Thermo-mechanical Analysis of Functionally Graded Material for Cylindrical Shell Subjected to Internal Pressure for Varying Volume Gradation

Vyom Shah, Darshita Shah, and Dhaval Shah

Abstract Functionally Graded Material (FGM) is a distinctive type of material in which properties vary steadily and smoothly across any direction which gives overall high performance in terms of mechanical and thermal aspects. Functionally graded material (FGM) application has a wide scope in modern tools and technology, which includes aerospace, pressure vessel of a nuclear reactor, High end cutting tools, etc. The main prominence of this paper is to present the structural mechanical and thermal analysis of cylindrical shells made up of ceramic and Aluminum used for fabrication of pressure vessels and reactors in heavy engineering applications. The FGM cylindrical is subjected to different boundary conditions and internal pressure. The material properties of FGM are pondered over the thickness utilizing power-law using MATLAB programming. The novel methodology for finite element analysis is applied for different volume gradations using MACROS in ANSYS APDL. The analysis outcomes for functionally graded materials are matched with a composite sandwich cylinder for the same boundary conditions. It was discovered that Von-Mises stress created in FGM is 22.23% less in comparison with sandwich structure, the stress in x, y, and z-direction is 22.23, 23.97, and 37.92% less and deflection is 18.11% less compared to sandwich cylindrical shell of aluminum and ceramic.

Keywords Functionally graded material · Thermo-Mechanical analysis · Finite element analysis

1 Introduction

The demand for special-purpose material has been increased with the advancement in technology which can fulfills and withstand rigorous thermo-mechanical load and meets product requirement. The conventional use of metals and alloy has its limitation whereas, in composites material, properties change suddenly which leads to

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the problem of delamination and stress concentration between interfaces. Functionally graded materials (FGMs) added much popularity as properties vary smoothly and continuously across the direction as well as property can be tailored according to the desired purpose. The FGM has an extra edge over conventional materials from automobiles to aviation and other industries due to its advantages over metals and composites. The cylindrical shells from FGM can be fabricated using powder metallurgy, laser cladding, and sintering process. Many researchers worked on the analysis of functionally graded material. Sarathchandra et al. [1] analyzed a functionally graded cylindrical shell made up of tungsten carbide and structural steel. Lin et al. [2] found the application of functionally graded material in civil structure to harvest solar energy more efficiently. Paulion et al. [3] explained an overview of functionally graded materials and their manufacturing processes like vapor decomposition technique, powder methodology, and solid freeform fabrication (SFF) method. The various mechanics model like a self-consistent model, Mori-Tanaka model, Tamura-Tomota-Ozawa (TTO) and Hashin-Shtrikman method for analytical analysis of functionally graded materials was also explained. Mohammad et al. [4] found the effect of parameters on asymmetric deformation for FG pressure vessels under thermo-mechanical loading conditions using numerical methods. Parida and Jena [5] described the overview of modeling, designing, and various manufacturing techniques used in the current stage of industrial application for FGM. Sondhi et al. [6] performed linear elastic analysis for variable thickness clamped rotating FGM disks of different profiles and found that disks of the variable profile have a significant reduction of stress compared to uniform profile. Bhandari and Purohit [7] analyzed the tailoring property of a functionally graded plate under transverse loading conditions. Shahzamanian et al. [8] found the application of functionally graded material for disc brake where heat is generated due to contact friction between brake pad and disc, Coulomb friction is taken as a source of heat. Bayat et al. [9] performed a thermo-mechanical analysis of a non-uniform functionally graded disc under a steady temperature field and observed, a disc with a concave thickness profile is the lightest and the one with a uniform profile is the heaviest in weight. Cooley [10] developed a patch to retrofit a cracked aircraft wash structure and reduce thermal-induced cracking. Wasim et al. [11] analyzed a rectangular plate made up of functionally graded material with the help of ANSYS parametric design language (APDL) in which material properties vary according to an exponential law. Ramu et al. [12] presented a modal analysis of functionally graded material plates to find its natural frequencies and different mode shapes and compared it with a plate of a homogeneous material. Hedia [13] used functionally graded material in dental implants and observed that the optimal material for the dental implant was functionally graded material of hydroxyapatite and titanium. Belhocine [14] analyzed a solid disc subjected to a thermal load for various material gradations during design and compared the result for the best optimum material.

In this research paper, a proper and user-friendly analysis methodology is defined for a cylindrical shell made of functionally graded materials. Conventionally various laws are directly used to calculate material properties of FGM after which an analysis is performed. The varying material properties across the thickness are calculated according to power law using MATLAB programming. The steady-state thermal and static structural analysis is performed using ANSYS APDL for functionally graded material made up of aluminum and ceramic. The analysis results are compared with a sandwich cylinder of aluminum and ceramic. MACROS is defined for steady-state thermal and static structural analysis of cylindrical shells.

2 Methodology

2.1 Mathematical Modelling of FGM

There are many theories available to define the material properties of FGM through or across any direction. Theoretical laws, for example, exponential law, power law, vogit models, Mori Tanaka law, and so forth are accessible. The cylindrical shell is split into numerous layers of extremely thin thickness and presume that every layer will have a homogeneous property. The property of each layer is governed by one of the theoretical models. The exponential law and power law gives the best outcome in terms of calculating material property. Exponential law is utilized in analytical analysis while power law gives the best result for practical and simulations.

Materials property like density (ρ), modulus of elasticity (E), the coefficient of heat conduction (K), and coefficient of thermal expansion (α), etc. are defined by power-law using Eq. (1) for FGMs.

$$P(z) = (P_t - P_b)\left(\frac{z}{h} - 0.5\right)^N + P_b \tag{1}$$

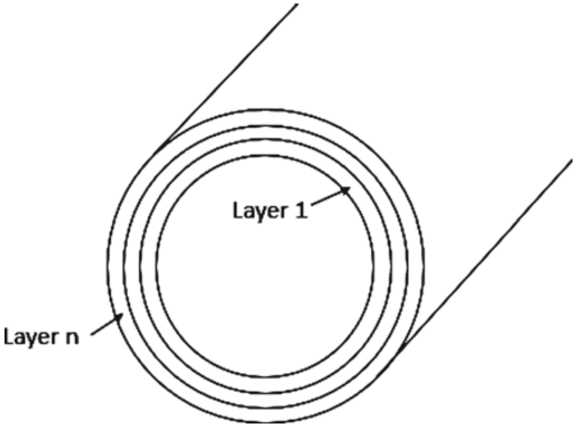
where, $P_{(z)}$ = Property of material, P_t = Top plane material, P_b = Bottom plane material, N = Volume gradient, Z = Height at which material property is defined and h = Distance from neutral axis.

2.2 Variation of Modulus of Elasticity (E) and Thermal Conductivity (K)

Functionally graded material (FGM) because of its distinctive nature material properties vary steadily across any given direction and so it is required to define to code a program that can calculate material property at any point across the thickness. The MATLAB software is utilized for programming to decide material properties according to the power law. Program is prepared in user-friendly aspect in which the user has to input some governing parameters such as thickness, number of layers and for that volume gradation material properties are required to define, it will split the material into predefined layers and property of each layer will be allocated at the center of that layer.

Figure 1 shows the FGM cylinder divided into n number of layers. Modulus of elasticity (E) and thermal conductivity (K) of functionally graded material with a thickness of 1 mm and 10 division is defined for different volume gradation (n). Figure 2 shows a variation of modulus of elasticity for $0.1 \leq n \leq 10$ and Fig. 3 shows a variation of thermal conductivity for $0.1 \leq n \leq 10$.

Fig. 1 FGM cylinder with ceramic at inner diameter and aluminium at an outer diameter



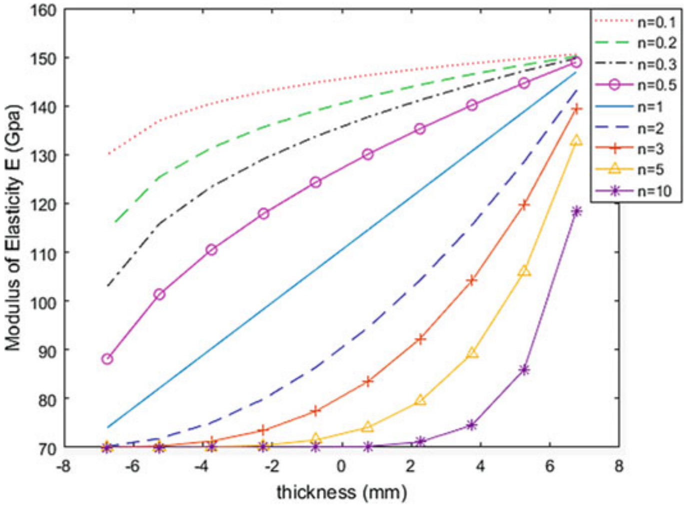


Fig. 2 Variation of modulus of elasticity for different volume gradation across thickness

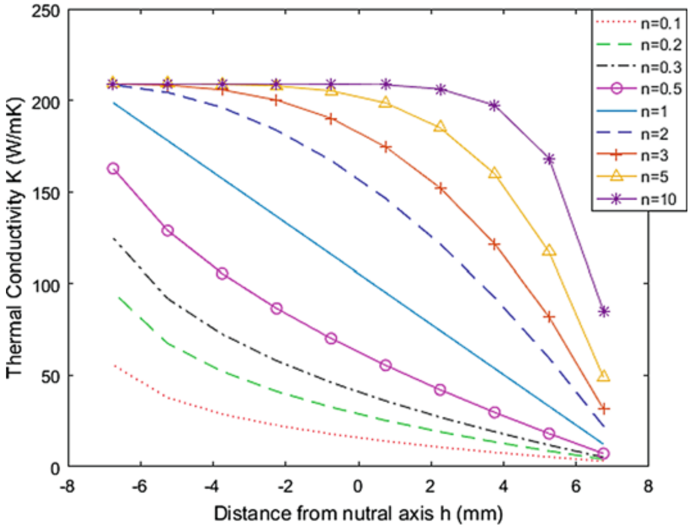


Fig. 3 Variation of thermal conductivity for different volume gradation across thickness

3 Finite Element Analysis

Finite element analysis of functionally graded cylindrical shells is performed. The first step is to model a functionally graded cylinder followed by meshing and applying boundary conditions.

3.1 *Modeling of FGM Cylinder*

A rectangular cylindrical shell of 1500 mm length, 1500 mm diameter, and thickness of 15 mm is considered for analysis purposes. The cylindrical shell is modeled in ANSYS using ACP tool system as shown in Fig. 4. The cylindrical shell is divided into 10 layers of equal thickness with aluminium at outer diameter and ceramic at the inner diameter. The meshing is generated using SHELL 181 element shown in Fig. 5. The static structural and steady-state thermal analysis is performed after applying proper boundary conditions.

3.2 *Analysis Procedure*

The cylinder is fixed at one edge and uniform pressure of 1.2 MPa is applied to at internal surface. After that proper boundary condition and loading condition analysis

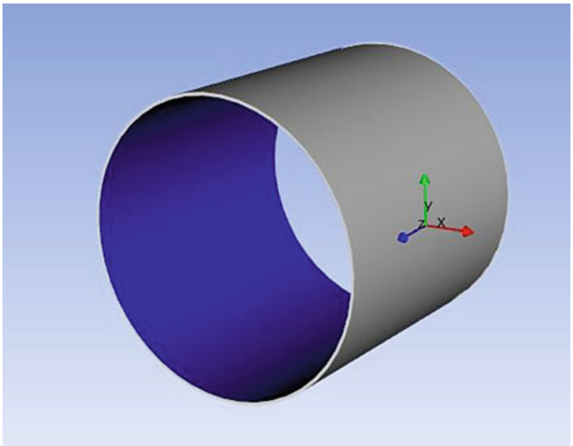


Fig. 4 Layering of functionally graded cylinder

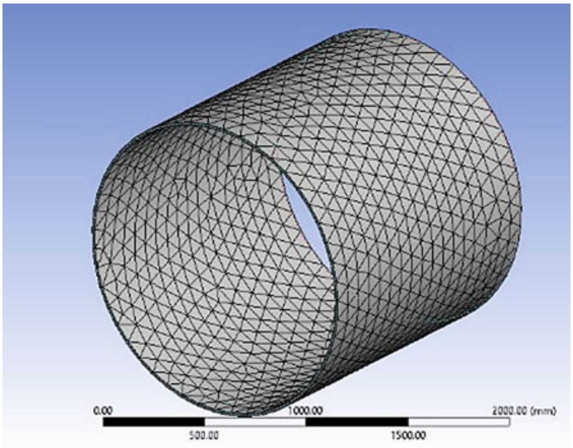


Fig. 5 Meshing of FGMs cylinder

are performed. Various material properties are defined in Table 1. Flow chart approach of methodology for the analysis of functionally graded material is illustrated in Fig. 6.

Table 1 Material properties of aluminium and ceramic

Material properties	Aluminium (6061-T6)	Ceramic (partially stabilized zirconia)
Modulus of elasticity (E) [MPa]	70×10^3	151×10^3
Density (ρ) [Kg/m^3]	2700	5700
Coefficient of thermal expansion (α) [°C^{-1}]	23×10^{-6}	10×10^{-6}
Thermal conductivity (K) [W/mK]	209	2

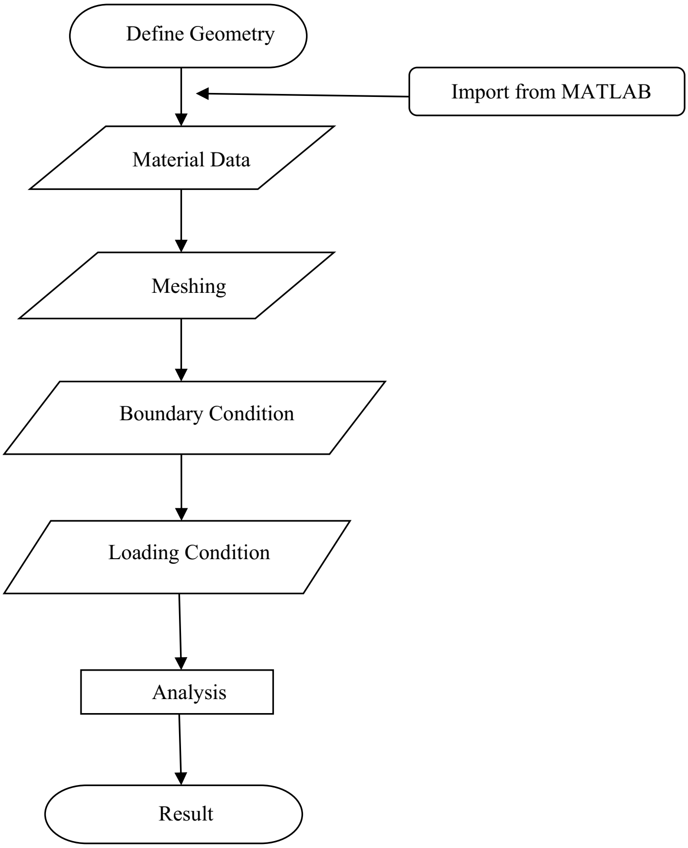


Fig. 6 Flow chart for methodology of analysis of functionally graded material

Figure 7 shows Von-Mises stress generated in cylinder for $n = 0.2$, Fig. 8 shows stress in deflection for $n = 0.2$, Fig. 9 shows stress in x-direction for $n = 0.2$ and Fig. 10 shows thermal flux for $n = 0.2$. Similarly analysis of cylindrical shell for different volume gradient ($n = 0.1$, 0.2, 0.3, 0.5, 1, 2, 3, 5 and 10) and for sandwich cylinder is performed.

4 Result and Discussion

4.1 Effect of Stresses with Respect to Volume Gradation

As volume gradation 'n' tends to move from 0.1 to 10, the percentage of constituents of aluminium in functionally graded cylindrical shells increases. Von-Mises stress

Fig. 7 Von-Mises for n = 0.2

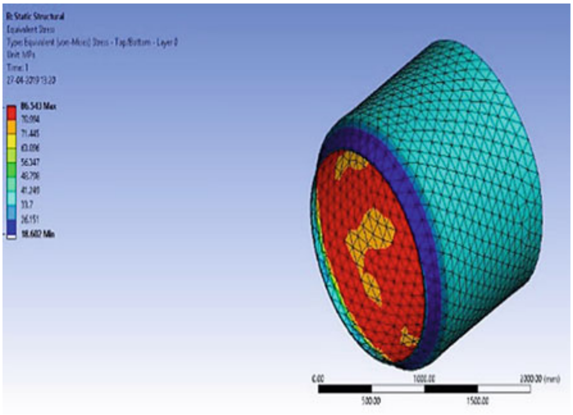


Fig. 8 Deflection for n=0.2

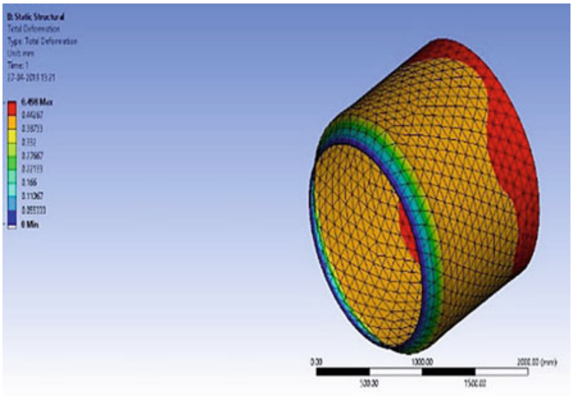
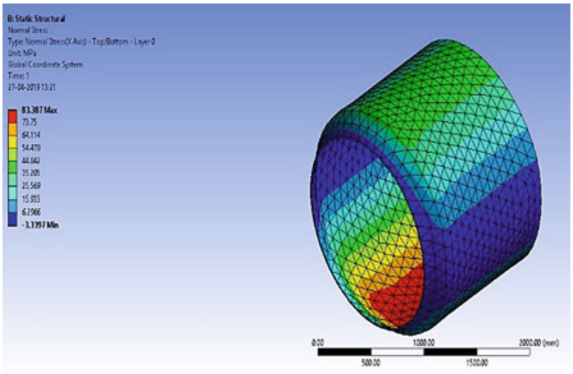


Fig. 9 Stress in X-direction for n = 0.2



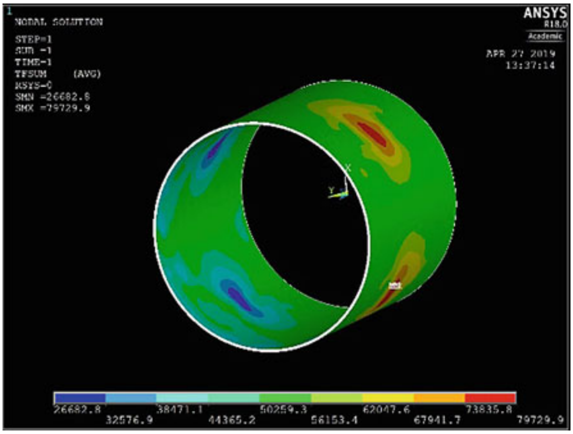


Fig. 10 Thermal Flux for n = 0.2

generated is minimum for the functionally graded shell for $n = 0.1$, which is 22.23% less than stress generated in aluminium/ceramic sandwich cylinder. Stress in X, Y, and Z direction for $n = 0.1$ in the functionally graded shell is 22.23, 23.97, and 37.92% less than that of stress generated in the sandwich shell. Figure 11 illustrates the comparison of various stress generated for different volume gradations and aluminium/ceramic sandwich cylinder. Various stresses generated in the shell for $n = 0.1$ are less it is because constituent of ceramic is dominant over aluminium so overall modulus of elasticity is more compared to $n = 10$.

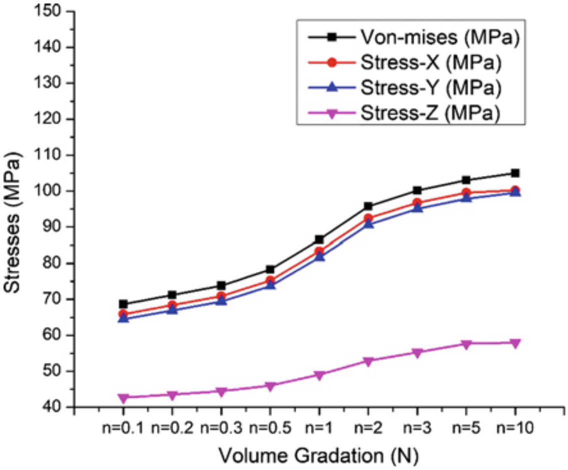
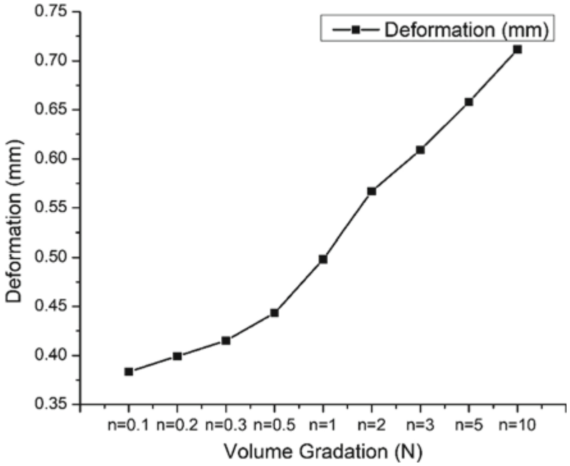


Fig. 11 The comparison of various stress generated for different volume gradation and for aluminium/ceramic cylindrical shell

Fig. 12 Variation of thermal flux and deflection for various volume gradation



4.2 Effect of Deflection with Respect to Volume Gradation

Functionally graded cylindrical shell has minimum deflection for $n = 0.1$ which is 33% lower compared to deflection generated in a sandwich cylindrical shell. Thermal flux increases as volume gradation increases from 0.1 to 10. The constituent of aluminium for $n = 10$ is more compare to $n = 0.1$ which results in increasing thermal conductivity and thus thermal flux increases. Figure 12 illustrates the graphical representation of the variation of deflection and thermal flux.

4.3 Variation of Temperature for Different Volume Gradation

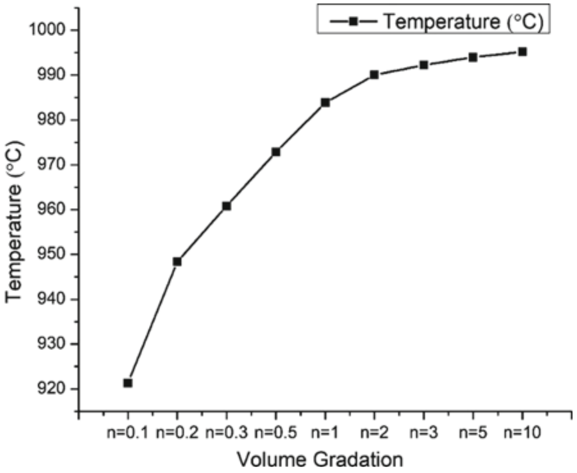
As volume gradation, 'n' tends to move from 0.1 to 10 temperature at the outer surface of the cylinder also increases. Figure 13 illustrate the variation of temperature at the outer surface of the cylinder for different volume gradation.

5 Conclusions

Functionally graded material has better thermal and mechanical performance than composites and alloys. The steady-state thermal and static structural analysis has been performed based on a novel methodology for FGM cylindrical shells.

- The von-Mises stress developed in FGM is 22.23% less than compared to the sandwich structure.

Fig. 13 Variation of temperature for different volume gradation



- Stress in *x, y,* and *z*-direction is 22.23, 23.97, and 37.92% less and deflection is 18.11% less than the sandwich structure of ceramic and aluminum.
- Thermal gradation across the thickness also increases as volume gradation increases. As volume gradation increases from 0.1 to 10 density of aluminum particles in FGM increases, so for better thermal performance, it is desired to have more density of ceramic. Material properties vary linearly when $n = 1$ which also gives better mechanical and thermal performance than the sandwich cylinder.

The different thermal and mechanical properties gradually change in the case of functionally graded materials. This FGM has a wide scope in modern tools and technology, which includes aerospace, pressure vessel of a nuclear reactor, high end cutting tools, etc. The thermo-mechanical analysis can be performed for different components for varying volume gradation.

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