

A Hybrid ANP-DEA Approach for Vulnerability Assessment in Water Supply System

C. Zhang and X. Liu

Abstract Vulnerability reflects the potential of disrupting the whole system to some extent when the system is exposed to hazard. One of the most important issues of the indicator-based vulnerability assessment problem is to determine the weights of vulnerability indicators, especially when they are correlated with each other in multiple dimensions (i.e., physical, functional and organizational). In this paper, a framework for assessing vulnerability of critical infrastructure system is identified and applied to the evaluation in a water supply system. A complete critical infrastructure system vulnerability index is developed, which contains dimensions of “protection and defense”, “quick response after disaster”, “maintenance and recovery capacity” and “possible damage to system”. A quantitative method, integrating analytic network process (ANP) and game cross-efficiency data envelopment analysis (DEA) model, is proposed to analyze the vulnerability of interdependent infrastructures. Finally, the assessed vulnerability level of each infrastructure in water supply system is graded into four classes.

Keywords Vulnerability · Assessment · Data envelopment analysis · Analytic network process

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1 Introduction

Since the interconnections (both physical and logical) between modern infrastructures become more complex, lifeline systems are more vulnerable to disasters. So it is important for the government to select critical elements of the system and protect them with highest priority. However, how to assess the vulnerability of these elements is still a huge challenge for researchers.

Vulnerability is an important attribute of critical infrastructure systems. The concept of vulnerability is still evolving and has not yet been established. Different researchers have different definitions on vulnerability. TurnerII et al. (2003) defines it as the degree to which human and environmental systems are likely to experience harm due to a perturbation or stress, Aven (2011) defines it as the manifestation of the inherent states of the system that can be subjected to a natural hazard or be exploited to adversely affect that system. Aggregating the existing definitions (TurnerII et al. 2003; Barbat and Carreño 2010), we define vulnerability in this study as follows: vulnerability reflects the potential of disrupting the whole system to some extent when the system is exposed to hazard.

There are various approaches for characterizing vulnerability. The first one is a multi-dimensional indicator framework which needs weights assignment, such as expert decision and analytic network process (ANP) (Aven 2011; Grubestic and Matisziw 2007; Piwowar et al. 2009). The second one is network modeling approaches (Qiao et al. 2007; Scaparra and Church 2008a, b), which is mainly based on the network topology of infrastructures, such as maximal flow model, shortest path model and network flow model. The third one is probabilistic modeling (Sultana and Chen 2009; Doguc and Ramirez-Marquez 2009), which is usually used when analyzing inter-dependencies and cascading failures.

In this paper, a hybrid ANP and Cross-Efficiency DEA approach is proposed to solve this multi-criteria assessment problem. The remainder of this paper is organized as follows. In Sect. 2, the indicator system for vulnerability evaluation and value functions of selected indicators are described. Section 3 presents the proposed hybrid ANP-DEA approach for vulnerability assessment. Section 4 illustrates the proposed evaluation framework via a case study. Finally Sect. 5 concludes this paper.

2 Indicators for Vulnerability Assessment

Four dimensions are considered for constructing the vulnerability index in this study: (1) Protection and Defense (reflecting the exposure of system to disasters); (2) Quick Response after disaster; (3) Maintenance and recovery capacity; (4)

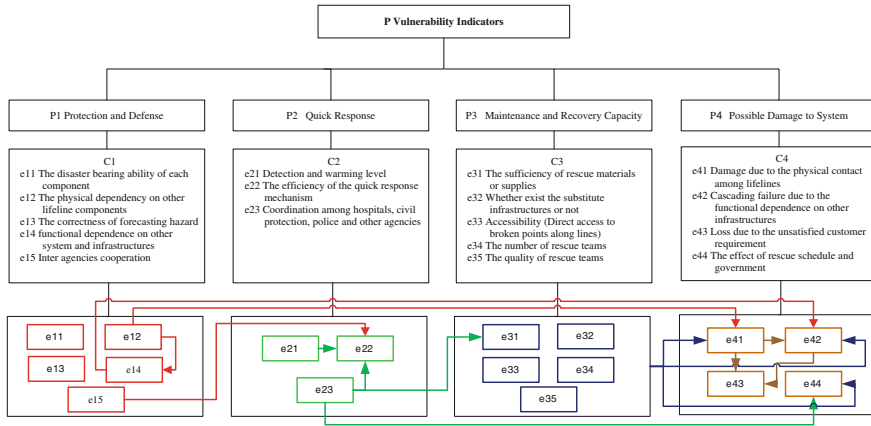


Fig. 1 The ANP decision structure for vulnerability evaluation

Possible damage to system (Ezell 2007). The vulnerability indicators are demonstrated in Fig. 1.

Values of qualitative indicators are acquired by ranking or categorizing performed by experts. For clearness, selected parameters need to be explained especially. “Functional or logical interdependency” values are obtained from the correlation matrices of infrastructures and “the sufficiency of rescue materials” are calculated using the following function: “the realistic storage of rescue materials/ the expected storage of rescue materials”.

3 The Proposed Methodology

The vulnerability evaluation processes are shown in Fig. 2.

3.1 Bounds of weights for each indicator by ANP

Variables:

- 1) CI_i ($i = 1, 2, \dots, n$) means the i th in infrastructure to be evaluated;
- 2) The total number of DMs (Decision makers) involved in the evaluation is K and $k = 1, 2, \dots, K$ expresses the k th expert;
- 3) The weight of the i th cluster, the j th element of the k th expert is represented by \hat{w}_{ijk} ($i = 1, 2, \dots, 4, j = 1, 2, \dots, 4, k = 1, 2, \dots, K$).

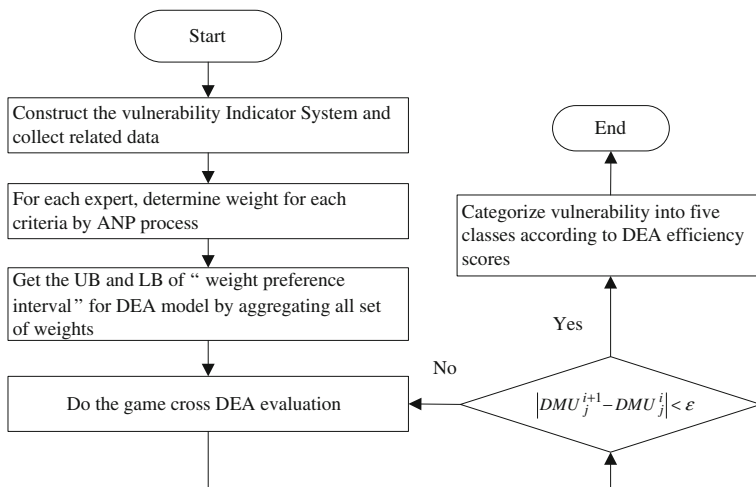


Fig. 2 Hybrid ANP-DEA vulnerability evaluation process

Analytic network process (ANP) is used for determining the upper and lower limit of weights. It is a relatively simple and systematic approach that can be used by decision makers (Khadivi and Fatemi Ghomi 2012). It allows both interaction and feedback within clusters of elements and between clusters. Such feedback well captures the complex effects of interplay in human society, especially when risk and uncertainty are involved. The Decision structure of this process is shown as Fig. 1 and Super Decision 1.6.0 can be used to complete the ANP process for each expert and get the weight of each parameter, referred to as \hat{w}_{ijk} ($i = 1, 2, \dots, 4, j = 1, 2, \dots, 4, k = 1, 2, \dots, K$).

Then, to eliminate the impact of subject bias of single expert, weights from all DMs should be aggregated. By calculating the mean value \bar{w}_{ij} and variance E_{ij}^2 , the LB and UB of weight interval can be given, as is shown in Eq. 1.

$$\hat{w}_{ij} \in [\bar{w}_{ij} - 1.96\delta_{ij}, \bar{w}_{ij} + 1.96\delta_{ij}] = [\hat{w}_{ijmin}, \hat{w}_{ijmax}] \tag{1}$$

3.2 DEA Game Cross-Efficiency Model

Charnes et al. (1978) proposed the initial DEA model (CCR model) for evaluating the relative efficiencies of a set of decision making units (DMUs). The application of DEA as an alternative multi-criteria decision making (MCDM) tool has been

gaining more attentions in the literatures because it can find optimal weights for all relevant inputs and outputs of each DMU in a relatively objective and fair way (Wu et al. 2009). The DEA game cross-efficiency model, which was proposed by Wu and Liang (2012), can obtain unique game cross-evaluation scores which constitute a Nash equilibrium point. This model is used for evaluating the vulnerability index here.

In the model, the input and output indicators are represented by m_i and s_j respectively. The indicators which have positive correlations with vulnerability are defined as “output” while others are defined as “input”. The i th input and r th output of DMU_j are represented as x_{ij} and y_{rj} respectively.

The steps of this DEA game cross-efficiency method are listed as follows.

Step 1: Add the bound of weights to the initial CCR model as constraints and solve the model.

The bounds of input and output indicators in DEA model are represented by $[w_{imin}, w_{imax}]$ and $[\mu_{imin}, \mu_{imax}]$ separately, and the constraint set of each indicator in this step is derived from LB and UB. The game cross DEA model is illustrated as model 2. Let $t = 1, \alpha_j = \alpha_j^1 = \bar{E}_j$, and solve the model.

$$\begin{aligned}
 &max \sum_{r=1}^s \mu_r y_{rd} = \theta_d \\
 &s.t \sum_{i=1}^m w_{ij}^d x_{id} - \sum_{r=1}^s \mu_{rj}^d y_{rd} \geq 0 \quad l = 1, 2, \dots, n \\
 &\sum_{i=1}^m \omega_i x_{id} = 1, \\
 &w_i - \frac{w_{imin}}{w_{lmin}} w_1 \geq 0, \quad w_i - \frac{w_{imax}}{w_{lmax}} w_1 \leq 0, \\
 &\mu_i - \frac{\mu_{imin}}{\mu_{lmin}} \mu_1 \geq 0, \quad \mu_i - \frac{\mu_{imax}}{\mu_{lmax}} \mu_1 \leq 0, \\
 &\omega_i \geq 0, \quad i = 1, 2, \dots, m, \quad \mu_r \geq 0, \quad r = 1, 2, \dots, s.
 \end{aligned} \tag{2}$$

Model 2 is solved n times (one for each d) for each alternative i , thus each DMU corresponds to a set of optimal weights: $w^*_{1d}, w^*_{2d}, \dots, w^*_{md}, \mu^*_{1d}, \mu^*_{2d}, \dots, \mu^*_{sd}$. Calculate the game cross-evaluation score for each DMU by Eq. 3.

$$\bar{E}_j = \frac{1}{n} \sum_{d=1}^n E_{dj} = \frac{1}{n} \sum_{d=1}^n \left(\frac{\sum_{r=1}^s \mu_{rd}^* y_{rj}}{\sum_{i=1}^m w_{id}^* x_{ij}} \right) \quad d, j = 1, 2, \dots, n \tag{3}$$

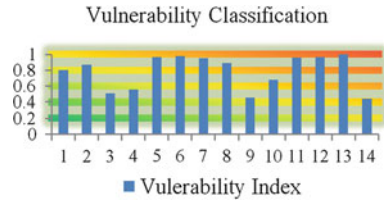
Table 1 Organizational structure of water system

Organizational structure of water system		Transmit		Treat and store		Distribution		Control					
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Raw water resources	Reservoir	Power supplies	Pipelines	Power supplies	Tanks	Settling pond	Facilities	Power supplies	Distribution network	Valves	Pump station	Power supplies	SCADA

Table 2 Score of vulnerability evaluation

Infrastructure no.	1	2	3	4	5	6	7
Vulnerability index	0.805	0.870	0.506	0.557	0.966	0.983	0.955
Infrastructure no.	8	9	10	11	12	13	14
Vulnerability index	0.893	0.456	0.680	0.958	0.964	1	0.442

Fig. 3 The grade of infrastructure



Step 2: Add a new constraint, as is shown in Eq. 4, to the DEA model and solve this model again. Then, define $\alpha_j^{t+1} = \frac{1}{n} \sum_{d=1}^n \sum_{r=1}^s \mu_{rj}^{d*}(\alpha_d^t) y_{rj}$, where $\mu_{rj}^{d*}(\alpha_d^t)$ represents the optimal value of μ_{rj}^d .

$$\alpha_d \times \sum_{i=1}^m w_{ij}^d x_{id} - \sum_{r=1}^s \mu_{rj}^d y_{rd} \leq 0 \tag{4}$$

Step 3: If $|\alpha_j^{t+1} - \alpha_j^t| \geq \varepsilon$ for some j , where ε is a specified small positive value, then let $\alpha_j^t = \alpha_j^{t+1}$ and return to Step 2, else if $|\alpha_j^{t+1} - \alpha_j^t| < \varepsilon$ for all j , then stop and α_j^{t+1} is the final game cross-evaluation score given to DMU_j . Finally rank the vulnerability according to the final efficiency scores.

4 Numerical Example

In this section, we examine a numerical example using the above hybrid ANP-DEA method to illustrate its validity in evaluating vulnerability for water supply system. Assume that City X locates at earthquake-prone regions, now the vulnerability of its water system against earthquake need to be assessed for prevention of disasters. As is shown in Table 1, the organizational structure of water system is composed of 14 infrastructures which need to be evaluated.

The data and specific calculation process are omitted due to the limited space, only the final DEA efficiency score of each DMU is presented in Table 2.

During the process, two virtual infrastructures, which are used as the benchmark for the most and the least vulnerable infrastructures are introduced into the analysis. In this study, the UB and LB of the vulnerability index are 1 and 0.2 and accordingly, the vulnerability is categorized into four levels. Figure 3 shows the ultimate ranking of each infrastructure.

5 Conclusions

A hybrid ANP-DEA approach for infrastructure vulnerability assessment is proposed in this paper. The difficulty of infrastructure vulnerability evaluation mainly comes from the complexity of lifeline system and the multi-dimensional nature of vulnerability. The incorporation of ANP and DEA game cross efficiency method make the evaluation more objective and fair. Since disasters are always dynamic process and evaluation indicators may keep changing with the evolvement of disasters, the suggestion for future researches is that a dynamic model can be considered for treating the uncertainties and dynamics.

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References

- Aven T (2011) On some recent definitions and analysis frameworks for risk, vulnerability, and resilience. *Risk Anal* 31:515–522
- Barbat AH, Carreño ML (2010) Seismic vulnerability and risk evaluation methods for urban areas. A review with application to pilot area. *Struct Infrastruct Eng* 6:17–19
- Charnes A, Cooper WW, Rhodes E (1978) Measuring the efficiency of decision-making units. *Eur J Oper Res* 2:429–444
- Doguc O, Ramirez-Marquez JE (2009) A generic method for estimating system reliability using Bayesian networks. *Reliab Eng Syst Saf* 94:542–550
- Ezell BC (2007) Infrastructure vulnerability assessment model (I-VAM). *Risk Anal* 27(3):571–583
- Grubestic TH, Matisziw TC (2007) A typological framework for categorizing infrastructure vulnerability. *GeoJournal* 78:278–301
- Khadivi MR, Fatemi Ghomi SMT (2012) Solid waste facilities location using of analytical network process and data envelopment analysis approaches. *Waste Manage (Oxford)* 32:1258–1265
- Piwowar J, Châtelet E, Laclémence P (2009) An efficient process to reduce infrastructure vulnerabilities facing malevolence. *Reliab Eng Syst Saf* 94:1869–1877
- Qiao J, Jeong D, Lawley M, Richard JPP, Abraham DM, Yih Y (2007) Allocating security resources to a water supply network. *IIIE Trans* 39:95–109
- Scaparra MP, Church RL (2008a) A bilevel mixed-integer program for critical infrastructure protection planning. *Comput Oper Res* 35:1905–1923

- Scaparra MP, Church RL (2008b) An exact solution approach for the interdiction median problem with fortification. *Eur J Oper Res* 189:76–92
- Sultana S, Chen Z (2009) Modeling flood induced interdependencies among hydroelectricity generating infrastructures. *J Environ Manage* 90:3272–3282
- TurnerII BL, Kasperson RE et al (2003) A framework for vulnerability analysis in sustainability science. *Proc Natl Acad Sci USA* 100:8074–8079
- Wu J, Liang L (2012) A multiple criteria ranking method based on game cross-evaluation approach. *Ann Oper Res* 197:191–200
- Wu J, Liang L, Chen Y (2009) DEA game cross-efficiency approach to Olympic rankings. *Omega* 34:909–918