2020 in two mathematics education journals (Educational Studies in Mathematics [ESM], and International Journal of Science and Mathematics Education [IJSME]), that are respected in the field of mathematics at an international level, and Turkish publications that were indexed in SSCI. In this way, this study aims to compare the trends of Turkish mathematics education researches with international mathematics education researches. Answers to the following questions were sought:

What are the similarities and differences between Turkish journals and international journals in terms of research content in mathematics education?

What are the similarities and differences between Turkish journals and international journals in terms of research methods and techniques in mathematics education?

3 Method

3.1 Research Design

The present study is a descriptive research based on document review (Seixas et al., 2018). Documents determined within the scope of the study were analyzed by content analysis. This analysis was carried out using six steps as follows: Step (1) Preparing data; Step (2) Creating main categories; Step (3) Coding according to main categories; Step (4) Compiling texts according to main categories and creating sub-categories inductively; Step (5) Conducting analysis based on categories and presentation of results; and Step (6) Reporting and documentation (Kuckartz, 2019). Content analysis, sometimes referred to as document analysis, includes methods and techniques that researchers use to examine, analyze and make inferences about their human communications (e.g., printed or written text, photographs, cartoons, illustrations, publications, and verbal interactions) (Julien, 2008).

3.2 Reviewed Documents

This section is limited to a review of articles of five journals which are indexed in SSCI, three of them based in Turkey origin, one in Europe, and one in Asia. The journals considered within this study were determined as follows: (a) Articles published in the Turkish journals, which have been indexed in SSCI in the last five years and (b) Articles published in ESM and IJSME. Within the scope of the research, three education journals originating from Turkey and indexed in SSCI in the last five years were identified as Education and Science (E&S), Educational Sciences-Theory and Practice (ESTP), and Hacettepe University Journal of Education (HJE). Some of the journals examined within the scope of the present study have varied history of publishing (ESM started its publication in 1968, Education and Science in 1976, HJE in 1986, IJSME in 2003, and ESTP in 2001). Brief information about these five journals examined is as follows:

Educational Sciences-Theory and Practice (ESTP) publishes articles in the field of education and educational research. It became one of the SSCI journals in 2007 in Turkey. The impact factor of ESTP is 0.532 in the year 2018, and the impact factor quartile of ESTP was Q4.

Education and Science (E&S) publishes articles in the field of education and educational research. It became one of the SSCI journals in 2007 in Turkey. The impact factor of Education and Science is 0.486 in the year 2020 and the impact factor quartile of Education and Science is Q4.

Hacettepe University Journal of Education (HJE) publishes articles in the field of education and educational research. It became one of the SSCI journals in 2007 in Turkey. The impact factor of Hacettepe University Journal of Education was 0.141 in the year 2015 and the impact factor quartile of Education and Science was Q4.

Educational Studies in Mathematics (ESM): ESM publishes articles in the field of mathematics education. It became one of the SSCI journals in 2009. The impact factor of ESM is 2.402 in the year 2020 and the impact factor quartile of ESM is Q2.

International Journal of Science and Mathematics Education (IJSME): It was launched in 1993 to provide both science and mathematics educators an opportunity to publish their papers in this journal sponsored by the National Science Council in Taiwan. The impact factor 2020 of IJSME is 2.073. The impact factor quartile of IJSME is Q3.

All the articles about mathematics education published by the journals analyzed during the years they were indexed in SSCI were included. In this sense, all articles published on mathematics education were examined during the specified years of the following journals: ESM and IJSME journals between 2009 and 2020 years, ESTP journal between 2007 and 2018 years, E&S journals between 2007 and 2020, and finally HJE journals between 2007 and 2015 years. As the IJSME journal has been indexed in SSCI since 2009, the articles published from 2009–2020 in this journal were examined. As the ESTP journal started to be indexed in SSCI in 2007, 2007 was determined to be the starting year, and 2018 was the ending year since it was not indexed in SSCI after 2018.

3.3 Data Analysis and Process

The process followed while applying descriptive content analysis is explained in detail in this section:

Step (1) Preparing Data: As mentioned above, articles of each journal related to mathematics education were included in the study. For example, since the ESM journal is only on mathematics education, all published articles from 2009, the year it was indexed in the SSCI, to the present day were examined. As IJSME accepts articles from science and mathematics. Only articles related to mathematics education were

Table 1 Number of publications of journals

	ESM Europa 2009–2020	IJSME Asia 2009–2020	E&S Turkey (2007–2020)	ESTP Turkey (2007–2018)	HJE Turkey 2007–2015	Total
Number of published articles	793	926	1131	1080	758	4688
Number of analyzed articles	793	361	105	79	48	1386

chosen for the study. Similarly, since E&S, ESTP, and HJE journals accept articles from all educational research fields, those related to mathematics education were selected from among the articles published in these journals. The articles related to mathematics education in these journals were first selected by two researchers. Then a consensus was reached by checking whether these selected articles were related to mathematics education. In addition, book reviews in journals were not included in this study and a total of 1386 articles were examined (see Table 1).

The full texts of the articles in each issue were obtained online from the web pages of the five journals examined within the scope of the study. Researchers used their own universities' databases to access the articles in the journals examined. Articles in the reviewed journals were coded according to year, volume, number, and order of the article in the web page. For example, an article published in ESM journal in 2018, which was published in the second volume, third issue, and fourth place according to the web page of that issue was coded as ESM-2018-2-3-4. In this way, the articles to be examined were arranged separately for each journal.

Step (2) Creating main categories: Creating main categories is explained in "2.a. Research Contents" and "2.b. Methods and Techniques". The data were analyzed using content analysis. It is the process of creating a category to explore the main subject categories at the core of the data analyzed and the specific sub-categories that these areas contain. In this sense, in this chapter, *research contents, methods, and techniques* were the main categories. The specific sub-categories for the research contents based on the framework of Chiu et al. (2016) were 11 sub-categories: *teacher education, teaching, learning (cognitive), learning (affective), goals, policy and curriculum, evaluation and assessment, cultural, social, and gender issues, history, philosophy, epistemology, and nature of mathematics, educational technology, informal learning, textbook,* and *text analysis.* Similarly, the specific sub-areas for the methods and techniques were four sub-categories: *research design, participants, data collection tools, and data analysis methods.* Table 2 summarizes general categories and sub-categories of mathematics education research.

Brief explanations for each general category and sub-category were given below:

Table 2 General categories and sub-categories of mathematics education research

General categories	Sub-categories
Research content	Teacher education, teaching, learning (cognitive), learning (affective), goals, policy and curriculum, evaluation, and assessment, cultural, social, and gender issues, history, philosophy, epistemology, and nature of mathematics, educational technology, informal learning
Methods and techniques	Research design, data collection tools, data analysis methods, participants

2.a. Research Contents

The 11 sub-categories examined under this category and their short explanations are below:

Teacher education: This sub-category covers prospective teachers and teacher education issues (teacher and prospective teachers' cognitive dimensions, teaching style, subject and pedagogical content knowledge etc.), affective dimensions (i.e., teacher and prospective teacher attitudes, values, beliefs, etc.) and professional development of teachers (e.g., lesson study), etc.

Teaching: This sub-category includes teaching theories, methods, techniques, etc. For example, discovery learning, component display theory, problem solving methods, etc.

Learning cognitive: It focuses on the cognitive dimension of students' learning. For example, concept learning, learning styles, mathematical process skills (problem solving, communication, reasoning, and proof etc.), argumentation, and metacognitive strategies, etc.

Learning affective: It includes the affective dimension of students' learning and classroom atmosphere. For example, values, beliefs, emotion, attitude, peer interactions, individual differences, etc.

Goals, policy, and curriculum: It includes education policies and aims, identifying effective schools, curriculum policy and reform and evaluation, etc.

Evaluation and assessment: It focuses on different evaluation and assessment approaches, development and implementation of questionnaire, educational measurement, etc.

Cultural, social, and gender issues: It includes cultural differences (multiculturalism, international comparative research, bilingualism etc.), socio-economic dimensions, gender differences, etc.

History, philosophy, epistemology, and nature of mathematics: This sub-category deals with the nature, history, philosophy, psychology, moral and ethical issues, literacy, and theory of mathematics, etc.

Educational technology: It comprises information and communication technologies.

Informal learning: This sub-category includes informal contexts such as museums, outdoor settings (e.g., street mathematics), and public awareness of mathematics.

Curriculum, textbook, and text analysis: It includes the analysis of mathematics curriculum, textbooks, and texts etc.

2.b. Methods and Techniques

The four sub-categories examined under this category and their short explanations are below:

Research design: It includes studies using quantitative, qualitative, mixed methods, theoretical and review articles, and documents etc.

Participants: It includes students with grade levels, pre-service teachers, teachers (in-service), parents, and mathematics education experts (having Ph.D. degree).

Data collection tools: It includes surveys, interviews, observations, documents (e.g., textbooks, texts, materials, etc.)

Data analysis methods: It covers quantitative (e.g., descriptive, and inferential statistics), qualitative (e.g., constant comparative analysis, content analysis), and combinations of the two analyzes.

Considering the possibility of missing the meaning of the sentence in the analysis of words and paragraphs (Yıldırım & Şimşek, 2008) in this study paragraphs were chosen as our analysis unit. For example, in some articles, the method is not clearly stated (e.g., case study); if the section/paragraph in which the working process is explained is examined, it is understood that the method used is qualitative or quantitative. Thus, the paragraphs are used as the unit of analysis of this research.

Step (3) Coding according to main categories: After the articles to be examined were determined, each article was coded by two researchers separately. Articles were coded according to the main categories at this stage. Analyzes were done with computer-assisted qualitative data analysis software (e.g., ???).

Step (4) Compiling texts according to main categories and creating sub-categories inductively: In the fourth step, after the data coded according to the main categories were reviewed, they were analyzed more deeply and recoded according to the sub-categories. For example, let's assume that a section in the article was coded as "methods and techniques", which is one of the main categories, in the third step. In this case, in this step, this section, which was coded as the methods and techniques main category, was examined and re-coded as the data collection category under the methods and techniques which is one of the main categories.

Step (5) Conducting analysis based on categories and presentation of results: The data obtained at this stage can be presented quantitatively or qualitatively (Kuckartz, 2019). In this research, the large data obtained were interpreted quantitatively. The data were converted into quantitative data using computer-assisted qualitative data analysis software, and the results were interpreted by the researchers and presented with tables and graphics.

Step (6) Reporting and documentation: The results and comments on the content analysis of the articles of each journal were reported by the researchers in the results section using frequency and percentage tables and graphs according to the main categories and subcategories.

3.4 Data Analysis Process and Trustworthiness of the Study

After determining the articles to be examined, the researchers checked the suitability of the coding scheme by coding 10 articles separately before proceeding to the main coding stage. After the suitability of the coding scheme was decided, the main coding phase was initiated. The coding phase lasted 4 months. The researchers analyzed the articles according to the coding scheme and reviewed these coding every three weeks. The reliability of data was ensured with peer review (Lincoln & Guba, 1985). The coefficient of concordance among the researchers was calculated as 0.93. This value indicates that there is a significant agreement in coding between researchers (see Landis & Koch, 1977). Since articles are complex in terms of *research contents*, *method, and technique*, an article can be coded according to more than one sub-category scheme.

In the case of a disagreement between the coding, the codes in disagreement were reviewed. The categories according to which these disagreed codes should have been recoded were determined by consensus of the two researchers.

4 Results and Discussion

The results of the study are presented in two separate titles according to the research problems: Research contents, methods, and techniques.

(a) **The trend of mathematics education researches according to research contents**

The trends of the journals examined within the scope of the current study for the research contents were examined in 11 sub-categories (see Fig. 1).

As can be seen from Fig. 1, the publications in the journals examined within the scope of this study are mainly for *teacher education* and *learning cognitive* categories. After these two categories, studies are published mainly about *teaching* category. In addition, it is seen that the articles are similar in these aforementioned three categories. On the contrary, very few studies have been published in all journals focusing on *curriculum, textbook,* and *text analysis*. In addition, unlike articles in IJSME and Turkish journals, more studies are published under the category of h*istory, philosophy, epistemology*, and *nature of mathematics* in ESM. Additionally, with respect to *cultural, social, and gender issues; educational technology; goals,*

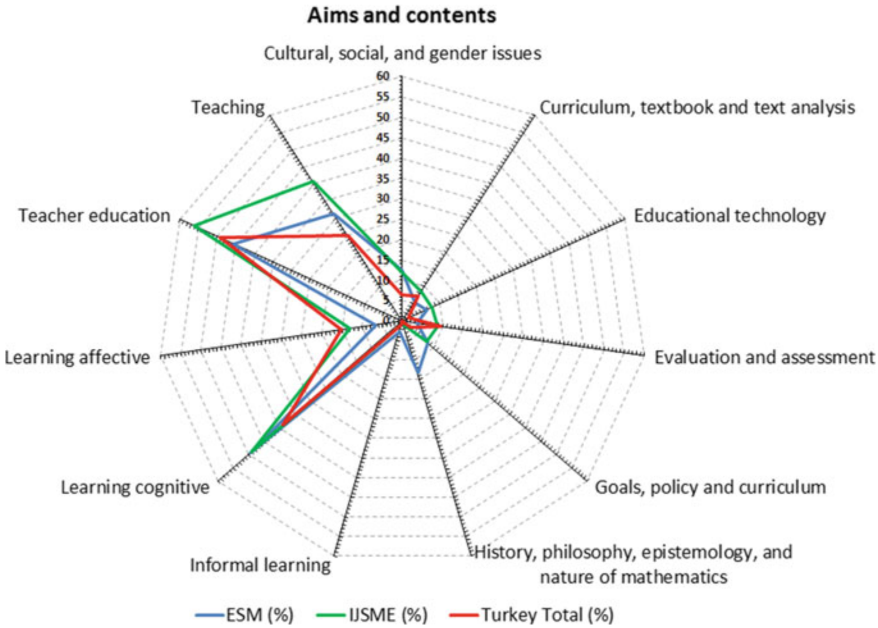

Fig. 1 Trends of mathematics education studies according to subcategories of research contents

policy, and curriculum; and professional development of teachers and *teaching*, it was determined that more articles were published in ESM and IJSME than in Turkish journals. More articles about *evaluation and assessment*, *learning affective* and *affective dimensions of teachers, and preservice teachers* have been published in the Turkish journals than in ESM and IJSME. Finally, it was determined that, in all journals examined within the scope of this study, very few articles about *informal learning* were published.

(b) **Trends of mathematics education research according to Methods and Techniques**

The trends of the journals examined within the scope of the current study for methods and techniques were divided into four sub-categories, and each category was divided into sub-categories.

In this section, articles in journals are analyzed according to sub-categories of *research design, data analysis methods, data collection tools* and *participants*.

As seen in Fig. 2, *quantitative methods* are dominant in Turkish journals in terms of research design, while *qualitative methods* are dominant in ESM and IJSME. The mixed method is preferred in Turkish journals more than in ESM and IJSME. On the other hand, in ESM and IJSME *design-based* researches and *theoretical and review* researches are preferred more than the Turkish journals. As can be seen from Fig. 2, in terms of *data analysis methods*, in accordance with the research design, *quantitative data analysis* is more prominent in Turkish journals, while *qualitative* *data analysis* is more prominent in ESM and IJSME. In terms of the articles in which the two types of analyses (Qual and Quan) are used together, it is seen that all journals are similar. As data collection tools depend on the research method chosen, as seen in Fig. 2, in ESM and IJSME journals where *qualitative research methods* are dominant, interview, observation, and videotapes data collection tools are more preferred than in Turkish journals. In Turkey, where *quantitative research methods* are more dominant, it is seen that *questionnaire* and *test* are preferred more than in ESM and IJSME. Finally, in terms of participants, it is seen that there are more studies with *kindergarten, grade 1–5 students, expert, parents'* participant groups in ESM and IJSME compared to journals in Turkey (see Fig. 2). As for the journals in Turkey, it is seen that studies are carried out with *preservice teachers* and *grade 6–8 students* at a higher rate than in ESM and IJSME. It is seen that all the journals in Turkey and the ESM and IJSME journals carried articles about grade 9–12 students at similar rates.

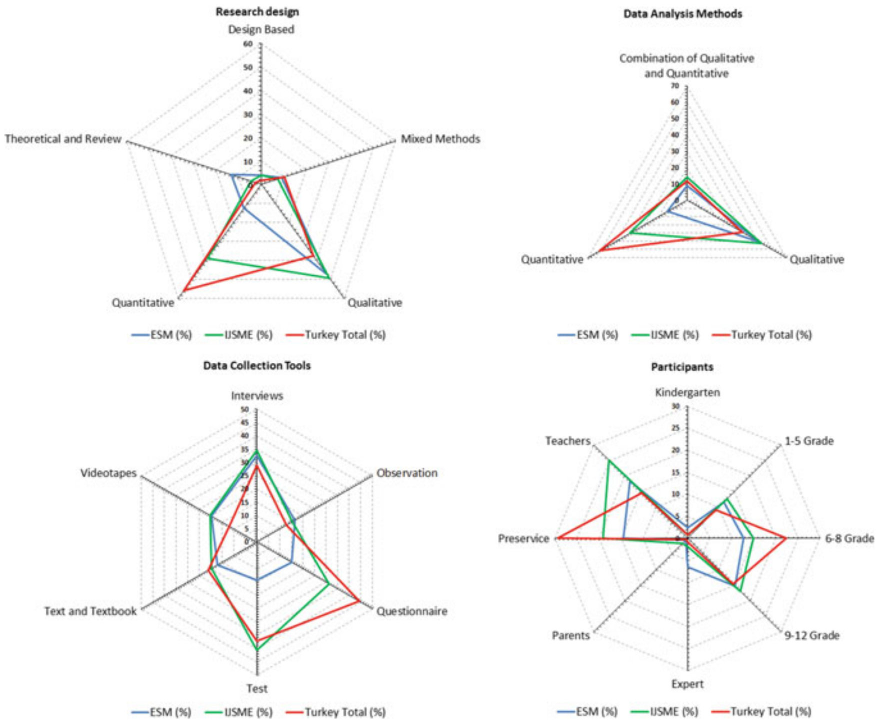

Fig. 2 Trends of mathematics education studies according to sub-categories of methods and techniques

4.1 Results and Discussion About Research Contents

In the context of the current study, it has been determined that more articles about *cultural, social, and gender perspectives* have been published in ESM and IJSME journals compared to Turkish journals (E&S, ESTP, and HJE). This result coincides with the results of studies by Ulutaş and Ubuz (2008). This is because Inglis and Foster (2018) examined the trend of research published in ESM journal in the last 50 years and revealed that the studies on mathematics education in the socio-cultural field have increased considerably. Similarly, in the current study, it has been found that there is a considerable amount of socio-cultural mathematics education studies in ESM and IJSME journals (especially in ESM) compared to the journals in Turkey (E&S, ESTP, and HJE), which are considered within the scope of the study. Also, similar to the results of the present study, Ulutaş and Ubuz (2008) found that very few articles related to the cultural and socio-cultural dimension of mathematics education were published. One reason for this may be that the international recognition of ESM and IJSME journals is much higher than the journals in Turkey (E&S, ESTP, and HJE), and the researchers publishing in ESM and IJSME journals may tend to publish more sociocultural studies. In addition, it is seen that the articles published in this category in the journals in Turkey (E&S, ESTP, and HJE) are mostly gender related.

It is seen that there are more published articles in the field of *educational technology* in the journals of ESM and IJSME than in Turkey. More importantly, in two journals (ESTP and HJE) in Turkey, there were no published articles in the field of *educational technology* in mathematics education in the years when they were indexed in the SSCI, while E&S journal has relatively few articles compared to other categories. This result is consistent with Chiu et al. (2016), Ulutaş and Ubuz (2008), and Baki et al. (2011). In their study, Chiu et al. (2016) found that few articles were published in the field of *educational technology* in the journals they examined. In addition, in the theses which have been examined in the study of Baki et al. (2011), it is seen that the categories that include the use of technology in mathematics education and teaching remain at a minimum rate compared to other categories. Furthermore, Ulutaş and Ubuz (2008) determined that there are very few articles about *educational technology* in the journals they have examined. Despite the huge incentive for the use of technology in education in Turkey (e.g., Movement of Enhancing Opportunities and Improving Technology (in Turkish: FATİH), interactive boards, tablets, etc.), the publishing of a few articles on the use of technology in mathematics education in the Turkish journals when they were indexed in the SSCI may have resulted from the fact that the researchers sent their publications about *educational technology* to journals that were specific only on the field of *educational technology* or the articles submitted in this field did not go through review processes or the journal publishing policies of the relevant years or the editors' preferences. It can be said that this situation may have been reflected in the articles examined in this study, and therefore, few articles have been published in the journals in Turkey about educational *technology category*.

It is seen that more articles about the evaluation and assessment dimensions have been published in the Turkish journals examined within the scope of the current study compared to the ESM and IJSME. This is similar to the results of the study conducted by Doğan and Tok (2018). They examined the articles about educational sciences published in the E&S journal and found that scale development is the third most studied subject in the field of educational sciences. In addition, the results of the present study show that most of the articles published in the examined Turkish journals about *evaluation and assessment* category focus on the scale development about affective dimension rather than using a measurement technique in the field of mathematics education. One consequence of this is that there are more articles about affective dimension in the Turkish journals (E&S, ESTP, and HJE) than ESM and IJSME.

In the category of *goals, policy, curriculum*, it is seen that more articles are published in ESM and IJSME journals. Baki et al. (2011) found that a small number of the thesis were carried out in the field of mathematics education curriculums, and in particular, it is observed that there is almost no thesis about this subject before 2005, when the mathematics curriculum reform has been made in Turkey, and after 2005, there has been a considerable increase in the curriculum area until 2007. In addition to this, as it can be understood from the results of this study, it can be said that although the curriculum updates were made in Turkey repeatedly in 2013 and 2018, the increase that stated in their study and the other updates made in the curriculum were not reflected in the articles published in the Turkish journals (E&S, ESTP, and HJE).

In the category of *history, philosophy, epistemology*, and *nature of mathematics*, no articles have been published in the Turkish journals (E&S, ESTP and HJE), whereas there are a few articles in EJSME and a large number of articles in ESM. This result coincides with the results of studies by Foster and Inglis (2019), Inglis and Foster (2018), and Ulutaş and Ubuz (2008). In the studies of Foster and Inglis (2019) and Inglis and Foster (2018), it is seen that few articles have been published in the journals examined about the category of *history, philosophy, epistemology, and nature of mathematics* compared to other fields. In contrast, similar to the results of this study, Ulutaş and Ubuz (2008) found that no articles were published in the journals they examined in Turkey about the category of history, philosophy, epistemology, and nature of mathematics. The study by Ulutaş and Ubuz (2008) included the years 2000–2006 and the E&S and HJE journals that were also examined in the current study. Although this study is up-to-date, there have been no publications in the categories mentioned above in journals indexed in SSCI in Turkey since 2006. One reason may be that Turkish mathematics education researchers do not pay enough attention to the categories of *history, philosophy, epistemology, and nature of mathematics*.

It is seen that the articles published about *teacher education*, *learning cognitive,* and *teaching* categories in the examined journals are on an equal basis, but in the affective dimension, more articles are published in Turkish journals (E&S, ESTP, and HJE) compared to ESM and IJSME journals. This result is consistent with the results of the studies by Inglis and Foster (2018), Selçuk et al. (2014), and Ulutaş and Ubuz (2008). Inglis and Foster (2018) found that fewer articles about *affective dimension* were published in ESM journal compared to other research areas. Selçuk et al. (2014) and Ulutaş and Ubuz (2008) determined that the most publications about the *affective dimension* were made in Turkish journals (E&S and HJE) which were also examined within the scope of this study. It may be due to the fact that, as mentioned above, researchers in Turkey are mostly inclined to conduct studies on scale development. Despite the fact that the number of publications about the *teacher education* category is similar, if the participant groups are taken into consideration, the frequent selection of pre-service in the researches in the Turkish journals constitutes evidence that the studies about *teacher education* in Turkey are mostly done with pre-service teachers. Therefore, it can be said that more studies about professional development of in-service teachers are carried out in ESM and IJSME journals than in Turkey journals (E&S, ESTP, and HJE). This result is consistent with the results of Baki et al. (2011), Doğan and Tok (2018), Inglis and Foster (2018), Ulutaş and Ubuz (2008). Ulutaş and Ubuz (2008) found that there are very few publications about in-service *teacher education.* In addition, Baki et al. (2011) found that there is a limited number of thesis about in-service teacher education. As is evident from the results of the current study, publications about *teacher training* in recent years have remained almost similar over the examined years. In the study of Inglis and Foster (2018), it can be seen that there is an increasing number of publications about in-service *teacher education* in the ESM in recent years.

4.2 Results and Discussion About Methods, Techniques, and Participants

In the journals examined in the current study, it is seen that in the Turkish journals (E&S, ESTP, and HJE), researchers tend to use *quantitative methods*, whereas in ESM and IJSME, researchers tend to use *qualitative methods*. This result can be said to be similar to the results in the studies of Inglis and Foster (2018), which reveal that experimental studies have decreased gradually in the ESM journal in the recent years; and Doğan and Tok (2018), Baki et al. (2011), Selçuk et al. (2014), and Ulutaş and Ubuz (2008), who revealed that the *quantitative methods* were dominant in the articles they examined. One of the reasons for the greater preference of *quantitative methods* in Turkish journals as suggested by Selçuk et al. (2014) is that quantitative research is easier and faster than qualitative methods in terms of feasibility and data analysis, and that sampling is easy to reach. Doğan and Tok (2018) stated that this may be because qualitative research requires more time and effort than quantitative research. In addition, some of the reasons why researchers prefer methods that requires less effort and time may be that researchers want academic titles quickly or that their institutions are forcing researchers to do many studies. It can be seen that researchers used the mixed method more frequently in articles published in Turkish journals (E&S, ESTP, and HJE) compared to ESM and IJSME (see Fig. 2). If the results of the current study are compared with the results of Selçuk et al. (2014), and Doğan and Tok (2018), it is found that there is an increase in the trends of researchers to use *mixed method* in the articles published in Turkish journals. The fact that ESTP has been accepting only *mixed method* researches since 2016 as the publication policy can be considered one of the reasons why mixed methods research appears more in Turkish journals than ESM and IJSME. In addition, *design-based researches* in ESM and IJSME is seen to be higher than the Turkish journals (E&S, ESTP, and HJE). In the *theoretical and review* category, it is seen that the articles about this category in Turkish journals (E&S, ESTP, and HJE) are less than the ESM and IJSME journals; in other words, the trends of the Turkish researchers are not in this direction. This result overlaps with the studies of Doğan and Tok (2018), Selçuk et al. (2014), which reveal that the researchers for the *review* category have very little place in the articles of the journals they examined. For example, as already mentioned, researchers in Turkey tend to use quantitative methods, therefore preferred data analysis methods and data collection tools that are in a way compatible with quantitative methods (see Fig. 2). This also applies to the trends of research, research and data analysis methods and data collection tools in ESM and IJSME. In studies published in the journals of ESM and IJSME, researchers generally tended to use qualitative methods, therefore the data collection tools and data analysis methods they used were in line with this qualitative method they preferred (see Fig. 2). Similar results can be seen in the studies conducted by Chiu et al. (2016), Doğan and Tok (2018), Selçuk et al. (2014), and Ulutaş and Ubuz (2008). Since the quantitative method was more dominant in these studies, data collection tools and data analysis methods were also quantitatively weighted.

As for the participant group, it is observed that researchers who publish articles in Turkish journals (E&S, ESTP, and HJE) tended to work with pre-service teachers. In contrast, the working group of articles in ESM and IJSME are generally teachers. In addition, next to pre-service teachers, in Turkey, studies were mostly carried out with 6-8th grade students as a participant group. These results coincide with the results of studies conducted by Doğan and Tok (2018), Selçuk et al. (2014), and Ulutaş and Ubuz (2008). Doğan and Tok (2018), Selçuk et al. (2014), and Ulutaş and Ubuz (2008) also found that researchers in Turkey generally worked with pre-service teachers and then conducted their studies with 6-8th grade students. The fact that pre-service teachers are easily accessible and the need to reach more sample groups in quantitative researches (e.g. scale development, survey methods, etc.) due to the greater preference of quantitative methods in Turkish studies may have been a factor in selecting more of pre-service teachers as research groups in Turkish studies. This may also be an indication that institutions do not devote much time and resources to researchers to conduct in-depth studies on issues related to primary and secondary school students. This situation is also compatible with the work of Ulutaş and Ubuz (2008). In addition, it was stated above that there are many studies about the professional development of teachers in ESM and IJSME journals. As an expected result of conducting studies about professional development of teachers, more selection of teachers as a study group can be seen as a common occurrence. In Turkey, the number of studies involving families as a participant group remained very limited. While there were no studies in ESTP and HJE in which families were participants, there were very few studies in E&S. There are more studies in ESM and IJSME where families are included as working group compared to Turkish journals. Although parents in Turkey are very fond of their children and follow their education and training processes very closely, conducting very small number of studies involving parents may be seen as a lack of researches and may be due to the researchers' preference in this direction. In addition, it is seen that kindergarten has very little place as a study group in researches in Turkey. One reason for this can be the small number of researchers with a Ph.D. in mathematics education in kindergarten.

To sum up, it is expected that the contributions of this study for researchers in Turkey may be interesting and relevant in terms of Turkish (local) and to the international context. In this sense, in terms of the local and international context, for mathematics education researchers in Turkey, this study may present a framework specific to Turkey, which will guide future studies on which direction the trends are, which topics need to be emphasized more, or which topics are lacking in mathematics education. In addition, mathematics education researchers can develop their research by comparing them with the trend of international studies; thus, this can increase the international acceptability of researchers' publications in Turkey. In this way, this could contribute significantly to research in mathematics education and thus to practice in teaching settings in Turkey. Eventually, the available results may also provide elements to specific critical and cultural perspectives for environment of mathematics education in Turkey.

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Research and Research Culture in Mathematics Education: The Case in Macao, China

Kwok-Cheung Cheung, Chunlian Jiang, and Lianghuo Fan

Abstract Macao, a Special Administration Region of China (Macao SAR), is unique in that its mathematics education practice and research have integrated both the eastern and western traditions. Its participation in the Programme for International Student Assessment (PISA) since 2003 inspired the research culture in mathematics education in Macao from international perspectives. In this article, we report on a survey of three kinds of research done in mathematics education in Macao: (1) research related to PISA, in particular PISA 2012 and PISA 2018; (2) research done for master's and Ph.D. theses/dissertations in higher institutions in Macao; and (3) articles published in educational journals, particularly in mathematics education in Macao. The survey reveals emerging research cultures in mathematics education over the past decade—the interests of researchers and practitioners in topics such as comparative studies, lesson studies, mathematical problem-solving, and the use of information technology in mathematics teaching and learning. Lastly, we summarize what Macao has done well and what it needs to do better for further development of Macao's research culture within the global trend of literacy-based mathematics education as modeled by the PISA.

Keywords Research culture · Mathematics education · Macao · PISA

1 Introduction

Macao,[1] a Special Administration Region of China (Macao SAR) since 1999, has been a hub for cultural interflow and mingling between the West and the East for

[1] Macao is the English spelling of the name, and Macau is its Portuguese spelling. Internationally, the two spellings are used interchangeably.

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over 400 years. Related to this long history, the education system in Macao has creatively integrated the western and eastern traditions and has been unique in many respects. Compared with the Chinese mainland, Hong Kong, and Taiwan, Macao is not competitive for high school graduates to succeed in applying for a higher education institution as the transition rate from secondary to tertiary education has been more than 90% in recent years (DSEJ, 2019a). Mathematics education in Macao, on the one hand, has maintained the Chinese tradition focusing on the importance of mathematics for children's future development and the role of practice in their mathematics learning. On the other hand, it is also open to the rapid development of mathematics curriculum, assessment, information and communication technology (ICT), and their implications to the research and classroom practice in mathematics education. Therefore, by studying research culture in mathematics education in Macao, we hope that it can also provide meaningful insight into the global development of mathematics education.

In this chapter, we first briefly introduce the structure of the education system and mathematics education in Macao for readers to understand the contexts of the study. Then, we survey the research in mathematics education in Macao in three areas: (1) research related to the Programme for International Student Assessment (PISA), which is the main research topic that mathematics educators in Macao have done since 2003 and that has helped policy-makers to improve mathematics education practice in schools; (2) research done for the master's and Ph.D. degrees in Macao; and (3) articles published in educational journals in Macao in the past decade, though the majority of these articles were not research-based, rather they were practice-oriented. Including the last collection of articles aims to identify what reflective mathematics educators did at the school level, what they focused on in real mathematics classrooms, and what are the areas that they need to pay attention to. Many of the last set of articles were written jointly by researchers and practitioners, discussing the learning difficulties that students might experience in mathematics classroom and providing better lesson plans to help students overcome their difficulties. This kind of researcher–practitioner partnership for conducting research has been advocated by mathematics educators (Cai et al., 2017). Therefore, we include them in the chapter. Based on the survey, we shall be able to describe the overall characteristics of research culture in mathematics education in Macao and the challenges that it is facing.

2 Characteristics of Macao's Research Culture in Mathematics Education

To cut the story short, education in Macao is characterized by a diverse pluralistic system, allowing the co-existence of various forms of education (Wang, 2009). For example, there are Chinese-medium schools, English-medium schools, and Portuguese-medium schools. The variety of school systems provides alternatives for parents and students. Unlike the Chinese mainland, Hong Kong, and Taiwan where public education is prevailing, private schools are the mainstay of Macao education (Wang, 2009). Based on the education statistics of 2018–2019, there are 74 schools in total in Macao and 77% of them are private schools. About 96.5% of the students were enrolled in private schools (DSEJ, 2019b). For this reason, the Education and Youth Affairs Bureau (DSEJ,[2] abbreviation in Portuguese), which is the administration department of Macao government for education, gives schools great autonomy in determining and selecting instructional medium, teaching materials and contents, and even developing school-based curricula to cater for students' individual and diversified needs. In addition, the DSEJ has initiated a series of educational reforms to promote the development of Macao's basic education since the handover to China in 1999. For instance, in 2006, the Non-tertiary Education Law was promulgated, aiming at protecting the rights of children for a fifteen-year (K1-3 plus 12 years of formal schooling) free compulsory education. Within ten years, the Macao government's investment in non-tertiary education increased from MOP22,819 (about USD2860) per student in 2007 (DSEJ, 2009) to MOP88,555 (about USD11,090) in 2017 (DSEJ, 2019b). With the financial support from the DSEJ, the class size has been reduced from more than 45 to 25–35 (Vong, 2013). The primary characteristics of Macao's research culture in mathematics education are that it is mostly school-based in the private schools and carried out by the autonomous educational practitioners, taking advantage of the Macao Government's investment in basic education and school sponsorship to achieve the ideal of Education-for-All in the first two decades of the new century.

DSEJ invited mathematics educators from local and the Chinese mainland to develop the Basic Academic Attainments (BAA) for major school subjects at the formal education level. Regarding mathematics, the following three documents were released: BAA in Mathematics for Primary Schools (BAAMPS, 小学数学基本学力要求) (Tam, 2016), BAA in Mathematics for Middle Schools (BAAMMS, 初中教育阶段数学基本学力要求) (Tam, 2017b), and BAA in Mathematics for High Schools (BAAMHS, 高中教育阶段数学基本学力要求) (Tam, 2017a). They stipulated the basic proficiency in knowledge, skills, abilities, attitudes, and values upon the completion of primary, middle, and high school mathematics education. The three BAAs in mathematics were implemented since 2016–2018 after being piloted in several schools. They spell out the minimum standards at various stages instead of the "ceilings" which allow schools to develop their own curricula based on their educational visions, missions, and students' abilities (Wong et al., 2015). There are only one series of mathematics textbooks for primary schools and one series for middle schools written by local mathematics educators. However, they are not widely used in local schools. Most Chinese-medium schools actually adopt textbooks used in the Chinese mainland, while most English-medium schools use textbooks from Hong Kong (Tang, 1999). The implementation of different mathematics curricula brings about different teaching practices in schools (Oliveira et al., 2015). However, many Macao teachers tend to teach mathematics in a traditional way emphasizing exercises

[2] Education and Youth Affairs Bureau (DSEJ) and the Higher Education Bureau (DSES) were merged into the Education and Youth Development Bureau (DSEDJ) in February 2021.

with variations and controlling classroom activities although they also encourage students to be engaged in the process of learning (Huang & Leung, 2004). In order to create an atmosphere of research on teaching among school teachers, DSEJ has also initiated the Award Scheme on Instructional Design (ASID) to encourage in-service teachers to develop lesson plans as a teaching unit or a course/program spanning a semester or an academic year, to do action research in their own classrooms and run open classes for peers to observe and for the reviewers to evaluate. The designs that are awarded will be uploaded to the DSEJ website for other teachers to use (DSEJ, 2020). It is hoped that teachers' teaching abilities will be enhanced in the process.

Starting from 2009, the Macao Mathematics Education Research Association has been organizing groups of high school students to attend the International Regions Mathematics League (IRML), which is a part of the American Regions Mathematics League (ARML). Macao teams were the winner of the Onsite Division of IRML in 2009, 2015, and 2018 (ARML, 2019; Macao Daily, 2018). All these programs and efforts have brought about observable achievements in education in Macao, particularly in mathematics education. For instance, the mathematics performance of Macao students in Programme for International Student Assessment (PISA) showed big progress in mathematics literacy, improving its ranking from 12th in 2009 to 3rd in 2018. Table 1 revealed that (a) Macao students' performance (scores) consistently improved from 2003 to 2018, in particular, in the past 10 years; and (b) the performance of the female students improved a lot and the gender differences decreased consistently (Cheung et al., 2020).

Thus, another key characteristic of Macao's research culture is that the government has been increasingly exercising tighter control and monitoring over the quality of academic provision to the students, and this has immense implications on the kind of curriculum and instruction research that the schools and teachers embarked in order to improve student performance in mathematical literacy.

Table 1 Mathematical literacy performance of Macao's 15-year olds in PISA (2003–2018)

PISA cycle	Rank	Score	SD	Males' average	Females' average	Difference (F − M)
2018	3	558	81	560	556	− 4
2015	3	544	80	540	548	8
2012	6	538	95	540	537	− 3
2009	12	525	86	531	520	− 11
2006	8	525	84	530	520	− 10
2003	9	527	87	538	517	− 21

3 Research and Research Culture in Mathematics Education in Macao

Culture is defined differently in different areas of research (Townes, 2018). The one that is closely related to the current study is "*the total range of activities and ideas of a group of people with shared traditions, which are transmitted and reinforced by members of the group*" (Townes, 2018, p. 56). Therefore, it is helpful to examine what mathematics educators including school mathematics teachers, who are the implementers of the intended curriculum and the practitioners of mathematics education, do in the recent years for describing the characteristics of research culture in mathematics education in Macao. In this section, we shall look at the research in mathematics education in Macao in the following three aspects: (1) research related to PISA; (2) research done for master's and Ph.D. degrees in higher institutions in Macao; and (3) articles published in educational journals in Macao. It will help us to know what had been done, which provides a base for us to describe the characteristics of mathematics education and its emerging research culture in Macao.

3.1 Research Related to Macao-PISA

PISA does not only test students' proficiencies in mathematics, science, and reading, but also does collect data through questionnaires from several stakeholders. The data allow us to describe Macao students' mathematics learning characteristics in more details and to identify factors that affect students' performance including quality and equity in mathematics education. In recent cycles, almost all the 15-year olds in Macao are participated in PISA; therefore, the Macao-PISA data are a rather complete set of data and results obtained from PISA are used for DSEJ's policymaking.

3.1.1 Secondary Analysis of the PISA Data

In the past decade, three lines of research are exemplary of the emerging research culture on mathematics education due to the secondary data analysis of the Macao-PISA data. The first line of research is the identification of factors affecting mathematical literacy performance, and after identification of key issues of upmost concern by Macao educational community, mechanisms and processes of the influencing factors cast on the stakeholders need to be clarified. The past two decades also witnessed a shift of assessment from one essentially centered on outcomes of schooling to the processes of classroom instruction effecting student academic performance. In line with this shift, there is a change from print-based to digital assessment, necessitating research into analysis of this kind of process data collected in the assessment platforms.

1. *Identification of factors affecting mathematical literacy performance.* It is important to identify pertinent factors (e.g., mathematics efficacy, mathematics interests, mathematics anxiety, etc.) affecting variations in mathematics performance of the full cohort or a subpopulation of it (e.g., low-achievers) (Cheung, 2015; Cheung et al., 2013, 2014; Sit et al., 2017). For instance, Sit et al. (2017) studied the prediction of digital problem-solving performance of the low-achievers in Macao by perseverance, openness, and use of ICT. Ieong et al., (2016a, 2016b) have investigated the effects of home socioeconomic status on mathematics performance through multilevel mediation analysis of self-regulated learning processes of Macao students in PISA 2012. Their research findings help understand why Macao is rated as a "high-performing high-equity" basic education system among all PISA participating countries/economies.
2. *Clarification of mechanisms and processes of key issues of upmost concern by the Macao educational community.* In the past decade, two thorny issues lingered in the minds of Macao's educational practitioners. Foremost is why Macao is having very high proportion of grade-repeated students and yet its performance in mathematics is very favorable. Sit et al.'s (2015) analysis on studying problems faced by the adolescent grade repeaters in Macao throws lights on this issue uncovering three underlying mechanisms based on evidences from the PISA 2012 data. This research ushered the local government urging schools to curb the rate of grade repetition in schools. Another issue is that the sense of school belonging of Macao students is among the worst in all participating countries/economies. Cheung (2010) identified that factors for remediation and policies were introduced in the subsequent years. Cheung et al. (2018) made another attempt to resolve the attitude-achievement paradox based on anchoring vignettes by evidences from PISA 2012 mathematics data. The conflicting results stemming from data aggregation with regard to attitudinal factors affecting achievement when the secondary data analysis is conducted at the student and system levels are now clarified so as to inform policymaking.
3. *New methodologies in the secondary analysis of the big process data in the information era—Shift of assessment mode from print-based to digital problem-solving in 2012.* The log file data captured by the test delivery system allow researchers to analyze relationships between problem-solving patterns and the cognitive/affective/behavioral learner characteristics (e.g., Ieong et al., 2016a, 2016b; Jin et al., 2016), as well as task- and non-task-specific factors classifying problem-solving experts and novices (e.g., Jin et al., 2017).

3.1.2 Analysis of Macao's Quality and Equity in Mathematics Education from a Comparative Education Perspective

One important aim for Macao to participate in PISA is to assist Macao schools to monitor the quality and equity of the basic education. Figure 1 shows the relationship between mathematical literacy performance of Macao students and their home economic, social, and cultural status (ESCS) in PISA 2003–2018. Compared

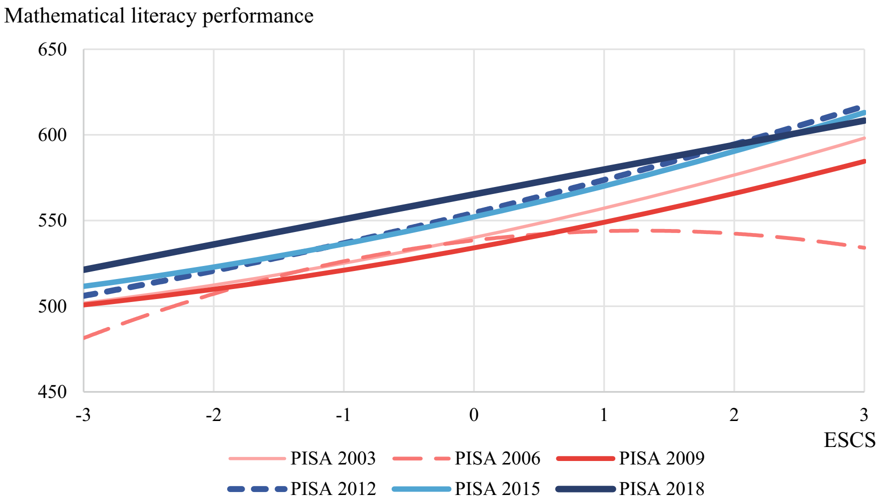

Fig. 1 Relationship between mathematical literacy performance and economic, social, and cultural status (ESCS) in six cycles of Macao-PISA (2003–2018)

with the three earlier ones (i.e., PISA 2003–2009), the intercepts of the regression lines in the latest three cycles (i.e., PISA 2012–2018) are higher indicating a higher level of quality of mathematics education and the slopes are so gentle that performance increase due to one unit increase of ESCS is the lowest in the world. We can see that one unit increase in ESCS is still associated with about 15 score points' increase in mathematical literacy, amounting to approximately to one-third of a grade of schooling among the participating countries of Organization for Economic Co-operation and Development. We still need to pay close attention to it.

Results obtained from PISA reveal that the education system in Macao is the most equitable in the world (IBP, 2016). However, as the insiders, as envisaged in Fig. 1, we know that inequality still exists. Concerning research in mathematics education, overcoming inequity and maintaining quality have becoming two of the most important goals of education in Macao as in other economies in the world (Li & He, 2010; UNESCO, 2008).

3.2 Research Done for Master's and Ph.D. Degrees

We have also tried to search for the master's and Ph.D. thesis to see what research the postgraduate students in universities in Macao did for their degrees.

After checking the master and Ph.D. programs offered at the universities in Macao, we found that only the University of Macau (UM) offered mathematics education programs at the postgraduate level; therefore, we shall only need to look at theses and dissertations in the Faculty of Education (FED) of UM. The Ph.D. dissertations on mathematics education in FED/UM were all about PISA under the supervision of the first author of this chapter. Therefore, our focus will be on the master theses in Curriculum and Instruction strand in FED/UM. Similarly, our focus will be on the research done in the past decade, i.e., starting from 2010. The theses after 2017 have not been uploaded to the UM library system; therefore, they were excluded in the current analysis. From 2010 to 2017, in total, 19 students received their master's degrees supervised by six professors in the field of mathematics education, ICT, and educational psychology. We would like to look at these theses from the research methods perspective, then get into the details under each category.

We commonly classify the methods used in educational research into quantitative and qualitative (Mertens, 2015). It is so interesting to find that all the researches done for the master's degrees all used quantitative methods. Educational research is generally classified into three categories, i.e., descriptive, associational, and intervention (Hoy & Adams, 2016). Among the 19 theses, seven used descriptive methods, four used associational methods, and eight used intervention methods.

Descriptive studies. Among the seven descriptive studies, four focused on students' learning, two focused on teachers' teaching, and one involved both teachers and students.

Among the four studies focusing on students, three used PISA data. Two used the Macao data of PISA 2003 (Lam, 2012; Loi, 2011), and one used the Macao and Singapore data of PISA 2012 (Chan, 2017). Lam (2012) used Chi-square tests to compare learning characteristics of students at different problem-solving proficiency levels. Loi (2011) explored the mathematical literacy performance of Macao students in the four content areas, gender differences, and performance differences among students at different grade levels. Chan (2017) compared the performance of Macao and Singapore students and their attributions of failure in mathematics. Chan found that Macao students with low mathematics literacy levels were more likely to attribute their failure in mathematics to external factors like bad luck, wrong guesses, and the difficult nature of mathematics contents.

The study that did not use PISA data was conducted by Leong (2016), who investigated students' geometrical thinking skills, in particular, eighth and ninth grade students' understanding of congruent triangles based on van Hiele model of geometrical thinking. Leong found that about 75% of the participants could reach Level 3; however, only about 30% of them could reach Level 4. About 10% of the participants were at the transit stage. Leong also identified the main difficulties that the students had: low visualization ability, the use of geometric symbols, the use of conditions for determining whether two triangles are congruent, and proof writing skills.

Both Li (2010) and Shi (2012) investigated teachers' teaching strategies. Li (2010) compared the difficulty levels of problems used by three teachers teaching the topic "Pythagorean theorem" in a secondary school in China and those of problems presented in mathematics textbooks. Li found that the number of the problems used in real classrooms is 3–5 times as those presented in mathematics textbooks, and nearly half of the worked examples used in classrooms are from materials other than textbooks. Based on the difficulty model developed by Bao (2002) for textbook analysis, Li found that the problems used in classroom are more difficult than those presented in textbooks in all the five aspects (i.e., investigation, context, computational complexity, reasoning, and topic coverage). Shi (2012) compared the conceptions between five expert and five novice teachers on the use of "One Problem with Multiple Solutions (OPMS)". Shi found that expert teachers did not only hold a positive attitude toward OPMS, but also realized its positive effect on students' learning and actively used it in their classroom teaching. Whereas the novice teachers held a negative attitude toward OPMS, they did not think that it might have a positive effect on students' learning. As a sequence, they thought that it was a waste of time to use OPMS in their classes.

Han (2012) examined what mathematics teachers and their students thought as important for effective mathematics teaching and learning. She found that both teachers and students thought "knowledge and skills", "mathematical thinking", "problem solving", and "emotions and attitudes" important. In terms of classroom activities for effective mathematics teaching and learning, both teachers and the students thought "students' behavior" (e.g., hands-on activities) followed by "teacher's behavior" (e.g., teacher's explanations) and lastly "teacher-student collaborative behavior" (e.g., making a rhyme together) are important.

Associational studies. Among the four associational studies, two investigated the factors that are associated with students' mathematics achievement (Cheong, 2010; Lao, 2016). Cheong (2010) used Academic Help-Seeking Questionnaire and Mathematical Self-Efficacy Questionnaires to collect data from students at grades 7–9. Cheong found that mathematics achievement is positively associated with students' mathematics self-efficacy and adaptive help-seeking, but negatively associated with dependent help-seeking and avoidance of help-seeking. Lao (2016) compared the mathematics achievement and attitudes toward mathematics between students of different cognitive styles (i.e., field-independent vs. field-dependent) in an interactive whiteboard instruction environment. It was found that students in a field-independent style performed better in mathematics and had a more positive attitude toward mathematics than those in a field-dependent style. Lao also found that students' mathematics achievement was positively correlated with their attitudes toward mathematics. Unlike these two studies, Lam (2016) used grade retention as an indicator of students' performance in schools (not mathematics achievement) and investigated the relationship between parental involvement and their self-regulated learning in mathematics and their experience of grade retention. Lam collected data from students at Grades 8 and 11 and found that students, who experienced grade retention, reported less parent–child communications and lower parents' school involvement.

Lei (2011) used the Macao data from PISA 2003 and structural equation modeling to investigate how Macao students' learning characteristics (including motivations in mathematics learning, sense of belonging to the school, mathematics self-efficacy, preference for competitive/co-operative learning situations) affect their learning strategies (including control, elaboration, and memorization strategies). Lei found that all the four characteristics positively are related to students' learning strategies directly or indirectly through intermediate factors.

Interventional studies. All the eight interventional studies used quasi-experimental design. Among them, three had the experimental group working in a Dynamic Geometry Software (DGS)-supported environment (Hu, 2014; Ke, 2014; Zhou, 2017) and one had mathematical games integrated (Ho, 2017). Hu (2014) designed a four-lesson experiment to teach Grade-10 students the translation of quadratic function's graphs based on Action–Process–Object–Scheme (APOS) theory and Relationship–Mapping–Inversion (RMI) principle in a DGS-supported environment. In the post-test, the experimental group outperformed the control group not only in items of point translations and quadratic functions, but also in terms in an extended area including complex functions and circles. Ke (2014) investigated the effects of using a DGS-based instruction on secondary students' achievement in linear functions. The experimental group worked in small groups with DGS to solve inquiry-based tasks, while students in the control group received traditional instruction. Ke found that students in the experimental group showed better improvement in items involving multiple representations of linear functions like $y = kx + b$ but not in items testing students understanding of basic concepts and their application. Zhou (2017) investigated the effect of cooperative learning in a DGS-supported environment on the ninth-grade students' mathematics performance in inverse proportional functions and attitudes. Zhou found that the experimental group had significantly better performance than the control group who received traditional instruction of the same topic, but there were no differences in attitudes toward mathematics between the two groups. Ho (2017) investigated the effect of integrating games into teaching the topic of "Directions" on the fourth-grade students' mathematics performance and attitudes. Ho found that there was no performance or attitude difference between the two groups. However, the students in the experimental group had better improvement in problem-solving abilities and self-confidence dimensions.

Two interventional studies were designed based on Polya's four-stage model (Si, 2014; Wu, 2016). Si (2014) modified the first three steps of Polya's four-stage model for the teaching of geometry and investigated the effect of the modified model on students' performance in geometric proof and their mathematics attitudes. The attitude questionnaire includes a section of self-conceptions on the use of Polya's four-stage approach. Si found that the students in the experimental group did not only perform better in the post-test in geometric proof, but also had an increased self-conception of the use of Polya's four-stage model. Wu (2016) integrated mathematical problem posing into the "Look back" stage of Polya's four-stage model and investigated its effect on students' problem-solving and problem-posing skills. The experiment lasted four lessons on "simultaneous linear equations in two variables". Wu found that the experimental group did not perform better than the control group in mathematical problem-solving, but the experimental group did make better improvement in mathematical problem posing.

Xiao (2010) experimented the dialogic teaching in high school mathematics classes and found that the experimental group could significantly better than the control group for which the instruction was teacher-centered in the post-test though there was no performance difference in the pre-test. Xiao also gave verbal evidence of students' improvement in problem-solving, reasoning and proof, communication, connections, and representation.

Ngai (2012) compared the effect of three kinds of layouts (i.e., dense, moderate, and sparse-layouts) of worked examples on the seventh- and eighth-grade students' mathematics achievements and cognitive load and found that moderate and sparse layouts had lower cognitive load on students than dense layout when the content to learn is at a high difficulty level. The moderate and sparse-layouts made learning easier for the students when the content is simple.

The review of the research done for master's and Ph.D. degrees in mathematics education reveals that the research in mathematics education in Macao is very limited in terms of both numbers and scope. Over eight years, only 19 students finished their master thesis in mathematics education. Research areas covered in these studies are from students' learning characteristics, learning strategies, and their learning difficulties in different topics, to teachers' teaching supported by DGS software and games, based on a modified Polya's four-stage model, dialogic teaching, and to study factors (e.g., parental involvement) that might affect students' mathematics learning and achievement. However, their scope is still very limited. The methods used for data collection and analysis are relatively simple; however, the results obtained in experimental studies are promising because they did help students in some ways.

3.3 Articles Published in Local Educational Journals/Magazines

As in many other countries/economies, there are several educational journals in Macao publishing articles mainly contributed by Macao mathematics educators to share their ideas and experiences. They are: Mathematics Education in Macao (MEM, 澳门数学教育), Teacher Magazine (TM, 教师杂志), and Education in Macao (EM, 澳门教育). Among these three journals/magazines, MEM is a yearly journal with a primary focus on mathematics education. We could not find a full copy of its first ten volumes of MEM; however, we did find two collective volumes with selected articles published in the first ten volumes. Therefore, we use these two volumes and the latest six volumes in the ensuing content analysis of the articles. Of note is that the other two (i.e., TM and EM) publish articles in all areas of education. All the articles related to mathematics education published in 2009–2018 were included in the current analysis. In total, there were 264 articles published, 115 (43.6%) in MEM, 76 (28.8%) in TM, and 73 (27.7%) in EM. After a quick reading of these journals/magazines, we would like to look at the articles from the following aspects:

(1) Authors and their categories (i.e., school teachers, administrators, and professors in mathematics education). The authors are the subjects who are involved in mathematics education research.

(2) Grade levels (kindergarten, lower primary, upper primary, middle school, high school, and higher education institutions).
(3) Content areas (numbers and operations, algebra, geometry, statistics and probability, etc.).
(4) Topic studies (mathematics curriculum, mathematics instruction, mathematics learning, mathematical problem-solving, etc.).

Authors and their categories. About 100 authors contributed articles to the three journals. Among the 264 articles, 211 (79.9%) were contributed by school teachers, 32 (12.1%) were contributed by professors including retired professors, 20 (7.6%) were contributed jointly by school teachers and professors, and 1 (0.4%) was contributed by a research center. Nearly 80% are written by school teachers, and they are about various ways to solve mathematical problems and alternative approaches to teach a particular topic. The one author (including joint writing) whose name appeared most frequently (in 25 articles) is a secondary school teacher. He received his Ph.D. in mathematics education from East China Normal University and now is actively involved in mathematics education research in his school and Macao-wide. He often represented the Macao Association of Research in Mathematics Education to act as one of the ASID reviewers. The one whose name appeared the second most frequently (in 21 articles) is the former head of mathematics department in a secondary school. He has led the mathematics team in his school to write a full serial of middle school mathematics textbooks (Mathematics Division of Hou Kong Middle School, 1995a, 1995b), which were very popular in the 1990s. These two authors also published books individually and jointly (Cheang, 2012, 2015; Cheang & Tang, 2017; Tang, 2011, 2013). In terms of contributions to the mathematics education practice in Macao, they have played a very important leading role.

Less than 10% of the articles are jointly written by school teachers and professors, which indicate that the collaboration between researchers and practitioners is not as common as anticipated. The second author of this chapter often visits schools, does classroom observations, discusses students' learning difficulties with team of mathematics teachers in the school, modifies their original lesson plans, and finally writes articles for these journals to share the instruction ideas with teachers in other schools. The above statistics reveals that more effort is needed, in particular, for lower grade levels as shown below.

Grade levels. We categorize the grade levels that the 264 articles focused on as kindergarten, lower primary level (Grades 1–3), upper primary (Grades 4–6), middle school (Grades 7–9), high school (Grades 10–12), and tertiary education levels. Some of the articles are more general and can be applied to the full primary education level (Grades 1–6) or the full secondary education level (Grades 7–12); some even covered all the 12 years of schooling. Only one grade-level category is determined for one article. The distribution is shown in Table 2. It is interesting to note that about half of the articles focused on mathematics education at the high school levels, the crucial stage before the tertiary education. It is also astonished to see that a bit more than 25% of the articles focused on the primary level, which builds the foundation for

Table 2 Distribution of articles in terms of educational levels

Grade levels	Frequency	Percent
K1–K3	1	0.4
1–6	23	8.8
1–12	20	7.6
7–12	15	5.7
1–3	13	4.9
4–6	17	6.4
7–9	44	16.7
10–12	131	49.6
Total	264	100

children's future development in mathematics. Only one article was for mathematics education at the kindergarten level, which is really very sparse.

Content areas. The content areas involved in the articles were classified into numbers and operations, algebra, geometry, measurement, statistics, and probability as Cai and Jiang (2017) did. Some articles concerned two of the four areas, and some covered all the four areas as PISA. These two categories are listed as a separate category each. The frequencies and percentages are shown in Table 3. About two-fifth of the articles are related to algebra, the main content in the middle and high school mathematics. Comparatively, fewer articles are related to the other four content areas. In particular, only five articles are related to data analysis and probability, which are relatively new to the teachers and students (Cai & Jiang, 2017). It is encouraging to find that about 10% of the articles are related to both algebra and geometry, some of which focused on the teaching of these two main topics. Some focused on the use of combination methods of number and shape, which is an effective problem-solving method that was highly recommended and widely applied in solving mathematical problems.

Topic studies. The topics that the articles focused on were classified into mathematics curriculum, mathematics instruction, mathematics learning, mathematical problem-solving, etc. Only one topic category is determined for one article based on its primary

Table 3 Distribution of articles in terms of content areas

Content areas	Frequency	Percent
Numbers and operations	24	9.1
Algebra	107	40.5
Geometry	38	14.4
Measurement	5	1.9
Data analysis and probability	5	1.9
Algebra and geometry	26	9.8
All	59	22.3
Total	264	100.0

focus. To Chinese mathematics educators, it is a tradition to integrate mathematical problem-solving into mathematics curriculum and instruction (Cai & Nie, 2007). Problems including worked examples and exercises are one of the main components in the mathematics textbooks (Fan & Zhu, 2007). Research on mathematical problem-solving often focuses on the study of multiple solutions of one problem (一题多解), the use of multiple problems to teach one concepts (多题一解), and the discussion of multiple changes of one problem (一题多变) to help students which forms a coherent scheme of an important concept like speed (Cai & Nie, 2007; Jiang et al., 2014). Therefore, we put all the articles on mathematical problem-solving as a single category.

Table 4 shows the frequencies and percentages of articles in different topics. It is not surprising to see that nearly half of the articles focused on mathematical problem-solving. As to the three vertices of the didactic triangle, mathematics instruction caught the most attention from the authors of these articles. School teachers wrote articles to make reflections on their lesson designs and their implementations in real classrooms and to make suggestions for future instruction. Although there is no teaching research office in DSEJ likened that in the Chinese Mainland (Yang & Ricks, 2013), there does exist teaching research offices at the individual school level in Macao. Teachers teaching the same level mathematics have weekly meetings to report on the progress in the current week and discuss their plan for the next week. They also organize open-class activities from time to time to do classroom observations and reflections. The second author of this chapter brings her undergraduate students in secondary mathematics education program to join in local schools' open-class activities twice a year. She also suggested the visited schools to arrange the so-called "Same Content Different Designs" (*Tongke Yigou*) activity (Yuan & Li, 2015) as one of the open-class forms. *Tongke Yigou* helped the prospective mathematics teachers to understand that there are different ways to teach the same content, and it is necessary to select the more appropriate way based on the students' realities and the supporting resources the school can provide. The second author of this chapter also tried to point out the most crucial steps in the different designs. For example, how to show the monotonic property of a function using dynamic geometry software (DGS) is the most important part in teaching the properties of inverse functions (Cheong et al., 2018).

As aforementioned DSEJ released BAAMs for different key stages in 2016–2017, about 16% of the articles discussed what is new in the curriculum standards, what impact the BAAMs will have on classroom teaching. Only 30 articles are concerned about students' mathematics learning, which is the starting point of instructional design. Our serial studies revealed that Macao students' learning is very similar to their peers in the Chinese mainland, Hong Kong, and Taiwan (Jiang et al., 2016, 2019). However, Macao students did not think that One Problem Multiple Solutions (OPMS) are important for their mathematics learning (Jiang et al., 2016, 2019). Our further investigation found that OPMS was not a common practice to mathematics teachers in Macao (Jiang et al., 2017). Furthermore, Macao students thought that memorizing was important to their mathematics learning. In particular, they thought that it is important not only to memorize the definition of mathematical

Table 4 Distribution of articles in terms of topics

Topics	Frequency	Percent
Mathematics curriculum	41	15.5
Mathematics instruction	60	22.7
Mathematics learning	30	11.4
Mathematical problem-solving	128	48.5
Teacher's professional development	2	0.8
Use of ICT in mathematics education	1	0.4
Others	2	0.8
Total	264	100.0

terms, formulae, and mathematical theorems, but also important to memorize exercises finished before the examinations, and the problems where they often make errors (Cheong et al., 2017). Very few articles are concerned about teachers' professional development and use of ICT in mathematics teaching and learning.

In summary, among the 264 articles, nearly 80% were contributed by school teachers, from which two contributed more than 20 articles each. As to the grade levels, about 50% of them are for high school levels, only one for kindergarten level. Concerning the content areas, about two-fifth of the articles are related to algebra, the main content in the middle and high school mathematics. About 10% of the articles are related to both algebra and geometry, and only five articles are related to data analysis and probability. Regarding the topics covered, nearly half focused on mathematical problem-solving, followed by mathematics instruction, then mathematics curriculum, and lastly the mathematics learning.

It seems that the results are conflicting with each other. On one hand, nearly half of the articles are related to mathematical problem-solving, which includes OPMS. On the other hand, OPMS is not considered as important for Macao students' learning. One possible explanation is that OPMS is an important component of teaching practice of very few mathematics teachers to be conducted in a limited number of schools only. Therefore, not too many students could have a lot of experiences in their classroom learning. We need to encourage more mathematics teachers to use OPMS in their classes because OPMS is useful in helping students to make connections among different kinds of problems, different forms of solutions, and different ways/types of approaches/strategies used.

4 Conclusions and Future Directions

In this chapter, we firstly introduce the general information about the education and mathematics education in Macao as a background, secondly describe the two key characteristics of Macao's research culture in mathematics education, that is, it has been mostly school-based in private schools and carried out by autonomous educational practitioners, and meanwhile, the government has been increasingly exercising tighter control over the education, which has shaped the development of mathematics education research at the school level, and finally report the three lines of research done in mathematics education in Macao.

The chapter highlighted several characteristics of mathematics education research in Macao. First, much and important research work has been done around the PISA. This line of research is not only highly policy-oriented and interpretation-oriented, but also an indication of the important influence of outside and international environment on Macao's mathematics education. This is because PISA has been used by Macao government to monitor the schools to be responsible for the academic standards as well as futuristic direction of the basic education they provided to their students.

Second, in terms of the topic areas in the field of mathematics education, the area that has received most attention by Macao's mathematics education researchers is problem-solving, followed by mathematics instruction, mathematics curriculum, and mathematics learning. Some other areas such as teacher education, teacher professional development, ICT, and mathematics education are largely ignored.

Third, in terms of the grade levels, Macao's mathematics education researchers have focused most on senior high school and then junior high school levels, much less on primary level and almost none on kindergarten level. It appears that this characteristic reflects the fact that research culture in junior and especially senior high school levels is much more active than in primary and kindergarten levels.

Fourth, in terms of the authorship, most (about 80%) of the publications in the field of mathematics education were authored by school teachers, which explains the importance of the idea of teachers as researchers in mathematics education research culture in Macao, and in addition, only a small portion (about 20%) was co-authored, which in a sense indicates that mathematics education research in Macao has been done more individually rather than collaboratively. Caveats must be made here to caution the readers that most publications are not research articles as usually stated, but by and large are action research or experience sharing of application of research knowledge to practice for purposes of improvement of curriculum and instruction.

Finally, most publications in mathematics education in Macao are practice-oriented and, to a large extent, based on teachers' own teaching experiences, observations and reflections. While the value of such publications and the related work should never be underestimated for the purpose of sharing for teachers, the weakness in terms of methods for data-collection or evidence gathering from a research perspective should be clearly noted.

To conclude, while the achievement of mathematics education research in Macao should be duly commended, there are also issues and challenges concerning Macao's mathematics education. For the future directions of mathematics education research in Macao, we suggest the following:

First, mathematics educators in Macao should pay more attention to under-researched yet important areas in mathematics education, in particular, teacher education and professional development, and the integration of ICT in mathematics education.

Second, more collaborations between different organizations, between different regions, and between different researchers (especially between school practitioners and university-based researchers) should be encouraged and promoted in order to utilize the knowledge, skills, and experiences of different parties and to make research more concerted.

Third, more research work should also be undertaken with focus on lower grade levels (particularly, junior and primary school levels), given their importance for students' learning of mathematics, and accordingly, related research culture and skills should be developed.

Fourth, there is a need for mathematics educators in Macao, as a research community, to further broaden the scope of research (e.g., go beyond interpretation-oriented, policy-oriented, and practice-oriented types of research) and enhance the capacity of research, in particular, in terms of research methods (e.g., using more qualitative and mixed methods).

Fifth, the educational context in Macao provides a good place to compare the effect of the implementation of different mathematics curricula on teaching practices in schools using different media of instruction. It merits further investigation in this direction.

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Trends in Mathematics Education Research in Indonesia

Nurwati Djam'an, Neni Mariana, and Mangaratua M. Simanjorang

Abstract Various efforts have been made in Indonesia to improve the quality of education, including improving the quality and productivity of educational research. Integrating research results into classroom learning is expected to enhance the quality of education. The government's commitment to research can be seen in increased research funding through several existing research schemes and mandatory output demands from funded research. This chapter reviews research in mathematics education funded by National Competitive Research in Indonesia according to research types, topics dealt with, research focus, research outputs, and target audiences. This review provides an overview of the dominant research trends and absences toward more diverse and effective education research to improve mathematics education in Indonesia. We hope that this chapter will stimulate discussions among researcher administrators, educators, and all who are concerned about quality education in Indonesia.

Keywords Mathematics education research · Trends research in Indonesia · Quality education

1 Introduction

Numerous calls and proposals have been made and many projects have been implemented by the government to improve the quality of mathematics education in Indonesia. Particularly, one of the efforts to improve mathematics education quality

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is through research activities. Improving the quality of research increases competitiveness and strengthens the Indonesian National Qualifications Framework (KKNI), which focuses on the outcomes of the national educational system in the country. Mathematics education research could enrich content, teaching strategies, media, and evaluation techniques for learning mathematics. Moreover, the implementation of research results in the classroom could improve the quality of school mathematics education.

In addition, many Indonesian scholars have obtained research degrees overseas and expectedly must have gained insights from international research in mathematics education. National and international conferences, seminars, and workshops provide a key role in disseminating research findings of mathematics education researchers on the current trends, issues, and innovations in mathematics education and research. However, Atweh (2007) argues that "there still remains the concern about whose knowledge is bring represented and who is benefiting from such [international contacts]."

2 Research Grants to Improve Research Productivity

The efforts to improve the quality and quantity of research and community service in higher education in Indonesia are continuously carried out by the Directorate General of Research and Development Strengthening at the Ministry of Research, Technology, and Higher Education (Ristekdikti). The significant increase in Indonesian researchers' publications related to mathematics education in various international journals may indicate Indonesia's effort and achievement in improving the mathematics research quality. In the Indonesian context, the focus on increasing research productivity and quality in the country requires the research to produce innovative products and respond to local educational needs. Likewise, the Indonesian government also aims that research results also need to be directed toward obtaining protection of Intellectual Property Rights (IPR), both in Copyright and Industrial Property Rights. In short, the number of publications and IPR are two directions taken by Indonesian authorities as mandatory research outcomes that guide the attempts to improve research quality in Indonesia.

The Indonesian government is committed to improving the quality and quantity of academic publications. Funding support for research is expressly stated in Law Number 12 of 2012 concerning Higher Education Article 89. Universities that receive State Higher Education Operational Assistance (BOPTN) are expected to allocate at least 30% of their funding for research activities. BOPTN is a cost aid provided by the government to finance deficiencies in operational costs, for example, in dealing with the high cost of education. In addition to funding for research through BOPTN, the Indonesia Directorate General of Higher Education (DGHE) provides broader authority in research management to universities through decentralized research and community service programs. Universities are expected to manage their research agendas in line with the increasing funding support from the government.

There are two main funding research schemes for lecturers. The first is directly managed by universities. The second scheme is at a national level called the National Competitive Research Grant, with the aim to improve national excellence in research, including National Strategic Superior Research, University and Industry Priority Research, Foreign Cooperation Research and International Publication, Competency Research, National Strategic Research, Acceleration of Master Plan Research, and the Expanding of Indonesia's Economic Development (Direktorat Riset dan Pengabdian Masyarakat, 2018). The national scheme is more competitive than the decentralized scheme. However, several lecturers have applied for university funded research grants, while many lecturers do not attempt to apply for the National Competitive Research grants. One of the reasons might be the general perception among such lecturers that the application process is too bureaucratic. The funds are sourced from the annual state budget, disbursed, and audited by the Ministry of Finance.

Based on R&D Magazine Survey data, funding for research in Indonesia in 2018 was estimated at around USD 10.23 billion or 0.91% of GDP. This figure puts Indonesia at 28th out of 116 countries in terms of numbers, yet when viewed from the ratio to GDP, this portion still looks relatively small. However, in 2019, the budget for the research fund increased dramatically to approximately USD 69.4 million.

3 Mathematics Education System in Indonesia

According to Law No. 2/1989 on the National Education System, the objectives of the National Education System are two-fold. First is to develop a high-quality and self-reliant human being whose values are based on Pancasila, i.e., State ideology, spelled out in the five basic principles of the Republic of Indonesia: belief in one God; just and civilized humanity, including tolerance to all people; unity of Indonesia; democracy led by the wisdom of deliberation among representatives of the people; and social justice for all. Second, education plays a significant role in supporting Indonesian society, people, and the state. In the broader context of social and national development, the aim of education is, on the one hand, to keep and maintain Indonesia's cultural background and, on the other, to generate the knowledge, skills, and scientific progress that will keep the nation abreast of developments in the twenty-first century.

However, there is ample evidence that Indonesia still needs to improve the quality of its education. PISA tests conducted by the OECD in 2015 showed that Indonesian students were performing at lower levels than the OECD average in all areas of science, mathematics, and reading. Further, in PISA 2018, Indonesia ranked 74th out of 79 countries. Its PISA average for mathematical literacy was 379. The ability of Indonesian children's students is below the average OECD country participant (489). In addition, the OECD demonstrated that there is a significant disparity between regions in Indonesia. PISA's focus on mathematical literacy reflects increasing concern about how well students can apply mathematics to solve a real-life problem (OECD, 2009). In response to this case, more emphasis and policy on mathematics literacy are needed. Moreover, the PISA results highlight mathematical literacy issues, the difficulties students face in solving PISA test items, teachers' quality, and equity.

In this chapter, the researchers analyzed the mathematics education research articles, which were funded by National Competitive Research. This chapter also attempts to relate to the type of research, research topics, focus, type of schools, and research outputs in Indonesia.

4 Research Procedure

To investigate the data regarding trends in mathematics education research, the data used in this study were drawn from 295 articles in 2019, and 381 articles in 2020 were published in national and international indexed journals and international conference proceedings, which are the output of funded research in those two years. This study used a random sampling technique to target 40 full papers included in international journals out of those 676 articles. The final sample consisted of 20 articles each year. The authors obtained the list of mathematics education lecturer's research that has been funded by the grants of Ristekdikti and has been published in international research journals. Furthermore, the authors analyzed the full text of sample articles published related to the funded research.

This study reviews the type of research, the topic of research, the focus of research, the targeted educational level of the study, and the outputs of research commonly adopted in the publication. The type of study refers to methods that are employed in the study. The research topic provides an overview of variables in the study (independent, dependent, and control variables). This study also identifies the educational level of the participants or the site of data collection was studied. In addition, reflect upon the outputs of research obtained over the sample articles.

5 Type of Research

The different research types gathered from the sample are the following: (1) research and development (R&D), a type of research in which a certain product is developed and later tested to determine its effectiveness; (2) survey, consisting of predetermined sets of questions for collecting data using a representative sample by interview, phone, or face to face; (3) experiment, wherein the treatment is administered to the subjects and the outcomes measured and analyzed; (4) literature reviews, which are conducted to provide or evaluate an overview of knowledge on a particular topic; (5) case study, which is an intensive study in which researchers focus on a unit of study, for instance, individual teachers, a classroom, etc.

Fig. 1 Type of research

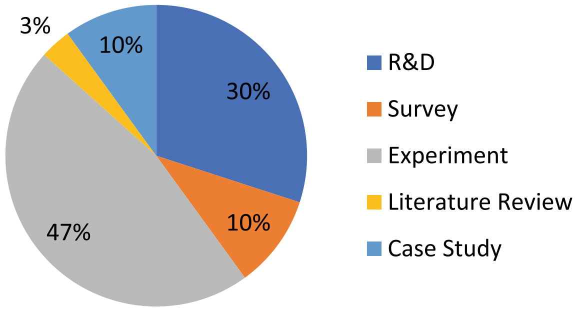
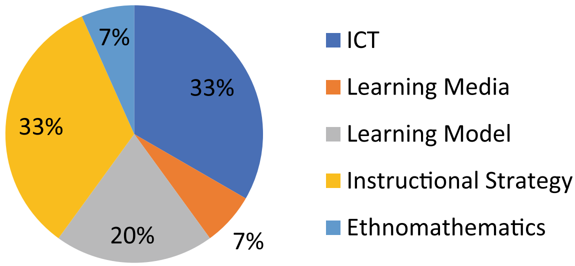


Fig. 2 Topics of research



From the overall data collected, 46.7% of the sample were experiments and then followed by Research and Development (R&D) 26.67%, survey and case study as many as 10%. The rest, 3.3%, are literature review and instructional design (Fig. 1).

Most researchers used the experimental design or mixed methods in this study. Experiment research is the most familiar type of research design for mathematics education researchers in Indonesia. In this research design, the treatment to be implemented is a model of teaching and learning, and one or more dependent variables are examined to measure the impact of the model. On the other hand, R&D, which follows after experimental design, serves to refine the practices and examine their impact (Fig. 2).

6 Topics of Research

Some research topics identified in this study include ICT, learning media, learning model, instructional strategy, and ethnomathematics. Learning media refer to equipment that can be a component for implementing active learning. Media could be manipulative and virtual media.

The learning model is designed to develop curricula, materials, and guidelines in class and outside the classroom, which consists of four things, namely: (a) syntax, which contains a teacher's steps in carrying out learning activities; (b) social system, which describes the role and relationship between students and teachers while the learning process is ongoing; (c) the principle of reaction is a picture of the teacher's role during the learning process; and (d) support system, which consists of all the means that support the implementation of learning (Joyce & Weil, 2009), while an instructional strategy is a technique used by teachers to deliver the materials and assist in the comprehension of the subject.

The percentage distribution of the use of both learning models is about 20%, and instructional strategy is around 33%. The trend of research topics in mathematics education in Indonesia is increasing in information communication technology (ICT) in mathematics education, such as e-learning and mobile technology by around 33%. In addition, increasing attention is being paid to ethnomathematics in mathematics education research in Indonesia (about 7%). Ethnomathematical approaches to mathematics education research seek to understand the roles of mathematics in different ethnic groups and nations. It presents mathematical concepts of the school curriculum in a way that relates these concepts to the students' cultural and daily experiences, thereby enhancing their abilities to elaborate meaningful connections and deepening their understanding of mathematics. For example, one article titled "The Development of Geometrical Learning Devices Based on Rumah Gadang Ethnomathematics for Grade VII Junior High School." In this study, the researcher developed the Rumah Gadang ethnomathematics-based geometrical learning devices in the form of the lesson plan and student activities.

An important change in mathematics learning needs to be realized to accommodate the ongoing and current changes in the demographics of learners in mathematics classrooms. Several scholars have developed a culturally relevant pedagogical theory that examines the teaching and learning process in a critical paradigm and through explicit connections between the culture of learners and their school subjects (Rosa & Orey, 2011). From an ethnomathematics perspective, it is necessary to integrate a culturally relevant curriculum into the existing mathematics curriculum. Based on the views of Torres-Velasquez and Lobo (2004), this perspective is an essential component of culturally relevant education because this perspective proposes teachers' contextualize mathematics learning by linking mathematical contents to the culture and real-life experiences of students.

7 Focus of Research

The data show that about 29.4% of the mathematics education research variable were related to learning achievement. While 11.8% focus on problem-solving, 5.9% is a type of higher-order thinking and creativity, and the rest is a type on mathematical communication, confidence, mathematics abstraction, and emotional intelligence. More details about these percentages are presented in Fig. 3. As some researchers perceive, the aims of education are that mathematics education needs to build students' capacity for problem-solving, reasoning, communicating, and intelligence. The data are consistent with Kilpatrick and Findel (2001), who argued that

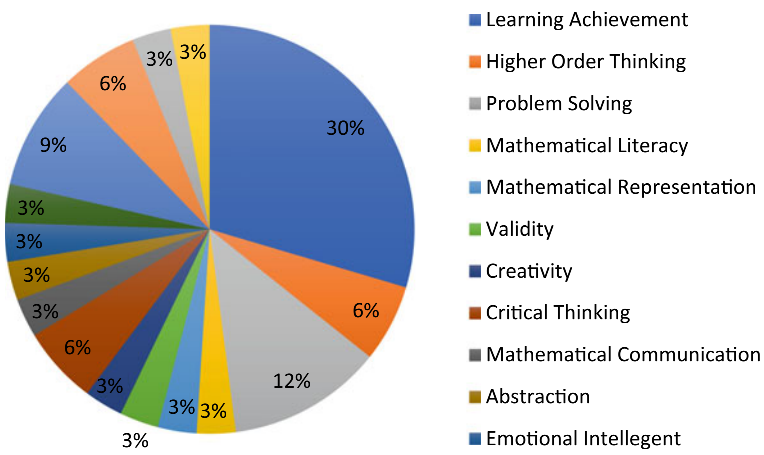


Fig. 3 Focus of research

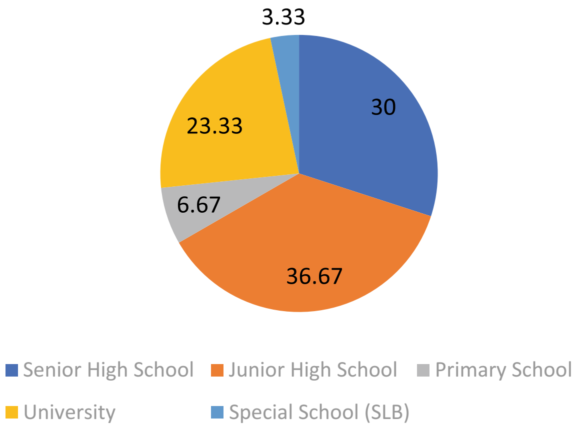
mathematics education is the means to instruct the students in understanding mathematical concepts, operations, and relations, formulizing problems mathematically, and devising strategies for solving it, seeing mathematics as useful.

One parameter of the success of an education system is to look at the results of learning achievements. As mentioned before, the type of research that was mostly used in mathematics education research in Indonesia is experimental research designs, including evaluation of the implementation teaching approach. More focus is on classroom pedagogy. The aims of mathematics education research in this study are mostly to promote creativity, critical thinking, problem-solving, mathematical literacy, communication, mathematical representation, and confidence. Soedjadi (2000) states that teaching mathematics needs to promote mathematics values. This is in accordance with competencies demanded in school mathematics learning from elementary to high school (Midgett & Eddins, 2001). Particularly, mathematics learning activities are oriented not only to mastery of mathematical material alone but also to mathematics as a tool and means for students to achieve other competencies.

8 Type of Schools

This research indicates that different types of schools are targeted by mathematics education researchers in Indonesia, including primary school, junior high school, senior high school, university, and special school. These classes include junior high school, about 36.67%, senior high school around 30%, university about 23.33%, and primary school about 6.67%. However, the data analysis returned very little research on mathematics education in a special school for learning disabilities, only

Fig. 4 Type of schools



about 3.33%. The distribution of this data shows the tendency of researchers' interest in finding an alternative solution to improve the quality of mathematics education, mainly at the secondary school level (junior and senior high school), then followed by tertiary level (university) with less concern so far to a primary and special school for learning disability.

Research that focuses on inclusive mathematics education is still low, especially the study to examine disability issues in mathematics education. Further investigation shows that the tertiary level is most likely related to teacher training universities (Fig. 4).

9 Outputs of the Research

According to the data obtained in this study, the product of research identified in the sample is categorized such as instructional material, learning models, and media. Instructional material mentioned here is about the research on the use of instructional materials for effective teaching and learning, which also discovered the reluctance and perceived effects of it. On the other hand, the research-based learning model is one of the researches that is expected to improve students' ability, including learning outcomes. The research aims to develop a learning model by combining two or three teaching approaches or strategies. In the end, the research examines the effectiveness of the model. Moreover, media in this study are related to the design of mathematics learning media using such as Research and Development (R&D); the Analysis, Design, Development, Implementation, and Evaluations (ADDIE) development model; and the effects of using media in mathematics learning.

Furthermore, handouts as part of media that arise from the research conducted define as mathematics learning material in the form of a book or module, for example, the article with the title: Development of reflective module based on child-friendly

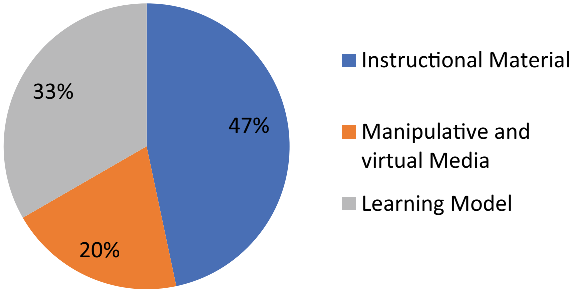


Fig. 5 Outputs of the research

school to improve numeracy and confidence. This study aims to: (1) determine the process of developing a reflective module based on child-friendly schools, (2) produce a child-friendly school-based reflective module that meets the eligibility criteria, and (3) reveal the effectiveness of the resulting reflective module to improve students' numeracy literacy skills and self-confidence of fifth-grade elementary school (Fig. 5).

The output of the research of the articles, namely: instructional material, manipulative and virtual media, and learning model. As concerned with experimental research design, some articles contain learning models about 33%. The experiment research aims to establish the cause-and-effect relationship between the implementation of the learning model and the dependent variable. With increasing interest by mathematics education researchers in Indonesia in the implementation of ICT, the research product identified was media, including manipulative and virtual, which is about 20%.

The fact that most of the research products (47%) are related to instructional material shows that there is a significant concern among Indonesian researchers to approach alternative solutions for improving mathematics education quality from theoretical aspects, which later may be used as the foundation of teaching and learning practice in the mathematics classroom. In addition to theoretical concern, there is also another concern about media used in teaching and learning, which is not far less than an instructional material. This data exposure may express that most researchers in Indonesia tend to seek the way to improve mathematics education from instructional material aspect and classroom implementation aspects, especially in learning media. These two concerns cover most of the research outputs.

10 Discussion

Educators and researchers in mathematics education in Indonesia have adopted a variety of perspectives to understand and study mathematics education issues. Mathematics education research in Indonesia by conducting R&D, experiment, case study, survey, and literature review tried to provide innovative mathematics learning, study the development of cognitive levels such as higher-order thinking and critical thinking skills, students' creativity in solving a mathematics problem.

Based on the data of the most trending types of research, experimental studies are on the top of the list. It indicates that most mathematics educational research in Indonesia tends to try out a certain learning model, media, and strategies used in mathematics learning. The second place was the R&D research type. This type of study is also another layer of research that also includes experimental study. Moreover, this demonstrates how mathematics education researchers in Indonesia primarily work with statistical data. In other words, the hegemony of positivism and post-positivism research paradigms is going on in Indonesia. Why do these two types of research dominantly funded in Indonesia?

Research for Indonesians might be a means to prove the hypotheses of the researchers, and thus, the researchers need to do it objectively supported by statistics. The objectivity plays most important role in such worldviews. Even if we do mixed methods such as R&D, we keep going by proving the product of the developmental process using experimental studies. At this point, we at last do the "real" research. Furthermore, most research funding goes to these types of research. Mostly, the funders reckon that research should have a clear outcome. Therefore, R&D has become the second most adopted research type.

Regarding learning mathematics in the twenty-first century, the few promising developments in mathematics education research in Indonesia are increasingly concerned about technology. No doubt much more research and reflection are needed about competencies to evaluate mathematical applications and ICT and the possible usefulness or its problematic effects. This is in accordance with Taguma (p. 42) who points out that "ICT can foster many benefits, including helping children visualize abstract issues or learn how to read. Besides, it fosters children's technological skills. Since computers are increasingly being used in households and schools and are becoming a more important part of people's everyday lives." Particularly, mathematics education research which appears in this study also has a concern with learning achievement but has no concern to connect or speak about economic factors behind mathematics achievement. As OECD (2013) points out a sobering thought that economics, income inequality, or socioeconomic status (SES) is more significant in explaining differences in mathematics achievement than gender and race.

There are more non-PGSD (Primary School Teacher Education) mathematics education lecturers than PGSD applying for the national research grant specifically designed to prepare the graduate students for teaching in junior and senior high schools. On the other hand, thematic learning is being implemented at all grade levels in elementary schools in Indonesia. Thus, elementary school teachers generally teach all fields of study. Mathematics education researchers tend to focus on their own expertise. Therefore, it makes sense that the research setting for mathematics education in Primary School is less likely to disperse to other types of schools.

Moreover, there may be other reasons for this phenomenon than to uncover the reason that needs more study, which is not covered by the focus of this study. However, high qualifications and status of primary education and kindergarten or preschool should be increased attention. Early childhood and primary education are essential to prepare pupils for the secondary level of their education. Another important thing, especially regarding mathematics education, is not to allow students to fall behind in mathematics in their mastery of the subject. Thus, Van de Walle (2007) suggests that when students who have not mastered facts are engaged in exciting and meaningful experiences, they are motivated to learn facts and real opportunities to develop relationships that can aid in that endeavor.

As mentioned in the type of school part, research attempting to study related students with special needs is still rarely the focus of mathematics education research in Indonesia. However, mathematics education research will need to focus on the conceptualization of equity and improving equity and related values such as inclusion. There is a need to promote equity, and quality issues in mathematics education arise when individual students engage in the collective activity of learning mathematics at the level of the classroom (Atweh, 2011). To deal with equitable access and distribution of quality issues in the mathematics classroom, the teacher has access to many possible practices, such as differentiation of instruction and developing high expectations of achievement from students. However, teachers' practices to promote equity and quality mathematics education are constrained, among other things, by school policies regarding reward structure, teacher professional development, improved technology, or attention to social circumstances. Thus, teachers' practices in this regard are shaped by school policies to a large extent.

Furthermore, Stephan et al. (2015) identified the three specific biggest challenges in mathematics education: changing perceptions about what it means to do mathematics, changing the public's perception about the role of mathematics in society, and achieving equity in mathematics education. In achieving equity in mathematics education, there are challenges in helping people see that doing mathematics is about problem-solving, reasoning, curiosity, and enjoyment and not about following procedures to get "the answer" or just about doing. Particularly, Bigelow (2001) suggests that teachers need to get students to begin to look critically at the many unequal power relations in our society. According to Osler (2007), mathematics teachers need to create lessons around issues and questions that students have raised and are interested in learning about; create projects that challenge students to suggest just and mathematically sound solutions to the problems that they identified; provide the opportunity to the students to present and share their work; scaffold students' understanding of both the mathematics concepts and the issues they are studying; and allow the assessment to determine what students have learned about the mathematics concepts that were in the lesson or the projects.

The new challenge of mathematics education research in Indonesia is determining social, cultural, and political views about mathematics and mathematics education: Is mathematics really for all? Social justice and mathematics education? In the increasing professionalization of mathematics researchers, the growth of collaborative research within the mathematics education community is needed. Challenges and perils of globalization in international collaboration. There is a need to further enrich the types of research to be implemented in mathematics education because other types of studies might be more useful.

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Correction to: Critical Analysis of Mathematics Education Doctoral Dissertations in the Philippines: 2009–2021

Bill Atweh, Minie Rose C. Lapinid, Auxencia A. Limjap, Levi E. Elipane, Michel Basister, and Rosie L. Conde

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In the original version of the chapter "Critical Analysis of Mathematics Education Doctoral Dissertations in the Philippines: 2009–2021" the author "Dr. Michel Basister" name was incorrectly published as "Dr. Michael Basister". This has now been rectified and the author's name has been corrected.

The correction chapter and the book has been updated with the changes.

The updated version of this chapter can be found at
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