An Optimal Ordering Policy of the Retailers Under Partial Trade Credit Financing and Restricted Cycle Time in Supply Chain

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Abstract The traditional EOQ (Economic Order Quantity) model assumes that retailers' capitals are unrestricting and the retailer must pay for items as soon as the retailer receives them from suppliers. However, this may not be true. In practice, the supplier will offer the retailer a delay period. This period is known as the trade credit period. Previously published papers assumed that the supplier would offer the retailer a delay period and the retailer could sell goods and earn interest or investment within the trade credit period. They assumed that the supplier would offer the retailer a delay period but the retailer would not offer the trade credit period to his/her customer. We extend their model and construct new ordering policy. In this paper, the retailer will also adopt the partial trade credit policy to his/her customer. We assume that the retailer's trade credit period offered by the supplier is not shorter than his/her customer's trade credit period offered by the retailer. In addition, they assumed the relationship between the supplier and the retailer is one-to-one. One thing we want to emphasize here is that the supplier has cooperative relations with many retailers. Furthermore, we assume that the total of the cycle time is restricted. Under these conditions, we model the retailers' inventory system to determine the optimal cycle times for *n* retailers.

Keywords EOQ model · Partial trade credit · Supply chain

1 Introduction

Inventory management is to decide appropriate times and quantities to produce goods. It has a significant impact on the costs and profitability of many organizations. In general, the EOQ model is still a widely used model to guide the

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management of inventories in many industrial enterprises and service organizations. The EOQ captures the trade-off between inventory carrying cost and ordering cost for each item with accuracy.

Until now, the EOQ model assumes that the retailer's capitals are unrestricting and the retailer must pay for the items as soon as the retailer receives the items from the supplier. However, this may not be completely true. In practice, the supplier will offer the retailer a delay period. This period is called the trade credit period. During this period, the retailer can sell goods and earn interest or investment (Chang et al. 2001; Chen and Chuang 1999; Kim et al. 1995; Hwang and Shinn 1997).

We extend their model and construct new ordering policy. We reconstruct an ordering policy to stimulate his/her customer demand to develop the retailers' replenishment model. The retailer will adopt the partial trade credit policy to his/her customer (Huang and Hsu 2008). We assume that the retailer's trade credit period M offered by the supplier is not shorter than his/her customer's trade credit period N offered by the retailer ($M \geq N$). Moreover, the problem we want to emphasize here is that the supplier has cooperative relations with n retailers. Furthermore, we assume that the total of the cycle time offered for n retailers is restricted. Under these conditions, we model an optimal ordering policy to determine the optimal cycle times for n retailers. This model can be formulated as a mathematical problem.

2 Model Formulations

2.1 Notation

The notations used in this paper are as follows: D: demand rate per year; A: ordering cost per order; c: unit purchasing price per item; h: unit stock holding cost per item per year excluding interest charges; α: customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer, $0 \leq \alpha \leq 1$, I_e: interest earned per \$ per year; I_k: interest charged per \$ in stocks per year by the supplier; M: the retailer's trade credit period offered by supplier in years; N: the customer's trade credit period offered by retailer in years; T: the cycle time in years; $TCV(T)$: the annual total inventory cost, which is a function of T; T^*: the optimal cycle time of $TCV(T)$.

2.2 Assumptions

(1) Demand rate is known and constant; (2) shortages are not allowed; (3) time period is finite; (4) the lead time is zero; (5) $I_k \geq I_e,\ M \geq N$; (6) when $T \geq M$, the account is settled at $T = M$ and the retailer starts paying for the interest charges on the items in stock with rate I_k. When $T \leq M$, the account is settled at $T = M$ and

the retailer does not need to pay any interest charges; (7) the retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of the trade credit.

2.3 Model Formulation

From the above notation and assumptions, we construct a model. First, we must consider all the inventory costs. That is, the annual total inventory cost consists of the following four elements.

1. Annual ordering cost is as follow: $\frac{A}{T}$.
2. Annual stock holding cost (excluding interest charges) is as follow: $\frac{DTh}{2}$.
3. According to Assumption 6, there are two cases to occur in costs of interest charges for the items kept in stock per year. Annual interest payable is as follows

$$\frac{cI_kD(T-M)^2}{2T},\ M\le T,\quad 0,\ N\le T<M,\ T\le N$$

4. According to Assumption 7, there are three cases to occur in interest earned per year. Annual interest earned is as follows:

$$\frac{sI_eD[M^2-(1-\alpha)N^2]}{2T},\ M\le T,\quad \frac{sI_eD[2MT-(1-\alpha)N^2-T^2)}{2T},\ N\le T<M$$

$$sI_eD\left[M-(1-\alpha)N-\frac{\alpha T}{2}\right],\quad 0\le T<N.$$

From the above arguments, the annual total inventory cost for the retailer can be expressed as $TCV(T)$ = ordering cost + stock-holding cost + interest payable cost − interest earned. That is, it is formulated as follows:

$$TCV(T)=\begin{cases} TCV_1(T) & T\ge M,\\ TCV_2(T), & N\le T\le M,\\ TCV_3(T), & 0<T\le N,\end{cases}$$

$$TCV_1(T)=\frac{A}{T}+\frac{DTh}{2}+\frac{cI_kD(T-M)^2}{2T}-\frac{sI_eD[M^2-(1-\alpha)N^2)}{2T},$$

$$TCV_2(T)=\frac{A}{T}+\frac{DTh}{2}-\frac{sI_eD[2MT-(1-\alpha)N^2-T^2]}{2T},$$

$$TCV_3(T)=\frac{A}{T}+\frac{DTh}{2}-sI_eD\left[M-(1-\alpha)N\frac{\alpha T}{2}\right].$$

Since $TCV_1(M) = TCV_2(M)$ and $TCV_2(N) = TCV_3(N)$, $TCV(T)$ is continuous and well-defined. All $TCV_1(T), TCV_2(T), TCV_3(T)$ and $TCV(T)$ are defined on $T > 0$. Also, $TCV_1(T), TCV_2(T)$ and $TCV_3(T)$ are convex on $T > 0$. Furthermore, we have $TCV_1'(M) = TCV_2'(M)$ and $TCV_2'(N) = TCV_3'(N)$. Therefore, $TCV(T)$ is convex on $T > 0$.

5. According to Assumption 7, there are three cases to occur in interest earned per year. Annual interest earned is as follows:

$$\frac{sI_eD[M^2-(1-\alpha)N^2]}{2T},\ M \leq T, \quad \frac{sI_eD[2MT-(1-\alpha)N^2-T^2)}{2T},\ N \leq T < M,$$

$$sI_eD\left[M-(1-\alpha)N-\frac{\alpha T}{2}\right], \quad 0 \leq T < N.$$

From the above arguments, the annual total inventory cost for the retailer can be expressed as $TCV(T)$ = ordering cost + stock-holding cost + interest payable cost − interest earned. That is, it is formulated as follows:

$$TCV(T) = \begin{cases} TCV_1(T), & T \geq M, \\ TCV_2(T), & N \leq T \leq M, \\ TCV_3(T), & 0 < T \leq N, \end{cases}$$

$$TCV_1(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_kD(T-M)^2}{2T} - \frac{sI_eD[M^2-(1-\alpha)N^2)}{2T},$$

$$TCV_2(T) = \frac{A}{T} + \frac{DTh}{2} - \frac{sI_eD\,[2MT-(1-\alpha)N^2-T^2]}{2T},$$

$$TCV_3(T) = \frac{A}{T} + \frac{DTh}{2} - sI_eD\left[M-(1-\alpha)N\frac{\alpha T}{2}\right].$$

Since $TCV_1(M) = TCV_2(M)$ and $TCV_2(N) = TCV_3(N)$, $TCV(T)$ is continuous and well-defined. All $TCV_1(T), TCV_2(T), TCV_3(T)$ and $TCV(T)$ are defined on $T > 0$. Also, $TCV_1(T), TCV_2(T)$ and $TCV_3(T)$ are convex on $T > 0$. Furthermore, we have $TCV_1'(M) = TCV_2'(M)$ and $TCV_2'(N) = TCV_3'(N)$. Therefore, $TCV(T)$ is convex on $T > 0$.

3 An Optimal Ordering Policy of the Retailers Under Partial Trade Credit Financing and Restricted Cycle Time

We consider an optimal ordering policy of the retailers for the following problem: (1) inventory problem we propose here is in the EOQ model; (2) the number of the retailers is n; (3) the summation of each cycle time t_i $(i = 1, 2, \ldots, n)$ is restricted to T_0; (4) we allocate the restricted cycle time T_0 to n retailers to minimize the summation of total inventory cost.

To solve the problem, we use the inventory model we have already discussed in previous section. First, we formulate the cost function for each retailer. Next, we lead to all retailers' total cost function.

Let t_i be a cycle time for retailer i $(i = 1, 2, \ldots, n)$. Then we obtain the following total inventory cost function for retailer i:

$$TCV_i(t_i) = \begin{cases} TCV_{1i}(t_i), & t_i \geq M_i, \\ TCV_{2i}(t_i), & N_i \leq t_i \leq M_i, \\ TCV_{3i}(t_i), & 0 < t_i \leq N_i. \end{cases}$$

Here, new subscript i represents the retailer's number i $(i = 1, 2, \ldots, n)$. From the above expression, the summation of total inventory cost is shown as follows:

$$\sum_{i=1}^{n} TCV_i(t_i), \quad i = 1, 2, \ldots, n, \tag{1}$$

Therefore, an optimal ordering policy of the retailers under trade credit financing and restricted cycle time we propose here is represented as follows:

$$Minimize \quad \sum_{i=1}^{n} TCV_i(t_i), \tag{2}$$

$$subject\ to \quad \sum_{i=1}^{n} t_i \leq T_0, \quad t_i > 0 \quad (1 = 1, 2, \ldots, n), \tag{3}$$

Now, our aim is to minimize the objective function [Eq. (2)] under the constraints [Eq. (3)]. We differentiate Eq. (2) with respect to t_i. We obtain as follows:

$$TCV'(t_i) = \begin{cases} TCV'_{1i}(t_i) = -\left[\frac{2A_i + c_i D_i M_i^2 I_{ki} - s_i D_i I_{ei}(M_i^2 - (1-\alpha_i)N_i^2)}{2t_i^2}\right] + D_i\left(\frac{h_i + c_i I_{ki}}{2}\right), & t_i \geq M_i, \\ TCV'_{2i}(t_i) = -\left[\frac{2A_i + s_i D_i (1-\alpha_i) N_i^2 I_{ei}}{2t_i^2}\right] + D_i\left(\frac{h_i + s_i I_{ei}}{2}\right), & N_i \leq t_i < M_i, \\ TCV'_{3i}(t_i) = -\frac{A_i}{t_i^2} + D_i\left(\frac{h_i + s_i \alpha_i I_{ei}}{2}\right), & 0 \leq t_i < N_i \end{cases} \tag{4}$$

Here, we define Δ_{1i} and Δ_{2i} respectively.

Table 1 The retailer's optimal cycle time $\tilde{t}_i^*$ for Lagrange's multiplier λ (Case 1)

The range of λ	Optimal cycle time $(\tilde{t}_i^*)$
$\lambda \geq 0$	$\sqrt{\frac{2A_i}{D_i(h_i+s_i\alpha_i I_{ei})}}$
$-\infty < \lambda < 0$	$\sqrt{\frac{2A_i}{D_i(h_i+s_i\alpha_i I_{ei})-2\lambda}}$

Table 2 The retailer's optimal cycle time $\tilde{t}_i^*$ for Lagrange's multiplier λ (Case 2)

The range of λ	Optimal cycle time $(\tilde{t}_i^*)$
$\lambda \geq 0$	$\sqrt{\frac{2A_i+s_iD_i(1-\alpha_i)N_i^2 I_{ei}}{D_i(h_i+s_i I_{ei})}}$
$\Delta_{2i} \leq \lambda < 0$	$\sqrt{\frac{2A_i+s_iD_i(1-\alpha_i)N_i^2 I_{ei}}{D_i(h_i+s_i I_{ei})-2\lambda}}$
$-\infty < \lambda < \Delta_{2i}$	$\sqrt{\frac{2A_i}{D_i(h_i+s_i\alpha_i I_{ei})-2\lambda}}$

Table 3 The retailer's optimal cycle time $\tilde{t}_i^*$ for Lagrange's multiplier λ (Case 3)

The range of λ	Optimal cycle time $(\tilde{t}_i^*)$
$\lambda \geq 0$	$\sqrt{\frac{2A_i+c_iD_iM_i^2 I_{ki}-s_iD_iI_{ei}[M_i^2-(1-\alpha_i)N_i^2]}{D_i(h_i+c_i I_{ki})}}$
$\Delta_{1i} \leq \lambda < 0$	$\sqrt{\frac{2A_i+c_iD_iM_i^2 I_{ki}-s_iD_iI_{ei}[M_i^2-(1-\alpha_i)N_i^2]}{D_i(h_i+c_i I_{ki})-2\lambda}}$
$\Delta_{2i} \leq \lambda < \Delta_{1i}$	$\sqrt{\frac{2A_i+s_iD_i(1-\alpha_i)N_i^2 I_{ei}}{D_i(h_i+s_i I_{ei})-2\lambda}}$
$-\infty < \lambda < \Delta_{2i}$	$\sqrt{\frac{2A_i}{D_i(h_i+s_i\alpha_i I_{ei})-2\lambda}}$

$$\Delta_{1i} = \frac{-2A_i + D_iM_i^2(h_i + s_iI_{ei}) - s_iD_i(1-\alpha_i)N_i^2 I_{ei}}{2M_i^2},\ \Delta_{2i} = \frac{-2A_i + D_iN_i^2(h_i + s_i\alpha_iI_{ei})}{2N_i^2}$$

where $\Delta_{1i} \geq \Delta_{2i}$ $(i = 1, 2, \ldots, n)$. To solve the problem, we use Lagrange's multiplier λ. The optimal solution $\tilde{t}_i^*$ for our problem is obtained from Tables 1, 2, 3 using $\sum_{i=1}^n t_i \leq T_0$.

For each retailer, we rearrange $\{\Delta_{1i} | 1 \leq i \leq n\}$, $\{\Delta_{2i} | 1 \leq i \leq n\}$, and $\{0\}$ small order, $B_1 \leq B_2 \leq \cdots \leq B_n \leq \cdots \leq B_{2n} \leq B_{2n+1}$. If $B_k \neq B_{k+1}$, the optimal Lagrange's multiplier λ^* only exists in the interval $B_k \leq \lambda^* < B_{k+1}$ $(0 \leq k \leq 2n+1)$ because the objective function is convex, where $B_0 = -\infty$ and $B_{2n+2} = \infty$. In this interval, we allocate the restricted cycle time T_0 to n retailers. Then, the summation of each cycle time has to be equal to T_0. That is to say, using $\sum_{i=1}^n t_i = T_0$, we can find the optimal Lagrange's multiplier λ^*.

Table 4 The parameters for each retailer

Retailer no.	D_i	A_i	c_i	h_i	α_i	I_{ei}	I_{ki}	M_i	N_i
1	6,100	160	100	6	0.3	0.13	0.15	0.11	0.08
2	2,600	100	90	4	0.5	0.12	0.15	0.10	0.06
3	1,500	80	80	3	0.2	0.10	0.15	0.10	0.04
4	6,300	100	110	7	0.3	0.12	0.15	0.11	0.08
5	2,650	75	92	4.5	0.5	0.12	0.15	0.10	0.06
6	1,300	130	79	3	0.25	0.10	0.15	0.10	0.045
7	5,000	100	140	7	0.4	0.09	0.15	0.10	0.08
8	2,650	98	94	4	0.5	0.12	0.15	0.10	0.065
9	1,480	75	80	3.5	0.5	0.10	0.15	0.10	0.06
10	5,950	190	98	6.5	0.5	0.13	0.15	0.11	0.08

Table 5 The optimal cycle time $\tilde{t}_i^*$ for each retailer and the total inventory cost $TCV_i(\tilde{t}_i^*)$

Retailer No.	$\tilde{t}_i^*$	$TCV_i(\tilde{t}_i^*)$
1	0.065281	139.9007
2	0.063961	381.1834
3	0.057087	903.6561
4	0.069594	785.8357
5	0.062986	377.0067
6	0.056952	913.8913
7	0.052168	211.1396
8	0.063189	340.6989
9	0.058609	862.7761
10	0.070176	88.1847
Total	0.620000	5,004.2731

4 Numerical Example

In this section, we show the numerical examples. Here, there are 10 retailers in our model. The parameters for each retailer are shown in Table 4.

4.1 The Optimal Retailers' Ordering Policy with the Restricted Cycle

We consider the case of the given T_0 is equal or less than the total of optimal cycle time (0.942389). For example, we set $T_0 = 0.62$. In Table 5, the optimal cycle time $\tilde{t}_i^*$ and the total inventory cost $TCV_i(\tilde{t}_i^*)$ for each retailer are shown. The optimal Lagrange's multiplier $\lambda^* = -19,244.2$. The summation of the total inventory costs are 5,004.2731.

5 Conclusion

This note is a modification of the assumption of the trade credit policy in previously published results to reflect realistic business situations. We assumed that the retailer also adopts the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model. In addition, their model assumed the relationship between the supplier and the retailer is one-to-one. In this paper, we assumed the supplier has cooperative relations with many retailers for more suiting and satisfying the real world problems and assumed that the summation of each retailer's cycle time is restricted. Under these conditions, we have constructed the retailers' inventory system as a cost minimizing problem to determine the n retailers' optimal ordering policy.

A future study will further incorporate the proposed model more realistic assumptions such as probabilistic demand, allowable shortages and a finite rate of replenishment.

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