Chapter 7
Predictions of Angular Observables for $\bar{B}_s \to K^*\ell\ell$ and $\bar{B} \to \rho\ell\ell$ in Standard Model

Bharti Kindra and Namit Mahajan

Abstract Exclusive semileptonic decays based on $b \to s$ transitions have been attracting a lot of attention as some angular observables deviate significantly from the Standard Model (SM) predictions in specific q^2 bins. B meson decays induced by other Flavor Changing Neutral Current (FCNC), $b \to d$, can also offer a probe to new physics with an additional sensitivity to the weak phase in Cabibbo–Kobayashi–Masakawa (CKM) matrix. We provide predictions for angular observables for $b \to d$ semileptonic transitions, namely $\bar{B}_s \to K^*\ell^+\ell^-$, $\bar{B}^0 \to \rho^0\ell^+\ell^-$, and their CP-conjugated modes including various non-factorizable corrections.

7.1 Introduction

Experimental evidence of new physics has been found in the channels involving FCNC $b \to s\ell^+\ell^-$ and charged current $b \to c\ell\nu$. However, the $b \to d$ counterpart of the weak decay, i.e., $b \to d\ell^+\ell^-$, has not caught much attention perhaps because of low branching ratio. The weak phases incorporate CKM matrix elements $\xi^i_q = V^*_{qi}V_{qb}$, where $q \in \{u, c, t\}$ and $i \in \{s, d\}$. For $b \to s\ell\ell$ transition, $\xi^s_{c,t} \sim \lambda^2$ and $\xi^s_u \sim \lambda^4$ where $\lambda = 0.22$. Since $u\bar{u}$ contribution introduces CKM phase which is negligible for $b \to s\ell\ell$, CP violating quantities are very small in SM. On the other hand, since $\xi^d_u \sim \xi^d_c \sim \xi^d_t \sim \lambda^4$ for $b \to d\ell\ell$, the B decays mediated through this transition allow for large CP violating quantities. Also, leading order contribution in this case is smaller than the leading contribution in $b \to s\ell\ell$ which makes it more sensitive to new particles and interactions. In this work, we focus on two such decay channels, $B_s \to \bar{K}^*\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$[1].

B. Kindra (✉) · N. Mahajan
Physical Research Laboratory, Ahmedabad, India
e-mail: Bharti.kindra04@gmail.com; bhrt.kndr56@gmail.com

B. Kindra
Indian Institute of Technology, Gandhinagar, India

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7.2 Decay Amplitude

We follow the effective Hamiltonian approach as used in [2] to write the Hamiltonian and decay amplitude. Th amplitude is written as a product of short-distance contributions through Wilson coefficients and long-distance contribution which is further expressed in terms of form factors,

$$\mathcal{M} = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{td}^* \Big\{ \Big[\langle V|\bar{d}\gamma^\mu (C_9^{\text{eff}} P_L) b|P\rangle - \frac{2m_b}{q^2} \langle V|\bar{d}\; i\; \sigma^{\mu\nu} q_\nu (C_7^{\text{eff}} P_R) b|P\rangle \Big] (\bar{\ell}\gamma_\mu \ell) + \langle V|\bar{d}\gamma^\mu (C_{10}^{\text{eff}} P_L) b|P\rangle (\bar{\ell}\gamma_\mu\gamma_5 \ell) - 16\pi^2 \frac{\bar{\ell}\gamma^\mu \ell}{q^2} \mathcal{H}_\mu^{\text{non-fac}} \Big\}. \quad (7.1)$$

Wilson coefficients ($C_i's$) are computed upto next-to-next-to leading order (NNLO) [3] and form factors are computed using the method of Light Cone Sum Rules (LCSR) and QCD lattice calculation [4]. $\mathcal{H}_\mu^{\text{non-fac}}$ represents the non-factorizable contribution of non-local hadronic matrix element. This results from four quark and chromomagnetic operators combined with virtual photon emission which then decays to lepton pair through electromagnetic interaction. These corrections are given in terms of hard-scattering kernels ($\mathcal{T}_a^q s$), where $a \in \{\perp, \|\}$ and $q \in \{u, c\}$, which are convoluted with B(B_S)-meson and $\rho(\bar{K}^*)$ distribution amplitudes. The non-factorizable corrections included here are spectator scattering $\mathcal{T}_a^{q,\text{spec}}$, weak annihilation $\mathcal{T}_a^{q,\text{WA}}$, and soft-gluon emission $\Delta C_9^{q,\text{soft}}$. These corrections have been computed in [5–7] except charm loop corrections corresponding to up quark in the loop. For present work, we are assuming that its contribution is less than 10% of C_9: $\Delta C_{9,u}^{\text{soft}} = a e^{i\theta}$; $|a| \in \{0, 0.5\}$, $\theta \in \{0, \pi\}$.

These corrections are then added to transversity amplitudes in the following way:

$$A_{\perp L,R}(q^2) = \sqrt{2\lambda}\, N \Big[2\frac{m_b}{q^2} (C_7^{\text{eff}} T_1(q^2) + \Delta T_\perp) + (C_9^{\text{eff}} \mp C_{10} + \Delta C_9^1(q^2)) \frac{V(q^2)}{M_B + M_V} \Big] \quad (7.2)$$

$$A_{\| L,R}(q^2) = -\sqrt{2} N (M_B^2 - M_V^2) \Big[2\frac{m_b}{q^2} (C_7^{\text{eff}} T_2(q^2) + 2\frac{E(q^2)}{M_B} \Delta T_\perp) + (C_9^{\text{eff}} \mp C_{10} + \Delta C_9^2(q^2)) \frac{A_1(q^2)}{M_B - M_V} \Big] \quad (7.3)$$

$$A_{0L,R}(q^2) = -\frac{N}{2M_V\sqrt{q^2}} \Big[2m_b \big((M_B^2 + 3M_V^2 - q^2)(C_7^{\text{eff}} T_2(q^2)) - \frac{\lambda}{M_B^2 - M_v^2} (C_7^{\text{eff}} T_3(q^2) + \Delta T_\|) \big) + (C_9^{\text{eff}} \mp C_{10} + \Delta C_9^3) \big((M_B^2 + M_V^2 - q^2)(M_B + M_V) A_1(q^2) - \frac{\lambda}{M_B + M_V} A_2(q^2) \big) \Big] \quad (7.4)$$

$$A_t(q^2) = \frac{N}{\sqrt{s}} \sqrt{\lambda} 2\, C_{10}\, A_0(q^2) \quad (7.5)$$

where,

$$\Delta T_\perp = \frac{\pi^2}{N_c}\frac{f_P f_{V,\perp}}{M_B}\frac{\alpha_s C_F}{4\pi}\int\frac{d\omega}{\omega}\Phi_{P,-}(\omega)\int_0^1 du\ \Phi_{V,\perp}(u)(T_\perp^{c,\text{spec}} + \frac{\xi_u}{\xi_t}(T_\perp^{u,\text{spec}})) \tag{7.6}$$

$$\Delta T_\parallel = \frac{\pi^2}{N_c}\frac{f_P f_{V,\parallel}}{M_B}\frac{M_V}{E}\sum_{\pm}\int\frac{d\omega}{\omega}\Phi_P(\omega)\int_0^1 du\ \Phi_{V,\parallel}(u)\big[T_\parallel^{c,WA} + \frac{\xi_u}{\xi_t}T_\parallel^{u,WA}$$
$$\frac{\alpha_s C_F}{4\pi}(T_\parallel^{c,\text{spec}} + \frac{\xi_u}{\xi_t}T_\parallel^{,\text{spec}})\big] \tag{7.7}$$

$$\Delta C_9^i = \Delta C_{9,c}^{i,soft} + \Delta C_{9,u}^{i,soft} \tag{7.8}$$

7.3 Observables

The angular decay distribution of $B \to V(\to M_1 M_2)\ell^+\ell^-$ is given in terms of angular functions $(I_i(q^2, \theta_V, \theta_l, \phi)$, the value of which can be obtained by integrating data over specific values of the parameters. We consider an optimized set of observables constricted choosing specific combinations of these angular functions. The observables considered here are

– Form Factor Dependent observables.

$$\frac{d\Gamma}{dq^2} = \frac{1}{4}(3I_1^c + 6I_1^s - I_2^c - 2I_2^s) \qquad A_{FB}(q^2) = \frac{-3I_6^s}{3I_1^c + 6I_1^s - I_2^c - 2I_2^s}$$
$$F_L(q^2) = \frac{3I_1^c - I_2^c}{3I_1^c + 6I_1^s - I_2^c - 2I_2^s} \tag{7.9}$$

– Form Factor independent observables.

$$P_1 = \frac{I_3}{2I_2^s},\ P_2 = \beta_l\frac{I_6^s}{8I_2^s},\ P_3 = -\frac{I_9}{4I_2^s},\ P_4' = \frac{I_4}{\sqrt{-I_2^c I_2^s}}$$
$$P_5' = \frac{I_5}{2\sqrt{-I_2^c I_2^s}},\ P_6' = -\frac{I_7}{2\sqrt{-I_2^c I_2^s}},\ P_8' = -\frac{I_8}{2\sqrt{-I_2^c I_2^s}} \tag{7.10}$$

– Lepton Flavor Universality violating observables.

$$R_{K^*}^{B_s} = \frac{\left[\mathcal{BR}(B_s \to \bar{K}^*\mu^+\mu^-)\right]_{q^2\in\{q_1^2,q_2^2\}}}{\left[\mathcal{BR}(B_s \to \bar{K}^*e^+e^-\right]_{q^2\in\{q_1^2,q_2^2\}}} \tag{7.11}$$

These observables are valid for $B_s \to \bar{K}^*\ell\ell$. For the CP-conjugate process, the I_i are replaced by $\tilde{I}_i \equiv \xi_i \bar{I}_i$, where $\bar{I}_i$ are I_s only with weak phase conjugated and $\xi_i = 1$ for $i = \{1, 2, 3, 4, 7\}$ and -1 for $i = \{5, 6, 8, 9\}$. For $B \to \rho\ell\ell$, angular functions are replaced with time-dependent angular functions, since the final state in this case is self conjugate [1]. Thus, observables are sensitive to $B^0 - \bar{B}^0$ oscillations in this case and the $I_i's$ are replaced by $J_i's$ in the definition of observables, where $J_i's$ are given as [9],

$$J_i(t) + \tilde{J}_i(t) = e^{-\Gamma t}[(I_i + \bar{I}_i)\cosh(y\Gamma t) - h_i \sinh(y\Gamma t)] \quad (7.12)$$

$$J_i(t) - \tilde{J}_i(t) = e^{-\Gamma t}[(I_i - \bar{I}_i)\cosh(y\Gamma t) - s_i \sinh(y\Gamma t)] \quad (7.13)$$

where $x = \Delta m/\Gamma$, $y = \Delta\Gamma/\Gamma$, and $\tilde{J}_i \equiv \xi_i \bar{J}_i$. The extra terms h_i and s_i are the cross terms because of meson mixing [9]. These are time-dependent angular functions. To construct time-independent observables, these are integrated over a range of time which is $t \in \{-\infty, \infty\}$ in the case of LHCb and $t \in [0, \infty\}$ in case of Belle. Because of this difference, the integrated angular functions are slightly different for Belle and LHCb. We have taken this into account and given the prediction of angular observables separately.

7.4 Results

The binned values for the decay modes in study are listed in Tables 7.1 and 7.2, where the first uncertainty is due to form factors and scond uncertainty is due to soft-

Table 7.1 Observables for $\bar{B}_s \to K^*\mu^+\mu^-$ and $B_s \to \bar{K}^*\mu^+\mu^-$ using form factors based on LCSR and QCD lattice calculation

Observable	$\bar{B}_s \to K^*\mu^+\mu^-$		$B_s \to \bar{K}^*\mu^+\mu^-$	
	[0.1-1] GeV^2	[1-6] GeV^2	[0.1-1] GeV^2	[1-6] GeV^2
P_1	$0.017 \pm 0.132 \pm 0.001$	$-0.096 \pm 0.128 \pm 0.005$	$0.015 \pm 0.135 \pm 0.001$	$-0.087 \pm 0.118 \pm 0.005$
P_2	$0.122 \pm 0.013 \pm 0.001$	$0.026 \pm 0.081 \pm 0.036$	$0.114 \pm 0.012 \pm 0.001$	$0.054 \pm 0.081 \pm 0.034$
P_3	$0.001 \pm 0.003 \pm 0.0$	$0.004 \pm 0.009 \pm 0.002$	$0.001 \pm 0.006 \pm 0.0$	$0.004 \pm 0.009 \pm 0.002$
P_4'	$-0.704 \pm 0.063 \pm 0.009$	$0.543 \pm 0.167 \pm 0.014$	$-0.736 \pm 0.064 \pm 0.008$	$0.453 \pm 0.176 \pm 0.016$
P_5'	$0.437 \pm 0.044 \pm 0.016$	$-0.422 \pm 0.124 \pm 0.046$	$0.445 \pm 0.045 \pm 0.016$	$-0.377 \pm 0.130 \pm 0.047$
P_6'	$-0.091 \pm 0.005 \pm 0.016$	$-0.087 \pm 0.010 \pm 0.002$	$-0.048 \pm 0.004 \pm 0.001$	$-0.064 \pm 0.004 \pm 0.002$
P_8'	$0.027 \pm 0.007 \pm 0.016$	$0.042 \pm 0.010 \pm 0.017$	$0.048 \pm 0.009 \pm 0.016$	$0.036 \pm 0.008 \pm 0.019$
$R_{K^*}^{B_s}$	$0.945 \pm 0.008 \pm 0.001$	$0.998 \pm 0.004 \pm 0.0$	$0.944 \pm 0.007 \pm 0.001$	$0.998 \pm 0.004 \pm 0.0$
$BR \times 10^9$	$4.439 \pm 0.648 \pm 0.086$	$8.251 \pm 1.872 \pm 0.357$	$5.082 \pm 0.699 \pm 0.101$	$8.763 \pm 1.959 \pm 0.375$
A_{FB}	$-0.048 \pm 0.008 \pm 0.001$	$0.001 \pm 0.021 \pm 0.009$	$-0.047 \pm 0.007 \pm 0.001$	$-0.012 \pm 0.020 \pm 0.009$
F_L	$0.576 \pm 0.066 \pm 0.014$	$0.872 \pm 0.035 \pm 0.007$	$0.553 \pm 0.065 \pm 0.014$	$0.862 \pm 0.035 \pm 0.007$

Table 7.2 Binned values of observables for the process $B \to \rho\mu^+\mu^-$ and $\bar{B} \to \rho\mu^+\mu^-$ for tagged events to be measured at Belle. Form factors are based on LCSR form factors

	$B \to \rho\mu^+\mu^-$		$\bar{B} \to \rho\mu^+\mu^-$	
	[0.1-1] GeV^2	[1-6] GeV^2	[0.1-1] GeV^2	[1-6] GeV^2
$\langle P_1 \rangle$	$0.009 \pm 0.177 \pm 0.001$	$-0.065 \pm 0.116 \pm 0.003$	$0.010 \pm 0.175 \pm 0.001$	$-0.069 \pm 0.120 \pm 0.003$
$\langle P_2 \rangle$	$0.082 \pm 0.0 \pm 0.001$	$0.021 \pm 0.056 \pm 0.023$	$0.076 \pm 0.008 \pm 0.0$	$-0.042 \pm 0.050 \pm 0.024$
$\langle P_3 \rangle$	$0 \pm 0.005 \pm 0.0$	$0.001 \pm 0.005 \pm 0.002$	$0.001 \pm 0.001 \pm 0.0$	$0.002 \pm 0.005 \pm 0.002$
$\langle P_4' \rangle$	$-0.724 \pm 0.081 \pm 0.047$	$0.508 \pm 0.161 \pm 0.029$	$-0.703 \pm 0.080 \pm 0.046$	$0.569 \pm 0.154 \pm 0.017$
$\langle P_5' \rangle$	$0.276 \pm 0.004 \pm 0.027$	$-0.270 \pm 0.083 \pm 0.085$	$0.246 \pm 0.003 \pm 0.030$	$-0.321 \pm 0.074 \pm 0.098$
$\langle P_6' \rangle$	$-0.043 \pm 0.003 \pm 0.001$	$-0.061 \pm 0.004 \pm 0.002$	$-0.075 \pm 0.005 \pm 0.001$	$-0.073 \pm 0.010 \pm 0.002$
$\langle P_8' \rangle$	$0.025 \pm 0.005 \pm 0.016$	$0.025 \pm 0.005 \pm 0.018$	$0.031 \pm 0.005 \pm 0.007$	$0.030 \pm 0.006 \pm 0.017$
$\langle R_\rho \rangle$	$0.936 \pm 0.008 \pm 0.001$	$0.997 \pm 0.003 \pm 0.0$	$0.950 \pm 0.167 \pm 0.002$	$1.064 \pm 0.392 \pm 0.0$
$\langle BR \rangle \times 10^9$	$5.233 \pm 0.711 \pm 0.080$	$8.714 \pm 1.668 \pm 0.366$	$4.736 \pm 0.656 \pm 0.077$	$8.414 \pm 1.649 \pm 0.365$
$\langle A_{FB} \rangle$	$-0.038 \pm 0.005 \pm 0.001$	$-0.007 \pm 0.019 \pm 0.007$	$-0.034 \pm 0.005 \pm 0.001$	$0.014 \pm 0.022 \pm 0.006$
$\langle F_L \rangle$	$0.495 \pm 0.067 \pm 0.014$	$0.813 \pm 0.037 \pm 0.007$	$0.514 \pm 0.072 \pm 0.014$	$0.838 \pm 0.046 \pm 0.006$

gluon emission from up quark. Moreover, the full branching ratio for $B_s \to \bar{K}^*\ell\ell$ is $(3.356 \pm 0.814) \times 10^{-8}$ which is consistent with the recent measurement [8].

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