Long Vehicle Turning

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Abstract Safety and efficiency are two major issues when it comes to long vehicle driving and operating, particularly those with very large lateral and longitudinal sizes. Trajectory planning for long vehicle tuning always plays an important role to secure safety and efficient operation, especially when turning in narrow spaces limited by surrounding objects such as buildings. This chapter discusses a trajectory calculation method for long vehicle turning, based on a set of differential equations. The solution can be numerically obtained for any trajectory under different circumstances. We develop a generalized and systematic mathematical approach to determine the trajectories swept by each wheel and other related components of the vehicle. The envelope of the trajectories of the vehicle can then be derived according to geometric relationships and characteristics. Based on numerical analysis results, a 3D simulation is developed in this work for different types of long vehicles along with different given turning roads surrounded by buildings and other objects. This way we are able to do trajectory planning for long vehicle turning.

Keywords Vehicle turning · Simulation · Trajectory planning

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1 Introduction

1.1 Background

Safety and efficiency issues of long vehicle turning can be attributed to a number of factors, including vehicle shape, its control system, driving skills of the operators, roads, and nearby environment. Long vehicles are difficult to maneuver and sometimes are even dangerous especially in a complex environment due to the space limitation. For example, at intersections, roundabouts, or bus terminals, the accessibility and safety need to be analyzed and evaluated before maneuvering the long vehicles. It is not easy to have long vehicles changing lanes, reversing, or parking. To secure a safety driving, planning the collision-free turning for long vehicles under different conditions is necessary and important. The procedures of analysis and evaluation include:

(1) Compute the path swept by the long vehicles along the proposed trajectory;
(2) Determine whether there are intersections between the swept area and the surrounding obstacles;
(3) Modify the pre-planned path until no intersections are found in procedure (2).

To provide a mathematical description of the path swept by a moving vehicle, the maximal displacement of the rear end of the bus is analyzed for studying the driving hazard issues (Baylis 1973, Bender 1979). After that, the solution to the motion equation of midpoint of the rear axle is investigated, and then applied to different types of motions including straight line motion, circular motion, and others. Also, several applications such as turning around a corner, changing lanes, and traffic circles are discussed based on the calculated results (Freedman and Riemenschneider 1983). The limitation of these methods is that the implicit solution is obtained with the assumption that the motion of the front of the bus can be described as a solution of the second order linear differential equation. Numerical computing and analysis are introduced to simulate the maneuvering situations using several simplified models of truck braking and handling (Fancher and Balderas, 1987). The models involving vehicle dynamics concepts are used for preliminary analyses of off-tracking, straight line braking, steady turn, etc. In addition to normal vehicles, off-tracking of multiple unit vehicle combinations is discussed using a bicycle wheel model (Sayers 1986, Erkert et al. 1989). Mathematical methods are increasingly used for vehicle path planning and navigation. For purpose of autonomous path-tracking, the curvature that will drive the vehicle to the goal point is calculated geometrically with tracking algorithms such as pure pursuit and its variations (Coulter 1992, Hellström and Ringdahl 2005). Based on the model adapted from J. Baylis, the maximum rightward displacement of the vehicle depends on the relationship between the turning angle of the front wheel and the angle between the vehicle and the direction of the roadway (Bender 2000).

On the other hand, interactions between vehicles and roadway geometry are summarized and discussed from the perspectives of the comprehensive truck and roadway separately (Battelle Team 1995). Such studies focus more on the vehicle performance characteristics such as low-speed off-tracking, rather than investigating the whole scenario of the trajectories swept by the vehicles. To address the issue of the track of a bicycle back tire, the matrix Riccati equations are derived with the assumption that the tangent vector to the path of the rear tire always points to the contact point on the path of the front tire (Dunbar et al. 2001). To provide a close to reality simulation, the behavior of the vehicle turning is always analyzed in terms of kinematics and kinetics (Sweatman et al. 1991, Farmer 2008). However, it needs accurate and detailed kinematic parameters and related sensors are required. Also, some efforts have been made by applying intuitive techniques such as representing the transition curves with polynomials (Fioretti et al. 2008). It does not reflect the actual motion of the vehicle, although the computation is time efficient. Therefore, developing a generalized mathematical representation and solution will be necessary and effective, especially for those low-speed vehicles.

1.2 Objective

This chapter focuses on the trajectory calculation for long vehicles. To make trajectory planning, we propose a systematic and applicable approach for long vehicle turning. We attempt to develop a mathematical method for trajectory calculation without considering driving speed and angular turning velocity. The calculation results will be used for a 3D simulation of maneuvering long vehicles such as turning and parking in relation to the given surrounding buildings and other objects.

2 Mathematical Modeling

We start from a four-wheel long vehicle with an Ackermann steering geometry, which can be represented as a box (Fig. 1). Throughout the whole calculation and the simulation process, all of the wheels are assumed to have no slippage or skidding, and contact area between the tire and the ground is zero.

Based on the assumptions above and the fact that the rear axle is fixed on the vehicle body and each wheel rotating around the rear axle cannot change the direction on its own, it is almost as if there is an imaginary steered wheel located at point P (the midpoint of the front axle) with point Q (the midpoint of the rear axle) pointing to it. Thus the path of point Q and the line of centers PQ should be tangent to each other at point Q. φ denotes the angle between PQ and the vertical line while θ is the angle between PQ and front wheels (turning angle). With the

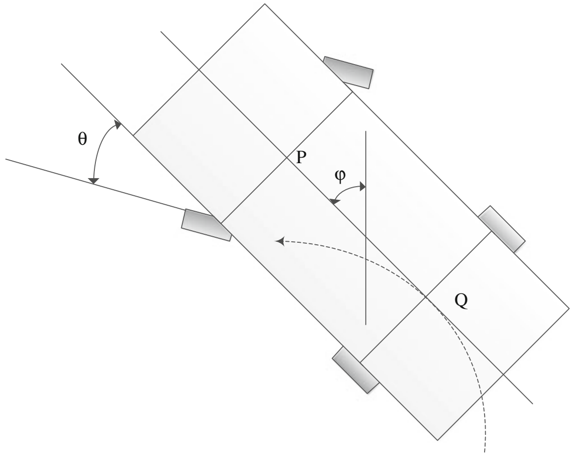


Fig. 1 Bottom view of a long vehicle with four wheels

assumption of no sideslip, there are three principles the steering process has to follow:

(1) The distance between midpoint of front axle *P* and midpoint of rear axle *Q* keeps constant and equal to the wheelbase;
(2) The direction of the velocity of midpoint of rear axle *Q* should point toward the midpoint of front axle *P*;
(3) The direction of the velocity of *Q* is always tangent to its trajectory.

2.1 Stable Circular Movement

Let us start with a simple question: how do we know the collision occurred between the vehicle and the walls (or other objects)? A straightforward solution is to find all the trajectories formed by the vehicle outline sweeping along the path; i.e., the trajectory profile or envelope of trajectories. If the envelopes of the trajectories and surrounding objects have no overlaps (intersections), the safety can be ensured for the vehicle to turn around the corner following the given path. Trajectories of front wheels are easy to get because these two wheels are totally controllable, consequently trajectories of these two points can be conveniently changed or modified with the aid of control system of the vehicle. The remaining problem is how to determine the trajectories of rear wheels with known parameters and conditions.

First, consider a simple case in which the vehicle is driven on a circular road, i.e., the front wheels are moving along a circular path. From common experience we know that the trajectories of rear wheels are also in the form of a circle.

To verify, a mathematical model (Prince and Dubois 2009) can be used for describing and solving this problem.

Vector-valued functions which depend on a single parameter t, representing time, can be used to determine the positions of P and Q, respectively:

$$\begin{aligned}\dot{\vec{Q}}(\mathrm{t}) &= \gamma(t)\left(\vec{P}(t) - \vec{Q}(t)\right)\\ \vec{B}(t) &= \vec{Q}(t) - \vec{P}(t)\end{aligned} \tag{1}$$

The scalar function $\gamma(t)$ represents the speed of point Q. Also, ψ is the angle between the horizontal line and the vector PQ, and it is equivalent to $\frac{\pi}{2} - \varphi$ in the case shown in Fig. 1. The Length of PQ is denoted as L while coordinates of point P are (P_1, P_2).

The general equation can be obtained:

$$\frac{d\psi}{d\mathrm{t}} = \frac{\sin(\psi)\dot{P}_1 - \cos(\psi)\dot{P}_2}{L} \tag{2}$$

For a circular path with radius R, we write it in parametric form and substitute it into Eq. (2), a differential equation with initial conditions can be obtained:

$$\begin{aligned}\vec{P}(t) &= R(\cos(\Omega(t)), \sin(\Omega(t)))\\ \frac{d\psi}{dt} &= -\dot{\Omega}\frac{R}{L}\cos(\psi - \Omega)\\ \psi(0) &= \psi_0,\ \Omega(0) = \Omega_0\end{aligned} \tag{3}$$

Actually, the equation above has a constant solution without applying the initial conditions, which shows that the trajectory of point Q is a circle of radius $\sqrt{R^2 - L^2}$ (if $R > L$) centered at the origin:

$$\cos(\psi - \Omega) = -\frac{L}{R} \tag{4}$$

2.2 General Form of the Trajectory

However, Eq. (4) indicates the final (stable) motion status of the vehicle, and it does not deliver any information about trajectory of Q at the very beginning when starting from the original stationary state to gradually merge onto the circular trajectory. We are interested in the locus curve of Q when the initial position of Q is not right on the inner circle. When steering around a corner, the movement of point P consists of several stages, as illustrated in Fig. 2, which shows the process of a long vehicle turning around a corner with 90°. R_{P} and R_{Q} represent the radii of the trajectories of P and Q, respectively, with the common center O. The initial position of vehicle body PQ is indicated in bold, while $P'Q'$ is its final position.

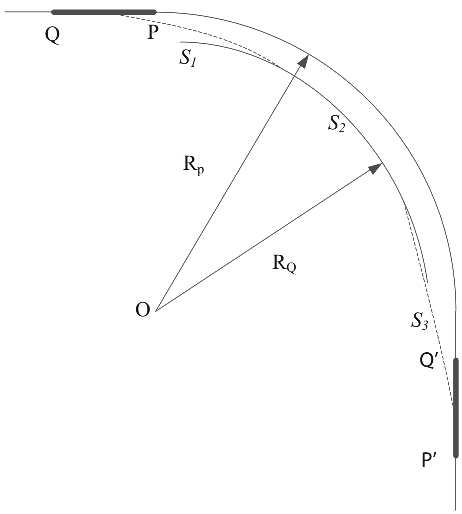


Fig. 2 Steering around a corner with 90°

(1) S_1: The dashed line S_1 denotes the trajectory of Q when P enters the circular path with radius R_P from the straight line motion. Although P can exactly follow the predefined trajectory, the path of Q is not a circle because Q cannot change its position from the straight line to the inner circle immediately.
(2) S_2: When Q enters its circular path, the vehicle body constrains Q along the inner circle with radius R_Q just by keeping P moving along the outer circle and this process has been illustrated in Eq. (4).
(3) S_3: The dashed line S_3 indicates the process when the vehicle begins to exit the circular arc. The trajectory of Q changes from a circular arc to a straight line gradually.

With S_2 easy to be calculated, S_1 and S_3 remain to be solved. A possible way to solve this problem is to obtain the trajectory of point Q depending on the velocity and acceleration of point P, by assuming that the motion of the vehicle can be represented as a solution of a second order linear differential equation, to obtain the trajectory of point Q depending on the velocity and accelerate of point P. However, it is not always applicable to set up such a differential equation sometimes. Here, following the three principles mentioned before, we attempt to determine the trajectory of midpoint of rear axle irrespective of kinetic and kinematic parameters. Concentrating on the components those play major roles in the motion while ignoring other irrelevant mechanisms, we can get a simplified model as shown in Fig. 3. Every point has its own behavior which should obey all the three principles. Point P can move freely in any direction while the locus curve of Q is determined by P: at any point on the curve, the tangent vector points to a corresponding point on the curve formed by P, and the distance between these two points is fixed.

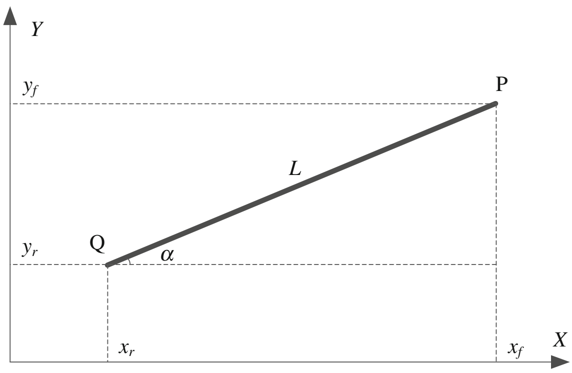


Fig. 3 Relationship between P and Q

If we express the trajectory coordinates of P and Q in parametric form, we can obtain the unit vector of velocity at point Q (serves as an intermediate variable):

$$\frac{\vec{v}}{\|\vec{v}\|} = (\cos\,\alpha,\, \sin\,\alpha) = \left(\frac{x_{\mathrm{r}}'}{\|\vec{v}\|}, \frac{y_{\mathrm{r}}'}{\|\vec{v}\|}\right) = \left(\frac{x_{\mathrm{f}} - x_{\mathrm{r}}}{L}, \frac{y_{\mathrm{f}} - y_{\mathrm{r}}}{L}\right) \tag{5}$$

According to the first principle, the length L keeps constant so that PQ can be considered as a rigid rod. The constraint imposed here is that the components of the velocities along the rod at these two points should be equal. Note that the resultant velocity at point Q is also along the rod. Hence,

$$\|\vec{v}\| = x_{\mathrm{f}}' \cos\,\alpha + y_{\mathrm{f}}' \sin\,\alpha \tag{6}$$

Replacing (6) in (5), we have:

$$\begin{aligned} x_{\mathrm{r}}' &= \frac{1}{L^2}\left[x_{\mathrm{f}}'(x_{\mathrm{f}} - x_{\mathrm{r}})^2 + y_{\mathrm{f}}'(x_{\mathrm{f}} - x_{\mathrm{r}})(y_{\mathrm{f}} - y_{\mathrm{r}})\right] \\ y_{\mathrm{r}}' &= \frac{1}{L^2}\left[x_{\mathrm{f}}'(x_{\mathrm{f}} - x_{\mathrm{r}})(y_{\mathrm{f}} - y_{\mathrm{r}}) + y_{\mathrm{f}}'(y_{\mathrm{f}} - y_{\mathrm{r}})^2\right] \end{aligned} \tag{7}$$

Because there is a possibility that the vehicle body is so long that the stage two does not exist at all for a 90° corner. In this case, stage three takes place after stage one or even during stage one. To make the process of stage one clear, we can investigate a complete circle. Then, the midpoint of the front axle is set to move along the circle with radius R, as shown in Fig. 4 below.

The parametric equation of P and initial conditions are as follows:

$$\begin{aligned} x_{\mathrm{f}}(t) &= R\,\sin(\omega t) \\ y_{\mathrm{f}}(t) &= R\,\cos(\omega t) - R \\ x_{\mathrm{r}}(0) &= -L,\ y_r(0) = 0 \end{aligned} \tag{8}$$

Because the equation set (7) is difficult to solve analytically, numerical method is an alternative way with the help of Matlab. Figure 5 illustrates the motion at three different times, and it shows how trajectory changes from linear motion to

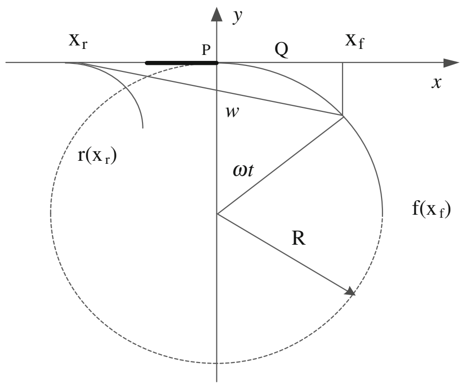


Fig. 4 Circular motion of the long vehicle

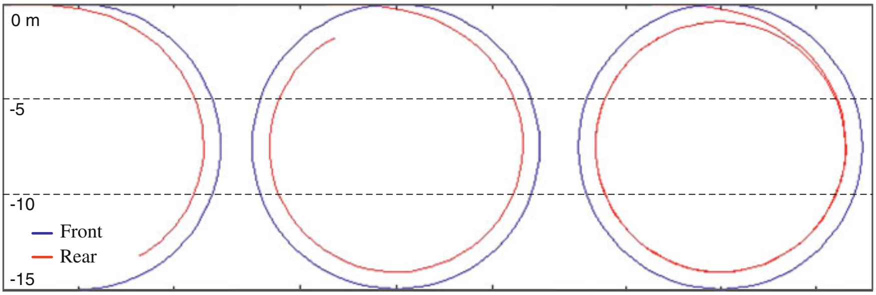


Fig. 5 Motion of Q at different times

circular motion. Notice that from now on, the trajectory of Q is denoted in red while the trajectory of P is in blue. Once Q merges onto the inner circle, it will not deviate from the path as long as P moves along the outer circle.

The parametric function has velocity and time t as its variables, seemingly depending on kinetic parameters. Also, it seems as if this method has a limitation that we have to write the locus curve of P in parametric form, which changes its value over time t. In addition to the shape of the curve of P, are these motion parameters such as velocity, angular velocity necessary to be specified before we calculate the trajectory of Q? The answer is no. Although the Eq. (7) are written in differential form, however, the independent variable here does not have to be time t. During the derivation, the purpose of using differentiation is to obtain the tangent at a specific point on the curve, which is in parametric form and the parameter does not necessarily denote time. Furthermore, the motion of P is not required to move at constant velocity. Consequently, it is very convenient and efficient for trajectory calculation. For this reason, what we need is just the shape of the curve formed by P. So, the parametric equation of P is written using only one parameter later in this article.

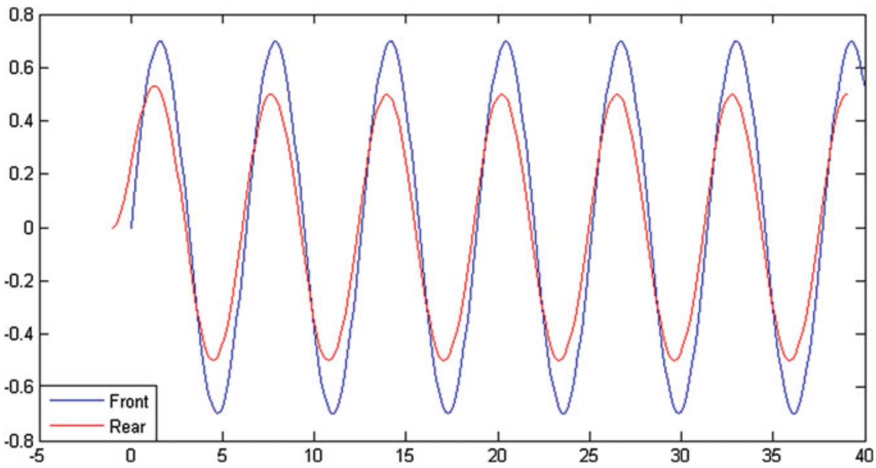


Fig. 6 An example of sinusoidal trajectory

To show that this method can be extended to other types of curves, not just for circular motion, here is another example. As shown in Fig. 6, the motion of P is described as a sinusoidal function (with $L = 1$, $A_f = 0.7$):

$$\begin{aligned} x_f(t) &= t \\ y_f(t) &= A_f \sin(t) \\ x_r(0) &= -1, \, y_r(0) = 0 \end{aligned} \tag{9}$$

With Matlab's ODE45, we can solve it numerically. Figure 6 shows that the trajectory of Q also has a sinusoidal-like shape. Actually, if the trajectory of P is given in an arbitrary shape expressed as discrete coordinates, the analytical expression can be obtained by piecewise fitting methods. For example, piecewise parametric cubic polynomial approximations can be generated automatically for computing from the sampled trajectory data (Plass and Stone, 1983).

2.3 Trajectory of Articulated Vehicles

Articulated vehicles generally include many kinds of vehicles, such as heavy equipment, buses, and even trams and trains. Here, we refer to those vehicles that do not need to run on tracks. Generally, the trailer has one or more axles, while two types of trailer axle placement are popular: single axle and double axle placement, and in most cases, the double axle setup can be treated as equivalent to the single axle mathematically during the calculation of trajectory. For the articulated vehicle shown in Fig. 7, there are more than one trailers towed by a tractor unit, which has a smaller size compared with the trailers. For the reason that it still

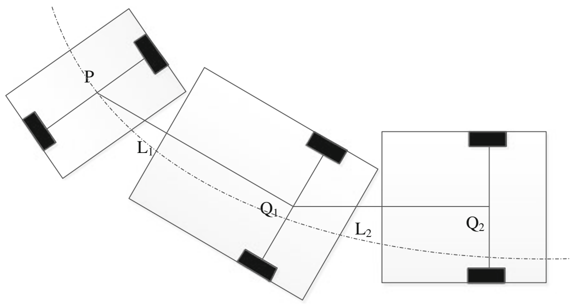


Fig. 7 The scheme of the articulated vehicle

follow the principles described before, the mathematical model still holds true for articulated vehicles. PQ_1, Q_1Q_2, etc., are always perpendicular to the direction of the rear axles of the trailers respectively. Each section, for example, PQ_1, can be treated as a mathematical model described in Fig. 3. In this way, the trajectory of each midpoint of the axle can be calculated one by one by using the method above repeatedly.

2.4 Other Trajectory Analysis Methods

There are various kinds of commercial programs for swept path analysis, and most of them are required to work with AutoCAD for two-dimensional simulation. The swept path analysis software usually works as a plugin for AutoCAD, and allows the customer to draw a user-defined path, and with the preloaded vehicle models in its library and its path calculation algorithm applied (though some simulation algorithms are heuristic, e.g., AutoTURN), the vehicle swept path can be generated accordingly. This way it is capable to calculate and analyze the movement and path of different parts of a vehicle when a turning maneuver happens. The results include the path taken by each wheel and the space needed by the vehicle body during the turn.

Here, we take TURN.LSP project as an example, which is licensed as free software under the terms of the GNU GPL. The basic theory of the program is that a vehicle wheel pair (front and rear) is assumed to be traveling in a circular motion for each computing step. The paths of front and rear wheels are two concentric circles, with the tangent line of the inner circle formed by rear wheel pointing to the front wheel, as Fig. 8 shows.

The initial locations of the front and rear wheels given (P_0 and Q_0), and the final location of the front wheel is known (P_1). The turning angle of rear wheel keeps constant in a circular motion for each step (see Fig. 9).

According to the geometric constrains, the following equations can be derived.

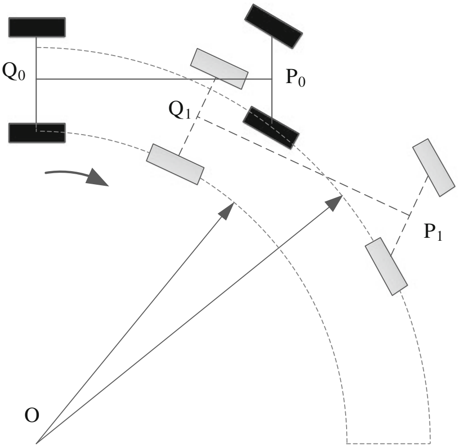


Fig. 8 Relationship between a vehicle wheel pair in each computing step

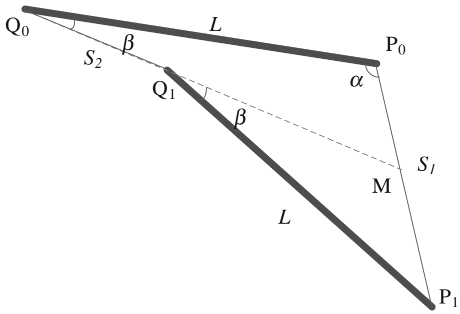


Fig. 9 Mathematical model of vehicle turning in each computing step

$$S_1 \sin(\alpha + \beta) = 2L \sin \beta$$
$$\beta = \tan^{-1}\left(\frac{\sin \alpha}{\frac{2L}{S_1} - \cos \alpha}\right) \tag{10}$$

where S_1, S_2 are the distances between P_0P_1 and Q_0Q_1, respectively, and L is the wheelbase, α and β are the angles as shown in Fig. 9.

From these formulas, obviously point M is the midpoint of P_0P_1, the line between the initial and final location of the front wheel. So, an efficient implementation of this scheme leads to the prediction algorithm, which is also adopted by AutoTrack. To obtain the new position of the rear wheel, first of all, the midpoint of the line P_0P_1 is calculated. Then draw a line connecting this point to the old position of rear wheel, and the new position of the rear wheel should be located along this line. The exact position of it is the intersection of this line with the circle formed by the rear wheel. Its position can be easily determined by calculating the turned angle β. To compare the results obtained from this prediction algorithm, we applied the same parameters and constrains to TURN.LSP.

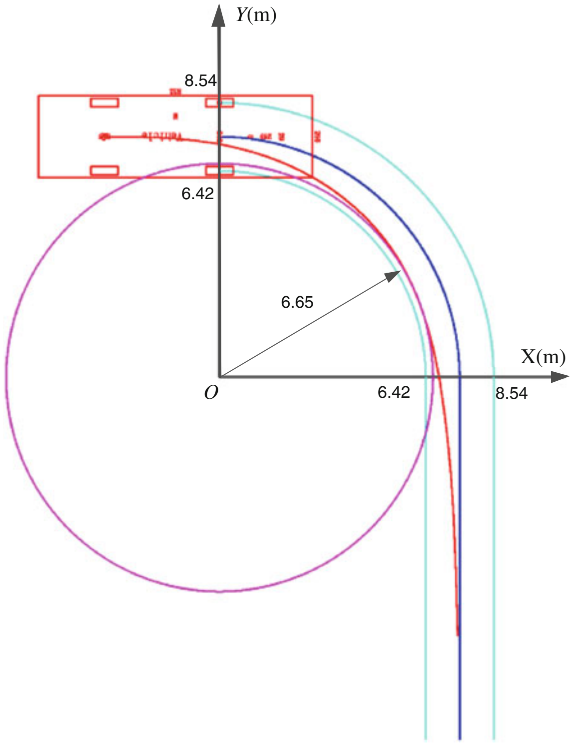


Fig. 10 The trajectory of rear midpoint using TURN.LSP

In AutoCAD we created a block to represent the long vehicle, and default settings were used for computing. Finally the path of the midpoint of the rear axle was generated, as shown in Fig. 10. It is clear that a segment of this curve has the same osculating circle at different positions, which indicates the inner circle formed by the rear midpoint. Here the radius of the inner circle is 6.57, which is different from the prior result due to different algorithms applied.

Because of the assumptions of circular motion and equal turning angle for each step, it is an approximate approach and only deals with constant velocity problem, thus does not reflect the actual situation. However, it is efficient for computation and effective for simulation and can provide visualization for trajectory planning with no strict requirements.

2.5 Reversing of Long Vehicles

In some cases, the vehicle needs to be driven in the reverse direction. Reversing a vehicle serves purposes of changing lanes in traffic, parking, etc. So we require the rear wheels to follow a predefined path instead of steering the vehicle with its minimum turning radius. Now the question becomes how to determine the trajectories of front wheels and other parts of the vehicle with the trajectories of rear wheels given. With no slippage and other similar assumptions, it also follows the three principles described in Sect. 2.

$$\begin{aligned} \frac{y_r'}{x_r'} &= \frac{y_f - y_r}{x_f - x_r} \\ L^2 &= (x_f - x_r)^2 + (y_f - y_r)^2 \end{aligned} \tag{11}$$

By solving these two equations with two unknowns, x_f and y_f, the coordinates of front midpoint can be determined. In this case it does not need any initial conditions because there is a one-to-one correspondence between the coordinates of rear midpoints and those of front midpoints with the direction of the movement is supposed. Once the trajectories of rear midpoint determined, other related trajectories of "feature points" can also be obtained using the same method as the calculating the trajectories of the crane.

3 Application: Mobile Crane

As an application of trajectory planning and simulation for long vehicles, the trajectory of a two-axle mobile crane is analyzed when it is turning around a corner with 90°. The specific model adopted here is LTM1040-2.1, which is made by Liebherr and has all-terrain capability, shown in Fig. 11.

In order to determine whether the mobile crane is allowed to pass through this area and whether the collision will happen throughout the turning process, we set

Fig. 11 Liebherr mobile crane LTM1040-2.1

the front midpoint P moving along the smallest circular turn that the mobile crane is capable of making, i.e., the turning circle. According to the technical data sheet, the turning circle of LTM1040-2.1 has a radius $R = 7.48$ m, with its body length $L = 3.58$ m.

Here, the initial conditions are $x_r(0) = -\mathrm{L}$ and $y_r(0) = 0$. Considering that the path of P consists of two parts: circular motion and linear motion, the simulation can be done in two steps, as Fig. 12 shows. The final trajectory is obtained by combining two parts. The radius of the inner circle can be easily determined by checking the data acquired in the second step. Then the radius equals 6.57 m, which is consistent with the geometric relationships.

It is not enough to determine the practical collision situations given only the trajectories of two midpoints of the axles. Ideally, if we know the trajectory profile or envelope of trajectories of the outer outline of the vehicle body when it moves along the predefined path, it is very easy and intuitive to get detailed information and make corresponding decisions. We can choose several "feature points" along the outline, as Fig. 13 shows. These points are chosen because they have a determinant influence on formation of the trajectory envelope. By calculating the trajectories of these points, we can draw a trajectory profile, which can be used to show if the collision will happen by check whether there are intersections between the trajectory profile and the surrounding objects.

Here, the sketch of the mobile crane bottom view just serves as an example to explain how it works, thus not necessarily accurate. We choose two positions at the boom head A, B, and other two positions representing the corners of rear end C, D. Also, more points can be chosen for further precise calculation and verification. The trajectories of these points can be calculated once a series of positions of Q are known. Dimensions marked in Fig. 13 is the geometric relationships between these points and rear midpoint Q. Figure 14 gives us a visualized description of trajectories formed by these chosen points throughout the steering process. According to the diagram, trajectories of point A and D determine the collision situation, and these two lines form an area representing the path swept by the

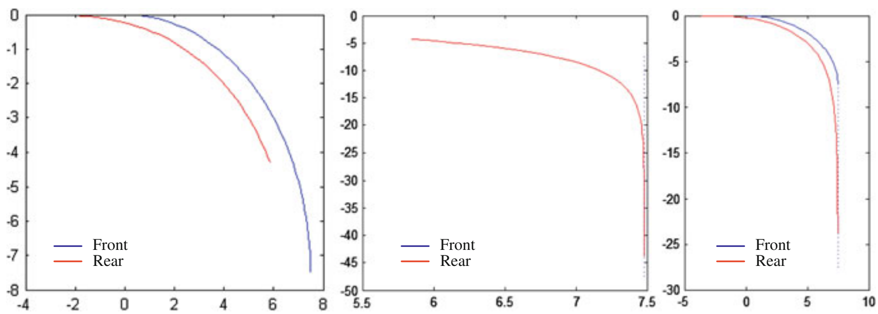


Fig. 12 Numerical simulation of the trajectory

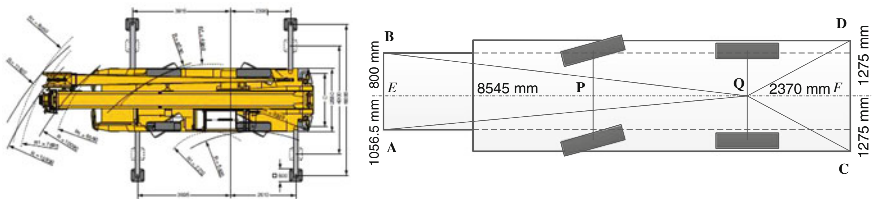
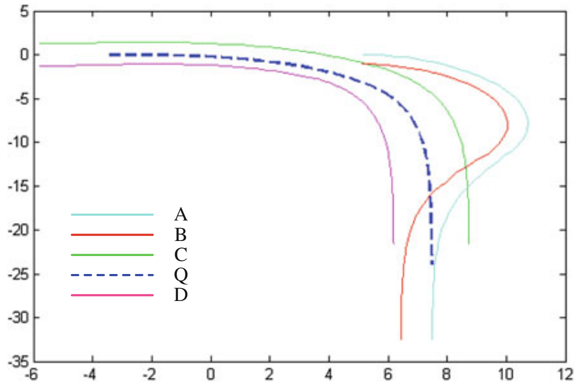


Fig. 13 Several important dimensions of the mobile crane

Fig. 14 Trajectories of several feature points



vehicle body. If the surrounding objects, such as barriers, walls, curbs, etc., have intersections with the area between these two lines, the crane will have difficulties passing through the area safely.

4 3D Simulation

For a vehicle with front wheel steering system, the turning sequence around a corner with 90° can be described in Fig. 15.

When the vehicle approaches a corner, it will steer near the outer edge of the incoming road in order to take advantage of the road width. For a long vehicle, the tail out-swing could be significant and this clearance should be consider to prevent collision at the tail of the vehicle. This clearance can be calculated from the vehicle geometry against its wheel position at minimum turning radius.

The position where vehicle should start to steer depends on the front clearance of the vehicle with the outer edge of the outgoing road. There should be a clearance to avoid any collision at the front edge of the vehicle with the surrounding objects. At this point, check should be made whether the inner wheel of the vehicle is able to clear the inner road corner, and this factor is dependent on the width of the outgoing road. With the geometric data of the vehicle and the terrain, the position where driver should start to steer can be calculated.

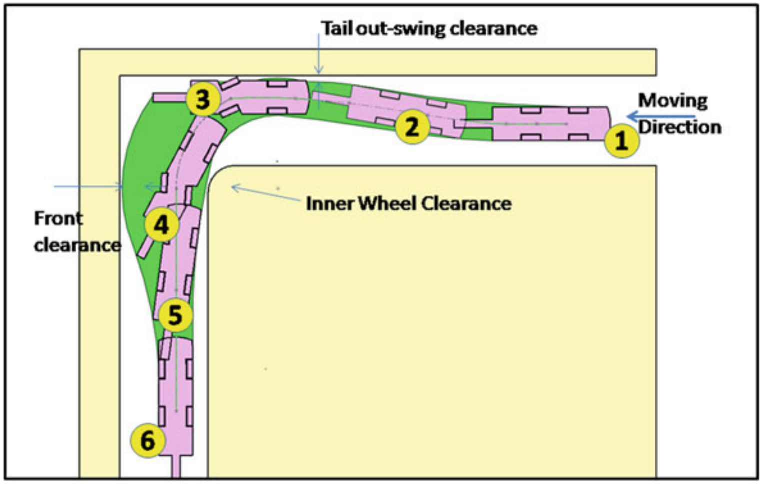


Fig. 15 The turning sequence around a corner with 90°

After the corner, the front steering is release to follow the direction of the outgoing road. The rear wheels will follow the path as discussed earlier in Sect. 2.

The swept area required for the vehicle to make through a corner can be determined from the following:

(1) The vehicle geometry (size, shape, axle centers);
(2) The minimum turning radius of the vehicle (i.e., the maximum steering angle of front wheels);
(3) The rear wheel path of the vehicle, which can be calculated for front wheel steering vehicle. For all wheel steering vehicles, the wheel path solely depends on the driver input and the control system of the vehicle.

Combining this swept area with the road terrain and optimizing with the required clearances as discussed earlier, the possibility of collision can be determined. If there is no overlap, the vehicle is able to turn around the corner. Any overlaps will mean that the vehicle is not able to make through the corner with the given surroundings.

For the mobile crane model adopted in the study, the swept area can be determined by using geometric parameters. Figure 16 illustrates the swept area by the mobile crane under two situations: front wheel steering and all wheel steering.

With the mathematical model described, the problem can be simulated using 3D CAD software with the following input:

(1) The geometry of the mobile crane,
(2) The terrain where the vehicle to be operated, and
(3) The desired path determined with the mathematical mode.

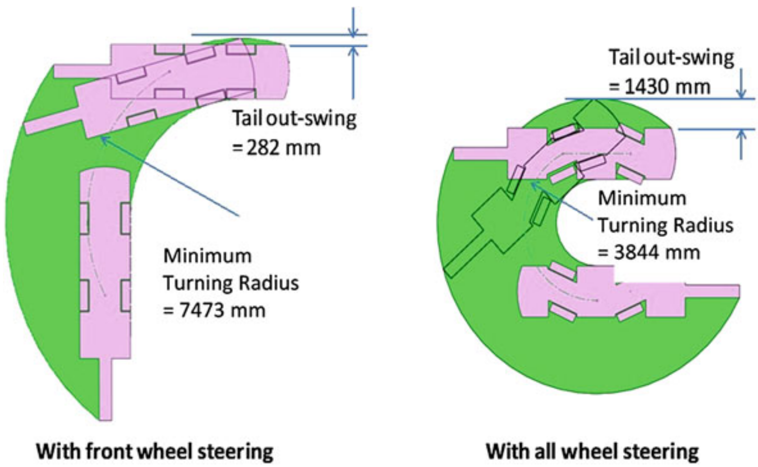


Fig. 16 Maximum displacement of the tail

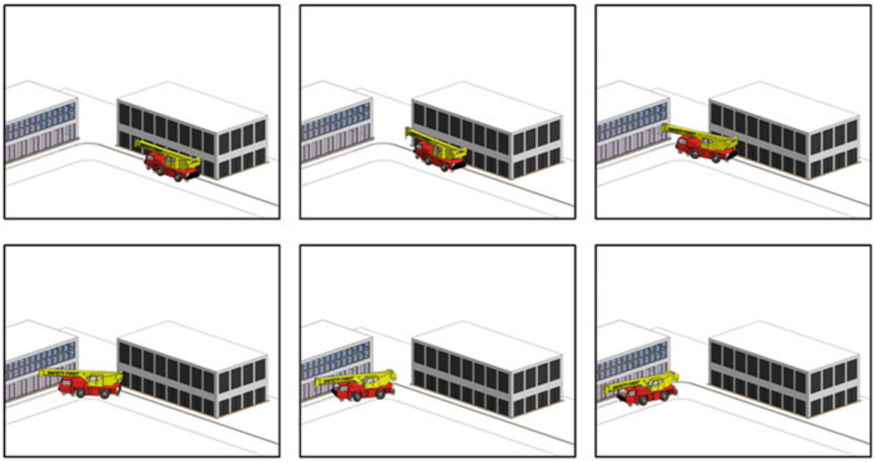


Fig. 17 Simulation of the mobile crane turning around a corner

Two examples of simulation are illustrated with the following figures. Figure 17 is the 3D simulation of the steering process around a corner with 90°. Here there are two buildings serving as the objects. However, in a real world situation, the operator probably encounters more complex environments, scattered with protective edges, barriers, peripheral walls, other vehicles, etc. Figure 18 illustrates the process of maneuvering the mobile crane out of the parking lot. Also, this simulation procedure is reversible and can be used to indicate the whole process of parking the crane.

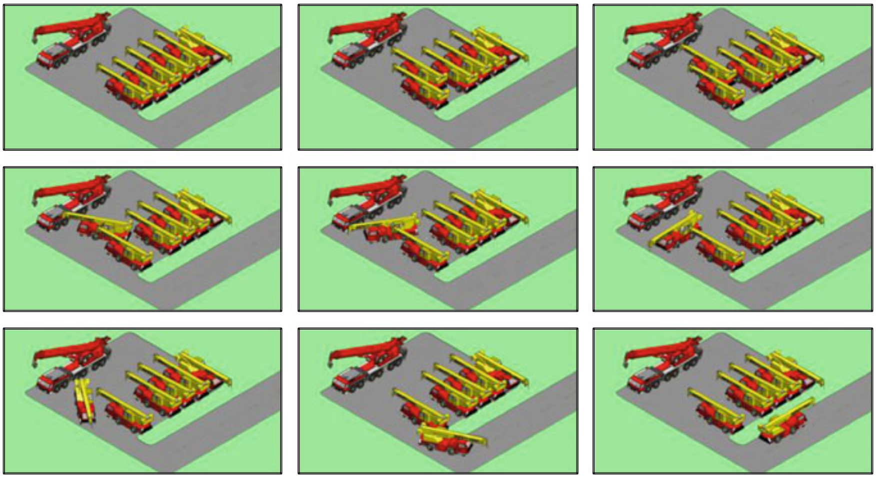


Fig. 18 Maneuvering of mobile crane in parking lot

5 Conclusion

In this chapter, through investigating different mathematical methods for vehicle trajectory calculation, we develop a systematic scheme for modeling long vehicle turning and trajectory calculation using geometrical parameters. Based on the data acquired, 2D numerical analysis and 3D simulation of long vehicles in different environments are conducted to provide accurate and intuitive information for trajectory planning. Also, by comparing with exist swept path analysis method, it shows a better accuracy and agree with the theoretical results, though less time efficient. Considering more realistic factors such as the slippage of the wheel and the road-tire contact, further studies will be carried out to provide more reliable, interactive simulation, as well as optimizing the computations.

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