The Joint Determination of Optimum Process Mean, Economic Order Quantity, and Production Run Length

Chung-Ho Chen

Abstract In this study, the author proposes a modified Chen and Liu's model with quality loss and single sampling rectifying inspection plan. Assume that the retailer's order quantity is concerned with the manufacturer's product quality and the quality characteristic of product is normally distributed. Taguchi's symmetric quadratic quality loss function will be applied in evaluating the product quality. The optimal retailer's order quantity and the manufacturer's process mean and production run length will be jointly determined by maximizing the expected total profit of society including the manufacturer and the retailer.

Keywords Economic order quantity · Process mean · Production run length · Taguchi's quadratic quality loss function

1 Introduction

The supply chain system is a major topic for the manufacturing industries in order to obtain the maximum expected total profit of society including the manufacturer and the retailer. The manufacturer's objective needs to consider the sale revenue, the manufacturing cost, the inspection cost, and the inventory cost for having the maximum expected profit. The retailer's objective needs to consider the order quantity, the holding cost, the goodwill loss of cost, and the used cost of customer for having the maximum expected profit. How to get a trade-off between them should be available for further study. Chen and Liu (2007) presented the optimum profit model between the producers and the purchasers for the supply chain system with pure procurement policy from the regular supplier and mixed procurement

C.-H. Chen (✉)
Department of Management and Information Technology, Southern Taiwan University of Science and Technology, Tainan, Taiwan
e-mail: chench@mail.stust.edu.tw

Y.-K. Lin et al. (eds.), *Proceedings of the Institute of Industrial Engineers Asian Conference 2013*, DOI: 10.1007/978-981-4451-98-7_141,

policy from the regular supplier and the spot market. Chen and Liu (2008) further proposed an optimal consignment policy considering a fixed fee and a per-unit commission. Their model determines a higher manufacturer's profit than the traditional production system and coordinates the retailer to obtain a large supply chain profit.

In Chen and Liu's (2008) model with traditional production system, they neglected the effect of product quality on the retailer's order quantity and only considered the order quantity obeying the uniform distribution. In fact, the retailer's order quantity is concerned with product quality. Chen and Liu's (2008) model with simple manufacturing cost did not consider the used cost of customers in traditional production system. Hence, the modified Chen and Liu's (2008) model needs to be addressed for determining the optimum process parameters. Chen (2010) proposed a modified Chen and Liu's (2008) model with quality loss and single sampling plan based on the dependent assumption of the retailer's order quantity and manufacturer's product quality. However, Chen (2010) neglected manufacturer's inventory cost and the cost for the non-conforming products in the sample of accepted lot.

Hanna and Jobe (1996) discussed the quality characteristic of product and quality cost on the effect of lot size. They determine the optimal order quantity lot when the model with quality cost evaluation based on 100 % inspection, sampling inspection, and no inspection for products. Jaber et al. (2009) considered entropic order quantity model when product characteristic is not perfect. Their results suggested that larger quantities should be ordered than those of the classical economic order quantity model. Economic selection of process mean is an important problem for modern statistical process control. It will affect the expected profit/cost per item. Recently, many researchers have addressed this work. Both 100 % inspection and sampling inspection are considered for different models. Taguchi (1986) presented the quadratic quality loss function for redefining the product quality. Hence, the optimum product quality should be the quality characteristic with minimum bias and variance. Recently, his quality loss function has been successfully applied in the problem of optimum process mean setting.

In this paper, the work will propose a modified Chen and Liu's (2008) model with quality loss, manufacturer's inventory cost, and single sampling rectifying inspection plan. Assume that the retailer's order quantity is concerned with the manufacturer's product quality and the quality characteristic of product is normally distributed. The non-conforming products in the sample of accepted lot are replaced by conforming ones. If the lot is rejected, then all of the products are rectified and sold at the same price as the products of accepted lot. Taguchi's (1986) symmetric quadratic quality loss function will be applied in evaluating the product quality. The optimal retailer's order quantity and the manufacturer's process mean and production run length will be jointly determined by maximizing the expected total profit of society including the manufacturer and the retailer. The motivation behind this work stems from the fact that the neglect of the quality loss within the specification limits and manufacturer's inventory cost should have the overestimated expected total profit of society.

2 Modified Chen and Liu's (2008) Traditional System Model

Taguchi (1986) redefined the product quality as the loss of society when the product is sold to the customer for use. The used product for customer maybe occur a lot of costs including maintenance, safety, pollution, and sale service. Chen and Liu's (2008) model also did not consider the used cost of customers. The neglect of the quality loss within the specification limits should have the overestimated expected profit per item for the retailer.

Assume that the quality characteristic of Y is normally distributed with unknown mean $\mu_y$and known standard deviation σ_y, i.e., $Y \sim N\left(\mu_y, \sigma_y^2\right)$ and $X|Y \sim N(\lambda_1 + \lambda_2 y, \sigma^2)$, where λ_1, λ_2, and σ^2 are constants. Hence, we have $X \sim N\left(\lambda_1 + \lambda_2\mu_y, \lambda_2^2\sigma_y^2 + \sigma^2\right)$ and $Y|X \sim N\left(\frac{\lambda_2\sigma_y^2(x-\lambda_1)+\mu_y\sigma^2}{\lambda_2^2\sigma_y^2+\sigma^2}, \frac{\sigma_y^2\sigma^2}{\lambda_2^2\sigma_y^2+\sigma^2}\right)$.

Taguchi (1986) proposed the quadratic quality loss function for evaluating the product quality. If the product quality characteristic is on the target value, then it has the optimum output value. However, we need to input some different resource in the production process. Hence, the process control needs to obtain minimum bias and variance for output product. According to Taguchi's (1986) definition for product quality, the retailer's expected profit should subtract the used cost of customer for product in order to avoid overestimating retailer's expected profit. Hence, the author proposes the following modified Chen and Liu's (2008) model.

The retailer's profit is given by

$$\pi_{PS}^{R} = \begin{cases} RX - WQ - H(Q - X) - X \cdot Loss(Y), & X <; Q, \ -\infty <; Y <; \infty \\ RQ - WQ - S(X - Q) \cdot Loss(Y), & X \geq Q, \ -\infty <; Y <; \infty \end{cases} \tag{1}$$

where X is the consumer demand which is an uniform distribution, $X \sim U[\mu_x - (\sigma_x/2), \mu_x + (\sigma_x/2)]$, μ_x is the mean of X, σ_x is the variability of X, and $f(x)$ is the probability distribution of X; R is a retailer purchasing a finished product from a regular supplier and reselling it at this price to the end customer; C is the regular manufacturer produces each unit at this cost; W is the regular manufacturer and the retailer entering into a contract at this wholesale price; Q is the regular manufacturer setting the wholesale price to maximize his expected profit while offering the buyer this specific order quantity; S is a goodwill loss for the retailer when realized demand exceeds procurement quantity; H is a carrying cost for the retailer when realized demand is less than procurement quantity; Y is the normal quality characteristic of product, $Y \sim N\left(\mu_y, \sigma_y^2\right)$; μ_y is the unknown mean of Y; σ_y is the known standard deviation of Y; $Loss(Y)$ is Taguchi's (1986) quadratic quality loss function per unit, $Loss(Y) = k(Y - y_0)^2$; k is the quality loss coefficient; y_0 is the target value of product.

The retailer's expected profit includes the sale profit when the demand quantity of customer is less than order quantity, the sale profit when the demand quantity of customer is greater than order quantity, the carrying cost when the demand quantity of customer is less than order quantity, and the goodwill loss when the demand quantity of customer is greater than order quantity. From Chen (2010), we have the expected profit of retailer as follows:

$$E\left(\pi_{PS}^{R}\right) = E(\pi_1) + E(\pi_2) - E(\pi_3) - E(\pi_4) \tag{2}$$

where

$$E(\pi_1) = (R+H)\left\{\mu_k \Phi\left(\frac{Q-\mu_k}{\sigma_k}\right) - \sigma_k \phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right\} - (W+H)Q\Phi\left(\frac{Q-\mu_k}{\sigma_k}\right) \tag{3}$$

$$\begin{aligned} E(\pi_2) =& (R-W+S)Q\left[1-\Phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right] \\ & - S\left\{\mu_k[1-\Phi\left(\frac{Q-\mu_k}{\sigma_k}\right)] + \sigma_k \phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right\} \end{aligned} \tag{4}$$

$$\begin{aligned} E(\pi_3) = & kA^2\left\{\mu_k^3 \Phi\left(\frac{Q-\mu_k}{\sigma_k}\right) + 3\mu_k^2 \sigma_k\left[-\phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right]\right. \\ & + 3\mu_k \sigma_k^2\left[-\frac{Q-\mu_k}{\sigma_k} \cdot \phi\left(\frac{Q-\mu_k}{\sigma_k}\right) + \Phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right] + \sigma_k^3\left[-\left(\frac{Q-\mu_k}{\sigma_k}\right)^2 \cdot \phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right. \\ & \left.\left. -2\phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right\} + 2kAB\left\{\mu_k^2\left[\Phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right] - 2\mu_k \sigma_k \phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right. \\ & \left. + \sigma_k^2\{-\frac{Q-\mu_k}{\sigma_k}\phi(\frac{Q-\mu_k}{\sigma_k})\}\right. + k\left(B^2+C_0\right)\left\{\mu_k \Phi\left(\frac{Q-\mu_k}{\sigma_k}\right) - \sigma_k \phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right\} \end{aligned} \tag{5}$$

$$\begin{aligned} E(\pi_4) =& kA^2 Q\left\{\mu_k^2\left[1-\Phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right] + 2\mu_k \sigma_k \phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right. \\ & \left. + \sigma_k^2\left\{\left[\left(\frac{Q-\mu_k}{\sigma_k}\right)\phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right] + \left[1-\Phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right]\right\}\right\} \\ & + 2kQAB\left\{\mu_k\left[1-\Phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right] + \sigma_k \phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right\} \\ & + kQ\left(B^2+C_0\right)\left\{1-\Phi\left(\frac{Q-\mu_k}{\sigma_k}\right)\right\} \end{aligned} \tag{6}$$

where $\mu_k = \lambda_1 + \lambda_2 \cdot \mu_y$; $\sigma_k = \sqrt{\lambda_2^2 \sigma_y^2 + \sigma^2}$; $A = \frac{\lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}$; $B = \frac{\mu_y \sigma^2 - \lambda_1 \lambda_2^2 \sigma_y^2}{\lambda_2^2 \sigma_y^2 + \sigma^2} - y_0$; $C_0 = \frac{\sigma_y^2 \sigma^2}{\lambda_2^2 \sigma_y^2 + \sigma^2}$; $\Phi(\cdot)$ is the cumulative distribution function of standard normal random variable; $\phi(\cdot)$ is the probability density function of standard normal random variable.

Assume that the retailer's order quantity is equal to the lot size of single sampling rectifying inspection plan. If the lot is accepted, then the selling price of product per unit is W. The non-conforming products in the sample of accepted lot are replaced by conforming ones. Let R_I denote the cost of replacing a defective item by an acceptable item in the accepted lot. If the lot is rejected, then all of the products are rectified and sold at a price W. Let R_L denote the expected cost of replacing all rejected items found in a rejected lot. Hence, the manufacturer's profit under adopting single rectifying inspection plan for determining the quality of product lot is given by

$$\pi_{PS}^{S} = \begin{cases} WQ - ni - DR_I - Qc\mu_y, & D \le d_0 \\ WQ - Qi - R_L - Qc\mu_y, & D > d_0 \end{cases} \tag{7}$$

where n is the sample size; c is the variable production cost per unit; i is the inspection cost per unit; d_0 is the acceptance number; D is the number of non-conformance in the sample; c is the cost of processing per unit; i is the inspection cost per unit; $R_L = R_I \cdot d_{rl}; d_{rl}$ is the expected number of defective items in a rejected lot (= the expected number of defectives found in the sample, given that the lot was rejected + the expected number of defectives in the non-sample portion of the lot),

$d_{rl} = E(D|D > d_0) + p(Q - n) \quad E(D|D > d_0) = \dfrac{np\left[1 - \sum_{x=0}^{d_0-1} \frac{e^{-np}(np)^x}{x!}\right]}{1 - \sum_{x=0}^{d_0} \frac{e^{-np}(np)^x}{x!}}$; p is the probability of a defective item$\left(= 1 - \left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]\right)$; L is the lower specification limit of product; U is the upper specification limit of product; $\Phi(\cdot)$is the cumulative distribution function of the standard normal random variable.

The manufacturer's expected profit for the product lot is

$$\begin{aligned} E_1\left(\pi_{PS}^{S}\right) &= \left(WQ - ni - DR_I - Qc\mu_y\right)P_1 + \left(WQ - Qi - R_L - Qc\mu_y\right)(1 - P_1) \\ &= [R_L + (Q - n)i]P_1 - R_I npP_0 + \left(WQ - R_L - Qi - Qc\mu_y\right) \end{aligned} \tag{8}$$

where

$$P_1 = \sum_{d=0}^{d_0} \frac{e^{-np} \cdot (np)^d}{d!} \tag{9}$$

$$P_0 = \sum_{d=0}^{d_0-1} \frac{e^{-np} \cdot (np)^d}{d!} \tag{10}$$

The manufacturer should consider the inventory cost if the product is produced and unsold before the retailer's order. Hence, the expected total profit for the manufacturer with imperfect quality of product is that the expected total profit for the product lot subtracts the total inventory cost including the set-up cost and the holding cost as follows:

$$E\left(\pi_{PS}^{S}\right) = E_1\left(\pi_{PS}^{S}\right) - S_1 \cdot \frac{Q}{I_1 T} - \frac{B_1(I_1 - O_1)T}{2} \tag{11}$$

where Q is the order quantity from the retailer; O_1 is the demand quantity in units per unit time; S_1 is the set-up cost for each production run; I_1 is the production quantity in units per unit time; B_1 is the holding cost per unit item per unit time; T is the production run length per unit time.

The expected total profit of society including the retailer and the manufacturer is

$$ETP\left(Q,\ \mu_y, T\right) = E\left(\pi_{PS}^{R}\right) + E\left(\pi_{PS}^{S}\right) \tag{12}$$

In Chen and Liu's (2008) model, the retailer determines the order quantity and the manufacturer sequentially determine the wholesale price for maximizing respective objective function. Their solution is based on independence between order quantity and wholesale price. However, the dependence exists in the modified Chen and Liu's (2008) model because the order quantity is related with the manufacturer's product quality characteristic. Hence, we need to solve Eq. (13) to simultaneously obtain the optimal retailer's order quantity (Q^*), the optimal manufacturer's process mean $\left(\mu_y^*\right)$, and the optimal production run length (T^*) with the maximum expected profit for the retailer and the manufacturer.

It is difficult to show that Hessian's matrix is a negative definite matrix for Eq. (13). One cannot obtain a closed-form solution. To decrease decision variables in solving the optimization problem, we consider maximizing the expected total profit of society, partially differentiating Eq. (13) with respect to T and equaling to zero:

$$\frac{\partial ETP\left(Q, \mu_y, T\right)}{\partial T} = \frac{S_1 Q}{I_1 T^2} - \frac{B_1(I_1 - O_1)}{2} = 0 \tag{13}$$

From Eq. (13), we get an explicit expression of T in terms of order quantity Q:

$$T = \sqrt{\frac{2S_1 Q}{I_1(I_1 - O_1)B_1}} \tag{14}$$

The heuristic solution procedure for the above model (13) is as follows:

Step 1. Set maximum $Q = Q_{\max}$.
Step 2. Let $Q = 1$
Step 3. Compute $T = \sqrt{\frac{2S_1 Q}{I_1(I_1 - O_1)B_1}}$.
Step 4. Let $L < \mu_y < U$. One can adopt direct search method for obtaining the optimal μ_y^* with the maximum expected total profit of society for Eq. (13) with the given order quantity Q and production run length T.

Step 5. Let $Q = Q + 1$. Repeat Steps 3–4 until $Q = Q_{\max}$. The combination $\left(Q^*,\ \mu_y^*, T^*\right)$ with maximum expected total profit of society is the optimal solution.

3 Numerical Example and Sensitivity Analysis

Assume that some parameters are as follows: $R = 100$, $W = 40$, $S = 3$, $H = 2$, $\lambda_1 = 100$, $\lambda_2 = 0.8$, $n = 16$, $d_0 = 1, \sigma = 2$, $y_0 = 10$, $\sigma_y = 0.5$, $i = 0.05$, $k = 50$, $c = 0.5$, $L = 8$, $U = 12$, $R_I = 1, I_1 = 10, O_1 = 8, S_1 = 2$, and $B_1 = 4$. By solving Eq. (13), one obtains the optimal process mean $\mu_y^* = 10.08$, the optimal order quantity $Q^* = 112$, and the optimal production run length $T^* = 2.37$with retailer's expected profit $E\left(\pi_{PS}^R\right) = 5029.82$, manufacturer's expected profit $E\left(\pi_{PS}^S\right) = 3895.79$, and expected total profit of society $ETP\left(Q,\ \mu_y\right) = 8925.61$.

We do the sensitivity analysis of some parameters. From Table 1, we have the following observations:

1. The order quantity, the process mean, and the production run length almost is constant as the sale price per unit (R) increases. The retailer's expected profit, the manufacturer's expected profit, and the expected total profit of society increase as the sale price per unit increases. The sale price per unit has a have a major effect on the retailers' expected profit and the expected total profit of society.
2. The order quantity increases, the process mean is constant, and the production run length increases as the intercept of mean demand of customer (λ_1) increases. The retailer's expected profit, the manufacturer's expected profit, and the expected total profit of society increase as the intercept of mean demand of customer increases. The intercept of mean demand of customer has a have a major effect on the retailers' expected profit, manufacturer's expected profit, and the expected total profit of society.

Table 1 The effect of parameters for optimal solution

R	Q	μ_y	T	$E\left(\pi_{PS}^R\right)$	$E\left(\pi_{PS}^S\right)$	$ETP\left(Q,\ \mu_y, T\right)$	Per
80	111	10.08	2.36	2904.38	3860.91	6765.29	−24.20
120	112	10.08	2.37	7190.67	3895.79	11086.47	24.21
λ_1	Q	μ_y	T	$E\left(\pi_{PS}^R\right)$	$E\left(\pi_{PS}^S\right)$	$ETP\left(Q,\ \mu_y, T\right)$	Per
80	92	10.08	2.14	4067.35	3198.36	7265.71	−18.60
120	132	10.08	2.57	5992.24	4593.37	10585.60	18.60

Note Per $= \frac{ETP\left(Q, \mu_y, T\right) - 8925.61}{8925.61} \cdot 100\,\%$

4 Conclusions

In this paper, the author has presented a modified Chen and Liu's (2008) traditional system model with quality loss of product. Assume that the retailer's order quantity is concerned with the manufacturer's product quality and the quality characteristic of product is normally distributed. The quality of lot for manufacturer is decided by adopting a single sampling rectifying inspection plan. The process mean of quality characteristic, the production run length of product, and the order quantity of retailer are simultaneously determined in the modified model. From the above numerical results, one has the following conclusion: The sale price per unit has a have a major effect on the retailers' expected profit and the expected total profit of society and the intercept of mean demand of customer has a have a major effect on the retailers' expected profit, manufacturer's expected profit, and the expected total profit of society. Hence, one needs to have an exact estimation on these two parameters in order to obtain the exact decision values. The extension to integrated model with 100 % inspection may be left for further study.

References

Chen CH (2010) The joint determination of optimum process mean and economic order quantity. Paper presented at the 2010 international conference in management sciences and decision making, Tamsui, Taiwan, pp 285–292

Chen SL, Liu CL (2007) Procurement strategies in the presence of the spot market-an analytical framework. Prod Plann Control 18:297–309

Chen SL, Liu CL (2008) The optimal consignment policy for the manufacturer under supply chain coordination. Int J Prod Res 46:5121–5143

Hanna MD, Jobe JM (1996) Including quality costs in the lot-sizing decision. Int J Qual Reliab Manag 13:8–17

Jaber MY, Bonney M, Rosen MA, Moualek I (2009) Entropic order quantity (EnOQ) model for deteriorating items. Appl Math Model 33:564–578

Taguchi G (1986) Introduction to quality engineering. Asian Productivity Organization, Tokyo