



Optimal Parameter Estimation for High Dynamic Acquisition of Non-coherent DSSS

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Abstract. The phase of the modulation data symbol inversion is irrelevant to the pseudorandom noise code, which could seriously destroy the periodic characteristic of the pseudorandom noise code and decrease the acquisition performance of conventional receiver. In view of this, square operation is proposed to add in a commonly used acquisition algorithm. Compared with delay multiplication operation, the probability density function of amplitude is studied. Finally, the constraints and optimal value of the critical parameter are evaluated based on the high dynamic characteristics and the acquisition performance of non-coherent direct sequence spread spectrum system. According to the randomness of the modulation data symbol inversion, the detection probability and SNR_{out} is estimated adopting statistical method on the conditions of different SNR_{in} and information rates. The simulation results show that the acquisition sensitivity of the square operation is 5 dB higher than that of delay multiplication operation on the same detection probability.

Keywords: Optimal parameter · High dynamic · Non-coherent DSSS

1 Introduction

Direct sequence spread spectrum (DSSS) signal system is widely used in the field of measurement, control and communication because of its anti-jamming ability and high ranging accuracy, and is divided into non-coherent DSSS and coherent DSSS. The phase between the modulation data symbol inversion and the pseudorandom noise (PN) code is fixed in coherent DSSS system, while the phase obeys random distribution in non-coherent DSSS. According to the above characteristics, the non-coherent DSSS system can flexibly achieve various modulation data rate without changing the PN code rate and the bandwidth. However, the non-coherent modulation mode could seriously destroy the periodic characteristics of the PN code, and decrease the acquisition performance of conventional receiver, which would not track signal properly.

Existing strategies for the acquisition of non-coherent DSSS system signal had present. In [1], the complex signal was construct, which would eliminate the impact of modulation data symbol inversion on FFT (Fast Fourier Transform) performance. In [2], the region of the modulation data symbol inversion is estimated, then FFT and incoherent accumulation are adopted in the valid data sections of each estimation

branch to acquire accumulated detection value of each frequency point. However, this algorithm has low acquisition sensitivity. Delay conjugate multiplication is presented in [3] and FFT is used to estimate the PN code phase and Doppler frequency shift. In conclusion, delay multiplication operation is commonly used in engineering design to eliminate the impact of modulation data symbol inversion. However, there has no optimization analysis of critical parameter such as correlation integration time on the condition of high dynamic and different modulation data rate.

In this paper, square operation is proposed to add in common acquisition algorithm. Compared with delay multiplication operation, the probability density function (PDF) of amplitude is studied. The constraints and optimal value of the critical parameter are evaluated based on the high dynamic characteristics and the acquisition performance. According to the randomness of the modulation data symbol inversion, the detection probability is estimated using statistical method under different SNR_{in} . Simulation results presented later will verify the optimal value.

The treatise is organized as follows: Sect. 2 depicts the acquisition algorithm principle, including the square operation and delay multiplication operation. In Sect. 3, constraints and optimal value of the critical parameter are evaluated. Section 4 presents some numerical simulation results with discussion the paper is summarized in Sect. 5.

2 Acquisition Algorithm Principle

The block diagram of the conventional acquisition algorithm is shown in the solid line in Fig. 1. In order to eliminate the impact of modulation data symbol inversion on conventional acquisition algorithm, the proposed Square operation (M1) and delay multiplication operation (M2) are added, as shown in the dotted lines in Fig. 1 [4].

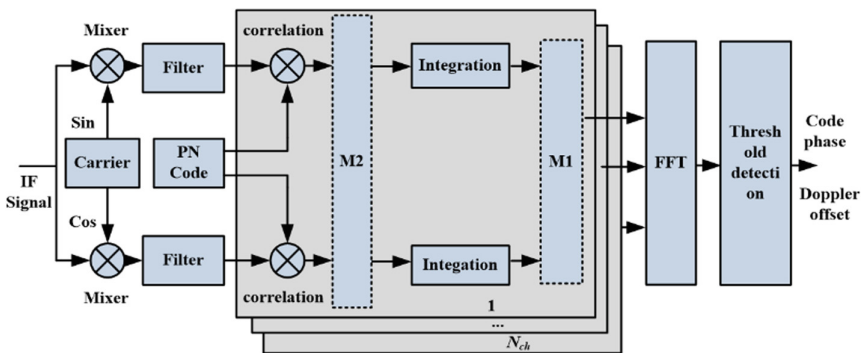


Fig. 1. Block diagram of acquisition algorithm

Step 1: Assume that there are N_{ch} parallel channels for code phase search, and the search interval of each channel is half code chip. The correlation results of baseband signal multiplying by the locally generated PN code can be expressed as:

$$y_2(n) = I_2(n) + i * Q_2(n) \quad (1)$$

where $I_2(n) = AD(n)R(\tau_p(n))\cos(2\pi f_e t + \theta_e) + n'_i(n)$, $Q_2(n) = AD(n)R(\tau_p(n))\sin(2\pi f_e t + \theta_e) + n'_q(n)$. $R(\tau_p(n))$ represents the correlation value of the PN code; $\tau_p(n)$ represents the phase difference between the local PN code and the received PN code; $n'_i = n_i * C_{local}(n)$, $n'_q = n_q * C_{local}(n)$. If $\tau_p(n) \geq 1/f_{code}$ or no signal input, $R(\tau_p(n))$ is approximately 0. n'_i and n'_q equivalent to zero mean Gaussian white noise, $y_2(n) \sim (0, \sigma^2)$. If $\tau_p(n) < 1/f_{code}$, $y_2(n) \sim N(AD(n)R(\tau_p(n))e^{2\pi f_e t + \theta_e}, \sigma^2)$.

Step 2: To simplify the analysis, θ_e would not be taken into account temporarily. Integral result in integration time T_{coh} can be simplified as:

$$corr'_1(m, n) = \{\text{sum}(y_2(k)) | k = t_0 + m * T_{coh} \sim t_0 + (m + 1) * T_{coh}\} \quad (2)$$

where, $n = 0, \dots, N_{ch}$, $m = 0, \dots, M - 1$. The total coherent integration time is $t_{tot-coh} = T_{coh} * M$. If $\tau_p(n) \geq 1/f_{code}$ or no signal input, $corr'_1(m, n) \sim (0, N_{coh}^2 \sigma^2)$. If $\tau_p(n) < 1/f_{code}$, $corr'_1(m, n) \sim (A_{s1}(\alpha, m, n), N_{coh}^2 \sigma^2)$, $N_{coh} = T_{coh}/f_s$. α is the relative position of data inversion in integration time T_{coh} , $0 \leq \alpha \leq 1$.

The modulation data can be regarded as a random variable after being encoded and encrypted. Due to the data inversion and its randomness, the amplitude of the correlation accumulation values exists in three cases >0 , $=0$, <0 . If the correlation accumulation values are directly analyzed in the frequency domain, the low-frequency component will be introduced into the magnitude frequency characteristics, which will affect the identification of acquisition results.

Step 3: Square operation (M1) or delay multiplication (M2) of correlation value is used to eliminate the influence of data inversion.

MI: Square Operation

Square of Eq. (2) is given by:

$$corr_2(m, n) = (corr'_1(m, n))^2 \quad (3)$$

It can be seen from Eq. (3) that after the square operation, the amplitude of correlation data is greater than or equal to 0, and the data inversion has been eliminated. In case of $\tau_p(n) \geq 1/f_{code}$ and no signal input, $corr_2(m, n)$ is in the form of central chi-square distribution, $corr_2(m, n) \sim \chi^2(k, \beta, \lambda)$. Where, k represents the degrees of freedom (DOF), it value relate to number of independent Gaussian variables, and here $k = 1$. β denotes the coefficient of proportionality, it accounts for the common variance of the Gaussian variables, and here $\beta = N_{coh}^2 \sigma^2$, $N_{coh} = T_{coh} * f_s$. λ stands for the non-centrality parameter, and here $\lambda = 0$. If $\tau_p(n) < 1/f_{code}$, $corr_2(m, n)$ is non-central chi-square distribution, $corr_2(m, n) \sim \chi^2(1, N_{coh}^2 \sigma^2, A_{s1}(\alpha, m, n))$ [7].

M2: Delay Multiplication

(1) Delay multiplication is defined by:

$$y_3(n) = y_2(n) * y_2(n + n_\tau) \approx D(n) D(n + n_\tau) R(\tau_p(n))^2 e^{2\pi j e t + \theta e} + n'(n) \quad (4)$$

where, n_τ is the delay time, $n_\tau \geq 1/f_{code}$. It can be seen that the value of $D(n) * D(n + n_\tau)$ is equal to 1 except in several sampling period the value is -1 . When $\tau_p(n) \geq 1/f_{code}$ and no signal input, $y_3(n)$ follows central chi-square distribution, $y_3(n) \sim \chi^2(1, \sigma^2, 0)$.

(2): The coherent integration result Integral result in integration time T_{coh} is given by:

$$corr_3(m, n) = \{\text{sum}(y_3(k)) | k = t_0 + m * T_{coh} \sim t_0 + (m + 1) * T_{coh}\} \quad (5)$$

when $\tau_p(n) \geq 1/f_{code}$ and no signal input, $corr_3(m, n)$ follows central chi square, $corr_3(m, n) \sim \chi^2(N_{coh}, \sigma^2, 0)$. When $\tau_p(n) < 1/f_{code}$, $corr_3(m, n)$ follows non-central chi-square distribution, $corr_3(m, n) \sim \chi^2(N_{coh}, \sigma^2, A_{s2}(\alpha, m, n))$. It is obviously from Eq. (4)–(5) that after delay multiplication and integration, the effect of data inversion has been eliminated.

Step 4: The output of K point FFT using the accumulated value obtained in step 4 padding 0 can be expressed as:

$$X(k, n) = \text{fft}(x(m, n)) \quad (6)$$

$|X(k, n)|$ is the amplitude of $X(k, n)$. If the amplitude exceeding the threshold value V_t , the process of acquisition will be end, otherwise jump to step 2.

For M1, if $\tau_p(n) \geq 1/f_{code}$, $x(m, n) \sim \chi^2(1, N_{coh}^2 \sigma^2, 0)$, the PDF of $|X(k, n)|$ can be obtained by statistical method, and suppose $E[|X(k, n)|] = \sigma_{m1}^2$ [8]. If $\tau_p(n) < 1/f_{code}$, $x(m, n) \sim \chi^2(1, N_{coh}^2 \sigma^2, A_{s1}(\alpha, m, n))$, according to the property of chi-square distribution, when N is large enough, the PDF of $|X(k, n)|$ approximately follows normal distribution, $|X(k, n)| \sim N(m_{X12}(k, n), \sigma_{m1}^2)$ [9, 10]. For M2, if $\tau_p(n) \geq 1/f_{code}$, $x(m, n) \sim \chi^2(N_{coh}, \sigma^2, 0)$, the PDF of $|X(k, n)|$ can be obtained by statistical method, where $E[|X(k, n)|] = \sigma_{m2}^2$. If $\tau_p(n) < 1/f_{code}$, $x(m, n) \sim \chi^2(N_{coh}, \sigma^2, A_{s2}(\alpha, m, n))$, when N is large enough, the PDF of $|X(k, n)|$ approximately follows normal distribution, $|X(k, n)| \sim N(m_{X22}(k, n), \sigma_{m2}^2)$.

3 Optimal Parameter Estimation

3.1 Correlation Loss Analysis

Suppose the Doppler frequency shift is 120 kHz, the change curve of correlation loss with integration time of square operation and delay multiplication operation is shown in Fig. 2. When the correlation integration time T_{coh} is 0.004 ms, the change curve of the

correlation loss with the Doppler frequency shift of square operation and delay multiplication operation is shown in Fig. 3. As is shown that the shorter the coherence integration time is, the smaller the correlation loss is, and the correlation loss of square operation is lower than that of delay multiplication operation. When the Doppler frequency shift is 120 kHz and $T_{coh} = 0.004$ ms, compared with conventional acquisition algorithm, the correlation loss of square operation increases 3 dB, the correlation loss of delay multiplication is more high.

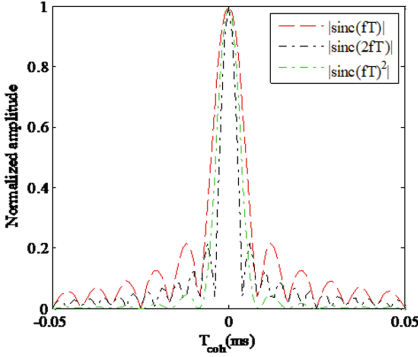


Fig. 2. Correlation loss @ $f_e = 120$ kHz

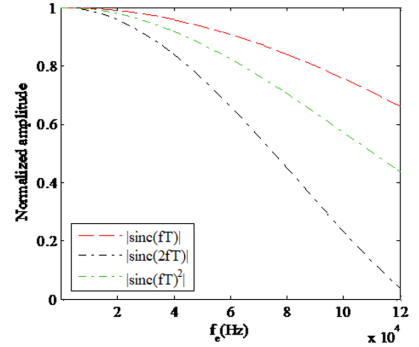


Fig. 3. Correlation loss @ $T_{coh} = 0.004$ ms

3.2 Optimal Parameter Estimation

The capture time T_{acq} of N chips is less than $M * T_{coh} * N * N_{nc}/N_{ch}$, where N_{ch} is the number of parallel search channels. The capture time has reached ms level, and is related to engineering design. Therefore, the capture time is not considered as the constraint of optimal design in this paper. On the contrary, due to the high dynamic characteristics of the carrier and the signal characteristics of the non-coherent DSSS, the acquisition sensitivity of the receiver is constrained, which refers to the minimum signal power that the receiver can detect.

The purpose of optimal parameter design in this paper is to improve the acquisition sensitivity of the receiver. The critical parameters of the design are coherent integration time T_{coh} and non-coherent integration number N_{nc} . The non-coherent integrals number N_{nc} affects the acquisition time, and this parameter is designed under the condition that one noncoherent integral can achieve the sensitivity according to the acquisition sensitivity requirements. Therefore, the mathematic description about the optimization problem is to estimate the optimal parameter T_{coh} subject to the condition $N_{nc} = 1$, so as to obtain the maximum acquisition sensitivity SNR_{in} [11, 12]. As follows:

$$\max SNR_{in}, \text{ s.t. } \{f_e * T_{coh} \leq 0.443 \text{ and } f_e \leq f_{dopp_request} \text{ and } N_{nc} = 1\} \quad (7)$$

4 Simulation

The dynamic characteristics and waveform parameters are list in Table 1, and the modulation data is set to 1 and 0 alternately. The Doppler shift is between -120 kHz and 120 kHz, the information rate is 4 Kbps, the PN code rate and cycle is n 3.069 Mbps and 1023 .

4.1 The Relationship Between α and Detection Probability

α is the relative position of data inversion in integration time T_{coh} . The PN code rate parameter $n = 3$. By statistical method, Table 1 shows the detection probability of M1 and M2 operation under different SNR_{in} conditions. On $SNR_{in} = -22$, the PDF of M2 algorithm is shown in Fig. 4. On $SNR_{in} = -26$, the PDF of M1 algorithm is shown in Fig. 5. It can be seen from the results that under the condition that the acquisition detection probability P_d is greater than 90% , the acquisition SNR_{in} of M1 is higher than that of M2 method by 5 dB.

Table 1. Detection probability under different SNR

SNR_{in}	P_d		SNR_{in}	P_d	
	M1	M2		M1	M2
-19	100%	99.75%	-24	100%	28.78%
-20	100%	97.02%	-25	99.01%	11.66%
-21	100%	89.83%	-26	97.27%	3.23%
-22	100%	82.38%	-27	79.16%	0.50%
-23	100%	65.01%	-28	61.29%	0.00%

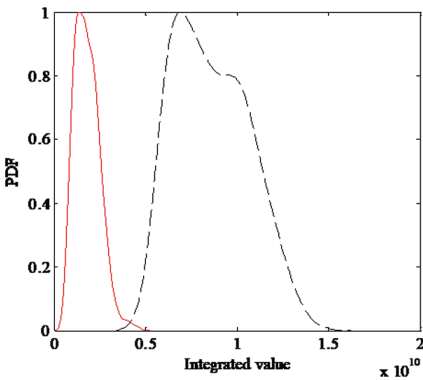


Fig. 4. PDF@ $SNR_{in} = -22$ &M2

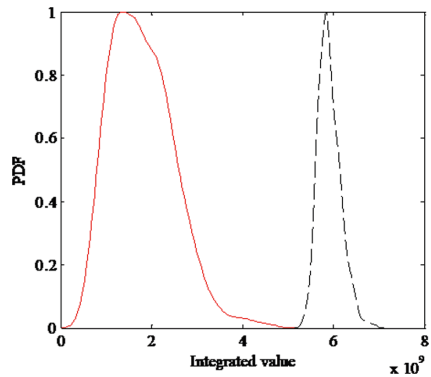


Fig. 5. PDF@ $SNR_{in} = -26$ &M1

4.2 Detection Probability Under Different Modulation Data Rates

On the same dynamic characteristics, the carrier characteristics determine the correlation integration time, which is independent of the modulation information rate. Analyze the acquisition detection probability under different information rates such as 3 Kbps, 5 Kbps, 6 Kbps and 7 Kbps on $SNR_{in} = -20$ and $SNR_{in} = -25$. The simulation results are shown in Table 2. It can be seen from the simulation results that the acquisition detection probability P_d of M1 is still greater than 90% on the condition of $SNR_{in} = -25$.

Table 2. Acquisition probability under different PN code rate

f_{code}	P_d		f_{code}	P_d	
	$SNR_{in} = -20$			$SNR_{in} = -25$	
	M1	M2		M1	M2
3 Kbps	100%	99.75%	3 Kbps	100%	16.38%
5 Kbps	100%	98.01%	5 Kbps	99.01%	8.19%
6 Kbps	100%	97.52%	6 Kbps	97.52%	13.15%
7 Kbps	100%	96.28%	7 Kbps	99.5%	8.44%

4.3 Acquisition Performance in Coherent DSSS System

On the same dynamic characteristics, the coherent DSSS system is compared with the acquisition SNR_{out} without M1/M2 operation and with M1 or M2 operation. The simulation results under different SNR_{in} are shown in Fig. 6 and 7. It can be seen from the simulation results that the acquisition performance of M1 is the same as the algorithm without M1/M2 on $SNR_{in} = -20$, and the acquisition performance of the algorithm without M1/M2 has a higher SNR_{out} on $SNR_{in} = -25$.

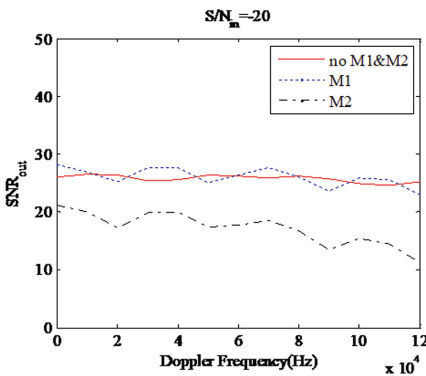


Fig. 6. Acquisition SNR_{out} @ $SNR_{in} = -20$

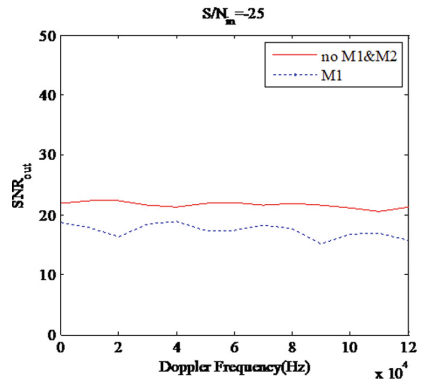


Fig. 7. Acquisition SNR_{out} @ $SNR_{in} = -25$

5 Conclusion

The acquisition algorithm presented in this paper adopts the square operation to construct the data, and effectively solves the impact of data jump on acquisition, while completing the code phase search, completes the Doppler frequency shift search. Through the optimization estimation of coherent integration time parameters in the acquisition process, on the one hand, it improves the acquisition sensitivity under the condition of non-coherent integrals number $N_{nc} = 1$ and high dynamic, on the other hand, it can be applied to the acquisition of non-coherent and coherent DSSS under the condition of various data rates, which enhances the universality of the algorithm, and has good theoretical research value and broad engineering application prospects.

References

1. Sun, G.L., Li, Y.Q., Zhang, X.L.: FFT-based acquisition method to eliminate influence caused by data modulation. *J. Beijing Univ. Aeronaut. Astronaut.* **34**, 262–266 (2008)
2. Xiong, Zh.L., An, J.P.: A low complexity acquisition algorithm for DSSS signal with low SNR and non-coherent data modulation. *Acta Electronica Sin.* **44**, 753–760 (2016)
3. Zhao, Ch.Y., Cui, W.: A DSSS signal acquisition algorithm for overcoming the impact of non-coherent data modulation. *Acta Electronica Sin.* **39**, 1941–1946 (2011)
4. Wang, C., Fei, G.: Low complexity DSSS acquisition method of LEO satellite communication ASIC. In: 2019 IEEE 2nd International Conference on Electronic Information and Communication Technology (ICEICT), pp. 84–88. IEEE (2019)
5. Yuan, Q., Ju, W., Wu, S.L., Jun, W.: Direct acquisition method of GPS P code and its parameter optimization. In: *Communications and Mobile Computing*, Shenzhen, pp. 469–475 (2010)
6. Spangenberg, S.M., Scott, I., McLaughlin, S., Povey, G.J.R., Cruickshank, D.G.M., Grant, P.M.: An FFT-based approach for fast acquisition in spread spectrum communication systems. *Wirel. Pers. Commun.* **13**, 27–55 (2000)
7. Huillery, J., Millioz, F., Martin, N.: On the description of spectrogram probabilities with a chi-squared law. *IEEE Trans. Sig. Process.* **56**, 2249–2258 (2008)
8. Lancaster, H.O., Seneta, E.: Chi-square distribution. In: *Encyclopedia Biostatistics*, vol. 2 (2005)
9. D’Ambra, L., Amenta, P., D’Ambra, A.: Decomposition of cumulative chi-squared statistics, with some new tools for their interpretation. *Stat. Methods Appl.* **27**(2), 297–318 (2018)
10. Hashemi, N., Ruths, J.: Generalized chi-squared detector for LTI systems with non-Gaussian noise. In: 2019 American Control Conference (ACC), pp. 404–410. IEEE (2019)
11. Sinclair, J., et al.: Weak-value amplification and optimal parameter estimation in the presence of correlated noise. *Phys. Rev. A* **96**(5), 052128 (2017)
12. Honório, L.M., Costa, E.B., Oliveira, E.J., de Almeida Fernandes, D., Moreira, A.P.G.: Persistently-exciting signal generation for optimal parameter estimation of constrained nonlinear dynamical systems. *ISA Trans.* **77**, 231–241 (2018)