Finite Element Modeling of Composite Beam with Transverse Cracks Under Free Vibration with Experimental Validation

Priyadarshi Das, Vedant D. Chakurkar, and Shishir Kr. Sahu

Abstract Laminated composite beams are greatly increased used in engineering sectors such as aerospace, marine, structural civil engineering, and high-speed machinery, due to composite materials are application-oriented and cost-effective in long run as well as for their improved tailor properties, high ratios of stiffness-to-weight, good corrosion resistivity, thermal insulation, and fatigue. Among the structural damages, crack regarded as the most critical damage because under progressive dynamic loading it amplifies the vibration amplitude that greatly affects the strength and stiffness of the structural component and ultimately led to the catastrophic failures. So, the technical importance of dynamic exploration of laminated cracked composite beams is significant and thus, the research interest of the present study. In the present research, the industry-driven bi-directional laminated glass/epoxy beam with pre-defined transverse cracks is used to record the changes observed in natural frequencies under free vibration through numerical analysis with experimental validation. The numerical analysis is done using the Finite element method (FEM) based software ABAQUS. The free vibration responses of the cracked composite beam are obtained experimentally via FFT analyzer. The recorded natural frequencies from the numerical study agree fairly well with the test results. The variations of natural frequencies concerning crack depth and fiber-orientation are observed. The experimental study results and finite element predictions exhibited that the crack position and relative depth of crack have considerable influences on the frequencies under free vibration of the laminated fiber glass/epoxy beam.

Keywords Vibration · Laminated composite beam · Crack

P. Das (✉) · V. D. Chakurkar · S. Kr. Sahu
Department of Civil Engineering, National Institute of Technology, Rourkela, Odisha 769008, India
e-mail: priyadarshicivil@gmail.com

P. Das
Department of Civil Engineering, Institute of Technical Education and Research, Siksha 'O' Anusandhan University, Bhubaneswar, Odisha 751030, India

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1 Introduction

In past decades, laminated composite beams (LCB) have a wide range of applications in aerospace, civil construction area, machine element structures, and related structural applications, primarily due to their good applicability, long-run utility, high specific stiffness, cost-effectiveness, and good tailorability. Unlike metallic structures, sandwich/composite materials are anisotropic, and under vibration, more complex failure modes can result. Among structural damages, crack is an inevitable failure and its presence greatly reduces the strength and stiffness of the structures. Because of the complex nature of composite structures, laminated beams are vulnerable to cracks and its frequency analysis is significant in structural health monitoring. In spite of the technical significance of vibration of cracked composite structures, relatively very less amount of researches is done for frequency assessment of multi-cracked LCB.

The frequency-based-crack analysis is an efficient and cost-effective non-destructive test to assess the structural health of the system and studies regarding this was summarized by Dimarogonas [1] through 1996. Teotia and Soni [2] summarized the developments regarding the complex damage behavior of laminated glass composite using FEM through 2018. Cawley and Adams [3] done a vibration analysis on carbon fiber reinforced plastic and aluminum plates for damage assessment using finite element analysis. The significance of local compliance of laminated cracked composite beam was presented by Nikpour and Dimarogonas [4] using experimental dynamic analysis. The effect of a single transverse crack on the natural frequency of unidirectional laminated graphite-fiber reinforced beam was investigated by Krawczuck and Ostachowicz [5] via FEM. Ghoneam [6] found that the variations in natural frequencies of graphite fiber polyamide due to cracks in absence and presence by both and numerical and experimental investigation. Kisa [7] used the component mode synthesis method and FEM for dynamic analysis of a composite cantilever beam made from graphite fiber with multiple transverse cracks. Loya et al. [8] considered a cracked Timoshenko beam for dynamic analysis and assessed the bending vibration frequencies using the perturbation method. Through experimental investigation, Manivasagam and Chandrasekaran [9] calculated the variational changes concerning the crack effects in the first natural frequency of laminated composite. Utilizing FEM, Jena et al. [10] investigated the influence of the effect of an open transverse crack of a unidirectional laminated fiber glass/epoxy composite beam on the vibration frequencies with experimental validation. Based on FEM, through the application of Newmark's integration method, the variations in vibration frequencies are demonstrated by Kim and Kim [11] for a rotating composite beam due to a single crack. Wang et al. [12] analytically studied the frequency changes of an LCB with a single open crack in Clamped-Free condition under free vibration by classical lamination theory. Using FFT analyzer, the experimental analysis for single cracked laminated beam was done by Kumar et al. [13] under free vibration. Daneshmehr et al. [14] utilized the generalized differential quadrature (GDQ) method based on FSDT to verify the crack effects of an open cracked composite beam exposed to the couple bending-torsion loading.

Crack is an inevitable failure in laminated composite structures. The presence of crack greatly reduces the stiffness and strength of the structures and under vibration increases the amplitude, causing the failure of the structure. Despite the practical significance of cracked laminated beams under vibration, the dynamic exploration of cracked composite laminated beams by experimental and numerical analysis is succinct in literature. Hence, the present study focused on the free vibration of industry-driven bi-directional glass/epoxy LCB with open transverse cracks. Different fiber-orientations with crack depths are the concern parameters to investigate the variations in natural frequencies due to multiple transverse cracks. Numerical analysis is done using finite element (FE) based software ABAQUS and the results are validated with the test results recorded from the FFT analyzer.

2 Mathematical Formulations

The geometrical cross-section of a laminated composite beam (LCB) with beam geometries Length '*L*', width '*b*', and depth '*h*' is shown in Fig. 1. The transverse open crack is defined on the beam is of depth '*a*' and placed at a distance '*x*' from the left end as given away in the figure.

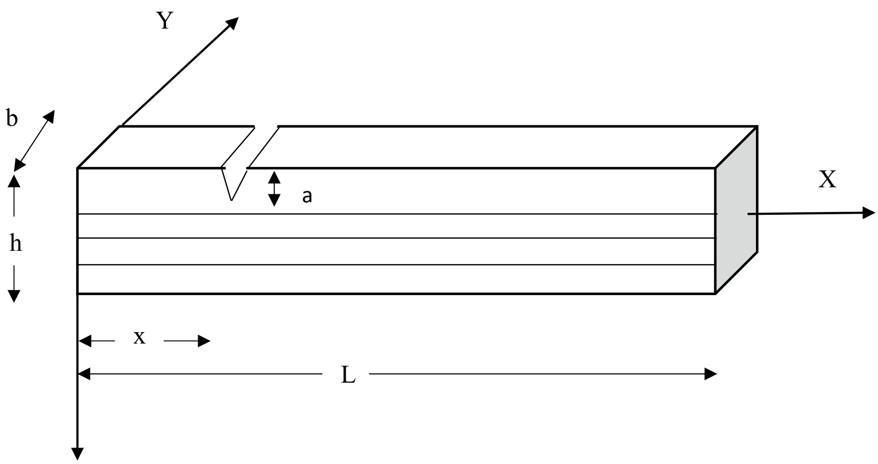

Fig. 1 Laminated composite beam geometry with a transverse crack

2.1 Intact Composite Beam Modeling

In order to get the numerical solution of the problem, a one-dimensional beam element having 3 nodes with 5 degrees of freedom is considered. The assumed displacement field for a laminated composite beam based on first-order shear deformation theory (FSDT) can be written as [15]:

$$u(x, y, z) = u_0(x, y) + z\theta_x \tag{1}$$

$$v(x, y, z) = v_0(x, y) + z\theta_y \tag{2}$$

$$w(x, y, z) = w_0(x, y) \tag{3}$$

where *u, v* and *w* are the displacements in the *x*, *y*, and *z*-directions respectively u_0, v_0 and w_0 are the associated mid plane displacements and θ_x and θ_y are the rotations in the *x*–*z* and *y*–*z* planes.

The relationship of strain-displacement is given by

$$U = \frac{1}{2}\int [\varepsilon]^T [B][\varepsilon] \mathrm{d}V \tag{4}$$

where V is the volume of the beam, $[\varepsilon]$ is the strain tensor, and $[B]$ is the material property matrix. In the analysis of beam under free vibration using the FEM, the element stiffness matrix $[k_\mathrm{e}]$ and mass matrix $[m_\mathrm{e}]$ of the intact composite beam is written as:

$$[K_\mathrm{e}] = \int_{-1}^{1} [B]^T [D][B] \mathrm{d}\xi \tag{5}$$

$$[M_\mathrm{e}] = \int_{-1}^{1} [N]^T [\rho][N] \mathrm{d}\xi \tag{6}$$

where

$[B]$ Strain displacement relation matrix
$[D]$ constitutive relation matrix.

And,

$$[\rho] = \begin{bmatrix} I & 0 & 0 & P & 0 \\ 0 & I & 0 & 0 & p \\ 0 & 0 & I & 0 & 0 \\ P & 0 & 0 & Q & 0 \\ 0 & P & 0 & 0 & Q \end{bmatrix}$$

$$(I, P, Q) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(1, z, z^2\right)\rho(z)\mathrm{d}z$$

2.2 *Composite Cracked Beam Modeling*

The compliance matrix for the cracked section is written in line with Nikpour and Dimarogonas [4]:

$$[C] = \begin{bmatrix} 2D_1 & 1.5D_1 & 0 & 12\frac{D_1}{W} & 12\frac{D_1}{B} \\ 1.5D_1 & 4.5D_2 & 0 & 9\frac{D_{12}}{W} & 9\frac{D_{12}}{B} \\ 0 & 0 & 4.5D_3 & 0 & 0 \\ 12\frac{D_1}{W} & 9\frac{D_{12}}{W} & 0 & 72\frac{D_1}{W^2} & 36\frac{D_1}{BW} \\ 12\frac{D_1}{B} & 9\frac{D_{12}}{B} & 0 & 36\frac{D_1}{BW} & 96\frac{D_1}{B^2} \end{bmatrix} * \frac{\prod a^2}{BW^2} \tag{7}$$

where

D_1 $-A_{11}/2$
D_2 $A_{11}/2$
D_{12} A_{11}
D_3 $(A_{44} * A_{55})^{0.5}$
B crack width
W crack depth
A crack location.

The stiffness matrix for the cracked element is given by

$$[K_{\text{crack}}] = \begin{bmatrix} [C^{-1}] & -[C^{-1}] & 0 \\ -[C^{-1}] & 2[C^{-1}] & -[C^{-1}] \\ 0 & -[C^{-1}] & [C^{-1}] \end{bmatrix}_{15\times 5} \tag{8}$$

The stiffness matrix for the cracked beam element is given by,

$$[K] = [K_{\text{e}}] - [K_{\text{crack}}] \tag{9}$$

The governing equation for a composite beam with crack is

$$\left([K] - \omega^2[M]\right)\{\Delta\} = 0 \tag{10}$$

where

$[K]$ stiffness matrix for cracked beam
$[M]$ Mass matrix of the cracked beam
ω Natural frequency.

3 Experimental Program

In the present study, industry-driven woven roving (WR) fiber E-glass sheets and epoxy resin are used for the fabrication of test specimens of laminated beams. The WR glass fibers and the matrix (epoxy resin) are balanced through a ratio of 50:50 by weight. The matrix is prepared by using 90% epoxy (Lapox L-12, Atul ltd., India) and 10% hardener (K-6, Atul ltd., India) by weight. The hand lay-up technique is employed for combining WR plies in the pre-defined order of orientation. The fabrication is done on a wooden platform and before placing the fiber sheets according to their order of orientation, a thin film of plastic is placed as mould releasing sheet with a mild spray of silicon as releasing agent. After this, the epoxy matrix is applied over the plastic sheet and then the fiber sheets are placed alternatively. On each layer of glass fiber sheet, epoxy is applied and alternative layers are fabricated by using a steel roller. Under heavy flat iron loads, the laminated test specimens are cured for 72 h at room temperature and after removal of loads, a smooth finished composite plate is produced. The LCB specimens are carved from the plate mechanically via a diamond cutter. On the beam samples, the cracks are created in a transverse direction by means of a hack saw. The beam geometries are measured as length = L = 25 cm, width = b = 2.5 cm and thickness = h = 0.24 cm.

3.1 Determination of Material Constants from Tensile Test

The tensile test is performed on the test specimens using universal testing machine (UTM) INSTRON setup according to ASTM-D3039/D3039M-14. The material elastic properties are acquired from the unidirectional tensile tests on beam samples in accordance with ASTM D 3039/D 3039 M. The cross-section of the beam samples is rectangular throughout in all the cases. The elastic moduli 'E_{11}', 'E_{22}' (in this case $E_{11} = E_{22}$), and Poisson's ratio 'ν_{12}' of WR glass/epoxy composite beams are recorded from the tensile test while shear modulus 'G_{12}' is determined in accordance with Jones [15].

Table 1 Material properties of WR laminated composite beam

Material property	Test results	
E (Young's modulus)	E_{11}	15.94 GPa
	E_{22}	15.94 GPa
	E_{45}	2.94 GPa
ν (poison ratio)	ν_{12}	0.3
	ν_{13}	0.3
	ν_{23}	0.3
G (shear modulus of rigidity)	G_{12}	3.57 GPa
	G_{13}	3.57 GPa
	G_{23}	3.57 GPa
ρ (density)	ρ_{m}	1630 kg/m^3

$$G_{12} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\nu_{12}}{E_1}} \tag{11}$$

The material properties are given in Table 1.

3.2 Experimental Free Vibration Test

Frequency-based experimental test is an efficient non-destructive method of testing for dynamic analysis of structures. The following mechanical devices are employed for experimental test of beams under free vibration.

1. B & K-3560 B model FFT analyzer
2. Model-B & K-Type 4507 Accelerometer
3. B & K-Type 2302-5 Modal impact hammer
4. Laptop/PC with PULSE software
5. Composite beam sample for test.

The laminated beam test samples are placed as per the desired end support conditions in the pre-fabricated iron frame. After the specimens are arranged in the iron frame as per required boundary conditions, the accelerometer is placed over the beam near to the crack location and through the modal hammer, excited 5-times to get the frequency responses. The signal received by the accelerometer is digitized through the FFT analyzer and the average of five frequency response functions (FRF) are displayed as the spectrum. A coherence closer to unity is observed for the FRF spectrum which indicates more accurate results are obtained for vibration frequency. The complete set of experimental programs is shown in Fig. 2.

Fig. 2 Experimental setup

4 Finite Element Modeling and Modal Analysis Using ABAQUS

Abaqus mainly consist of three-step as follows:

1. Modeling of problem
2. Processing of problem
3. Output of problem.

In the modeling part, structural geometry and material properties are analyzed. For the present investigation, the structure is idealized as a beam. In this step, assembling is done for composite beam by assigning the beam geometry and elastic properties to each layer of LCB. In the next stage method of analysis, modeling of crack, application of boundary conditions, and meshing of the structure is done. After adopting the method of modal analysis, crack simulation is applied to the location where the transverse cracks are made. In the third step, the different modular frequencies and mode shapes are obtained after the successful running of the problem as shown in Fig. 3.

5 Results and Discussions

In the present study, the industry-driven-WR laminated fiber glass/epoxy beam is examined under free vibration with pre-defined cracks via FE analysis and experimental study. The material elastic constants are taken for analysis as summarized in Table 1. Recorded geometric configurations of the laminated beam are 250 mm, 25 mm, and 2.4 mm for length (L), width (b), and thickness (h) respectively. In this study, the discussions are made on the crack effects on the beam vibration frequencies with respect to fiber-orientation and relative crack depth.

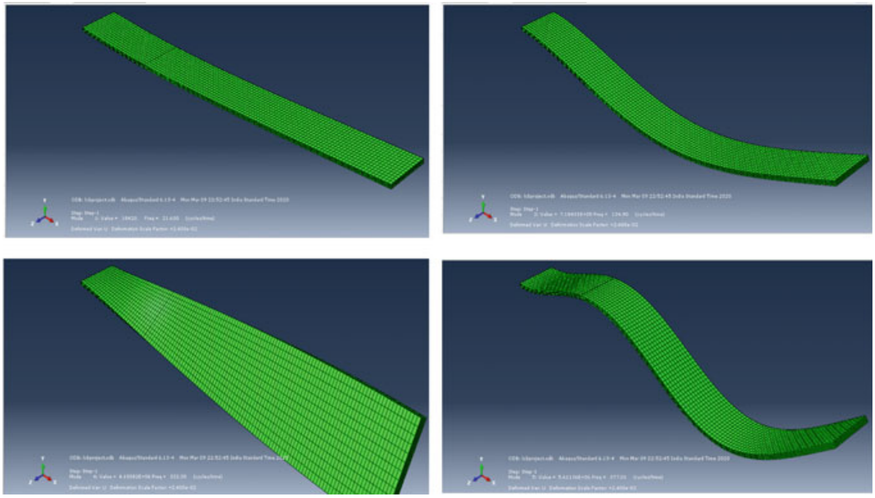

Fig. 3 Mode shapes and modular frequencies

5.1 Crack Depth Effect on Beam Vibration Frequency

Five relative crack depths (*a*/*h*) 0, 0.125, 0.25, 0.375, 0.5 and 0.625 are defined to verify the crack effect on natural frequencies. The variations of natural fundamental frequency due to multiple cracks for these relative crack depths (rcd) are summarized in Fig. 4. It is noted from Fig. 4, the first natural frequency shows a decreasing pattern for an increase in crack depth. A significant variation in vibration frequency

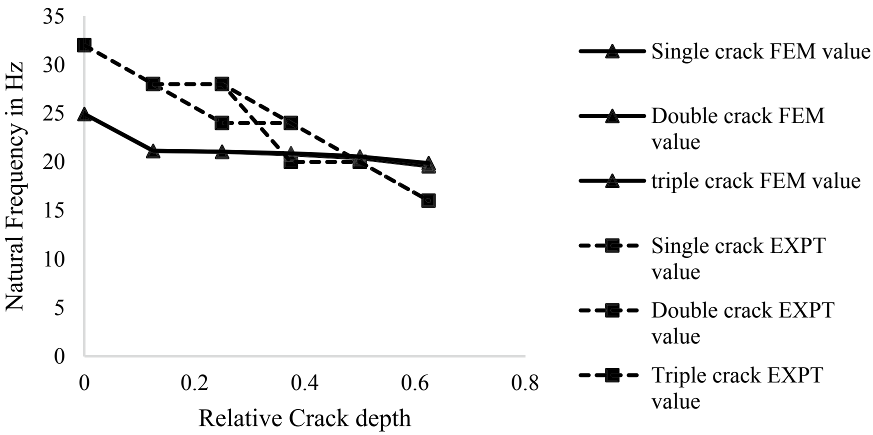

Fig. 4 Effect of crack depth on natural frequency

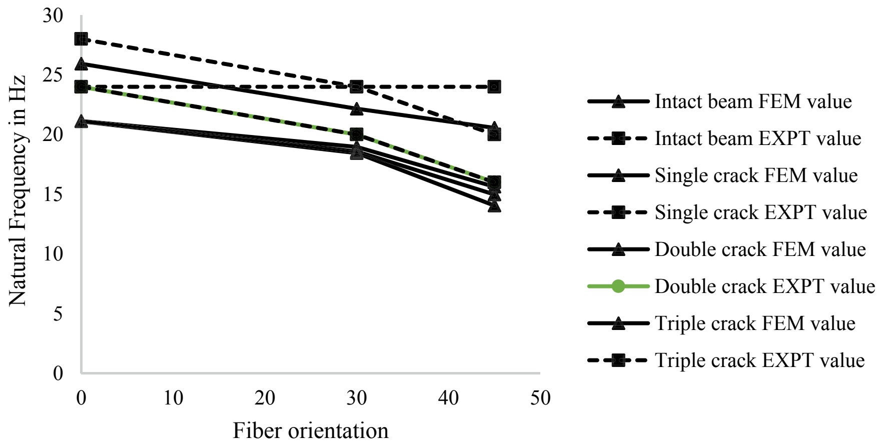

Fig. 5 Fiber angle of orientation effect on natural frequency

is observed for rcd greater than 0.5. 50% reduction in first natural frequency is noticed for the LCB with CF boundary condition for rcd 0.625 with respect to the intact beam.

5.2 Effect of Fiber Orientation on Beam Vibration Frequency

In order to check the fiber angle of orientations effect on the vibration frequencies of the composite beam, an 8-layered cantilever composite beam made of woven Glass fiber with three fiber orientations, 0°, 30°, and 45°, is considered. The fundamental frequency changes for an LCB having transverse cracks with CF boundary condition for different fiber-orientation is presented in Fig. 5. The finite element predictions fared well with the test results. The fundamental frequency decreased by 16.74%, 18.6%, and 31.6% for 0°, 30°, and 45° respectively with multiple cracks with respect to the intact beam. This approves the natural frequency of cracked LCB varies inversely to the ply-orientation of the beam.

Each sample with respective boundary conditions is excited with the modal hammer and the average of 5 FRFs is displayed as the spectrum. The vibration test on cracked beam samples is repeated on 3-days and 7-days intervals. The obtained frequencies are showing high reproducibility. The experimental results are in fine agreement with the finite element predictions. For a typical composite cantilever beam of 0° ply orientation, taking single, double, and triple crack, the deviation of the frequency of vibration between experimental and numerical results varies from 3.1 to 7.9%. This percentage of error doesn't affect significantly in the analysis of crack effects on the natural frequency of the laminated composite beam.

6 Conclusion

In this study, the vibration behavior of industry-driven fiber glass/epoxy LCB with multiple transverse cracks is investigated through experimental and numerical analysis. Numerical predictions made using finite element-based software ABAQUS are validated with the experimental results from the FFT analyzer. From the recorded results discussed above, it is concluded that the vibration characteristics are greatly influenced by cracks in the composite structure. Resulting conclusions are made from this study are,

- The finite element predictions are validated with the experimental results.
- Significant variation in vibration frequencies observed for the increase in crack depth.
- Significant variation in the higher mode frequencies is observed with the increase in fiber angle of orientation.
- Both experimental results as well as the finite element predictions show that the size of the crack along with the boundary conditions considerably affects the modal parameters of the laminated composite beam.
- The boundary conditions considerably affect the modal parameters of the laminated composite beam.

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