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**Abstract** The treatment of corona virus disease is not possible without any vaccine. However, spreading of the deadly virus can be controlled by various measures being imposed by Government like lockdown, quarantine, isolation, contact tracing, social distancing and putting face mask on mandatory basis. As per information from the Department of Medical Health and Family Welfare of Rajasthan on 19 September 2020, corona virus COVID-19 severely affected the state of Rajasthan, resulting in cumulative positive cases 113,124, cumulative recovered 93,805 and cumulative deaths 1322. Without any appropriate treatment, it may further spread globally as it is highly communicable and because potentially affecting the human body respiratory system, which could be fatal to mankind. Therefore, to reduce the spread of infection, authors are motivated to construct a predictive mathematical model with sustainable conditions as per the ongoing scenario in the state of Rajasthan. Mathematica software has been used for numerical evaluation and graphical representation for variation of infection, recovery, exposed, susceptibles and mortality versus time. Moreover, comparative analysis of results obtained by predictive mathematical model has been done with the exact data plotting by curve fitting as obtained from Rajasthan government website. As a part of analysis and result, it is noted that due to the variation of transmission rate from person to person corresponding rate of infection goes on increasing monthly and mortality rate found high as shown and discussed numerically. Further, we can predict that the situation will become worse in the winter months especially in month of December due to unavailability of proper

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vaccine. This model may become more efficient when the researchers, experts from medical sciences and technologist work together.

Keywords Covid-19 · Corona virus · Curve fitting · Modelling

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### Introduction

Firstly, this pandemic COVID-19 has been identified in Wuhan city situated in China in the month of December 2019 and is an infectious disease. Yang and Wang [1] developed a mathematical model for the novel coronavirus epidemic in Wuhan city, China. Day by day, this pandemic spread due to the high transmission rate because no vaccine at present available. Some renowned researchers have dedicated nice work based on COVID-19 disease. COVID-19 disease came in the state of Rajasthan on 2nd March and identified in the SMS hospital which is situated in Jaipur city. Initially, the pandemic was slow increasing due to slow transmission rate. Nowadays, this disease is rapidly distributing to all over Rajasthan. The government organizations have taken some nice primary decision and tried to control it with the implementation of lockdown as well as manage the social distancing. Guckenheimer and Holmes [2] proposed a mathematical model along with nonlinear oscillation, dynamical systems and bifurcations of vector field. Driessche and Watmough [3] presented a mathematical model with the help of reproduction factor and endemic equilibriums for disease transmission. Mizumoto and Chowell [4] have explained a mathematical model based on potential of novel corona virus. Rothan and Byrareddy [5] have presented a mathematical model concern with the epidemiology and pathogenesis coronavirus disease (COVID-19). Sohrabi et al. [6] developed a mathematical model based on COVID-19 disease using data of World Health Organization. They reviewed the latest situation. World Health Organization declared the highly infectious COVID-19 as a global pandemic in March 2020. Thevarajan et al. [7] explained the symptoms of COVID-19 which includes high fever, cough and difficulties in inhalation. Riou and Althaus [8] studied a mathematical model in which they discussed that people may get infected while breathing in an infected environment or touching any infected surface or by coming in contact of an infected person. Some authors like Khot and Nadkar [9], Wang [10], have contributed their work wherein they studied that aged people, children and adults with low persons immunity are prone to catch infection and may be severely affected from this disease, even lead to death. Some experts like Sahin et al. [11], Cheng and Shan [12] considered the risk factor in their study as people are unaware of reason behind the spreading of virus.

All mathematical models discussed above are predictive model and have not given an exact solution. The above said references motivate us to develop a mathematical model under different strategies to discuss COVID-19 pandemic situation in Rajasthan. The present mathematical model explains the transmission of COVID-19

disease in the cold season and sensitivity analysis has been studied with the help of modelling graph under different cases. Moreover, a comparative analysis of results obtained by predictive mathematical model has been developed with the exact data plotting by curve fitting as obtained from Rajasthan government website. We have taken some input data from the site of Rajasthan, India (as shown in Annexures 5.1, 5.2, 5.3 and 5.4).

The spread of COVID-19 is found to be pandemic in nature for the state of Rajasthan. From the graphical plotting of the last four months date (June, July, August and September), it is noted that there is drastic change in variation of infection, recovery and mortality observed. Here, the input data used for the analysis is taken from the Rajasthan government website and as reflected in Annexures 5.1, 5.2, 5.3 and 5.4.

From the graphical plotting as shown in Figs. 5.1, 5.2 and 5.3 for the month of June 2020 in the state of Rajasthan, it is clearly noted that there is rapid growth in



Fig. 5.1 Variation of infection versus time for the month of June 2020. Source own



Fig. 5.2 Variation of recovery versus time for the month of June 2020. Source own



Fig. 5.3 Variation of death versus time for the month of June 2020. Source own

the rate of infection but corresponding recovery rate also found significant but rate of deaths increases as time increases. The commutative infection reaches to 18,000 approximately whereas total recovery observed close to 14,000 and commutative deaths touch 400 till end of the June month.

From Figs. 5.4, 5.5 and 5.6, it is observed, for the month of July 2020 for the state of Rajasthan, growth in the rate of infection is double as compare to June month but there is small decay in corresponding recovery rate. Also, infection outbreak is noted near the end of the month as well as death rate increases with increase in time. The commutative infection reaches more than 40,000 whereas total recovery observed close to 29,000 and commutative deaths touch 680 till end of the July month.



Fig. 5.4 Variation of infection versus time for the month of July 2020. Source own



Fig. 5.5 Variation of recovery versus time for the month of July 2020. Source own



Fig. 5.6 Variation of death versus time for the month of July 2020. Source own

From Figs. 5.7, 5.8, 5.9, 5.10, 5.11 and 5.12, a similar behaviour is noted for the overall variation of infection, recovery and mortality in the month of August and September, respectively. Infection touches 80,000 in August, which is double as happened in July month. Close 110,000 infected people noted in September, but corresponding recovery is not significantly increasing, and death rates become high.

#### **Mathematical Modelling of COVID-19**

By looking the overall scenario, here we construct a mathematical modelling of COVID-19 for the state of Rajasthan by making use of the parameters described in Table 5.1. We divide the total population of Rajasthan into mainly five compartments, the susceptible (denoted by S), the exposed (denoted by E), the infected (denoted by



Fig. 5.7 Variation of infection versus time for the month of August 2020. Source own



Fig. 5.8 Variation of death versus time for the month of August 2020. Source own

*I*), and the recovered (denoted by R) and mortality (denoted by M). In Fig. 5.13, we describe the flowchart presentation of model.

Here it is assumed that infected individual class has fully developed disease symptoms and has the capacity to infect others. Also, exposed class individuals are those who are in incubation period and they do not show symptoms but still capable to pass infection to others. In this model, *E* and *I* compartment can be interpreted as they contain asymptomatic and symptomatic infected individuals.

The mathematical model suits the current COVID-19 situation to describe the transmission dynamics represented as:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \Lambda - C_1 S E - C_2 S I \tag{5.1}$$



Fig. 5.9 Variation of recovered versus time for the month of August 2020. Source own



Fig. 5.10 Variation of infection versus time for the month of September (up to 19th September 2020). *Source* own



Fig. 5.11 Variation of death versus time for the month of September (up to 19 September 2020). *Source* own



Fig. 5.12 Variation of recovered versus time for the month of September (up to 19 September 2020). Source own



Fig. 5.13 Mathematical representation of model. Source own

Parameter	Description
- araineter	
Λ	Population parameter
S	Susceptible
Ε	Exposed
Ι	Infected
R	Recovery
М	Mortality rate
$\frac{1}{\alpha}$	Incubation period between the infection and the onset of symptoms
r	Recovery rate
<i>C</i> <sub>1</sub>	Represent the direct human-to-human transmission rates between the exposed and susceptible individuals
<i>C</i> <sub>2</sub>	Represent the direct human-to-human transmission rates between the infected and susceptible individuals

 Table 5.1
 Notation and its description

$$\frac{\mathrm{d}E}{\mathrm{d}t} = C_1 SE + C_2 SI - \alpha E \tag{5.2}$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \alpha E - (m+r)I \tag{5.3}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = rI \tag{5.4}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = mI \tag{5.5}$$

Here,  $C_1$  and  $C_2$  are assumed as non-increasing functions, given that higher values of E and I would motivate stronger control measures that could reduce the transmission rates.

All the initial conditions of the system are assumed non negative as  $S(0) \ge 0$ ,  $E(0) \ge 0$ ,  $I(0) \ge 0$ ,  $R \ge 0$  and  $M(0) \ge 0$ .

### **Numerical Analysis**

The basic input parameters of this model are physical in nature and taken by looking the current situation as per the data shown in annexure.

Figures 5.14, 5.15 and 5.16 indicate the variation of infection versus time for the month of June, July and August, respectively. From the figures, it is observed that infection goes on increasing with increase in time. Also, growth of infection



Fig. 5.14 Variation of infection versus time in the month of June 2020. Source own



Fig. 5.15 Variation of infection versus time in the month of July 2020. Source own



Fig. 5.16 Variation of infection versus time in the month of August 2020. Source own

found triple in the month of July and five times in the month of August as compared to infection in June. This variation of infection indicates that the virus growth as discussed above by taking exact data using curve fitting (Figs. 5.1 and 5.4) is nearly similar to the analysis done by our mathematical model. Hence, we can say that our developed mathematical model is efficient and works parallel to the exact situation of spread of infection as recorded in Rajasthan state.

Figures 5.17, 5.18 and 5.19 show the variation of recovery versus time for the month of June, July and August, respectively. It is observed that recovery goes on increasing with increase in time and found significant in the month of June but there is small decay noted in the month of July and August as compared to corresponding rate of infection. The same behaviour of recovery rate was observed by curve fitting as shown in Figs. 5.2 and 5.5 for the exact data, hence our mathematical model work exactly parallel in case of rate of recovery also and gives efficient results.

From Figs. 5.20, 5.21 and 5.22, it is clear that rate of mortality goes on increasing with increase in time but at a steady rate, also as compare to growth in infection the rate of mortality is very low. A similar characteristic of mortality was found during curve fitting for the exact recorded data.



Fig. 5.17 Variation of recovery versus time in the month of June 2020. Source own



Fig. 5.18 Variation of recovery versus time in the month of July 2020. Source own



Fig. 5.19 Variation of recovery versus time in the month of August 2020. Source own



Fig. 5.20 Variation of mortality versus time in the month of June 2020. Source own

Figures 5.23, 5.24 and 5.25 show the variation of exposed versus time. It is noted that with the passage of time, number of exposed goes on increasing. In the June month, number of exposed was nearly 500,000 but in month of July and August, it increases from 800,000 to 1,000,000. It shows that rate of exposed increases rapidly day by day.

Figures 5.26, 5.27 and 5.28 clearly agree with the condition that with the increase in infection corresponding decay is susceptible observed. Variation of susceptible corresponding to different months shown versus time and completely agrees with current scenario of COVID-19 in Rajasthan state.



Fig. 5.21 Variation of mortality versus time in the month of July 2020. Source own



Fig. 5.22 Variation of mortality versus time in the month of August 2020. Source own

Figures 5.29, 5.30, 5.31, 5.32 and 5.33 show the overall variation of infection, mortality, recovery, exposed and susceptibles versus for the June to December month. After observing overall plotting, it is clearly visible that the impact of corona virus will increase significantly in the winter months especially in December 2020. It is forecast that till the end of December month infected population will touches 178,000 and becomes steady, mortality may nearly 3500; recovery may approximately 120,000 (which is insignificant variation as compared to infection), number of exposed may increases 200,000 (double as in August month). The impact can be reduced only if the guideline provided by the Government of India should be strictly followed by entire population with the arrangement of proper vaccination.



Fig. 5.23 Exposed versus time in the month of June 2020. Source own



Fig. 5.24 Exposed versus time in the month of July 2020. Source own

### Conclusion

In the present study, a comparison model has been constructed to examine the transmission of dynamic COVID-19 for the state of Rajasthan, India. Numerical results obtained by the mathematical modelling which have been compared with exact curve fitting method, where data is taken from government website of Rajasthan state. Numerical plotting in mathematical and curve fitting shows same characteristic in the case of infection, recovery, mortality, susceptible and exposed. The presented mathematical model is hypothetical in nature but for some particular functions and parameters, it is feasible for the real-world problems. Finally, on the basis of the presented model, we can predict that the situation will become worse in the month of December due to unavailability of proper vaccine. Hence, to control the spreading of



Fig. 5.25 Exposed versus time in the month of August 2020. Source own



Fig. 5.26 Susceptible versus time in the month of June 2020. Source own

virus infection, it is very much essential to follow all the restrictions like lockdown, social distancing, contact tracing, mask cover, etc., as laid down by Government of Rajasthan and India.



Fig. 5.27 Susceptible versus time in the month of July 2020. Source own



Fig. 5.28 Susceptible versus time in the month of August 2020. Source own



Fig. 5.29 Overall variation of infection versus time from June to December 2020. Source own



Fig. 5.30 Overall variation of mortality versus time from June to December 2020. Source own



Fig. 5.31 Overall variation of recovery versus time from June to December 2020. Source own



Fig. 5.32 Overall variation of susceptible versus time from June to December 2020. Source own



Fig. 5.33 Overall variation of Exposed versus time from June to December 2020. Source own

## Annexure 1

Date (June)	Cumulative infected	New infected	Cumulative death	New death	Recovered
1	8831	214	194	1	6032
2	9100	269	199	5	6213
3	9373	273	203	4	6435
4	9652	279	209	6	6744
5	9862	210	213	4	7104
6	10,084	222	218	5	7359
7	10,337	253	231	13	7501
8	10,599	262	240	9	7754
9	10,876	277	246	6	8117
10	11,245	369	255	9	8328
11	11,600	355	259	4	8569
12	11,838	238	265	6	8775
13	12,068	230	272	7	9011
14	12,401	333	282	10	9337
15	12,694	293	292	10	9566
16	12,981	287	301	9	9785
17	13,216	235	308	7	9962
18	13,542	326	313	5	10,467
19	13,857	315	330	17	10,742

The status of COVID-19 disease in the month of June

Date (June)	Cumulative infected	New infected	Cumulative death	New death	Recovered
20	14,156	299	333	3	10,997
21	14,555	399	337	4	11,274
22	14,930	393	349	12	11,597
23	15,232	302	356	7	11,910
24	15,627	395	365	9	12,213
25	16,009	382	375	10	12,611
26	16,296	287	379	4	12,840
27	16,660	364	380	1	13,062
28	16,944	284	391	11	13,367
29	17,271	327	399	8	13,611
30	17,660	389	405	6	13,921

Source [13]

# Annexure 2

The status of COVID-19 disease in the Month of July

Date (July)	Cumulative infected	New infected	Cumulative death	New death	Recovered
1	18,014	354	413	8	14,220
2	18,312	298	421	8	14,574
3	18,662	350	430	9	14,948
4	19,052	390	440	10	15,281
5	19,532	480	447	7	15,640
6	20,164	632	456	9	15,928
7	20,688	524	461	5	16,278
8	21,404	716	472	11	16,575
9	22,063	659	482	10	16,866
10	22,563	500	491	9	17,070
11	23,174	611	497	6	17,620
12	23,748	574	503	6	17,869
13	24,392	644	510	7	18,103
14	24,936	544	518	8	18,630
15	25,571	635	524	6	19,169
16	26,437	866	530	6	19,502
17	27,174	737	538	8	19,970

Date (July)	Cumulative infected	New infected	Cumulative death	New death	Recovered
18	27,789	615	546	8	20,626
19	28,500	711	553	7	21,144
20	29,434	934	559	6	21,730
21	30,390	956	568	9	22,195
22	31,373	983	577	9	22,744
23	32,334	961	583	6	23,364
24	33,220	886	594	11	23,815
25	34,178	958	602	8	24,547
26	53,298	1120	613	11	25,306
27	36,430	1132	624	11	25,954
28	37,564	1134	633	9	26,834
29	38,636	1072	644	11	27,317
30	39,780	1144	654	10	28,309
31	40,936	1156	667	13	29,231

Source [13]

# Annexure 3

Date (August)	Cumulative infected	New infected	Cumulative death	New death	Recovered
1	42,083	1147	680	13	29,845
2	43,243	1160	694	14	30,668
3	44,410	1167	706	12	31,216
4	45,555	1145	719	13	32,051
5	46,679	1124	732	13	32,832
6	47,845	1166	745	13	33,849
7	48,996	1151	757	12	35,131
8	50,157	1161	767	10	36,195
9	51,328	1171	778	11	37,163
10	52,497	1169	789	11	38,235
11	53,670	1173	800	11	39,060
12	54,887	1217	811	11	40,399
13	56,100	1213	822	11	41,648

The status of COVID-19 disease in the month of August

Date (August)	Cumulative infected	New infected	Cumulative death	New death	Recovered
14	57,414	1264	833	11	41,819
15	58,692	1278	846	13	43,897
17	61,296	1317	876	14	46,604
18	62,630	1334	887	11	47,654
19	63,977	1347	898	11	48,960
20	65,289	1312	910	12	49,963
21	66,619	1330	921	11	51,190
22	67,954	1335	933	12	52,496
23	69,264	1310	944	11	54,144
24	70,609	1345	955	11	55,324
25	71,955	1346	967	12	56,600
26	73,325	1370	980	13	58,126
27	74,670	1345	992	12	59,579
28	76,015	1345	1005	13	60,585
29	77,370	1355	1017	12	62,033
30	78,777	1407	1030	13	62,971
31	80,227	1450	1043	13	65,093

Source [13]

# Annexure 4

The status of COVID-19 disease up to 19 September 2020

Cumulative infected	New infected	Cumulative death	New death	Recovered
83,163	1470	1069	13	68,124
84,674	1511	1081	12	70,674
86,227	1553	1095	14	71,220
87,797	1570	1108	13	71,899
89,363	1566	1122	14	73,245
90,956	1593	1137	15	74,861
92,536	1580	1151	14	76,427
94,126	1590	1164	13	77,872
95,736	1610	1178	14	79,450
	Cumulative infected 83,163 84,674 86,227 87,797 89,363 90,956 92,536 94,126 95,736	Cumulative infected         New infected           83,163         1470           84,674         1511           86,227         1553           87,797         1570           89,363         1566           90,956         1593           92,536         1580           94,126         1590           95,736         1610	Cumulative infectedNew infectedCumulative death83,1631470106984,6741511108186,2271553109587,7971570110889,3631566112290,9561593113792,5361580115194,1261590116495,73616101178	Cumulative infectedNew infectedCumulative deathNew death83,163147010691384,674151110811286,227155310951487,797157011081389,363156611221490,956159311371592,536158011511494,126159011641395,7361610117814

Date (September)	Cumulative infected	New infected	Cumulative death	New death	Recovered
10	97,376	1640	1192	14	80,482
11	99,036	1660	1207	15	81,970
12	100,705	1669	1221	14	82,902
13	102,408	1703	1236	15	84,518
14	104,138	1730	1250	14	86,162
15	105,898	1760	1264	14	87,873
16	107,680	1782	1279	15	89,352
17	109,473	1793	1293	14	90,685
18	111,290	1817	1308	15	92,265
19	113,124	1834	1322	14	93,805

Source [14]

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