Water Demand as Fuzzy Random Variable in the Analysis of Water Distribution Networks

Prerna Pandey, Shilpa Dongre, and Rajesh Gupta

Abstract The analysis of water distribution networks (WDNs) involves number of parameters, some of which are uncertain and may affect the system performance. The deterministic approach considers such uncertain parameters as precisely known in the crisp value analysis. Among the various uncertain parameters, water demand at nodes which shows the major variation over the design life due to various factors including increase in population, improved way of living, fire demand, thefts, and leakages. The variation is due to combined effect of its random nature and imprecise knowledge at its values which is taken fuzzy. The various approaches in the literature consider water demand as either random or fuzzy. The present study aims to incorporate water demand as a fuzzy random variable (FRV) in the analysis of WDNs. The water demand at each node is assumed to be normally distributed with a fuzzy mean and standard deviation of $\pm 10\%$. The water demand uncertainty is represented by a triangular membership function having random demand at its kernel and $\pm 5\%$ variation as its support. The methodology is illustrated through an example network from literature. The results obtained using fuzzy random approach are compared with those obtained by fuzzy approach.

Keywords Water distribution networks · Fuzzy–random approach · Uncertainty · Water demand · Fuzzy mean

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R. Al Khaddar et al. (eds.), *Advances in Energy and Environment*, Lecture Notes in Civil Engineering 142, https://doi.org/10.1007/978-981-33-6695-4_10

1 Introduction

In the analysis of water distribution networks (WDN), it involves several uncertain parameters such as: (1) Pressure head requirement and future water demand at nodes which is difficult to analyze correctly. (2) Pipe roughness value. (3) Economic and environmental factors, i.e., discount rate and cost of repair of failed components etc [3]. Precise values of the parameters like future water demand and pipe roughness can be obtained when new. However, due to aging process quantifying, these values are difficult and causes some uncertainty. Hence, it is very much necessary to incorporate these uncertainties for a reliable design of WDNs.

In general, there exist two types of uncertainties, i.e., random and fuzzy. The uncertainty of random nature is caused due to natural variability and it is termed as irreducible or aleatoric uncertainty. Similarly, the uncertainty due to lack of knowledge or fuzziness is referred as reducible or epistemic uncertainty. The major difference between the two approaches is that the prior one is a probabilistic or stochastic approach while the later is possibilistic approach. The probabilistic approach is mainly statistical in nature and large reliable data is needed to define the probability distribution function (PDF) of any uncertain variable, which is treated as random variable. The approach thus suited best for such uncertain events/ processes whose occurrence is uncertain, i.e., whether the event/ process can occur or not and is based on binary characterization (either 0 or 1). However, the possibility-based approach considers uncertain parameter as fuzzy parameter and it is based upon possibility approach, thus considers, the possibility (between 0 and 1) of occurrence of an event. The uncertainty associated with lack of information is appropriate to handle with fuzzy approach. While the fuzzy random approach holds good for simultaneous representation of randomness and fuzziness for uncertainty analysis. Such uncertain parameters are together known as fuzzy random variables (FRVs).

The probabilistic approach represents the uncertain parameters by means of probability distribution function (PDF) or cumulative distribution function (CDF). From the known PDF of uncertain parameter, PDF for output variable is generated using Monte Carlo simulation (MCS), Latin hypercube sampling (LHS), etc. [2]. This requires evaluations of thousands of alternatives making the process computationally intensive. Lansey et al. [9] used chance constrained nonlinear programming to formulate single objective design problem considering nodal demands, required pressures, and roughness coefficients as an uncertain parameter. Kapelan et al. [7] proposed a robust nondominated sorting genetic algorithm II (RNSGAII), for multi-objective design considering uncertainties in nodal demands, and pipe roughness. Xu and Goulter [17] solved its model by integrating the first-order reliability method (FORM) and GRG2 optimization program, hence provided an improved design solution. Babayan et al. [1] replaced MCS by integration-based uncertainty quantification technique and solved using modified genetic algorithm (GA).

To study the effect of imprecision and uncertainty in pipe roughness coefficients and nodal demands on system performance, fuzzy sets theory adopted [13]. Gupta and Bhave [6] have shown fuzzy uncertainty modeling to analyze the effect of uncertainty in nodal demands and pipe friction factors on pipe flows in the WDN using impact table approach. Karmakar [8] carried out hydraulic analysis with different possible combination. From these solutions, maximum and minimum values of dependent parameter were selected to form the membership function. Sivakumar et al. [18] adopted similar approach and named it as vertex approach, but this remains feasible for smaller network. Shibu and Reddy [17] used cross-entropy (CE) method for WDN analysis under fuzzy demand. Spiliotis and Tsakiris [19] opted Newton–Raphson method with fuzzy demands for WDN analysis, and Dongre and Gupta [4] obtained a reliable solution by transforming a fuzzy constrained optimization model into a deterministic model by taking the relationship between fuzzy demands and fuzzy nodal heads. Moosavian and Lence [13] opted an approximate approach for the analysis. The paper suggested that as nodal demand increases, pressure head decreases, and based on this, the maximum and minimum values of nodal demand were obtained.

For uncertainty analysis of WDN, the uncertain parameters are either treated as random or fuzzy. However, as the water demand is uncertain due to both its random nature and insufficient knowledge about the same, recently, some of the studies have considered simultaneous representation of randomness and fuzziness in the consolidate framework [5, 16]. Fuzzy random theory is observed to be one of advanced method that has been appeared as a valuable tool to deal with probabilistic problems involving fuzzy data [8]. Recently, Fu and Kapelan [5] and Shibu and Reddy [16] considered water demand as FRV in multi-objective optimization problem of minimizing the cost and maximizing the reliability. While Fu and Kapelan [5] opted GA and Shibu and Reddy [16] opted CE as optimization tool. The methodology resulted in cost-effective reliable design.

The aim of this paper is to present the uncertainty analysis of WDN through fuzzy random approach. The nodal demand is considered to be uncertain parameter and a benchmark network of two loops is solved.

2 Methodology

2.1 Basic Concept of Fuzzy Random Approach

The FRV was first discovered by Kwakernaak [9]. The outcome of the random experiments is considered as fuzzy number instead of the crisp real value. He observed that both fuzzy and random approaches are fundamentally different from each other, as fuzzy is due to lack of data while random is based upon huge statistical information. The approach holds good for uncertainties which are partially random and partially fuzzy in nature. Hence, a lot of efforts have been made [11, 12] for an individual but simultaneous representation of both the uncertainties and is referred as fuzzy random approach and such variables are termed as fuzzy random variables (FRV) [14].

The concept of FRV is developed by combining the concept of both random and fuzzy approaches. The approach can be considered as a mapping from probability space to fuzzy numbers. This can be shown as

$$\xi : \Omega \rightarrow F(\Re) \tag{1}$$

In case of random approach, uncertain parameter x is shown either by probability distribution function (PDF) or cumulative distribution function (CDF). These PDF/CDF can be obtained using various approaches such as MCS, LHS, FOSM, FORM as shown in Fig. 1a. While in case of fuzzy approach, any uncertain parameter x is represented by the membership function $\mu(x)$ where its values ranges between 0 and 1. The most commonly used membership function is triangular one and is mostly represented by triplet (a, b, c), where $[a, c]$ represents the support and $[b]$ represents the kernel of membership function. The α cut of uncertain parameter is expressed as set containing all values x having membership degree between $\alpha \in [0,1]$ as shown in Fig. 1b.

The approach first uses the probabilistic approach in order to get the improved cumulative probability spread. These new probability is spread is now been projected

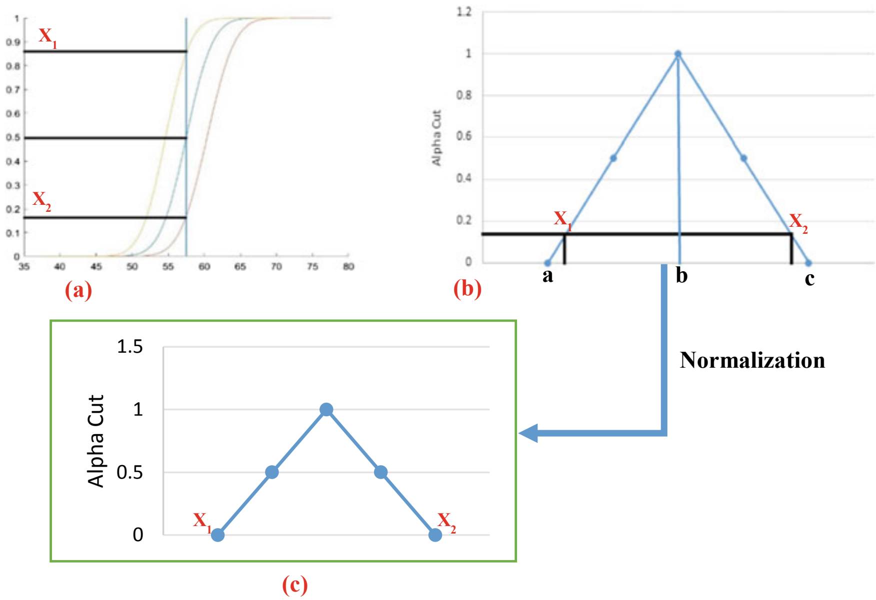

Fig. 1 Layout of fuzzy random representation. **a** CDF of input parameter drawn with the help of MCS. **b** Membership function of uncertain parameter drawn with impact table approach and the improved probability plot are shown in the same. **c** Normalized/new membership function plot considering both randomness and fuzziness

to fuzzy membership function. After the normalization, the output of the dependent parameter is achieved. The layout of same is explained in Fig. 1.

2.2 Application of Fuzzy Random Approach for Future Water Demand

Water demand being the highly uncertain parameter involved in the analysis and design of WDN, as it is affected by various factors such as population growth, climate change, socio-economic changes, etc. [14]. These factors affect the water demand forecasting accuracy. Hence, correct prediction of water demand both in short-term or long-term time horizon has become really a challenge.

Uncertainty in the water demand has to be considered while designing the system so as to have more reliable design. Probabilistic approaches considers only the random uncertainty in water demand such as, Lansey et al. [10], Kapelan et al. [7], Xu and Goulter [20] and Babayan et al. [1] used the used the probabilistic approach considering normally distributed PDF. Later, Revelli and Ridolfi [15], Gupta and Bhave [6], used fuzzy approach where water demand is expressed using triangular or trapezoidal membership function. Both these approaches hold good only for individual uncertainty, i.e., either random or fuzzy. But the water demand becomes uncertain due to both random and fuzzy events. Hence, considering both simultaneously is of great importance.

The use of FRV to characterize the nodal demand can be defined as fuzzy number serves as prior knowledge in determining the parameter of PDF or CDF. For instance, the mean of the CDF of future water demand cannot be known accurately; hence, it is better shown by a fuzzy number. While solving the problem, the nodal demand at each node is considered to be normally distributed with fuzzy mean and standard deviation as 10% original demand. The fuzzy mean is represented by the triangular membership function, where kernel is having the original demand and certain deviation of this original demand as support. As shown in Fig. 1, the core of the CDF is the fuzzy mean and the lower and upper envelops are the probability bands. These probability bands provide the cumulative probability spreads by combining both the uncertainty. With this new probability spread, the membership function is normalized again and the output of dependent parameter is obtained at various α-cut values.

A C programme is developed to carry out the analysis and it is linked to EPANET as hydraulic solver.

3 Methodology

- Decide the uncertain parameter and its uncertainty to be considered. Also, the mean and standard deviation of input uncertain parameter.
- With this mean and % uncertainty, plot the membership function for water demand.
- Plot the CDF for the water demand with the given fuzzy mean and standard deviation along with two probability bounds, through MCS or LHS. (Core of the CDF family shows the highest possibility ($\alpha = 1$) and the lower and upper bounds reveal the envelop of CDF ($\alpha = 0$), as shown in Fig. 1a
- Obtained cumulative probability spread at most likely value 0.5 is then projected on the fuzzy membership function.
- Represent the fuzzy sets for the obtained probability spread in triangular form (a,b,c) and normalize with maximum value at 1 and minimum at 0, as shown in Fig. 1b.
- The membership function for water demand is modified with the obtained probability spread after normalization as shown in Fig. 1c.
- Obtain the membership function for output parameter (nodal heads and pipe flows) through impact table approach at different α-cut (i.e., 5 α-cut values, 0, 0.2, 0.4, 0.6, 0.8, 1).

4 Example Network

A Two-loop WDN used for the study is opted from Revelli and Ridolfi [15] as shown in Fig. 2. The network consists of a source node fixed at 100 m head and three demand nodes 2, 3, and 4. The network parameter values are given in Table 1. The direction of flow assumed is also shown in Fig. 1. The source head, nodal head, and roughness coefficient of pipes have precise value with no uncertainty. The nodal demand (q) is considered to be the uncertain. The future water demand at each node is assumed to follow normal probability distribution with fuzzy mean and standard deviation as

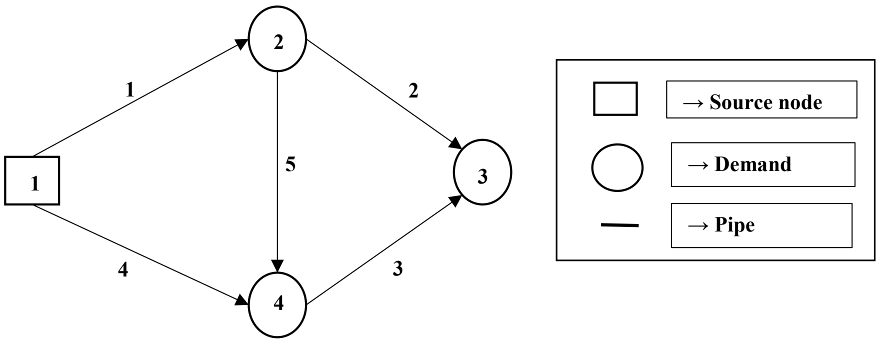

Fig. 2 Layout of two-loop water distribution network

Table 1 Pipe data of two-loop WDN

Pipe	Length (m)	Diameter(m)
1	1200	0.50
2	1100	0.50
3	1500	0.50
4	900	0.35
5	1000	0.35

Table 2 Node data of two-loop WDN

Node No.	Nodal demand (cumec)			Heads (m)
	Min	Nor	Max	
1	0	0	0	100
2	0.135	0.15	0.165	0
3	0.27	0.3	0.33	0
4	0.207	0.23	0.253	0

$\pm 10\%$ of original demand. The fuzzy mean represented as a triangular fuzzy number with kernel at its original demand and $\pm 5\%$ variation at its support, respectively.

Input Parameters

See Table 2.

5 Results and Discussion

The output of the analysis is shown in Tables 3 and 4. Table 3 shows the fuzzy and FRV values of pipe flow for all the pipes and Table 4 shows the fuzzy and FRV values of head for every demand nodes for each α-cut level. These values are used to form the membership function of the respective parameters. On comparing the results obtained by fuzzy and FRV approaches, results are found to be different. FRV considers both the uncertainty, i.e., fuzzy and random together, which leads to reduction in the spread of uncertainty. This proportionally leads to reduction in the values of dependent parameter when compared with fuzzy.

The comparison of membership functions of the discharge in all the pipes and head at demand nodes for case study are shown in Figs. 3 and 4. It can be observed

Table 3 Comparison of fuzzy and FRV values of pipe flow (lps) for different α-cut values

α-cut	Fuzzy		FRV		
	Max	Min	Max	Min	
0	Q[1] = 497.548	Q[1] = 407.085	Q[1] = 474.932	Q[1] = 429.700	
	Q[2] = 248.156	Q[2] = 194.734	Q[2] = 234.796	Q[2] = 208.083	
	Q[3] = 102.515	Q[3] = 53.463	Q[3] = 90.672	Q[3] = 66.168	
	Q[4] = 250.452	Q[4] = 204.915	Q[4] = 239.068	Q[4] = 216.300	
	Q[5] = 92.174	Q[5] = 69.458	Q[5] = 86.540	Q[5] = 75.187	
0.2	Q[1] = 488.501	Q[1] = 416.131	Q[1] = 470.409	Q[1] = 434.224	
	Q[2] = 242.812	Q[2] = 200.073	Q[2] = 232.124	Q[2] = 210.753	
	Q[3] = 97.809	Q[3] = 58.584	Q[3] = 88.272	Q[3] = 68.671	
	Q[4] = 245.899	Q[4] = 209.469	Q[4] = 236.791	Q[4] = 218.576	
	Q[5] = 89.924	Q[5] = 71.754	Q[5] = 85.410	Q[5] = 76.328	
0.4	Q[1] = 474.027	Q[1] = 430.605	Q[1] = 463.172	Q[1] = 441.461	
	Q[2] = 234.261	Q[2] = 208.617	Q[2] = 227.849	Q[2] = 215.027	
	Q[3] = 90.192	Q[3] = 66.670	Q[3] = 84.409	Q[3] = 72.650	
	Q[4] = 238.613	Q[4] = 216.755	Q[4] = 233.148	Q[4] = 222.219	
	Q[5] = 86.314	Q[5] = 75.415	Q[5] = 83.600	Q[5] = 78.151	
0.6	Q[1] = 461.001	Q[1] = 443.632	Q[1] = 456.658	Q[1] = 447.974	
	Q[2] = 226.567	Q[2] = 216.309	Q[2] = 224.002	Q[2] = 218.873	
	Q[3] = 83.245	Q[3] = 73.838	Q[3] = 80.909	Q[3] = 76.206	
	Q[4] = 232.055	Q[4] = 223.312	Q[4] = 229.870	Q[4] = 225.498	
	Q[5] = 83.056	Q[5] = 78.697	Q[5] = 81.968	Q[5] = 79.788	
0.8	Q[1] = 454.053	Q[1] = 450.579	Q[1] = 453.185	Q[1] = 451.448	
	Q[2] = 222.463	Q[2] = 220.412	Q[2] = 221.950	Q[2] = 220.925	
	Q[3] = 79.502	Q[3] = 77.621	Q[3] = 79.033	Q[3] = 78.092	
	Q[4] = 228.558	Q [4] = 226.810	Q[4] = 228.121	Q [4] = 227.247	
	Q[5] = 81.315	Q[5] = 80.443	Q[5] = 81.097	Q[5] = 80.661	
1	**Q[1]= 452.316**	**Q[2]= 221.437**	**Q[3]= 78.563**	**Q[4]= 227.684**	**Q[5]= 80.87**

from these figures that the results obtained through FRV are n lower side as fuzzy. The FRV is more appropriate as it combines random and fuzzy uncertainty together.

Discussion

The paper presents the uncertainty-based analysis of WDNs using fuzzy random approach. The uncertainty in the nodal demand is shown by the fuzzy random variable, where it merges the fuzzy and random uncertainty of demands. The MCS method is used for the random sampling of demand and the membership functions are characterized using impact table approach. After combining both the uncertainties, it is observed that there is reduction in the final output values of pipe flow and nodal heads when compared with fuzzy. Hence, this leads to more economical and

Table 4 Comparison of fuzzy and FRV values of head (m) for different α-cut values

α-cut	Fuzzy		FRV	
	Max	Min	Max	Min
0	H[2] = 90.000	H[2] = 85.498	H[2] = 88.946	H[2] = 86.695
	H[3] = 87.557	H[3] = 81.957	H[3] = 86.247	H[3] = 83.446
	H[4] = 88.046	H[4] = 82.665	H[4] = 86.787	H[4] = 84.096
0.2	H[2] = 89.584	H[2] = 85.983	H[2] = 88.730	H[2] = 86.929
0.2	H[3] = 87.041	H[3] = 82.560	H[3] = 85.978	H[3] = 83.737
	H[4] = 87.549	H[4] = 83.245	H[4] = 86.528	H[4] = 84.376
0.4	H[2] = 88.903	H[2] = 86.742	H[2] = 88.379	H[2] = 87.299
	H[3] = 86.193	H[3] = 83.505	H[3] = 85.542	H[3] = 84.197
	H[4] = 86.736	H[4] = 84.152	H[4] = 86.110	H[4] = 84.818
0.6	H[2] = 88.273	H[2] = 87.409	H[2] = 88.060	H[2] = 87.628
	H[3] = 85.410	H[3] = 84.334	H[3] = 85.144	H[3] = 84.606
	H[4] = 85.983	H[4] = 84.950	H[4] = 85.728	H[4] = 85.211
0.8	H[2] = 87.931	H[2] = 87.758	H[2] = 87.888	H[2] = 87.801
	H[3] = 84.984	H[3] = 84.769	H[3] = 84.930	H[3] = 84.823
	H[4] = 85.574	H[4] = 85.367	H[4] = 85.522	H[4] = 85.419
1	**H**[2]**= 87.845**	**H**[3]**= 84.876**	**H**[4]**= 85.470**	

cost-effective design even after considering the uncertainty in the input parameter. The methodology is observed to be very much useful in engaging the effect of both the uncertainties in the analysis of WDNs.

6 Conclusion

- Method is a hybrid uncertainty characterization approach considering both fuzzy and random uncertainty.
- Such hybrid approach leads to cost-effective design while considering reliability aspect.
- It can be helpful in long-term planning and design of WDN.
- One drawback of the methodology is its computational burden and excess time requirement.

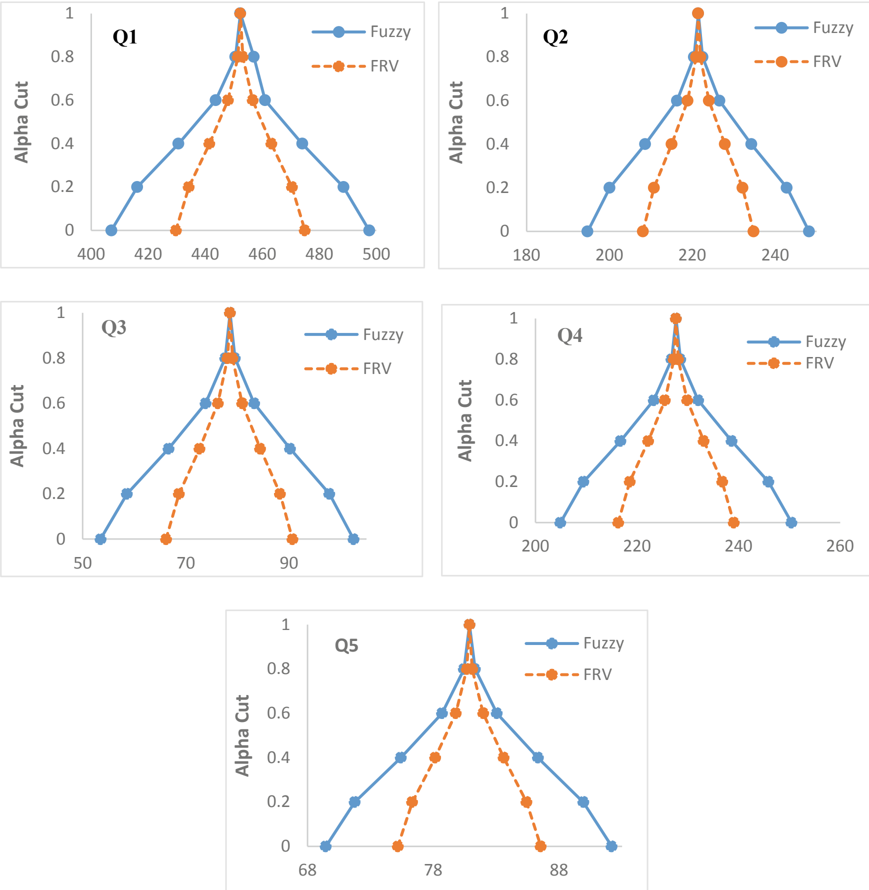

Fig. 3 Membership functions of pipe flows (lps) using fuzzy and FRV

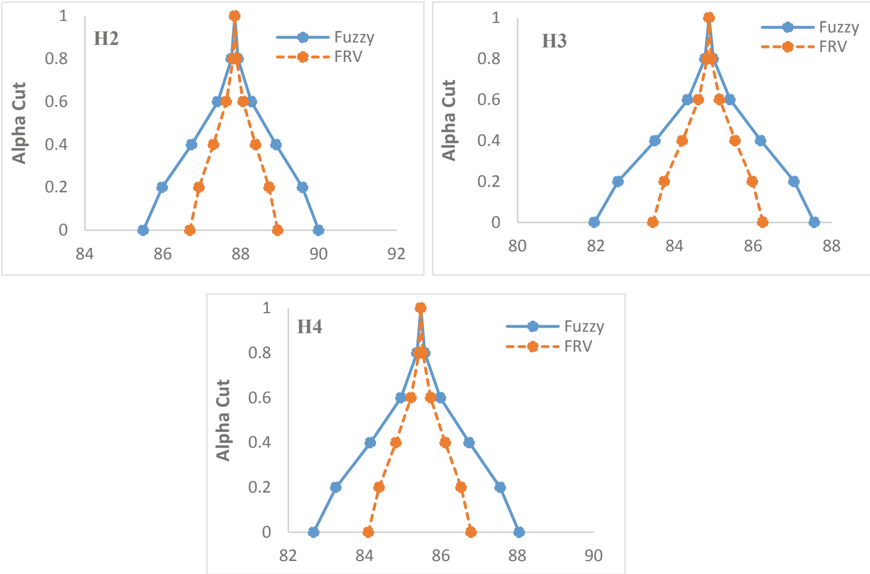

Fig. 4 Membership function of head (m) at every nodes using fuzzy and FRV

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