Impact of Wind on the Predictability and Uncertainty Management of 4D-Trajectories

Á. Rodriguez-Sanz and M. Terradellas Canadell

Abstract The future Air Traffic Management (ATM) system will depend on Trajectory Based Operations (TBO) to accommodate the growing demand in air traffic. This system will expect aircraft to follow an assigned 4D-trajectory with high precision, meeting arrival times over established checkpoints with great accuracy. These time-constraints are called Target Windows (TWs). Wind is one of the greatest sources of uncertainty and, consequently, a key point for the improvement of predictability and, ultimately, the implementation of 4D-trajectories. The main aim of this paper is to develop a methodology to characterize these TWs and to assess the uncertainty on the evolution of 4D-trajectories due to the effect of wind. For such purpose, 4D-trajectories are modelled deterministically, using a point mass model and the BADA (Base of Aircraft Data) methodology of EUROCONTROL. In parallel, wind is modelled with a hybrid approach, where the stochastic component captures the error associated with weather forecasts. Through Monte Carlo Simulation, the variability of the trajectory´s parameters is evaluated under different atmospheric scenarios. Using these results, TWs are defined along the different stages of flight, quantifying the uncertainty associated with the aircraft´s position under the effect of wind.

Keywords ATM · Predictability · Uncertainty · 4D-trajectories · Time constraint · Target windows · Monte carlo simulation

1 Introduction and Justification

The recent rise in demand for air traffic poses challenging operational conditions for the existing Air Traffic Management (ATM) system [1]. While air traffic continues to grow, achieving reliable trajectory predictions is a critical prerequisite for the accurate identification and resolution of potential conflicts [2]. To ensure sustainable

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support of airspace usage, SESAR (Single European Sky Air Traffic Management Research), CARATS (Collaborative Actions for Renovation of Air Traffic Systems) and NextGen (Next Generation Air Transportation System) are changing the ATM framework [3–5]. Enhanced predictability and reliability is one of the 11 objectives for the Global ATM Operational Concept as established in ICAO (International Civil Aviation Organization) Doc 9854 [6]. The optimization of trajectory synchronization and conflict detection/resolution is also one of the requirements of SESAR, NextGen and CARATS.

In this context, the upcoming ATM system is based on the Trajectory Based Operations (TBO) concept. TBO requires separating aircraft by defining a strategic (long-term) trajectory, instead of the tactical (short-term) conflict resolution traditionally practiced [6]. Under the TBO framework, airspace users (AUs) will negotiate a trajectory with Air Navigation Service Providers (ANSPs) and airport operators (AOs) [7]. Aircraft systems will exchange information with ground systems, revising the evolution of the trajectory and the planned airspace capacity to ensure that flights meet the assigned Controlled Time of Arrival (CTA) [4, 7, 8]. While this approach allows operators to choose a practically unrestricted, optimal trajectory (with the associated benefits in efficiency, reliability, sustainability and cost-effectiveness of aircraft operations [3, 9]), it also requires that aircraft do not deviate significantly from their agreed reference trajectory and therefore are kept within very small volumes around this trajectory [10]. The goal behind this condition is to ensure that safety separation standards are met. Consequently, a fundamental requirement to TBO operations is to achieve a greater precision on the real-time position of aircraft. To this purpose, SESAR, NextGen and CARATS support the 4D-trajectory operational concept. 4D-trajectories integrate time into the 3D aircraft trajectory, meaning that each point on the flight track is defined by position (latitude, longitude and flight level) and time. In exchange for a more optimal 3D flight path, the aircraft would be obliged to fulfil with great precision an arrival time or position over a specified four-dimensional checkpoint. Such time constraints or spatial constraints are called target windows (TWs) and require the ability to produce accurate and reliable predictions of trajectories [11]. Nevertheless, uncertainties such as the actual aircraft performance or atmospheric/weather conditions affecting the flight, have a great influence in this process [12, 13]. If not corrected, these uncertainties and its associated disruptions can result in the deterioration of the intended trajectory, a degradation that increases over time [14], directly impacting on the reliability and safety of operations. Thus, uncertainty management becomes a keypoint in the future air traffic operations, as adjustments in the trajectory need to be coordinated to ensure reliability. Meteorological circumstances (wind, temperature), aircraft performance (phase of flight, weight, speed), navigational constraints (holdings) and initial conditions are the factors with the largest impact on the evolution of trajectories [15]. Nonetheless, the effect of wind shear on optimum performance [16] has been identified as one of the most important uncertainties in path deviation [17, 18]. For this reason, the quantification of the uncertainty brought by wind constitutes the main focus of this paper.

Several studies have addressed the prediction of trajectories in different phases of flight [19–22]. Most of the methods used in the trajectory prediction (TP) problem can be categorized as either deterministic or probabilistic [23, 24]. The traditional approach is deterministic and deals with TP as a mathematical problem that explains aircraft motion. This approach is heavily and fundamentally constrained by the accuracy of the models that describe the actual behavior of the aircraft and by the quality and consistency of the inputs [25]. In addition, the assumptions and hypotheses of the model might introduce potential inaccuracies or errors in the prediction, i.e. sources of uncertainty that are not considered explicitly by such deterministic methods. Where external factors or parameters (like aircraft performance, environmental conditions, accuracy of navigation systems or traffic regulations) are uncertain or cannot be reliably measured, the probabilistic approach turns the deterministic problem into a stochastic one [12, 13].

The CATS (Contract-Based Air Transportation) project developed the idea of 4D Target Windows (TWs), which the aircraft needs to reach during the flight execution phase, as a way of managing uncertainty [10]. The multiple stakeholders involved in the operation of a flight agree on the target windows definition and location, usually in the areas of transfer of responsibility [26]. Han et al. [27] confirmed that, by incorporating TWs at intermediate locations along a 4D-trajectory rather than just at sector boundaries, these TWs can help in the management of en-route punctuality and uncertainty. Additionally, TWs provide a useful balance between predictability and maneuverability of air traffic [12]. In terms of the TWs geometry, while some studies consider TWs to be circular cross-sections labelled with the expected times of arrival [27], others model them as rectangles with time or space characteristics [2, 28]. When considering the safety requirements associated to the 4D-trajectory operational framework, it is more reasonable and more practical to predict space intervals than exact aircraft positions [22]. This study focuses on TWs with a time control: times of arrival at certain points are fixed, stating the space intervals where the aircraft should be found when reaching these times (scheduled milestones). The idea for reducing uncertainty about the future evolution of a flight is therefore associated to the imposition of spatial constraints at various sections of the trajectory, i.e. TWs that each aircraft will have to meet. Therefore, these constraints or TWs will help to increase punctuality and safety during the flight [28]. Instead of precise and concrete 4D points, a TW is defined as a spatial window or interval, where the times of checkpoints are given as a series of constant values. Hence, uncertainty management is discussed in terms of spatial variability, and the analysis is approached as a spatial reachability problem.

When associated to TBO and RBTs (Reference Business Trajectories), TWs should be large enough to allow AUs and ANSPs to react flexibly to a variety of flight conditions but small enough to improve certainty and increase capacity [2, 7]. An experiment with an Airbus A320 test aircraft flying from Toulouse to Stockholm was performed by EUROCONTROL in 2012 and defined an achievable tolerance window of between -2 min and +3 min over the route and ±30 s for CTA [29]. Moreover, pilots were subjected to conditions where the aircraft deviated from the expected course. Results showed that compliance with 4D-trajectories (adherence to planned paths) in the cruise phase was feasible, whereas the TW for CTA was more difficult to accomplish and needed additional cooperation between pilots and controllers [29].

The main goal of this paper is to design a methodology for characterizing TWs and for managing the uncertainty associated with the evolution of 4D-trajectories due to wind impact. The study uses a simplified flight path, which includes all phases (take off, climb, cruise, descent, final approach and landing). In this study, it is proposed to manage time-related uncertainty by setting multiple intermediate locations (checkpoints) along a trajectory, where TWs can constrain space variability. 4D-trajectories are modelled using a point mass approach and the Base of Aircraft Data (BADA) methodology of EUROCONTROL [30, 31]. The BADA aircraft model relies on a mass-varying, kinetic and kinematic view to aircraft performance modelling. Despite knowing the physics of the wind and its condition as one of the most influential agents in the degradation of the trajectory of an aircraft, many studies do not include it in the equations of motion. However, they do include other atmospheric variables such as temperature, pressure or density. This is due to the difficulty of modeling this phenomenon of changing nature. For this reason, it was decided to focus on the predictability implications of this less explored phenomenon, which is modelled as a stochastic variable while the rest of agents are modelled deterministically. Several wind models were developed with increasing complexity, culminating in a hybrid model in which the deterministic component is wind forecast data and the stochastic component captures the error associated with those weather forecasts. Through Monte Carlo simulation, the variability of the trajectory parameters in different atmospheric scenarios is evaluated. Based on the results of the simulation, TWs are defined for several checkpoints (time-milestones) along the trajectory to estimate and quantify the uncertainty associated with the position of an aircraft under the effect of the wind. Consequently, this will enable us to provide the probability of an aircraft achieving the TW constraint as a function of a space interval. Results are analyzed to draw lessons regarding 4D-trajectories predictability and uncertainty management.

The key contribution of this study is the provision of a model to address uncertainty in TP and improve predictability of flights, whilst offering a methodology to evaluate the robustness and reliability of 4D-trajectories, by quantifying one of its main perturbations (the impact of wind). The proposed methods may be applied in a predictive manner, hence being able to foresee and anticipate the degradation of the expected trajectory, in order to plan appropriate corrective actions. These models improve traffic synchronization and potentially ease conflict resolution in 4D-trajectories, which are cornerstones in future airspace operational environments.

This section presented the problem and its characteristics and reviewed how previous studies have approached this issue. The remainder of the paper is organized as follows. First, we develop a 4D-trajectory model for the specific scenario of study (problem statement). This model is validated using actual flight data, obtained from EUROCONTROL. Subsequently, we propose a model for the wind, as the objective of the study is to understand its impact on 4D-trajectory prediction. The wind model is calibrated with data obtained from the NOAA (National Oceanic and Atmospheric Administration). We then use a stochastic approach to simulate

4D-trajectories that allows us to evaluate variability in the control parameters (input variables that affect the model outputs). This conceptual framework is the basis for identifying TWs (space intervals) along different checkpoints (time constraints). It represents a methodology to characterize uncertainty in 4D-trajectories due to wind. Finally, results are reviewed, and novel insights related to 4D-trajectories predictability and management are proposed. Please see Annex I for the meaning of the acronyms presented throughout the paper.

2 Methodology

2.1 Scenario Characterization

The first assumption of this study is that the aircraft is flying a 4D-trajectory under the new SESAR operational concept and its associated systems and functionalities [2, 32]. This implies that the aircraft follows an optimized path, avoiding the complexity of traditional trajectories, which must use predefined airways, holding patterns and structured operational procedures for take-off and landing. In this framework, the trajectory consists on the following phases: take off, climb, cruise, descent and final approach, and landing. The take-off phase (1) is initiated with the aircraft cleared for take-off by the control tower, and the landing phase (11) finalizes after the aircraft has fully decelerated at the end of the runway. This is justified by the fact that, during the taxi phase, wind has little influence over uncertainty compared to other variables such as traffic congestion. As the climb and descent phases have 3 distinct sections (before passing the transition altitude and when changing to clean or non-clean configuration), they have been modelled taking into account the different performance equations under each of these conditions. The climb flight phase (2) starts at 35 ft and finishes when the aircraft reaches FL360. The cruise flight phase includes 4 stabilized horizontal flight sections (3, 5, 7, 9), an en-route climb section (4) and a descent section (6) that represent a flight level change between FL360 and FL380, and a levelled heading change at FL360 (8). The descent flight phase (10) starts at FL360 and initiates the landing phase (11) at a height of 50ft.

Sections 3, 7 and 9 have a length of 50km. Section 5 length is 100km. The length of the other sections is determined by the aircraft performance. Figure 1 and Table 1 schematize the described trajectory. Please note that the term “phase” refers to the 5 flight phases (take off, ascent, cruise, descent and landing), while the term “section” is used to describe the different segments of the trajectory model.

The aircraft selected to model and simulate the flight was the Boeing 737-900ER, as it is one of the most frequent aircraft used in short- medium range flights in Europe [33] (similar routes to the one modelled in the study).

The atmospheric variables are modelled in accordance to the International Standard Atmosphere (ISA) model [34]. Pressure and density are calculated as a function of the temperature, which is estimated from the flight altitude.

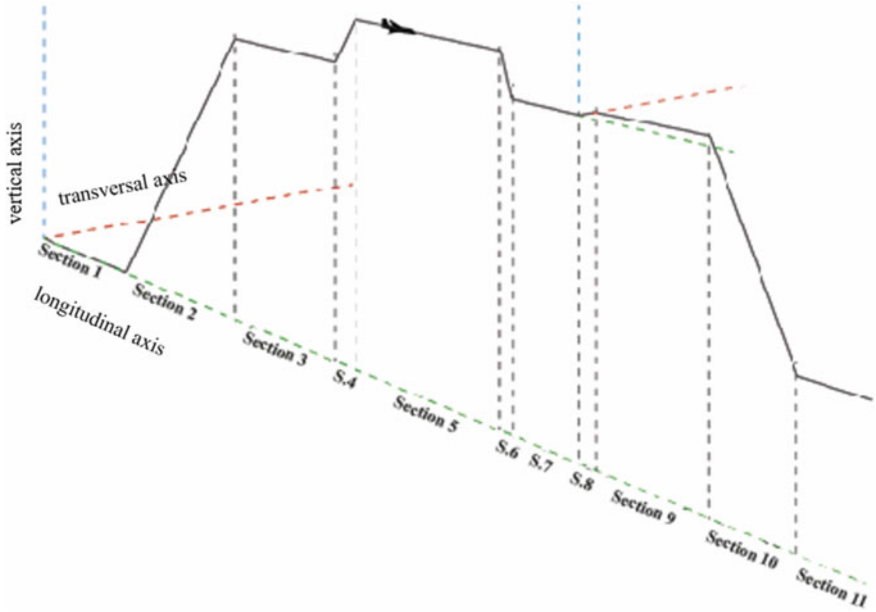


Fig. 1 Simulated flight profile

Table 1 Scenario of study

Flight phase	Section	Description
Take-off	1	Take-off run
Climb	2	35 ft → FL360
Cruise	3	Stabilized horizontal flight section
	4	En-route climb section (FL360 → FL380)
	5	Stabilized horizontal flight section
	6	En-route descent section (FL380 → FL360)
	7	Stabilized horizontal flight section
	8	Heading change
	9	Stabilized horizontal flight section
Descent and final approach	10	FL360 → 50 ft height (threshold)
Landing	11	Landing run

2.2 *The 4D-Trajectory Model*

The 4D-trajectory is modelled using EUROCONTROL's BADA 4.0 methodology [30], which is developed based on the aircraft´s kinetic and kinematic parameters.

BADA 4.0 employs the so-called Total Energy Model (TEM) to determine the performance of the aircraft. It applies the equations of classic flight mechanics with some coefficients which are specific to the aircraft type and to the flight envelope at each phase of the flight [2].

The BADA aircraft model is structured on a mass-varying, kinetic approach to aircraft performance modelling [35]. It can be considered as being a reduced point-mass model. TEM equates the rate of work done by forces acting on the aircraft to the rate of increase in potential and kinetic energy [30]. It is organized in three parts or blocks: (a) Aircraft Performance Model (APM), that provides complete information on the theoretical aircraft performance parameters for a number of different aircraft types; (b) Airline Procedure Model (ARPM), that provides nominal speeds for the climb, cruise and descent phases, assuming normal aircraft operations as provided in the aircraft manufacturers' documentation; and (c) Aircraft Characteristics Model (ACM), that provides a set of coefficients which represent characteristics that are intrinsic to the aircraft. These three elements, together with the Atmosphere model (AM), represent the Aircraft Dynamic Model (ADM), which determines the interdependencies between the modelling parameters. Therefore, each aircraft type in BADA 4.0 is characterized by a group of coefficients, called Aircraft Characteristics (included in ACM), which are used by the APM and ARPM [30]. These blocks allow us to estimate aerodynamic and propulsive variables from the input/control parameters (including mass) with the functional relationships shown in Table 2 (based on [2, 24, 30]).

The functional relationships and structural interdependencies between the parameters that shape the trajectory are directly obtained from the BADA manual [30]. A series of simplifications are performed to adapt the generic trajectory model to our scenario characteristics:

- The aircraft is considered as a point mass with three-degrees-of-freedom (3DoF) [12, 23, 36, 37]. Variation in mass is due to fuel consumption only. The flight is assumed to be symmetrical with all forces acting on the center of mass and included in the plane of symmetry, except during the heading change. The rotational equations are decoupled, the angular speeds are small, and the lifting surfaces do not affect the forces [24].
- We estimate that the aircraft's initial mass is 10% lower than the aircraft's MTOW (Maximum Take-Off Weight), following operational data and past studies [2].
- The maneuver in section (8) (levelled heading change) consists of two consecutive heading changes of 90° at a constant bank angle, μ, which is easily derived from the equations of motion of the aircraft:

$$\mu = \tan^{-1}\left(\frac{V_{\mathrm{TAS}}^2}{Rg}\right) \tag{1}$$

$$\boldsymbol{R} = \frac{\boldsymbol{V_{TAS}}}{\dot{\boldsymbol{\chi}}\frac{\pi}{180}} \tag{2}$$

Table 2 Modelling parameter

Block	Parameter	Dependencies	Description
Atmosphere Model (AM)	Pressure	$p = f[T(h), \rho(h)]$	T (temperature), ρ (density) and h (altitude)
	Speed of sound	$a_0 = f[k, R, T, M]$	M (flight Mach), R (universal gas constant) and k (adiabatic air coefficient)
	Wind	$w = f[\varphi, \lambda, h]$	φ (latitude) and λ (longitude)
Aerodynamic forces model (AFM)	Lift coefficient	$C_L = f[\delta, p_0, k, S, M, m, \varphi, g_0]$	δ (pressure ratio), p_0 (pressure at mean sea level), S (wing surface area), m (aircraft mass) and g_0 (acceleration of gravity at mean sea level)
	Lift	$L = f[\delta, p_0, k, S, M, C_L]$	–
	Drag coefficient	$C_D = f[C_L, \delta, d_1 \ldots d_{15}, M_{\max}, p_0, k, S, M, m, \varphi, g_0]$	$d_1 \ldots d_{15}$ (characteristic parameters of aircraft)
	Drag	$D = f[\delta, p_0, k, S, M, C_D]$	–
Propulsive forces model (PFM)	Thrust coefficient	$C_T = f[t_{i1} \ldots t_{i12}, a_1 \ldots a_{36}, M, \delta, \delta_T]$	$t_{i1} \ldots t_{i12}$ and $a_1 \ldots a_{36}$ (characteristic parameters of aircraft) and δ_T (throttle ratio)
	Thrust	$T_h = f[\delta, m_{ref}, W_{mref}, C_T]$	m_{ref}, W_{mref} (aircraft reference mass and weight)
	Fuel consumption coefficient	$C_F = f[\delta, \theta, M, f_{i1} \ldots f_{i9}, C_T]$	$f_{i1} \ldots f_{i9}$ (characteristic parameters of aircraft) and θ (temperature ratio)
	Fuel consumption	$F = f[\delta, \theta, m_{ref}, W_{mref}, a_0, L_{hv}, C_F]$	$f_{i1} \ldots f_{i9}$ (characteristic parameters of aircraft)

where V_{TAS} is the aircraft's true airspeed at the moment of calculation, R is the turn radius, $g = 9.81$ m/s^2 is the acceleration of gravity and $\dot{\chi} = 1.5$ °/s is the turn rate.

- The selected aircraft type (Boeing B737-900ER) uses a turbofan engine, with idle rating configuration for descent phase and non-idle rating for the rest of the flight. During climb phases, thrust is estimated with the *maximum available thrust in climb* (MCMB) while for the rest of the phases, *thrust for maximum cruise* is used (MCRZ).

- To determine the aerodynamic configuration of the aircraft at each stage of flight, as well as the speed regime in each of the phases, the Airline Procedure Model (ARPM) is used.
- The algorithm for trajectory prediction uses the ground speed of the aircraft (V_{gs}), which is the aircraft's horizontal speed relative to the ground. V_{gs} can be calculated, using vector addition, from wind speed (w), wind direction, heading angle ψ and the aircraft's true airspeed (V_{TAS}).

The model considers deviation control measures along the transversal axis. The automatic control will correct the lateral speed with the last measurement available of transversal wind. Similarly, during cruise phases, the vertical component of wind is compensated by the automatic pilot.

Considering these simplifications and operational adjustments, the 4D-trajectory model was generated using the MATLAB software [38], allowing us to compute theoretical 4D-trajectories. The model's accuracy, was checked by performing a validation test which compares the MATLAB simulated trajectory with real data flights extracted from the EUROCONTROL's database and scenario-based modelling tool DDR2–NEST (Demand Data Repository–Network Strategic Tool) [39, 40]. The convergence between the modelled trajectory and real flights is evaluated using different intra-European routes; particularly, the flights chosen for comparison were those that present similar characteristics to the studied scenario (flight level changes and rectilinear sections). The test error regarding time and position achieves an average value of 7%, reaching less than 5% during the stabilized flight level sections, which is in line with past studies [2, 24, 41]. As an example of the validation procedure, Fig. 2 presents a real trajectory that was flown by a Boeing B737-900ER between Madrid and Cologne; this trajectory is used to test and validate the model [39, 40]. The vertical profile of the trajectory and the section selected for validation are given in Fig. 3 [39, 40]. The vertical axis shows the altitude (FL) and the horizontal axis depicts the range (NM).

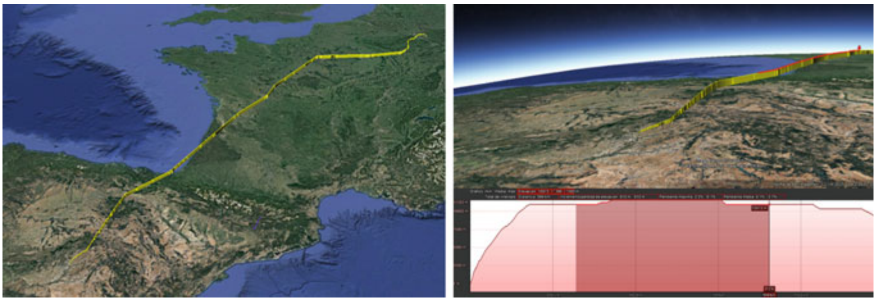


Fig. 2 Real flight between Madrid and Cologne by a Boeing B737-900ER used to validate the model

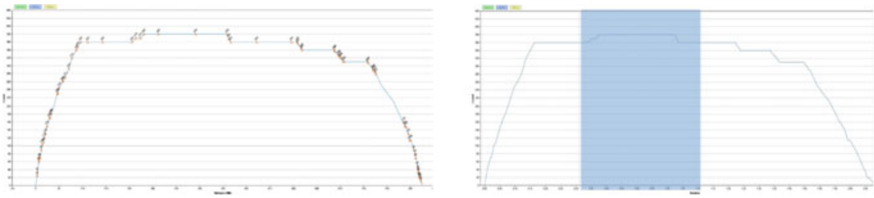


Fig. 3 Real trajectory (vertical profile) of a B737-900ER flight between Madrid and Cologne (left) and section of the trajectory used to validate the model (right)

2.3 The Wind Model

In this study, the wind has also been modelled in 4D. Space has been discretized in N layers of height, defining in each of these layers a grid in the horizontal plane composed of $C_x \times C_y$ cells. Time has also been discretized in order to capture the temporal variations of the wind in magnitude and direction. At any given time and position in space not corresponding to a grid note, the model linearly interpolates between the cells, layers and closest times to obtain the best wind approximation at the desired point. The result of this interpolation is a wind vector specific to the 4D-position of the aircraft in its trajectory.

To define the components of the wind vector, the model implements a hybrid approach, meaning that each vector component is the result of summing a deterministic and a stochastic wind value. The deterministic wind value is the weather forecast for the negotiated 4D-trajectory. Then, the intrinsic uncertainty associated with the wind is introduced by adding a stochastic variable that quantifies the expected error in the weather forecast.

The deterministic wind component is, as mentioned, the weather forecast for the negotiated 4D-trajectory. It is assumed that in the context of 4D-trajectory operation, weather forecast detailing the predicted en-route wind by region will be available during the trajectory planning phase, and it is reasonable to assume that this information will be used to choose the optimal flight path. Therefore, the model developed in this study uses real weather forecast data. In particular, wind data is obtained from the RAP (Rapid Refresh), the NOAA (National Oceanic and Atmospheric Administration) wind prediction tool for North America. This tool is selected because it stores wind data in a convenient format: it is updated every hour and generates a weather forecast stored in a 3D grid, with a resolution of 13 km and a vertical resolution of 50 mb. As the purpose of this study is exemplifying the potential methodology, the specific magnitude of the wind is not relevant and data for a random region and date is used.

In the wind model generated, the size of the grid and the cells adapts to that of the RAP, as well as the temporal variability, which adjusts to the frequency of updating of the aforementioned wind data tool. The code developed for this model loads the RAP results for 3 consecutive hours and stores the wind speeds corresponding to the heights of interest, between sea level and 14,000 m altitude, in a network formed by grids at various heights. As the first layer of height available in RAP is at 12 m altitude, the hypothesis that this layer contains data at sea level has to be made. In addition, as the RAP does not consider vertical speeds, since it focuses on levelled flights, it has been decided to also take this approach and consider the vertical wind, ω_z, void [42].

The stochastic wind component is modelled as a random field, $\omega : \Re \times \Re^3 \rightarrow \Re^2$, where each value is calculated considering the correlation in space and in time of the forecast data [42]. If $\omega(t, P)$ is the wind at a point $P \in \Re^3$ at the time $t \in \Re$, we assume that $\omega(t, P) \in \Re^2$ is gaussian, with void mean and with covariance matrix defined by (3).

$$R\left(t, P, t', P'\right) \in \Re^{2x2} \tag{3}$$

The fact that the mean is zero reflects the hypothesis that all deterministic wind information is contained in the weather forecast. In addition, it is assumed that the wind field is isotropic (invariant to rotations) and that the north-south and east-west components of the wind are not correlated. Under these hypotheses, the covariance matrix R can be expressed by (4), with correlation given by (5).

$$R\left(t, P, t', P'\right) = E\left[\omega(t, P)\omega^T\left(t', P'\right)\right] = \begin{bmatrix} r\left(t, P, t', P'\right) & 0 \\ 0 & r\left(t, P, t', P'\right) \end{bmatrix} \tag{4}$$

$$r\left(t, P, t', P'\right) = \sigma(Z)\sigma\left(Z'\right)r_t\left(\left|t - t'\right|\right)r_{XY}\left(\left\| \begin{matrix} X - X' \\ Y - Y' \end{matrix} \right\|\right) \cdot r_Z\left(\left|p(Z) - p\left(Z'\right)\right|\right) \tag{5}$$

$p(Z)$ is the atmospheric pressure at a height Z and $\sigma(Z)$ is the standard deviation of the wind in m/s at height Z. The functions $r_t(s)$, $r_{XY}(s)$ and $r_Z(s)$ can be obtained from the analysis developed by Cole et al. [43]. If $s \geq 0$, then:

$$r_t(s) = c_t + (1 - c_t - d_t)e^{-\frac{s}{G_t}} + d_t \cos\left(2\pi \frac{(s - e_t)}{g_t}\right) \tag{6}$$

$$r_{XY}(s) = c_{XY} + (1 - c_{XY})e^{-\frac{s}{G_{XY}}} \tag{7}$$

$$r_{Z(s)} = c_Z + (1 - c_Z)e^{-\frac{s}{G_Z}} \tag{8}$$

According to these equations, the correlation between points (t, P, t', P′) decreases exponentially with the horizontal distance and with the difference in height and time of the points. Cole et al. [43] defined the correlation parameters (c_t, d_t, G_t, g_t, e_t, c_{XY}, G_{XY}, b_{XY}, c_Z, G_Z) on Eqs. (6–8) as given by Tables 3, 4 and 5, where a distinction is made between the correlation on the longitudinal component of the wind (ω_x) and the transversal component of the wind (ω_y). The parameters of correlation in the horizontal plane are given in Table 3.

Table 3 Parameters of horizontal correlation

r_{XY}	ω_x	ω_y
c_{XY}	0.05	−0.06
G_{XY} [km]	311	363

Table 4 Parameters of vertical correlation

r_Z	ω_x	ω_y
c_Z	−0.016	−0.041
G_Z [mb]	153	273

Table 5 Parameters of temporal correlation

r_t	ω_x	ω_y
c_t	0.14	0.10
G_t [min]	141	254
g_t [min]	1275	935
e_t [min]	97	447
d_t	0.06	0.05

The parameters of correlation in the vertical plane are defined in Table 4.

The parameters of correlation for the time domain are defined in Table 5.

These parameters allowed us to adjust the RUC (Rapid Update Cycle) prediction tool properties to different functions: correlation data in horizontal and vertical planes is best fitted by an exponential curve (Eqs. 7 and 8 with parameters in Tables 3 and 4), while time correlation is best fitted by a sinusoidal function (Eq. 4 with parameters in Table 5).

Even though the parameters were calculated for RUC data (the predecessor to RAP), this resulted in an acceptable approximation given the nominal variations used in the simulation (of the order of seconds and less than 1km) versus the variations used in the correlation (of the order of thousands of seconds and hundreds of km). The value of the parameters suggests a strong correlation between the wind forecast error for points in the same horizontal plane, a very strong correlation in time and a weaker correlation between points at different heights. For variations in time of between 30 s and 1 h, it is acceptable to simplify the temporal correlation (6) as described by (9) [43].

$$r_t(s) = e^{-\frac{s}{G_t}} \tag{9}$$

With this approximation, the covariance matrix becomes constant over time, and then the wind matrices (W_x, W_y) can be expressed with a linear Gaussian model with the structure given by (10)–(11).

$$W_x(0) = \hat{Q} v_{X(0)} \quad W_X(k+1) = a \cdot W_x(k) + Q \cdot v_x(k) \tag{10}$$

$$W_y(0) = \hat{Q} v_{Y(0)} \; W_Y(k+1) = a \cdot W_Y(k) + Q \cdot v_Y(k) \tag{11}$$

In these equations, $v_x(k)$ and $v_y(k)$ are random, independent and standard Gaussian variables, meaning that they follow a normal distribution with zero mean and identity covariance matrix. These two random variables will be different in every simulation, making the variable stochastic. k is the current time step and a is a parameter given by (12). Q and $\hat{Q}$ are derived from Cholesky decomposition from the covariance matrix $\hat{R}$ [44].

$$a = e^{-\frac{d_t}{G_t}} \tag{12}$$

The stochastic component of the wind can now be calculated by implementing the previous equations in MATLAB. Figure 4 shows the evolution of the wind error between two samples.

Then, when this stochastic component is added to the meteorological forecast data (the deterministic component), the total wind that the model assumes is acting at each point is obtained. Figure 5 illustrates the calculated error of the wind in a given simulation -the stochastic component—(right) and the total corrected wind when this error is added to the forecast data -hybrid approach—(left).

The wind model here presented acts as an input for the remainder of the study. As the study deals with a prediction problem, weather forecasted data is used, since projected data is what would be available in the pre-tactical time horizon. The more accurate this weather forecast is, the more precise the model results will be. To validate the wind model, the forecasted wind values were compared with actual data and it was found that the maximum error is less than 10% in magnitude and direction. Therefore, the wind data used is accurate enough for the model to provide useful scenarios.

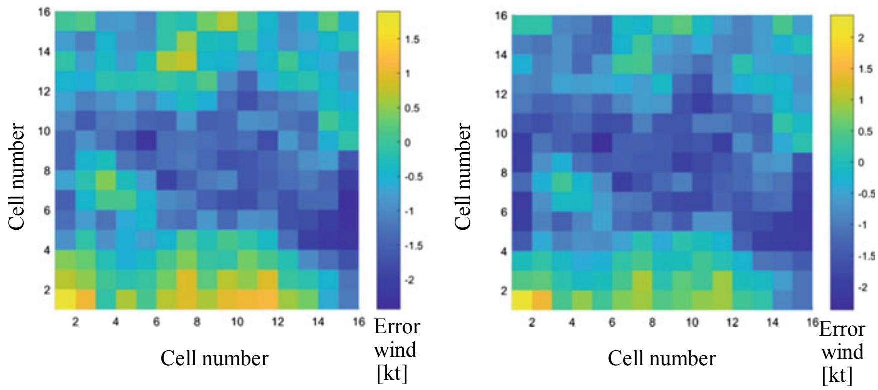


Fig. 4 Wind forecast error at the beginning (left) and at the end (right) of a one-hour period

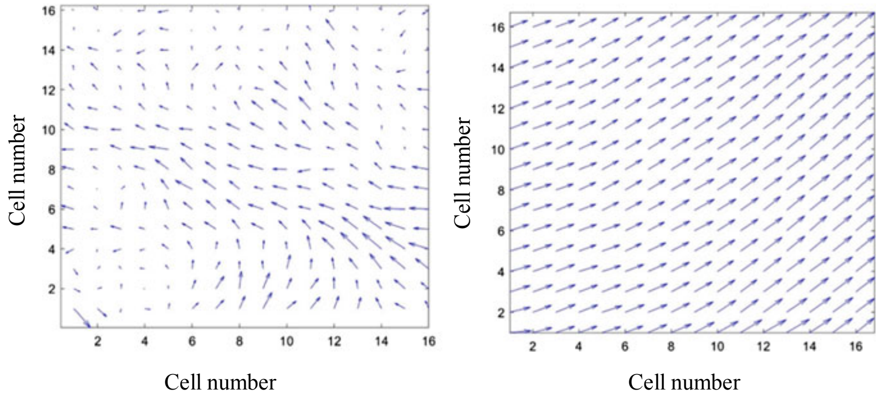


Fig. 5 Modeled error of the wind (left) and total corrected wind (right)

2.4 Monte Carlo Simulation

Monte Carlo simulation is a statistical technique to model the probability of a specific result, in non-deterministic processes where randomness intervenes [45]. This technique is based on the generation of a set of runs (simulations) which rely on the variability of probabilistic inputs. These inputs are randomly produced from a probability distribution that shapes the uncertainty associated with the defining parameters of the process (control variables) [2, 24]. For each set of inputs, the deterministic problem is solved, obtaining a bunch of outputs that are aggregated to obtain the stochastic solution [2]. This methodology can handle many random variables in a single model structure, several types of statistical distributions and non-linear dynamic models [2, 12]. Monte Carlo simulation completes a random sampling and eases the achievement of a large number of numerical experiments, which is essential in problems where extensive physical experimentation is not feasible [46]. The Monte Carlo technique has been widely used and proved effective in air traffic control for 4D-trajectories management [2, 24, 41], conflict resolution [47], safety verification [48], and to estimate the impact of wind uncertainty [49, 50].

In this specific study, we apply the Monte Carlo simulation technique to obtain a set of possible trajectories by varying the wind input in 1,000 consecutive simulations.

The first step of the Monte Carlo simulation consists on the determination of the statistical distribution of the input variable (the wind, in this case). In previous studies [13, 14, 25], a detailed analysis of the variables with the largest impact on 4D-trajectories (mass, temperature, pressure, wind and navigation systems precision) was carried out. Results indicated that wind is one of the parameters with the greatest influence. Therefore, in this study, to isolate the effect of the wind on the degradation of the negotiated trajectory, we consider a deterministic approach for all the other variables and modify the wind values. Therefore, aircraft positions are the parameters resulting in the use of different sets of weather forecast. As indicated previously, our intention is to capture the wind disturbance on the motion of the aircraft with a stochastic dynamic model (aircraft positions are the output parameters resulting in the use of different sets of weather forecast). Specifically, the wind has been modeled as the sum of a nominal component that contains the weather forecast and a stochastic component that quantifies errors in the weather forecast. As described in Sect. 2.3 'The wind model', the stochastic part of the wind is modeled as the correlation between two 4D-points derived from the covariance matrix, multiplied by a random variable that follows a normal distribution of null mean and standard deviation 1; i.e. $N(0, 1)$. By means of this random variable, the wind input will take a different value in each simulation.

Figure 6 shows the wind acting along the entire trajectory for the 1000 simulations. Then, the mean of the wind acting on each simulation is calculated, and then this mean is approximated to a normal distribution, which expresses the mean wind parameters for the set of experiments. With this, it is obtained that the input variable of the model follows a normal distribution of mean $\mu = 9.34$ m/s and standard deviation $\sigma = 1.02$ m/s. This distribution is represented in Fig. 7.

The stochastic approach used to forecast the impact of wind on the trajectory introduces variability in the deterministic model. Figure 8 represents the set of trajectories resulting from the 1000 simulations. Because of the different wind scenario on each of the simulations, a dispersion is observed in the trajectory followed by the aircraft. In the following section, trajectory degradation is quantified through the estimation of TWs.

The variance of the variables estimated by the Monte Carlo technique converges to the inverse square root of the number of runs (N) [45]. Consequently, this method has an absolute error for the estimation that decreases like $1/\sqrt{N}$.

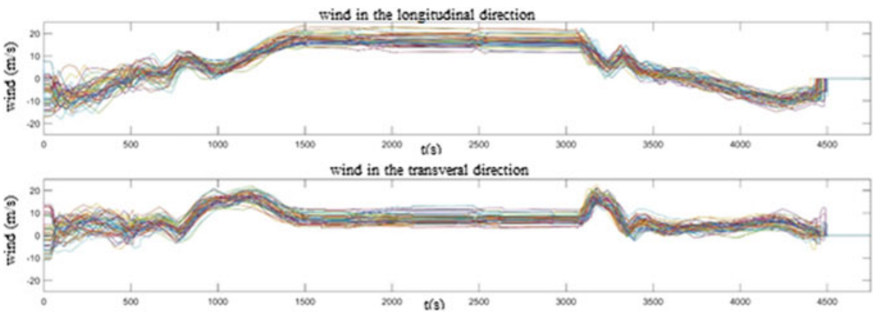


Fig. 6 Wind speed for the N simulations

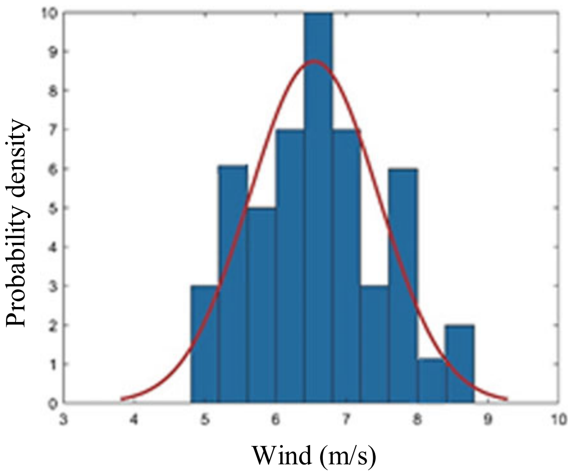


Fig. 7 Normal distribution of the mean wind speed for the *N* simulations

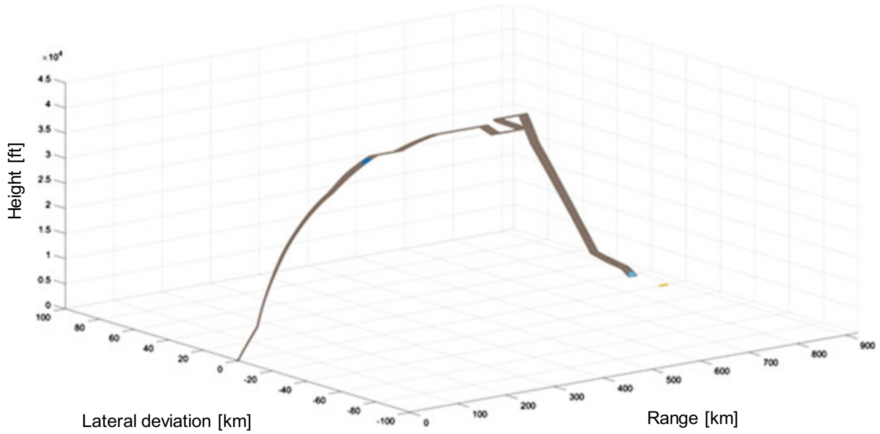


Fig. 8 Representation in 3D of the trajectories for *N* simulations

2.5 The Estimation of Target-Windows

The 4D-trajectory notion aims to ensure a practically unrestricted, optimum trajectory for a flight (if possible), in exchange for enforcing the aircraft to meet with great accuracy an arrival time over a designated control position (milestone) or checkpoint (CP). These time constraints are evaluated in this section by defining TWs or spatial limitations, where the aircraft is required to be found at specific flight times. The Monte Carlo methodology is applied, having as input variable the error in the weather forecast and as output variable the arrival position at each CP. Results reflect the stochastic and time-changing nature of the progress of the flight. Moreover, the variability in the output variable adds a more realistic approach to the deterministic model by including actual uncertainties in trajectory prediction [2, 24]. The required TWs can be defined at all CPs of the agreed trajectory once the simulations are performed and the width of the spatial constraints is established; thereby, providing AUs, ANSPs and AOs with a framework for traffic synchronization and conflict detection and resolution. For the practical application of the TW concept, the size of these constraints on a RBT should at least represent the spatial interval within which any aircraft arriving at the checkpoint can avoid conflicts with other aircraft [27]. The first step to estimating the TW is setting the time (t) of the checkpoints (CP) which represent the CTAs. Each aircraft must hit them while holding a position versus the negotiated trajectory within the required precision [12]. Figure 9 shows the position of these CPs over the modelled trajectory.

Checkpoint 1 determines the deviation at the end of the take-off phase ($t = 41$ s). Similarly, Checkpoint 7 controls the deviation at the beginning of the landing phase ($t = 4441$ s). Checkpoints 2 and 6 aim to determine the deviation on the phases of climb ($t = 750$ s) and descent ($t = 3350$ s) respectively. Finally, checkpoints 3 ($t = 1640$ s), 4 ($t = 2240$ s) and 5 ($t = 2800$ s) will help us study the deviation from the negotiated trajectory during the course of the cruise phase.

After several simulations, these arrival intervals conform a histogram that can be fitted on a normal distribution for each checkpoint, given by the probability density function (13) which provides the probability of an aircraft achieving the

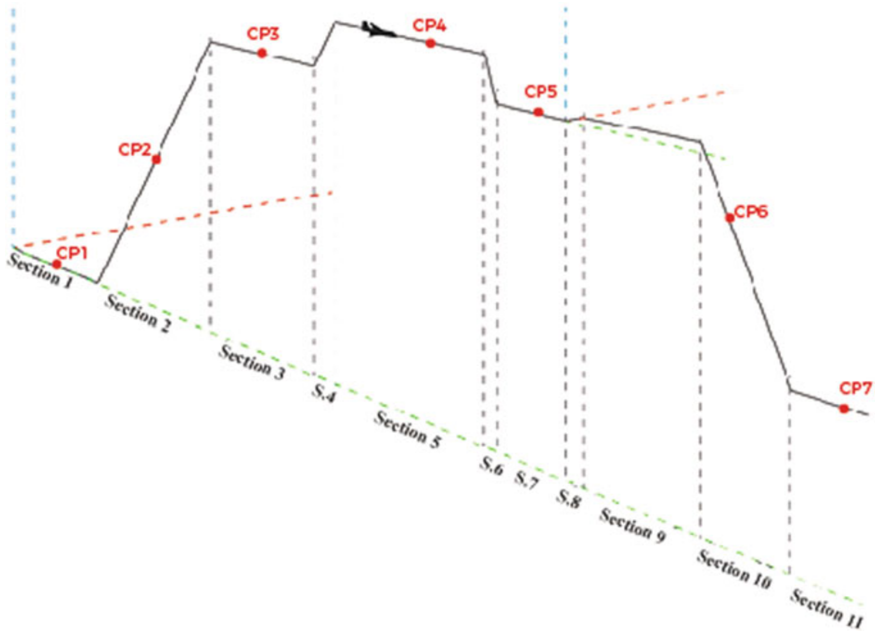


Fig. 9 Checkpoints over the vertical plane of a simulated trajectory

TW constraint as a function of a space interval centered at μ and the level of accuracy σ. Then, different TWs or intervals can be defined depending on the precision requirements established. The width of the TW is an indication of how predictable a flight is and how its progress can be managed. Setting longer space intervals increases predictability and reduces uncertainty, although this can lead to a less efficient time management. For this study, a sigma level of $\pm 2\sigma$ is used.

$$f_{\mathrm{NORMAL}}(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot e^{-\left(\frac{x-\mu}{2\sigma}\right)^2} \tag{13}$$

3 Results and Conclusions

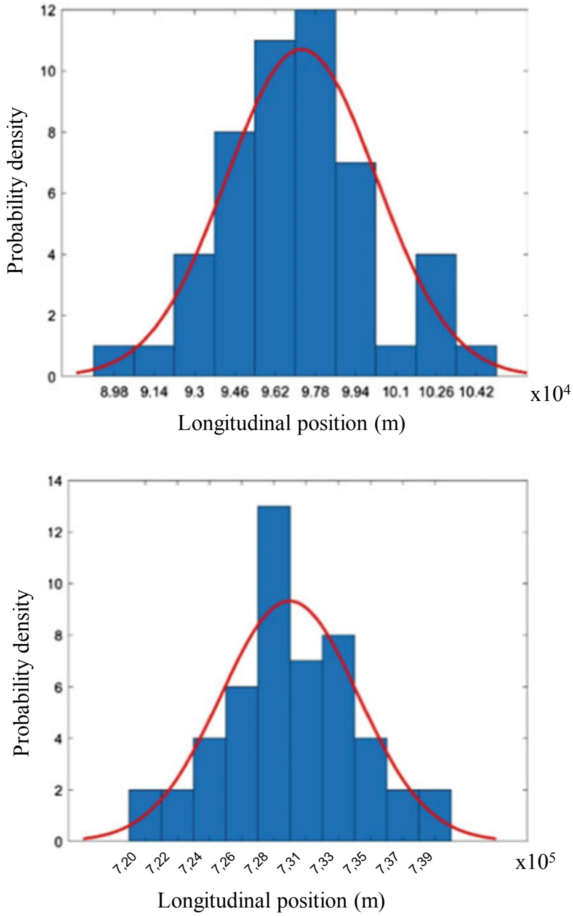
The fitted probability curve for the longitudinal arrival position at checkpoints 2 and 6 is shown in Fig. 10.

The results for CP2 imply that there is a 95.44% ($\pm 2\sigma$) probability that an aircraft will be found at Checkpoint 2 (750s) within a TW of ±6km centered at 97km. Similarly, the arrival positions at Checkpoint 6 (3,350s) can be fitted to a normal distribution with mean $\mu = 729$ km and a standard deviation of $\sigma = 4.5$ km. There is a 95.44% ($\pm 2\sigma$) probability that an aircraft will reach CP6 within a TW of ±9 km. The values of σ and the Interquartile Range (IQR) are higher for CP6 than for CP2. This first result implies that, as the flight advances, the uncertainty and data dispersion are greater. The TWs calculated for the rest of these CPs can be found in Table 6.

The integration of time into the 3D trajectory becomes tangible by setting an instant of time and estimating the 3D position of the aircraft (Fig. 11 shows the aircraft's potential positions around the different control points, represented along the path followed in one of the simulations). However, it is necessary to note that, in each simulation, the aircraft will follow a unique trajectory, as shown in Fig. 8. In this context, the results of the work show that, for a time of 750 s, the aircraft will be found, with a 95.44% probability, within an ellipsoid defined by a longitudinal deviation of ±6000 m, a lateral deviation of ±4 m and a height range of ±276 m. Ultimately, the windows on the three axes define some ellipsoids around the aircraft. The volume of those ellipsoids may be used by ATM service providers to define security minimums, improve synchronization and anticipate the resolution of conflicts to the pre-tactical phase, increasing the predictability of the aircraft in its monitoring of the contracted trajectory.

In general, the trajectory will experience gradual nonlinear degradation over time. Results show a greater dispersion during the turn maneuver and the descent phase. On the other hand, the dispersion is very low at the end of the take-off stage, and at the beginning of the landing phase the dispersion is also reduced with respect to that presented in descent maneuver. Extending the estimation of TWs (space intervals) to several points of the trajectory shows that the relationship of the longitudinal position window with the flight time is monotonously increasing, implying that the aircraft

Fig. 10 Statistical distribution of the aircraft´s longitudinal position in m at CP2 (upper) and CP6 (lower)



suffers a gradual degradation of its ability to follow the contracted trajectory. This will have direct consequences on the operational procedures of the SESAR concept and a maximum amplitude of the position window will have to be specified in accordance with the requirements of the airspace. The dimension of the time window will then determine the maximum flight time before updating the flight data.

Other conclusions can be extracted by evaluating the sensitivity of the results to changes in the parameters that shape the case study (the modelled scenario). On the one hand, the lateral deviations are small in relation to the longitudinal deviation, since a simple control system that simulated the autopilot of the aircraft was applied in this dimension; this system compensated in each instant of time the wind measured in the immediately previous interval. However, even with this measure, the aircraft experiences lateral deviations of up to 50 m. Therefore, it is necessary to study the degradation of the trajectory, with the purpose of proposing the corrective measures that guarantee that the aircraft does not deviate excessively from the

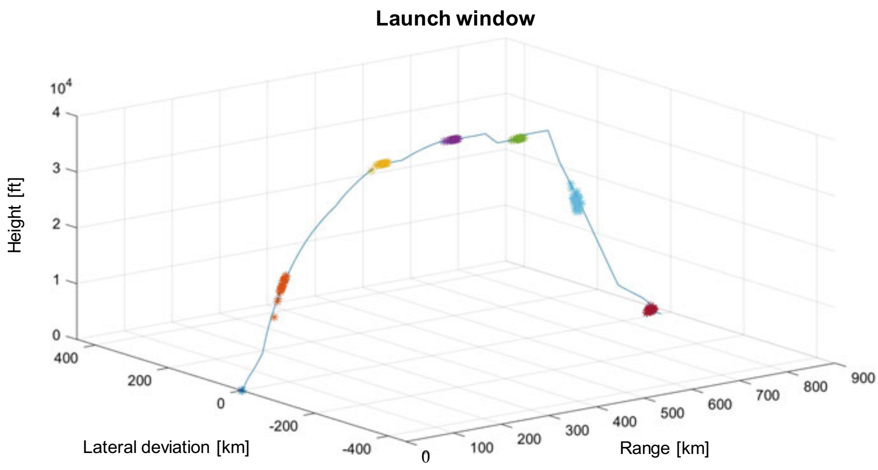


Fig. 11 3D Position of the aircraft in the time-constraint control points

planned trajectory. Finally, the deviations in altitude are smaller than those suffered in the longitudinal direction, although of similar relative magnitude when considering the distances travelled in the respective axes. From this observation, future works will conduct a sensitivity and causal relationships analysis, for example, through the application of Bayesian Networks [24], in order to quantify the dependence of the results obtained from the parameters defined in the model.

The presence of substantial uncertainties in the systems and models required for trajectory prediction represents a major challenge for the future TBO concept. Weather, and particularly wind, can be considered as one of the most relevant sources of trajectory degradation. Understanding and managing the impact of wind is hence necessary to increase the predictability of the ATM system. This study has presented preliminary results on trajectory prediction in which wind is assumed to be the unique source of uncertainty.

The main contribution of this paper regarding uncertainty management is the provision of a methodology to generate TWs (ellipsoids around the aircraft) at

Table 6 Results summary

	Window-x (km)	Window-y (km)	Window-z (km)
CP1	0.514	0.010	0.004
CP2	5.966	0.004	0.276
CP3	7.240	0.002	0.000
CP4	7.700	0.002	0.000
CP5	8.006	0.050	0.000
CP6	8.982	0.034	0.438
CP7	6.500	0.030	0.072

different checkpoints (time constraints) of the 4D-trajectory. These TWs are defined considering wind variations and allow us to determine, with a 95.44% probability, the position of the aircraft. Therefore, it increases the predictability of the aircraft flight and enhances the trajectory robustness when assessing its evolution.

Key findings of the paper are concrete values (Table 6) for space variability due to wind impact, which covers a gap in current literature and provides a rule of thumb for airspace users and network planners when evaluating 4D-trajectory potential deviations. The method also allowed us to appraise 4D-trajectories sensitivity to wind variations.

In the future TBO concept, the RBT is the trajectory which the AUs agree to fly and ANSPs and airports agree to facilitate (subject to separation provision) [3]. Therefore, an RBT is the representation of an AU's intention with respect to a given flight. This trajectory may be modified during the execution phase when constraints are to be changed due to separation of traffic or weather hazards, i.e. if the airspace's requirements regarding safety, regularity and efficiency are not achieved. The provision of TWs eases the definition of trajectory requirements when significant wind impacts are projected. The method can be practically applied in a predictive way to anticipate the trajectory degradation and determine potential corrective actions. This represents a move from reactionary (tactical) interventions to preventive (strategic) interventions.

Moreover, in a pre-tactical phase, this methodology could be used as an input for synchronization measures, and conflict detection and resolution algorithms. TWs offer pilots and air traffic controllers a better awareness of the positions that aircraft are projected to reach during the flight. It also provides intermediate objectives for a flight execution, while TWs were traditionally understood as boundary objects for coordinating timing between adjacent sectors in order to handle punctuality of aircraft as they transit between these sectors, we have extended this operational concept and presented a more flexible approach. We propose TWs distributed across the entire 4D-trajectory of an aircraft's flight, and not just at sector boundaries. However, this will demand additional procedures and tools to share the necessary information and enable smooth coordination between pilots and controllers, as a higher number of checkpoints and TWs will likely increase the amount of coordination required between them.

Our results suggest that the definition of TWs associated to 4D-trajectory management will offer a promising balance between predictability and maneuverability. It is concluded that uncertainty (in this case due to wind) can not only be quantified, but also managed and reduced by establishing TWs. The proposed methodology could prove useful for both airspace users and networks managers for the design of a more resilient and robust ATM system.

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ANNEX I: List of Acronyms, Abbreviations and Parameters for Calculations and Equations

Acronym	Meaning
ACM	Aircraft characteristics model
ADM	Aircraft dynamic model
AFM	Aerodynamic forces model
AM	Atmosphere model
ANSPs	Air navigation service providers
AOs	Airport operators
APM	Aircraft performance model
ARPM	Airline procedure model
ATM	Air traffic management
AUs	Airspeed users
BADA	Base of aircraft data
C	Cells in the wind model
CARATS	Collaborative actions for renovation of air traffic systems
CATS	Contract-based air transportation
CP	Checkpoint
CTA	Controlled time of arrival
EUROCONTROL	European organisation for the safety of air navigation
FL	Flight level
g	Acceleration of gravity
ICAO	International Civil Aviation Organization
MCMB	Maximum thrust in climb available
MCRZ	Thrust for maximum cruise
MTOW	Maximum take-off weight
NextGen	Next generation air transportation system
NOAA	National Oceanic and Atmospheric Administration
P	Position
PFM	Propulsive forces model
R	Turn radius
RAP	Rapid refresh
RBTs	Reference business trajectories
RUC	Rapid update cycle
SESAR	Single European Sky Air Traffic Management Research
t	Time
TBO	Trajectory based operations

(continued)

(continued)

Acronym	Meaning
TEM	Total energy model
TP	Trajectory prediction
TWs	Target windows
V_{gs}	Aircraft's ground speed
w	Wind speed
V_{TAS}	Aircraft's true airspeed
$\dot{\chi}$	Turn rate
ψ	Heading angle
μ	Bank angle

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