Chapter 9
Water Resource Management Aided by Game Theory

Icen Yoosefdoost, Taufik Abrão, and Maria Josefa Santos

Abstract Game theory is a theoretical framework for conceiving social situations among competing players. In some respects, game theory is the science of strategy, or at least the optimal decision-making of independent and competing actors in a strategic setting. Using game theory, real-world scenarios for such situations as pricing competition and product releases (and many more) can be laid out, and their outcomes predicted. In this chapter, after introduction and discussion about game theory, the main reason for applying game theory and the area of using this approach explained. In the next step, applying game theory in various fields, game-theoretical models, Game Classification are discussed. After that, the application of Game Theory in Water Resources Management and Useful definitions of applied GP in water resources conflict introduced. Three examples of game theory in water Allocation, water Costs, and groundwater conflicts are introduced and discussed in the final section. the result of these studies illustrates the noticeable performance of game theory approaches in water resources problems.

Keywords Game theory · Water resources management · Water allocation · Water costs · Groundwater

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9.1 Introduction

9.1.1 Game Theory

Game theory (GT) is essentially the mathematical study of competition and cooperation. It illustrates how strategic interactions among players result in overall outcomes with respect to the references of those players. Such outcomes might not have been intended by any player (Snyder 2017).

Game theory can be used to predict how people behave, following their own interests, in conflicts. In a typical game, decision makers (players), with their own goals, try to outsmart one another by anticipating each other's decision. The game is resolved as a result of the players' decisions. GT analyses the strategies players use to maximize their payoffs. A solution to a game prescribes decisions the decision makers might make and describes the game's outcome. GT was established in 1944 with the publication of von Neumann and Morgenstern's "Theory of Games and Economic Behavior" book, which mainly dealt with quantitative game theory methods. After World War II, most scholars worked on developing quantitative game theory methods; and this trend still persists today (Hipel and Obeidi 2005).

Every baby realizes what games are. When a person overreacts, we sometimes tell, "it's just a game." Games are often not earnest. Mathematical games are different. It was the main goal of game theory GT from its beginnings in 1928 to be applied to serious situations in economics, politics, business, and other areas. Even we can analyze war by using mathematical game theory. Let's describe some ingredients of a mathematical game:

Rules: Mathematical games have strong rules. They determine what is allowed and what isn't. Although many real-world games allow for finding new moves or ways to act, games that can be analyzed mathematically have a strict set of possible steps, generally, all known in advance.

Outcomes and payoffs: people play games just for fun. Mathematical games may have several likely outcomes, each producing payoffs for the players. The payoffs possibly are monetary, or they may represent satisfied.

Uncertainty of the Outcome: In most cases, a mathematical game is "thrilling" because the result cannot be forecast in advance. As its rules are stable, this implies that a game must either have some accidental parameters or own more than a single player.

Decision making: A game with no resolve possibly is annoying, at least mentally. Running a 100 m race does not need mathematical proficiency; it needs just fast legs. However, most sports games also involve decisions, and can, therefore, at least partly be analyzed by game theory. No cheating in real-life games, fraud is possible. Cheating means not playing by the rules. If, when your chess rival is distracted, you take your queen and put it on a better square, you are cheating. Game theory doesn't even acknowledge the being of cheating. We will learn how to win without cheating.

9.2 Definition and Terms

9.2.1 Game Definition

Let's define the tuple:

$$G_T \triangleq \langle N, A, P, I, O \rangle$$

where:

G_T—the game, which exist manly in two forms: Strategic (or Normal) games, denoted by G, and Extensive games, denoted by Γ;

N—set of players. N = {1, 2, …, n} is a finite set. Every player is denoted by i; the other n − 1 players or i's opponents in some senses, denoted by −i; ∀ i, −i ∈ N;

A—the profile of action (or move) of the players. An action carried out by player is a variable of his decision, which is denoted by ai. The set of A_i, i = $\{a_i\}$ is player's action set, i.e. the entire set of actions available to him. The ordered set $a_i = \{a_i\}$, i ∈ {1, 2,…, n}, is an action combination for each of the n players in a game. In the action set, S is the strategy set (called strategies space) of the players. Strategy is the rule to choose actions. The strategy space of player *i*, denoted by S_i is the set of all the strategies which player *i* can choose from.

P—payoff (or utility). A payoff is the value of the outcome to the players. It refers to both actual payoff and expected payoff. Payoffs are based on benefits and costs of actions and outcomes of each player. $u_i(s_i, s_{-1}-)$ means player *i* 's payoff function, which is determined by the strategies chosen by himself and the other players;

I—information set. It is players' knowledge about another player, such as the characteristics, action profile, and payoff function in the game. If the payoff function of every player is a common knowledge among all the players, then it is complete information. Otherwise, it is called incomplete information. If the information is complete and perfect, it means that the players know well the former process of the game before he chooses his next move at each step. If the player who will choose his next move does not know the prior processing of the game at some steps, it is called complete but not perfect information;

O—outcomes of the game. An outcome is a strategy profile rusting from the action/moves combination chosen by all the players at the end of a game;

E—equilibrium or equilibria. In the equilibrium, each of the players can maximize his payoff. $s^* = \{s_1^*, s_2^*, \ldots s_n^*\}$ is a best strategy combination of the *n* players. For player *i*,s_i^* is the player's best response to the strategies specified for the *n-1* other players, i.e. $u_i\left[s_i^* - s_i^*\right] \geq u_i\left\{s_i - s_i^*\right\}$.

Generally speaking, the elements of game theory includes ***N***—Players, ***A***—Action, ***P***—Payoff, ***I***—Information, ***O***—Outcome, and ***E***—Equilibrium, i.e., NAPI-OE. NAPI are concertedly known as the rules of a game and *OE* are the game results. The main task of constructing a game model is to define the rules (NAPI) in mathematical language and get the solution from OE. The detailed GT can be referred by

(Friedman 1998; Gibbons 1992; Kreps 1990; Straffin and Philip 1993; Gardner 1995; Myerson 2013; Stahl 1999; Osborne 2004; Gintis 2001). Every player has different strategies; however, the optimal strategy for an individual player is to maximize his benefits by using the game rules; while the optimal strategy for the player of a society as whole is to maximize the common welfare of the society through the rules. GT models involve the following conditions and assumptions:

- Players in the game models are regarded as "intelligent and rational". Rational payer means that each player will choose an action or strategy which can maximize his expected utility given he thinks what action other players will choose. Intelligent player means that each player understands the situation, and he knows the fact that others are intelligent and rational;
- Each player considers not only his own knowledge and behavior but also others' during pursuing exogenous aims;
- Each player has more than one choice or sequence ("plays");
- All possible combinations of choices or plays result in a well-defined outcome: win or lose, or mutual gains and losses.
- The players are aware of the rules of the game and the options of other players, but they do not know the real decisions of other players in advance. Therefore, every player has to choose options based on his assumption of what other player will choose;
- Each player knows that his actions can affect the others, and the actions of others affect him;
- Each player makes the best possible move, and he knows that his opponent is also making the best possible move (Wei 2008).

9.3 Basic Principles and Logics

9.3.1 Game Theoretical Models

In the last years, game theoretical modelling is becoming an indispensable approach to analyze, understand, and solve many water problems around the world. Like other sciences, game GT itself is comprised of a collection of models. There are different methods to classify these models. In general, they are summarized as follows:

- Binding agreements: non-cooperative and cooperative games;
- Numbers of players: single player game (decision problem), two-persons game and multi-person game;
- Order of actions (moves): static and dynamic games;
- Elements of actions (moves) set: finite and infinite games;
- Sum of payoffs: zero-sum and non-zero-sum games;
- Information set: complete information and incomplete information games;
- Numbers of the same play in a game: one-shot game and repeated game (Wei 2008).

9.3.2 Some More Definitions: Game, Play, Action

The entire collection of rules describes a game. A play is a sample of the game. In specified situations, called positions, a player has done make a decision, called a **move** or an **action**. This is not the same as a strategy. A strategy is a plan that expresses to the player what move to choose in every feasible position. Rational behavior is almost supposed for all players. That is, players have priority, beliefs about the world (including the other players) and try to optimize their individual payoffs. Besides, players are aware that other players are trying to maximize their payoffs (Dinar and Hogarth 2015).

9.3.3 Game Classification

Game can be categorized according to several criteria, including:

Number of players

Usually there should be more than one player. However, you can play roulette alone—the casino doesn't count as player since it doesn't make any decisions. It collects or gives out money. Most references on GT do not treat one-player games, but (Dinar and Hogarth 2015) discuss one-player games provided they contain elements of randomness.

Simultaneous or sequential play

In a simultaneous game, each player has just one move, and all movements are made simultaneously. In a sequential game, it is forbidden two or more players move at the same time, and players possibly have to move multiple times. Some games are neither simultaneous nor sequential (Dinar and Hogarth 2015).

The game with random moves

Games possibly can contain random events which can influence its outcome. They are called random moves (Dinar and Hogarth 2015).

Players with perfect information

A sequential game has perfect information if every player, when about to move, knows all previous moves (Dinar and Hogarth 2015).

Players with complete information

This means that whole players are aware of the structure of the game. The order in which the players move, all thinkable moves in every position, and the payoffs for all outcomes. Actual world games generally do not have complete information. In our games, we consider complete information in most cases, since games of incomplete information are more difficult to analyze (Dinar and Hogarth 2015).

Zero-sum game

Zero-sum games possess the property that the sum of the payoffs to the players equals zero. A player can have a positive payoff just if the other has a negative payoff. Poker and chess can be great examples of zero-sum games. Actual-world games are scarcely zero-sum (Dinar and Hogarth 2015).

Permitted communication

Sometimes the relationship between the players is allowed before the game starts and between the moves and sometimes it is not (Dinar and Hogarth 2015).

The cooperative and non-cooperative game

Game theoretical models are usually divided broadly into two branches, either noncooperative game or cooperative game. It does not mean that these two branches are applied to analyze different kinds of games, but they are just two ways to view the same game (Gibbons 1992; Wei 2008).

Even if players converse, the question is whether the results of the negotiations can be performed. If not, a player can always move differently from what was promised in the talks. Then the relationship is named "cheap talk." A cooperative game is one where the outcome of the negotiations can be put into a contract and be performed. There should also be a method of distributing the payoff among the members of the coalition (Dinar and Hogarth 2015).

Non-cooperative game can be defined from the following aspects:

- modelling the situation of lacking binding agreements;
- what actions (moves) that players can take;
- how players interact with each other to maximize individual welfares;
- solutions concepts: Nash equilibrium, sub-game perfect Nash Equilibrium, Bayesian Nash Equilibrium and perfect Bayesian (sequential) Equilibrium;
- mainly stressing individual rationality, individual optimal strategies and payoff;
- the results may be efficient and maybe not (Wei 2008).

Cooperative game can be defined by:

- modelling the situation of binding agreements;
- what coalitions forms that players can use to maximize the collective welfare of all the players;
- how the available total value split in a satisfactory way;
- most popular solution concepts include: the stable set, equity-based rule, the core Shapley value, as well as the Nash bargaining solution;
- Stressing mainly collective rationality, efficiency and fairness;
- the results are usually social optimum (Wei 2008).

In summary, the non-cooperative and cooperative game theories are similar to the positive and normative approaches that economists use. In economics, the positive approach describes what the real world is, and it usually deals with analyzing and prediction. However, normative approach deals with what the world should be, and it focuses on the methods to change the world. Moreover, noncooperative game theory is a strategy-oriented game, and it studies what players expect to do and how they do it. On the other hand, cooperative game theory establishes what the players can achieve and how they can achieve it (Wei 2008).

9.3.4 Why Game Theory?

"Game theory is essentially the mathematical study of competition and cooperation. It illustrates how strategic interactions among players result in overall outcomes with respect to the preferences of those players." The objective is to predict how people behave, trying to achieve their goals while in conflict. It includes decision makers (players) trying to outsmart one another by anticipating each other's decision. The game is resolved as a byproduct of the players' decisions. The resolution of the 'game' leads to optimal decision making and describes the game's outcome. The application of GT in the industry could result in a revolutionary change in process efficiency and optimality, saving significant amounts of the essential scarce resource water.

9.3.4.1 GT Features

- Game theory creates a realistic simulation of stakeholders' interest-based behavior. The self-optimizing behavior of players and stakeholders often results in non-cooperative behaviors, although cooperative competition could be a win-win situation.
- The model can create planning, policy, and design insights that would be unavailable from other traditional systems and engineering methods.
- Another advantage of GT over traditional methods is its ability to simulate different aspects of the conflict, incorporate various characteristics of the problem, and predict the possible resolutions in absence of quantitative payoff information.
- Often non-cooperative GT methods can help resolve the conflict based on the qualitative knowledge about the players' payoffs. This enables to handle the socio-economic aspects of conflicts and planning, design, and policy problem when quantitative information is not readily available available (Madani 2010; Jhawar et al. 2018).

9.3.4.2 Challenges in Applying GT to Water Resource Management

- The complexity in the economics that comes with large-scale water resources projects are challenging; the industry impacts different strata of the society and varied geographical locations differently.

– The unpredictability of natural and climatic conditions makes creating forecasts even more challenging. However, the implications would be significant in extent and variety.
– The large number of decision variables involved, stochastic nature of the inputs, and multiple objectives makes this sector an obstacle course towards optimality using the given model (Datta 2005). Therefore, we can see the relevance, challenge, and important resource incorporating operations research into the management of water resources (Jhawar et al. 2018).

9.4 Importance and Necessity of Game Theory

GT was established in 1944 with the publication of von Neumann and Morgenstern's "Theory of Games and Economic Behavior" book, which mainly dealt with quantitative game theory methods. After World War II, most scholars worked on developing quantitative GT methods; this trend still persists today (Hipel and Obeidi 2005). Besides, over the years, GT applications have been developed for several water sectors. Many researchers have attempted water conflict resolution in different area of studies in a game-theoretic framework.

9.4.1 Application of Game Theory

9.4.1.1 Water Resource Management

Leoneti and Pires (2017a) conducted research which the main goal was to supply a review of the literature from the field of decision sciences to the area of *water resource management*.They discuss the application of multi-criteria methods, including Analytical Hierarchy Process, Measuring Attractiveness by a Categorical Based Evaluation Technique, Multi-attribute Utility Theory, Elimination and Choice Translating Reality, Preference Ranking Organization Method for Enrichment Evaluations, Technique for Order Preference by Similarity to Ideal Solution, and GT, containing cooperative and non-cooperative bargaining games. Their numerical results illustrate that, these techniques are useful for creation and comparison of scenarios, decrease the time needed to achieve a solution for complicate problems containing a large number of criteria and agents. It besides shows the benefits of creating greater transparency in the decision-making process, thus improve the potential for a solution acceptable to all the parties involved. Although still little explored, discussions of sanitation problems can and must be raised with the use of techniques and methods of decision sciences, while multi-criteria and game theory techniques are particularly suitable for this task.

9.4.1.2 Water Resource Conflict of Interest

Varouchakis et al. (2018) in one study tried to help people who live in the city of Chania, Greece, the resident have asked for a fairer tariff policy and represent the purpose to save water under the performance of stricter measures. A two-person zero-sum game was proposed, including a conflict of interest among the Municipal Enterprise for Water and Sewage of Chania (MEWS) (Player 1) and the city's near 108,000 residents (Player 2). Three scenarios for the gradual decrease for the fixed charges and the continuous growth for volumetric charges were developed, assuming various degrees of change in water use behavior by each consumption block of consumers. The payoff matrices, for each scenario, incorporated two clear cost strategies for Player 1, in terms of changing the current tariff policy, and four clear cost strategies for Player 2, regarding the change in consumers' behavior. The optimal decision for both players, derived from the identification of the equilibrium point, demonstrated that domestic water consumption might be reduced by up to 4.6% while maintaining the MEWS's profit. The proposed model can provide a guide for other similar applications.

Conflicts in Irrigation Area and Drainage Network

Gholami et al. (2017). In the study with the purpose of analyze and to provide solutions for solving the conflicts in Sefidrud irrigation and drainage network the GT approach was employed. For modeling and analysis of the conflict, Graph Model for Conflict Resolution (GMCR) were used. After determining players and options and inserting them into the model, 64 states were created in this conflict arena. Using non-cooperative solution concepts with regard to prioritize strategies by decision-makers, 4 situations were identified as equilibrium points. After the final analysis of the strongest points of equilibrium, status quo and the base status form one of the points of equilibrium. The other equilibrium point was situations that farmers take alternative irrigation tout for the favorable situation in the future. Therefore, it is essential to train farmers as a primary player and involve them in decisions and to form water user associations to improve the condition of their participation in the management of water resources. This would reduce the conflict using stability of the resources.

Irrigation Water Conflict in Southeastern Brazil

Getirana and de Fátima Malta (2010) apply GT approaches to conflict among irrigators among the Coqueiros Canal water users, located in the Campos dos Goytacazes municipality, in the northern region of the State of Rio de Janeiro, and a canal in Rio de Janeiro State in southeastern Brazil. The Graph Model for Conflict Resolution (GM-CR) tool, which is able to solve Non-Cooperative Games (NCG) based on graph theory has been applied to efficiently solve such water resource dispute. The authors developed six scenarios pertaining to the decision makers, and their options and strategies. Then they identified two possible roles for the managing institution: (a) the conflict resolution managing institution takes into the account the fact that it has no explicit preferences for any of the outcomes; (b) the managing institution explicitly demonstrates preference for those scenarios and solutions that provide more income taxes. The results suggest a solution to the conflict among the irrigators, with the demand for irrigation water affecting the priorities in attaining possible equilibria.

9.4.1.3 Relicensing Process with Bargaining Solutions

Madani (2011) developed a method based on Nash and Nash–Harsanyi bargaining solutions to illustrate the Federal Energy Regulatory Commission (FERC) relicensing process, in which owners of non-federal hydropower projects in the United States must negotiate their suitable operations, with other interest groups (mainly environmental). In this study, the connection of games to develop the possible solution range and the "strategic loss" concept are considered, and a FERC relicensing bargaining model is expanded for studying the bargaining stage (third stage) of the relicensing process. According to the suggested solution method, how the lack of incentive for cooperation results in a long delay in FERC relicensing in practice is explained. Further, the potential impacts of climate change on the FERC relicensing are express, and how climate change may provide an incentive for collaboration between the parties to hasten the relicensing is discussed. An "adaptive FERC license" framework is proposed, according to cooperative game theory, to increase the efficiency and competitivity of the system to future changes with no cost to the FERC in the meat uncertainty about future hydrological and ecological conditions.

9.4.1.4 Multiple-Reservoir Cooperation in Hydropower System in China

In a methodology namely Progress Optimality Algorithm based on Discrete Differential Dynamic Programming (POA-DDDP) and implemented by the Multidimensional Search Algorithm (PDMSA) combined to game theory is proposed to address the challenge of fairly allocate the incremental benefits of cooperation among all hydropower plants participants/players. In this study, The PDMSA expands to define optimal operation decisions, obtaining a multi-yearly average earning under all feasible coalitions of plants. Then, the collaboration benefit can be correctly calculated based on the differences of generation production revenue among various alliances. In addition, the game-theoretic Shapley method is used to discover the suitable share of each plan cooperator from overall cooperation benefits. The cooperative core based on a set of necessary conditions helps to choose a possible, stable allocation plan, while their stability is evaluated by the propensity to disrupt (PTD). The proposed methodology is used to a multiple-reservoir hydropower system on the Lancang River, which is one of 14 largest hydropower bases in China. This case illustrates that the method provides the most stable incremental allocation project by comparison with different generally used methods.

9.4.2 An Overview to Studies, Applying Game Theory Approach in Water-Resources Systems

Publications dealing with Game theory have covered several domains of water-resources systems. We can categories these studies into eleven typologies that mark an essential contribution to the literature in the last fifty years, such as **Urban water supply and sanitation** (Barreiro-Gomez et al. 2017; Leoneti and Pires 2017b; An et al. 2018; Zarei et al. 2019a; Goksu et al. 2019; Chhipi-Shrestha et al. 2019). **Irrigation** (Omidvar et al. 2016; Xing and Yuan 2017; Mukherjee 2017; Chhipi-Shrestha et al. 2019; Ristić and Madani 2019; Hone et al. 2020). **Hydro-electric power** (Gately 1974a, b; Anderson 2016; García Mazo et al. 2020). **Water pollution control** (Adhami and Sadeghi 2016; Guo 2016; Shi et al. 2016; Yong et al. 2017; Xu et al. 2017; William et al. 2017; Zhang et al. 2019; Hu et al. 2019). **Groundwater** (Huang et al. 2016; Gao et al. 2017; López-Corona et al. 2018; Tian and Wu 2019; Tian and Wu 2019; Ghadimi and Ketabchi 2019; Nazari and Ahmadi 2019; Nazari et al. 2020). **Allocation issues** (Xiao et al. 2016; Shi et al. 2016; Oftadeh et al. 2017; Yuan et al. 2017; Han et al. 2018; Degefu et al. 2016; Zarei et al. 2019a). **International/transboundary water** (Menga 2016; Li et al. 2016, 2019; Fu et al. 2018; Khachaturyan and Schoengold 2019; Zhang et al. 2019; Tayia 2019; Zeng et al. 2019; Janjua and Hassan 2020). **Water conflict and negotiations** (Mehrparvar et al. 2016; Zomorodian et al. 2017; Oftadeh et al. 2017; Zanjanian et al. 2018; Mogomotsi et al. 2019; Zeng et al. 2019; Yu et al. 2019). **Water and ecological systems** (Dinar et al. 2013; Hachoł et al. 2019). **Watershed management and regulation/river basin planning** (Girard et al. 2016; Hui et al. 2016; Jeong et al. 2018a; Rahmoun and Rahmoun 2019; Andik and Niksokhan 2020; Lee et al. 2020; Adhami et al. 2020; Janssen et al. 2020). **Multipurpose water projects** (Kahil et al. 2016; Jeong et al. 2018b; Alahdin et al. 2018; Alaghbandrad and Hammad 2018; Zarei et al. 2019b; Chhipi-Shrestha et al. 2019).

9.5 Methodology

9.5.1 Game Theory Approach in Water Resources Management

Game theory began as applied mathematics and microeconomic theory, but it serves here as a modelling approach to manage water resources. The questions arose in the game modelling of water resources management are as follows:

- What kind(s) of the game (games) can water resources management be modeled as? In other words, what kind(s) of the game (games) is (are) involved in water resources management? Can the rational choices of multi-stakeholders be translated into a mathematical or/and economic problems? Can the rational outcome be as the “solution” to the game?
- How to translate a case of water resources management into a game in mathematic or/and economic language? In details, what is (are) the player(s)? What are the strategies available to each player? What is payoff that each player can obtain from the combination of strategies chosen by the players? what methods can be used to solve for the Nash Equilibria of strategies?
- What is the strategy space? In which condition does a player use pure strategies or mixed strategies? How to choose dominated strategy (strategies)?
- What does it mean complete and incomplete information in the games of water resources management? What uncertainties or risks are there in a game of water resources management? How to predict them?
- How to value the problems and benefits in payoff terms? How to value the payoff and make right decisions? If the game is cooperative one, how to divide the joint payoff? (Wei 2008).

9.5.2 Types of Games

From the game theoretical point of view, there are full of games in human society and nature. Figure 9.1 depicts the nature and human society from a game point of view, and each interacting and interdependent group or/and individual can be modelled as game(s). For examples, the game can be between human and rain, rivers, lakes and animals, and between animals and animals, plants and plants, animals, plants and their habitats, human and human, and so on (Wei 2008).

The game components/elements for Fig. 9.1 could define as:

N (set of players), In this picture according to each game, could be different, for example in HH-G it can be several countries, people, company, etc. In NN-**G** it is different species of animals in front of each other, river and animal, rain, animal, and, ... In HN-G The players are human-rain, human-force, human-water, human-animal, etc.

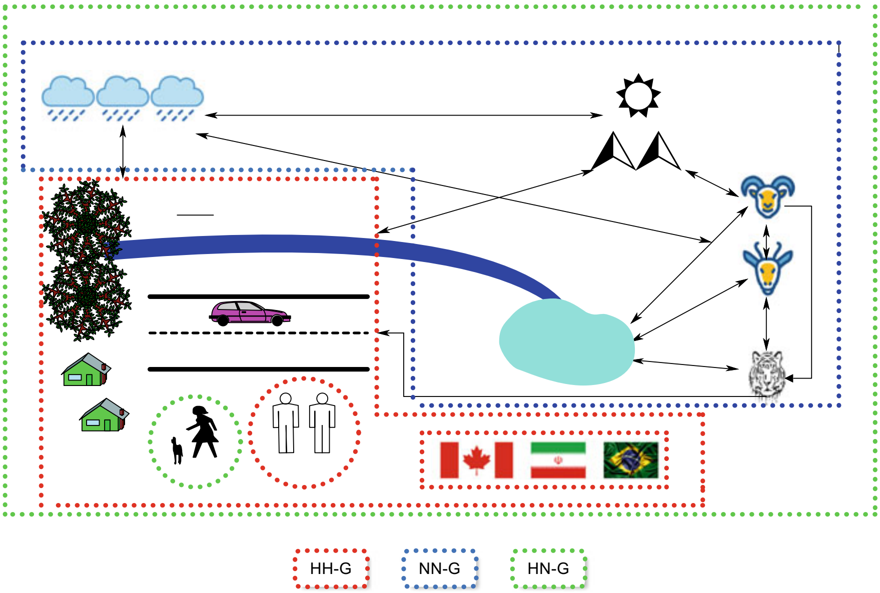

Fig. 9.1 Nature and human society from a game theoretical perspective (Wei 2008)

A (the profile of action (or move) of the players). For example, in HH-G, It can the way two countries treat each other, respectfully, aggressive, and so on. Or in HN-G, between human and groundwater resources, the action of humans is excessive, improper use of groundwater sources, land subsidence is nature's action. In NN-G, for example, hunting is an action for a predator, and camouflaging is an action of prey.

I (information set) is player knowledge about another player. For example, information which two countries have from each other, or The knowledge that prey and predator have from each other, in the subject of characteristics, action profile, and payoff function in the game.

O (outcomes of the game), In HH-G in two countries game, it can result in a loser country. Become a colony of another country or in NN-G; it can be a Successful predator to prey or escape prey from and reduce the number of one of them due to lack of food or hunting.

Vrieze (1995) classified environmental games in two ways: society's game and game of exhaustion, while (Kelly 2003) classified into games of skill, games of chance and games of strategy. In this study, the games involved in water or other nature resources management is classified into the following three kinds:

HH-G: Human and human games, the games played among human beings, including different countries, world regions, or areas within regions;

HN-G: Human and nature games, the games played between human beings and the nature;

NN-G: Nature and nature games, the games in nature itself (Wei 2008).

In definition, HH games are similar to society's game and games of strategy, and HN games are similar to the game of exhaustion and the combination of games of chance and games of skill. HN game is a close relative of decision theory. Parson and Wooldridge (2002) stated that decision theory could be considered to be the study of games against nature, where nature acts randomly. In the literature of game theory, nature usually is regarded as a pseudo player entering the game. Some people maybe do not believe that nature can be players because they cannot move. However, there are so many examples to show that nature really moves and strict back when humans use it improperly, such as pollution, the greenhouse effect, and so on. If so, the question is what their strategies and payoffs are since they are players. For the NN games, there are very few studies comparing with the former two kinds. Smith (1982) analyzed the NN games in his book Evolution and Theory of Games (Wei 2008).

9.5.3 A Game Theoretical Approach to Solve Conflicts

The question is how to construct a game model. Figure 9.2 depicts the process of using game theoretical approach to solve conflicts. Generally speaking, the process of game theoretical modeling approach can be divided into four steps (Wei 2008).

Step 1: Defining the game

Defining the players
Defining their payoff functions f
Defining their moves (strategies)
Defining information set

Step 2: Setting up game models

Non-cooperative game models
Cooperative game models

Step 3: Analyzing the game models

Getting the possible game outcomes
Comparing these outcomes

Step 4: Solving the game

Getting the equilibrium of non-cooperative games
Getting the trade-off point to share the benefit obtained from cooperative games.

This flow can be shortly summarized into the following questions:

Fig. 9.2 General flow chart of game theoretic approach to solve conflicts (Wei 2008)

- Who involves in the conflict?
- What are their actions (strategies)?
- How to form the payoff function of each player?
- How does every player know the payoff function of others?
- Is the game one-time game, continuous game, finite game or infinite one?
- How to compute the equilibrium/equilibria of the game(s) in the case of a noncooperative game?
- Is every player better off if he cooperates with others?
- How to deal with the amount of benefit derived from cooperative games among the players? (Wei 2008).

9.5.4 Useful Definitions on GT Applied to Water Resources Conflicts

In a water conflict, several interest groups or persons can be modeled as decision-makers (players), where each decision-maker can make choices unilaterally, and the combined decisions of all players together determine the possible outcomes of the conflict. Instead of unilaterally moving, decision-makers also may decide to cooperate or form coalitions leading to Pareto-optimal outcomes. GT techniques create an efficient and *accurate language* for discussing *specific water conflicts*. A systematic study of a strategic water dispute provides insights about how the conflict can be better resolved and may suggest innovative solutions. Many researchers have attempted water conflict resolution using a game-theoretic framework. We provide a menu of several water resource allocation schemes available in the literature. In the next subsections, we provide a theory review behind each of these allocation schemes by visiting the sources provided for each resource allocation scheme. While we use cost allocation schemes, the reader can convert them very easily to benefit/profit allocation schemes as well. The examples are taken from (Dinar and Howitt 1997) and (Kreps 1990).

In Mehrparvar et al. (2016), cooperative game theory (CGT) approaches were used to water allocation in a river basin with attention to equity benefit shares between stakeholders. Firstly, to allocate water between competing users, an optimization model is developed based on the containing industrial, agricultural, and environmental users and their economic objectives. The model is elaborated to determine water shares for different likely coalitions among water users. Then, CGT approaches, including Shapely, Nucleolus, and Nash-Harsanyi methods, were used for reallocating net profits to the users as a solution to encourage them to participate in equitable cooperation. Then, the results from different game-theoretic approaches are evaluated by using the *stability index* and *voting* methods, such as *social choice* and *fallback bargaining*. This study was proceeded in the Zayandehrood River basin located in Iran, which struggles with water scarcity. The different CGT approaches applied to two predefined real-life scenarios in the basin under study, and their performance have been investigated. The results indicate the proper performance of both Nash-Harsanyi and Shapely methods for pessimist and optimistic scenarios, respectively. It is also found that the application of the proposed methodology effectively increases the users' benefits in the study region through optimal water allocation and reallocation of benefits.

9.5.5 Non-GT Cost Allocation Schemes Used in GT Studies

There are a wide variety of cost allocation schemes for joint operation of facilities proposed in the accounting and engineering literature (Biddle and Steinberg 1985) and (Alchian 1965), providing a comprehensive review, from which we use three main types:

– an engineering approach where the cost allocation is proportional to the physical use of the facility;
– marginal cost analysis based on economic efficiency principles;
– the separable cost remaining benefit (SCRB) principle, where the allocation of the fixed investment is based on an equitable division of the cost.

In the following subsection, the terms "player" and "user" are used interchangeably (Dinar and Hogarth 2015).

9.5.5.1 Resource Allocation Based on Pollution Generation

This water resource allocation scheme simply suggests that each user of the joint facility will be charged in proportion to the services the facility provides for this player (e.g., volume of pollution it generates that is treated in the joint facility). In summary, the cost allocated to user *j* is:

$$P_j = f^N \cdot \frac{q_j}{\sum_{i \in N} q_i}$$

where, f^N is cost of the joint facility and q_j is the quantity of pollution generated by user *j*. This scheme allocates all of the joint cost among all *N* users (Dinar and Hogarth 2015).

9.5.5.2 Allocation Based on Marginal Cost

The allocation based on the marginal cost of the joint facility takes into account marginal quantities generated by each potential user. Since economies of scale in the joint cost function exist, the revenues generated by this allocation scheme will not cover the total cost. Therefore, an additional procedure is necessary to account for the remaining uncovered costs. Usually, this can be done using any *proportional rule*, such as pollution volume, or volume of production. The allocation in terms of joint cost for the *j*-th user is defined as:

$$b_j = \left\{ \frac{\partial f^N}{\partial q_j} + f^N \left[1 - \sum_{i \int N} \frac{\partial f^N}{\partial q_i} \right] \right\} \frac{q_j}{\sum_{i \int N} q_i}$$

where q_j is the quantity of pollution generated by j-th user; $\frac{\partial f^N}{\partial q_j}$ is the marginal cost associated with the use of user j; and $f^N[1 - \sum_{i \int N} \frac{\partial f^N}{\partial_{q_i}}]$ is the remaining uncovered cost, which is now included in the allocation scheme (Dinar and Hogarth 2015).

9.5.6 *Separable Cost Remaining Benefit (SCRB)*

The separable cost of user $J \in N$ is the incremental cost:

$$m_j = f^N - f^{N-\{j\}}.$$

The alternate cost for j is the cost $f^{\{j\}}$ it bears while acting alone, and the *remaining benefit* to j (after deducting the separable cost) is $r_j = f^{\{j\}} - m_j$. The SCRB assigns the joint cost according to:

$$k_j = m_j + \frac{r_j}{\sum_{i \in N} r_i} \left\{ f^N - \sum_{i \in N} m_i \right\}^+.$$

where the operator $\{x\}^+ = \max\{0, x\}$. In other words, each user pays their separable cost m_j, while the "non-separable costs" $f^N - \sum_{j \in N} m_j$ are then allocated in proportion to the remaining benefits, assuming that all remaining benefits r_j are nonnegative for each player (Dinar and Hogarth 2015).

9.5.6.1 Game Theory Cost Allocation Solutions

Given the initial conditions of voluntary collective action, and the prior establishment of independent resource management institutions among the users (a region, river basin, etc.), the problem of allocating the joint costs of a joint water facility (joint well, treatment facility, reservoirs, hydropower generation) is modeled as a game among the players. Based on the empirical situation, it can be assumed that institutional regulations facing the players are already in place and that the players agree to consider them. If a player chooses not to cooperate (not to participate in the investment and the operation of a joint facility), it faces a certain outcome resulting from the operation of a private facility or alternative measures needed to meet the regulations. If the players choose to cooperate, they may benefit from economies of scale embodied in the larger capacity of the joint facility with lower average treatment costs compared to the cost in private actions. Some players may cooperate while others may choose not to cooperate, depending on the degree to which they can reduce their cost under cooperation. As a result, the larger the economies of scale, the bigger the incentive for cooperation. the following is based (Martin Shubik 1982a, b, c; Shubik 1982a; Shapley 1952). Let N be the set of all players in the region, S (S $\subseteq$ N), the set of all feasible coalitions in the game, and s (s ∈ S) a feasible coalition in the game. The non-cooperative coalitions are {j}, j = 1, 2, …, n, and the grand coalition is {N}.

Assuming that the players' objective is to minimize their cost, let f^s be the cost of coalition s, and $f^{\{j\}}$ be the cost of the *j*-th member in non-cooperation. A necessary condition for *regional cooperation* is that the joint cost will be less than the sum of the individual costs:

$$f^s \leq \sum_{j \in S} f^{\{j\}}, \forall s \in S \subseteq \mathrm{N}.$$

The *joint savings* that are allocated among the players are defined simply by

$$\sum_{j \in S} f^{\{j\}} - f^{\{s\}} \geq 0, \forall s \in S \subseteq \mathrm{N}.$$

The above inequality can be interpreted as a cooperative game, with side payments, and can be described in terms of a characteristic function. The value of a characteristic function for any coalition expresses the coalition expenses, or profit, in the case of a benefit game:

$$\vartheta(s) = f^s, \forall_s \in s \subseteq N :$$

for details, see (Owen, n.d.).

We will consider four GT allocation schemes that have been widely used in water resources: the Core, the Shapley Value, the Nucleolus, and the Nash–Harsanyi allocation schemes (Dinar and Hogarth 2015).

9.5.6.2 The Core Allocation Scheme

The Core of an *n* player-cooperative game in the characteristic function form is a set of game allocation increasing that is not dominated by any other allocation set. The Core game theory (CGT) provides a locus for the maximum (or minimum in terms of cost) allocation each player may request. In this respect, it is an overall solution for several allocation schemes that are contained within the Core. The CGT scheme fulfills requirements for individual and group rationality, and for joint efficiency (Martin. Shubik 1982b).

CGT scheme is conducted under the assumption that the players in the game are economically rational. This means that the decision of each player to join a given coalition is voluntary, and it is based on the minimal cost they bear by joining that coalition.[1] Let ω_j be the *j*-th Core player allocation for the cost from the game. In case of a cost allocation game, the CGT equations can be defined as:

[1]In a benefit game it is the incremental benefit that such players gain.

$$\omega_i \leq \vartheta(\{j\}), \forall_j \in N,$$

$$\sum_{j \in s} \omega_i \leq \vartheta(\{s\}), \forall_s \in S,$$

$$\sum_{j \in N} \omega_i = \vartheta(N).$$

The first inequality in the CGT resource allocation scheme fulfills the conditions for individual rationality, i.e., the cooperative solution for each player is preferred to the non-cooperation case. The second inequality fulfills the group rationality conditions, meaning that the cooperative allocation to any combination of players is preferred regarding any allocation in any sub-coalition the player could establish. The third inequality fulfills the efficiency condition, which is the joint cost to be fully covered by the grand coalition participants. The system of these three inequalities has more than one allocation solution. A method of calculating the extreme points of the Core (Shapley 1971) provides the incremental contributions of each player when joining any existing coalition, and assigns these contributions to that player. Thus, having a non-empty Core allocation for a cooperative game provides the necessary condition for a solution that will be acceptable to the players (Dinar and Hogarth 2015).

9.5.6.3 The Nucleolus Allocation Scheme

The Nucleolus (Schmeidler 1969a) is a single point solution that always exists (if the Core is non-empty) and minimizes the dissatisfaction of the most dissatisfied coalition. To obtain the Nucleolus, we define the ε-core of the game to be the set of allocations that would be in the Core if each coalition given a subsidy at the same level of ε. By varying ε one finds the smallest non-empty ε-core, namely the *least* Core. The least Core is the intersection of all ε-cores. The least Core for a cost allocation game satisfies:

$$\text{minimize } \varepsilon$$

$$s.t. \sum_{j \in N} \omega_j \leq v(s) + \varepsilon, \forall s \subseteq S$$

$$\sum_{j \in N} \omega_j = v(N),$$

$$\varepsilon \lesseqgtr 0.$$

The solution to the minimization problem above may provide the Nucleolus (as a single solution) but it may also provide several individual cost allocations ω_j for the same value of ε for each coalition s. In this case, we define the *excess function* e(ε, s) for each s, that measures how much less a coalition costs to act alone, and in a lexicographical process (Schmeidler 1969b) obtain the Nucleolus, for which the value of the smallest excess e(ε, s) is as large as possible. The interpretation of ε is interesting. It can be used as a tax or a subsidy to change the size of the Core. If the Core is empty, then e ($\varepsilon < 0$) is an "organizational fee" for the players in sub-coalitions, causing them to prefer the grand coalition. If the Core is too big, ε might reduce it ($\varepsilon > 0$) by subsidizing sub-coalitions. The Nucleolus is always in the Core if it exists (Dinar and Hogarth 2015).

9.5.6.4 The Shapley Value Allocation Scheme

The Shapley Value (Shapley 1952) resource allocation scheme allocates θ_j to each player based on the weighted average of their contributions to all possible coalitions and sequences. In the calculation of the Shapley Value, an equal probability is assigned to the formation of any coalition of the same size, assuming all possible sequences of formation. The Shapley value can be calculated as (Dinar and Hogarth 2015):

$$\theta_j = \sum_{s \subseteq S j \in s} \frac{(n - |s|)!(|s| - 1)!}{n!}[v(s) - v(s - \{j\})], \forall j \in N,$$

where n is the number of players in the game, $|s|$ is the number of members in coalition s, i.e., the cardinality of the subset s; the function $v(.)$ is a characteristic function. It mean that, if s is a coalition of players, then $v(s)$ representing the worth of coalition s, describes the total expected sum of payoffs of the members of S; indeed, the payoffs maximization can be obtained by cooperation (Dinar and Hogarth 2015).

9.5.6.5 The Nash–Harsanyi (N–H) Allocation Scheme

The N–H Solution (Harsanyi 1958; Dinar and Hogarth 2015) for an n-person bargaining game is a modification of the 2-player Nash Solution (Nash 1953). This solution concept maximizes the product of the grand coalition members' additional utilities (income or savings) from cooperation compared to the non-cooperation case, subject to Core conditions, by equating the utility gains of all players. The N–H solution satisfies the Nash axioms (Nash 1953); it is unique and it is contained in the Core (if it exists). The solution might provide unfair allocations if there are big utility differences between the players, e.g., very rich player and very poor player.

The N–H solution for the j-th player, h_j, is calculated as:

$$\text{maximize} \prod_{j \in N} \left(f^{j} - h_{j} \right)$$

$$s.t. h_{j} = f^{N}, \forall_{j} \subseteq N,$$

$$\sum_{j \epsilon s} h_{j} \leq f^{s}, \forall_{s} \subseteq S,$$

$$\sum_{j \epsilon N} h_{j} = f^{N}$$

where h_j is the N–H allocation that satisfies efficiency and individual rationality conditions.

The fulfillment of the Core conditions for an allocation scheme is a necessary condition for its acceptability by the players. Thus, solutions not included in the Core are also not stable. Although an allocation scheme may fulfill the Core requirements for the regional game, it still may not be accepted by some players that might view it as relatively unfair compared to another allocation. Allocations that are viewed as unfair by some players are less stable. Some players might threaten to leave the grand coalition and form sub-coalitions because of their critical situation in the grand coalition. The consistency of any solution is essential given the existence of constants investments, and a more fix solution might be preferred even if it is harder to perform. We do not discuss coalitional stability here. The reader is referred for more reading to accessing (Shapley and Shubik 1954) and (Loehman et al. 1979), who used a measure of power in voting games. This power index is also used in (Williams 1988). Another measure of stability was introduced in (Dermot Gately 1974b) as the "propensity to disrupt" the grand coalition and was modified and applied considering N > 3 by (Straffin and Heaney 1981) to the case of the Tennessee Valley (Dinar and Hogarth 2015).

9.5.7 A Strategy for Water Resources Management Using Game Theory

For water resources management using GT approach on a river basin scale, mainly includes three important steps (Fig. 9.3), as follows:

(1) First important step is to decompose the river system and define the conflicting areas and/or bodies. After the players are defined, their moves (or action) and strategies, as well as their information set, and their payoff function can be defined.
(2) In the second step is defined how each player optimizes water quantity in order to maximize his payoff. Rather, this step includes the socio-macroeconomic

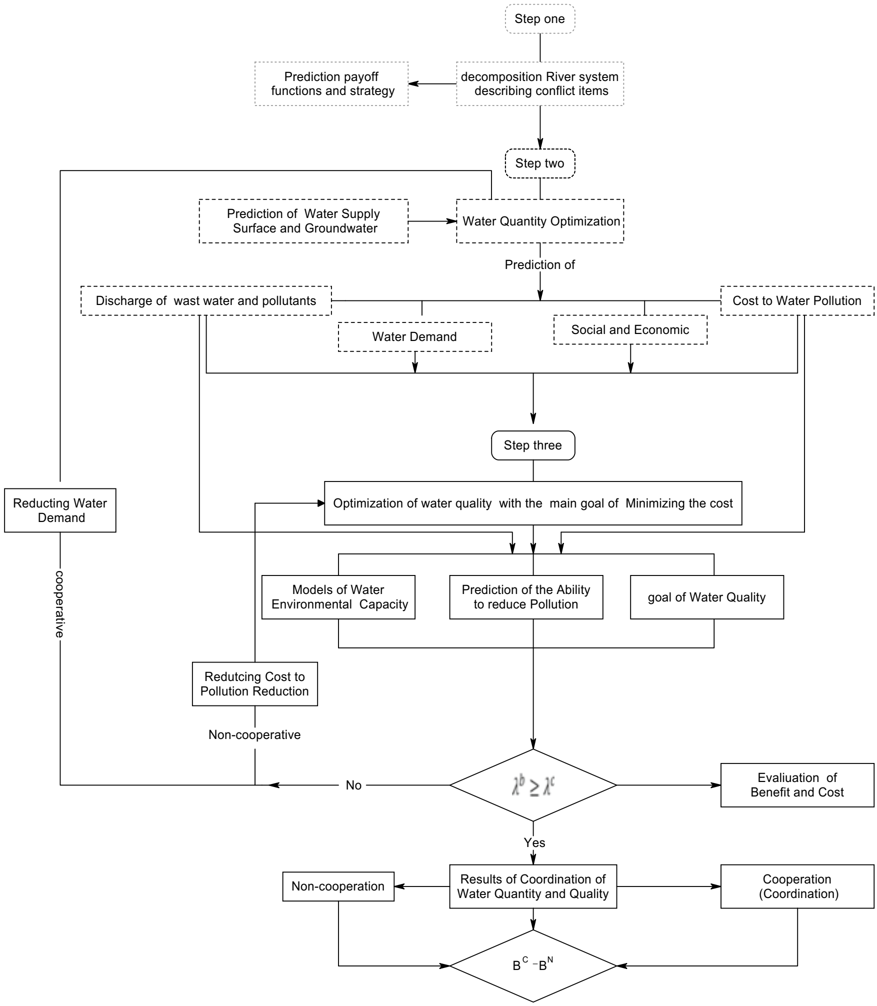

Fig. 9.3 A strategy for water resources management using game theory (Wei 2008)

predictions,[2] water supply and water demand predictions for different players, wasting water and pollutants predictions discarded by different players, as well as the cost of each player investment to treat his sewage. Step 2 is the benefiting process in which each player usually maximizes the output values per unit water (Wei 2008).

(3) Third step is to optimize water quality so that every player can maximize his payoff. This step consists of setting up models of pollutant capability in different

[2]Such as population, GDP, output values of agriculture and industry and the net incomes of household.

river sections, predicting each player's ability to reduce wasting water discharge and treat water pollution, while setting a target for water quality or water quality standard. In this step each player decides if they impose cost to reduce waste. The rational players will make planning by calculating the benefits and costs. From an economic point of view, waste production or pollution is public good or bad. In the non-cooperative situation, each player usually cut the waste treatment cost because he can freeride on other players' achievement of waste reduction. If all the players choose the strategies of free riding, equilibrium of prisoner dilemma will be reached. In the cooperative situation, the players will maximize their welfare by efficient water use (Wei 2008).

9.6 Practical Examples

9.6.1 Cooperative Water Allocation: A Cooperative GT Approach

Water allocation is essentially a practice in allocating available water to different demanding users. Water allocations merely based on a water rights approach, always do not make efficient use of water for the whole river basin. Meanwhile, an economy. Efficient water allocation plan cannot be well implemented if the involved participants or stakeholders do not regard it as being fair. in the study which is done by (Wang 2003), an equitable and efficient cooperative allocation approach had been proposed to solve water allocation problems in two steps. Water rights are initially allocated to water stakeholders and users based on existing water rights systems or agreements, and then the water had been reallocated to achieve efficient use of water through water transfers. The associated net benefit reallocation had been carried out by the application of cooperative game theory. The integrated cooperative water allocation modeling approach had been designed to promote and guide equitable water transfers and cooperation of relevant stakeholders to achieve optimally economic and environmental values of water, subject to hydrological and other constraints. A cooperative game theoretic approach which is proposed to solve water allocation problems is in two steps (Wang 2003):

(1) **initial water rights allocation** to water consumer or stakeholders based on existing water rights systems or agreements
(2) **reallocation of water to achieve** efficient consumption of water by water transfers.

A rational example is utilized to show the effectiveness and potential benefit of this approach.

9.6.1.1 Initial Water Rights Allocation

Generalized transboundary water allocation principles for sharing the water resources of international river basins between countries include:

(1) absolute sovereignty,
(2) absolute riverine integrity,
(3) limited territorial sovereignty
(4) economic criteria (Dinar and Wolf 1994; Wang 2003).

The seemingly fair and simple principles or guidelines of reasonable and equitable use are difficult to be applied in practice, especially for an inter-country river basin. Measurable criteria and models for water allocation need to be constructed and used to achieve fair apportionment of water (Seyam et al. 2000; Van Der Zaag et al. 2002).

In a water allocation problem, resource users have heterogeneities arising from physical resource characteristics, users' technologies and skill levels, and institutional arrangements. An institution can cause heterogeneities in pricing, property rights and political power (Schlager and Resource, n.d.).The water rights are allocated according to a legal intra-country water rights system and water policies or inter-country agreements before moving to the second gaming stage of the cooperative water allocation model (Wang 2003).

9.6.1.2 Cooperative Water Allocation Game

Recall that $N = \{1, 2, \ldots, n\}$ is the set of water stakeholders or *players* competing for water allocations in the concerned river basin or sub-watershed, and iN a typical stakeholder. A group of stakeholders SN entering a cooperative agreement and working together is called a coalition. N itself is called the grand coalition, the coalition consisting of all stakeholders (Wang 2003).

A coalition structure is a partition $\pi = \{s_1, s_2, \ldots, s_m\}$ of the n stakeholders, in which $\cup_i^m s_i = N$ and for all i $\neq$ j, $S_i - S_j = \emptyset$. For a game with n players, 2^n coalitions are possible, or $2^n - 1$ if the null coalition is excluded. The expression $v(S)$ is used to represent the aggregate payoff to the members of coalition S, while the *payoffs* to individual stakeholders acting in isolation are represented as $v(\{1\}), v(\{2\}), \ldots, v(\{n\})$. In a cooperative water allocation game, the generic notations of *payoffs* $v(\{i\})$ and $v(S)$ are interpreted specifically as the net benefits by the following definitions. The payoff $v(\{i\})$ of a stakeholder i is the maximum total net benefit $NB(i)$ that stakeholder i can gain based on its water rights over the entire planning period, subject to not decreasing the water flows and not increasing the pollutant concentrations in the flows to other stakeholders. Thus, the *payoff* $v(\{i\})$ is normally greater than the total net benefit $NB(i)$ gained with the initial water rights since there is additional value for the internal cooperation among the uses and users within stakeholder i (Wang 2003). Thus, the *payoff* $v(\{i\})$ optimization problem can be formulated as the maximization problem of the *total net benefit* (NB) subject to the water balance and hydrological constraints:

$$\vartheta(\{i\}) = \text{maximize} NB(i) = \sum_{t \in T} NB_{i,t} = \sum_{t \in T} \sum_{j \in U_i} NB_{i,j,t}$$

$$\text{s.t.} Q(k, j, t) \geq Q_R(K, j, t), \forall k \in V, \forall j \in U \; and \; j \notin U_i$$

$$S(j, t) \geq S_R(j, t), \forall k \in V, \forall j \in RES \; and \; j \notin U_i$$

$$\mathrm{C}_p(k, j, t) \geq C_{PR}(K, j, t), \forall k \in V, \forall j \in U \; and \; j \notin U_i$$

$$\mathrm{C}_p(j, t) \geq C_{PR}(j, t), \forall k \in V, \forall j \in RES \; and \; j \notin U_i$$

where, RES is the set of reservoirs.

Moreover, the payoff $v(S)$ of a coalition S is the maximum total net benefit $NB(S)$ that coalition S can gain based on coalition members' water rights over the entire planning period, subject to not decreasing the water flows and not increasing the pollutant concentrations in the flows to other stakeholders not taking part in coalition S (Wang 2003). This total net benefit maximization from coalition S subject to the water balance and hydrological constraints can be formulated as:

$$\vartheta(S) = \text{maximize NB(S)} = \sum_{i \in T} \sum_{i \in s} NB_{i,t} = \sum_{t \in T} \sum_{i \in s} \sum_{j \in U_i} NB_{i,j,t}$$

$$\text{s.t.} Q(k, j, t) \geq Q_R(K, j, t), \forall k \in V, \forall j \in U \; and \; j \notin U_s$$

$$S(j, t) \geq S_R(j, t), \forall k \in V, \forall j \in RES \; and \; j \notin U_s$$

$$\mathrm{C}_p(k, j, t) \geq C_{PR}(K, j, t), \forall k \in V, \forall j \in U \; and \; j \notin U_{is}$$

$$\mathrm{C}_p(j, t) \geq C_{PR}(j, t), \forall k \in V, \forall j \in RES \; and \; j \notin U_{is}$$

where, $U_S = \underset{i \in S}{-} U_i$, and $NB_{i,j,t}$ is the net benefit function of stakeholder i' s water demand node j during time step t, given by:

$$NB_{i,j,t} = f_{i,j,t} Q(k_1, j, t), C_P(k_1, j, t), S(j, t), C_P(j, t), \\ Q(j, k_2, t), C(j, k_2, t), \; (k_1, j) \in A, (j, k_2)$$

The net benefit function and cost function for demand node j is determined by:

$$f_{i,j,t}(.) = B_{i.j,t}(.) - C_{i.j,t}(.),$$

where $B_{i.j,t}(.)$, and $C_{i.j,t}(.)$ is the benefit function and cost function for demand node j, respectively.

Notice that $f_{i,j,t}(.)$ can be estimated from historical data statistics and simulation or obtained through optimization with control variables such as use type, area, user's technology and skill level, price, and other economic and policy factors. Note that in the latter case, $Q(k_1, j, t), C_P(k_1, j, t), S(j, t), C_P(j, t), Q(j, k_2, t)$, and $C(j, k_2, t)$ are the control variables deployed in searching for $v(S)$, as well as parameters in searching for the optimal value of $f_{i,j,t}(.)$ (Wang 2003).

A "*solution*" to a game is a vector of the payoffs received by each stakeholder. This payoff or reward vector after a trade can be written as $x = \{x_1, x_2, \ldots, x_n\}$. This trade process to achieve a cooperative water allocation under certain water balance and hydrological constraints is essentially a cooperative water allocation game. The payoff vector is called an imputation to the cooperative game, and meets the conditions of individual rationality, group rationality and joint efficiency (Young et al. 1982; Tisdell and Harrison 1992; Wang 2003), i.e.:

Individual rationality: $x_i \geq \upsilon(\{i\})$

Group rationality: $\sum_{i \in S} x_i \geq \vartheta(S)$

Joint efficiency: $\sum_{i \in N} x_i = \vartheta(N)$

$Let\ x(S) = \sum_{i \in S} x_i$, then the above three conditions can be reduced to:

Individual and group rationality: $x(S)(S)$, for all $S \subset N$

Joint efficiency: $x(N) = \upsilon(N)$

The set of reward payoff vectors that satisfy the conditions of individual rationality, group rationality and joint efficiency forms the *core of a cooperative game*. The core of a cooperative game may not always exist. If it exists, there is no guarantee that it has a unique feasible solution. Core-based and non-core-based resource allocation concepts may be applied to reduce it to a unique one (Dinar et al. 1986). Nucleolus and related solutions are listed in Table 9.1. The nucleolus minimizes the maximum excess $e(S, X) = v(S) - x(S)$ of any coalition S lexicographically (Schmeidler 1969a; Wang 2003):

Table 9.1 Nucleolus and related solutions (Wang 2003)

Solution concepts	Net benefit excess	Individual and group rationalities
Nucleolus	$e = v(S) - x(S)$	$min\ e$ $Subject\ to$ $x(S) + e \geq e = v(S)\ for\ all\ S \subset N$
Nucleolus	$e_w = {(v(S) - x(S))}/{\lvert S \rvert}$	$min\ e_w$ $Subject\ to$ $x(S) + e_w \lvert S \rvert \geq v(S)\ for\ all\ S \subset N$
Proportion nucleolus	$e_P = {(v(S) - x(S))}/{v(S)}$	$min\ e_P$ $Subject\ to$ $x(S) + e_P v(S) \geq v(S)\ for\ all\ S \subset N$
Normalized nucleolus	$e_n = {(v(S) - x(S))}/{x(S)}$	$min\ e_n$ $Subject\ to$ $x(S) + (1 + e_n) \geq v(S)\ for\ all\ S \subset N$

$$\text{minimize } \upsilon(S) - \sum\nolimits_{i \in S} x_i$$

$$\text{s.t.} : x(S) + e \geq \upsilon(S) \text{ for all } S \subset N$$

$$x(N) = \upsilon(S)$$

Application of this optimizing algorithm narrows the core solution space. Successive applications of the algorithm involve setting aside coalitions for which $e(S, X)$ equals the *critical excess* e_{crit} value found at each step and running the optimization program for remaining coalitions. Each iteration further constrains the solution space until a unique point is ultimately reached. The excess e can be interpreted as *subsidies* ($e \geq 0$) or *tax* ($e < 0$) to the water stakeholders. The weak nucleolus concept (Young et al. 1982) replaces the excess e by the *average excess* e_{avg}.

Proportional nucleolus (Young et al. 1982) replaces excess e by the ratio of excess to net benefit of coalition S; while the normalized nucleolus replaces excess e with the ratio of excess to imputation of coalition S. The nucleolus and related variation approaches can reduce or expand the core to obtain a unique solution in both cases of large core and empty core (Dinar et al. 1986). Applying the Shapley value solution concept, each stakeholder's reward or value to the game should be equal a weighted average of the contributions the stakeholder makes to each coalition of which he or she is a member. The weighting depends on the number of total stakeholders and the number of stakeholders in each coalition. The Shapley value gives the *payoff* to the i-th stakeholder such that (Shapley 1971; Wang 2003):

$$\mathrm{X}_i = \sum_{\substack{S \subseteq N \\ i \in s}} \frac{((|S| - 1)!(|N| - |S|)!}{|N|!} [\mathrm{V}(S) - \mathrm{V(S)} - \{i\}]$$

$$= \sum_{\mathrm{s}=1}^{\mathrm{n}} \frac{((\mathrm{s}-1)!(\mathrm{n}-\mathrm{s})!)}{\mathrm{n}!} [\mathrm{v(s)} - \mathrm{v(s} - \{i\}], \text{ for all } i \in N$$

where $|S|$ is the cardinality of coalition S.

By using the above description, a two-step cooperative water allocation approach can be formulated, which consists of an initial water rights allocation and a cooperative water reallocation game. Water rights are initially allocated based on existing water rights systems or agreements, while the cooperative water reallocation game is formulated by using net benefits as a stakeholder's payoff. The cooperative water reallocation game can be solved by solution concepts such as the nucleolus, weak nucleolus, proportional nucleolus, normalized nucleolus and Shapley value. Since the model performs initial water rights allocation and subsequent reallocation based on existing water rights systems or agreements, and it utilizes the node-link river basin network, water balance and hydrological constraints, with a time step length of Δt during a planning period, the model realistically takes into account knowledge and sub-models from hydrology, economics and cooperative GT. This makes it possible to reach fair and efficient water allocation among competing uses with multiple stakeholders in an operational way. The methodology can be applied to an entire river basin or a sub-watershed (Wang 2003).

9.6.2 Water Costs Allocation in Complex Systems Using a Cooperative GT

Sechi et al. (2013) present a methodology to allocate water service charges in a water resource system between several users that attempts to fulfill the WFD requirements. The method is according to Cooperative Game Theory (CGT) techniques, while the related characteristic function definition deploys a mathematical optimization approach. The CGT provides the facilities that are essential to analyze condition that needs a cost-sharing rule. The CGT approach can describe efficient and fair solutions that supply the appropriate incentives between the parties involved. So, the water system value allocation has been costed as a game in which it is essential to determine the right payoff for each player, in this case water costumers. To use the CGT principles in a water resources system, the specific function needs to be defined and evaluated using enough modeling approach; in this study, it is evaluated using the WARGI optimization model. The so-called "core" represents the game-solution set. It represents the area of the possible cost allocation values from which the borders on the cost values for each player can be provided. Within the core lie, all of the allocations must satisfy the principles of equity, fairness, justice, performance, and that guarantee cost recovery. The core of a cooperative game as a reliable support in water resource management to attain the economic analysis required with WFD. This methodology was applied to a multi-reservoir and multi-demand water system in Sardinia, Italy (Sechi et al. 2013).

9.6.2.1 Cost Allocation Problem and Cooperative Game Theory

The CGT belongs to the Game Theory (GT) scientific area developed in the first half of the last century (Von Neumann and Morgenstern 1953). In GT, conflict situations are analyzed and competitive and/or cooperative solutions among participants sought. In the literature, many cost allocation problems have been analyzed using CGT principles; however, the approaches vary significantly in the different research fields. The CGT principles have also been applied in studies related to water resources (Authority 1938; Young et al. 1980a; Lippai and Heaney 2000; Andreu et al. 2009). One of the most important aspects of the methodology is the definition of the game's characteristic function (CF), which is the focus of this study. To define the CF of the cooperative game for a water system the following definitions, which were described extensively by (Barile and Stoner 1994) are required. $N = (1, 2, .., n)$ refers to a set of players that are participating in the game. Each subset $S \subseteq N$ is defined as a "coalition", while the Grand Coalition (GC) occurs when $S = N$. The players can represent real subjects, such as the users of a water system, or members of a more abstract set, such as the sector of a company, or different planning alternatives that can be realized together or separately. The *stand-alone cost*, given as $c(i)$, represents the cost that is connected to the i-th user when the user is considered independent of the other players. The cost linked to the coalition S, i.e., the cost commonly sustained by all of the users that belong to S is represented as $c(S)$. Besides, the cost associated with the GC, i.e., the common cost sustained by all participants of the game (all users in the water system) is represented as $c(N)$. Finally, the cost linked to an empty coalition is zero by convention,($c(\Phi) = 0$ (Sechi et al. 2013).

As defined previously, an allocation is a vector $x = [x_1, x_2, \ldots x_n]$, where x_i is the amount charged to the *i*-th player. The cost associated with a generic coalition, which can be formed by either a player, a partial coalition, or even all participants of the game, must represent the lowest cost of serving the coalition in the most efficient way, i.e., the minimum cost necessary to satisfy all of the players in such coalition. Moreover, the discrete function that is formed by the costs of every coalition is called the *Characteristic Function* (CF), which is the key element setting a cooperative game (Sechi et al. 2013).

If for every pair of disjoint coalitions *S*' and *S*",

$$c\left(S' \cup S''\right) \leq c\left(S'\right) + c\left(S''\right)$$

then, the CF and the related game are sub-additives. In this case, the players cooperate because the unions of the two groups of players will determine a cost that is lower than the sum of the autonomous costs. Because a game with a sub-additive CF will be characterized by economies of scale, the GC should be the most efficient alternative. This is the case when it is economically more convenient to realise a common project than independent projects. For players that cooperate and collectively accomplish a project, the principle of stand-alone cost test, commonly called the *rationality principle*, must be guaranteed (Dinar and Hogarth 2015). However, it can be extended to each player and thus, it is also referred to as *individual rationality*. The principle of *individual rationality* can refer to an individual player or to a coalition, satisfying (Sechi et al. 2013):

$$x_i \leq c(i) \forall i \in N$$

$$\sum_{i \in s} x_i \leq c(S) \forall \mathrm{S} \subseteq N$$

where x_i is the amount of total game cost that is assigned to a given player. According to this principle, no player or group of players that forms a coalition would accept a cost assignment that is higher than the cost that it/they would sustain when participating autonomously, i.e., greater than its/their own opportunity cost.

Another principle is the so-called *marginality principle* or incremental cost test. In general, the incremental cost or the marginal cost of a coalition S is defined as $c_m = c(N) - c(N - S)$. According to the marginality principle, the following inequality:

$$\sum_{i \in s} x_i \geq c(N) - c(N - S) \forall \mathrm{S} \subseteq N$$

must be verified (Sechi et al. 2013). Each player or set of players will have to sustain at least its/their own marginal cost c_m when joining a coalition. Otherwise, the coalition of pre-existing players will be inefficient because it has to finance the entry of the new player or set of players. The *rationality principle* produces an incentive for the players to cooperate voluntarily, while the *marginality principle* supplies the equity conditions in such game (Peter et al. 1994).

Furthermore, the cooperative-based games support the solutions that include all players, and therefore, the majority of the CGT solving methods are able to divide completely the cost among all of the game participants. Taking into account these aspects, a generic solution is defined by a vector $x = [x_1, x_2, \ldots, x_n]$, such that (Sechi et al. 2013):

$$\sum_{i \in N} x_i = c(N)$$

where x_i is the payoff assigned to the ith player. This generic solution satisfies the *efficiency principle;* besides, in such configuration, the *marginality* and *rationality* principles are equivalent. In terms of *individual rationality*, the amount i saves by cooperating rather than going alone is given by $v_i = c(i) - x_i$. As a result, one can define the *group rationality* saving:

$$v(S) = c(S) - \sum_{i \in s} x_i \forall S \subseteq N$$

The main application problems come out evaluating the *characteristic function*: each potential coalition must be defined and assessed. Therefore, the number of players affects the complexity of the problem; hence, for n players, there are $2^n - 1$ coalitions that must be analyzed (Sechi et al. 2013).

The *game solutions* can be grouped into two branches:

set-theoretical solutions: identify a set of vectors that shares the value of the game among all players, as the core;

point solutions: define only one division and are more similar to the classic idea of a unique problem solution, as the Shapley value or the so-called nucleolus (Schmeidler 1969; Sechi et al. 2013).

The adopted game solution should guarantee an acceptable cost allocation considering the particular characteristics belonging to the water system and requirements given by the water authorities and other decision makers. In (Sechi et al. 2013), the core of the solutions is used under de assumption that the core is a closed, compact, convex subset in R^N. Unfortunately, it may be empty, even if c is sub-additive. Moreover, inside the core, there are several cost allocations, which respect the efficiency and equity principles while incentivize cooperation among the players. Consequently, the decision maker is provided with an admissible, potentially easily acceptable range of alternatives for defining water rates. These aspects will be examined in the following revisited case study (Sechi et al. 2013).

9.6.2.2 Water Cost Allocation Methodology

The CGT approach is particularly appropriate for water services, in which it is important to define the agreements, encourage cooperation among the stakeholders and achieve more efficient solutions by determining a fair cost allocation. The proposed methodology, which uses CGT techniques to allocate the costs in a complex water resources system, consists of the following steps, further detailed in (Andreu et al. 2009; Andreu et al. 2009; Sechi et al. 2013):

(1) *Water system analysis*: functional definition of the water system and evaluation of its different aspects, e.g., hydrologic, hydraulic, infrastructural and economic;
(2) *Cooperative game definition*: identification of players and coalitions, analysis of priorities and so on;
(3) *Characteristic function* (CF) *evaluation*: set up of the optimization model, calculation of the minimum cost associated with each coalition potential solution, and game's CF evaluation;
(4) *Game solution*: application of the CGT methods necessary to share the costs among the players.

First step: the hydrologic, hydraulic and infrastructural aspects of the description and the characterization of the water system must be identified in this step. During this phase, the costs that characterize the water system, which are to be shared among the water users must be defined. In fact, the majority of European water systems are almost entirely equipped and new important works are rarely expected. Therefore, it is mainly the management costs of the existing infrastructure that need to be allocated. In this case, the ordinary and supplementary maintenance costs, the adaptation and substitution infrastructure costs and the energy costs of the pumping stations need to be considered (Sechi et al. 2013).

Second step: identifies the players and which cooperative game is set up. The players can represent individual users, sets of users or more abstract memberships, such as sectors of water services, agricultural associations and city services. In the following real case application, the elements of the players' set consist of users belonging to a unique macro-demand having the same interests and priorities of irrigational, industrial and urban municipal demands.

Third step: the CF of the game is defined. According to its definition, the CF consists of a set of minimum costs associated with all of the possible coalitions. The need to value each coalition's minimum costs is a key feature of CGT. Even if the costs of GC are to be shared, each coalition needs to be valued in order to estimate the parameters for efficient cost sharing among the players. Nevertheless, in CGT applications, the coalition's minimum cost is defined as the sum of the management costs in the "minimum" infrastructures set necessary to completely satisfy the water request of the players included in that coalition. This modelling method has significant differences from that described by Deidda et al. (2009). Actually, the approach considered herein specifically refers to water system management and the CF evaluation is reached using the optimization modelling tool WARGI-DSS. The WARGI. tool allows apply Linear (LP) and Quadratic (QP) Programming models, to obtain the optimum system infrastructures set definition and the optimum system performance to be achieved for each coalition. Indeed, the optimization model can be easily built using the WARGI graphical interface and solved using Cplex optimizer tool. Depending on the system infrastructure, on the system sources, as well as on the demand characterization, the number and the typology of the potential infrastructures should be varied for each coalition game. Consequently, we can evaluate the management costs of the entire system referring to the optimal flows assessment given by the WARGI optimization tool and the solution values that are associated to a specific coalition solution. In this manner, the *least cost* (optimal system assessment) of each coalition can be defined, and the CF of the cooperative game can be established (Sechi et al. 2013).

Last Phase: In the game solution, the water system costs are allocated among the players using CGT techniques. In this way, CGT gives an admissible range providing an easily acceptable tool to the decision maker for defining water costs allocation (Sechi et al. 2013).

9.6.2.3 Result

The core solutions-set of the game is graphically represented in Fig. 9.4. In the triangle, the heights are proportional to the cost of the Grand Coalition and each internal point represents a possible cost allocation between the macro-users defined as players in the game. Every side represents a player and the distance between the side and the point inside the triangle provides the cost amount that is assigned to the player. The barycenter is the point at which the costs are equally shared, whereas the vertices correspond to the situation in which the total cost is assigned to one user. The dashed lines represent the maximum and the minimum costs that are sustainable by each player according to below equation and the painted area represents the core solution of the game. The analytical formulation for the core deploying GC can be defined by the following cost boundaries:

$$civil + Irrigation + Industrial = 293.36$$

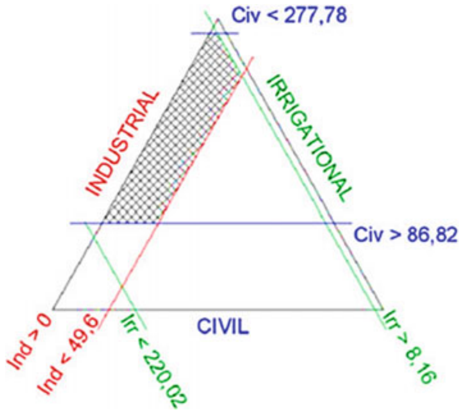

Fig. 9.4 Core of the game (Sechi et al. 2013)

$$86.82 \leq Civil \leq 277.78$$

$$8.16 \leq Irrigation \leq 220.02$$

$$0.00 \leq Industrial \leq 49.60$$

Each cost allocation that verifies the above boundaries also satisfies the rationality and marginality principles and guarantees a total cost recovery (Sechi et al. 2013).

The results presented in (Sechi et al. 2013) demonstrated that the evaluation of the CGT core of solutions represents the *set of admissible cost allocation* and supplies the boundary values for each player. Inside the core, each allocation satisfies the *marginality* and *rationality* principles; hence, the stakeholders should recognize equity and fairness. Moreover, the total cost recovery can be realized (Sechi et al. 2013).

9.6.3 GT Application for Groundwater Conflicts Resolution

In another study carried out by, game theory was applied to a multi-objective conflict problem for the Alto Rio Lerma Irrigation District, located in the state of Guanajuato, in Mexico, where economic benefits from agricultural production should be balanced with associated negative environmental impacts. The short period of rainfall in this area, combined with high groundwater withdrawals from irrigation wells, has produced severe aquifer overdraft. In addition, current agricultural practices of applying high loads of fertilizers and pesticides have contaminated regions of the aquifer. The net economic benefit to this agricultural region in the short-term lies with increasing crop yields, which require large pumping extractions for irrigation, as well as high chemical loading. In the longer term, this can produce economic loss due to higher pumping costs, i.e., higher lift requirements, or even loss of the aquifer as a viable source of water. Negative environmental impacts include continued diminishment of groundwater quality, and declining groundwater levels in the basin, which can damage surface water systems that support environmental habitats. The two primary stakeholders or players, the farmers in the irrigation district and the community at large, must find an optimal balance between positive economic benefits and negative environmental impacts. In GT was applied to find the optimal solution between the two conflicting objectives among twelve alternative groundwater extraction scenarios. Different attributes were used to quantify the benefits and costs of the two objectives; hence, following the Pareto frontier generation (trade-off curve), four conflict resolution methods have been identified and applied accordingly.

Step 1: water management problem

Modeling real water management problems, possible groundwater extraction scenarios have been proposed. The environmental and economic impacts of each groundwater scenario were measured by using the Groundwater Lading Effects of Agricultural Management Systems (GLEAMS), and then the identified water resource optimization problems solved by linear programming. Finally, conflict resolution methodology is applied to identify compromise solutions, which balances the economic and environmental concerns of the region.

Step 2: attributes estimation

Different groundwater extraction scenarios were proposed. For each groundwater extraction scenario, we have conflicting economic and environmental objectives. The economic attributes are the net income generated in the linear program and the pumping cost described below. The environmental attributes include nutrients and pesticides associated with irrigation runoff and percolation and a measure of groundwater depletion. Figure 9.5 shows the general hierarchy of the criteria. Table 9.2

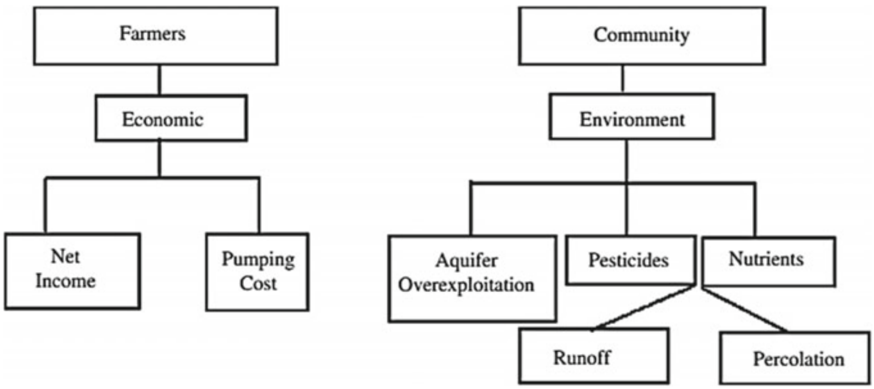

Fig. 9.5 Conflict resolution scheme

Table 9.2 List of attributes

Attribute	Estimation procedure
Economics Net benefits (106 $)	Generated by the linear program for each alternative
Pumping cost (106 $)	Calculated separately using data from ARLID and subtracted as production cost
Environmental Nitrogen in runoff (103 kg) Nitrate in percolation (10 crop 3 kg) Pesticides in runoff (103 g) Pesticides in percolation (103 g)	Output from GLEAMS for each crop
Aquifer overexploitation	Evaluated for each groundwater supply

presents a list of the alternatives and attributes, and show their estimation procedure, respectively.

Net Income: The farmers' net benefit for each groundwater extraction scenario was estimated by assuming that the farmers utilized crops selection optimally according to market prices and water availability. This can be formulated as a linear programming problem, where the total net benefit maximization can be defined as:

$$\text{maximize } NB = \sum_{j=1}^{n}\left[Y_j P_{cj} - C_j\right]A_{cj}$$

where the NB is defined as $\left(profit{-}cost\$\right)$, the A_{cj} holds for the *j*-th crop area (ha), Y_j is the yield of the *j*-th crop j (ton/ha), P_{cj} the price of crop j($/ton), the term C_j represents the production cost of crop j ($/ha), which also include pumping cost, and n is the number of crops. Regarding the constraints of the aforementioned net benefit maximization linear programming.

Pumping cost: The average pumping cost in month *k* is given as Cp Qk, where:

$$C_P = \left[\frac{h \times \text{a}}{E_p}\right]C_e + C_{r,}$$

where a is the energy required to lift 1 m^3 of water to 1 m height (kWh/m^4), the variable h is the total head (m), C_e represents the average annual energy cost ($/kWh), E_p the pump efficiency, and C_r the repair cost ($/ha m).

Environmental attributes estimation: The environmental objective can be computed as a weighted sum of nitrates and pesticides in runoff and percolation, as well as aquifer overexploitation, which depend on crop volumes and water usage:

$$Env = Z \times 0.25 + P \times 0.25 + AO \times 0.50$$

where Z is the normalized measure of nitrates in runoff and percolation; P the normalized measure of pesticides in runoff and percolation and AO the aquifer overexploitation coefficient.

Step 3: conflict resolution methodology

Considering two conflicting objectives, the *feasible payoff* and the *worst payoff,* both conflicting objectives can be normalized in a way that zero value corresponds to the worst case and unit value to the best outcome. Hence, both objectives are now maximized. This conflict is mathematically defined by a pair (S, d), where $S \subseteq R^2$ is the *feasible payoff* set an $d \in R^2$ has the *worst* possible *payoff* values in its components. This vector is also known as the "nadir". The players want to increase their payoff values from these minimal values as much as possible. In the case of normalized objectives, $d_1 = d_2 = 0$. It is assumed that the Pareto frontier is given by the graph of a strictly decreasing concave function $g(.)$ defined in interval $\left[d_1, f_1^*\right]$, where $g\left(f_1^*\right) = d_2$, as depicted in Fig. 9.6. Herein, we also use the notation $\left(f_2^*\right) = g(d_1)$.

In many applications vector d is selected as the *current payoff vector* (called the "status quo" point), or the "*disagreement payoff*" vector, the components of which give the payoffs of the players in the case when they are unstable to reach an agreement. In such cases, the *feasible payoff* set S is restricted to the set.

$$S_+ = \{f = (f_1, f_2)/f \in S, f \geq d\},$$

since no rational player accepts an agreement which is worse than the outcome without an agreement or worse than the current situation. If vector d is selected as the nadir, then $S_+ = S$. The Nash-solution selects the unique point of the Pareto frontier, which maximizes the product of the gains from the disagreement payoff values. That is, the Nash solution is the unique solution of the following optimization problem (Optz-I):

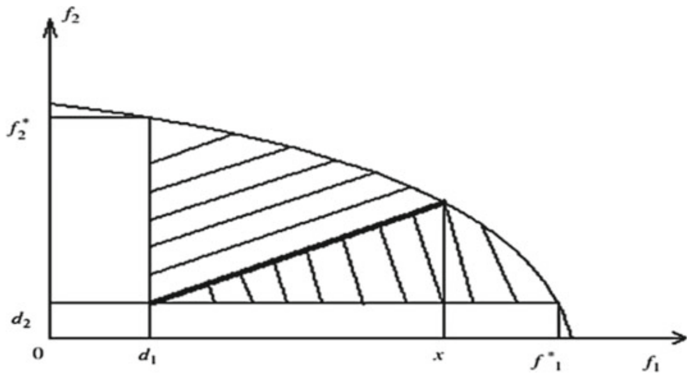

Fig. 9.6 The area monotonic solution

$$\text{maximize}\,(f_1 - d_1)(f_2 - d_2)$$

$$\text{s.t.} d_1 \ll f_1 \ll f_1^*$$

$$f_2 = g(f_1).$$

Notice that at $f_1 = d_1$, and also at $f_1 = f_1^*$, the objective function is zero, and it is positive for all $f_1 \in \left(d_1, f_1^*\right)$. Therefore, the optimum is interior. The second constraint allows us to solve a single-dimensional problem:

$$\text{maximize}\,(f_1 - d_1)(g(f_1) - d_2)$$

$$\text{s.t.} d_1 \ll f_1 \ll f_1^*,$$

where a simple one-dimensional search algorithm can be used, or a single-dimensional monotonic equation can be solved based upon the first-order condition.

Four conflict resolution methods have been discussed in (Raquel et al. 2007b). These optimization methods are briefly revisited in the next subsections.

Method 1: Non-symmetric Nash Solution

The non-symmetric Nash solution is the unique optimal solution of the problem

$$\text{maximize}\ (f_1 - d_1)^{w1}(f_2 - d_2)^{w2}$$

$$\text{s.t.} d_1 \ll f_1 \ll f_1^*$$

$$f_2 = g(f_1),$$

where w_1 and w_2 are the powers of the two players, or the importance factors of their objectives. Clearly, it is a straightforward generalization of the previous formulation (Optz-I) but with unequal weights.

Method 2: Kalai–Smorodinsky Formulation

Method 2 uses the Kalai–Smorodinsky formulation, described as follows. Consider the linear segment between the disagreement point (d_1, d_2) and the "ideal" point $\in \left(f_1^*, f_2^*\right)$; then the solution is the unique intercept of this segment with the Pareto frontier. Hence, we have to compute the unique solution of equation

$$d_2 + \left\{\left(f_2^* - d_2/f_1^* - d_1\right)\right\}(f_1 - d_1) - g(f_1) = 0$$

in the interval $\left[d_1, f_1^*\right]$. If both objectives are normalized, then $(d_1 = d_2 = 0)$, and $\left(f_1^* = f_2^* = 1\right)$; so, along the linear segment connecting the disagreement and ideal points, the two objective $\bar{f}_1$ and $\bar{f}_2$ increase at the same rate. If the objectives have different importance weights, then the more important objective has to be improved more rapidly. This idea leads to the nonsymmetric Kalai–Smorodinsky solution that computes the unique intercept between the Pareto frontier and the straight line

$$\bar{g}\left(\bar{f}_1\right) = (w_1/w_2)\bar{f}_1,$$

where the two coordinate directions are the normalized objective functions.

Method 3: Area monotonic solution

The area monotonic solution is based on a linear segment starting at the disagreement point that divides S_+ into two subsets of equal area. If the conflict is not symmetric, meaning that $w_1 \neq w_2$, then we might define the non-symmetric area monotonic solution by requiring that the ratio of the areas of the two subsets be w_1/w_2. Hence, the first coordinate of the solution is the root of the nonlinear equation.

$$w_2\left[\int_{d_1}^{x} g(t)dt - 1/2(x - d_1)(g(x) + d_2)\right] = w_1\left[\int_{x}^{f_1^*} g(t)dt - \left(f_1^* - d_1\right)(g(x) + d_2)\right]$$

in the interval $\left(d_1, f_1^*\right)$, as it is illustrated in Fig. 9.6.

Method 4: Equal loss solution

The equal loss solution was also originally developed for the symmetric case, when both payoffs are relaxed simultaneously with equal speed until an agreement is reached. If $w_1 \neq w_2$, then we may generalize this concept by requiring that the more important objective is relaxed slower than the other by assuming that the ratio of the relaxation speeds be equal to w_1/w_2. Therefore, one can determine a point $(x, g(x))$ on the Pareto frontier such that (Raquel et al. 2007b):

$$\left(f_1^* - x\right)w_1 = \left(f_2^* - g(x)\right)w_2.$$

Notice that similarly to the other three methods, this is also a nonlinear equation for the single unknown x, which can be easily solved by using standard optimization methodology.

9.7 Summary

The results for the conflict resolution methods discussed in (Raquel et al. 2007b) demonstrate that the implemented linear programming model for each groundwater extraction scenario has produced less water extraction corresponds to less net income. The last result shows that farmers have the option of growing the most profitable crops if they know the future prices for the next season. The results obtained from the linear model also suggest leaving some land idle (without crops and irrigation) in order to maximize net income when water availability decreases.

The computed compromise solutions between the economic and environmental objectives applying different formulation methods in (Raquel et al. 2007b) have evidenced that the Kalai–Smorodinsky solution generated more uniformly distributed points along the Pareto frontier, demonstrating efficiency and effectiveness of the Method 2 in providing a multitude of solutions in the Pareto frontier. As expected, the more weight given to protecting the environment, the lower the optimal groundwater extraction volume for agricultural irrigation, with all four formulations exhibiting such basic feature.

The results of net income obtained in the four methods with different weights show that when applying Method 4, the net income increases linearly with increasing economic weight, while the remaining methods exhibit nonlinear behavior. When economic benefit is considered as the only objective, the optimal groundwater withdrawal attains its maximum level. At the other extreme, when only environment is considered, the optimal groundwater scenario is to extract the minimum volume of groundwater via the irrigation wells. When environment and economics are assigned equal importance one can provide water resource allocation solutions considering equilibrium among environment protection and the economic activities viability.

In conclusion, the results of the above and other studies indicated the noticeable performances of Game Theory approaches in solving water resources management issues. Therefore, we can consider this method as a strong tool in solving water management problems.

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