A Novel Robust Finite-Time Control for Active Suspension Systems with Naturally Bounded Inputs

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Abstract. This paper focuses on a novel robust finite-time control for active suspension systems with external disturbance where the control inputs are naturally bounded by a prior known range to avoid input saturation. To achieve finite-time stability, a novel nonsingular terminal sliding mode variable with an integral term is designed. More importantly, the control inputs are naturally bounded all the time due to the characteristics of hyperbolic tangent functions such that extra saturation compensation methods can be avoided. A disturbance compensation technique is deliberately designed in the proposed control to enhance the robustness of the system while the bounded property can be ensured simultaneously. The whole control structure is relatively simple yet effective compared with existing control techniques which is much easier to implement in practical active suspension systems. Additionally, the overall stability of the closed-loop system is verified by the Lyapunov theorem. Various simulation results are provided to demonstrate the robustness and effectiveness of the proposed control design.

Keywords: Active Suspension Systems · Robustness · Finite-Time Control · Bounded Inputs

1 Introduction

Vehicle suspension systems, which play an important role in vehicle ride comfort, driving safety, and maneuverability, arouse great attention in recent years [1–3]. Besides, vehicle suspension systems are used to support the weight of the vehicle and passengers, deal with the vibration between the vehicle body and wheels, and maintain contact between tires and the ground [4]. Normally, vehicle suspension systems can be divided into three categories including passive suspensions, semi-active suspensions, and active suspensions [5].

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The passive suspension systems consist of dampers and springs whose coefficients are fixed during the whole time, which usually leads to the unsatisfactory anti-vibration performance of ride comfort and road handling abilities [6]. To address the disadvantages of passive suspensions, variable dampers are considered a great improvement in semi-active suspension systems by adjusting the rate of energy dissipation [7]. Furthermore, compared with passive and semi-active suspensions, active suspension systems (ASSs) are able to achieve a better suspension response due to the extra actuator assembled between the vehicle body and the wheel axle which can effectively add and dissipate energy from the system [8]. Hence, various advanced control techniques are adopted for ASSs including the H-infinity control [9], adaptive control [10], back-stepping control [11], etc. Specifically, the sampled-data method was integrated with the robust H-infinity control for ASSs where the whole controller was converted to a convex optimization problem [9]. An adaptive trajectory tracking control was designed for ASSs while the actuators suffer from unknown dead zones and hysteresis nonlinearities [10]. Furthermore, a back-stepping tracking control was constructed for uncertain ASSs while the safety constraints can be ensured simultaneously [11]. Despite the satisfactory performance that can be obtained by these controllers, more efforts should be devoted to the control design of ASSs to improve the anti-vibration ability and ride comfort.

Considering the physical limitations of the actuators in ASSs, only limited forces can be provided by the actuators which leads to the input saturation problem. Excessive control signals may cause damage to the actuators or the instability of the systems. Therefore, the control input signals are constrained within the predefined upper and lower bounds. However, the resulting inputs are non-smooth when saturation occurs, and extra saturation compensation techniques should be integrated to enhance the system stability. Some naturally bounded functions, such as the inverse tangent function *arctan*, signum function *sign* and the hyperbolic tangent function *tanh*, are widely used in recent controllers to deal with the input saturation problems. For instance, a robust proportional-derivative sliding mode control was proposed for ASSs where the control signals were bounded utilizing the *arctan* and *sign* functions [12]. An adaptive neural network control was developed for uncertain robots where the control inputs are asymmetrically bounded using the *tanh* functions [13]. Besides the results above, the robust controllers for ASSs with bounded inputs are rarely considered and need more effort.

One should also note that the aforementioned controllers with naturally bounded inputs can only obtain asymptotic stability with a slow convergence rate. The finite-time stability of the closed-loop system can provide a higher convergence speed while the robustness and disturbance rejection ability can be improved. Some efforts have been made in the control theory community to achieve finite-time stability with naturally bounded inputs for various mechanical systems. An adaptive finite-time control was designed for Euler-Lagrange systems with prior known bounded inputs generated by *tanh* and *sign* functions [14]. Tian et al. proposed a continuous finite-time bounded control for double integrator systems using the extension of super-twisting algorithms [15]. Furthermore, the finite-time trajectory tracking control was proposed for rigid robots with bounded inputs and terminal sliding mode techniques [16]. A novel cluster formation control was designed for networked marine surface vehicles with a predefined time estimator and bounded input constraints [17]. However, most of these controllers are complex and difficult to be implemented in practical applications. Therefore, it is meaningful to propose an easy yet effective finite-time control for ASSs with prior known bounded inputs.

To effectively address the issues above, a novel robust finite-time control is proposed for ASSs with bounded control inputs. The characteristics of hyperbolic tangent functions and the adaptive technique with bounded outputs are investigated such that the whole control inputs are constrained within the prior known bounds. Furthermore, the newly designed nonsingular integral terminal sliding mode variable is constructed for ASSs to achieve finite-time stability while the singular phenomenon can be effectively avoided. This paper can provide a simple yet effective controller design for practical ASS applications.

The rest of this paper is organized as follows. In Sect. 2, the problem formulation including the system description and the control objects is stated. The main results of the robust finite-time control design with bounded inputs are presented in Sect. 3. After that, some comparative simulations are provided in Sect. 4 to illustrate the effectiveness and robustness of the closed-loop system. Finally, Sect. 5 concludes the main results of this paper.

2 Problem Formulation

2.1 System Description

Without loss of generality, a quarter-vehicle active suspension system is considered in this paper shown in Fig. 1 borrowed from [18]. The main components of the system are the masses, springs, dampers, and an active actuator. The dynamic equations of the ASS can be written as [19, 20]

$$\begin{aligned} m_s\ddot{D}_s &= -R_s(D_s, D_u) - R_d(\dot{D}_s, \dot{D}_u) + F + \Delta \\ m_u\ddot{D}_u &= R_s(D_s, D_u) + R_d(\dot{D}_s, \dot{D}_u) - R_t(D_u, D_r) - R_b(\dot{D}_u, \dot{D}_r) - F \end{aligned} \tag{1}$$

with

$$\begin{aligned} R_s(D_s, D_u) &= c_{s1}(D_s - D_u) + c_{s2}(D_s - D_u)^3 \\ R_d(\dot{D}_s, \dot{D}_u) &= c_{d1}(\dot{D}_s - \dot{D}_u) + c_{d2}(\dot{D}_s - \dot{D}_u)^2 \\ R_t(D_u, D_r) &= c_t(D_u - D_r) \\ R_b(\dot{D}_u, \dot{D}_r) &= c_b(\dot{D}_u - \dot{D}_r) \end{aligned} \tag{2}$$

where the definitions of system parameters and variables for the ASS are provided in Table 1.

The following state variables are defined $x = \left[x_1\ x_2\ x_3\ x_4\right]^T$ where $x_1 = D_s$, $x_2 = \dot{D}_s$ denote the vertical displacement and velocity of the sprung mass, respectively; $x_3 = D_u$, $x_4 = \dot{D}_u$ represent the vertical displacement and velocity of the unsprung mass, respectively. Then the system dynamic equations of the ASSs can be rewritten as follows.

$$\begin{cases} \dot{x}_1 = x_2 \\ m_s\dot{x}_2 = P(x) + F + \Delta \\ \dot{x}_3 = x_4 \\ m_u\dot{x}_3 = Q(x) - F \end{cases} \tag{3}$$

with

$$P(x) = -c_{s1}(x_1 - x_3) - c_{s2}(x_1 - x_3)^3 - c_{d1}(x_2 - x_4) - c_{d2}(x_2 - x_4)^2 \tag{4}$$

$$Q(x) = c_t(x_3 - D_r) + c_b(x_4 - \dot{D}_r). \tag{5}$$

In practice, the system parameters are difficult to obtain due to system nonlinearities and external disturbance. Besides, an unstable environment may affect the accuracy of the values of parameters, which should be carefully considered. Therefore, control robustness is required for the uncertain ASSs to achieve satisfactory anti-vibration performance.

Table 1. The definitions of system parameters and variables for the ASS

Parameter/variable	Definition
m_s	Sprung mass
m_u	Unsprung mass
D_s	Vertical displacement of the sprung mass
D_u	Vertical displacement of the unsprung mass
D_r	Road input profile
c_{s1}, c_{s2}	Stiffness coefficients of the nonlinear spring
c_{d1}, c_{d2}	Damping coefficients of the damper
c_t	Stiffness coefficient of the tire
c_b	Damping coefficient of the tire
F	Control signal
Δ	Unknown external disturbance

Some useful assumptions are made for the ASSs to facilitate the controller design.

Assumption 1: The unknown disturbance Δ and its first-time derivative $\dot{\Delta}$ are bounded.

Assumption 2: The road input profile D_r and its first-time derivative $\dot{D}_r$ are bounded by $|D_r| \le \delta_{r1}$ and $\left|\dot{D}_r\right| \le \delta_{r2}$ with $\delta_{r1}, \delta_{r2} > 0$.

Remark 1: Assumptions 1 and 2 are common assumptions in the control design for nonlinear ASSs shown in [21, 22] which are reasonable under normal working conditions.

2.2 Control Objects

In this paper, the main control objects are to design a robust finite-time control with bounded control inputs such that the vibration of the vehicle body can be effectively

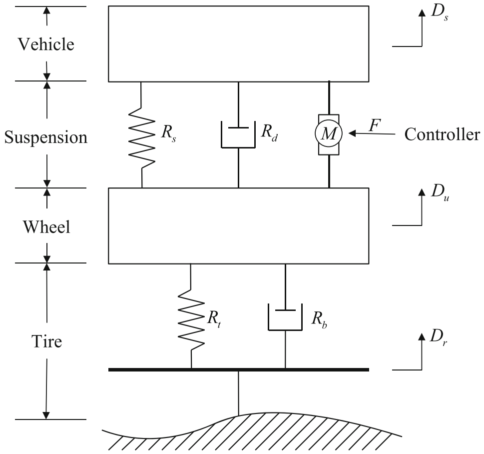

Fig. 1. Quarter-vehicle ASS model

suppressed, and the tire load $R_{dyn} = R_t + R_b$ and suspension space $D_p = D_s - D_u$ are bounded and remain within the permitted ranges for driving safety given by

$$\left|R_{dyn}\right| \leq (m_s + m_u)g = R_{sat} \tag{6}$$

$$\left|D_p\right| \leq D_{\max} \tag{7}$$

where g is the gravitational constant, R_{sat} means the static tire load, D_{max} denotes the maximum suspension space.

3 Main Results

In this section, a robust finite-time control design with naturally bounded inputs is proposed for the ASSs.

In order to achieve finite-time stability of the closed-loop system, a novel nonsingular terminal sliding mode variable with the integral term is described as

$$s = x_2 + \int_0^t \mu_1[x_1]^p + \mu_2[x_2]^q d\tau \tag{8}$$

where $0 < p < 1$, $q = 2p/(1+p)$, μ_1, $\mu_2 > 0$ are positive design constants, $[x]^*$ denotes the abbreviation of $|x|^*\text{sgn}(x)$.

An auxiliary velocity signal is defined as

$$\vartheta = -\int_0^t \mu_1[x_1]^p + \mu_2[x_2]^q d\tau - \lambda_1\zeta \tag{9}$$

where $\lambda_1 > 0$ is a design constant, and ζ is an auxiliary state variable generated by

$$\dot{\zeta} = \lambda_2 s + \tanh(s),\ \zeta(0) = 0. \tag{10}$$

Next, a new state variable can be constructed as

$$\chi = x_2 - \vartheta. \tag{11}$$

Then the state variable becomes

$$\begin{aligned}\chi &= x_2 + \int_0^t \mu_1[x_1]^p + \mu_2[x_2]^q d\tau + \lambda_1\zeta \\ &= s + \lambda_1\zeta\end{aligned} \tag{12}$$

Based on the previous definitions, the robust finite-time control is given by

$$F = -\lambda_3 \tanh(\chi/\epsilon_1) + \Omega \tag{13}$$

where $\lambda_3 > 0$ is a design parameter, Ω is the adaptive variable compensating for the external disturbance denoted by

$$\dot{\Omega} = -\lambda_4 \tanh(\chi/\epsilon_2) - \lambda_5\Omega\ , \quad \Omega(0) \le \lambda_4/\lambda_5 \tag{14}$$

with λ_4, λ_5 are positive design constants, ϵ_1 and $\epsilon_2 > 0$ are used to adjust the convergence rate.

Theorem 1. For the nonlinear suspension system (1), the proposed controller (13) with the nonsingular integral terminal sliding mode surface (8) and the adaptive variable (14) can guarantee that the signals are bounded all the time, D_s can converge to zero in a finite time, the control input of the closed-loop system can be naturally bounded by

$$|F| \le \lambda_3 + \lambda_4 \big/ \lambda_5. \tag{15}$$

and the tire load $R_{dyn} = R_t + R_b$ and suspension space D_p can keep within the following ranges

$$\left|R_{dyn}\right| \le (m_s + m_u)g = R_{sat} \tag{16}$$

$$\left|D_p\right| \le D_{\max} \tag{17}$$

Proof: The detailed proof of Theorem 1 is omitted which is available upon request.

4 Simulation Results and Analysis

In this section, some comparative simulation results on a quarter-vehicle suspension system are presented to demonstrate the effectiveness and robustness of the proposed control. The values of the system parameters for the ASS are given in Table 2.

Three different suspension systems are evaluated in this paper.

- Passive: Passive suspension without force inputs;
- SPDSM: Active suspension system using the saturated PD sliding mode control [12];
- Proposed: Active suspension system utilizing the proposed robust finite-time control with bounded inputs.

The control gains for the SPDSM control are tuned as $\lambda = 40$, $k_p = 300$, $k_d = 300$ and $k_s = 10$. Besides, the parameter values of the proposed control are designed as $\lambda_1 = \lambda_2 = 10$, $\lambda_3 = 50$, $\lambda_4 = 6$, $\lambda_5 = 2$, $\mu_1 = 10$, $\mu_2 = 1$, $p = 0.9$ and $\epsilon_1 = \epsilon_2 = 0.1$. The external disturbance for three suspensions is selected as $\Delta = 0.2\sin(0.1t)$.

In this paper, two different cases are considered for the suspension systems including the sinusoidal and random road inputs presented in Fig. 2. In Case 1, the sinusoidal road input is designed as $D_r = 0.005\sin(6\pi t)$. In Case 2, a random road profile is provided. The simulation results are depicted in the following Figs. 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Table 2. Parameters of the active suspension system

Symbols	Values	Symbols	Values
m_s	2.45 kg	m_u	1 kg
c_{s1}	900 N/m	c_{s2}	10 N/m^3
c_d	8 Ns/m	c_t	1250 N/m
c_b	5 Ns/m		

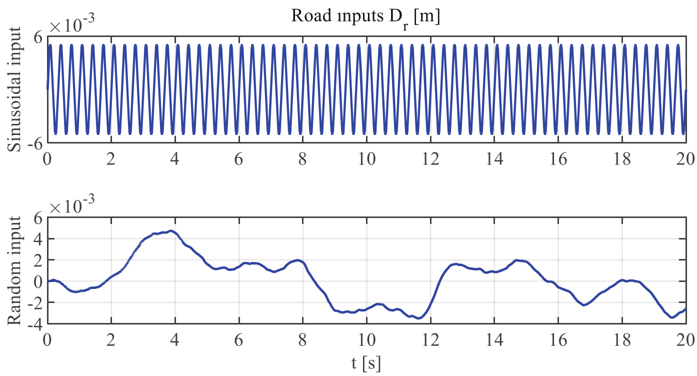

Fig. 2. Road inputs in two Cases

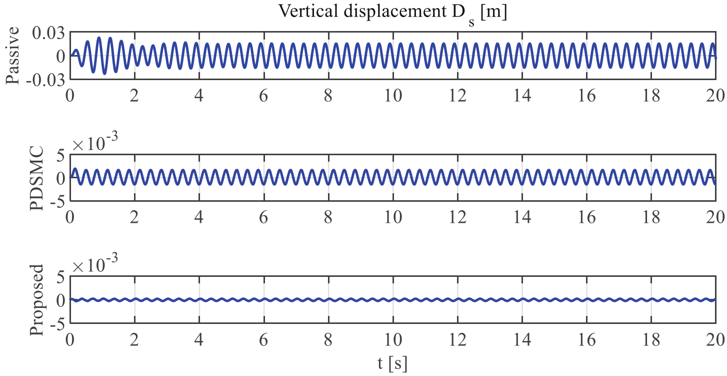

Fig. 3. Vertical displacement D_s in Case 1

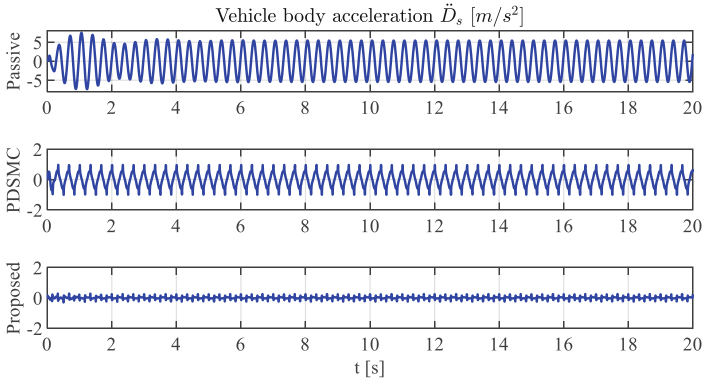

Fig. 4. Vehicle body acceleration $\ddot{D}_s$ in Case 1

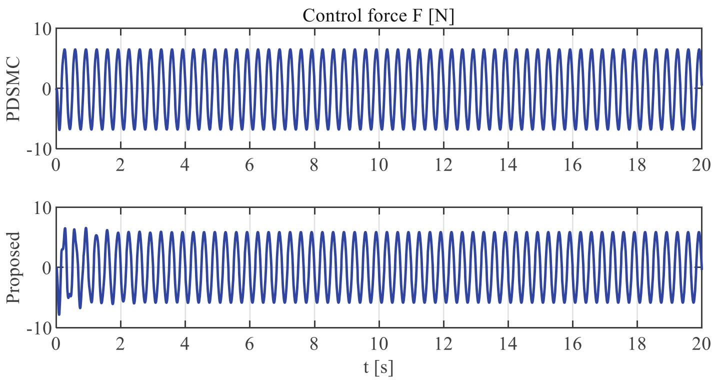

Fig. 5. Control force input F in Case 1

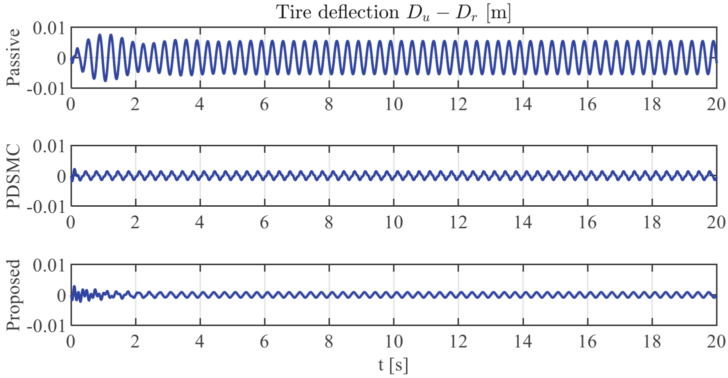

Fig. 6. Tire deflection $D_u - D_r$ in Case 1

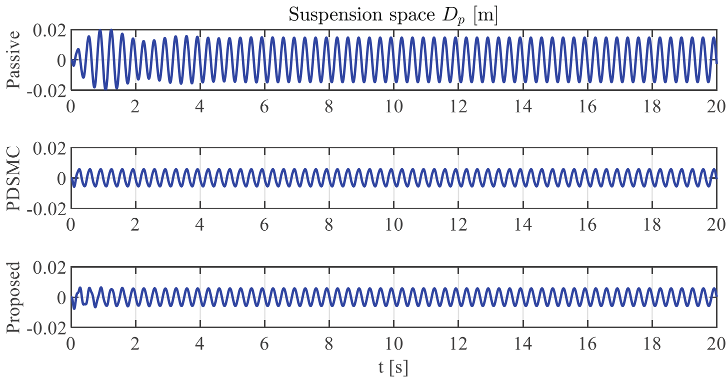

Fig. 7. Suspension space D_p in Case 1

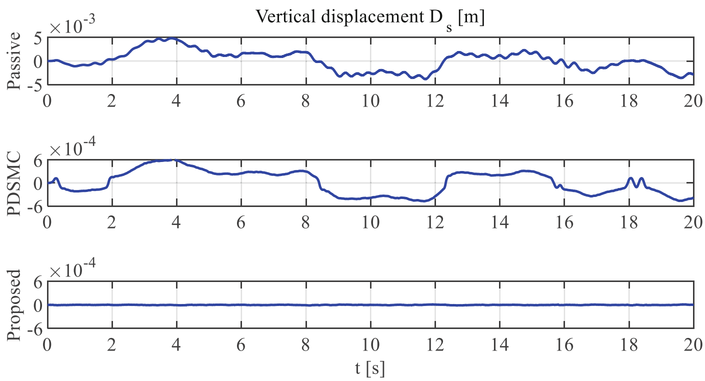

Fig. 8. Vertical displacement D_s in Case 2

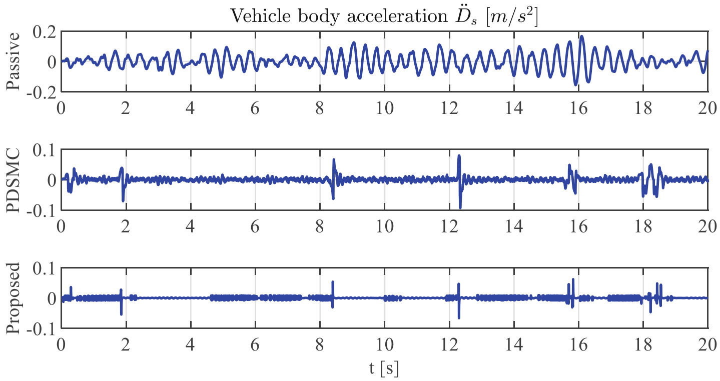

Fig. 9. Vehicle body acceleration $\ddot{D}_s$ in Case 2

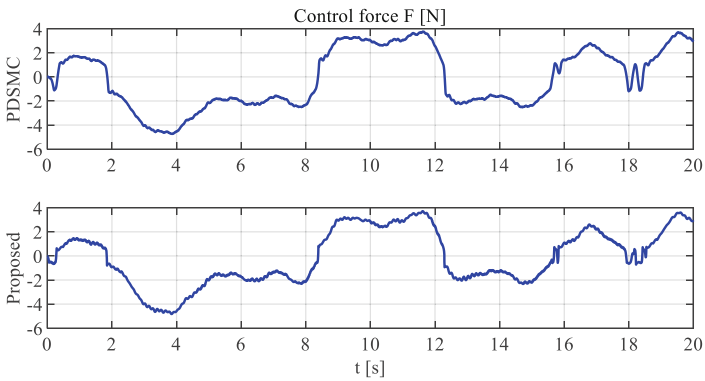

Fig. 10. Control force F in Case 2

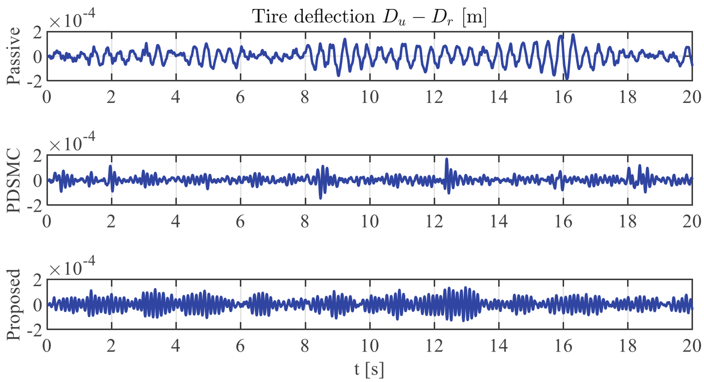

Fig. 11. Tire deflection $D_u - D_r$ in Case 2

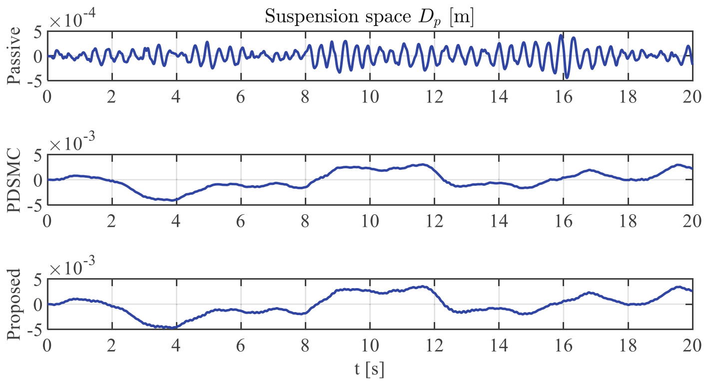

Fig. 12. Suspension space D_p in Case 2

The vertical displacement results are presented in Fig. 3 and Fig. 8, which present the satisfactory control performance of the SPDSM control and the proposed control compared with the passive suspension. However, the proposed control method shows superior control performance with smaller vertical displacement of the sprung mass, thus it can provide better anti-vibration results compared with the existing SPDSM control. Furthermore, these figures also indicate that the proposed control can achieve finite-time stability, that is to say, the convergence rate is higher than the SPDSM method.

The vehicle body acceleration results are recorded in Fig. 4 and Fig. 9, which are important evaluation indicators of ride comfort. As is obviously shown, the amplitudes of the vehicle body acceleration of the proposed control under two cases are smaller than those of the passive suspension system and the SPDSM method. Thus, the higher ability of the proposed control in improving ride comfort can be demonstrated effectively.

Furthermore, the control forces given in Fig. 5 and Fig. 10 depict that the control inputs in both active suspensions under two different cases are always bounded while the proposed control has a smaller prior known upper bound of the control force. For system stability and driving safety, the tire load and suspension space should be restricted in permitted ranges. According to the simulation results shown in Fig. 6 and Fig. 11, the tire deflections $D_s - D_u$ for three different suspensions are always bounded and kept in a reasonable set. Besides, as observed from Fig. 7 and Fig. 12, the suspension space values behave within the allowable ranges while the maximum suspension space is designed to be 3.8cm according to [23]. All these results demonstrate the robustness and effectiveness of the proposed control method.

5 Conclusions

In this paper, a novel robust finite-time control with prior known bounded inputs is proposed for ASSs in the presence of unknown external disturbance. First, a nonsingular integral terminal sliding mode surface is defined in the control to achieve finite-time stability. Then the control inputs are constructed based on the hyperbolic tangent functions and the newly designed adaptive technique which are constrained in the prior known range. One should note that the input saturation problem can be avoided without any extra saturation compensation designs. The robustness and effectiveness of the proposed control are verified by various simulation results.

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