Analysis of Hugging Flow Through the Powerful Technique of Homotopy Asymptotic Method (HAM)

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Abstract. A hugging flow occurs when a fluid is hugged between two undistinguishable plates fronting one another. To this end, the hugging flow between two disks in drive of unstable nature is of immense importance for its technical and scientific uses such as, molding of fluids, the study of fluid machinery such as pumps, fans, blowers, windmills, air compressors, heat exchangers, jet & rocket engines, gas turbines, power plants, pollution control equipments, air-conditioning equipments, heating & ventilation systems, breathing aids, heat-lungs machines, among others makes fluid mechanics of massive importance to Mechanical Engineers. Similarly, expansion and contraction in blood flow, piston motion, brakes, and in cooling towers among others. The aim of this research is to investigate Newtonian fluid between two porous time-varying plates in hugging flow. The impact variable, magnetic field is taken into account. The treatment of obtained system of equations is done by HAM (Homotopy Asymptotic Method). After a comparative analysis between The results obtained through HAM and Numerical methods showed a great agreement of harmony. Variation in the flow fields is presented with the help of figures. The residual errors for fluid flow fields are calculated and shown with the help of table. All the computational work has been done with the help of computer software Mathematica Software.

Keywords: Non-Newtonian Fluid · Two Plates · HAM Solution

1 Introduction

Non-Newtonian fluid flow has established the researchers' interest in modern age as they are taking interest due to its excessive applications like pollution control equipments, air-conditioning equipments, heating & ventilation systems, breathing aids, heat-lungs machines and other nano-fluids are examples. Numerous scientists and engineers have gone through and scrutinized this concept from innumerable rheological viewpoints. A fluid is a non-Newtonian liquid with exceptional features. As for example Mohmand et al. [24–27] have thrown light on the graphical solution of fluid flow. Geniuses have been successfully applied to a number of nonlinear problems arising in the science and engineering by various researchers [3–5, 10–16, 18, 20, 33–35]. Shah et al. [17–19] have

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investigated the behavior through graphical representation. Khan et al. [1, 2] discussed flow between rotating stretchable disks. What's more, Khan et al. [1] found that when both the discs rotate in the same sense then the fluid in the disks rotates with an angular velocity. Khan et al. [3] discussed Dynamics with Cattaneo–Christov heat and mass flux theory of bioconvection Oldroyd-B nanofluid. Notwithstanding, Khan et al. [1–3] further explored that the their study provides the best solutions and it has been proved that its solution is close to exact solution. Khan et al. [4] discussed Rotating flow assessment of magnetized mixture fluid suspended with hybrid nanoparticles and chemical reactions of species. Rasheed et al. [5] discussed Computational analysis of hydromagnetic boundary layer stagnation point flow of nano liquid by a stretched heated surface with convective conditions and radiation effect. Mohmand et al. [24–27] scrutinized oscillating and porous, and flow with heat transfer effect as well as vibratory flow. Usman et al. [6] discussed Computational optimization for the deposition of bioconvection thin Oldroyd-B nanofluid with entropy generation. Khan et al. [7] explored Lorentz forces effects on the interactions of nanoparticles in emerging mechanisms with innovative approach. Khan et al. [8] analyzed Solution of magnetohydrodynamic flow and heat transfer of radiative viscoelastic fluid with temperature dependent viscosity in wire coating analysis. Khan et al. [9] investigated A Framework for the Magnetic Dipole Effect on the Thixotropic Nanofluid Flow Past a Continuous Curved Stretched Surface. Khan et al. [10] studied Analytical solution of UCM viscoelastic liquid with slip condition and heat flux over stretching sheet: Galerkin Approach. Shah et al. [17–19] have explored the transient flow, with unsteady stretching surface and accompanied by Soret and Dufour effects. Khan et al. [11] discussed Mechanical aspects of Maxwell nanofluid in dynamic system with irreversible analysis. Khan et al. [12] studied Numerical simulation of double-layer optical fiber coating using Oldroyd 8-constant fluid as a coating material. Khan et al. [8, 10, 12] have also discussed heat and heat transfer. Shah et al. [13, 18] analyzed Gravity Driven Flow of an Unsteady Second Order Fluid as well as Heat transfer rate of the fluid at the belt is also calculated. Khan et al. [14] discussed Investigation of wire coating using hydromagnetic third-grade liquid for coating along with Hall current and porous medium. Khan et al. [15] studiedAnalytical Solution of the MHD Viscous Flow over a Stretching Sheet by Multistep Optimal Homotopy Asymptotic Method. Fiza et al. [16] explored Modifications of the multistep optimal homotopy asymptotic method to some nonlinear KdV-equations. Shah et al. [17]discussed. The ADM solution of MHD non-Newtonian fluid. Khan et al. [1–4] have discussed solution through tables. Shah et al. [18] studied the Heat transfer and hydromagnetic effects on the unsteady thin film flow of Oldroyd-B fluid over an oscillating moving vertical plate. Shah et al. [19] explored for Soret and Dufour effect on the thin film flow over an unsteady stretching surface. Khan et al. [14, 20, 21] discussed for Mechanical aspects of Maxwell nanofluid in dynamic system with irreversible analysis as well as the impact of emerging parameters involved in the solutions are discussed through graphs on the velocity and temperature profiles in detail. Khan et al. [14] researched on the investigation of Wire Coating using Hydromagnetic Third-Grade Liquid for Coating along with Hall Current and Porous Medium. Khan et al. [21] discussed the Analytical Solution of UCM Viscoelastic Liquid with Slip Condition and Heat Flux over Stretching Sheet: The Galerkin Approach. Mohmand et al. [22] discussed the Engineering Investigations of Dufour and Soret effect on MHD heat and mass transfer with radiative heat flux in a liquid over a rotating dick. Mohmand et al. [23] explored the Engineering applications and analysis of vibratory motion fourth order fluid film over the time dependent heated flat plate. Mohmand et al. [24] analyzed for Time dependent Oldroyd-B liquid film flow over an oscillating and porous vertical plate with the effect of thermal radiation. Mohmand et al. [25] studied Time dependent second grade fluid between two vertical oscillating plates with heat transfer effect. Mohmand et al. [26] investigated the Vibratory motion of fourth order fluid film over a unsteady heated flat. Mohmand et al. [27] discussed Engineering applications and analysis of vibratory motion fourth order fluid film over the time dependent heated flat plate. Mohmand et al. [28] explored for Heat transfer and hydromagnetic effects on the unsteady thin film flow of Oldroyd-B fluid over an oscillating moving vertical plate. Shah et al. [29] discussed Soret and dufour effect on the thin film flow over an unsteady stretching surface. Likewise Khan et al. [20] have also discussed about the Brownian motion and thermophoresis with thermal radiation and buoyancy effects are encountered in the governing equations. The oscillating parallel plates were discussed by Shah et al.[13]. Shah et al. [17–19] have also explored some more properties of fluids. Similarly, Fiza et al. and Khan et al. [14, 16] have discussed flow through numerical results (Runge-Kutta method). Rasheed et al. [30] and Shah et al. [29] have discussed fluid flow through shooting technique and numerical approach. Moreover, flow of Oldroyd-B fluid over an oscillating was discussed by Shah et al. [28]. Likewise Mohmand et al.[27] have discussed time-dependent heated plate. Moreover, Rasheed et al. [31, 32] have discussed the fluid flow. To this end, Khan et al. [2] have investigated pressure distribution and entropy generation rate and then their solution through HAM approach. Moreover, Khan et al. [3–5] Darcy–Forchheimer law is used to study heat and mass transfer flow and microorganisms motion in porous media as well as flow of Maxwell nanofluid induced by two parallel rotating disks were analyzed. Furthermore Rasheed et al.[30–32] have discussed fluid motion with thermal analysis. Shah et al. [33] have also discussed MHD flow. Khan et al. [1–4] have analyzed the fluid motion through graphs. Similarly, Shah et al. [17–19] have also investigated the MHD fluid motion through graphs. Likewise, Khan et al. [7–12] have also discussed fluid flow through graphs and found excellent harmony with the already published works. Shah et al. [33, 34] have scrutinized a mathematical and computational analysis of MHD fluid with heat source effect and the chemically reactive Casson fluid to add knowledge to the existing one. OHAM technique was also applied by Shah [35] and as concern of PDE or ODE was discussed by Khan et al. [36].

The aim of the author in this study is to examine the non-Newtonian fluid model, which includes two dimensional fluid flow of a viscous, incompressible, and electrically conducting fluid in two parallel plates, and to represent the obtained results in graphical as well as in a tabulated form.

2 Problem Formulation and Solution

Consider the two dimensional fluid flow of a viscous, incompressible, and electrically conducting fluid in two parallel plates. The temperature of the lower and upper plates are T0 and T1. The lower plate is hugging in the direction of lower plate with velocity ($\frac{dh(t)}{dt}$). The two-dimensional coordinate system is taken as that the x-axis is along the lower plate while y-axis is normal to it. The velocity is represented by $\vec{u} = \vec{u}(u(x, y, t))$, temperature is shown by $T = T(x, y, t)$, and magnetic field is considered by $T = T(x, y, t)$

. The magnetic field is considered in both directions. In view of these assumptions, the leading equations are in Fig. 1:

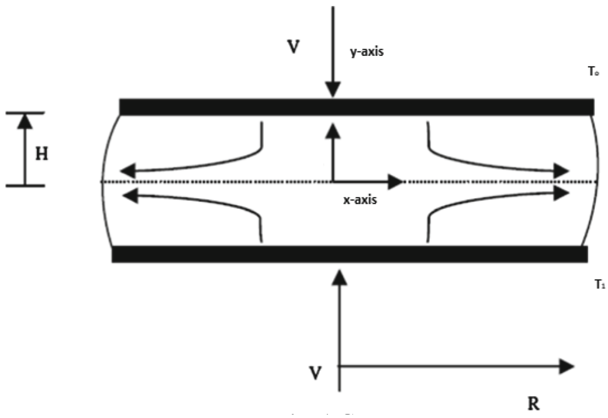

Fig. 1. Geometry

$$\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x}\upsilon\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2}\right) - \frac{\sigma B_0}{\rho}(uB_0 + vb), \tag{2}$$

$$v\frac{\partial u}{\partial y} + u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x}\upsilon\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2}\right) - \frac{\sigma B_0}{\rho}(uB_0 - vb), \tag{3}$$

$$v\frac{\partial T}{\partial y} + u\frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} = \sigma\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right), \tag{4}$$

$$\frac{\partial b}{\partial t} - \delta B_0\frac{\partial u}{\partial x} - \delta u\frac{\partial B_0}{\partial y} + \delta b\frac{\partial v}{\partial y} + \delta v\frac{\partial b}{\partial y} - \frac{1}{\delta\mu_c}\frac{\partial^2 b}{\partial y^2} = 0 \tag{5}$$

$$\frac{\partial B_0}{\partial t} - \delta B_0\frac{\partial u}{\partial x} - \delta b\frac{\partial v}{\partial x} + \delta b\frac{\partial v}{\partial y} = -\frac{1}{\delta\mu_c}\frac{\partial^2 B_0}{\partial y^2} - \delta V\frac{\partial b}{\partial y} \tag{6}$$

with

$$\tilde{u} = 0, \tilde{v} = -v_w = -\frac{a^{\cdot}}{c}, T = 0, at\, \tilde{y} = h(t), \tag{7}$$

$$\tilde{u} = 0, \tilde{v} = 0, T = T_0\left(1 - e^{-\eta t}\right)\ldots\mathrm{AT}\ldots y = 0, \tag{8}$$

By these transformations, the leading equations are reduced to,

$$S\left(f''' + 3f'' + f'f' - ff'''\right) = A\left(2f'GH + 2FG^2\right) \\ +f'''' - A\left(2f'HH + H^2f' + f'HG + fHG' + fH'G\right), \tag{9}$$

$$2\theta'' - f\theta l^2 - \theta'\eta l^2 = 0 \tag{10}$$

$$G'\eta - 2G - \delta Hf'' - \delta f'H' - \delta f'H' - \delta Gf' - fG' - G''M = 0, \tag{11}$$

$$H''Q + H'\eta + H\left(1 - f'\right) = 0, \tag{12}$$

3 Results and Discussions

Figure 2 portrays the complete harmony of MAM and numerical method for f(η). Notwithstanding, table shows a very small error which lead us to believe on the authenticity of our current research.

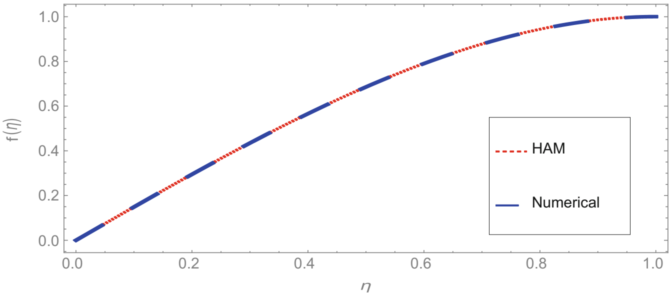

Fig. 2. AS and NS comparison for $f(\eta)$

Table 1 shows the individual Residual Error of $f'(\eta)$, $\theta(\eta)$, and $H(\eta)$.

Table 1. Errors among different quantities

No	$f(\eta)$	$\theta(\eta)$	$G(\eta)$	$H(\eta)$
2	3.45099×10^{-4}	2.79452×10^{-11}	0.00675467	2.7712924
4	1.00488×10^{-3}	1.77377×10^{-13}	0.000406648	2.43464×10^{-3}
6	1.11916×10^{-4}	1.64768×10^{-14}	9.33238×10^{-6}	2.3744×10^{-4}
8	1.11381×10^{-5}	1.61982×10^{-15}	2.40518×10^{-6}	1.44139×10^{-4}
10	1.2227×10^{-6}	1.88767×10^{-15}	1.00453×10^{-7}	1.76503×10^{-6}
12	1.12164×10^{-7}	1.6313×10^{-16}	2.11491×10^{-8}	1.47125×10^{-6}
14	1.23155×10^{-8}	1.0007×10^{-18}	1.23349×10^{-9}	1.07647×10^{-7}
16	1.95402×10^{-9}	1.17776×10^{-18}	1.01639×10^{-10}	1.0624×10^{-8}
18	1.10497×10^{-10}	1.23983×10^{-21}	1.11093×10^{-11}	1.20031×10^{-9}
20	1.23668×10^{-11}	1.19905×10^{-20}	1.0113×10^{-12}	6.65142×10^{-11}
22	1.10174×10^{-12}	1.25915×10^{-22}	1.00199×10^{-13}	4.16438×10^{-11}
24	1.56872×10^{-13}	1.12765×10^{-22}	1.0015×10^{-14}	4.00878×10^{-13}
26	1.22234×10^{-13}	1.27271×10^{-23}	1.20496×10^{-15}	4.00156×10^{-14}
28	1.00903×10^{-15}	1.13626×10^{-24}	1.01007×10^{-17}	4.03467×10^{-15}
30	1.00566×10^{-15}	1.27023×10^{-25}	1.00222×10^{-17}	4.01952×10^{-16}

4 Conclusion

Concluding remarks of the current study are listed as under: a. The current study has found that both techniques are agreed to treat the modeled system of equations in a best possible way. b. It is logical that the larger values of S show falling influence on $f'(\eta)$. c. It is noticed that the upper values of P show declining impact on $f'(\eta)$. d. It is witnessed that the higher values of R show the heightening impact on $\theta(\eta)$. e. The current analysis suggests that the growing values of R enhances $G(\eta)$ and vice versa.

5 Future Work

For the upcoming researchers, one can reformulate this mathematical model for compressible flow.One can calculate the results by finite element method among others to open more doors of intriguing knowledge in this area of research for other scientists in the field.

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