# **An Optimistic Planning Approach for the Aircraft Landing Problem**



**S. Ikli, C. Mancel, M. Mongeau, X. Olive, and E. Rachelson**

**Abstract** The Aircraft Landing Problem consists in sequencing aircraft on the available runways and scheduling their landing times taking into consideration several operational constraints, in order to increase the runway capacity and/or to reduce delays. In this work we propose a new Mixed Integer Programming (MIP) model for sequencing and scheduling aircraft landings on a single or multiple independent runways incorporating safety constraints by means of separation between aircraft at runways threshold. Due to the NP-hardness of the problem, solving directly the MIP model for large realistic instances yields redhibitory computation times. Therefore, we introduce a novel heuristic search methodology based on Optimistic Planning that significantly improve the FCFS (First-Come First-Served) solution, and provides good-quality solutions in reasonable computational time. The solution approach is then tested on medium and large realistic instances generated from real-world traffic on Paris-Orly airport to show the benefit of our approach.

**Keywords** Aircraft landing problem · Mixed integer programming · Optimistic planning

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## **1 Introduction**

The International Air Transport Association (IATA) expects 7.8 billion passengers to travel in 2036, which represents nearly double the passengers recorded in 2016 [\[1](#page-14-0)]; this increasing demand on air transportation exposes the available infrastructure to a risk of saturation. Constructing new infrastructures (runways, airports) is a solution to increase the capacity, however, it may not always be feasible due to the huge cost incurred. The alternative is to optimize the use of current infrastructure, especially the runway which is recognized to be the bottleneck of the whole Air Traffic Management (ATM) system.

Since the runway sequence is one of the key factors that determines runway capacity [\[2\]](#page-14-1), several researchers were interested in the optimization of runway sequences, which corresponds in the literature to the following problems:

- The Aircraft Landing Problem (ALP) aims at sequencing arriving aircraft on the available runways and scheduling their landing times taking into consideration several operational constraints.
- The Aircraft Take-off Problem (ATP) consists in scheduling take-off slots to departing aircraft
- The Aircraft Scheduling Problem (ASP) consists in sequencing and scheduling simultaneously departing and arriving aircraft.

According to the survey [\[3](#page-14-2)] by Bennell *et al.*, the ALP received much more attention in the literature than the ATP or the ASP. Several approaches are proposed in the literature for the three above-mentioned problems, and can be classified in two main categories:

- Exact approaches, mainly MIP-based approaches [\[4](#page-14-3)[–9\]](#page-14-4) and Dynamic Program-ming [\[6](#page-14-5), [10](#page-14-6), [11\]](#page-14-7)
- Heuristic approaches [\[4,](#page-14-3) [9](#page-14-4), [12](#page-14-8)] and Meta-heuristics, such as Simulated Annealing [\[13](#page-15-0)[–16](#page-15-1)], Tabu Search [\[8,](#page-14-9) [17\]](#page-15-2), Genetic Algorithms [\[18](#page-15-3), [19](#page-15-4)], Ant Colony Optimization [\[20](#page-15-5), [21](#page-15-6)], and Variable Neighborhood Search [\[13,](#page-15-0) [22\]](#page-15-7).

Interested readers may refer to [\[3](#page-14-2)] for a comprehensive review of existing approaches to the ALP.

In this work, we are interested in sequencing and scheduling aircraft landings at the runway threshold. Each aircraft has a target landing time and an authorized landing time window, expressed as an earliest and a latest acceptable landing time based on fuel considerations. Deviations from the target times will cause a cost that depends on each aircraft, and the aim is to minimize the total deviations from target times, which is more general than minimizing only total schedule tardiness. To model the problem, we propose a novel MIP formulation that takes into consideration safety constraints by imposing separation between aircraft at the runway threshold (Table [1\)](#page-2-0). The proposed formulation is adapted to airports that involve multiple independent runways. Due to the NP-hardness of the problem [\[4](#page-14-3)], solving directly the MIP model for large realistic instances leads to redhibitory computation times, which

		Following aircraft			
		H	M		
Leading aircraft	$\mathbf{H}$	96	157	196	
	M	60	69	131	
		60	69	82	

<span id="page-2-0"></span>**Table 1** Final-approach separation matrix (in seconds) according to ICAO's basic wake-turbulence categories (*source* [\[10](#page-14-6)])

is unsuited for the dynamic nature of the problem that requires air-traffic controllers to make quick but good decisions. Therefore, we introduce a novel heuristic search methodology based on Optimistic Planning [\[23](#page-15-8)], that provides good-quality solutions in a negligible time. We then evaluate empirically our approach on realistic instances generated from real-traffic data from Paris-Orly airport.

The remainder of this paper is organized as follows. In Sect. refsec:problem we describe the ALP and highlight the operational constraints. Next, Sect. [3](#page-4-0) presents our proposed MIP formulation and the constraints taken into account. Then, in Sect. [4,](#page-6-0) we explain our proposed solution approach. Section [5](#page-8-0) presents computational results that show the benefits of our approach, and finally in Sect. [6](#page-13-0) we summarize the contributions of this work and suggest some perspectives for future research work.

#### **2 Problem Description**

Given a set of aircraft near the terminal area of an airport, the ALP consists in mapping each aircraft to a landing time such that a given criterion is optimized while operational constraints are satisfied. When the airport has more than one runway, a decision with respect to the landing runway has to be made by controllers; the runway assignment depends on several factors such as the airport configuration and the direction of arriving aircraft.

The most common approach used by controllers to sequence aircraft is the First-Come First-Served (FCFS) rule, where aircraft land according to the order of the scheduled times of arrival at the runway, and air-traffic controllers ensure only the minimum separation requirements. This FCFS heuristic is easy to implement and guarantees equity between aircraft. However, it is rarely optimal in terms of runway throughput, especially in congested airports [\[10](#page-14-6)], simply consider the large separation requirement in some scenarios where a heavy aircraft is followed by a light aircraft (Figure [1\)](#page-3-0). This motivates the development of efficient methods that compute optimal sequences while satisfying several operational constraints such as minimum separation, authorized time windows and constrained-position shifting.



<span id="page-3-0"></span>**Fig. 1** Comparison of three landing sequences. Case 1 illustrates the FCFS sequence. In case 2, the landing sequence is optimized with respect to wake turbulence separation, and in case 3 the landing sequence is further optimized by runway assignment

- The **minimum separation** constraint guarantees that no aircraft is affected by the wake-vortex turbulence generated by a leading aircraft, especially during takeoffs and landings. The International Civil Aviation Organization (ICAO) classifies aircraft in three main categories, namely Heavy (H), Medium (M) and Light (L), and the separation requirements are defined depending on the category of both the leading and the following aircraft. Separation requirements are given in Nautical Miles (NM), but can be converted to seconds as explained in [\[2](#page-14-1)] and summarized in Table [1.](#page-2-0)
- **Time-windows** constraints are defined by an earliest and a latest possible landing times, based on fuel availability or on possible speed-ups. Indeed, once an aircraft arrives at the boundary of airport control centers, decision-support tools compute an Estimated-Time of Arrival (ETA) at the runway threshold. If the aircraft speeds up, the Actual Landing Time (ALT) may be earlier than the ETA. On the other hand, aircraft may be delayed after entering the radar range and, in this case, the ALT will be later that the ETA and the latest possible landing time is limited by the available fuel [\[24\]](#page-15-9).
- The **Constrained-Position Shifting** (CPS) constraint limits the deviation from the FCFS sequence for equity reasons. This constraint ensures that an aircraft is not deviated from its initial position in the FCFS sequence by more than a given number of positions called maximum position shifting and denoted *m*, which is

usually small;  $m = 3$  or  $4 \lfloor 10 \rfloor$ . This constraint does not only ensure equity between aircraft, but it also reduces the complexity of the problem.

In the following section, we introduce a MIP formulation for the ALP involving one or multiple runways, and we show how we can incorporate different operational constraints in the model.

#### <span id="page-4-0"></span>**3 Mathematical Modeling**

Runway assignment and scheduling aircraft at each runway is formulated as a MIP model which decides the landing dates at each runway threshold, while respecting safety requirements so as to optimize a given objective. We leave the control problem, i.e. how aircraft can be controlled so as to implement the solution of our decision problem, for future research work.

#### *3.1 Input Data*

Consider a set of arriving aircraft  $A = \{1, 2, ..., N\}$ , and a set of available runways  $K = \{1, 2, ..., R\}$ . Without loss of generality, let us assume that each aircraft index *i* ∈ *A* represents its position in the FCFS sequence. Then, for each flight  $i \in \mathcal{A}$ , the given input data are presented in Table [2.](#page-4-1)

For each aircraft  $i \in A$ , the earliest acceptable landing times  $E_i$  is chosen to be 60 seconds before the target time  $T_i$ , because it is the most economic for arriving aircraft according to  $[25]$  $[25]$ . The latest landing time  $L_i$  is set to 1800 s after the target time due to the limited fuel on board [\[25](#page-15-10)].

## *3.2 Decision Variables*

Our proposed model involves binary optimization variables for sequencing and runway assignment, and continuous optimization variables for assigning times at the runway threshold. The binary variables are defined as follows:

Notation	Parameter
$T_i$	Target landing time
$[E_i, L_i]$	Landing time window ( $L_i > E_i$ )
$S_{ij}$	Minimum separation time ( $\geq 0$ ) between aircraft i and j, where i lands before j
$c_i^-$	Penalty cost ( $\geq$ 0) per time-unit for landing <i>before</i> the target time $T_i$
$\overline{c_i^+}$	Penalty cost ( $\geq$ 0) per time-unit for landing <i>after</i> the target time $T_i$

<span id="page-4-1"></span>**Table 2** List of input data

\n- \n
$$
a_{ik} =\n \begin{cases}\n 1 & \text{if aircraft } i \text{ is assigned to runway } k, \\
0 & \text{otherwise,} \\
1 & \text{if aircraft } i \text{ and } j \text{ are assigned to runway } k, \text{ and } i \text{ lands before } j, \\
0 & \text{otherwise,} \\
0 & \text{otherwise,}\n \end{cases}
$$
\n
\n- \n
$$
y_{ij} =\n \begin{cases}\n 1 & \text{if aircraft } i \text{ lands before } j, \\
0 & \text{otherwise,} \\
0 & \text{otherwise,}\n \end{cases}
$$
\n
\n

For each aircraft  $i \in A$ , the continuous variables are:

- $\bullet$   $t_i$  : landing time
- $t_i^-$ ,  $t_i^+$ : deviations from the target landing time  $T_i$  (before and after  $T_i$ , respectively).

### *3.3 MIP Model*

Our objective minimizes the total deviation cost from target times  $(T<sub>i</sub>)$  which is more general than minimizing only the total schedule delay. The complete model is given by  $(1)$ – $(12)$ 

$$
\min_{\delta, y, a, t} \sum_{i \in \mathcal{A}} c_i^{-} \overbrace{\max(0, T_i - t_i)}^{t_i^-} + c_i^+ \overbrace{\max(0, t_i - T_i)}^{t_i^+} \tag{1}
$$

$$
t_i = T_i - t_i^- + t_i^+ \qquad i \in \mathcal{A}
$$
 (2)

<span id="page-5-4"></span><span id="page-5-3"></span><span id="page-5-2"></span><span id="page-5-0"></span>
$$
E_i \le t_i \le L_i \qquad i \in \mathcal{A} \tag{3}
$$

<span id="page-5-5"></span>
$$
y_{ij} + y_{ji} = 1 \qquad i, j \in \mathcal{A}: i < j \tag{4}
$$

<span id="page-5-6"></span>
$$
\sum_{k \in \mathcal{K}} a_{ik} = 1 \qquad i \in \mathcal{A} \tag{5}
$$

<span id="page-5-7"></span>
$$
\sum_{k \in \mathcal{K}} \delta_{ijk} + \delta_{jik} \le 1 \qquad i, j \in \mathcal{A}: i < j \tag{6}
$$

<span id="page-5-8"></span>
$$
\delta_{ijk} + \delta_{jik} \ge a_{ik} + a_{jk} - 1 \quad i, j \in \mathcal{A} : i < j, k \in \mathcal{K} \tag{7}
$$

<span id="page-5-9"></span>
$$
2(\delta_{ijk} + \delta_{jik}) \le a_{ik} + a_{jk} \ \ i, j \in \mathcal{A} : i < j, k \in \mathcal{K} \tag{8}
$$

<span id="page-5-10"></span>
$$
t_j \ge t_i - M_1(1 - y_{ij}) \qquad i, j \in \mathcal{A} : i \ne j \tag{9}
$$

<span id="page-5-11"></span>
$$
t_j \ge t_i + S_{ij} - M(1 - \delta_{ijk}) \quad i, j \in \mathcal{A} : i \ne j \tag{10}
$$

<span id="page-5-1"></span>
$$
i - m \le N - \sum_{j \in A, j \ne i} y_{ij} \le i + m \quad i \in \mathcal{A}
$$
 (11)

$$
\delta_{ijk}, y_{ij}, a_{ik} \in \{0, 1\} \quad i, j \in \mathcal{A} : i \neq j, k \in \mathcal{K}
$$
 (12)

In the above formulation, constraints [\(2\)](#page-5-2) are introduced to linearize the objective function; constraints [\(3\)](#page-5-3) represent the time window restrictions; constraints [\(4\)](#page-5-4) enforce the order precedence relationship between flights *i* and *j* at landing; con-

straints [\(5\)](#page-5-5) ensure that an aircraft is assigned to exactly one runway; constraints [\(6\)](#page-5-6) enforce the order precedence relationship between flights landing on the same runway; constraints [\(7\)](#page-5-7) and [\(8\)](#page-5-8) enforce the logical relationship between δ*ijk* and  $a_{ik}$ ; constraints [\(9\)](#page-5-9) relates precedence relationships between landings to landing times; constraints [\(10\)](#page-5-10) ensure the separation requirements between aircraft landing at the same runway; constraints  $(11)$  impose the CPS constraint, and constraints  $(12)$ enforce the binary restrictions of our discrete variables.

Before reporting numerical results obtained with this formulation, we shall first present a novel alternate methodology to solve the ALP, since solving directly the MIP leads to redhibitory computation times, as we shall show in Section [5.](#page-8-0)

#### <span id="page-6-0"></span>**4 Optimistic Planning**

The dynamic nature of the ALP requires air-traffic controllers to make quick but good decisions; the computation time of any solution is thereby a critical issue. Given the complexity of the problem, the computation time to find an optimal solution either with our MIP model or with other exact approaches is unsuited for real-time applications. Therefore, we introduce a novel heuristic search approach based on the *Optimistic Planning* (OP) paradigm [\[23,](#page-15-8) [26](#page-15-11)], capable of computing solutions that do not deviate too much from the FCFS solution sequence and that are relatively close to optimal solutions, within an acceptable computational time.

Our approach models the ALP as an environment defined by *states*, *transitions*, *actions*, and *costs* where:

- each *state* denoted *x*, is a partition  $(I, \overline{I})$  of the set of aircraft, where  $\overline{I}$  is the (ordered) set of aircraft that have already landed, and *I* is the set of aircraft that have not landed yet.
- each *action* denoted *u* is an aircraft index  $i \in I$  that we decide to land, while satisfying the CPS constraints.
- each *transition* is defined as follows. If we execute action  $u = i \in I$  from a given state  $x = (I, I)$ , then the system generates the unique next state  $x' = (I', I')$ , where  $I' = I \setminus \{i\}$ , and  $\overline{I'} = \overline{I} \cup \{i\}$  (aircraft *i* landed).
- when the environment transits from the state  $x$  to the new state  $x'$  defined above, the estimated value *c* (*cost*) of the the new state is defined by

<span id="page-6-1"></span>
$$
c = f(\overline{I}') + g(I'),\tag{13}
$$

where  $f(I')$  is the delay cost of the (landed) sequence  $I'$ . Indeed, aircraft in the set  $\bar{I}$  are already sequenced. Thus, computing the landing times for aircraft in this



<span id="page-7-1"></span>**Fig. 2** An OP tree illustration for  $|A| = 4$  and  $m = 1$ 

sequence is straightforward, and  $f(\bar{I})$  is simply the weighted <sup>[1](#page-7-0)</sup> sum of aircraft delays.  $g(I')$  is a function that estimates the lowest cost among all sequences obtained from *I'* that satisfy the CPS constraints. In our numerical experiments, the FCFS rule is chosen as the estimation heuristic *g*, i.e.  $m = 0$ .

*Optimistic Planning* is the method that incrementally explores this search tree so as to identify an optimal branch as quickly as possible. Figure [2](#page-7-1) illustrates an example of this tree for 4 aircraft  $(A = \{1, 2, 3, 4\})$ , and a maximum position shifting of 1  $(m = 1)$ . Nodes are labeled by states, arcs are labeled by actions and costs. Near the nodes, the update process of the two sets  $\bar{I}$  and  $I$  and the estimated costs are highlighted. Remark that the values of the costs in this figure are randomly chosen for the sake of this illustration.

The algorithm starts from the initial state where the set  $\overline{I}$  is empty, and  $I = A$  (all aircraft available to land). At each iteration, its main loop seeks to determine which aircraft to land based on the optimistic evaluation  $c$ , and it updates  $\hat{I}$  by adding this aircraft, until a stopping criteria is met, i.e., all aircraft are landed or a time limit is reached. Only actions that satisfy the operational constraints are available in a given state.

<span id="page-7-0"></span><sup>&</sup>lt;sup>1</sup>These individual weights are provided with the data  $(e.g.,$  delay cost in Table [3\)](#page-9-0).

### <span id="page-8-0"></span>**5 Results and Discussion**

In this section we report the computational results of the MIP formulation  $(1)$ – $(12)$ and of our Optimistic Planning approach. Experiments are run on a personal computer under GNU/Linux operating system, processor Intel(R) Core(TM) i7-4700M with 8 GB of RAM. The MIP model was implemented in DoCplex, and solved using IBM CPLEX (version 12.8). Before reporting the computational results, we first present the test instances used in this paper.

#### *5.1 Test Instances*

Our test instances are generated from data sets from a benchmark test-problem set under construction at ENAC, obtained from real traffic in Paris-Orly Airport, that features two runways (06/24 and 08/26 as shown in Fig. [3\)](#page-8-1), which are considered independent (runway 02/20 is rarely used for commercial traffic).

The test-problem sets are constructed from two traffic days obtained from the OpenSky Network [\[27\]](#page-15-12): one in July 2018 containing mostly data about landed aircraft on runway 06/24, and one in April 2019 containing data about landed aircraft on runway 08/26. We merge these two traffic days and artificially add light aircraft to obtain larger and also more congested data sets.

We construct four data sets of 40 flights, named alp\_40\_1.txt, alp\_40\_2. txt, alp\_40\_3.txt, and alp\_40\_4.txt. They contain data about aircraft whose scheduled time of arrival (sta) lies between 07:00–08:10, 11:00–12:30, 15:00– 16:10, and 19:00–20:00. These data sets are available at [\[28](#page-15-13)]. A test instance of size |*A*| is obtained by considering the first |*A*| lines of data from one of these data sets.

Table [3](#page-9-0) shows an example from [\[28](#page-15-13)], which is the most congested data set among the four, named  $a1p_40_4$ .txt. In Table [3,](#page-9-0) the fourth and fifth columns, denoted "sta" and "sta\_s", indicate the scheduled time of arrival in HH:MM:SS format and in seconds respectively. The sixth column displays the delay cost per time unit of each aircraft, that we computed following a similar approach to that used in [\[8](#page-14-9)].

<span id="page-8-1"></span>**Fig. 3** A representation of Orly runways



Index	mdl	category	sta	sta_s	Delay cost
$\mathbf{1}$	<b>B738</b>	Medium	19:00:00	68,400	7
$\overline{c}$	A320	Medium	19:00:00	68,400	5
3	<b>B738</b>	Medium	19:00:00	68,400	7
4	$\overline{\phantom{0}}$	Light	19:00:00	68,400	$\mathbf{1}$
5	<b>B744</b>	Heavy	9:00:00	68,400	22
6	<b>B737</b>	Medium	19:05:00	68,700	6
7	A320	Medium	19:05:00	68,700	5
8	<b>B738</b>	Medium	19:05:00	68,700	7
9	<b>B738</b>	Medium	19:05:00	68,700	7
10		Light	19:10:00	69,000	$\mathbf{1}$
11	A319	Medium	19:10:00	69,000	$\overline{4}$
12	AT43	Medium	19:10:00	69,000	$\mathbf{1}$
13	A320	Medium	19:10:00	69,000	5
14	A320	Medium	19:10:00	69,000	5
15	B744	Heavy	19:10:00	69,000	22
16	A320	Medium	19:15:00	69,300	5
17	A321	Medium	19:15:00	69,300	7
18	<b>B738</b>	Medium	19:20:00	69,600	$\overline{7}$
19	A320	Medium	19:20:00	69,600	5
20	A318	Medium	19:25:00	69,900	3
21	AT45	Medium	19:25:00	69,900	$\,1$
22	A320	Medium	19:25:00	69,900	5
23	<b>CRJX</b>	Medium	19:25:00	69,900	$\overline{c}$
24	E145	Medium	19:30:00	70,200	$\mathbf{1}$
25	A319	Medium	19:35:00	70,500	$\overline{4}$
26	AT45	Medium	19:35:00	70,500	$\mathbf{1}$
27	A320	Medium	19:35:00	70,500	5
28		Light	19:35:00	70,500	$\mathbf{1}$
29	<b>B744</b>	Heavy	19:35:00	70,500	22
30	CRJ7	Medium	19:40:00	70,800	3
31	A320	Medium	19:40:00	70,800	5
32	<b>CRJX</b>	Medium	19:40:00	70,800	$\overline{c}$
33	<b>B738</b>	Medium	19:45:00	71,100	$\overline{7}$
34	E145	Medium	19:45:00	71,100	$\mathbf{1}$
35	<b>B744</b>	Heavy	19:50:00	71,400	22
36	$\overline{a}$	Light	19:50:00	71,400	$\mathbf{1}$
37	A321	Medium	19:50:00	71,400	7
38	<b>CRJX</b>	Medium	19:50:00	71,400	$\overline{c}$
39	A319	Medium	19:50:00	71,400	$\overline{4}$
40	A320	Medium	19:55:00	71,700	5

<span id="page-9-0"></span>**Table 3** Example of a data-set features with  $|A| = 40$  aircraft

In particular, the delay cost for each aircraft is obtained by multiplying the aircraft average pax number (in hundreds of seats) by its fuel consumption (in ton/hour), then round the results to the nearest integer. These individual delay costs are used as weights to compute the total cost of a given fixed sequence of aircraft. In particular, they are used in the computation of  $c$  from Eq.  $(13)$ , as well as in the computation of % improv in Eq.  $(14)$ .

#### *5.2 Computational Results*

We first report results obtained from implementing our MIP model involving a single runway ( $|K| = 1$ ), for different values of the maximum position shifting  $m = 2, \ldots, 6$ . Figure [4](#page-10-0) illustrates the evolution of the computation time in seconds for each value of *m* and for a set of 8 test instances of various sizes  $|A| = 16, 18, \ldots, 30$ , obtained by simply considering the first  $|\mathcal{A}|$  lines of the data set alp  $\_40\_4$ .txt, presented in Table [3.](#page-9-0) We impose a time limit of 1800 s (30 min) in CPLEX.

Figure [4](#page-10-0) exhibits the expected exponential growth of the computing time with the size of the instance,  $|A|$ , and with increasing values of *m*, (recall that the ALP is an NP-hard problem). The saturation effect than can be observed is simply due to our time limit.

Table [4](#page-11-1) reports the performance of the MIP model on various test instances, obtained this time from the four data sets of  $[28]$  $[28]$ , by considering the first  $|\mathcal{A}|$  lines of each of the four data sets. Results for each instance size are averaged over the four tests. Throughout Table [4,](#page-11-1) column " $|\mathcal{A}|$ " represents the size of the instance, column "*m*" shows the value of the maximum position shift parameter, column "% improv (MIP)" displays the percentage improvement of the MIP approach, and the



<span id="page-10-0"></span>**Fig. 4** Computational time of the MIP approach for different maximum position shifting values

$-$ $ \mathcal{A} $	m	%improv (MIP)	cpu (s)
$18\,$	$\sqrt{2}$	$22\,$	1.3
	$\overline{3}$	34	12
$20\,$	$\sqrt{2}$	21	$\overline{\mathbf{3}}$
	$\overline{3}$	32	$17\,$
$22\,$	$\sqrt{2}$	$28\,$	3
	$\overline{\mathbf{3}}$	37	26
$24\,$	$\overline{c}$	26	$\overline{4}$
	$\overline{\mathbf{3}}$	37	40
$26\,$	$\overline{c}$	25	5
		36	45
$28\,$	$\sqrt{2}$	26	8
	$\overline{\mathbf{3}}$	35	68
$30\,$	$\sqrt{2}$	27	$30\,$
	$\overline{3}$	37	164
$32\,$	$\sqrt{2}$	26	46
	$\overline{\mathbf{3}}$	37	410
$34\,$	$\sqrt{2}$	25	$70\,$
	$\overline{\mathbf{3}}$	36	550
36	$\sqrt{2}$	24	208
	$\overline{\mathbf{3}}$	34	974
$38\,$	$\sqrt{2}$	24	482
	$\overline{3}$	35	1110
$40\,$	$\sqrt{2}$	24	1090
	$\overline{3}$	33	1400

<span id="page-11-1"></span>**Table 4** MIP-approach performance on different instances from the four data sets [\[28](#page-15-13)]

last column reports the computing time in seconds. The percentage of improvement obtained by a method M (here  $M = MIP$ ) is computed with respect to the FCFS solution as:

<span id="page-11-0"></span>
$$
\%improv (M) = \frac{C_{FCFS} - C_M}{C_{FCFS}} \times 100,\tag{14}
$$

where  $C_{\text{FCFS}}$  and  $C_{\text{M}}$  are the cost of the FCFS sequence and that of the solution provided by the method M, respectively.

It can be concluded from Table [4](#page-11-1) that significant improvements can be obtained with the MIP approach, starting from  $m = 3$ , but this requires large computation times, which make it non-adapted to the dynamic nature of our problem, especially since the complexity scales exponentially with the number of aircraft, and since future ATM systems will have to deal with very large ALP instances.

$ \mathcal{A} $	% improv (OP) $m = 2$			% improv (OP) $m = 3$		
	2s	5s	15s	2s	5s	15s
18	12	12	12	$\overline{2}$	15	15
20	18	18	19	10	23	24
22	23	22	21	19	32	33
24	19	25	26	22	26	31
26	14	19	20	11	24	31
28	15	16	24	9	23	29
30	14	18	24	$\,8\,$	21	36
32	14	18	26	$\sqrt{2}$	22	37
34	14	12	25	6	23	$28\,$
36	13	13	25	$\overline{7}$	24	24
38	12	13	26	8	23	26
40	12	13	25	$\tau$	22	26
$\min$	12	12	12	$\mathbf{2}$	15	15
max	23	25	26	$22\,$	32	37
avg	15	16	20	28	23	29

<span id="page-12-0"></span>**Table 5** Algorithm performance details (average improvement) for one runway

We report the results of the OP approach on different instance sizes  $|A|$  =  $18, 20, \ldots, 40$  involving a single runway (Table [5\)](#page-12-0), and imposing each time a limited computational time-budget of  $\{2, 5, 15\}$  s. For each instance size, we evaluate the approach on different instances of the same size—generated from the four data sets [\[28\]](#page-15-13)—and report the average, minimum and maximum improvement over the FCFS solution.

Table [5](#page-12-0) shows the average percentage improvement of the total cost given by Eq. [\(14\)](#page-11-0) for two values of the maximum position shifting parameter,  $m = 2$  and 3. It can be observed that large instances of sizes greater than 30 can benefit from a significant improvement (on average more than 21%) for a maximum position shifting of  $m = 3$ , within only 5 seconds.

Finally, Table [6](#page-13-1) reports an example of the solution provided by our OP approach on the instance given in Table [3](#page-9-0) with  $|\mathcal{A}| = 22$  aircraft, and imposing a computational budget of 2 s. The first column displays the aircraft position in the solution sequence. The "index" columns corresponds to the aircraft index from Table [3](#page-9-0) occupying each position. The "landing" columns report the landing times. For this example, the percentage improvement of the FCFS sequence is 35%. Moreover, remark that for this scenario, the utilization of the runway in terms of the makespan i.e. length of the sequence is also optimized. Indeed, the last landing in the sequence for the FCFS is 7:41:20 while the last landing with our approach is at 7:40:45.

Position	<b>FCFS</b>		<b>OP</b>		
	Index	Landing	Index	Landing	
$\mathbf{1}$	1	7:00:00	3	7:00:00	
$\sqrt{2}$	$\sqrt{2}$	7:03:16	5	7:01:36	
$\mathfrak{Z}$	3	7:04:16	$\overline{4}$	7:03:12	
$\overline{4}$	$\overline{4}$	7:05:52	$\mathbf{1}$	7:04:48	
5	5	7:07:28	$\overline{c}$	7:08:04	
6	6	7:10:05	6	7:10:00	
$\overline{7}$	$\overline{7}$	7:15:00	8	7:15:00	
$\,8\,$	8	7:16:00	9	7:16:36	
9	9	7:17:36	$\overline{7}$	7:19:13	
10	10	7:20:13	10	7:20:22	
11	11	7:21:22	12	7:25:00	
12	12	7:25:00	13	7:26:09	
13	13	7:26:09	11	7:27:18	
14	14	7:30:00	15	7:30:00	
15	15	7:33:16	14	7:31:00	
16	16	7:34:25	17	7:33:37	
17	17	7:35:34	20	7:35:00	
18	18	7:36:43	21	7:36:09	
19	19	7:37:52	16	7:37:18	
20	20	7:39:01	18	7:38:27	
21	21	7:40:10	19	7:39:36	
22	22	7:41:20	$22\,$	7:40:45	

<span id="page-13-1"></span>**Table 6** Example of solutions provided by FCFS and by our optimistic approach

Our computational experiments on the MIP formulation and the heuristic search approach show that the latter is more suited and more promising to solve the ALP with large congested instances, since it can provide good solutions in short computation time.

## <span id="page-13-0"></span>**6 Conclusion**

Runway sequence optimization is an ongoing challenge for researchers and controllers due to the dynamic nature of the problem and to the various operational constraints that must be taken into consideration. In this work, we proposed an exact approach (MIP) to solve the deterministic case of the ALP as well as a novel method based on Optimistic Planning to solve medium and large challenging instances.

Our computational experiments show that computation times for our MIP model (and other exact approaches) are very high for large congested instances, which make them unsuited to the dynamic nature of ALP. With the constrained-position shifting restrictions, the complexity of the problem can be reduced, but the problem remains untractable for increasing values of the maximum position-shifting parameter and the number of aircraft. On the other hand, our proposed heuristic search approach based on optimistic planning is able to find good quality solutions that significantly improve the FCFS sequence within a limited time budget, making it a promising method for solving the ALP in real time.

In future studies, we plan to extend our heuristic search approach to the multiplerunway case. Furthermore, instead of using the First-Come First-Served rule as the estimation heuristic, we are planning to construct more accurate functions to help the search to explore the most promising nodes first, so as to identify an optimal branch faster. Taking into consideration uncertainty on the arrival times is also a future track of research, since the solutions of our two deterministic approaches (MIP and the heuristic search) cannot be straightforwardly applied in the presence of uncertainty.

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