

Provably Insecure Group Authentication: Not All Security Proofs are What they Claim to Be



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1 Introduction

A paper presented at ICICS 2019 [8] describes a protocol designed to enable members of a group to authenticate one another in a group-wise fashion. The paper also presents a formal security model for such ‘group authentication’ schemes, and provides proofs of security for the protocol. Unfortunately, as we describe in this paper, the protocol is completely insecure, allowing an outsider to masquerade as any group member and set up contradictory views of group authentication membership within a set of participating entities.

The fact that a fundamental flaw exists in a provably secure scheme is perhaps surprising. However, as we discuss in greater detail below, the examination of the main theorems reveals that they do not establish that the protocols are secure in any practical sense.

It turns out that the ICICS 2019 scheme is related to a scheme presented in 2013 [4] by one of the authors of the 2019 paper. As we discuss below, this earlier scheme is also completely insecure. Since the 2013 scheme is slightly simpler than the 2019 scheme, we present it and its flaws first, before doing the same for the 2019 scheme. We observe that the 2013 scheme has been cryptanalysed previously by Ahmadian and Jamshidpur [1], although the attack we describe here is very much simpler than the previously published attack.

The remainder of the paper is structured as follows. In Sect. 2, the 2013 protocol is described, and the goals of, and security claims for, the protocol are summarised. This leads naturally to Sect. 3 in which it is shown that the claimed security properties do not hold by describing a very simple attack; the ‘proofs’ of the failed theorems are also examined to see why an apparently provably secure scheme is fundamentally flawed. Section 4 presents the ICICS 2019 protocol, together with a summary of

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its design goals and security claims. This is followed by Sect. 5, where we show why it also possesses fundamental flaws; again the security theorems are examined. Conclusions are drawn in Sect. 6.

2 The 2013 Harn Scheme

2.1 Goals of the Scheme

In the context of the schemes considered in this paper, a *group authentication protocol* is one in which ‘each user acts both roles of the prover and the verifier, and all users in the group are authenticated at once’ [8]. The primary goal of such protocols is speed and efficiency, and not privacy (since all users in such a protocol are identified to each other). As discussed, for example, by Yang et al. [9], this contrasts with the use of the same or similar terms elsewhere in the literature, where protocols are considered which allow an entity to authenticate to another party as a member of a group, without revealing his or her identity.

The main goal for a group authentication protocol as considered here is to enable all members of a defined group to be given assurance, by executing the protocol, that the specified members are all present and actively involved in the protocol, and that no other parties are involved. A review of recent work on the design of such protocols can be found in Sect. 1.1 of Xia et al. [8].

Unfortunately, the precise threat model for which the protocol was designed is not clear from the 2013 paper [4]. References are made to both insider and outsider attacks, i.e. the protocol is intended to be secure against both of these classes of attacks. However, no reference is made to the trust assumptions for the broadcast channel used for communication between the parties. However, it is standard practice when analysing authentication protocols to assume that an attacker can manipulate the communications channel, including to intercept, delete, insert and modify messages (see, for example, Boyd et al. [2], Sect. 1.5.1). We therefore assume this in our analysis of the scheme. Indeed, it is hard to imagine a practical situation where it would not be possible for a determined attacker to modify messages; certainly, there are many real-world examples of message manipulation attacks on the broadcast channels used in mobile telephony—see, for example, the rich literature on IMSI catcher attacks [3, 5, 7].

2.2 Operation

Harn [4] actually presents three distinct protocols. The first, the ‘basic scheme’, is intended to demonstrate the main ideas; however, it requires information to be divulged simultaneously by all parties and hence would not be secure in practice.

The second and third schemes are elaborations of the basic idea designed to allow for asynchronous information release. In the second scheme, participant credentials can only be used once, whereas the third scheme allows multiple uses of credentials. However, since the second and third schemes are very similar in operation, for simplicity we focus here on the second scheme.

Initialisation This scheme, like all the schemes in both papers, involves a *Group Manager* (GM) trusted by all participants, which pre-equips all participants with credentials used to perform the group authentication process. We suppose that there are n participants $\mathcal{U} = \{U_1, U_2, \dots, U_n\}$.

To initialise the protocol, the GM performs the following steps.

- The GM chooses parameters t and k , where t determines the resistance of the scheme to insider adversaries—that is, the scheme is designed to be secure so long as at most $t - 1$ insiders collaborate. No explicit guidance on the choice of k is given except that it must satisfy $kt > n - 1$, and hence here we assume $k = \lceil n/t \rceil$.
- The GM chooses a large prime p . All calculations are performed in $\text{GF}(p) = \mathbb{Z}_p$.
- The GM chooses a cryptographic hash function H with domain \mathbb{Z}_p .
- The GM chooses a secret $s \in \mathbb{Z}_p$, and computes $H(s)$.
- The GM selects a set of k polynomials $\{f_1(x), f_2(x), \dots, f_k(x)\}$ over \mathbb{Z}_p of degree $t - 1$, where the coefficients are chosen uniformly at random from \mathbb{Z}_p .
- The GM selects two sets of k integers $\{w_1, w_2, \dots, w_k\}$ and $\{d_1, d_2, \dots, d_k\}$ with the property that

$$s = \sum_{j=1}^k d_j f_j(w_j),$$

where the values $\{w_1, w_2, \dots, w_k\}$ are all distinct.

- The GM computes a set of k tokens $\{f_1(x_i), f_2(x_i), \dots, f_k(x_i)\}$ for each participant U_i ($1 \leq i \leq n$), where $x_i \in \mathbb{Z}_p$ is a unique identifier for U_i .
- Using an out-of-band secure channel, the GM equips participant U_i ($1 \leq i \leq n$) with t, k, p, H , the identifiers $\{x_1, x_2, \dots, x_n\}$, the integers $\{w_1, w_2, \dots, w_k\}$ and $\{d_1, d_2, \dots, d_k\}$, $H(s)$, and the participant's collection of k secret tokens $\{f_1(x_i), f_2(x_i), \dots, f_k(x_i)\}$.

Group Authentication We now suppose that some subset $\mathcal{U}' \subseteq \mathcal{U}$ of the participants (where $|\mathcal{U}'| = m \leq n$) wish to authenticate each other in a group-wise fashion. Suppose $\mathcal{U}' = \{U_{z_1}, U_{z_2}, \dots, U_{z_m}\}$. We suppose every participant in \mathcal{U}' is aware of the membership of \mathcal{U}' . Each participant $u_{z_i} \in \mathcal{U}'$ now proceeds as follows.

- Compute

$$c_{z_i} = \sum_{j=1}^k d_j f_j(x_{z_i}) \prod_{\substack{r=1 \\ r \neq i}}^m \frac{(w_j - x_{z_r})}{(x_{z_i} - x_{z_r})}$$

- Broadcast c_{z_i} to all members of \mathcal{U}' .

- Once all the values $\{c_{z_1}, c_{z_2}, \dots, c_{z_m}\}$ have been received, compute

$$s' = \sum_{r=1}^m c_r.$$

- If $H(s') = H(s)$ then the protocol succeeds, i.e. all users have been successfully authenticated.

Note that the protocol can only be executed once per initialisation, as the secret s is revealed to anyone receiving the messages sent on the broadcast channel. The third scheme removes this limitation.

2.3 Security Claims

A number of claims are made with respect to the security properties of the protocol. In particular the *security* property is claimed, namely that any outside adversary cannot impersonate ... a member ... after knowing at most $n - 1$ values from other members. The meaning of impersonation in this context is not clear, but we assume that this means that, following the completion of the protocol, legitimate participants cannot end up with differing beliefs about who are the participants in a group authentication. Sadly, as we show below, this property does not hold.

3 Analysis of the 2013 Scheme

3.1 Previous Results

As noted in Sect. 1, this scheme has previously been cryptanalysed by Ahmadian and Jamshidpour [1]. Their approach involves performing computations using broadcast values intercepted during protocol execution, and requires certain conditions to be satisfied to succeed. The attack we describe below is almost trivially simple, and works regardless of group size.

3.2 Preliminary Observation

The attack we propose below relies on a very simple fact. From the description in Sect. 2.2, it should be clear that participant U_{z_i} will accept that the group authentication has succeeded if and only if the sum of the $m - 1$ received values c_{z_j} ($j \neq i$)

and the value c_{z_i} it computed is equal to s . That is, the correctness of individual c_{z_j} values is not checked.

3.3 *An Outsider Impersonation Attack*

Suppose an (insider) adversary controls the broadcast channel with respect to the ‘victim’ participant U_{z_i} , i.e. the adversary can (a) prevent messages sent by other legitimate participants from reaching U_{z_i} , and (b) send messages to U_{z_i} on this channel that appears to have come from other legitimate participants. Since the protocol makes no assumptions about the trustworthiness of the communications channels (see Sect. 2.1), this assumption is legitimate (indeed, if the broadcast channel was completely trustworthy, then the security protocol would not be needed).

The adversary does two things. Firstly, it legitimately engages in the protocol with an arbitrary subset \mathcal{U}'' of the legitimate participants, where $U_{z_i} \notin \mathcal{U}''$. As a result of completing this protocol, the adversary now knows s . During the execution of the protocol, the adversary prevents any of the broadcast messages from reaching U_{z_i} . The adversary now engages with the ‘victim’ participant U_{z_i} , suggesting that a group authentication is to be performed by the members of an arbitrary set of participants $\mathcal{U}' \subseteq \mathcal{U}$, where $U_{z_i} \in \mathcal{U}'$ and $|\mathcal{U}'| = m$, say. This may involve sending ‘fake’ messages to U_{z_i} that apparently originate from the other members of \mathcal{U}' .

The adversary now chooses values c_{z_j} ($j \neq i$) for $U_{z_j} \in \mathcal{U}'$, and starts sending them to U_{z_i} as if they come from the members of \mathcal{U}' . The only condition the values must satisfy is that they sum to $s - c_{z_i}$. Of course, this means that the adversary cannot send all $m - 1$ values to U_{z_i} until c_{z_i} is sent by U_{z_i} , but the protocol is meant to be used ‘asynchronously’, i.e. where not all participants send their messages at the same time. It should be immediately obvious that U_{z_i} will accept the success of the protocol, although clearly the group authentication that U_{z_i} believes has occurred has not actually occurred.

Note that the third scheme in the 2013 paper [4] suffers from a precisely analogous attack.

3.4 *What About the Security Theorems?*

The fact that the protocol is so fundamentally flawed is perhaps surprising given Theorem 2, [4], which asserts that the scheme ‘has the properties of the t -secure m -user n -group authentication scheme ... if $kt > n - 1$ ’. This appears to contradict the simple attack we have just described. The answer is simple—the ‘proof’ of Theorem 2 only attempts to show that an adversary cannot forge legitimate values c_{z_j} , but the attack does not require this. Thus it is clear that the ‘proof’ is making unwarranted assumptions about how an attack might be launched, and as such Theorem 2 is demonstrably not a theorem at all.

To be fair, this shortcoming was already noted by Xia et al. [8], who observe that the security properties of the 2013 scheme ‘are only justified by heuristic arguments rather than formal security proofs’. Unfortunately, despite a much more formal approach, we show below that the Xia et al. scheme is also completely insecure, and that the threat model underlying the security arguments is not adequate to reflect real-world attacks.

4 The Xia-Harn-Yang-Zhang-Mu-Susilo-Meng Scheme

4.1 Goals of the Scheme

The second protocol we consider here, [8], is also an example of a group authentication protocol in the sense given in Sect. 2.1. Xia et al. [8] go much further than much of the prior art in attempting to formalise the goals and security model for a group authentication scheme. However, even here the specific objectives of such a protocol are left a little vague. The following statement is the closest to a formal definition.

In general, a group authentication scheme works as follows. The group manager (GM) generates a number of credentials, and sends each of these credentials to a user in the group. In the authentication stage, every participating user uses her credential to compute a token and broadcasts it. Subsequently, every user can use the revealed information to verify whether all users are belonging to the same group.

However, as was the case with the 2013 paper analysed above, no explicit references are made to the trust assumptions applying to the channel used for communications. As a result, when analysing the protocol below, we make the same (standard) assumptions about this channel as were made for the 2013 protocol, namely that messages are subject to interception, insertion, deletion and/or modification. As stated in Sect. 2.1, it is hard to imagine a real-world deployment scenario where this would not be possible. However, as we discuss below, the security *proof* implicitly assumes that the attackers are restricted to being passive interceptors (‘honest but curious’), which is why it is possible to construct a security proof for a protocol that under reasonable real-world assumptions is subject to a fundamental attack.

4.2 Operation

As is the case for the 2013 protocol, the scheme can be divided into two phases: initialisation, when the GM equips each participant with the credentials needed to perform group authentication, and the group authentication phase where a subset of the participants simultaneously authenticate each other as a group.

Initialisation Again as before we suppose that there are n participants $\mathcal{U} = \{U_1, U_2, \dots, U_n\}$. To initialise the protocol, the GM performs the following steps.

- The GM chooses parameters t and ℓ , where the scheme is designed to be secure as long as at most $t - 1$ insiders collaborate, and ℓ determines the number of group authentication sessions that can be performed before new credentials need to be issued.
- The GM chooses a cyclic group G (expressed multiplicatively) with order a large prime q , and randomly selects g_1, g_2, \dots, g_ℓ to be ℓ independent generators of G .
- The GM chooses a cryptographic hash function H with domain G .
- The GM chooses a secret $s \in \mathbb{Z}_q$, and computes the ℓ values $H((g_i)^s)$, $1 \leq i \leq \ell$.
- The GM randomly selects a polynomial $f(x) = \sum_{i=0}^{t-1} a_i x^i$ over \mathbb{Z}_q of degree $t - 1$, where $a_0 = s$.
- The GM computes a credential $s_i = f(x_i)$ for each participant U_i ($1 \leq i \leq n$), where $x_i \in \mathbb{Z}_p$ is a unique identifier for U_i .
- Using an out-of-band secure channel, the GM equips participant U_i ($1 \leq i \leq n$) with t, G, q, H , the identifiers $\{x_1, x_2, \dots, x_n\}$, the generators $\{g_1, g_2, \dots, g_\ell\}$, the hash codes $\{H((g_1)^s), H((g_2)^s), \dots, H((g_\ell)^s)\}$ and the participant's own secret credential $s_i (= f(x_i))$.

Group Authentication Just as in the 2013 scheme, we now suppose that some subset $\mathcal{U}' \subseteq \mathcal{U}$ of the participants (where $|\mathcal{U}'| = m \leq n$) wish to authenticate each other in a group-wise fashion. Suppose $\mathcal{U}' = \{U_{z_1}, U_{z_2}, \dots, U_{z_m}\}$. We suppose every participant in \mathcal{U}' is aware of the membership of \mathcal{U}' . We further suppose that the set of participants has reached session number σ in the period of use of a particular credential set, where $1 \leq \sigma \leq \ell$. Note that each session must be conducted using a new value of σ , and σ determines which generator g_σ from the set of generators will be used in this particular protocol instance.

Each participant $u_{z_i} \in \mathcal{U}'$ proceeds as follows.

- Choose $u_{z_i} \in \mathbb{Z}_q$ uniformly at random, and broadcast it to all other participants.
- Once the set of values $\{u_{z_1}, u_{z_2}, \dots, u_{z_m}\}$ has been received, compute

$$\gamma_i = \prod_{\substack{j \in \{1, 2, \dots, m\} \\ z_j < z_i}} (g_\sigma)^{u_{z_j}} \prod_{\substack{j \in \{1, 2, \dots, m\} \\ z_j > z_i}} (g_\sigma)^{-u_{z_j}},$$

$$L_i = \prod_{\substack{j \in \{1, 2, \dots, m\} \\ z_j \neq z_i}} \frac{x_{z_j}}{x_{z_j} - x_{z_i}},$$

and

$$c_{z_i} = (g_\sigma)^{s_{z_i} L_i} (\gamma_i)^{u_{z_i}}.$$

- Broadcast c_{z_i} to all members of \mathcal{U}' .
- Once all the values $\{c_{z_1}, c_{z_2}, \dots, c_{z_m}\}$ have been received, compute

$$\prod_{r=1}^m c_{z_r}.$$

- If $H(\prod_{r=1}^m c_{z_r}) = H((g_\sigma)^s)$, then the protocol succeeds, i.e. all users have been successfully authenticated.

4.3 Security Claims

We first observe that Xia et al. [8] make the following statement about the assumed properties of the broadcast channel.

Note that the broadcast channel is only assumed to be asynchronous, such that messages sent from the uncorrupted users to the corrupted ones can be delivered relatively fast, in which case, the adversary can wait for the messages of the uncorrupted users to arrive, then decide on her computation and communication, and still get her messages delivered to the honest users on time.

The security model of Xia et al. [8] gives the *No impersonation* property as follows.

The outside adversary A_O cannot impersonate a group member without being detected, even if A_O computes her token after seeing all other users' tokens in the asynchronous networks.

5 Analysis of the ICICS 2019 Scheme

5.1 Preliminary Observation

We consider what can be learnt by observing a single value c_{z_i} in a single instance of the protocol, together with the initial broadcasts of the values $\{u_{z_1}, u_{z_2}, \dots, u_{z_m}\}$. We suppose that the (outside) observer has access to the system parameters, i.e. the values provided by the GM to all participants, namely t, G, q, H , the identifiers $\{x_1, x_2, \dots, x_n\}$, the generators $\{g_1, g_2, \dots, g_\ell\}$ and the hash codes $\{H((g_1)^s), H((g_2)^s), \dots, H((g_\ell)^s)\}$.

As defined in Sect. 4.2,

$$c_{z_i} = (g_\sigma)^{s_{z_i} L_i} (\gamma_i)^{u_{z_i}}.$$

Now, again as defined above

$$\gamma_i = \prod_{\substack{j \in \{1, 2, \dots, m\} \\ z_j < z_i}} (g_\sigma)^{u_{z_j}} \prod_{\substack{j \in \{1, 2, \dots, m\} \\ z_j > z_i}} (g_\sigma)^{-u_{z_j}},$$

i.e. computing γ_i does not involve any secret credential values and hence is simple to derive for anyone with access to the system credentials. Since we also supposed

that u_{z_i} has been intercepted, the observer can thus compute

$$c_{z_i} \cdot (\gamma_i)^{-u_{z_i}} = (g_\sigma)^{s_{z_i} L_i}.$$

Next observe that, yet again as defined above,

$$L_i = \prod_{\substack{j \in \{1, 2, \dots, m\} \\ z_j \neq z_i}} \frac{x_{z_j}}{x_{z_j} - x_{z_i}},$$

and hence L_i is also available to anyone with access to the system credentials.

Having derived L_i , the observer now computes a value M such that $ML_i \equiv 1 \pmod{q}$, a calculation which is simple to perform given that q is known (see, for example, Algorithm 2.142 of Menezes et al. [6]). Note that M is guaranteed to exist since q is prime (see, for example, Fact 2.119 [6]).

It then follows immediately that

$$[c_{z_i} \cdot (\gamma_i)^{-u_{z_i}}]^M = (g_\sigma)^{s_{z_i} L_i M} = (g_\sigma)^{s_{z_i}}.$$

That is, an observer of c_{z_i} and the values $\{u_{z_1}, u_{z_2}, \dots, u_{z_m}\}$ can compute $(g_\sigma)^{s_{z_i}}$.

5.2 An Outsider Impersonation Attack

The above observation leads to a very simple and powerful attack, enabling the impersonation of a participant in any group. The attack scenario is very similar to that described in Sect. 3.3. We suppose an (outsider) adversary controls the broadcast channel with respect to the ‘victim’ participant U_{z_i} , i.e. the adversary can (a) prevent messages sent by other legitimate participants from reaching U_{z_i} , and (b) send messages to U_{z_i} on this channel that appears to have come from other legitimate participants. Finally, we assume that it is ‘time’ for a session using the group generator g_σ .

We first suppose the adversary observes a group of participants $\mathcal{U}'' \subseteq \mathcal{U}$ (where $U_{z_i} \notin \mathcal{U}''$) engaging in the protocol. The adversary

- intercepts all the u_{z_j} and c_{z_j} values sent by each $U_{z_j} \in \mathcal{U}''$;
- uses these intercepted values, together with the system parameters, to compute $(g_\sigma)^{s_{z_j}}$ for each $U_{z_j} \in \mathcal{U}''$;
- prevents any of the messages reaching U_{z_i} .

We now suppose that the adversary persuades the victim participant U_{z_i} that it is being invited to join a group of participants $\mathcal{U}' \subseteq \mathcal{U}'' \cup \{U_{z_i}\}$, where $U_{z_i} \in \mathcal{U}'$, e.g. by sending ‘fake’ messages from members of \mathcal{U}' to U_{z_i} . The adversary chooses arbitrary values u_{z_j} for every $U_{z_j} \in \mathcal{U}' - \{U_{z_i}\}$, and sends these values to U_{z_i} as if they come from U_{z_j} . Once U_{z_i} sends its value u_{z_i} , the adversary can use the complete set of values

$\{u_{z_j}\}$ and the computed values $(g_\sigma)^{s_{z_j}}$ (which it has for every $U_{z_j} \in \mathcal{U}' - \{U_{z_i}\}$) to compute the ‘correct’ values c_{z_j} for every $U_{z_j} \in \mathcal{U}' - \{U_{z_i}\}$, which it sends to the victim participant U_{z_i} . Since all the received values are ‘correct’, the victim will falsely believe that it is part of a group authentication with a set of participants, of whom none believe they are being authenticated to the victim.

5.3 Other Possible Attack Scenarios

There are many other scenarios in which the observation in Sect. 5.1 could be used to launch an attack on the protocol. For example, if an attacker could control the broadcast network with respect to two victims, a range of conflicting beliefs about who has been authenticated to whom could be established. That is, once an attacker has observed a participant U_{z_j} output a value c_{z_j} , this can be used to impersonate U_{z_j} in any group the attacker chooses (assuming control over the broadcast channel).

5.4 What About the Proof of Security?

The attack described above clearly breaks the ‘no impersonation’ property given in Sect. 4.3. However, Theorem 4 [8] states that ‘The proposed group authentication scheme satisfies the no impersonation property, assuming that H is a preimage resistant hash function and the DDH assumption holds in G ’. The attack does not invalidate the assumptions of the theorem, and hence the theorem must be false.

How can this be true? Well, the examination of the proof of Theorem 4 suggests why. The proof apparently only deals with the ‘honest but curious’ case, where all participants are assumed to follow the protocol correctly. The sort of manipulation of messages and beliefs involved in the attack do not appear to be covered by the proof. That is, while the mathematics may be correct, the result does not establish that the protocol would actually be secure in a real-world deployment (which, of course, it is not).

Indeed, this is partly admitted by Xia et al. [8]. In the concluding section of their paper, it is stated that ‘There are two distinct approaches to defining security for cryptographic protocols: simulation proof and reduction proof. The former is more intuitive because it models security of the targeted problem via an ideally trusted third party. However, the definitions will become complicated once all details are filled in. In contrast, the reduction proof yields definitions that are simpler to describe and easier to work with. However, the adequacy for modelling the problem is less clear. In this paper, we followed the latter approach, and it is still open how to provide formal security treatment for group authentication using the simulation proof.’

6 Conclusions

We have examined two different group authentication protocols, and found that both possess fundamental flaws. Clearly, this means that neither of them should be used in practice. Fortunately, there are many well-established and relatively efficient means of performing authentication—see, for example, Boyd et al. [2].

The fundamental flaws in the protocols exist despite the fact that in both cases theorems are provided asserting their security. Indeed, in the more recent case, the theorems are given within the context of a formal security model. This is clearly worrying—modern cryptography takes as a fundamental tenet that ‘proofs of security’ are necessary, but clearly they are not of much value if the proofs are false.

Of course, mistakes in proofs are commonplace, but in these cases the issue is clearly not just a mistake. In the earlier paper, there is no formal security model, and the theorems are simply heuristic arguments. Even in the more recent paper, where the results may well be valid, the authors themselves admit that the security model used is not sufficient to establish security other than in a case where the attackers are restricted to behaving in an ‘honest’ fashion. This clearly suggests that reviewers need the time to carefully review proofs (and the precise details of claims of security) for adequacy. This flies in the face of the modern obsession with speedy publication, both for conferences and many journals (e.g. *IEEE Access* which allows referees only a week to complete a review). Perhaps we, as the research community, need to think more carefully about finding ways to allow reviewers the time and space to write carefully considered and detailed reviews.

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