



# The Influence of Controlled Vibration Effects on Fluid Flow in Technological and Engineering Processes

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**Abstract.** This article presents the results of studies demonstrating the influence of nonlinear effects of laminar flow under vibrational harmonic effects on fluid flow and heat transfer. The paper summarizes the results of research on the influence of vibrations in various fluid flow problems. The effect of periodic oscillations on the symmetrization of an asymmetric flow in a diffuser, on Rayleigh-Bernard convection and on the wide of boundary layers in various single crystal growth processes are shown.

**Keywords:** Vibrations · Fluid Flow · Symmetrization Flow · Heat Transfer · Boundary Layers · Crystal Growth · Rayleigh-Bernard Convection · Numerical Simulation

## 1 Introduction

During vibrational action on continuous media, their anomalous nonlinear peculiarities and resonant properties may manifest themselves [1–3]. Nonlinear peculiarities of the moving under vibration action are manifested not only in liquids, but also in the movement of bulk granular media [2]. The study of the effects of vibrations on liquid media has been carried out since the works of M. Faraday (1831) and L. Rayleigh (1883). Vibrational control of the heat exchange in the melt is more energy-efficient and simpler than controlling the melt flow by changing the gravitational or magnetic field. Therefore, the study of vibration effects on the hydrodynamics of the melt is an actual task. Reviews of works on vibrational convective flow can be found in [3–6]. Many theoretical papers [1–6] and experimental papers [7–10] have been devoted to the study of vibrations.

This paper presents and summarizes the results of mathematical modeling of the following problems: on flow symmetrization in a flat diffuser, on Rayleigh-Benard convection, and on the hydrodynamics of melt and heat and mass transfer in the processes Bridgman and Czochralski of crystal growth [10–18]. The results of numerical modeling have shown also that vibrations can reduce the thickness of dynamic and temperature boundary layers and increase the temperature gradient at the crystallization front, which can intensify heat and mass transfer and the rate of crystal growth [10–16]. The fact

of increasing the crystal growth rate up to four times under vibrational action on the crystal was discovered experimentally in [7], which is an experimental confirmation of an increase in the temperature gradient at the crystallization front. The paper [17] shows the change in the beginning time and in structure of Rayleigh-Benard convection under vertical vibrations in a long-confined layer heated from below (the Rayleigh-Benard problem with vibration of the heated wall).

The study of the problem symmetrization of asymmetric fluid flows by means of vibration action on the flow is also important in a lot of applications, for example, in mechanical engineering for fuel injection in engines, as well as in biomedicine when creating new technologies and methods for the precise targeted delivery of drugs to the necessary areas of organs in human treatment. This paper presents the results on the symmetrization of the flow in a flat diffuser (for (Jeffery-Hamel problem [19–23]) using two techniques of vibration action.

## 2 Mathematical Model

The mathematical model is based on the numerical solution of a system of non-stationary planar 2D Navier-Stokes equations for natural convection of an incompressible liquid in the Boussinesq approximation (1–3):

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\rho_0 d\mathbf{u}/dt + \nabla p = \nabla \cdot (\rho_0 \nu \nabla \mathbf{u}) - \rho_0 g \beta (T - T_0) \mathbf{e}_z \quad (2)$$

$$\rho_0 c_v dT/dt = \nabla \cdot (k_T \nabla T) \quad (3)$$

where traditional notation is used. The problems were considered for flat cases or for conditions of axial symmetry with or without rotation. Therefore, for a cylindrical coordinate system  $r, \theta, z$ , then  $u, v, w$  are radial, circumferential and axial velocity projections,  $\nu, k_T$  are kinematic viscosity, heat conduction coefficients,  $\beta$  is the buoyancy coefficient,  $T_0$  is a reference temperature,  $\rho_0$  is a reference density,  $g$  – acceleration of gravity opposite directed to the vertical coordinate axis ( $z$ ). The boundary conditions were as follows: for velocity - no friction on a free surface, no slip condition on solid surfaces and setting the velocity of the vibrator or moving at the vibrating wall (on law  $y(\text{or } z) = A \sin(2\pi f t)$  with a frequency  $f$  and an amplitude  $A$ ,  $Re_{vibr} = A^2 2\pi f / \nu$  – is vibration Reynolds number); for temperature - were conditions of the first kind or thermal insulation conditions and at the crystallization interface, either the crystallization temperature or the Stefan condition with latent heat release was set.

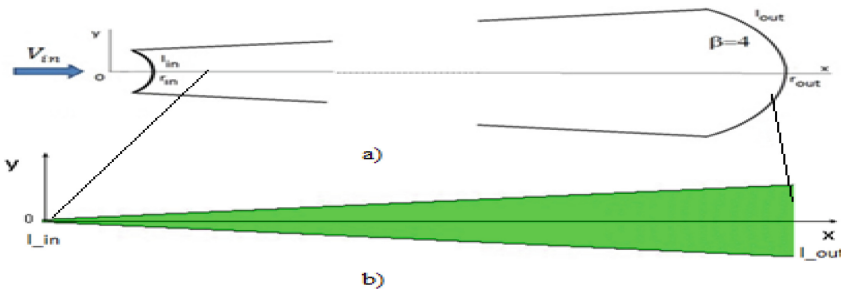
The results presented in this paper were obtained using different numerical methods: the finite-difference scalar method, the fully implicit matrix finite-difference and the finite element methods [24, 25]. The good accuracy of numerical results was confirmed by comparison with experimental data and comparison of numerical results obtained by various numerical models.

### 3 Result and Discussion

#### 3.1 Symmetrization of Fluid Flow in a Flat Diffuser by Vibrational Effect

The problem of the flow of a viscous incompressible liquid in a flat diffuser in the approximation of flow symmetry was solved by the authors of [19, 20]. It is known that when the Reynolds number increases above the critical  $Re^*$  number, the flow loses symmetry, staying steady state and laminar. [21–23]. This article shows two methods of symmetrization of the asymmetric flow of a viscous incompressible liquid in a flat diffuser using periodic vibration action: 1 - from the side of the input stream, 2 - from the side of the walls of the diffuser. The research was carried out based on solving the complete two-dimensional Navier-Stokes equations for an incompressible fluid (1, 2) for case  $g = 0$ . The harmonic effects of vibration effects (in the form of  $A\sin(2\pi ft)$ , where  $A$  and  $f$  are the amplitude and the frequency of the changing velocity) on velocity are considered.

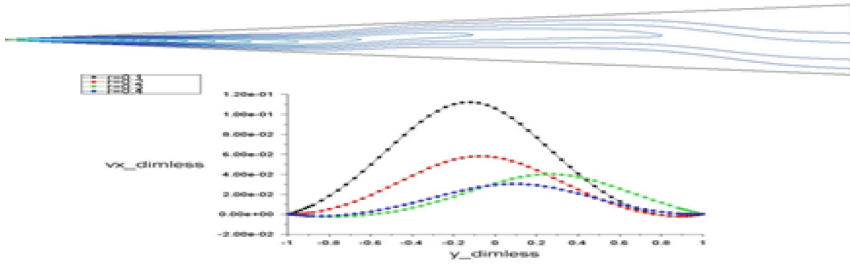
**The Problem Statements.** The laminar flow of a viscous incompressible fluid driven.



**Fig. 1.** Scheme of the computational domain for a flat diffuser: a) it is details of domain near the inlet and outlet of the diffuser; b) the numerical region with mesh ( $\beta = 4^\circ, L = 0.495$  m).

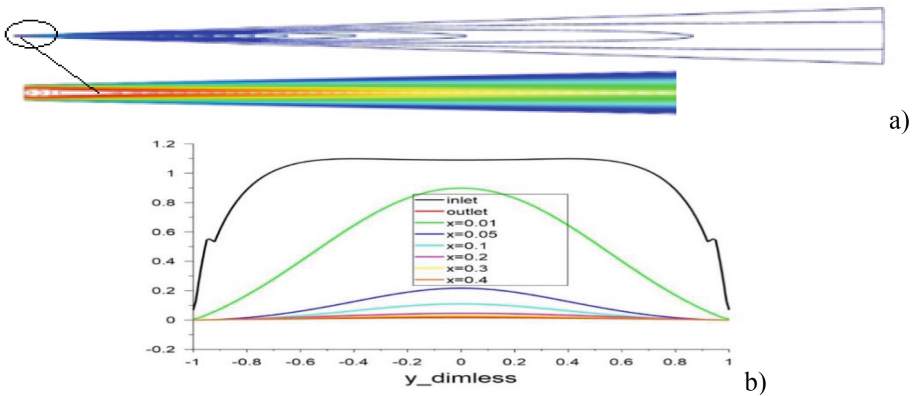
Through a channel bounded by two flat walls inclined towards each other at a small angle  $\beta$  is considered. In this paper we consider flat diffuser bounded by two arcs (“input” and “output” boundary) with the one center (Fig. 1a). The geometry of the mathematical model was chosen to be able to compare our results with the results of well-known works [19–22]. Geometric model of the diffuser is as follows: opening angle is  $\beta = 4^\circ$ , of an arc the input boundary has the form  $l_{in}(r_{in}=0.005$  m) where  $r$  is calculated by formula  $r^2 = x^2 + y^2$  (Fig. 1). The initial conditions are  $t = t_0 = 0, V(t_0) = 0, P = 0$ . The velocity scale is chosen by the velocity  $V_{in}$  and the Reynolds numbers are defined as  $Re = Re_{in} = V_{in}l_{in}/\nu, Re_{vibr} = Afl_{in}/\nu, y_{dimless} = y/r \sin(\beta/2), V_{x\_dimless} = V_x/V_{x\_in}, V_{y\_dimless} = V_y/V_{x\_in}$ .

**The Fluid Flows in the Diffuser Without Vibrations.** The results for the case asymmetric fluid flows ( $Re = 279$ ) in the diffuser without vibration effects are presented in Fig. 2 [23]. The results coincide with results of paper [22].



**Fig. 2.** The isolines and the profiles in vertical cross- sections of horizontal component  $V_x$  of velocity vector for the case of asymmetrical steady state fluid flows ( $Re = 279$ ).

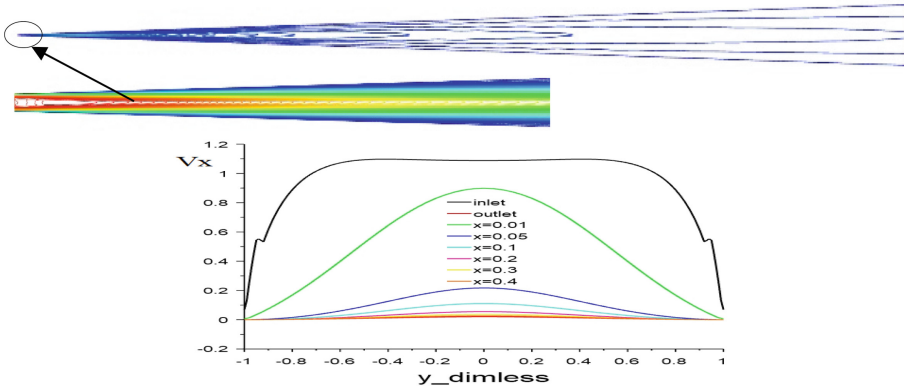
**Symmetrization of the Fluid Flow in the Diffuser Due to the Effect of Vibration on the Inlet Velocity.** The effect of a periodic vibrational disturbance  $V = V_{in} + A \sin(2\pi f)$  ( $f = 10 \text{ Hz}$ ,  $A = 0.1 \text{ m/s}$   $Re_{vibr} = 2.4$ ) on the basic flow with  $Re = 279$  are presented in Fig. 3.



**Fig. 3.** The isolines of the averaged longitudinal component of the velocity mean  $V_x$ , (the isolines of the mean  $V_x$  velocity near the entrance to the diffuser are shown below) (a), the profiles of the longitudinal component of the mean  $V_x$  velocity (b) for case  $V_{in} = 11.7 \text{ m/s}$ ,  $A = 0.1 \text{ m/s}$ ,  $f = 10 \text{ Hz}$  ( $Re = 279$ ,  $Re_{vibr} = 2.4$ )

Comparison of the results in Fig. 2 and Fig. 3 shows that the effect of vibrations ( $Re_{vibr} = 2.4$ ), even at amplitudes less than 1% of the velocity  $V_{in}$  ( $Re = 279$ ) can lead to symmetrization of the fluid flow in the diffuser.

**Symmetrization of the Fluid Flow in the Diffuser Due to the Effect of Vibration From the Walls.** An example second approach of symmetrization of the fluid flow velocity in a flat diffuser by vibration action along normal to the walls of the diffuser according to the harmonic law  $V_n = A \sin(2\pi f)$  with a small amplitude  $A$  and a frequency  $f$  is shown in Fig. 5. In Fig. 4 mean  $V_x$  – is the time-average velocity profiles for  $Re = 279$ ,  $A = 0.001 \text{ m/s}$ ,  $f = 10 \text{ Hz}$  ( $Re_{vibr} = 0.02$ ) are shown.



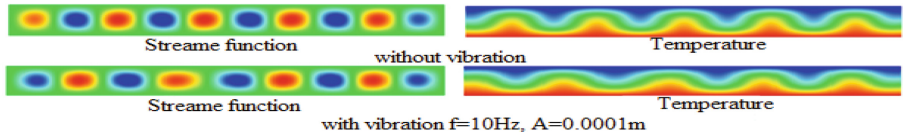
**Fig. 4.** The isolines and the profiles of time average velocity (mean\_  $V_x$ ) for fluid flow in a flat diffuser with vibration action from the walls of the diffuser for  $Re = 279$ ,  $A = 0.001\text{m/s}$ ,  $f = 10\text{Hz}$  ( $Re_{vibr} = 0.02$ ).

The results of numerical simulation have shown two ways of symmetrization of asymmetric laminar flows of viscous incompressible fluid in a flat diffuser: the first - due to a weak periodic effect on the flow velocity at the entrance to the diffuser and the second - due to vibration action from the walls of the diffuser. It is shown that the impact of vibration, even at amplitudes less than 1% of the velocity  $V_{in}$ , can lead to the symmetrization of the fluid flow in the diffuser.

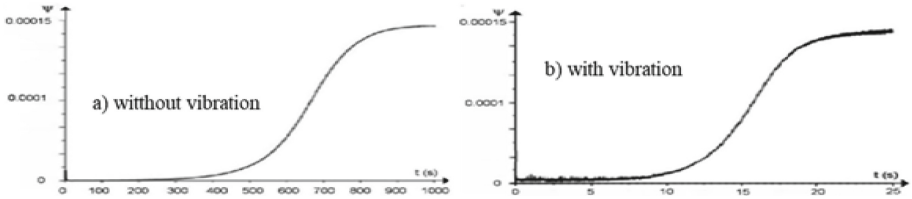
### 3.2 The Effect of Controlled Vibrations on Rayleigh-Benard Convection

The problem of convective flow in a horizontal layer heated from below is called the Rayleigh-Benard (R-B) problem. This problem has a threshold character of the occurrence of natural convection, which is determined by the critical Rayleigh number. R-B problem was considered for a horizontal layer with free top boundary with an aspect ratio of 1:10 and the Prandtl number  $Pr = 1$  in a gravity field with specified temperatures on horizontal walls and with thermally insulated vertical walls. The results of numerical simulation presented in Fig. 5 show the influence of the lower horizontal wall oscillations on the structure of the convective flow in the Rayleigh-Benard problem. The number of Rayleigh-Benard rollers decreases from 10 to 9 during vertical harmonic vibrations of the lower wall (on law  $y = A\sin(2\pi ft)$  with a frequency  $f = 10\text{ Hz}$  and an amplitude  $A = 10^{-4}\text{ m}$ ,  $Re_{vibr} = A^2 2\pi f / \nu = 0.007$ ), which indicates a decrease in the wave number of the periodic convective structure.

The simulation results also showed the possibility of a significant decrease in the critical Rayleigh number for the occurrence of R-B convection under vibration action. The time of occurrence and establishment of the quasi-stationary regime of convective flow is also significantly reduced. A decrease in the critical Rayleigh number and the time of occurrence of Rayleigh-Benard convection due to vertical vibrations of the lower wall was also shown in paper [17] (Fig. 6). This is important for boiling processes [18].



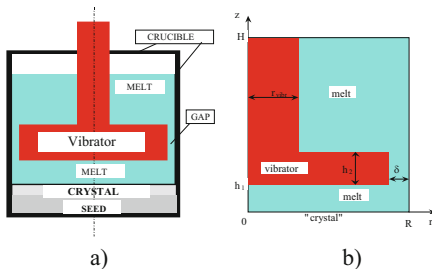
**Fig. 5.** Pictures of isolines of the stream function and isotherms with and without vibrations of the lower wall with  $Re_{vibr} = 0.007$ ,  $Ra = 4 \cdot 10^3$ ,  $Pr = 1$ .



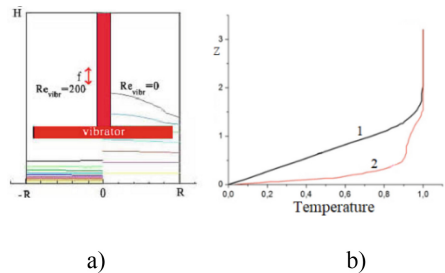
**Fig. 6.** The dependences of the maximum values of the stream function on time ( $Ra = 4 \cdot 10^3$ ,  $Pr = 1$ ): a) – without vibrations; b) – with vertical vibrations of the bottom wall with  $Re_{vibr} = 0.007$ [17].

### 3.3 The Effect of Vibrations in Crystal Growth Processes

Bridgman model. The calculation results were carried out for the following geometric configurations of crucibles for Bridgman method with submersible vibrators for a fixed flat and variable calculated shape of the crystallization front shown in Fig. 7. The area under consideration has the following dimensions:  $R = 1.6 \cdot 10^{-2}$ ;  $H = 3.2 \cdot 10^{-2}$ ;  $r_{vibr} = 4 \cdot 10^{-3}$ ;  $h_1 = 8 \cdot 10^{-3}$ ;  $h_2 = 8 \cdot 10^{-3}$ ;  $\delta = 10^{-3}$  (m) where  $R$  is the radius of the ampoule,  $H$  is the height of the ampoule,  $h_1$  is the distance between the vibrator and the solid-liquid interface,  $h_2$  is the thickness of the vibrator (the distance between its lower and upper surfaces), the gap  $\delta$  (the distance between the vibrator and the side wall of the crucible). The following variants with size values  $A = 5 \cdot 10^{-4}$  and  $10^{-4}$  m,  $f = 0-100$  (Hz) are calculated. The effect of vibrations on temperature boundary layers are shown in Fig. 8.

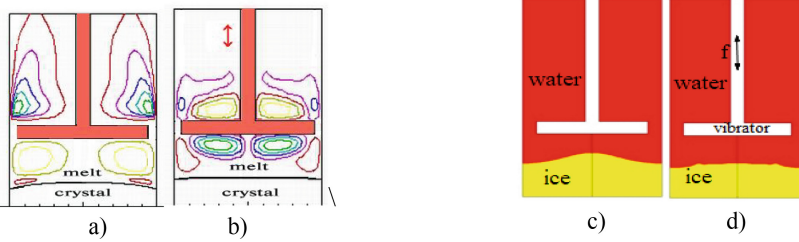


**Fig. 7.** The geometrical schemes for Bridgman crystal growth model with submerged vibrator, a) for Stefan problem, b) model with fixed flat shape of the melt-crystal interface



**Fig. 8.** a) - Isotherms in the melt ( $Pr = 5.43$ ) (on the right – without vibrations, on the left – with vibrations  $Re_{vibr} = 200$ ), b) - temperature profiles ( $r = 0.75$ ) (line 1 – without vibrations, 2 – with vibrations)

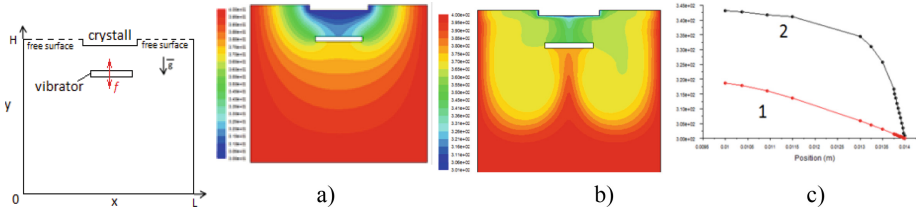
**The Effect of Vibrations on the Shape of the Crystallization Front.** Using the method of solving the Stefan problem described in [25], for the Bridgman method with a submerged vibrator (Fig. 9), a simulation of convective heat transfer was performed in order to determine the effect of vibrations on the shape of the crystallization front.



**Fig. 9.** The effect of vibrations on the shape of the front crystallization: a), b) – stream function in the melt of NaNO<sub>3</sub>, (a) without vibrations -  $f = 0$ ; b) - with vibrations  $A = 10^{-4}$  m,  $f = 50$  Hz), c), d) – water-ice interface, c) – without vibrations,  $f = 0$ , d) – with vibrations,  $A = 10^{-4}$  m,  $f = 30$  Hz)

**Czochralski Model with Submerged Vibrator.** The scheme of the computational domain is shown in Fig. 10. The computational domain is a square with sides  $L = H = 3$  cm crystal with a diameter of  $d = 1$  cm and immersed into the melt to a depth of 1 mm, the vibrator has a diameter of 0.8 cm and thickness 1 mm. It is assumed that the immersed vibrator is located under the crystal at a distance  $h$ . Irregular grid with refinement near the solid walls and the corners of the vibrator and the crystal were used in the calculations. The vibrator makes translational oscillatory movements along the vertical axis of the crystal according to the law:  $y = y_0 + A \sin(2\pi ft)$ , with frequency  $f$  and small amplitude  $A = 10^{-4}$  m,  $y_0$  is initial location of vibrator.

The isotherm and structure of the averaged flow is presented in Fig. 11(a, b). It is showing how the vibrating immersed activator leads to the mixing of the entire volume of the melt. In Fig. 11c presents temperature profiles on vertical cross section (on axis) that show the effect of vibration on the temperature boundary layer and the temperature gradient near the crystal-melt interface ( $Pr = 7$ ;  $Re_{vibr} = 1500$ ;  $h/d = 0.5$ ,  $A = 4 \cdot 10^{-4}$  m,  $f = 20$  Hz).



**Fig. 10.** Scheme of the computational domain

**Fig. 11.** Isotherms temperature: a) – without vibration, b) with vibrations, c) temperature profiles on the axis section: curve 1- is without vibration, curve 2 - is with vibration. ( $Pr = 7$ ,  $Re_{vibr} = 1500$ ,  $Ra = 0$ ).

## 4 Conclusion

It is possible to symmetrize the flow of viscous liquid in a flat diffuser using the effects of weak harmonic vibration from the inlet side or from the walls of the diffuser. It is also shown that the vibration effect can change the structure and time of occurrence of Rayleigh–Benard convection. This is important for boiling processes. By controlling the vibration effect on the convective fluid flow, the thickness of the boundary layers can be reduced. For the Bridgman model, it is shown that the surface of the crystallization front can be made flatter by means of vibration action. This is of fundamental importance in crystal growth and for controlling temperature gradients to control the kinetics and rate of crystal growth through vibration.

**Acknowledgements.** This work was supported by the Russian Science Foundation grant 24–29–00101.

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