Chapter 7
Effective Lockdown and Plasma Therapy for COVID-19

Nita H. Shah, Nisha Sheoran, and Ekta N. Jayswal

Abstract COVID-19 is a major pandemic threat of 2019–2020 which originated in Wuhan. As of now, no specific anti-viral medication is available. Therefore, many countries in the world are fighting to control the spread by various means. In this chapter, we model COVID-19 scenario by considering compartmental model. The set of dynamical system of nonlinear differential equation is formulated. Basic reproduction number R_0 is computed for this dynamical system. Endemic equilibrium point is calculated and local stability for this point is established using Routh-Hurwitz criterion. As COVID-19 has affected more than 180 countries in several ways like medically, economy, etc. It necessitates the effect of control strategies applied by various government worldwide to be analysed. For this, we introduce different types of time dependent controls (which are government rules or social, medical interventions) in-order to control the exposure of COVID-19 and to increase recovery rate of the disease. By using Pontryagins maximum principle, we derive necessary optimal conditions which depicts the importance of these controls applied by the government during this epidemic.

Keywords COVID-19 · Basic reproduction number · Local stability · Optimal control

Mathematics Subject Classification 37Nxx

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Introduction

As of 3 May 2020, the countries affected by COVID-19 are suffering major loss in terms of economy (globally) and also many workers are losing their jobs. So far, the number of cases reported on 3 May 2020 are more than 3.24 million across 187 countries and territories, resulting in more than 243,000 deaths [1]. COVID-19 is type of virus that infects the respiratory system of humans. It originated in Wuhan (China) on 31 December 2019. It is highly contagious with the reproduction number 6.47 calculated by Tang et al. [2] (as on 22 January 2020). Being a major public health threat declared by WHO [3], it is necessary to control the pandemic by understanding early dynamics of transmission of disease in china which has been discussed by Kucharski et al. [4]. Since no pharmaceutical treatment is available, interventions such as complete ban on air travel, shutting down of educational institutions, enforcing lockdown in the entire country, social distancing as studied by Prem et al. [5], random testing at large scale studied by Mueller et al. [6] and by isolating cases of COVID-19 and there contacts (Hellewell et al. [7]) have helped some of the countries like China, Hong Kong to control the transmission of COVID-19.

Further to understand the spread of COVID-19 and to study the effect of various interventions measures adopted by individuals and government, compartmental modelling is significant. Some authors like Toda [8] developed basic SIR model to study the effectiveness of social distancing in reducing the spread, Peng et al. [9] developed SEIR compartmental model to study epidemics of COVID-19 in China. Also Tang et al. [10] modified SEIR model for new prediction of COVID-19. Piguillem et al. [11] extended standard SIR model to study the importance of rigours testing and concluded mandatory quarantine can bring world close to what is considered as optimal. Some of the early research work with modelling of COVID-19 to understand disease dynamics in various countries includes: study by Sun et al. [12], discussed the various characteristic to COVID-19 situation in china which helps in understanding the fatality rate and transmission rate of COVID-19 so as to help in controlling the epidemic spread, the importance of travel quarantine or travel restriction in Wuhan was studied by Chinazzi et al. [13]. Other related researchers include Zhao and Chen [14], Xu et al. [15], Yang et al. [16] etc.

Now, as the world is very well aware of COVID-19 and everywhere the respective government is carrying out necessary measures to control the spread or human to human transmission of COVID-19. The best way to visualize the importance of measures been taken is to analyse it by introducing optimal control theory using Pontryagins maximum principle [17] into the model. Some of the previous research includes: Sharomi and Malik [18] have discussed very nicely optimal control in epidemiology by considering various compartmental models, Lemos-Paião et al. [19] have also applied optimal control theory showing treatment of cholera with quarantine effects, Tilahun et al. [20] applied optimal control to pneumonia disease, etc. Similarly, in COVID-19 scenario also optimal control is applied by various authors like Djidjou-Demassea et al. [21] formulated a model to minimize the death and the cost by applying control until the vaccines arrives as it will take near about 18 months.

Mallela [22] also applied optimal control theory by taking social distancing as the control in his model. Also, Tsay et al. [23] have modelled COVID-19 outbreak in USA with optimal control theory, etc.

In this chapter, our target is to predict the importance of various control strategies such as lockdown, curfew, viral load testing, plasma therapy, etc., adopted by the government in COVID-19 environment, by introducing these, measures as time dependent controls into the model and using Pontryagins theory, we will be obtaining optimal control conditions. We will also simulate through trajectories the situation with and without control in an exposed environment.

This chapter is organized as follows: Sect. Formulation of Mathematical Model describes the formulation of mathematical model and calculation of its equilibrium points. In Sect. Basic Reproduction Number, basic reproduction number is computed. In Sect. Stability Analysis, local stability of the equilibrium point is established. In Sect. Optimal Control, we develop optimal control theory by taking various controls into the model and calculate optimality conditions. The results of optimal control and other numerical simulation are discussed in Sect. Numerical Simulation. Finally, the findings are summarized with conclusion in last Sect. Conclusion.

Formulation of Mathematical Model

This study considers formulation of mathematical model of COVID-19 dividing human population into eight mutually exclusive compartmental model. The compartments taken into account are exposed class E_{CO}, identified population I_F, isolated population I_{SO}, test T_E—it is taken as the number of test done so far including both positive and negative test, population in COVID-19 care centre C, population with COVID-19 in hospital H, Home quarantined population Q and recovered population R.

The parametric definitions and values used in formulation of this dynamical system are given by Table 7.1.

Here, we develop a mathematical model starting with the exposure stage of COVID-19, i.e. individuals those who are exposed to COVID-19 or are in surrounding of COVID-19 infectives are considered to be in this compartment also new recruitments to this class occur at the rate B. Out of this exposed class, COVID-19 infected individuals (both symptomatic and asymptomatic, where asymptomatic are those with less clinical symptoms such as fever, fatigue etc.) are identified joining the compartment I_F at the rate β_1. After this, the identified population is isolated (I_{SO}) at the rate β_2. Isolation of asymptomatic infectives is a vital strategy in containing the spread of COVID-19. Next, isolated population is then tested through viral load test for COVID-19 by laboratories and this tested population is contained in T_E class at the rate β_3. Here, if the population is tested positive for COVID-19 then, we again sub-divide this positive tested population into two classes as population in COVID-19 care centre C and hospitalized population H. Here, we assume that if the positive tested population is not in need of emergency medical treatment and is not severe it

Table 7.1 Parametric definitions and its values [*source* own]

Notations	Description	Parametric values
B	Birth rate	0.01
β_1	Rate at which population exposed to COVID-19 is been identified	0.0009
β_2	Rate at which identified population is isolated	0.0086
β_3	Rate at which isolated population is tested	0.0059
β_4	Rate at which individuals joins COVID-19 care centre	0.0046
β_5	Rate at which individuals get admitted to hospital	0.0024
β_6	Rate at which individuals are quarantined after tested	0.0076
β_7	Rate at which individuals in COVID-19 care centre gets recovered	0.00006
β_8	Rate at which individuals in hospital gets recovered	0.007
β_9	Rate at which quarantined individual gets recovered	0.0001
μ	Natural morbidity rate	0.00009
μ_{CO}	Morbidity rate due to COVID-19	0.00029

goes to COVID-19 care centre with the rate β_4 and emergency situations get hospitalized at the rate β_5. The negative tested population is asked to home quarantine themselves (Q) at the rate β_6 which what the government is doing. Next, population from COVID-19 care centre, hospital and home quarantine are recovered at the rate β_7, β_8 and β_9, respectively. Also, μ, μ_{CO} are taken as the morbidity rates.

The following set of nonlinear differential equations is established form the Fig. 7.1.

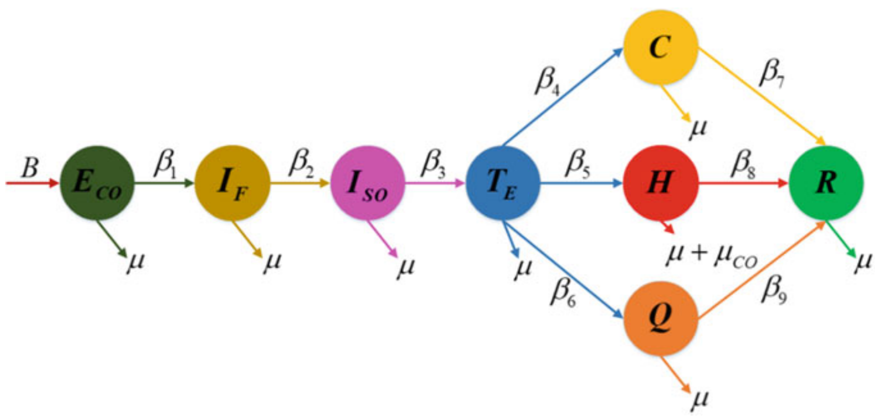


Fig. 7.1 Compartmental diagram showing flow of human population through different compartments [*source* own]

$$
\begin{aligned}
\frac{\mathrm{d}E_{CO}}{\mathrm{d}t} &= B - \beta_1 E_{CO} I_F - \mu E_{CO} \\
\frac{\mathrm{d}I_F}{\mathrm{d}t} &= \beta_1 E_{CO} I_F - (\beta_2 + \mu) I_F \\
\frac{\mathrm{d}I_{SO}}{\mathrm{d}t} &= \beta_2 I_F - (\beta_3 + \mu) I_{SO} \\
\frac{\mathrm{d}T_E}{\mathrm{d}t} &= \beta_3 I_{SO} - (\beta_4 + \beta_5 + \beta_6 + \mu) T_E \\
\frac{\mathrm{d}C}{\mathrm{d}t} &= \beta_4 T_E - (\beta_7 + \mu) C \\
\frac{\mathrm{d}H}{\mathrm{d}t} &= \beta_5 T_E - (\beta_8 + \mu + \mu_{CO}) H \\
\frac{\mathrm{d}Q}{\mathrm{d}t} &= \beta_6 T_E - (\beta_9 + \mu) Q \\
\frac{\mathrm{d}R}{\mathrm{d}t} &= \beta_7 C + \beta_8 H + \beta_9 Q - \mu R
\end{aligned}
\tag{7.1}
$$

where, $N = E_{CO} + I_F + I_{SO} + T_E + C + H + Q + R$.

The feasible region for the solutions of the system (7.1) is given by

$$
\Lambda = \left\{ \begin{array}{l} (E_{CO}, I_F, I_{SO}, T_E, C, H, Q, R);\ E_{CO} + I_F + I_{SO} + T_E + C + H + Q + R \le \dfrac{B}{\mu}, \\ E_{CO} > 0, I_F > 0, I_{SO} > 0, C > 0, H > 0, Q > 0, R > 0 \end{array} \right\}
$$

Equilibrium Solutions

Solving above system of equation, we get following equilibrium point

1. Disease-free equilibrium point $E^0\left(\frac{B}{\mu}, 0, 0, 0, 0, 0, 0, 0\right)$
2. Endemic equilibrium point $E^*(E^*_{CO}, I^*_F, I^*_{SO}, T^*_E, C^*, H^*, Q^*, R^*)$

where

$$E_{CO}^* = \frac{\beta_2 + \mu}{\beta_1}$$
$$I_F^* = \frac{B\beta_1 - \mu(\beta_2 + \mu)}{\beta_1(\beta_2 + \mu)}$$
$$I_{SO}^* = \frac{\beta_2(B\beta_1 - \mu(\beta_2 + \mu))}{\beta_1(\beta_2 + \mu)(\beta_3 + \mu)}$$
$$T_E^* = \frac{\beta_2\beta_3(B\beta_1 - \mu(\beta_2 + \mu))}{\beta_1(\beta_4 + \beta_5 + \beta_6 + \mu)(\beta_2 + \mu)(\beta_3 + \mu)}$$
$$C^* = \frac{\beta_2\beta_3\beta_4(B\beta_1 - \mu(\beta_2 + \mu))}{\beta_1(\beta_4 + \beta_5 + \beta_6 + \mu)(\beta_2 + \mu)(\beta_3 + \mu)(\beta_7 + \mu)}$$
$$H^* = \frac{\beta_2\beta_3\beta_5(B\beta_1 - \mu(\beta_2 + \mu))}{\beta_1(\beta_4 + \beta_5 + \beta_6 + \mu)(\beta_2 + \mu)(\beta_3 + \mu)(\beta_8 + \mu + \mu_{CO})}$$
$$Q^* = \frac{\beta_2\beta_3\beta_6(B\beta_1 - \mu(\beta_2 + \mu))}{\beta_1(\beta_4 + \beta_5 + \beta_6 + \mu)(\beta_2 + \mu)(\beta_3 + \mu)(\beta_9 + \mu)}$$
$$R^* = \frac{\begin{array}{c}\beta_2\beta_3(B\beta_1 - \mu(\beta_2 + \mu))((\beta_9 + \mu)(\beta_4\beta_7(\mu + \mu_{CO}) + \beta_5\beta_8(\beta_7 + \mu)) \\ +\beta_6\beta_9\mu_{CO}(\beta_7 + \mu) + (\beta_8 + \mu)(\beta_7\beta_9(\beta_4 + \beta_6) + \beta_6\beta_9\mu))\end{array}}{\beta_1\mu(\beta_4 + \beta_5 + \beta_6 + \mu)(\beta_2 + \mu)(\beta_3 + \mu)(\beta_7 + \mu)(\beta_8 + \mu + \mu_{CO})(\beta_9 + \mu)}$$

Basic Reproduction Number

Basic reproduction number R_0 is defined as number of secondary infections produced due to a single infection in a completely susceptible population. Basic reproduction number plays a significant role in determining the disease spread and in developing control strategies.

Basic reproduction is derived using next-generation matrix method by Diekmann et al. [24]. Here, F is the Jacobian matrix of the new recruitments in the population and V is the Jacobian matrix of the new transfer of exposed individuals from one compartment to another.

$$F = \begin{bmatrix} \beta_1 E_{CO} & 0\ 0\ 0\ 0\ 0\ 0 & \beta_1 I_F \\ 0 & 0\ 0\ 0\ 0\ 0\ 0 & 0 \\ 0 & 0\ 0\ 0\ 0\ 0\ 0 & 0 \\ 0 & 0\ 0\ 0\ 0\ 0\ 0 & 0 \\ 0 & 0\ 0\ 0\ 0\ 0\ 0 & 0 \\ 0 & 0\ 0\ 0\ 0\ 0\ 0 & 0 \\ 0 & 0\ 0\ 0\ 0\ 0\ 0 & 0 \\ 0 & 0\ 0\ 0\ 0\ 0\ 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} -\beta_2-\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\beta_2 & \beta_3+\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta_3 & \beta_4+\beta_5+\beta_6+\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_4 & \beta_7+\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_5 & 0 & \beta_8+\mu+\mu_{CO} & 0 & 0 & 0 \\ 0 & 0 & -\beta_6 & 0 & 0 & \beta_9+\mu & 0 & 0 \\ 0 & 0 & 0 & -\beta_7 & -\beta_8 & -\beta_9 & \mu & 0 \\ \beta_1 E_{CO} & 0 & 0 & 0 & 0 & 0 & 0 & \beta_1 I_F+\mu \end{bmatrix}$$

The reproduction number R_0 is the spectral radius of FV^{-1} evaluated at $E^0\left(\frac{B}{\mu}, 0, 0, 0, 0, 0, 0, 0\right)$ and is given by the expression $R_0 = \frac{B\beta_1}{(\beta_2+\mu)\mu}$.

Here, R_0 gives the number of newly exposed individual to COVID-19 due to single exposure in a population which has been calculated as 11.5 using data from Table 7.1. This shows that an individual who is exposed to COVID-19 through any mode of transmission of disease via infected individual exposes 12 more individuals. Here, if $R_0 < 1$, it means that the exposure to COVID-19 is deteriorating which indicates the die out situation of COVID-19. This is stage which the world has not yet achieved. And $R_0 > 1$ shows the existence of endemic equilibrium point. Which is the scenario as off 3rd May 2020. In the next section, we will discuss the local stability of endemic point only.

Stability Analysis

Here, we study local stability of endemic equilibrium point using Routh-Hurwitz criterion.

Theorem 1 *The equilibrium point E^* is locally asymptotically stable if $(\beta_2+\mu) > \beta_1 E^*_{CO}$.*

Proof The Jacobian matrix for the dynamical system (7.1) is given by

$$J^* = \begin{bmatrix} -a_{11} & -\beta_1 E^*_{CO} & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 I^*_F & -a_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & -a_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & -a_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_4 & -a_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_5 & 0 & -a_{66} & 0 & 0 \\ 0 & 0 & 0 & \beta_6 & 0 & 0 & -a_{77} & 0 \\ 0 & 0 & 0 & 0 & \beta_7 & \beta_8 & \beta_9 & -a_{88} \end{bmatrix}$$

where

$a_{11} = \beta_1 I^*_F + \mu, a_{22} = -\beta_1 E^*_{CO} + (\beta_2+\mu),$

$a_{33} = \beta_3 + \mu, a_{44} = \beta_4 + \beta_5 + \beta_6 + \mu, a_{55} = \beta_7 + \mu,$

$a_{66} = \beta_8 + \mu + \mu_{CO}, a_{77} = \beta_9 + \mu, a_{88} = \mu.$

Here, trace$(J^*) = -(a_{11} + a_{22} + a_{33} + a_{44} + a_{55} + a_{66} + a_{77} + a_{88}) < 0$ and det$(J^*) > 0$ if $(\beta_2 + \mu) > \beta_1 E^*_{CO}$. Hence, by Routh-Hurwitz criterion [25], the endemic equilibrium point is locally asymptotically stable if $(\beta_2 + \mu) > \beta_1 E^*_{CO}$.

Optimal Control

In this section, we consider different measures adopted by government as control in order to study the effectiveness of this model. Here, we apply Pontryagins maximum principle [17] in order to determine necessary conditions for optimality by introducing time dependent controls in the system (7.1). The introduced controls as follows can be observed in Fig. 7.2: u_1 and u_2 are taken as lock down and curfew control in order to restrict the exposure to COVID-19, u_3 as viral load test which detects and measures virus level consistently in COVID-19 infected patient, u_4 is a control which allows more and more individuals to opt for COVID-19 care centre, u_5 a control which allows only emergency medical patients get into the hospital and u_6 as plasma therapy in-order to increase recovery rate of hospitalized patients.

And the modified system (7.1) with controls is rewritten as

$$\begin{aligned}
\frac{\mathrm{d}E_{CO}}{\mathrm{d}t} &= B - \beta_1 E_{CO} I_F - \mu E + u_1 I_F \\
\frac{\mathrm{d}I_F}{\mathrm{d}t} &= \beta_1 E_{CO} I_F - (\beta_2 + \mu) I_F - u_1 I_F + u_2 I_{SO} \\
\frac{\mathrm{d}I_{SO}}{\mathrm{d}t} &= \beta_2 I_F - (\beta_3 + \mu) I_{SO} - u_2 I_{SO} - u_3 I_{SO} \\
\frac{\mathrm{d}T_E}{\mathrm{d}t} &= \beta_3 I_{SO} - (\beta_4 + \beta_5 + \beta_6 + \mu) T_E - u_4 T_E + u_5 H + u_3 I_{SO}
\end{aligned}$$

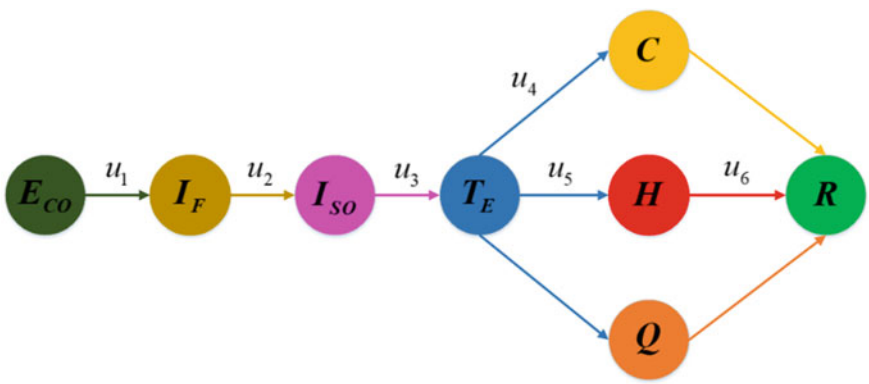


Fig. 7.2 Optimal controls applied to the Fig. 7.1 [*source* own]

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \beta_4 T_E - (\beta_7 + \mu)C + u_4 T_E$$
$$\frac{\mathrm{d}H}{\mathrm{d}t} = \beta_5 T_E - (\beta_8 + \mu + \mu_{CO} + u_5 + u_6)H$$
$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \beta_6 T_E - (\beta_9 + \mu)Q$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = \beta_7 C + \beta_8 H + \beta_9 Q - \mu R + u_6 H$$

For this, we consider following objective function

$$\begin{aligned} J(c_i, \Lambda) = \int_0^T \big(& W_1 E_{CO}^2 + W_2 I_F^2 + W_3 I_{SO}^2 + W_4 T_E^2 + W_5 C^2 \\ & + W_6 H^2 + W_7 Q^2 + W_8 R^2 + v_1 u_1^2 + v_2 u_2^2 \\ & + v_3 u_3^2 + v_4 u_4^2 + v_5 u_5^2 + v_6 u_6^2 \big) \, \mathrm{d}t \end{aligned}$$

The control functions u_1, u_2, u_3, u_4, u_5 and u_6 are bounded, Lebesgue integrable functions. Here, Λ denotes the set of all compartmental variables. The coefficients $W_1, W_2, W_3, W_4, W_5, W_6, v_1, v_2, v_3, v_4, v_5, v_6$ are the balancing cost functions.

Now, we seek to find out $u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*$ for the time $t = 0$ to $t = T$ such that

$$J(u_i(t)) = \min\{J(u_i^*, \Lambda)/(u_i) \in \phi\}, \quad i = 1, 2, 3, 4, 5, 6$$

where ϕ is a smooth function on the interval [0, 1].

Next, we introduce the Lagrangian function as follows

$$\begin{aligned} L(u, \lambda) = {} & W_2 I_F^2 + W_3 I_{SO}^2 + W_4 T_E^2 + W_5 C^2 + W_6 H^2 + W_7 Q^2 + W_8 R^2 \\ & + v_1 u_1^2 + v_2 u_2^2 + v_3 u_3^2 + v_4 u_4^2 + v_5 u_5^2 + v_6 u_6^2 \end{aligned}$$

To obtain the value of Lagrangian function, we define Hamiltonian H for the optimal control as

$$\begin{aligned} H = {} & W_2 I_F^2 + W_3 I_{SO}^2 + W_4 T_E^2 + W_5 C^2 + W_6 H^2 + W_7 Q^2 \\ & + W_8 R^2 + v_1 u_1^2 + v_2 u_2^2 + v_3 u_3^2 + v_4 u_4^2 + v_5 u_5^2 \\ & + v_6 u_6^2 + \lambda_1 (B - \beta_1 E_{CO} I_F - \mu E + u_1 I_F) \\ & + \lambda_2 (\beta_1 E_{CO} I_F - (\beta_2 + \mu) I_F - u_1 I_F + u_2 I_{SO}) + \lambda_3 (\beta_2 I_F \\ & - (\beta_3 + \mu) I_{SO} - u_2 I_{SO} - u_3 I_{SO}) \\ & + \lambda_4 (\beta_3 I_{SO} - (\beta_4 + \beta_5 + \beta_6 + \mu) T_E - u_4 T_E + u_5 H + u_3 I_{SO}) \\ & + \lambda_5 (\beta_4 T_E - (\beta_7 + \mu) C + u_4 T_E) \\ & + \lambda_6 (\beta_5 T_E - (\beta_8 + \mu + \mu_{CO} + u_5 + u_6) H) \end{aligned}$$

$$+ \lambda_7(\beta_6 T_E - (\beta_9 + \mu)Q)$$
$$+ \lambda_8(\beta_7 C + \beta_8 H + \beta_9 Q - \mu R + u_6 H)$$

Now using Pontryagins maximum principle and existence condition discussed by Fleming and Rishel [26], we obtain adjoint equations for the adjoint variable $\lambda_i = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8)$ associated with the state variables $(E_{CO}, I_F, I_{SO}, T_E, C, H, Q, R)$

$$\dot{\lambda}_1 = -2W_1 E_{CO} + (\lambda_1 - \lambda_2)\beta_1 I_F + \lambda_1 \mu$$

$$\dot{\lambda}_2 = -2W_2 I_F + (\lambda_1 - \lambda_2)\beta_1 E_{CO} + (\lambda_2 - \lambda_1)u_1 + (\lambda_2 - \lambda_3)\beta_2 + \lambda_2 \mu$$

$$\dot{\lambda}_3 = -2W_3 I_{SO} + (\lambda_3 - \lambda_2)u_2 + (\lambda_3 - \lambda_4)(\beta_3 + u_3) + \lambda_3 \mu$$

$$\dot{\lambda}_4 = -2W_4 T_E + \beta_5(\lambda_4 - \lambda_6) + \beta_6(\lambda_4 - \lambda_7) + (\lambda_4 - \lambda_5)(\beta_4 + u_4) + \lambda_4 \mu$$

$$\dot{\lambda}_5 = -2W_5 C + (\lambda_5 - \lambda_8)\beta_7 + \lambda_5 \mu$$

$$\dot{\lambda}_6 = -2W_6 H + (\lambda_6 - \lambda_4)u_5 + (\lambda_6 - \lambda_8)(u_6 + \beta_8) + (\mu + \mu_C)\lambda_6$$

$$\dot{\lambda}_7 = -2W_7 Q + (\lambda_8 - \lambda_7)\beta_9 + \lambda_7 \mu$$

$$\dot{\lambda}_8 = -2W_8 R + \lambda_8 \mu$$

The optimality conditions for control are given by
$u_1^* = \max\left(a_1, \min\left(b_1, \frac{I_F(\lambda_2-\lambda_1)}{2v_1}\right)\right)$, $u_2^* = \max\left(a_2, \min\left(b_2, \frac{I_{SO}(\lambda_3-\lambda_2)}{2v_2}\right)\right)$,
$u_3^* = \max\left(a_3, \min\left(b_3, \frac{I_{SO}(\lambda_3-\lambda_4)}{2v_3}\right)\right)$, $u_4^* = \max\left(a_4, \min\left(b_4, \frac{T_E(\lambda_4-\lambda_4)}{2v_4}\right)\right)$,
$u_5^* = \max\left(a_5, \min\left(b_5, \frac{H(\lambda_6-\lambda_4)}{2v_5}\right)\right)$ and $u_6^* = \max\left(a_6, \min\left(b_6, \frac{H(\lambda_6-\lambda_8)}{2v_6}\right)\right)$.
In next section, we will discuss these controls through plot.

Numerical Simulation

In this section, simulation is carried out to understand the compartmental model of COVID-19. Here, we also discuss the various controlling interventions applied to the system (7.1). The data taken in Table 7.1 has been calculated and assumed

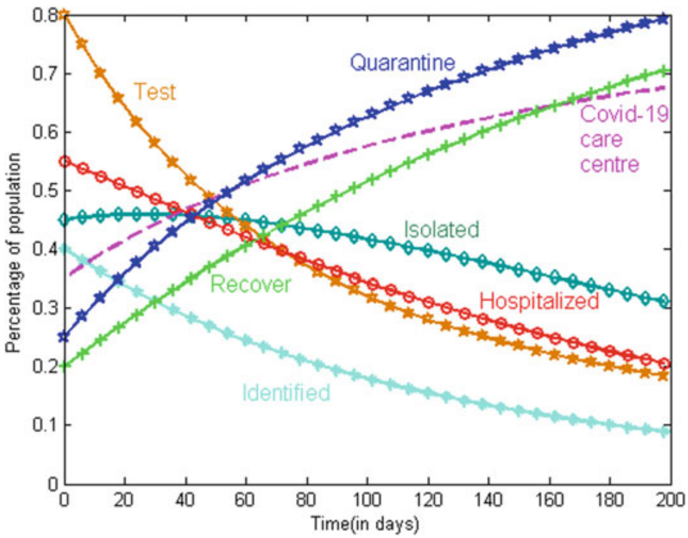


Fig. 7.3 Transmission dynamics of COVID-19 [*source* own]

accordingly to the current scenario and can be found on https://ourworldindata.org/covid-testing [27].

In Fig. 7.3, we plot trajectory of all the compartments taken in our model. The path of various compartment can be observed showing behaviour of the model. Numerically, we observe that approximately 40% of population exposed to COVID-19 when tested shows positive report and is hospitalized within 72 days. About 35% of identified population adopts quarantine law. It is also seen that individual hospitalized gets recover at a faster rate than the individuals in COVID-19 care centre. Again, approximately 49% of exposed population when tested, shows negative report and is asked to home quarantine themselves. From Fig. 7.3, we also observe with time hospitalization decreases and individuals in quarantine and care centres increases.

The system (7.1) is said to exhibit oscillatory behaviour observed in Fig. 7.4. We see that Fig. 7.4a–c, i.e. exposed to COVID-19, identified and isolation, respectively, these compartments gets stabilized in due course of time. Exposed population initially decreases as seen in Fig. 7.4a which leads increase in identified (Fig. 7.4b) and isolation (Fig. 7.4b) population. Figure 7.4d–f oscillates as number of tests, individuals in COVID-19 care centre and individuals in hospital fluctuates with time. Quarantine population with the fluctuation decreases with time observed in Fig. 7.4g and recovery rate increases with time as seen in Fig. 7.4h.

Next, we discuss the simulation of the model after applying various controls.

In this model, we have applied 6 controls namely $u_1, u_2, u_3, u_4, u_5, u_6$ as lock down control, curfew, viral load test, control to increase population in care centres and to decrease population in hospital and plasma therapy as a medical intervention, respectively. Optimal control conditions are developed using iterative method. We start with solving state equations with a guess for controls within a simulated time and

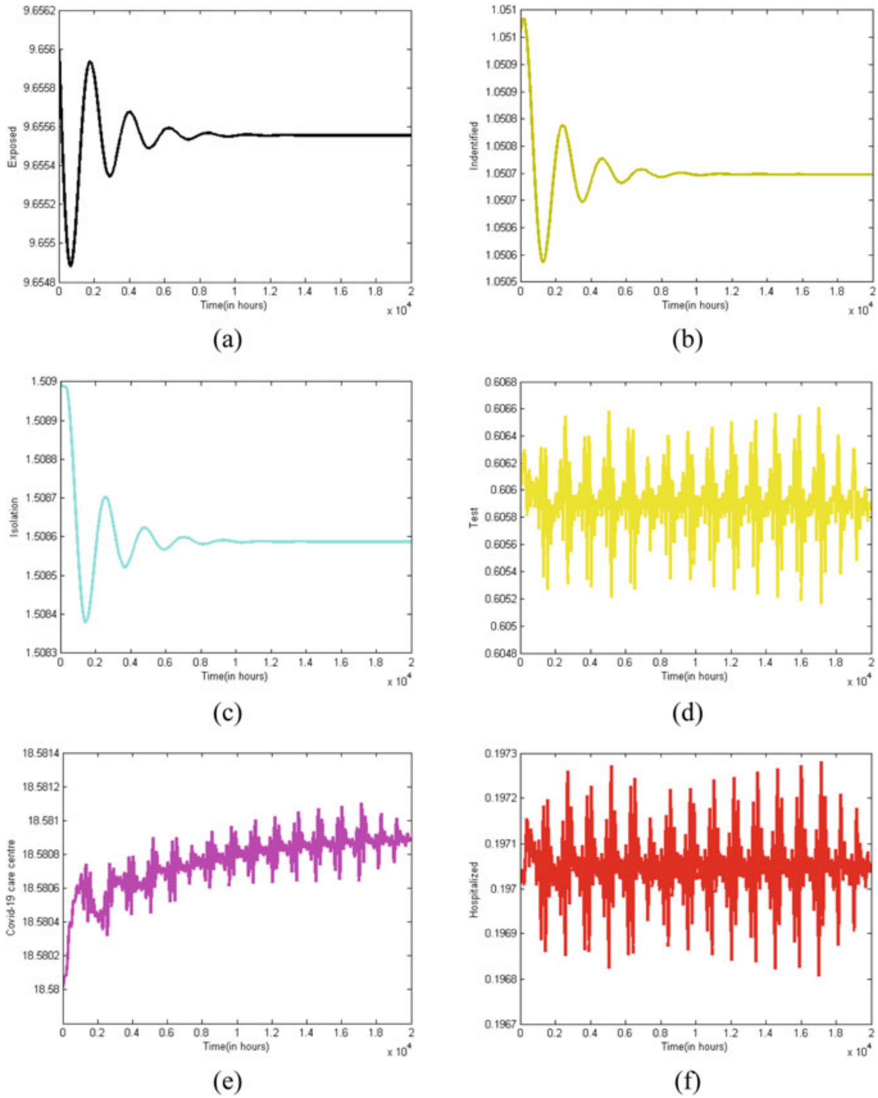
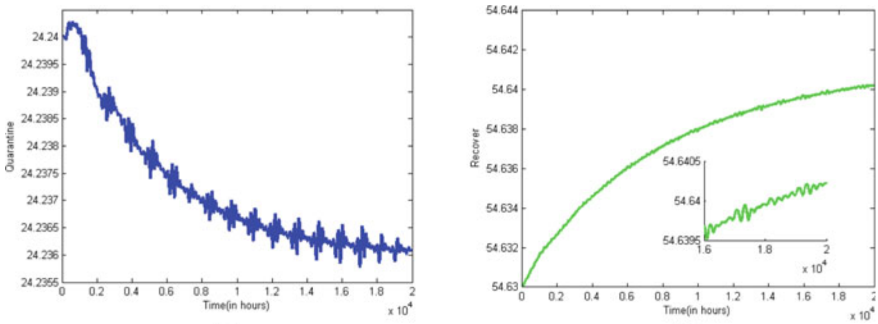


Fig. 7.4 Oscillation observed in different compartments [*source* own]

apply fourth order Runge–Kutta method. Here, we plot each compartment against all the controls applied to the system (7.1) which is observed in Fig. 7.5. From Fig. 7.5a, we observe decrease in exposure to COVID-19. Showing importance of lockdown and curfew control in the model. Similarly, identification of COVID-19 cases increases initially for 2 weeks but then with time and with all the controls it decreases as observed in Fig. 7.5b. Similarly, decrease in number of isolated cases is observed in Fig. 7.5c. After encouraging individuals for the test as shown in Fig. 7.5d,



(g) (h)

Fig. 7.4 (continued)

first 4 weeks helps to reduce catching infection. Again, number of individuals in COVID-19 care centre increases seen in Fig. 7.5e and number of COVID-19 cases decreases in hospital (Fig. 7.5f) under the influence of all the controls. Quarantine population decreases (Fig. 7.5g) when control is applied as compared to when no control is applied. This happens due to decrease in exposure cases which indirectly decreases identification and isolation population. Recovery (Fig. 7.5h) increases with the decrease in hospitalized human population which shows the positive effect of plasma therapy used as one of the controls.

Figure 7.6 shows the plot of objective function under the influence of all the controls taken in this chapter. Here, we observe that implementing all these controls strictly can end this pandemic within 2 months.

Conclusion

Observing current situation of the world, a huge proportion is infected with COVID-19. There are several social, medical interventions taken up by the government throughout the world to fight against the transmission of COVID-19 in absence of vaccination. In this chapter, we constructed a model considering exposure stage of COVID-19. Here we have computed basic reproduction number and showed it to be greater than 1 indicating current scenario of various countries. We have also calculated endemic equilibria and has shown it to be locally stable using Routh-Hurwitz criterion. Now until vaccine arrives, COVID-19 pandemic will have caused huge loss. Therefore, it is for our safety to follow various measures adopted by different government. In this chapter, we applied optimal control theory using social, medical measures as a control in a COVID-19 exposure scenario. It is observed that using all the controls strictly life can come back to normal within 2 months. This model suggest testing of COVID-19 at large scale also plays important role in combating

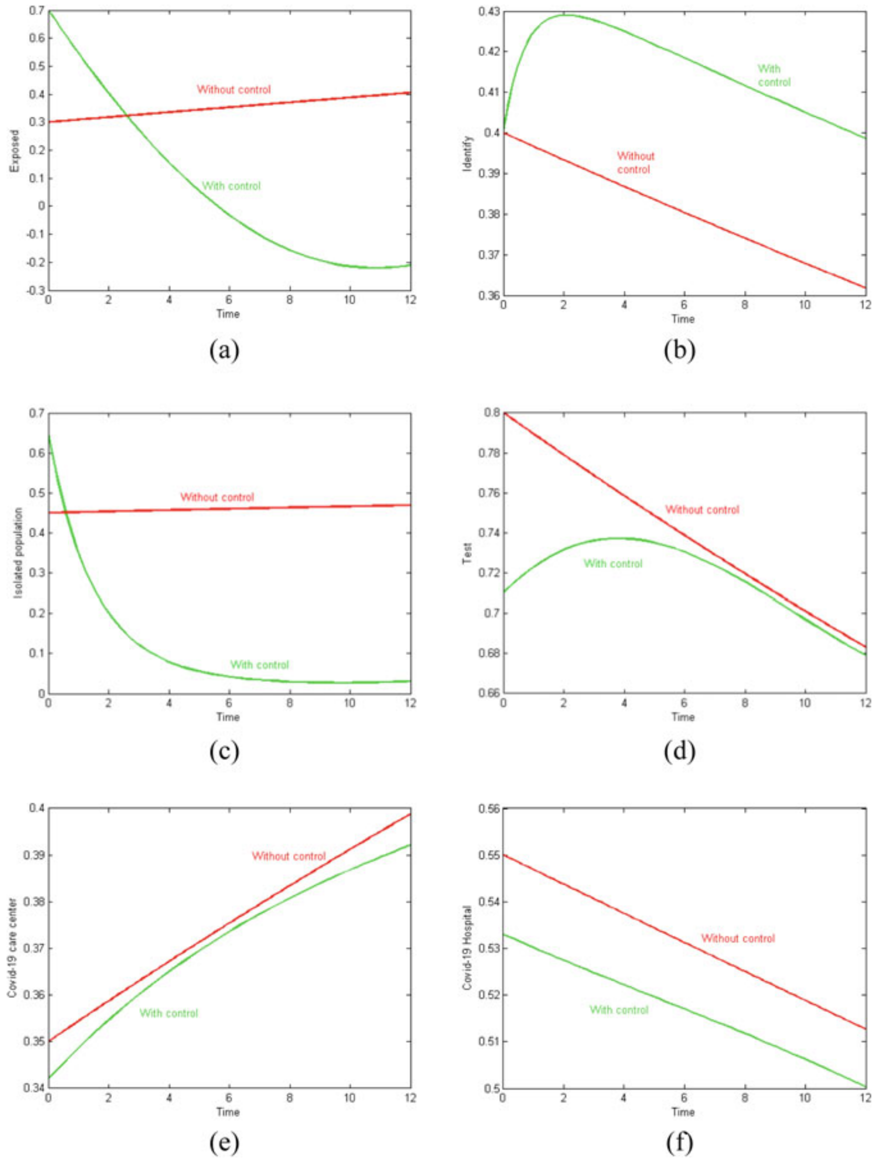
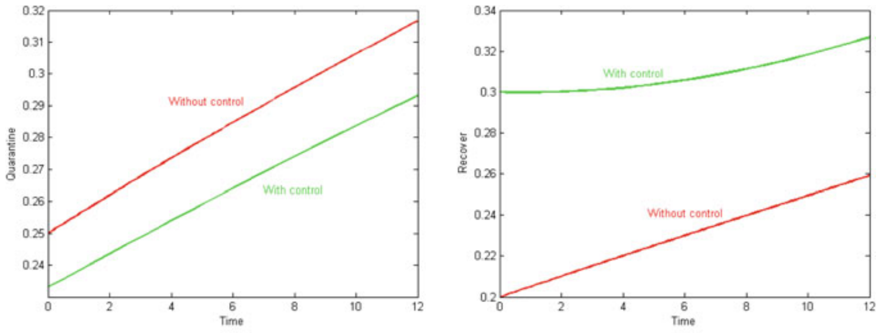


Fig. 7.5 Effect of various controls applied to the model [*source* own]

COVID-19. Also, plasma therapy used as one of the controls plays vital role in increasing recovery rate of infective's.



(g) (h)

Fig. 7.5 (continued)

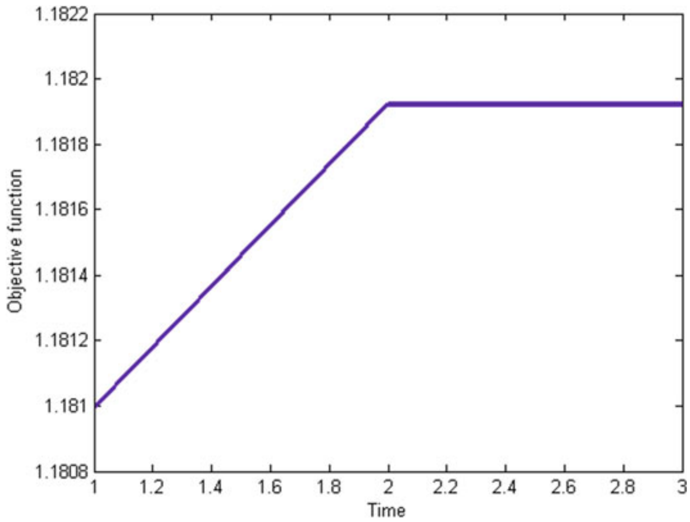


Fig. 7.6 Plot of objective function [*source* own]

One of the limitations of this model is we have not taken into account cost-effectiveness strategies while applying controls.

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Data Availability The data used to support the findings of this study are included within the article.

Conflict of Interest The authors do not have conflict of interest.

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