Applications of Optimal Homotopy Asymptotic Method (OHAM) to Tenth Order Boundary Value Problem

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Abstract. The aim of this paper is to apply the Optimal Homotopy Asymptotic Method (OHAM), a semi-numerical and semi-analytic technique for solving linear and nonlinear Tenth order boundary value problems. The approximate solution of the problem is calculated in terms of a rapidly convergent series. Two bench mark examples have been considered to illustrate the efficiency and implementation of the method and the results are compared with the Variational Iteration Method (VIM). An interesting result of the analysis is that, the OHAM solution is more accurate than the VIM. Moreover, OHAM provides us with a convenient way to control the convergence of approximate solutions. The obtained solutions have shown that OHAM is effective, simpler, easier and explicit.

Keywords: OHAM · Tenth Order Boundary Value Problems

1 Introduction

In this paper, it is observed that the OHAM is a powerful approximate analytical tool like HAM (Homotopy Asymptotic Method) that is simple and straight forward and does not require the existence of any small or large parameter as does traditional perturbation method. OHAM has been successfully applied to a number of nonlinear problems arising in the science and engineering by various researchers [3–5, 10–16, 18, 20, 34–36]. Shah et al. [17–19] have investigated the behavior through graphical representation. Whilst Khan et al. [1, 2] discussed flow between rotating stretchable disks. What's more, Khan et al. [1] found that when both the discs rotate in the same sense then the fluid in the disks rotates with an angular velocity. Khan et al. [3] discussed Dynamics with Cattaneo–Christov heat and mass flux theory of bioconvection Oldroyd-B nanofluid. Notwithstanding, Khan et al. [1–3] further explored that the their study provides the best solutions and it has been proved that its solution is close to exact solution. Khan et al. [4] discussed Rotating flow assessment of magnetized mixture fluid suspended with hybrid nanoparticles and chemical reactions of species. Rasheed et al. [5] discussed Computational analysis of hydromagnetic boundary layer stagnation point flow of nano liquid by a stretched heated surface with convective conditions and radiation effect. Mohmand et al. [25–28]

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scrutinized oscillating and porous, and flow with heat transfer effect as well as vibratory flow. Usman et al. [6] discussed Computational optimization for the deposition of bio-convection thin Oldroyd-B nanofluid with entropy generation. Khan et al. [7] explored Lorentz forces effects on the interactions of nanoparticles in emerging mechanisms with innovative approach. Khan et al. [8] analyzed Solution of magnetohydrodynamic flow and heat transfer of radiative viscoelastic fluid with temperature dependent viscosity in wire coating analysis. Khan et al. [9] investigated A Framework for the Magnetic Dipole Effect on the Thixotropic Nanofluid Flow Past a Continuous Curved Stretched Surface. Khan et al. [10] studied Analytical solution of UCM viscoelastic liquid with slip condition and heat flux over stretching sheet: Galerkin Approach. Shah et al. [17–19] have explored the transient flow, with unsteady stretching surface and accompanied by Soret and Dufour effects. Khan et al. [1–4] have discussed solution through tables. Khan et al. [11] discussed Mechanical aspects of Maxwell nanofluid in dynamic system with irreversible analysis. Khan et al. [12] studied Numerical simulation of double-layer optical fiber coating using Oldroyd 8-constant fluid as a coating material. Khan et al. [8, 10, 12] have also discussed heat and heat transfer. Shah et al. [13, 18] analyzed Gravity Driven Flow of an Unsteady Second Order Fluid as well as Heat transfer rate of the fluid at the belt is also calculated. Khan et al. [14] discussed Investigation of wire coating using hydromagnetic third-grade liquid for coating along with Hall current and porous medium. Khan et al. [15] studied Analytical Solution of the MHD Viscous Flow over a Stretching Sheet by Multistep Optimal Homotopy Asymptotic Method. Fiza et al. [16] explored Modifications of the multistep optimal homotopy asymptotic method to some nonlinear KdV-equations. Shah et al. [17] discussed The ADM solution of MHD non-Newtonian fluid with transient flow and heat transfer. Shah et al. [18] studied the Heat transfer and hydromagnetic effects on the unsteady thin film flow of Oldroyd-B fluid over an oscillating moving vertical plate. Shah et al. [19] explored forSoret and Dufour effect on the thin film flow over an unsteady stretching surface. Khan et al. [20–22] discussed for Mechanical aspects of Maxwell nanofluid in dynamic system with irreversible analysis as well as the impact of emerging parameters involved in the solutions are discussed through graphs on the velocity and temperature profiles in detail. Khan et al. [21] researched on the investigation of Wire Coating using Hydromagnetic Third-Grade Liquid for Coating along with Hall Current and Porous Medium. Khan et al. [22] discussed the Analytical Solution of UCM Viscoelastic Liquid with Slip Condition and Heat Flux over Stretching Sheet: The Galerkin Approach. Mohmand et al. [23] discussed the Engineering Investigations of Dufour and Soret effect on MHD heat and mass transfer with radiative heat flux in a liquid over a rotating dick. Mohmand et al. [24] explored the Engineering applications and analysis of vibratory motion fourth order fluid film over the time dependent heated flat plate. Mohmand et al. [25] analyzed for Time dependent Oldroyd-B liquid film flow over an oscillating and porous vertical plate with the effect of thermal radiation. Mohmand et al. [26] studied Time dependent second grade fluid between two vertical oscillating plates with heat transfer effect. Mohmand et al. [27] investigated the Vibratory motion of fourth order fluid film over a unsteady heated flat. Mohmand et al. [28] discussed Engineering applications and analysis of vibratory motion fourth order fluid film over the time dependent heated flat plate. Mohmand et al. [29] explored for Heat transfer and hydromagnetic effects on the unsteady thin film flow of Oldroyd-B fluid over an oscillating moving vertical plate. Shah et al. [30] discussed Soret and dufour effect on the thin film flow over an unsteady stretching surface. Likewise Khan et al. [20] have also discussed about the Brownian motion and thermophoresis with thermal radiation and buoyancy effects are encountered in the governing equations. The oscillating parallel plates were discussed by Shah et al. [13]. Shah et al. [17–19] have also explored some more properties of fluids. Similarly, Fiza et al. and Khan et al. [16, 21] have discussed flow through numerical results (Runge-Kutta method). Rasheed et al. [31] and Shah et al. [30] have discussed fluid flow through shooting technique and numerical approach. Moreover, flow of Oldroyd-B fluid over an oscillating was discussed by Shah et al. [29]. Likewise Mohmand et al. [28] have discussed time-dependent heated plate. Moreover, Rasheed et al. [32, 33] have discussed the fluid flow. To this end, Khan et al. [2] have investigated pressure distribution and entropy generation rate and then their solution through HAM approach. Moreover, Khan et al. [3–5] Darcy–Forchheimer law is used to study heat and mass transfer flow and microorganisms motion in porous media as well as flow of Maxwell nanofluid induced by two parallel rotating disks were analyzed. Furthermore Rasheed et al. [31–33] have discussed fluid motion with thermal analysis. Shah et al. [34] have also discussed MHD flow. Khan et al. [1–4] have analyzed the fluid motion through graphs. Similarly, Shah et al. [17–19] have also investigated the MHD fluid motion through graphs. Likewise, Khan et al. [7–12] have also discussed fluid flow through graphs and found excellent harmony with the already published works. Shah et al. [34] & [35] have scrutinized a mathematical and computational analysis of MHD fluid with heat source effect and the chemically reactive Casson fluid to add knowledge to the existing one. As regard to the validity solution of PDEs one can consult the research done by Khan et al. [36]. Similarly, Shah [37] has discussed the fluid motion through the treatment of OHAM. This paper is organized as follows. First, we formulate the problem. Then, we present basic principles of OHAM. The OHAM solution is also given. After that, we analyze the comparison of the solution using OHAM with existing solution of VIM and DTM. Last but not the least, drew the conclusion.

2 Formulation of the Problem

In the present paper, thirteen-order boundary value problems are solved using OHAM. The following thirteen-order boundary value problems are considered

$$\left.\begin{array}{c} u^{(13)}(x) = f(x, u(x)), a \le x \le b \\ u^{(i)}(a) = A_{i,} \\ u^{(j)}(b) = B_j \end{array}\right\} \tag{1}$$

where for i = 0,1, 2, 3, ..., 6 and j = 0,1, ..., 5 A'_is and B'_js are finite real constants. Also f(x, u(x)) is a continuous function on [a, b].

3 Fundamental Mathematical Theory of OHAM

Consider the differential equation of the following form:

$$\mathcal{A}(u(x)) + f(x) = 0, x \in \Omega \tag{2}$$

$$\mathcal{B}(u, \partial u/\partial x) = 0, x \in \Gamma \tag{3}$$

where $\mathcal{A}$ is a differential operator, u(x) is an unknown function, and x and t denote spatial independent variable, Γ is the boundary of Ω and f(x) is a known analytic function. $\mathcal{A}$ can be divided into two parts: $\mathcal{L}$ and $\mathcal{N}$ such that:

$$\mathcal{A} = \mathcal{L} + \mathcal{N} \tag{4}$$

$\mathcal{L}$ is the linear part of the differential equation which is easier to solve, and $\mathcal{N}$ contains the nonlinear part of $\mathcal{A}$.

According to OHAM, one can construct an optimal homotopy $\phi(x, p) : \Omega \times [0,1] \to \Re$ which satisfies:

$$H(\phi(x, p), p) = (1 - p)\{\mathcal{L}(\phi(x, p)) + f(x)\} - H(p)\{\mathcal{A}(\phi(x, p)) + f(x)\} = 0, \tag{5}$$

where the auxiliary function H(p) is nonzero for $p \neq 0$ and $H(0) = 0$. Equation (5) is called optimal homotopy equation. Clearly, we have:

$$p = 0 \Rightarrow H(\phi(x, 0), 0) = \mathcal{L}(\phi(x, 0)) + f(x) = 0, \tag{6}$$

$$p = 1 \Rightarrow H(\phi(x, 1), 1) = H(1)\{\mathcal{A}(\phi(x, p)) + f(x)\} = 0, \tag{7}$$

Obviously, when $p = 0$ and $p = 1$ we obtain $\phi(x, 0) = u_0(x)$ and $\phi(x, 1) = u(x)$ respectively. Thus, as p varies from 0 to 1, the solution $\phi(x, p)$ approaches from $u_0(x)$ to u(x), where $u_0(x)$ is obtained from Eq. (5) for $p = 0$:

$$\mathcal{L}(u_0(x)) + f(x) = 0,\ \mathcal{B}(u_0, \partial u_0/\partial x) = 0. \tag{8}$$

Next, we choose auxiliary function H(p) in the form

$$H(p) = p^1 C_1 + p^2 C_2 + \ldots \tag{9}$$

To get an approximate solution, we expand $\phi(x, p, C_i)$ by Taylor's series about p in the following manner,

$$\phi(x, p, C_i) = u_0(x) + \sum_{k=1}^{\infty} u_k(x, C_i)p^k, i = 1, 2, \ldots \tag{10}$$

Substituting Eq. (10) into Eq. (5) and equating the coefficient of like powers of p, we obtain Zeroth order problem, given by Eq. (8), the first and second order problems are given by Equations. (11–12) respectively and the general governing equations for $u_k(x)$ are given by Eq. (13):

$$\mathcal{L}(u_1(x)) = C_1\mathcal{N}_0(u_0(x)), \mathcal{B}(u_1, \partial u_1/\partial x) = 0 \tag{11}$$

$$\begin{gathered}\mathcal{L}(u_2(x)) - \mathcal{L}(u_1(x)) = C_2\mathcal{N}_0(u_0(x)) + \\ C_1[\mathcal{L}(u_1(x)) + \mathcal{N}_1(u_0(x), u_1(x))], \mathcal{B}(u_2, \partial u_2/\partial x) = 0\end{gathered} \tag{12}$$

$$\begin{aligned}\mathcal{L}(u_k(x)) - \mathcal{L}(u_{k-1}(x)) = C_k\mathcal{N}_0(u_0(x)) + \textstyle\sum_{i=1}^{k-1} C_i\big[\mathcal{L}(u_{k-i}(x)) \\ +\mathcal{N}_{k-i}(u_0(x), u_1(x), \ldots, u_{k-i}(x))\big], k = 2, 3, \ldots \mathcal{B}(u_k, \partial u_k/\partial x) = 0\end{aligned}, \tag{13}$$

where $\mathcal{N}_{k-i}(u_0(x), u_1(x), \ldots, u_{k-i}(x))$ are the coefficient of p^{k-i} in the expansion of $\mathcal{N}(\phi(x, p))$ about the embedding parameter p.

$$\mathcal{N}(\phi(x, p, C_i)) = \mathcal{N}_0(u_0(x)) + \sum\nolimits_{k\geq 1} \mathcal{N}_k(u_0, u_1, u_2, \ldots, u_k)p^k \tag{14}$$

It should be underscored that the u_k for $k \geq 0$ are governed by the linear equations with linear boundary conditions that come from the original problem, which can be easily solved.

It has been observed that the convergence of the series Eq. (10) depends upon the auxiliary constants $C_1, C_2, \ldots$. If it is convergent at $p = 1$, one has:

$$\tilde{u}(x, C_i) = u_0(x) + \sum\nolimits_{k\geq 1} u_k(x, C_i) \tag{15}$$

Substituting Eq. (15) into Eq. (1), it results the following expression for residual:

$$R(x, C_i) = \mathcal{L}(\tilde{u}(x, C_i)) + f(x) + \mathcal{N}(\tilde{u}(x, C_i)) \tag{16}$$

In actual computation $k = 1, 2, 3, \ldots, m$.

If $R(x, C_i) = 0$ then $\tilde{u}$ (x, C_i) is the exact solution of the problem. Generally it doesn't happen, especially in nonlinear problems.

For the determinations of auxiliary constants, $C_i = 1, 2, \ldots, m$, there are different methods like Galerkin's Method, Ritz Method, Least Squares Method and Collocation Method. One can apply the Method of Least Squares as under:

$$J(C_i) = \textstyle\int_a^b R^2(x, C_i)dx \tag{17}$$

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = \ldots = \frac{\partial J}{\partial C_m} = 0 \tag{18}$$

The mth order approximate solution can be obtained by these constants so obtained. The constants C_i can also be determined by another method as under:

$$R(h_1; C_i) = R(h_2; C_i) = \ldots = R(h_m; C_i) = 0, i = 1, 2, \ldots, m. \tag{19}$$

at any time t, where $h_i \in \Omega$.

The more general auxiliary function H(p) is useful for convergence, which depends upon constants $C_1, C_2, \ldots$ can be optimally identified by Eq. (18) and is useful in error minimization.

4 Solution of the Problem via OHAM

In this section we will apply OHAM to a linear boundary value problem and non-linear boundary value problem. Example 1: The linear thirteen-order BVP, is considered as

$$u^{(13)}(x) = cosx - sinx, 0 \leq x \leq 1, \tag{20}$$

Subject to the boundary conditions,

$$\left.\begin{array}{l} u(0) = 1, u(1) = \cos(1) + \sin(1), \\ u^{(1)}(0) = 1, u^{(1)}(1) = \cos(1) - \sin(1), \\ u^{(2)}(0) = -1, u^{(2)}(1) = -\sin(1) - \cos(1), \\ u^{(3)}(0) = -1, u^{(3)}(1) = -\cos(1) + \sin(1), \\ u^{(4)}(0) = 1, u^{(4)}(1) = \cos(1) + \sin(1), \\ u^{(5)}(0) = 1, u^{(5)}(1) = \cos(1) - \sin(1), \\ u^{(6)}(0) = -1, \end{array}\right\} \quad (21)$$

The exact solution of the problem is,

$$u(x) = cosx + sinx. \quad (22)$$

According to above equation, we have

$$\mathcal{L}(u(x)) = u^{(13)}(x); \mathcal{N}(u(x)) = 0; \text{ and } f(x) = -cosx + sinx; \quad (23)$$

5 Comparison of OHAM Solution with VIM Solution

Comparison of absolute error of OHAM with VIM in Table 1.

Table 1. Shows the comparison of absolute error of OHAM with VIM [10]

x	Exact	OHAM	Abs: Error in present method	Abs: Error in VIM
0	1.00000	1.00000	0.00000	0.00000
0.1	1.105170918	1.10517	4.32987×10^{-14}	4.17444×10^{-14}
0.2	1.221402758	1.2214	2.44249×10^{-14}	2.64144×10^{-12}
0.3	1.349858808	1.34986	5.99520×10^{-15}	2.99314×10^{-11}
0.4	1.491824698	1.49182	1.44329×10^{-14}	1.67101×10^{-10}
0.5	1.648721271	1.64872	2.64677×10^{-13}	6.30955×10^{-10}
0.6	1.8221188	1.82212	1.02807×10^{-12}	1.84757×10^{-9}
0.7	2.013752707	2.01375	8.95284×10^{-13}	4.47866×10^{-9}
0.8	2.225540928	2.22554	6.89315×10^{-12}	9.21592×10^{-9}
0.9	2.459603111	2.4596	2.60871×10^{-11}	1.58906×10^{-8}
1	2.718281828	2.71828	3.41771×10^{-12}	2.09057×10^{-8}

6 Conclusion of the Current Analysis

In the above table, it is clearly observed that OHAM is better than VIM and give wonderful results of Tenth order boundary value problems for both linear and nonlinear. Therefore, we conclude that OHAM is reasonably good method for any type of T order boundary value problem. Below Fig. 1: represents the exact solution of the Tenth order nonlinear boundary value problem, while Fig. 2: shows the OHAM solution of the Tenth order nonlinear boundary value problem.

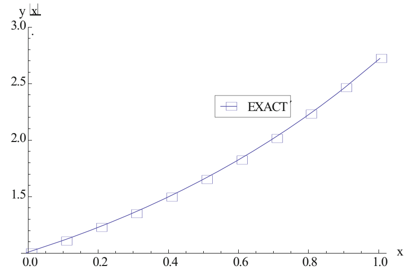

Fig. 1. Exact Solution

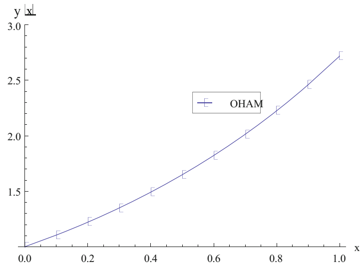

Fig. 2. OHAM Solution

References

1. Khan, N.S., Shah, Q., Bhaumik, A., Komam, P.: Entropy generation in bioconvection nanofluid flow between two stretchable rotating disks Nat.-Sci. Rep. **30**(01), 52–88 (2020)
2. Khan, N.S., Zuhra, S., Shah, Q.: Entropy generation in two phase model for simulating flow and heat transfer of carbon nanotubes between rotating stretchable disks with cubic autocatalysis chemical reaction. Appl. Nanosci. **9**(8), 1797–1822 (2019)
3. Khan, N.S., Shah, Q., Sohail, A.: Dynamics with Cattaneo–Christov heat and mass flux theory of bioconvection Oldroyd-B nanofluid. Adv. Mech. Eng. **12**(8), 168781402093046 (2020)
4. Khan, N.S., et al.: Rotating flow assessment of magnetized mixture fluid suspended with hybrid nanoparticles and chemical reactions of species. Sci. Rep. **11**(1), 11277 (2021)
5. Rasheed, H.U., et al.: Computational analysis of hydromagnetic boundary layer stagnation point flow of nano liquid by a stretched heated surface with convective conditions and radiation effect. Adv. Mech. Eng. **13**(10), 168781402110531 (2021)
6. Usman, A.H., et al.: Computational optimization for the deposition of bioconvection thin Oldroyd-B nanofluid with entropy generation. Sci. Rep. **11**(1), 1–23 (2021)
7. Khan, N.S., et al.: Lorentz forces effects on the interactions of nanoparticles in emerging mechanisms with innovative approach. Symmetry **12**(10), 1700 (2020)
8. Khan, Z., Khan, M.A., Siddiqui, N., Ullah, M., Shah, Q.: Solution of magnetohydrodynamic flow and heat transfer of radiative viscoelastic fluid with temperature dependent viscosity in wire coating analysis. PLoS ONE **13**(3), e0194196 (2018)
9. Khan, N.S., et al.: A framework for the magnetic dipole effect on the thixotropic nanofluid flow past a continuous curved stretched surface. Crystals **11**(6), 645 (2021)
10. Khan, Z., et al.: Analytical solution of UCM viscoelastic liquid with slip condition and heat flux over stretching sheet: the galerkin approach. Math. Prob. Eng. **2020**, 1–7 (2020)
11. Khan, N.S., Shah, Q., Sohail, A., Kumam, P., Thounthong, P., Muhammad, T.: Mechanical aspects of Maxwell nanofluid in dynamic system with irreversible analysis. ZAMM-J. Appl. Math. Mech./Zeitschrift für Angewandte (2021)

12. Khan, Z., Rasheed, H.U., Shah, Q., Abbas, T., Ullah, M.: Numerical simulation of double-layer optical fiber coating using Oldroyd 8-constant fluid as a coating material. Opt. Eng. **57**(07), 076104 (2018)
13. Shah, Q., Gul, T., Shah, R.A., Ullah, W., Idrees, M., Mohmand, M.I.: Gravity driven flow of an unsteady second order fluid between two parallel and vertical oscillating plates. J. Appl. Environ. Biol. Sci **5**(3), 213–220 (2015)
14. Khan, Z., et al.: Investigation of wire coating using hydromagnetic third-grade liquid for coating along with Hall current and porous medium. Math. Prob. Eng. **2020**, 1–8 (2020)
15. Fiza, M., Islam, S., Ullah, H., Chohan, F., Shah, Q.: Analytical solution of mhd viscous flow over a stretching sheet by multistage optimal homotopy asymptotic method. Int. J. Fluid Mech. Res. **45**(4), 369–375 (2018)
16. Fiza, M., Ullah, H., Islam, S., Shah, Q., Chohan, F.I., Mamat, M.B.: Modifications of the multistep optimal homotopy asymptotic method to some nonlinear KdV-equations. Eur. J. Pure Appl. Math. **11**(2), 537–552 (2018)
17. Shah, Q., Mamat, M.B., Gul, T., Tofany, N.: The ADM solution of MHD non-Newtonian fluid with transient flow and heat transfer. In: AIP Conference Proceedings, vol. 1775, no. 1, p. 030087 (2016)
18. Shah, Q, Mamat, M.B., Gul, T., Tofany, N: Heat transfer and hydromagnetic effects on the unsteady thin film flow of Oldroyd-B fluid over an oscillating moving vertical plate. In: AIP Conference Proceedings, vol. 1775, no. 1, p. 030085 (2016)
19. Shah, Q., Gul, T., Mamat, M.B., Khan, W., Tofany, N.: Soret and Dufour effect on the thin film flow over an unsteady stretching surface. In: AIP Conference Proceedings, vol. 1775, no. 1, p. 0300881 (2016)
20. Khan, N.S., Shah, Q., Sohail, A., Kumam, P., Thounthong, P.: Mechanical aspects of Maxwell nanofluid in dynamic system with irreversible analysis. Zamm J. Appl. Math. Mech. I.FACTOR **101**(12) (2021)
21. Khan, Z., et al.: Investigation of wire coating using hydromagnetic third-grade liquid for coating along with hall current and porous medium. Math. Probl. Eng. **2020**, 1–8 (2020). https://doi.org/10.1155/2020/4218717
22. Khan, Z., et al.: Analytical solution of UCM viscoelastic liquid with slip condition and heat flux over stretching sheet: the galerkin approach. Math. Prob. Eng. **2020**, 1–7 (2020). https://doi.org/10.1155/2020/7563693
23. Mohmand, M.I., Mamat, M.B., Shah, Q.: Engineering Investigations of Dufour and Soret effect on MHD heat and mass transfer with radiative heat flux in a liquid over a rotating dick (2018)
24. Mohmand, M.I., Mamat, M.B., Shah, Q.: Engineering applications and analysis of vibratory motion fourth order fluid film over the time dependent heated flat plate. In: AIP Conference Proceedings, vol. 1859. no. 1, p. 020073 (2017)
25. Mohmand, M.I., Shah, Q., Mamat, M.B., Shah, Z., Khan, A.S: Time dependent Oldroyd-B liquid film flow over an oscillating and porous vertical plate with the effect of thermal radiation. In: AIP Conference Proceedings, vol. 1846, no. 1, p. 050003 (2017)
26. Mohmand, M.I., Mamat, M.B., Shah, Q.: Time dependent second grade fluid between two vertical oscillating plates with heat transfer effect. In: AIP Conference Proceedings, vol. 1823, no. 1, p. 020093 (2017)
27. Mohmand, M.I., Mamat, M.B., Shah, Q., Gul, T.: Vibratory motion of fourth order fluid film over a unsteady heated flat. In: AIP Conference Proceedings, vol. 1823, no. 1, p. 020066 (2017)
28. Mohmand, M.I., Shah, Q., Mustafa, M: Engineering applications and analysis of vibratory motion fourth order fluid film over the time dependent heated flat plate. In: AIP Conference Proceeding (2017)

29. Shah, Q., Gul, T., Mamat, M.B., Tofany, N.: Heat transfer and hydromagnetic effects on the unsteady thin film flow of Oldroyd-B fluid over an oscillating moving vertical plate. In: AIP Conference Proceeding (2016)
30. Shah, Q., Gul, T., Mamat, M.B., Khan, W., Tofany, N.: Soret and dufour effect on the thin film flow over an unsteady stretching surface. In: AIP Conference Proceeding (2016)
31. Rasheed, H.U., Zeeshan, S.I., Ali, B., Shah, Q., Ali, R.: Implementation of shooting technique for Buongiorno nanofluid model driven by a continuous permeable surface. Heat Transf. **52**(4), 3119–3134 (2023)
32. Rasheed, H., Shah, Q., Khan, J., Abbas, T., Khan, W., Mohmand, M.I.: Analysis of 2D stagnation point micropolar fluid with MHD effect and modified heat and mass flux towards a permeable channel: a computational study. Heat Transf. (2023)
33. Rasheed, H.U., Shah, Q., Khan, J., Abbas, T., Khan, W., Mohmand, M.I.: Physical insight into thermal analysis of magnetohydrodynamic stagnation point flow of micropolar nanofluid across a flexible surface equipped with porous medium and Fourier and Fick's law. Heat Transf. **53**(2), 512–532 (2024)
34. Shah, Q., Rasheed, H., Khan, M.I: A mathematical and computational framework for MHD and heat source effect on nanofluid flow by a nonlinearly stretching sheet. Kongzhi yu Juece/Control and Decision (2023)
35. Shah, Q., Rasheed, H., Khan, M.I.: Analysis of MHD flow chemically reactive Casson liquid by an elongated permeable sheet with Lorentz force and heat reservoir effects. Kongzhi yu Juece/Control Decis. (2023)
36. Shah, Q.: Application of Optimal Homotopy Asymptotic Method (OHAM) to Tenth order Boundary Value Problem. Springer Conference Series (2024)
37. Khan, S., et al.: A well-conditioned and efficient implementation of dual reciprocity method for Poisson equation. AIMS Math. **6**, 12560–12582 (2021)