

## Chapter 10

# Impact of Nonleptonic $\bar{B}_{d,s}$ Decay Modes on $\bar{B} \rightarrow \bar{K}^* \mu^+ \mu^-$ Process



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**Abstract** We scrutinize the effect of nonleptonic  $B$  decay modes on the branching ratio and angular observables of  $\bar{B} \rightarrow \bar{K}^* \mu^+ \mu^-$  process involving  $b \rightarrow s$  quark level transition in the non-universal  $Z'$  model. The new couplings are constrained by using the experimental limits on the branching ratios of  $B_d \rightarrow \pi K$ ,  $B_d \rightarrow \rho K$ , and  $B_s \rightarrow \eta' \eta'$ ,  $K^* K^*$  nonleptonic processes. Using the allowed parameter space, we perform an angular analysis of the  $\bar{B} \rightarrow \bar{K}^* \mu^+ \mu^-$  process. We observe significant impact of nonleptonic decay modes on  $\bar{B} \rightarrow \bar{K}^* \mu^+ \mu^-$  observables.

## 10.1 Introduction

Although Standard Model (SM) is a successfully fundamental theory, it fails to explain the open puzzles such as matter–antimatter asymmetry, hierarchy problem, neutrino mass, dark matter, and dark energy. Thus, it implies the existence of new physics (NP) beyond it. In this regard, the study of rare  $B$  decays, which provide not only deep understanding on CP violation but also different anomalies both in nonleptonic as well as semileptonic sectors, is quite interesting. The decay rate and  $P'_5$  observable of  $\bar{B} \rightarrow \bar{K}^* \mu^+ \mu^-$  process have  $3\sigma$  [1] deviation from their SM results. The decay distribution of  $B_s \rightarrow \phi \mu^+ \mu^-$  also has tension [2]. Furthermore the lepton universality violating ratio,  $R_K = \text{Br}(B^+ \rightarrow K^+ \mu^+ \mu^-) / \text{Br}(B^+ \rightarrow K^+ e^+ e^-)$  disagrees with SM prediction at the level of  $2.5\sigma$  [3]. Discrepancy of  $2.2\sigma$  ( $2.4\sigma$ ) has been observed in  $R_{K^*}$  measurement by LHCb experiment [4]

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$$\begin{aligned}
R_{K^*}^{\text{Expt}} &= \frac{\text{Br}(B^0 \rightarrow K^{*0} \mu^+ \mu^-)}{\text{Br}(B^0 \rightarrow K^{*0} e^+ e^-)} = 0.66_{-0.07}^{+0.11} \pm 0.03, \quad q^2 \in [0.045, 1.1] \text{ GeV}^2, \\
&= 0.69_{-0.07}^{+0.11} \pm 0.05, \quad q^2 \in [1.1, 6] \text{ GeV}^2, \quad (10.1)
\end{aligned}$$

from their SM predictions [5]. Though the measurements on  $R_{K^*}$  by Belle Collaboration [6] is toward the SM results, the error values are comparatively higher than the previous LHCb result. Additionally, the mismatch between the measured data and the SM results are also observed in the two body hadronic decay processes like  $B \rightarrow PP, PV, VV$ , where  $P = \pi, K, \eta^{(\prime)}$  are the pseudoscalar mesons and  $V = K^*, \phi, \rho$  are the vector mesons. Inspired by these anomalies, we would like to see whether the new physics (arising due to an additional  $Z'$  boson) influencing the nonleptonic  $B$  decays also have significant impact on rare semileptonic  $B$  decay processes.

The paper is organized as follows. In Sect. 10.2, we discuss the effective Hamiltonian of  $b \rightarrow sl(\bar{q}\bar{q})$  processes in both SM and in  $Z'$  model. We also present the new physics contribution in this section. Section 10.3 describes the constraints on new parameters from the nonleptonic  $B$  modes. The impact of new couplings on  $\bar{B} \rightarrow \bar{K}^* \mu \mu$  is presented in Sects. 10.4 and 10.5 summarize our results.

## 10.2 Effective Hamiltonian

The generalized effective Hamiltonian for  $b \rightarrow sq\bar{q}$  process, where  $q$  is any light quark, is given as [7]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \sum_{p=u,c} \lambda_p (C_1 \mathcal{O}_1^p + C_2 \mathcal{O}_2^p) - \lambda_t \sum_{i=3}^{10} (C_i \mathcal{O}_i + C_{7\gamma} \mathcal{O}_{7\gamma} + C_{8\gamma} \mathcal{O}_{8\gamma}) \right] + h.c., \quad (10.2)$$

where  $G_F$  is the Fermi constant,  $\lambda_p = V_{pb} V_{ps}^*$ ,  $\lambda_t = V_{tb} V_{ts}^*$  are the product of CKM matrix elements. Here  $\mathcal{O}_{1,2}^p$  are left-handed current–current operators;  $\mathcal{O}_{3,\dots,6}$  and  $\mathcal{O}_{7,\dots,10}$  are QCD and electroweak penguin operators; and  $\mathcal{O}_{7\gamma}, \mathcal{O}_{8\gamma}$  are the electromagnetic and chromomagnetic dipole operators. The relevant  $\mathcal{O}_{7,\dots,10}$  operators are defined as

$$\mathcal{O}_{7(9)} = (\bar{s}b)_{V-A} \sum_q e_q (\bar{q}q)_{V+A(V-A)}, \quad \mathcal{O}_{8(10)} = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q e_q (q_\beta \bar{q}_\alpha)_{V+A(V-A)},$$

where  $V \mp A$  denotes  $\gamma^\mu P_{L(R)}$  with  $P_{L(R)} = (1 \mp \gamma_5)/2$  are the projection operators and  $e_q$  stand for the charge of  $q$  quark. The effective Hamiltonian for  $b \rightarrow sq\bar{q}$  transition in the  $Z'$  model is given by [8]

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{2G_F}{\sqrt{2}} \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 B_{sb}^L (\bar{s}b)_{V-A} \sum_q [(B_{qq}^L (\bar{q}q)_{V-A} + B_{qq}^R (\bar{q}q)_{V+A})], \quad (10.3)$$

where  $g_1(g')$  are the coupling constants of  $Z^{(\prime)}$  boson and  $B_{bs}^{L(R)}$ ,  $B_{qq}^{L(R)}$  are the new couplings. Now, assuming  $B_{uu}^{L(R)} \simeq -2B_{dd}^{L(R)}$  and comparing the Hamiltonian of  $Z'$  (10.3) with SM (10.2), we find an extra contribution to the electroweak penguin sector of nonleptonic decay modes as

$$\Delta C_9^{Z'} = \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \left( \frac{B_{sb}^L B_{dd}^L}{V_{tb} V_{ts}^*} \right), \quad \Delta C_7^{Z'} = \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \left( \frac{B_{sb}^L B_{dd}^R}{V_{tb} V_{ts}^*} \right). \quad (10.4)$$

The most general effective Hamiltonian describing  $b \rightarrow sl^+l^-$  processes in the SM is given by [9]

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left( \sum_{i=1, \dots, 10, S, P} C_i \mathcal{O}_i + \sum_{i=7, \dots, 10, S, P} C'_i \mathcal{O}'_i \right), \quad (10.5)$$

where  $V_{qq'}$  are the CKM matrix elements,  $\mathcal{O}_i$ 's are the effective operators and  $C_i$ 's are the corresponding Wilson coefficients. Though only  $\mathcal{O}_7$  and  $\mathcal{O}_{9,10}$  operators have contributions to the SM, additional  $\mathcal{O}_{9,10}^{(\prime)}$  can be generated due to the presence of  $Z'$  gauge boson, defined as

$$\begin{aligned} \mathcal{O}_7^{(\prime)} &= \frac{e}{16\pi^2} [\bar{s} \sigma_{\mu\nu} (m_s P_{L(R)} + m_b P_{R(L)}) b] F^{\mu\nu}, \\ \mathcal{O}_9^{(\prime)} &= \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma^\mu P_{L(R)} b) (\bar{l} \gamma_\mu l), \quad \mathcal{O}_{10}^{(\prime)} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma^\mu P_{L(R)} b) (\bar{l} \gamma_\mu \gamma_5 l), \end{aligned}$$

where  $\alpha_{\text{em}}$  denotes the fine structure. The effective Hamiltonian of  $b \rightarrow sl^+l^-$  in the  $Z'$  model can be written as [10]

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{Z'}(b \rightarrow sl^+l^-) &= -\frac{2G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left( \frac{g_2 M_Z}{g_1 M_{Z'}} \right)^2 \left[ -\frac{B_{sb}^L B_{ll}^L}{V_{tb} V_{tq}^*} (\bar{q}b)_{V-A} (\bar{l}l)_{V-A} \right. \\ &\quad \left. - \frac{B_{qb}^L B_{ll}^R}{V_{tb} V_{tq}^*} (\bar{s}b)_{V-A} (\bar{l}l)_{V+A} \right] + \text{h.c.}, \end{aligned}$$

which after comparing with (10.5) gives additional coefficients as well as new contributions to the SM Wilson coefficients ( $C_{9,10}^{Z'(\prime)}$ ) as

$$C_9^{Z'}(M_W) = -2 \left( \frac{g_2 M_Z}{g_1 M_{Z'}} \right)^2 \frac{B_{sb}^L}{V_{tb} V_{ts}^*} (B_{ll}^L + B_{ll}^R), \quad (10.6)$$

$$C_{10}^{Z'}(M_W) = 2 \left( \frac{g_2 M_Z}{g_1 M_{Z'}} \right)^2 \frac{B_{sb}^L}{V_{tb} V_{ts}^*} (B_{ll}^L - B_{ll}^R). \quad (10.7)$$

**Table 10.1** The experimental values and SM predictions on the branching ratio of nonleptonic  $B_{d,s}$  decay modes

| Decay processes                             | SM values              | Experimental values [11]        |
|---|------------------------|---------------------------------|
| $\bar{B}_d \rightarrow \pi^- K^+$           | $20.11 \times 10^{-6}$ | $(1.96 \pm .05) \times 10^{-5}$ |
| $\bar{B}_d \rightarrow \pi^0 K^0$           | $6.57 \times 10^{-6}$  | $(9.9 \pm .5) \times 10^{-6}$   |
| $\bar{B}_d \rightarrow \rho^0 K^0$          | $2.80 \times 10^{-6}$  | $(4.7 \pm .6) \times 10^{-6}$   |
| $\bar{B}_d \rightarrow \rho^- K^+$          | $2.77 \times 10^{-6}$  | $(7 \pm .9) \times 10^{-6}$     |
| $\bar{B}_s \rightarrow \eta' \eta'$         | $57.53 \times 10^{-6}$ | $(3.3 \pm .7) \times 10^{-5}$   |
| $\bar{B}_s \rightarrow K^{0*} \bar{K}^{0*}$ | $3.72 \times 10^{-6}$  | $(1.11 \pm .27) \times 10^{-5}$ |

### 10.3 Constraints on New Couplings

After getting an idea on new coefficients, we now proceed to constrain the coefficients by using the branching ratios of nonleptonic  $B$  decay modes. Using the CKM matrix elements, particles masses, life time of  $B_{d,s}$  meson from [11], the form factors, decay constants except  $f_\pi = .131$ ,  $f_K = .160$  from [12], the predicted SM branching ratios of  $B_d \rightarrow (\pi, \rho)K$ ,  $B_s \rightarrow \eta' \eta'$ ,  $K^* K^*$  decay modes, and their respective measured values are presented in Table 10.1.

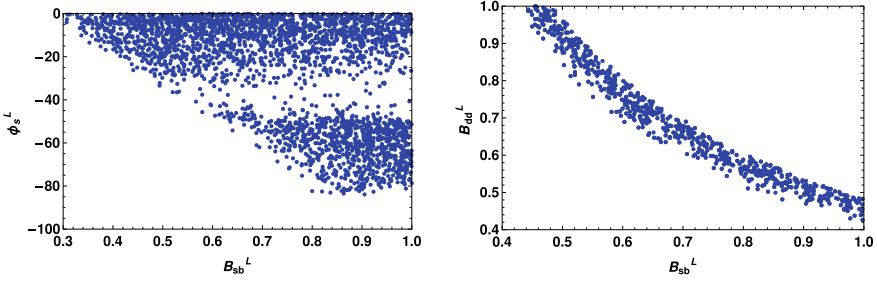
We consider two cases, (a)  $B_{dd}^R = 0$ , which implies  $\Delta C_7^{Z'} = 0$  (b)  $B_{dd}^R = B_{dd}^L$ , which implies  $\Delta C_7^{Z'} = \Delta C_9^{Z'}$  in order to constrain the new parameters. In this manuscript, we will only discuss the first case. Comparing the theoretical predictions from Table 10.1 with their experimental results, the constraints on  $B_{sb}^L - \phi_s^L$  (left panel) and  $B_{sb}^L - B_{dd}^L$  (right panel) planes for first case are shown in Fig. 10.1.

### 10.4 Impact on $\bar{B} \rightarrow \bar{K}^* \mu^+ \mu^-$ Decay Mode

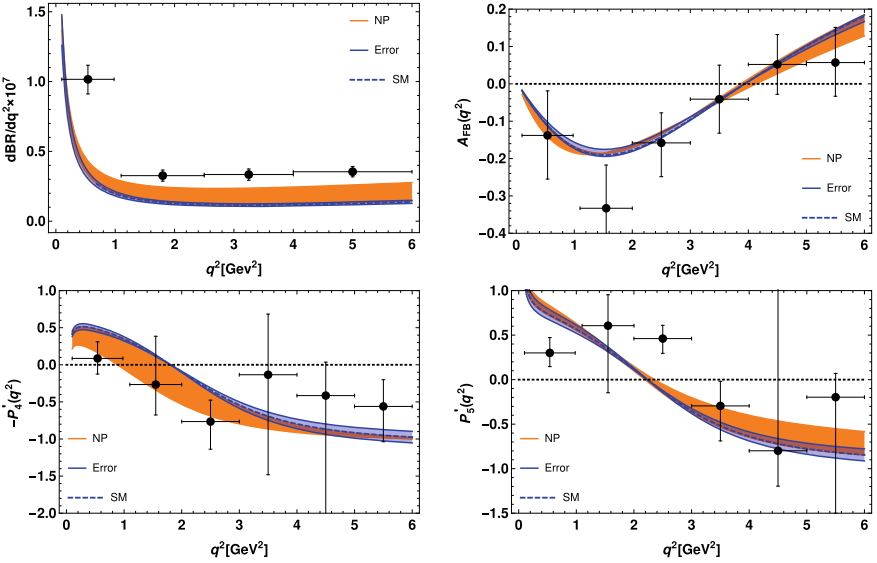
In this section, we present the impact of new parameters constrained from the nonleptonic  $B$  modes on the  $\bar{B} \rightarrow \bar{K}^* \mu^+ \mu^-$  process, which can be completely described in terms of only four kinematical variables; the lepton invariant mass squared ( $q^2$ ) and three angles  $\theta_l$ ,  $\theta_V$  and  $\phi$ , where  $\theta_l$  is the angle between  $l^-$  and  $B_{(s)}$  in the dilepton frame,  $\theta_V$  is defined as the angle between  $K^-$  and  $B_{(s)}$  in the  $K^- \pi^+$  ( $K^- K^+$ ) frame, the angle between the normal of the  $K^- \pi^+$  ( $K^- K^+$ ) and the dilepton plane is given by  $\phi$ .

The decay rate, forward-backward ( $A_{FB}$ ) asymmetry and  $P'_{4,5}$  observables are defined as [13]

$$\begin{aligned} \frac{d\Gamma}{dq^2} &= \frac{3}{4} \left( J_1 - \frac{J_2}{3} \right), \quad A_{FB}(q^2) = -\frac{3}{8} \frac{J_6}{d\Gamma/dq^2}, \\ P'_4 &= \frac{J_4}{\sqrt{-J_2^c J_2^s}}, \quad P'_5 = \frac{J_5}{2\sqrt{-J_2^c J_2^s}}, \end{aligned} \quad (10.8)$$



**Fig. 10.1** Constraints on new parameters from the branching ratios of nonleptonic  $B$  processes for  $B_{qq}^R = 0$  case



**Fig. 10.2** The  $q^2$  variation of branching ratio (top-left), forward-backward asymmetry (top-right),  $P'_4$  (bottom-left) and  $P'_5$  (bottom-right) observables of  $\bar{B} \rightarrow \bar{K}^* \mu \mu$  process. Here  $P'_4|_{\text{LHCb}} = -P'_4$

where  $J_i = 2J_i^s + J_i^c$  contain the transversity amplitudes which are the functions form factors and Wilson coefficients. All the input parameters are taken from [11] and the form factors from [14].

Using the allowed parameter space from Fig. 10.1, we show the variation of branching ratio (top-left),  $A_{FB}$  (top-right),  $P'_4$  (bottom-left) and  $P'_5$  (bottom-right) of  $\bar{B} \rightarrow \bar{K}^* \mu \mu$  with respect to  $q^2$  in Fig. 10.2. Here the dashed blue lines (light blue bands) represent the SM predictions (uncertainties arising due to the input parameters) and orange bands stand for the NP contributions. The experimental results are shown in black color [1]. We observe that NP contribution provide significant deviation from their SM results and can accommodate experimental data.

## 10.5 Conclusion

We have studied the rare semileptonic  $\bar{B} \rightarrow \bar{K}^* \mu \mu$  in a non-universal  $Z'$  model. We constrain the new parameters from the branchings ratios of nonleptonic  $B$  decay modes. We mainly check whether the new physics couplings influencing the nonleptonic modes also have impact on semileptonic processes. We found that the constraint from nonleptonic decays significantly affect the branching ratios and angular observables of  $\bar{B} \rightarrow \bar{K}^* \mu \mu$  process.

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