Bond Graph Modeling and Simulation of a Differential Drive Mobile Robot

Jyoti Joshi, Rushali Pant, Narendra Gariya, Avi Raj Manral, and Pushpendra Kumar

Abstract Wheeled mobile robots (WMRs) are being used in many walks of life due to their simple structure and easy control. Furthermore, a differential drive robot is the simplest form of a WMR, which is having two independent actuators to drive its two tractive wheels. The steering is performed using the difference of velocities of two wheels; hence, two wheels velocities are manipulated in order to control the velocity and orientation of the robot. Modeling is the first step for the control system design. A kinematic model is not enough to represent the real behavior of the robot. Therefore, this paper deals with the dynamic modeling of a differential drive mobile robot. The model considers the multi-domain nature of the robot and includes electrical actuators and mechanical dynamics. A multi-domain graphical modeling tool called bond graph (BG) is used to develop the robot's model. The developed BG model is verified through simulation and the robot's motion is analyzed on the surfaces with different coefficients of friction.

Keywords Differential drive · Mobile robots · Bond graph

1 Introduction

A mobile robot is the robot whose base is not fixed such as wheeled, legged, drones, underwater mobile robots, etc. Wheeled mobile robots (WMRs) have increasingly been used in many applications due to their simple design. Some applications include transportation, industries, and disaster missions. WMRs can have different number of wheels such as two, three, four, and many more. A WMR with two tractive wheels on the same axle line is called differential drive robot, where orientation of the robot is obtained by different angular velocities of the two wheels. In addition, there may be one or two passive supporting wheels. In this work, we focus on dynamic analysis of a differential drive mobile robot.

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P. Joshi et al. (eds.), *Advances in Engineering Design*, Lecture Notes in Mechanical Engineering, https://doi.org/10.1007/978-981-33-4684-0_48

Modeling is the first step in analyzing a robot; further it is necessary to develop some control strategies. In the existing literature, kinematic models of differential drive robots have been analyzed for different controls [1–3]. Furthermore, dynamic models of differential drive robots have been developed for controller design [4, 5]. In [6], a controller is designed to solve the problem of trajectory tracking of a differential drive robot, where both the kinematic and dynamic models are considered.

Besides analytical models, graphical models are of interest to cope up with the complexity of a system. Among those graphical models, bond graph (BG) is a suitable graphical tool to model multi-domain complex systems [7]. On contrary to other graphical tools, BG represents edges carrying power not signal. BG has been used to model mobile robots [8, 9]. In the present work, we focus on BG modeling of a differential drive mobile robot considering its chassis, wheels, and actuators dynamics. The developed model is analyzed through simulation over various surfaces with different coefficients of friction.

The rest of the paper is organized in the following sections. Section 2 presents an introduction of BG modeling. The BG model of differential drive robot is developed in Sect. 3. Simulation results are presented in Sect. 4. Finally, the paper is concluded in Sect. 5.

2 Introduction to Bond Graph Modeling

Bond graph (BG) is a graphical modeling approach based on power exchange phenomena between systems. The power exchange is represented by a half arrow called power bond. This power bond carries two unified variables of power (P) namely, effort (e) and flow (f). Mathematically, it is represented by the relation:

$$P = ef \tag{1}$$

Since BG is based on the energy exchange, it can be applied to the systems of multi-domain nature such as mechanical, electrical, and thermal. The applications of BG in various domains with corresponding effort and power variables are described in Table 1.

Refer to Table 1, BG has been applied to various domains of physics because power (P) is common to all the domains. Therefore, power is exchanged between multi-domain physical systems using power bonds in BG. The multi-domain physical systems are modeled using unified BG elements, which are single port (source of flow SF and source of effort SE, resistance R, compliance C, and inertia I), double port (transformer TF and gyrator GY), and multi-port (common effort 0-junction and common flow 1-junction) elements. There comes a term called causality, which helps to determine the direction of effort and flow in a power bond. The main advantages of BG modeling include (i) multi-domain modeling tool, (ii) graphical tool, (iii) suitable to model complex systems, and (iv) can be used for control as well [7]. Therefore, in this work BG is used for modeling the differential drive mobile robot.

Table 1 Bond graph in different domains

Domain	Effort (e)	Flow (f)	Power (P)
Mechanical translation	Force (F)	Velocity (v)	$P = Fv$
Mechanical rotation	Torque (τ)	Angular velocity (ω)	$P = \tau\omega$
Electrical	Voltage (V)	Current (i)	$P = Vi$
Thermal	Temperature (T)	Entropy flow ($\dot{s}$)	$P = T\dot{s}$
Hydraulic	Pressure (p)	Mass flow rate ($\dot{q}$)	$P = p\dot{q}$

3 Bond Graph Modeling of the Differential Drive Mobile Robot

3.1 Description of the Robot

In this work, a WMR called differential drive robot is considered, which is having two active wheels on the same axle line and one or more passive supporting castor wheels. The active wheels are actuated by two independent direct current (DC) motors. The schematic diagram of the quadcopter is shown in Fig. 1.

Refer to Fig. 1, there are two coordinate frames, first one is the inertial frame $E\{x, y\}$ fixed to the earth, while second frame $CM\{x_b, y_b\}$ is the body-fixed frame attached to the center of mass (CM) of the robot. The CM of the robot is assumed to be located at the center point of the line joining the rotation axes of the two wheels. The angular velocities of the two wheels are represented by ω_1 and ω_2, for the left and the right wheels, respectively. The wheels' radii are equal and denoted by r. The linear velocities of wheels are represented by v_1 and v_2. The velocity of robot's CM is denoted by v, while its orientation is θ. The robot is having three degrees of freedom

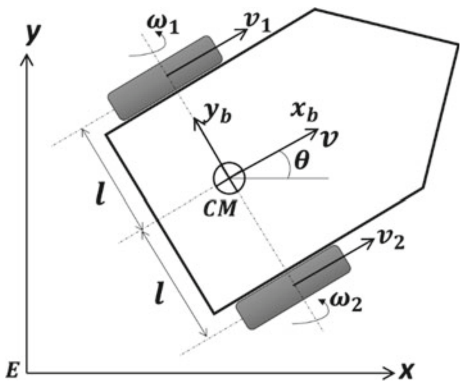

Fig. 1 Schematic diagram of the differential drive mobile robot

(DoF) in the two-dimensional (2D) plane. The three motions along the three DoF include longitudinal ($\dot{x}$), lateral ($\dot{y}$), and yaw ($\dot{\theta}$) motions. The steering of the robot is performed using the difference between the angular velocities of two wheels.

3.2 *Kinematics of the Robot*

The relationship between the body-fixed frame and the earth-fixed frame is given by the following transformation.

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{Bmatrix} = [R] \begin{Bmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{\theta} \end{Bmatrix} \tag{2}$$

where $[R]$ is the rotation transformation matrix between the inertial frame and the body-fixed frame, which is given by:

$$[R] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The nonholonomic constraint restricts the motion of the robot in y_b direction, hence, $\dot{y}_b = 0$. Also, we know $\dot{x}_b = v$. Finally, velocities in the inertial frame are given by:

$$\dot{x} = v\cos\theta \tag{4}$$

$$\dot{y} = v\sin\theta \tag{5}$$

The wheel velocities can be given by the following relations.

$$v_1 = v - l\dot{\theta} \tag{6}$$

$$v_2 = v + l\dot{\theta} \tag{7}$$

The wheels' linear velocities v_1 and v_2 are caused by the rotation of wheels with angular velocities ω_1 and ω_2.

3.3 Bond Graph Dynamic Modeling of the Robot

In this subsection, we will develop the dynamic BG model of the robot using the kinematic equations presented in the previous subsection. The kinematic constraints at some points automatically reflect in the BG dynamic model, because it is based on the energy flow between various points. In the present paper, the dynamic model of the robot considers the multi-domain components including electrical motors and the mechanical dynamics of the robot's CM. The robot's wheels are actuated by two independent DC motors. The following modeling assumptions are made: (i) the robot moves in a plane with three DoF, i.e., longitudinal, lateral, and yaw, (ii) rigid body dynamics is considered, (iii) since the robot moves in a known indoor environment, hence, suspension, roll, and pitch dynamics are ignored, and (iv) wheels are actuated by two independent DC motors. The BG model of the robot is developed in Fig. 2.

In Fig. 2, the half-headed arrows represent the power bonds in BG, while the full-headed arrows represent the measurements, say detector of flow (Df). Each Df is a sensor, which measures the flow at the junction where it is attached. Eight 1-junctions say, 1_v, 1_θ, 1_{i1}, $1_{\omega 1}$, 1_{v1}, 1_{i2}, $1_{\omega 2}$, and 1_{v2}, are corresponding to linear and angular velocities of CM, motor current, angular and linear velocities of the left and right motor-wheel systems, respectively. Two 0-junctions say, 0_{F1} and 0_{F2}, are corresponding to the left and right wheel forces, respectively. Theses wheels' forces F_1 and F_2 are transmitted to the robot's CM. Two 1-junctions say, 1_x and 1_y, are

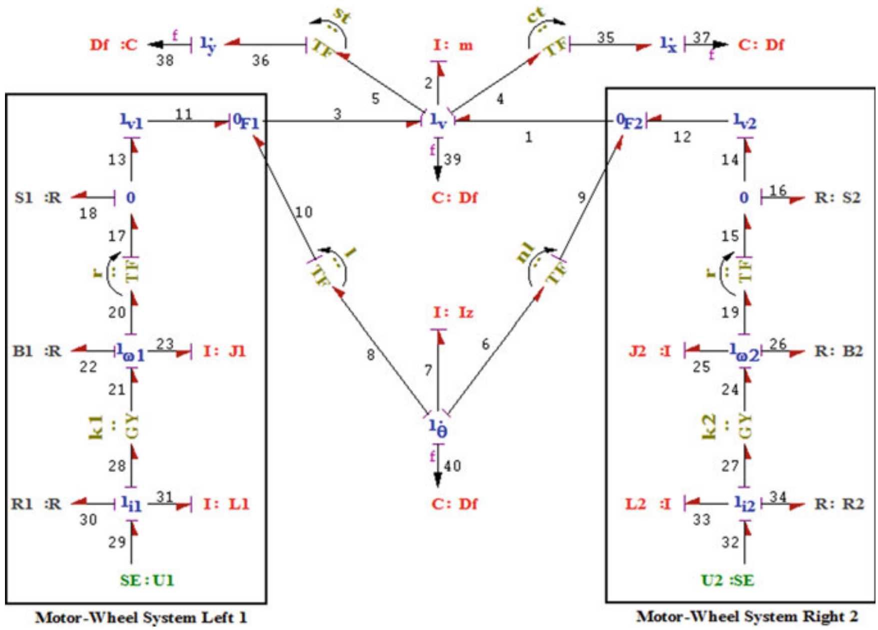

Fig. 2 Bond graph model of the differential drive mobile robot

corresponding to the longitudinal and lateral velocities of the robot in the inertial frame, the transformers TF moduli ct and st represent sin θ and cosθ, respectively.

Refer to BG in Fig. 2, there are various parameters related to the robot's dynamics. In CM dynamics, m and I_z are mass and rotary inertia of the robot; transformers TF moduli l and nl represent the robot's dimension, where nl means negative l. In the electrical part of the left motor-wheel system, U_1, i_1, L_1, R_1, and k_1 represent the input voltage, current, inductance, resistance, and motor constant, respectively. In the mechanical part of the left motor-wheel system, ω_1, J_1, and B_1 represent the angular velocity, rotary inertia, and viscous friction of the motor's rotor shaft. In the wheel-ground part of the left motor-wheel system, r and S_1 represent the wheels radius and the coefficient for the contact dynamics, which depends on the normal load and the friction between the wheel and ground. In the same way, different parameters in the right motor-wheel system can be defined. The expressions for the S_1 and S_2 are given as follows.

$$S_1 = S_2 = \frac{\mu m g}{2} \tag{8}$$

where μ is the friction coefficient between the wheel and ground. In this way, a complete dynamic model of the robot is developed which includes the actuators and the wheel-ground contact dynamics as well.

4 Simulation

The model is verified through simulation in a dedicated software for BG modeling and simulation called SYMBOLS [10]. Once BG model of a system is drawn in SYMBOLS, it automatically generates the equations of motion of the system, and simulation can be carried out (Fig. 3).

The aim of this simulation study is to verify the model and to analyze the dynamic behavior of the robot when it moves on various surfaces with different coefficients of friction. The following numerical values of the parameters are taken for simulation: $L_1 = L_2 = 0.05$ H, $R_1 = R_2 = 0.5\ \Omega$, $k_1 = k_2 = 0.1$ N-m/A, $J_1 = J_2 = 0.2$ kg/m^2, $B_1 = B_2 = 0.1$ N m s/rad, $r = 0.05$ m, $\mu = 0.6$, $m = 5$ kg, $l = 0.15$ m, and $I_z =$ 1.5 kg m^2. First, same input voltage is given to both the motors, say 10 V. The results are shown in Fig. 4, where it can be observed that the robot moves along X direction in the straight line and there is no motion along Y direction, which is an expected behavior for the equal angular velocities of the wheels.

Now, the robot's behavior is analyzed for the different angular velocities of the wheel, the left wheel's motor is excited with $U_1 = 10$ V and the right one with U_2 = 15 V. The results are shown in Fig. 5, where we can see the expected behavior of the robot with positive anticlockwise orientation and circular trajectory.

In order to analyze the behavior of the robot over various surfaces, we have simulated the robot with different coefficients of friction, i.e., 0.5, 0.6, 0.7, and 0.8.

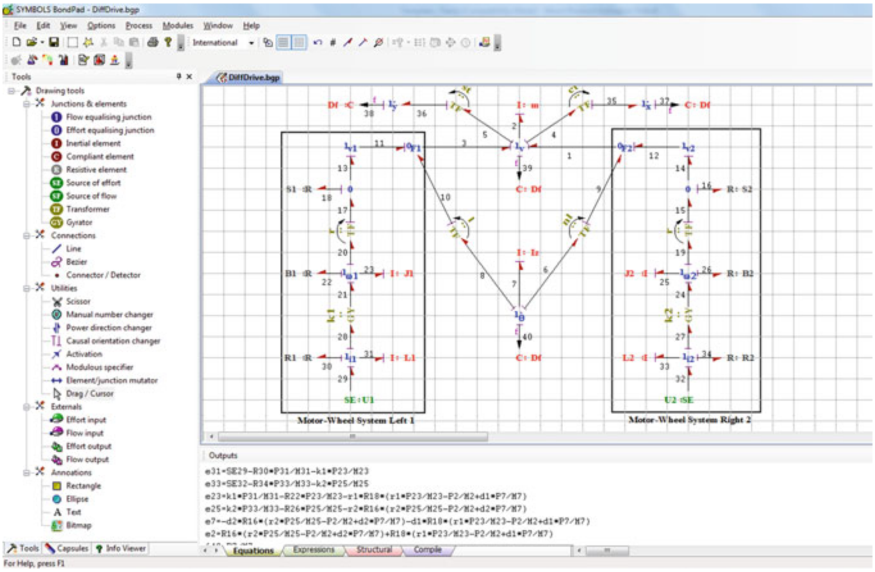

Fig. 3 BG modeling and simulation software SYMBOLS

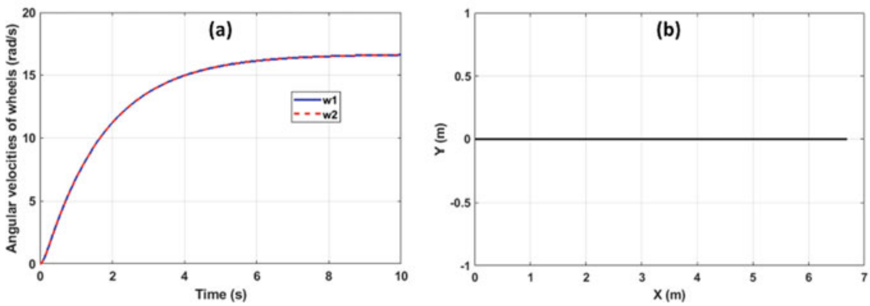

Fig. 4 Equal input voltage to both the motors: **a** angular velocities and **b** trajectory

The different input voltages are given to motors $U_1 = 10$ V and $U_2 = 15$ V. The results are shown in Fig. 6, where it can be seen that the changing values of friction coefficient affect the wheel forces, which leads to change in orientation and trajectory of the robot.

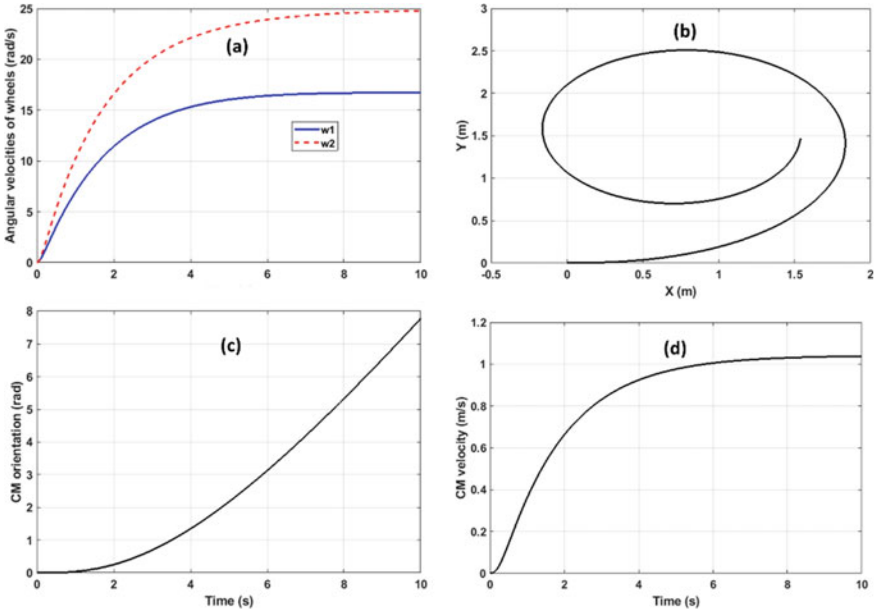

Fig. 5 Different input voltages to both the motors: **a** angular velocities, **b** trajectory, **c** orientation, and **d** velocity of the center of mass of the robot

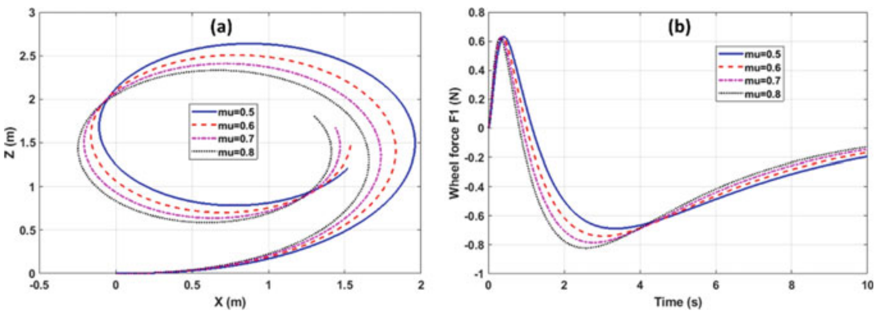

Fig. 6 **a** Trajectories and **b** left wheel forces for different values of coefficient of friction

5 Conclusion

This paper presents the dynamic model of a differential drive mobile robot using a graphical modeling tool called BG. The model considers the chassis, wheels, and actuators dynamics of the robot. The developed model is verified through simulation. The obtained simulation results show the expected dynamic behavior of the robot. Further, the robot's model is simulated over various surfaces with different friction coefficients. For future work, it is interesting to use the developed BG model for control design with some faulty components.

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