Cooperative Beam Assignment and Power Allocation in Phased Array Radar Network for Multi-target Localization

Weiwei Zhang, Chenguang Shi(✉), Jianjiang Zhou, and Ruiguang Lv

Key Laboratory of Radar Imaging and Microwave Photonics, Ministry of Education, Nanjing University of Aeronautics and Astronautics, Nanjing, China
scg_space@163.com

Abstract. Based on cooperative beam allocation and power assignment (CBAPA), this article proposes a resource allocation strategy to improve the low intercept probability (LPI) performance of the phased array radar network (PARN) for the application of multi-target localization. This strategy reduces the PARN total transmit power by optimally allocating the beams, transmit power under the constraints of system resources and target localization accuracy. Firstly, we derive the target localization Cramer-Rao lower bound (CRLB) expressions and exploit it as localization metric. Then, the CBAPA strategy is built as a mixed nonconvex Boolean problem. According to semi-definite programming (SDP) and particle swarm optimization (PSO), a two-step iterative algorithm is introduced to resolve the problem. At last, the s numerical simulation demonstrates the rationality and superiority of CBAPA strategy.

Keywords: Beam assignment · Power allocation · Multi-target localization · Low probability of intercept · Phased array radar network

1 Introduction

Phased array radar network (PARN) had attracted extensive attention due to its superior performance, such as beam agility [1, 2]. The resource management is viewed as the most vital operation for radar networks [3–5], which not only improve the task performance under fuel limitation [6, 7], but also enhance low probability of interception (LPI) performance. Therefore, there exist lots of research efforts relation to resource management of radar network. For example, Garcia et al., tried to enhance the target localization accuracy by optimizing the allocation of power and bandwidth [8]. Xie et al., proposed the optimization of power allocation and combined node selection to minimize the worst-case tracking error among all targets [9]. Furthermore, Yan and co-workers pointed out that achieving accurate target parameter estimation strategies not only depend on cooperative target assignment and dwell allocation [4] but also relied on assigning the resource of beam and power for each radar [10].

Additionally, another example aims at reducing the total transmit power under the constraints of the given target tracking/localizing accuracy. For instance, Godrich et al.,

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reported how to reduce the total transmit power under the constraints of target localization accuracy by power allocation [11]. In a previous research, Shi developed two LPI strategies: one relies on jointly optimizing the bandwidth, dwell time allocation, target assignment and revisit time control [5], and the other based on collaborative power and bandwidth allocation [12].

Reviewing recent progress, [10] indicates that optimal beam-target assignment can improve the target tracking accuracy. However, there have no report referring to beam-target assignment in the application of target localization. In view of these, to bridge this gap in target localization and improve the LPI performance of PARN, our present work proposed a resource management strategy which not only optimizes the allocation of power, but also optimizes the beam-target assignment. To demonstrate the superiority of our suggested scheme, herein the problem of cooperative beam allocation and power assignment (CBAPA) strategy, on the basis of multitarget localization in PARN, is firstly formulated. Afterwards, a two-step iterative optimization algorithm abased on semi-definite programming (SDP) and particle swarm optimization (PSO) is introduced to address this problem. At last, we demonstrate the superiority and rationality of the CBAPA strategy through simulation results by comparing with other existing resource allocation strategies.

2 System Model

2.1 Signal Model

Suppose a PARN with M monostatic phased array radars, which are arbitrarily deployed and locate at $\left(x_m^{\mathrm{E}}, y_m^{\mathrm{E}}\right)$, $m = 1, 2, \cdots M$. The Q targets with coordinate of $\left(x_q, y_q\right)$, $q = 1, 2, \cdots Q$ are widely separated, denoted by the state vector $\boldsymbol{\psi}_q = \left[x_q, y_q\right]^{\mathrm{T}}$. Each monostatic phased array radar has N array elements, emitting multi-beam to localize the targets. The echo signal baseband representation scattered from the qth target to the mth phased array radar is expressed as:

$$\mathbf{s}_{m,q}^{\mathrm{R}}(t) = \xi_{m,q}\sqrt{p_{m,q}\eta_{m,q}}\mathbf{a}_{\mathrm{r}}(\theta_{m,q})\mathbf{a}_{\mathrm{t}}(\theta_{m,q})^{\mathrm{T}}\mathbf{s}_{m,q}\left(t-\tau_{n_m,q}\right) + \mathbf{w}_{m,q}(t) \tag{1}$$

where $p_{m,q}$ is the power of the signal emitted by each element. $\eta_{m,q}$ is the propagation path attenuation coefficient in the signal strength from the mth phased array radar to Q target. $\mathbf{s}_{m,q}\left(t-\tau_{n_m,q}\right) = [s_{1,q}\left(t-\tau_{1,q}\right), s_{2,q}\left(t-\tau_{1,q}\right), \cdots\cdots, s_{n_m,q}\left(t-\tau_{n_m,q}\right)]^{\mathrm{T}}$, $s_{n_m,q}\left(t-\tau_{n_m,q}\right)$ is the signal with lowpass normalized energy emitted by the n_mth array element of the mth phased array radar. $\tau_{n_m,q}$ is the time delay from the qth target to the n_mth array element of the mth phased array radar. Here, the point target model assumption is invoked, which means the time delay form the qth target to every array element of the mth phased array radar are almost equal, that is, $\tau_{1,q} \approx \tau_{2,q} \approx \cdots \approx \tau_{n_m,q} \approx \tau_{m,q}$, where $\tau_{m,q}$ represents the time delay form the qth target to the mth phased array radar. $\tau_{m,q} = 2R_{m,q}/c$, , and the radar reflective cross section (RCS) is denoted by $\xi_{m,q}$. $\mathbf{a}_{\mathrm{t}}(\theta_{m,q}) = \mathbf{a}_{\mathrm{r}}(\theta_{m,q}) = [1, \mathrm{e}^{-\mathrm{j}\pi\sin\theta_{m,q}}, \cdots\cdots, \mathrm{e}^{-\mathrm{j}\pi(n_{m,q}-1)\sin\theta_{m,q}}]^{\mathrm{T}}$ are the transmit and receive steering vectors of the mth phased array radar, respectively, where $n_{m,q}$ stands for the array elements number belonging to the mth phased array radar. $\theta_{m,q}$ is the observation angle of the subarray also belonging to the mth phased array radar to illuminate the qth target. $\mathbf{w}_{n_m,q}(t) = [w_{1,q}(t), w_{2,q}(t), \cdots\cdots, w_{n_m,q}(t)]^{\mathrm{T}}$, where $w_{n_m,q}(t) \sim \mathcal{CN}(0, \sigma_v^2), \forall n_m$ denotes the zero-mean complex Gaussian white noise with the power of σ_v^2.

2.2 Estimation Metric

Inversing the Fisher information matrix (FIM) can obtain the CRLB which can be employed to evaluate the target localization performance. The FIM of the localization accuracy for the qth target is described as [3]:

$$\mathbf{J}(\boldsymbol{\psi}_q) = \mathbf{E}\left\{ \frac{\partial}{\partial \boldsymbol{\psi}_q} \log f(\mathbf{TA}_q|\psi_q) \left(\frac{\partial}{\partial \boldsymbol{\psi}_q} \log f(\mathbf{TA}_q|\psi_q) \right)^{\mathrm{T}} \right\} \tag{2}$$

where $f(\mathbf{TA}_q|\psi_q)$ represents the conditional probability distribution function (PDF) of the measurement vector $\mathbf{TA}_q = \left[\mathbf{T}_q^{\mathrm{T}}, \mathbf{A}_q^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\mathbf{T}_q = \left[\tau_{1,q}, \tau_{2,q}, \cdots \tau_{M,q}\right]^{\mathrm{T}}$ denotes the measurement vector of TOA, and $\mathbf{A}_q = \left[\theta_{1,q}, \theta_{2,q}, \cdots \theta_{M,q}\right]^{\mathrm{T}}$ is the measurement vector of DOA, Given the signal model expressed by (1), the conditional PDF can be given by:

$$\begin{aligned} &\log f(\mathbf{TA}_q|\psi_q) \\ &\propto \frac{-1}{\sigma_v^2} \sum_{m=1}^{M} \int \left| \mathbf{s}_{m,q}^{\mathrm{R}}(t) - \xi_{m,q}\sqrt{p_{m,q}\eta_{m,q}}\mathbf{a}_{\mathrm{r}}(\theta_{m,q})\mathbf{a}_{\mathrm{t}}(\theta_{m,q})^{\mathrm{T}}\mathbf{s}_{m,q}(t-\tau_{m,q}) \right|^2 dt \end{aligned} \tag{3}$$

The FIM is inferred as:

$$\mathbf{J}_{\text{TOA - DOA}}(\boldsymbol{\psi}_q) = \left(\frac{\partial \mathbf{TA}_q^{\mathrm{T}}}{\partial \boldsymbol{\psi}_q} \right) \mathbf{J}(\mathbf{TA}_q) \left(\frac{\partial \mathbf{TA}_q^{\mathrm{T}}}{\partial \boldsymbol{\psi}_q} \right)^{\mathrm{T}} \tag{4}$$

The Jacobian matrix can be written as:

$$\left(\frac{\partial \mathbf{TA}_q^{\mathrm{T}}}{\partial \boldsymbol{\psi}_q} \right) = \begin{bmatrix} \frac{\partial \tau_{1,q}}{\partial x_q}, \frac{\partial \tau_{2,q}}{\partial x_q}, \cdots, \frac{\partial \tau_{M,q}}{\partial x_q}, \frac{\partial \theta_{1,q}}{\partial x_q}, \frac{\partial \theta_{2,q}}{\partial x_q}, \cdots \frac{\partial \theta_{M,q}}{\partial x_q} \\ \frac{\partial \tau_{1,q}}{\partial y_q}, \frac{\partial \tau_{2,q}}{\partial y_q}, \cdots, \frac{\partial \tau_{M,q}}{\partial y_q}, \frac{\partial \theta_{1,q}}{\partial y_q}, \frac{\partial \theta_{2,q}}{\partial y_q}, \cdots, \frac{\partial \theta_{M,q}}{\partial y_q} \end{bmatrix} \tag{5}$$

The Jacobian matrix elements of (5) is described by

$$\alpha_{m,q} = \frac{\partial \tau_{m,q}}{\partial x_q} = \frac{2}{c}\left(\frac{x_q - x_m^{\mathrm{E}}}{R_{m,q}} \right) \tag{6}$$

$$\beta_{m,q} = \frac{\partial \tau_{m,q}}{\partial y_q} = \frac{2}{c}\left(\frac{y_q - y_m^{\mathrm{E}}}{R_{m,q}} \right) \tag{7}$$

$$\kappa_{m,q} = \frac{\partial \theta_{m,q}}{\partial x_q} = \frac{y_q - y_m^{\mathrm{E}}}{R_{m,q}^2} \tag{8}$$

$$\mu_{m,q} = \frac{\partial\theta_{m,q}}{\partial y_q} = \frac{x_q - x_m^{\mathrm{E}}}{R_{m,q}^2} \tag{9}$$

On the basis of the conditional PDF (3), $\mathbf{J}(\mathbf{TA}_q)$ is given as:

$$\mathbf{J}(\mathrm{TA}_q) = \begin{bmatrix} \mathbf{J}(\mathrm{TA}_q)_{\tau\tau} & \mathbf{J}(\mathrm{TA}_q)_{\tau\theta} \\ \mathbf{J}(\mathrm{TA}_q)_{\theta\tau} & \mathbf{J}(\mathrm{TA}_q)_{\theta\theta} \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} \mathbf{J}(\mathrm{TA}_q)_{\tau\tau} &= -\mathrm{E}\left[\frac{\partial^2 \log f(\mathrm{T}_q|\psi_q)}{\partial\tau_{m,q}\partial\tau_{m,q}}\right] \\ &= \frac{8\pi^2 B_{m,q}^2 |\xi_{m,q}|^2 p_{m,q} n_{m,q}^2}{R_{m,q}^2 \sigma_v^2} \end{aligned} \tag{11}$$

$$\begin{aligned} \mathbf{J}(\mathrm{TA}_q)_{\theta\tau} &= \mathbf{J}(\mathrm{TA}_q)_{\tau\theta} \\ &= -\mathrm{E}\left[\frac{\partial^2 \log f(\mathbf{R}_q|\psi_q)}{\partial\tau_{m,q}\partial\theta_{m,q}}\right] \\ &= \mathbf{0}_{M\times M} \end{aligned} \tag{12}$$

$$\begin{aligned} \mathbf{J}(\mathrm{TA}_q)_{\theta\theta} &= -\mathrm{E}\left[\frac{\partial^2 \log f(\mathbf{A}_q|\psi_q)}{\partial\theta_{m,q}\partial\theta_{m,q}}\right] \\ &= \frac{2\pi^2 |\xi_{m,q}|^2 p_{m,q} n_{m,q}^4}{3R_{m,q}^2 \sigma_v^2} \end{aligned} \tag{13}$$

then CRLB matrix can be expressed as follows:

$$\begin{aligned} \mathbf{c}_q(\mathbf{P}_q, \mathbf{B}_q, \mathbf{n}_q, \psi_q) &= \left[\mathbf{J}_{\text{TOA - DOA}}(\mathbf{P}_q, \mathbf{B}_q, \mathbf{n}_q, \psi_q)\right]^{-1} \\ &= \left\{ \begin{array}{l} \sum_{m=1}^{M} p_{m,q} B_{m,q}^2 n_{m,q}^2 \begin{bmatrix} a_{m,q} & c_{m,q} \\ c_{m,q} & b_{m,q} \end{bmatrix} \\ + \sum_{m=1}^{M} p_{m,q} n_{m,q}^4 \begin{bmatrix} d_{m,q} & f_{m,q} \\ f_{m,q} & e_{m,q} \end{bmatrix} \end{array} \right\}^{-1} \end{aligned} \tag{14}$$

where $a_{m,q} = \gamma_{m,q}\alpha_{m,q}^2$, $b_{m,q} = \gamma_{m,q}\beta_{m,q}^2$, $c_{m,q} = \gamma_{m,q}\alpha_{m,q}\beta_{m,q}$, $d_{m,q} = \upsilon_{m,q}\kappa_{m,q}^2$, $e_{m,q} = \upsilon_{m,q}\mu_{m,q}^2$, $f_{m,q} = \upsilon_{m,q}\kappa_{m,q}\mu_{m,q}$, $\gamma_{m,q} = \frac{8\pi^2|\xi_{m,q}|^2}{R_{m,q}^2\sigma_v^2}$, $\upsilon_{m,q} = \frac{2\pi^2|\xi_{m,q}|^2}{3R_{m,q}^2\sigma_v^2}$.

3 Proposed Strategy

3.1 Problem Formulation

Since the CRLB can be exploited as the localization accuracy metric, the CBAPA strategy is formulated as following.

$$\min_{\mathbf{P}_q,\mathbf{u}_q} \sum_{q=1}^{Q} \mathbf{1}^{\mathrm{T}}\mathbf{P}_q,$$
$$\text{s.t.}: \begin{cases} \operatorname{tr}\left(\mathbf{c}_q\left(\boldsymbol{\psi}_q, \mathbf{P}_q, \mathbf{u}_q\right)\right) \le \sigma_p^2, \forall q, \\ \sum_{q=1}^{Q} u_{m,q} = L, \forall m, \\ u_{m,q} = \{0, 1\}, \\ p_{m,q} \ge 0, \end{cases} \tag{15}$$

where L is the number of the beam, $\mathbf{1} = [1_1, 1_2, \cdots, 1_M]^{\mathrm{T}}$. σ_p is the predetermined MSE thresholds of target localization. The first constraint means that the target localization should achieve predetermined MSE thresholds. The second one suggests each phased array radar can generate L beams. The last two represents the natural constraints.

3.2 Solution Scheme

It is worth noting that the optimization problem (15) can be considered as a hybrid non-convex non-linear Boolean optimization formula, and belongs to the NP-hard problem. In this work, a two-step solution technique of SDP and PSO is proposed to solve the above problems.

First-Step: Regards the allocation of beam-target is fixed. Since there is no effect among different targets, minimizing the total transmit power is equivalent to summing the minimum power of localizing each target, that is, $\min_{\mathbf{P}_q,\mathbf{u}_q} \sum_{q=1}^{Q} \mathbf{1}^{\mathrm{T}}\mathbf{P}_q = \sum_{q=1}^{Q} \left(\min_{\mathbf{P}_q} \mathbf{1}^{\mathrm{T}}\mathbf{P}_q\right)$. Therefore, solving (15) is equivalent to solve q sub-problems for each target, shown as following:

$$\min_{\mathbf{P}_q} \mathbf{1}^{\mathrm{T}}\mathbf{P}_q,$$
$$\text{s.t.}: \begin{cases} \operatorname{tr}\left(\mathbf{c}_q\left(\boldsymbol{\psi}_q, \mathbf{P}_q\right)\right) \le \sigma_p^2, q = 1, 2, \cdots Q, \\ p_{m,q} \ge 0, m = 1, 2, \cdots M, q = 1, 2, \cdots Q. \end{cases} \tag{16}$$

Since the (16) is a convex optimization, there exist multiple solution techniques to tackle it. In this article, the problem is resolved by the SDP algorithm.

Second-Step: By achieving the minimum power of every target in the selection of distinct phased array radar, (15) is converted to a discrete optimization problem, resolved by PSO. The schematic illustration of this process is observed by Fig. 1.

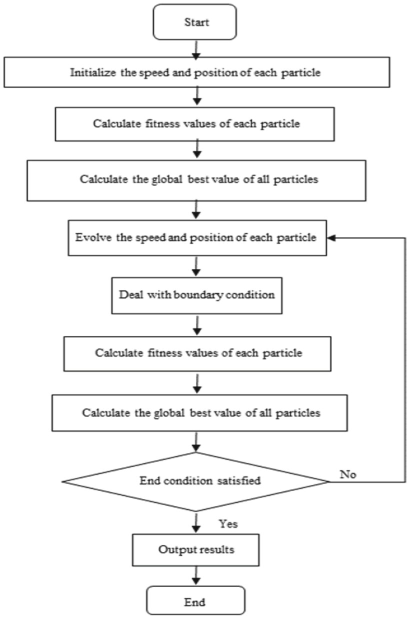

Fig. 1. The schematic illustration of step 2.

4 Numerical Results

In this section, we suppose there are 4 targets located by 3 phased array radars, as presented in Fig. 2. The array element number and beams are set as 4000 and 2, respectively. The RCS of every target, predefined localization accuracy and noise power of every phased array radar are set as 1 m^2, 10 m and 1 W, respectively.

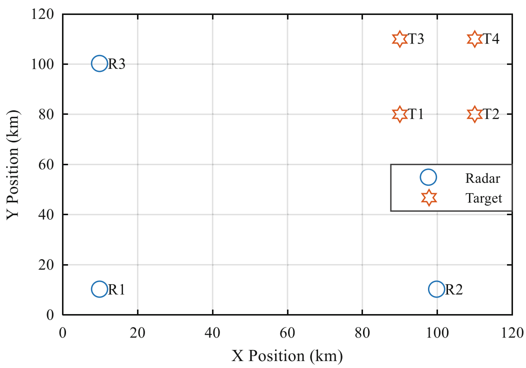

Fig. 2. Target deployment and phased array radars.

To reveal the superiority of the CBAPA strategy in the light of LPI performance, the total transmit power for the same predefined localization accuracy is compared with those of existing resource allocation strategies as follows:

- **Optimal Power Allocation** (OPA) [11]: Each phased array radar generates the same number of beams as the targets, then minimizes the total transmit power by optimally allocating power to each beam, while meeting the predetermine localization accuracy.
- **Optimal Power and Bandwidth Allocation** (OPBA) [7, 8]: Each phased array radar generates the same beam number as OPA, and then minimizes the total transmit power to a predefined threshold by optimizing the power and bandwidth allocation of each beam, subject to all target localization accuracy constraints.

Figure 3 presents the comparison of their total consumption power for the same localization accuracy. Obviously, the total transmit power of the CBAPA strategy is the least compared other two strategies. The proposed strategy lowers the total transmit power by 91 and 88% comparing with OPA and OPBA strategies. The comparison analysis indicates that our proposed strategy can greatly decrease the total transmit power comparing with existing resource allocation strategies, certainly possessing best LPI performance.

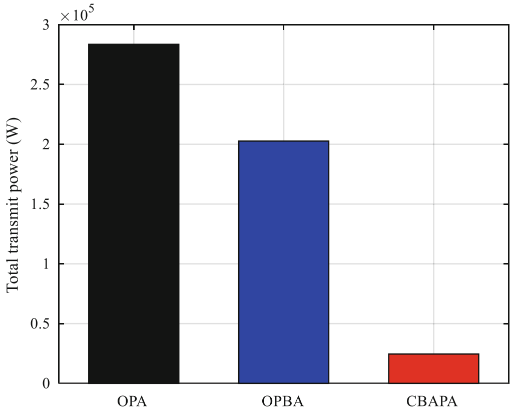

Fig. 3. The comparison about the total transmit power between the proposed strategy and the existing resource allocation strategies.

5 Concluding Remarks

In this article, the CRLB of muti-target localization is developed and applied as the localization index, then the CBAPA resource allocation strategy is defined as a hybrid nonconvex Boolean problem. For seeking a response to this problem, we design a two-step iterative algorithm in terms of SDP and PSO. Finally, the rationality and superiority of the proposed strategy can be certified via simulation results.

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