Interval Observers Design for Discrete-Time Linear Systems with Uncertainties

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Abstract. The paper studies the problem of designing interval observers for discrete-time linear dynamic systems under the external disturbances, measurement noises, and unknown system parameters. To construct such observers, we use not the original system but its reduced-order model of the original system of minimal dimension which is insensitive to the disturbances. The observers are designed in such a way to estimate the prescribed linear function of the original system state vector. Canonical form of the observer is used that allows to simplify the design procedure. The obtained results are displayed by example of the electric servoactuator.

Keywords: Linear systems · estimation · interval observers · uncertainties · canonical form

1 Introduction

In [1], a concept of intelligent control-emergency systems was proposed using knowledge bases for a comprehensive analysis of autonomous underwater vehicles malfunctions and developing solutions to adapt to them. Knowledge bases allow obtaining formal description of malfunctions and attributes to identify them. Besides, an ontological approach to construction of these databases makes it possible to unify their structure and include domain experts in the development process who provide its qualitative content. For realization of intelligent control-emergency systems, it is necessary to promptly receive information about the correct functioning and operability of individual subsystems of the underwater vehicles. This can be done by using interval observers.

The problem to estimate the system vector of state is very important for many practical applications. The main difficulties in designing an estimator are the system

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complexity and uncertainty (unknown parameters, measurement noises, and external disturbances). It is known that sliding mode observers can solve this problem [2–4]; under uncertainties, however, the estimation error is not equal to zero. Recently, this problem has been successfully solved on the basis of interval observers evaluating the state of system. An advantage of such observers is that they allow to consider many types of the system uncertainties: measurement noise, external disturbances, parametric uncertainties.

Such an approach can be used to deal with significant disturbances and provide component-wise information on possible solutions. Therefore, this approach is fundamentally different from conventional techniques of robust stability analysis or control law construction for different perturbed processes. An advantage of interval observer is that it allows to take into account many types of uncertainties in the system. The main peculiarity of interval observation is that it is necessary to ensure positivity of the estimation error dynamics in addition to their stability.

For many types of dynamic systems different interval observers have been developed in many papers: for linear and non-linear continuous-time [5–15], discrete-time linear and non-linear [16, 17], for time-delay systems [18, 19], for Takagi-Sugeno fuzzy systems [20], for switched systems [21–23], and singular systems [19]. Many practical problems also successfully have been solved [24–26]. Exhaustive reviews are in [18, 27, 28].

Note that the above-mentioned papers solve the problem of estimation of full state vector. Contrary to these papers, the interval observers in the present paper are designed to estimate the prescribed linear function of the original system state vector. Such a solution may be useful in some practical applications. Our approach is closed to that based on the functional interval observers developed in [29–31] which enable estimating some linear function of the state vector. In comparison with [30, 31], we take into account system parametric uncertainties; unlike [29], our approach considers measurement noise.

The main contribution of the paper is that interval observers estimate not the state vector but its some prescribed linear function. The suggested solution is based on the reduced order model of minimal dimension which is invariant with respect to the disturbances. This allows to reduce the interval width and the dimension of observer in comparison with [11, 16, 18] and similar papers. Besides, in Sect. 2 we design interval observers insensitive to the external disturbances and parametric uncertainties. In comparison with [32] where measurement noise and external disturbances are considered, the present paper takes into account parametric uncertainties additionally. Based on the reduced order model, one can obtain more precise estimates for the full state vector. Besides, a new method to design such observers on the basis of the identification canonical form is suggested. As a result, this enables designing such observers for broadened class of dynamic systems.

2 The Main Models

Consider the following model of a linear system

$$\begin{aligned} x(t+1) &= A(\mu)x(t) + Bu(t) + Q\rho(t), \\ y(t) &= Cx(t) + w(t) \end{aligned} \tag{1}$$

with states $x \in R^n$, inputs $u \in R^m$, and outputs $y \in R^l$; the matrix $A(\mu)$ is of the form $A(\mu) = (A + \Delta A(\mu(t)))$, where $A \in R^{n\times n}$ is constant matrix, the matrix function $\Delta A(\mu)$ is known for a given value of μ, $\mu(t) \in \Pi \subset R^s$ is the bounded vector of parameters where Π is known; it is assumed by analogy with [18] that the values of $\mu(t)$ cannot be measured. The matrices $B \in R^{n\times l}$, $C \in R^{l\times n}$, and $Q \in R^{n\times p}$ are known constant matrices; $\rho(t) \in R^p$ is a bounded unknown function with $\|\rho(t)\| \le \rho_*$ describing the unmatched disturbance; $w(t) \in R^l$ is a bounded unknown function with $\|w(t)\| \le w_*$ describing the measurement noise.

The objective is to develop the method of interval observer design generating functions $\underline{v}(t)$ and $\overline{v}(t)$ under the condition $\underline{v}(t) \le v(t) \le \overline{v}(t)$ for all $t \ge 0$ where $v(t)$ is such that $v(t) = Mx(t)$ for known matrix M. For matrices $A^{(1)}$, $A^{(2)}$ and vectors $x^{(1)}$, $x^{(2)}$, the inequalities $A^{(1)} \le A^{(2)}$ and $x^{(1)} \le x^{(2)}$ one understands elementwise.

In comparison with [11, 16, 18] where the problem of estimation of full state vector is studied, the suggested solution is based on the reduced order model of minimal dimension which is invariant with respect to the disturbances. This allows to reduce the interval width and the dimension of interval observer.

Note that the problem when the parametric uncertainties are absent, that is $\Delta A(\mu(t)) = 0$ was solved in [32]. Recall briefly the main results of this solution.

The solution is based on the reduced-order model of the initial system:

$$\begin{aligned} \eta(t+1) &= A_*\eta(t) + B_*u(t) + G_*Cx(t) + Q_*\rho(t), \\ v(t) &= C_*\eta(t) + P_*y_0(t). \end{aligned} \tag{2}$$

Here $\eta \in R^k$ is the vector of state, the matrix A_* is specified in the identification canonical form

$$A_* = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & & \dots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}, \tag{3}$$

the matrices B_*, G_*, Q_*, P_*, and C_* of the appropriate dimensions have to be determined, $y_0(t)$ will be determined later. Since the matrix A_* is stable and nonnegative, system (2) is cooperative [18].

Remark 1. The term $G_*Cx(t)$ in the model (2) is used instead of $G_*y(t)$ due to the necessity to account measurement noise since $y(t) = Cx(t) + w(t)$. We will use the term $G_*y(t)$ in the interval observers (10) and (17).

We assume that $x_*(t) = \Phi x(t)$ for some matrix Φ satisfying the conditions [4, 34]

$$\Phi A = A_*\Phi + G_*C,\; B_* = \Phi B,\; Q_* = \Phi Q. \tag{4}$$

The variable $y_0(t)$ in (2) must be free of the disturbance $\rho(t)$. The method to obtain such a variable was suggested in [32]; it is based on the equation

$$(N_1 \quad -N_2)\begin{pmatrix} Q_0 \\ C \end{pmatrix} = 0$$

for some matrices N_1 and N_2 of maximal rank. Solution of this equation produces $y_0 = N_2 y(t) = N_2 C x$. Here Q_0 is the maximal rank matrix satisfying the condition $Q_0 Q = 0$.

The relation $v(t) = Mx(t)$ and (2) result in

$$M = C_* \Phi + P_* N_2 H = (C_* \; P_*) \begin{pmatrix} \Phi \\ N_2 C \end{pmatrix}. \tag{5}$$

This equation is solvable if

$$rank \begin{pmatrix} \Phi \\ N_2 C \end{pmatrix} = rank \begin{pmatrix} \Phi \\ N_2 C \\ M \end{pmatrix}. \tag{6}$$

The best observer generating the minimal interval width is when the model is free of the parametric uncertainties $\Delta A(\mu(t))$ and the disturbance $\rho(t)$. Such a model can be designed based on the equation

$$\left(\Phi_1 \; -G_{*1} \; ... \; -G_{*k} \right) \left(W^{(k)} \; Q^{(k)} \; \Delta^{(k)} \right) = 0, \tag{7}$$

generalizing the equation suggested in [4, 34] where

$$W^{(k)} = \begin{pmatrix} A^k \\ CA^{k-1} \\ ... \\ C \end{pmatrix},$$

$$Q^{(k)} = \begin{pmatrix} Q & AQ & ... & A^{k-1}Q \\ 0 & CQ & ... & CA^{k-2}Q \\ ... & ... & ... & ... \\ 0 & 0 & ... & 0 \end{pmatrix},$$

$$\Delta^{(k)} = \begin{pmatrix} \Delta A(\mu) & A\Delta A(\mu) & ... & A^{k-1}\Delta A(\mu) \\ 0 & C\Delta A(\mu) & ... & CA^{k-2}\Delta A(\mu) \\ ... & ... & ... & ... \\ 0 & 0 & ... & 0 \end{pmatrix},$$

G_{*i} is the i-th row of the matrix G_*. The matrices $Q^{(k)}$ and $\Delta^{(k)}$ guarantee invariance of the model with respect to the disturbance and parametric uncertainties, respectively, $W^{(k)}$ enables to design the model (2). The Eq. (7) is solvable when

$$rank \left(W^{(k)} \; Q^{(k)} \; \Delta^{(k)} \right) < n + lk. \tag{8}$$

To design the model, one has to find minimal k from (8); the matrix $(\Phi_1 \; -G_{*1} \; ... \; -G_{*k})$ is determined from (7) and the rows of the matrix Φ are found from the equations

$$\Phi_i A = \Phi_{i+1} + G_{*i} C, \; i = 1, ..., k-1, \; \Phi_k A = G_{*k} C \tag{9}$$

which can be obtained based on (3) and (4). Finally, the matrices B_*, C_*, and P_* are found from (5) and (4).

Based on the model (2), the interval observer is designed:

$$\begin{aligned} \underline{\eta}(t+1) &= A_*\underline{\eta}(t) + B_*u(t) + G_*y(t) - |G_*|E_l w_* - |Q_*|E_p\rho_*, \\ \overline{\eta}(t+1) &= A_*\overline{\eta}(t) + B_*u(t) + G_*y(t) + |G_*|E_l v_* + |Q_*|E_p\rho_*, \\ \underline{\eta}(0) &= \underline{\eta}_0, \overline{\eta}(0) = \overline{\eta}_0. \end{aligned} \tag{10}$$

Here E_k is $(k \times 1)$-matrix:$E_k = (1\ 1\ \ldots\ 1)^T$, the matrix $|G_*|$ contains the absolute values of the corresponding entries of G_*.

Theorem 1. [32] When $\underline{\eta}(0) \le \eta(0) \le \overline{\eta}(0)$, then it follows for all $t \ge 0$.

$$\underline{\eta}(t) \le \eta(t) \le \overline{\eta}(t), \quad \underline{v}(t) \le v(t) \le \overline{v}(t), \tag{11}$$

where

$$\underline{v}(t) = C_*\underline{\eta}(t) + P_*y_0(t), \quad \overline{v}(t) = C_*\overline{\eta}(t) + P_*y_0(t) \tag{12}$$

if $C_* \ge 0$ *and*

$$\underline{v}(t) = C_*\overline{\eta}(t) + P_*y_0(t), \quad \overline{v}(t) = C_*\underline{\eta}(t) + P_*y_0(t) \tag{13}$$

if $C_* \le 0$.

Note that when (8) is true for some k, then $Q_* = 0$ in (10) and the interval width is minimal. If (8) is not true for all k, one has to check the condition

$$rank\left(W^{(k)}\ \ \Delta^{(k)}\right) < n + lk. \tag{14}$$

If it is satisfied, then $Q_* \ne 0$, that is the disturbance affects the model, and the interval width becomes greater. It can be reduced by applying robust method based on singular value decomposition described in [32]. If (14) is not satisfied for all k, one has to use the method developed in Sect. 3.

3 Parametric Uncertainty

Assume that (14) is not satisfied for all k and $\underline{\Delta A} \le \Delta A(\mu(t)) \le \overline{\Delta A}$ for all $\mu(t) \in \Pi$ with some $\underline{\Delta A}, \overline{\Delta A} \in R^{n\times n}$. The interval $\left(\underline{\Delta A}, \overline{\Delta A}\right)$ can be calculated for known Π and $\Delta A(\mu)$ [18]. To simplify the procedure, assume initially that $w = 0$ and $\rho = 0$.

The model estimating the variable $v(t)$ is based on the model

$$\begin{aligned} \eta(t+1) &= (A_* + \Delta A_*)\eta(t) + B_*u(t) + \left(G_* + G'\right)y(t), \\ v(t) &= C_v\eta(t) + Py_0(t). \end{aligned} \tag{15}$$

The matrices ΔA_* and G' can be found as follows. It follows from (1) and (15)

$$\Phi(A + \Delta A(\mu)) = (A_* + \Delta A_*)\Phi + G_* C + G' C.$$

Since Φ satisfies (4), one obtains $\Phi \Delta A(\mu) = \Delta A_* \Phi + G' C$, or

$$\Phi \Delta A(\mu) = \begin{pmatrix} \Delta A_* & G' \end{pmatrix} \begin{pmatrix} \Phi \\ C \end{pmatrix}. \tag{16}$$

After obtaining the matrix Φ we find ΔA_* and G' from (16). Generally, the matrix G' depends on ΔA. Assume for simplicity that G' does not depend on ΔA.

Assumption. $\underline{\Delta A}_* \le \Delta A_* \le \overline{\Delta A}_*$For some $\underline{\Delta A}_*$ and $\overline{\Delta A}_*$.

Given a matrix ΔA, define by analogy with [18] $\Delta A^+ = max(0, \Delta A)$ and $\Delta A^- = \Delta A^+ - \Delta A$; clearly, $\Delta A^+ \ge 0$ and $\Delta A^- \ge 0$.

Lemma 1. [19] *If* $\Delta A_* \in R^{k\times k}$ *is constant and* $\underline{\eta} \le \eta \le \overline{\eta}$, *then*

$$\Delta A_*^+ \underline{\eta} - \Delta A_*^- \overline{\eta} \le \Delta A_* \eta \le \Delta A_*^+ \overline{\eta} - \Delta A_*^- \underline{\eta}.$$

If $\underline{\Delta A}_* \le \Delta A_* \le \overline{\Delta A}_*$ *for some* $\underline{\Delta A}_*$, ΔA_*, $\overline{\Delta A}_*$ *and* $\underline{\eta} \le \eta \le \overline{\eta}$, *then*

$$\begin{aligned} &\underline{\Delta A}_*^+ \underline{\eta}^+ - \overline{\Delta A}_*^+ \underline{\eta}^- - \underline{\Delta A}_*^- \overline{\eta}^+ + \overline{\Delta A}_*^- \overline{\eta}^- \le \Delta A_* \eta \\ \le\ &\overline{\Delta A}_*^+ \overline{\eta}^+ - \underline{\Delta A}_*^+ \overline{\eta}^- - \overline{\Delta A}_*^- \underline{\eta}^+ + \underline{\Delta A}_*^- \underline{\eta}^-. \end{aligned}$$

The interval observer is given by

$$\begin{aligned} \underline{\eta}(t+1) &= A_* \underline{\eta}(t) + \left(\underline{\Delta A}_*^+ \underline{\eta}^+ - \overline{\Delta A}_*^+ \underline{\eta}^- - \underline{\Delta A}_*^- \overline{\eta}^+ + \overline{\Delta A}_*^- \overline{\eta}^- \right) \\ &\quad + B_* u(t) + \left(G_* + G'\right) y(t), \\ \overline{\eta}(t+1) &= A_* \overline{\eta}(t) + \left(\overline{\Delta A}_*^+ \overline{\eta}^+ - \underline{\Delta A}_*^+ \overline{\eta}^- - \overline{\Delta A}_*^- \underline{\eta}^+ + \underline{\Delta A}_*^- \underline{\eta}^- \right) \\ &\quad + B_* u(t) + \left(G_* + G'\right) y(t), \\ \underline{\eta}(0) &= \underline{\eta}_0,\ \overline{\eta}(0) = \overline{\eta}_0, \end{aligned} \tag{17}$$

The estimation errors are as follows:

$$\begin{aligned} \underline{e}_*(t+1) = \ & A_* \underline{e}_*(t) + \Delta A_* \eta(t) - (\underline{\Delta A}_*^+ \underline{x}^+(t) - \overline{\Delta A}_*^+ \underline{\eta}^-(t) \\ & - \underline{\Delta A}_*^- \overline{\eta}^+(t) + \overline{\Delta A}_*^- \overline{\eta}^-(t)), \\ \overline{e}_*(t+1) = \ & A_* \overline{e}_*(t) + (\overline{\Delta A}_*^+ \overline{\eta}^+(t) - \underline{\Delta A}_*^+ \overline{\eta}^-(t) - \overline{\Delta F}_*^- \underline{\eta}^+(t) \\ & + \underline{\Delta A}_*^- \underline{\eta}^-(t)) - \Delta A_* \eta(t). \end{aligned}$$

Theorem 2. *If* $\underline{\Delta A}_* \leq \Delta A_* \leq \overline{\Delta A}_*$ *and* $\underline{\eta}(0) \leq \eta(0) \leq \overline{\eta}(0)$, *then the relations* (11) *with* (12) and (13) *are true for the observer* (17) *and all* $t \geq 0$.

Proof. The condition $\underline{\eta}(0) \leq \eta(0) \leq \overline{\eta}(0)$ implies $\underline{e}_x(0), \overline{e}_x(0) \geq 0$. Since $A_* \geq 0$ and $\underline{e}_x(0) \geq 0$, it follows from Lemma that $\underline{e}_x(1) \geq 0$ that is $\underline{\eta}(1) \leq \eta(1)$. It can be shown by induction that $\underline{\eta}(t) \leq \eta(t)$ is true for all $t \geq 0$. The relation $\eta(t) \leq \overline{\eta}(t)$ one can prove analogously.

If $C_* \geq 0$, it follows from (12)

$$
\begin{aligned}
\underline{e}_v(t) &= v(t) - \underline{v}(t) = C_*\eta(t) + P_*y_0(t) - \left(C_*\underline{\eta}(t) + P_*y_0(t)\right) = C_*\underline{e}_*(t),\\
\overline{e}_v(t) &= \overline{v}(t) - v(t) = C_*\overline{\eta}(t) + P_*y_0(t) - (C_*\eta(t) + P_*y_0(t)) = C_*\overline{e}_*(t).
\end{aligned}
$$

Since $\underline{e}_x(t), \overline{e}_x(t) \geq 0$, one obtains $\underline{e}_v(t), \overline{e}_v(t) \geq 0$ which is equivalent to $\underline{v}(t) \leq v(t) \leq \overline{v}(t)$. If $C_* \leq 0$, one has from (13)

$$
\begin{aligned}
\underline{e}_v(t) &= v(t) - \underline{v}(t) = C_*\eta(t) + P_*y_0(t) - (C_*\overline{\eta}(t) + P_*y_0(t)) = -C_*\overline{e}_*(t),\\
\overline{e}_v(t) &= \overline{v}(t) - v_*(t) = C_*\underline{\eta}(t) + P_*y_0(t) - (C_*\eta(t) + P_*y_0(t)) = -C_*\underline{e}_*(t).
\end{aligned}
$$

Taking into account $C_* \leq 0$, the relations $\underline{e}_v(t), \overline{e}_v(t) \geq 0$ can be obtained as well. Theorem has been proved. □

The case when $w \neq 0$ and $\rho \neq 0$ can be taken into account by additional addends $-|G_*|E_l w_* - |Q_*|E_p\rho_*$ and $|G_*|E_l w_* + |Q_*|E_p\rho_*$ in (17) by analogy with (10).

4 Example

Consider the control system

$$
\begin{aligned}
x_1(t+1) &= \gamma_1 x_2(t) + x_1(t),\\
x_2(t+1) &= (\gamma_2 + \delta_1(t))x_2(t) + \gamma_3 x_3(t) + \rho(t),\\
x_3(t+1) &= \gamma_4 x_2(t) + (\gamma_5 + \delta_2(t))x_3(t) + \gamma_6 u(t),\\
y_1(t) &= x_1(t) + w_1(t),\\
y_2(t) &= x_3(t) + w_2(t).
\end{aligned}
\tag{18}
$$

Equation (18) constitute the sampled-data model of the robot electric servoactuator. The coefficients $\gamma_1 \div \gamma_6$ depend on the servoactuator parameters and the sampling time; the function $\rho(t)$ is induced by the external loading moment; the uncertainty $\delta_1(t)$ is due to change of inertia properties, $\delta_2(t)$ is due to change of active resistances.

The matrices describing the system are given by

$$
A = \begin{pmatrix} 1 & \gamma_1 & 0 \\ 0 & \gamma_2 & \gamma_3 \\ 0 & \gamma_4 & \gamma_5 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ \gamma_6 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$

$$
Q = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \Delta_1 A(t) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta_1(t) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 A(t) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \delta_2(t) \end{pmatrix}.
$$

The problem is to estimate the variables $x_1(t)$, $x_2(t)$, and $x_3(t)$ that is

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

It should be noted that to solve this problem, the approaches suggested in [11, 16, 18] and similar papers design full-order interval observer. Our approach allows reducing the observer dimension and interval width.

Since $y_1(t) = x_1(t) + w_1(t)$ and $y_2(t) = x_3(t) + w_2(t)$, then

$$\underline{x}_1(t) = y_1(t) - w_{*1}, \overline{x}_1(t) = y_1(t) + w_{*1}$$

and

$$\underline{x}_3(t) = y_2(t) - w_{*2}, \overline{x}_3(t) = y_2(t) + w_{*2}.$$

To estimate the variables $x_2(t)$, solve the Eq. (7) with $k = 1$:

$$(\Phi \quad -G_*) \begin{pmatrix} 1 & \gamma_1 & 0 \\ 0 & \gamma_2 & \gamma_3 \\ 0 & \gamma_4 & \gamma_5 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0.$$

Its solution is $\Phi = (1/\gamma_1 \quad -1/\gamma_2 \;\; 0)$ and $G_* = (1/\gamma_1 \quad -\gamma_3/\gamma_2)$ that gives $B_* = 0$ and $Q_* = -1/\gamma_2$. Clearly, $C_* = -\gamma_2$, $P_* = (\gamma_2/\gamma_1 \;\; 0)$, $\Delta A_{*1}(t) = \delta_1(t)$, $G'_1 = (-\delta_1(t)/\gamma_2 \;\; 0)$; $\Delta A_{*2}(t) = 0$ and $G'_2 = 0$ according to (16).

The reduced order model is given by

$$\begin{aligned} \eta(t+1) &= \delta_1(t)\eta(t) + (1/\gamma_1 - \delta_1(t)/\gamma_2)C_1x(t) - (\gamma_3/\gamma_2)C_2x(t) - \rho(t)/\gamma_2, \\ v(t) &= -\gamma_2\eta(t) + (\gamma_2/\gamma_1)y_1(t). \end{aligned}$$

The observer has the following description:

$$\begin{aligned} \underline{\eta}(t+1) &= \underline{\delta_1}\underline{\eta}(t) + \Big(1/\gamma_1 + \underline{Sg}/\gamma_2\Big)y_1(t) - (\gamma_3/\gamma_2)y_2(t) \\ &\quad -\Big(1/\gamma_1 + \overline{\delta_1}\Big)w_{*1} - (\gamma_3/\gamma_2)w_{*2} - \rho_*/\gamma_2, \\ \overline{\eta}(t+1) &= \overline{\delta_1}\overline{\eta}(t) + \big(1/\gamma_1 + \overline{Sg}/\gamma_2\big)y_1(t) - (\gamma_3/\gamma_2)y_2(t) \\ &\quad +\Big(1/\gamma_1 + \overline{\delta_1}\Big)w_{*1} + (\gamma_3/\gamma_2)w_{*2} + \rho_*/\gamma_2, \\ \underline{v}(t) &= -\gamma_2\overline{\eta}(t) + (\gamma_2/\gamma_1)y_1(t), \\ \overline{v}(t) &= -\gamma_2\underline{\eta}(t) + (\gamma_2/\gamma_1)y_1(t), \end{aligned} \tag{19}$$

where

$$\begin{aligned} \underline{Sg} &= 0.5\Big((1 - sign(y_1(t)))\underline{\delta_1} + (1 + sign(y_1(t)))\overline{\delta_1}\Big), \\ \overline{Sg} &= 0.5\Big((1 - sign(y_1(t)))\overline{\delta_1} + (1 + sign(y_1(t)))\underline{\delta_1}\Big). \end{aligned}$$

Comparing the obtained results and the results which can be obtained for this example by methods developed in [11, 16, 18] and similar papers, we may conclude that the dimension of the observer (19) is fewer than that in [11, 16, 18] and the suggested approach produces the estimations with smaller interval width since the ones for $x_1(t)$ and $x_3(t)$ do not contain the disturbance ρ_* and the uncertainties. Besides, the observer (19) does not contain the uncertainty $\delta_2(t)$.

For simulation consider the model (18) and the observer (19); the measurement noises $w_1(t)$ are $w_2(t)$ are random processes evenly distributed on $[-0.01,\ 0.01]$, the parametric uncertainty δ_1 is modeled as $\delta_1 = 0.03(1 + sin(10t))$. Set for simplicity $\gamma_1 = \gamma_2 = \gamma_5 = \gamma_6 = 1$, $\gamma_3 = \gamma_4 = -1$; set $\underline{\delta}_1 = 0$ and $\overline{\delta}_1 = 0.06$; $w_{*1} = w_{*2} = \rho_* = 0.01$. Figures 1 and 2 illustrate simulation results, where $\underline{v}(t)$, $\overline{v}(t)$, and $x_2(t)$ are presented for $\eta(0) = 0$, $\underline{\eta}(0) = -0.05$, and $\overline{\eta}(0) = 0.05$. In Fig. 1, the control $u(t) = 0.2$; in Fig. 2, $u(t) = 0.2sin(t/20)$.

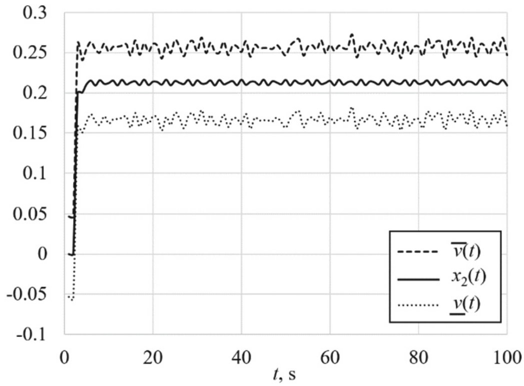

Fig. 1. Behavior of $x_2(t)$ and $\underline{v}(t)$ and $\overline{v}(t)$ with $u(t) = 0.2$

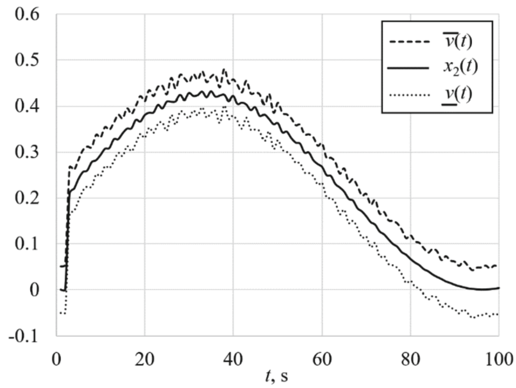

Fig. 2. Behavior of $x_2(t)$ and $\underline{v}(t)$ and $\overline{v}(t)$ with $u(t) = 0.2sin(t/20)$

5 Conclusion

The paper has studied the problem of interval observer design for linear discrete-time systems under the disturbance, measurement noise, and unknown parameters. The reduced-order model which does not depend on the disturbances or has minimal sensitivity to them and is realized in identification canonical form has been used to solve the problem. The designed interval observer has minimal dimension and estimates the prescribed linear function of the original system state vector with the reduced interval width. The limitation of the proposed approach is that it can be applied for limited class of nonlinear systems; nonlinearities should satisfy some requirements. A future research direction is the interval observer design for hybrid dynamic systems.

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