Integral Operators on a Class of Analytic Functions

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Abstract A class $SD(\alpha)$, $\alpha \geq 0$, of analytic functions is considered and functions in this class are shown to be univalent and starlike of order $(1 - \frac{1}{\alpha})$, for $\alpha \geq 1$. For functions $f(z)$ to belong to the class $SD(\alpha)$, a sufficient condition is obtained. For functions $f(z)$ satisfying this condition, the functions $F(z)$ defined by several integral operators on $f(z)$ are shown to be in the class $SD(\alpha)$. For a hypergeometric function to belong to the class $SD(\alpha)$, a sufficiency condition is also obtained.

Keywords Analytic functions · Starlike function of order α · Integral operators

1 Introduction

Let A be the class of analytic functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

defined on the unit disk $\Delta = \{z \in \mathcal{C} : |z| < 1\}$. Let $S \subset A$ be the class of analytic univalent functions. Ruscheweyh [22] considered a subclass $D \subset S$ consisting of

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convex functions f for which $Re\{f'(z)\} \geq |zf''(z)|$. Motivated by the class D, a family $UCD(\alpha), \alpha \geq 0$, was introduced in [21] connecting various subclasses of convex functions, especially, the subclass (UCV) of uniformly convex functions (see, for example, [2] for an excellent survey on UCV). A function f given by (1) is in $UCD(\alpha)$ if $Re\{f'(z)\} \geq \alpha|zf''(z)|,\ z \in \Delta, \alpha \geq 0$. A family $SD(\alpha)$ related to $UCD(\alpha)$ was introduced in [20]. Recently, this class has been studied in [9, 25]. Also several authors have considered different integral operators on functions in S and its subclasses (see, for example, [1, 4, 5, 7, 11–14, 16, 17, 26]). In this paper, functions in the class $SD(\alpha)$ are shown to be univalent and starlike of order $(1 - \frac{1}{\alpha})$, for $\alpha \geq 1$. For a function f to belong to the class $SD(\alpha)$ [20], a sufficient condition is obtained. For functions f satisfying this condition, it is shown that the functions $F(z)$ defined by various integral operators on $f(z)$ belong to the class $SD(\alpha)$. Also, for a hypergeometric function to belong to the class $SD(\alpha)$, a condition of sufficiency is obtained.

2 The Class $SD(\alpha)$

In [20, 21], a class $UCD(\alpha),\ \alpha \geq 0$, consisting of functions satisfying the condition $Ref'(z) \geq \alpha|zf''(z)|,\ z \in \Delta$ was introduced and various properties of this class were obtained. Subsequently, this class has been considered by several authors [4, 5, 10, 24] in the context of different studies. A related class $SD(\alpha)$ motivated by the class $UCD(\alpha)$ was considered in [21], which is recalled here.

Definition 1 [21] A function f of the form (1) is said to be in the class $SD(\alpha)$ if

$$Re\left\{\frac{f(z)}{z}\right\} \geq \alpha\left|f'(z) - \frac{f(z)}{z}\right| \tag{2}$$

for $\alpha \geq 0$.

We note that $f \in UCD(\alpha)$ if and only if $zf'(z) \in SD(\alpha)$.

Remark 1 Chichra [6, pp. 41 and 42] has considered a class $\mathcal{G}(\alpha)$ of analytic functions f of the form (1) satisfying the condition

$$Re\left\{(1-\alpha)\frac{f(z)}{z} + \alpha f'(z)\right\} \geq 0 \tag{3}$$

for $\alpha \geq 0$ and has shown that functions in $\mathcal{G}(\alpha)$ are univalent, if $\alpha \geq 1$. Hence the functions f in $SD(\alpha)$ are univalent for $\alpha \geq 1$, since $Re\left\{\frac{f(z)}{z}\right\} \geq -\alpha Re\left(f'(z) - \frac{f(z)}{z}\right)$ if $f \in SD(\alpha)$ so that $Re\left\{(1-\alpha)\frac{f(z)}{z} + \alpha f'(z)\right\} \geq 0$.

The class $S^*(\alpha)$ of starlike functions of order α [19] is well-known and consists of functions f satisfying the analytic condition $Re\{\frac{zf'(z)}{f(z)}\} > \alpha$, for

$0 \leq \alpha < 1,\ z \in \Delta$. The class $SD(\alpha)$ is now related with the class $S^*(\alpha)$ in the following theorem.

Theorem 1 $SD(\alpha) \subseteq S^*(1 - \frac{1}{\alpha}), \alpha \geq 1$.

Proof Let $f \in SD(\alpha),\ \alpha \geq 1$. Then f is univalent and

$$\left|\frac{f(z)}{z}\right| \geq Re\left\{\frac{f(z)}{z}\right\} \geq \alpha\left|f'(z) - \frac{f(z)}{z}\right| = \alpha\left|\frac{f(z)}{z}\right|\left|\frac{zf'(z)}{f(z)} - 1\right|$$

so that

$$\left|\frac{zf'(z)}{f(z)} - 1\right| \leq \frac{1}{\alpha}.$$

Now

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} = Re\left\{\left(\frac{zf'(z)}{f(z)} - 1\right) + 1\right\} \geq 1 - \left|\frac{zf'(z)}{f(z)} - 1\right| \geq 1 - \frac{1}{\alpha}.$$

Hence $f \in S^*\left(1 - \frac{1}{\alpha}\right)$.

The following theorem gives a sufficient condition for f of the form (1) to be in the class $SD(\alpha)$.

Theorem 2 *A function f of the form (1) is in the class $SD(\alpha)$ if*

$$\sum_{n=2}^{\infty}[1 + \alpha(n-1)]|a_n| \leq 1 \tag{4}$$

Proof For $|z| < 1$,

$$Re\left\{\frac{f(z)}{z}\right\} - \alpha\left|f'(z) - \frac{f(z)}{z}\right| \geq 1 - \left|\frac{f(z)}{z} - 1\right| - \alpha\left|f'(z) - \frac{f(z)}{z}\right|$$

$$\geq 1 - \sum_{n=2}^{\infty}|a_n| - \alpha\sum_{n=2}^{\infty}(n-1)|a_n| = 1 - \sum_{n=2}^{\infty}[1 + \alpha(n-1)]|a_n| \geq 0$$

by (4). Hence

$$Re\left\{\frac{f(z)}{z}\right\} - \alpha\left|f'(z) - \frac{f(z)}{z}\right| \geq 0$$

which implies that $f \in SD(\alpha)$.

Theorem 3 *Let $f \in A$ be given by (1) and satisfy the condition (4). Then the function*

$$F(z) = \frac{1+\gamma}{z^{\gamma}} \int_0^z f(t)t^{\gamma-1}dt, \ \gamma \geq -1$$

defined by the Bernardi operator belongs to $SD(\alpha)$ for all $\alpha \geq 0$.

Proof Since $f \in A$, $F(z) = z + b_2 z^2 + \cdots$ where $b_n = \frac{\gamma+1}{\gamma+n} a_n$, $n \geq 2$. Now

$$\sum_{n=2}^{\infty} [1 + \alpha(n-1)]|b_n| = \sum_{n=2}^{\infty} [1 + \alpha(n-1)] \left(\frac{\gamma+1}{\gamma+n}\right) |a_n|$$

$$< \sum_{n=2}^{\infty} [1 + \alpha(n-1)]|a_n| < 1, \text{ since } \gamma + 1 < \gamma + n.$$

Thus, by Theorem 2, $F \in SD(\alpha)$, for all $\alpha \geq 0$.

On substituting $\gamma = 0$ and $\gamma = 1$ in the Bernardi operator, we obtain the Alexander transformation and Libera operator, respectively, and so we have the following Corollary of Theorem 3.

Corollary 1 *Let $f \in A$ be given by (1) and satisfy the condition (4). Then*

1. *the function*

$$F(z) = \int_0^z \frac{f(t)}{t} dt$$

   defined by the Alexander transformation belongs to $SD(\alpha)$, for all $\alpha \geq 0$.
2. *the function*

$$F(z) = \frac{2}{z} \int_0^z f(t)dt$$

   defined by the Libera operator belongs to $SD(\alpha)$, for all $\alpha \geq 0$.

Theorem 4 *Let $f \in SD(\alpha)$ and*

$$F(z) = \frac{1+\gamma}{z^{\gamma}} \int_0^z f(t)t^{\gamma-1}dt. \tag{5}$$

Then

$$|\gamma F(z) + zF'(z)| \geq \alpha |z^2 F''(z) + \gamma z F'(z) - \gamma F(z)| \tag{6}$$

Proof By (5),

$$f(z) = \frac{\gamma}{\gamma+1} F(z) + \frac{zF'(z)}{1+\gamma} \tag{7}$$

and

$$f'(z) = F'(z) + \frac{1}{1+\gamma} zF''(z) \tag{8}$$

Since $f \in SD(\alpha)$,

$$Re\left\{\frac{f(z)}{z}\right\} \geq \alpha\left|f'(z) - \frac{f(z)}{z}\right|$$

which implies

$$\left|\frac{f(z)}{z}\right| \geq \alpha\left|f'(z) - \frac{f(z)}{z}\right| \tag{9}$$

Using (7) and (8) in (9), we obtain

$$\left|\frac{\gamma}{1+\gamma}\frac{F(z)}{z} + \frac{F'(z)}{1+\gamma}\right| \geq \alpha\left|\frac{\gamma F'(z)}{1+\gamma} + \frac{zF''(z)}{1+\gamma} - \frac{\gamma F(z)}{1+\gamma}\right|$$

which is equivalent to

$$|\gamma F(z) + zF'(z)| \geq \alpha|z^2F''(z) + \gamma zF'(z) - \gamma F(z)|$$

Corollary 2 *If $f \in SD(\alpha)$, then*

$$\left|\left(log\frac{f(z)}{z}\right)'\right| \leq \frac{1}{\alpha|z|} \; for \; all \; z \in \Delta.$$

Proof By Theorem 4, (6) can be written as

$$\left|\frac{z^2F''(z) + \gamma zF'(z) - \gamma F(z)}{\gamma F(z) + zF'(z)}\right| \leq \frac{1}{\alpha}$$

In terms of $f(z)$, the above inequality becomes

$$\left|\frac{f'(z)}{f(z)} - \frac{1}{z}\right| \leq \frac{1}{\alpha|z|}$$

which implies

$$\left|\left(log\frac{f(z)}{z}\right)'\right| \leq \frac{1}{\alpha|z|} \; for \; all \; z \in \Delta.$$

Theorem 5 *Let $f \in A$ be given by (1) and satisfy the condition (4). Then the function*

$$F(z) = \frac{a^\lambda}{\Gamma(\lambda)}\int_0^1 t^{a-2}\left(log\frac{1}{t}\right)^{\lambda-1} f(tz)dt, a > 0, \lambda \geq 0$$

defined by the Komatu operator belongs to $SD(\alpha)$, for all $\alpha \geq 0$.

Proof Here $F(z) = z + b_2z^2 + \cdots$ where $b_n = \left(\frac{a}{a+n-1}\right)^{\lambda} a_n, a > 0, \lambda \geq 0.$
Now

$$\sum_{n=2}^{\infty}[1+\alpha(n-1)]|b_n| = \sum_{n=2}^{\infty}[1+\alpha(n-1)]\left|\frac{a}{a+n-1}\right|^{\lambda}|a_n|$$

$$< \sum_{n=2}^{\infty}[1+\alpha(n-1)]|a_n| < 1, \forall\alpha \geq 0,$$

since $a < a+n-1 \; for \; n \geq 2$. Thus by Theorem 2, $F \in SD(\alpha)$ for all $\alpha \geq 0$.

Theorem 6 *Let $f \in A$ be given by (1) and satisfy the condition (4). Then the function*

$$F(z) = \frac{2^{\lambda}}{z\Gamma(\lambda)}\int_0^z \left(log\frac{z}{t}\right)^{\lambda-1} f(t)dt, \alpha > 0$$

defined by the Jung-Kim-Srivastava operator I belongs to $SD(\alpha)$ for all $\alpha \geq 0$.

Proof Here $F(z) = z + b_2z^2 + \cdots$ where $b_n = \left(\frac{2}{n+1}\right)^{\lambda} a_n, \alpha > 0.$
Now

$$\sum_{n=2}^{\infty}[1+\alpha(n-1)]|b_n| = \sum_{n=2}^{\infty}[1+\alpha(n-1)]\left|\frac{2}{n+1}\right|^{\lambda}|a_n|$$

$$< \sum_{n=2}^{\infty}[1+\alpha(n-1)]|a_n| < 1, \forall\alpha \geq 0,$$

since $n \geq 2$. Thus by Theorem 2, $F \in SD(\alpha)$ for all $\alpha \geq 0$.

Theorem 7 *Let $f \in A$ be given by (1) and satisfy the condition (4). Then the function*

$$F(z) = \frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+1)\Gamma(\beta+1)}\frac{\alpha}{z^{\beta}}\int_0^z \left(1-\frac{t}{z}\right)^{\alpha-1} t^{\beta-1} f(t)dt, \beta > 0,$$

defined by the Jung-Kim-Srivastava operator II belongs to $SD(\alpha)$ for all $\alpha \geq 0$.

Proof Here $F(z) = z + b_2z^2 + \cdots$ where $b_n = \frac{\Gamma(\beta+n)\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+\beta+n)\Gamma(\beta+1)}a_n$.
Now

$$\sum_{n=2}^{\infty}[1+\alpha(n-1)]|b_n| = \sum_{n=2}^{\infty}[1+\alpha(n-1)]\frac{\Gamma(\beta+n)\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+\beta+n)\Gamma(\beta+1)}|a_n|$$

$$= \sum_{n=2}^{\infty}[1+\alpha(n-1)]\frac{\Gamma(\beta+n)}{\Gamma(\beta)}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+n)}\frac{\alpha+\beta}{\beta}$$

$$< \sum_{n=2}^{\infty}[1+\alpha(n-1)]|a_n|\frac{(\beta+n-1)\cdots(\beta+1)\beta}{(\alpha+\beta+n-1)\cdots(\alpha+\beta)}\frac{\alpha+\beta}{\beta}$$

$$= \sum_{n=2}^{\infty}[1+\alpha(n-1)]|a_n|\frac{(\beta+n-1)(\beta+n-2)\cdots(\beta+1)}{(\alpha+\beta+n-1)(\alpha+\beta+n-2)\cdots(\alpha+\beta+1)}$$

$< 1,\ since\ \beta+i < \alpha+\beta+i,\ for\ i = 1, 2, 3, \cdots, n-1.$

Thus, by Theorem 2, $F \in SD(\alpha)$ for all $\alpha \geq 0$.

Several studies on the problem of deriving conditions for different forms of hypergeometric functions to belong to various subclasses of analytic functions in the unit disk have been done (see, for example, [3, 8, 15, 23, 24]). Here we consider the Gaussian hypergeometric function $F(\xi, \eta, \zeta; z)$ given by

$$F(\xi, \eta, \zeta; z) = \sum_{n=0}^{\infty}\frac{(\xi)_n(\eta)_n}{(\zeta)_n(1)_n}z^n,\ z \in \Delta \tag{10}$$

where ξ, η, ζ are complex numbers such that $\zeta \neq -n,\ n \in \{0, 1, 2, \cdots\}$, $(\xi)_0 = 1$, for $\xi \neq 0$ and for each positive integer n, $(\xi)_n = \xi(\xi+1)(\xi+2)\cdots(\xi+n-1)$ is the Pochhammer symbol. We derive a sufficient condition in terms of a hypergeometric inequality for $zF(\xi, \eta, \zeta; z)$ to belong to the class $SD(\alpha)$. We make use of the Gauss summation formula [18] given by

$$F(\xi, \eta, \zeta; 1) = \sum_{n=0}^{\infty}\frac{(\xi)_n(\eta)_n}{(\zeta)_n(1)_n} = \frac{\Gamma(\zeta-\xi-\eta)\Gamma(\zeta)}{\Gamma(\zeta-\xi)\Gamma(\zeta-\eta)}$$

if $Re(\zeta-\xi-\eta) > 0$.

Theorem 8 *Let ξ, η be two non-zero complex numbers and ζ be a real number such that $\zeta > |\xi| + |\eta| + 1$. Let $f \in A$ be of the form given by (1). Then $zF(\xi, \eta, \zeta; z) \in SD(\alpha)$ if the following hypergeometric inequality holds:*

$$\frac{\Gamma(\zeta-|\xi|-|\eta|-1)\Gamma(\zeta)}{\Gamma(\zeta-|\xi|)\Gamma(\zeta-|\eta|)}[(\zeta-|\xi|-|\eta|-1)+\alpha|\xi\eta|] < 2. \tag{11}$$

Proof In view of Theorem 2 and the series representation of $zF(\xi, \eta, \zeta; z)$ given by

$$zF(\xi, \eta, \zeta; z) = z + \sum_{n=2}^{\infty} \frac{(\xi)_{n-1}(\eta)_{n-1}}{(\zeta)_{n-1}(1)_{n-1}} z^n, \ z \in \Delta, \tag{12}$$

it is enough to prove that

$$S = \sum_{n=2}^{\infty} (1 + \alpha(n-1)) \left| \frac{(\xi)_{n-1}(\eta)_{n-1}}{(\zeta)_{n-1}(1)_{n-1}} \right| < 1. \tag{13}$$

Using the fact that $|(\xi)_n| \leq (|\xi|)_n$ and noticing that ζ is a positive real number, we have

$$\begin{aligned} S &\leq \sum_{n=2}^{\infty} (1 + \alpha(n-1)) \frac{(|\xi|)_{n-1}(|\eta|)_{n-1}}{(\zeta)_{n-1}(1)_{n-1}} \\ &= \sum_{n=2}^{\infty} \frac{(|\xi|)_{n-1}(|\eta|)_{n-1}}{(\zeta)_{n-1}(1)_{n-1}} + \alpha \sum_{n=2}^{\infty} (n-1) \frac{(|\xi|)_{n-1}(|\eta|)_{n-1}}{(\zeta)_{n-1}(1)_{n-1}} \\ &= \sum_{n=2}^{\infty} \frac{(|\xi|)_{n-1}(|\eta|)_{n-1}}{(\zeta)_{n-1}(1)_{n-1}} + \alpha \sum_{n=2}^{\infty} \frac{(|\xi|)_{n-1}(|\eta|)_{n-1}}{(\zeta)_{n-1}(1)_{n-2}} \end{aligned}$$

Thus using the property $(\xi)_n = \xi(1 + \xi)_{n-1}$, we have

$$\begin{aligned} S &\leq \sum_{n=2}^{\infty} \frac{(|\xi|)_{n-1}(|\eta|)_{n-1}}{(\zeta)_{n-1}(1)_{n-1}} + \alpha \frac{|\xi||\eta|}{\zeta} \sum_{n=2}^{\infty} \frac{(1 + |\xi|)_{n-2}(1 + |\eta|)_{n-2}}{(1 + \zeta)_{n-2}(1)_{n-2}} \\ &= F(|\xi|, |\eta|, \zeta; 1) - 1 + \alpha \frac{|\xi||\eta|}{\zeta} F(1 + |\xi|, 1 + |\eta|, 1 + \zeta; 1) \end{aligned}$$

An application of the Gauss summation formula in (9) yields the result.

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References

1. Ahuja, O.P.: Integral operators of certain univalent functions. Int. J. Math. Math. Sci. **8**, 653–662 (1985)
2. Ali, R.M., Ravichandran, V.: Uniformly convex and uniformly starlike functions. Ramanujan Math. Newslett. **21**(1), 16–30 (2011)
3. Aouf, M.K., Darwish, H.E.: Subclasses of analytic functions associated with the generalized hypergeometric function. Comput. Math. Appl. **57**, 356–366 (2009)
4. Breaz, D.: Integral operators on the UCD(α)-class. In: Proceedings of International Conference on Theory and Applications of Mathematics and Informatics (ICTAMI2003), Alba Lulia, pp. 61–65 (2003)
5. Breaz, D.: Integral operators on the TUCD(α)-class. In: Proceedings of International Conference on Theory and Applications of Mathematics and Informatics (ICTAMI2004), Thessaloniki, Greece, pp. 61–66 (2004)
6. Chichra, P.N.: New subclasses of the class of close-to-convex functions. Proc. American Math. Soc. **62**(1), 37–43 (1977)
7. Cho, N.E., Owa, S.: Integral operators preserving certain analytic functions. J. Math. Anal. Appl. **286**, 168–176 (2003)
8. Dziok, J., Srivastava, H.M.: Certain subclasses of analytic functions associated with the generalized hypergeometric function. Inte. Trans. Spec. Fun. **14**, 7–18 (2003)
9. Ezhilarasi, R., Sudharsan, T.V., Mohd, M.H., Subramanian, K.G.: Connections between certain subclasses of analytic univalent functions based on operators. J. Complex Anal. 5. Article ID 6104210 (2017). https://doi.org/10.1155/2017/6104210
10. Fadipe-Joseph, O.A., Oladipo, A.T., Ejieji, C.N.: Sufficient condition for a class of analytic and univalent functions of complex order. Acta Universitatis Apulensis **36**, 101–108 (2013)
11. Gordji, M.E., Ebadian, A., Aghalary, R., Darus, M.: Starlikeness and univalence criterion for non-linear operators. Math. Rep. **13**(63). No. 2, 141–149 (2011)
12. Li, J.L.: Some properties of two integral operators. Soochow J. Math. **25**(1), 91–96 (1999)
13. Liu, J.L.: Notes on Jung-Kim-Srivastava integral operator. J. Math. Anal. Appl. **294**, 96–103 (2004)
14. Miller, S.S., Mocanu, P.T.: Integral operators on certain classes of analytic functions. In: Univalent Functions, Fractional Calculus and their Applications, vol. 153–166. Halstead Press, Wiley (1989)
15. Murugusundaramoorthy, G., Magesh, N.: On certain subclasses of analytic functions associated with hypergeometric functions. Appl. Math. Lett. **24**, 494–500 (2011)
16. Noor, K.I., Noor, M.A.: On integral operators. J. Math. Anal. Appl. **238**, 341–352 (1999)
17. Ong, K.W., Tan, S.L., Tu, Y.E.: Integral operators and univalent functions. Tamkang J. Math. **43**, 215–221 (2012)
18. Rainville, E.D.: Special Functions. Chelsea Publishing Company, New York (1960)
19. Robertson, M.S.: On the theory of univalent functions. Annals Math. **37**, 374–408 (1936)
20. Rosy, T.: Studies on subclasses of starlike and convex functions. Ph.D. thesis, University of Madras (2001)
21. Rosy, T., Stephen, B.A., Subramanian, K.G., Silverman, H.: Classes of convex functions. Int. J. Math. Math. Sci. **23**(12), 819–825 (2000)
22. Ruscheweyh, S.: Extension of Szegö's theorem on the sections of univalent functions. SIAM J. Math. Anal. **19**(6), 1442–1449 (1988)
23. Silverman, H.: Starlike and convexity properties for hypergeometric functions. J. Math. Anal. Appl. **172**, 574–581 (1993)
24. Sivasubramanian, S., Sokol, J.: Hypergeometric transforms in certain classes of analytic functions. Math. Comput. Model. **54**, 3076–3082 (2011)
25. Varma, S.S., Rosy, T.: Certain properties of a subclass of univalent functions with finitely many fixed coefficients. Khayyam J. Math. **3**(1), 26–33 (2017)
26. Tehranchi, A.: Study on integral operators by using komato operator on a new class of univalent functions. J. Math. Res. **3** (2011)