Evaluating the Performance of Selected Mortality Forecasting Models: A Malaysia Case Study

Khairunnisa Mokhtar, Syazreen Niza Shair, and Norazliani Md Lazam

Abstract The study of human mortality is growing in Malaysia, as accurate mortality rates are classified important especially for social policy planning. This research aims at evaluating the performance of three selected mortality forecasting models, namely the Lee-Carter, CBD and M8 model in which the two latter models are from Cairns, Blake and Dowd. We applied the Malaysian central death rates and the number of mid-year exposures to the models and estimate the goodness of fits of all models using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). In addition, the 20-year out-samples forecast errors are estimated from 1999 to 2019 using the Root Mean Square Errors (RMSEs) and the Mean Absolute Percentage Errors (MAPEs). The findings of this study suggest that the M8 model is the best model for fitting Malaysian mortality data with minimum AIC and BIC values, and by far the most accurate model with the lowest out-sample errors, particularly for higher age category.

Keywords The Lee-Carter model · The Cairns, Blake and Dowd model · Mortality modeling

1 Introduction

In Malaysia, the study of population mortality is increasing over the past few years as mortality estimates are useful to pension and insurance industries [1]. The importance of the estimation of mortality rates derives from their potential in evaluating the liabilities of pension funding and insurance accurately as well as improving public's

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health. Statistics have shown that life expectancies in Malaysia have significantly increased from 62.8 in 1966 to 74.5 years old in 2019 [2]. The increase in life expectancy in Malaysia may be resulted from several factors such as better access to health care resources and higher economic status among population [3].

An increase in life expectancy practically gives a good perspective that Malaysian can live longer. However, this also can be directed to another problem known as longevity risk [4]. For example, longevity risk may put pressure to the governments in spending more on health and other social programs especially for elderlies. In terms of individual prospective, an increase in life expectancy provides serious economic consequences as when people get older, they tend to live with chronic health conditions hence will incur substantial costs of treatment. Thus, information on how long one is expected to live is important. To obtain an accurate life expectancy estimate, an accurate mortality forecasting model is much needed particularly for the Malaysian population.

To this date, the Lee-Carter model is known as the significant model which was widely used to forecast mortality rates [5]. This well-developed model is proposed by Ronald D. Lee and Lawrence Carter in 1992 [6]. This model comprises log-bilinear form of mortality, integrating two major effects which are age (a_x and b_x) and year (k_t) [7, 8]. To reduce the complexity in estimating the two factors (b_x and k_t), the model is reduced to a univariate time series forecasting using the Singular Values Decomposition (SVD) technique. Since k_t is the only parameter with time involved, the Autoregressive Integrated Moving Average (ARIMA) models are employed to predict the k_t parameter. The model assumes invariant age-component and a linear time-component reduction over time [6].

The Lee-Carter model not only proven to be well-developed, it has been widely applied in many countries [9, 10] including Malaysia. However, some researchers have proved that the model has some weaknesses such as the age component is assumed invariant over time [11]. An extension version of the Lee-Carter forecasting mortality model denoted as the CBD model has been developed by [12]. This model is unique due to the focus that is more on longevity risk factor. This model contributed a huge influence for financial sectors like retirement funds, life-insurance corporation and independent annuity suppliers [12]. The model accustoms to the development of mortality curve through time with two-factor stochastic model. Instead of one k_t, the model is extended to include ${k_t}^{(2)}$. The first k_t affects the dynamics of mortality at all ages in the same way as the Lee-Carter model while the second k_t affects the dynamics of mortality at higher ages [12].

The Lee-Carter model has initiated the development of all the CBD models. The model assumes there is a stationary age function, a_x with no cohort effects and named as the M1 model. Then, from the M1 model, the study from [13] extended the model to include the cohort effect and add one more age related parameter, ${b_x}^{(3)}$. This model was called M2 model. The model M3 is a special case of the model M2 with ${b_x}^{(2)} = 1$ and ${b_x}^{(3)} = 1$ parameter. In order to fit the mortality surface, the M4 model is proposed to include B-splines and P-splines smoothing techniques for age variables. A research from [14] revised the original CBD model to become the M5 model. The M5 model comprises two-period effects (${k_t}^{(1)}$ and ${k_t}^{(2)}$) including age effect and no component of cohort effect. Next M5 is expanded to the model M6—the first version that integrates cohort effects into the CBD model. The M6 model is extended to the second generation named as the M7 model. This model includes the quadratic term in age factors and the third period effects.

The M8 model is the recent extension of CBD model that incorporates two models which are the M2 and M5 models [15]. The current version of the CBD models are not widely applied by the researchers as they are new. Such models were tested for data from England and Wales and it showed that the M8 model was not only capable of capturing the cohort effect substantially but also had the lowest error [14]. In the comparison study done by utilizing the Italian death rates with regard to Lee-Carter and CBD model, a significant difference can be found between the two. While the CBD model is suitable for older age groups, the Lee-Carter model performs best throughout the whole analysis. These earlier studies show that mortality data from different countries will yield different results where the latest CBD model, the M8 model, is the most accurate model for forecasting mortality rates.

The design of the CBD model based on the risk of longevity suggests that the model might be suitable to apply to Malaysian data as it is predicted by 2030 Malaysia will become an aging nation. Malaysia is currently focusing on social protection arrangements for elderlies in order to prepare the nation with ageing issues [16]. There is a recent study from [17] that made a comparison between the Lee-Carter model and Cairns, Blake and Dows (CBD) model. The study found that, the evaluation between both models showed that there is no model better than the other. It is noteworthy that [17] did not include the most recent extended model which is the M8 model. Hence, this study would like to extend [17] by adding the M8 model and conduct this research using more up-to-date data from 1960 to 2019.

2 Methodology

2.1 Data

This section discusses on the selected mortality forecasting models specifically the Lee-Carter model, the CBD model a well as the M8 model. The central mortality rates by age groups for both female and male from 1966 to 2019 (54-year-data) was taken from the Department of Statistics Malaysia (DOSM). Meanwhile the mid-year exposures or number of population by age groups for both genders over the same years was retrieved from the World Population Prospect [18].

2.2 The Lee-Carter Model

The Lee-Carter model assumes there are fixed age function and a unique non-parametric age-period. This model is widely used by many researchers since it was proposed. The general Lee-Carter Model equation is given through:

$$\ln\left(m_{x,t}\right) = a_x + b_x k_t + \varepsilon_{x,t} \tag{1}$$

where,

$m_{x,t}$ central rate of death at age x in year t.
a_x average of log $[m_x, t]$ across years.
b_x relative speed of change at each age also known as age component.
k_t time-varying parameter.
$\varepsilon_{x,t}$ residual at age x and time t.

To get unique solutions, coefficient a_x, b_x and k_t are set $a_x = \frac{1}{T}\sum \ln(m_{x,t})$, $\sum_x b_x = 1$ and $\sum_t b_t = 1$. Then, adopting Singular Value Decomposition (SVD) method to acquire the estimation values of b_x and k_t. To estimate the value of k_t, Autoregressive Integrated Moving Average Model (ARIMA) is applied. The equation of the ARIMA model is given:

$$k_t = k_{t-1} + \phi_1 k_{t-1} - \phi_1 k_{t-2} + \varepsilon_t - \theta_1 \varepsilon_{t-1} \tag{2}$$

where k_t is denoted as the actual value at the time period, t, $\varepsilon_{x,t}$ denoted as random error at time period, t, and ϕ_1 and θ_1 denoted as parameters of the model.

2.3 The CBD Model

The Cairns, Blake and Down (CBD) model proposes a predictor structure with two age-periods. The age modulating parameters were specified as $b_x^{(1)} = 1$ and $b_x^{(2)} = x - \bar{x}$. Where $\bar{x}$ is denoted as the average age. This model assumes no static age function; the population is stationary and no cohort effect. The general equation for the CBD model is given:

$$logit\left(q_{x,t}\right) = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) \tag{3}$$

where $q_{x,t}$ is the probability of a person age x will die in t years and $k_t^{(1)}, k_t^{(2)}$ denoted as the period effects.

2.4 The M8 Model

The M8 model is the third generalization of the CBD model and this model is also based on the adjustment of [13]. Unlike the previous models, this model includes cohort effect in the calculation basis. This model proposed the impact of the effect $\gamma_{t-x}^{(3)}$ for any specific cohort reduces over time instead of remaining constant. The formula of the M8 model is given:

$$logit(q_{x,t}) = \beta_x^{(1)} k_t^{(1)} + \beta_x^{(2)} k_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)} \tag{4}$$

where the equation is derived by taking $\beta_x^{(1)} = 1$, $\beta_x^{(2)} = (x - \bar{x})$ and $\beta_x^{(3)} = (x_c - x)$. For some constant, the x_c in this study is the 80 years old. Therefore, the equation generalizes as:

$$logit(q_{x,t}) = k_t^{(1)} + \left(x - \bar{x}\right) k_t^{(2)} + (x_c - x)\gamma_{t-x}^{(3)} \tag{5}$$

To overcome the identifiable problems, new constraint is introduced by letting $\sum_{x,t} \gamma_{t-x}^{(3)} = 0$. In fitting the mortality models, namely the Lee Carter, CBD and M8 models, the packages of R programming software, known as *StMoMo* [14] and *Demography* [19] were used.

2.5 Evaluations of Mortality Forecasting Models

As defined by the Department of Social Welfare, older persons refer to those who are 60 years and above [19]. Therefore, this study fitted the ages between 60 and 80 years old. However, to prove that the CBD and M8 models are only applicable for higher ages (60–80), the evaluation for all ages (0–80) and lower ages (0–59) were also being carried out from 1966 to 2019 (54 years). Primarily, the data was separated into two parts namely the training and validation, with the percentage of training is 60% (1966–1998) and validation is 40% (1999–2019) from the available data.

Model Goodness of Fit

The goodness-of fit test usually used to check the fitted model residuals. Consistent residual patterns specify the model's incompetence for properly defining all of the data features. The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are two parameters chosen to check the fit-of-data quality. The lower the values of AIC and BIC the better the model fits to the data.

The Akaike Information Criterion (AIC) measures the probability penalty for each additional term that is included in the model. The equation is as follow:

$$AIC = 2k - 2\ln(L) \tag{6}$$

The Bayesian Information Criterion (BIC) is the best criterion to balance between the models' complexity and goodness-of-fit.

$$BIC = 2\ln(Nk) - 2\ln(L) \tag{7}$$

where k denoted as the number of parameters estimated in the model and L is the log-likelihood and N is the number of sample.

Out-Sample Error Evaluation

For 40% validation set, Root Mean Square Errors (RMSEs) and Mean Absolute Percentage Errors (MAPEs) are used to test the performance of all three selected models for three different age categories namely all age (0–80), lower age (0–59) and higher age (60–80). The Root Mean Square Errors (RMSEs) is commonly used and able to make an outstanding general purpose error metric for numerical predictions [20]. The general formula is as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\hat{y}_{x,i} - y_{x,i}\right)^2} \tag{8}$$

Mean Absolute Errors (MAPEs) are mean or average of the absolute percentage errors of forecasted values. By canceling each other out this approach will prevent the issue of both positive and negative errors. The formula as follows:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\left(y_{x,i} - \hat{y}_{x,i}\right)^2}{y_{x,i}}\right| \tag{9}$$

where $\hat{y}_{x,i}$ is the prediction central death rates for person age x in year i, $y_{x,i}$ is the observed central death rates for person age x in year i, and n is the number of sample in year.

3 Results and Discussions

This section discusses the results obtained from the selected mortality models used in the study. Due to some missing values from the given data, the interpolation process needs to be carried out in filling up the missing values.

Figure 1 demonstrates the plot for observation data which is Malaysian population pattern of logarithm of death rates according to age and time. Several behaviors are shown respectively for both male and female. As it can be seen, the mortality rates are increasing for both genders in all age's group, however male is slightly thinner compared to female and the presence of volatile accident humps in between the ages of 18–30 years old that is more visible in male than female.

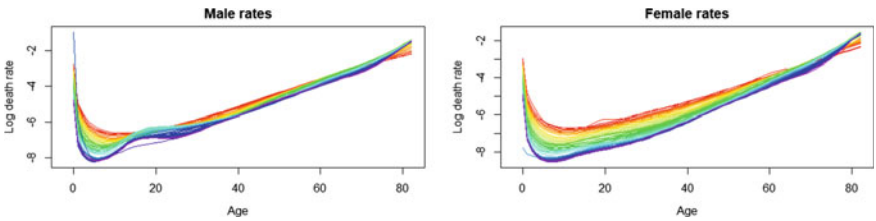

Fig. 1 Log death rates according to age for Malaysian male and female

3.1 Estimation of Models' Parameters

This section estimates the parameters for all the three models. Figure 2 shows the parameter shapes of the Lee-Carter model. As can be seen, the parameter a_x is increasing by age, whereby the parameter b_x is declining downward as it reacts towards the mortality changes over ages. Meanwhile, the k_t, parameter is decreasing by years, with the presence of humps visible for both genders during the year-range (1990–2000). As depicted in Fig. 3, the CBD model shows a declining trend of $kt^{(1)}$ for both genders. However, for mortality-rate dynamic at higher ages ${k_t}^{(2)}$ parameter,

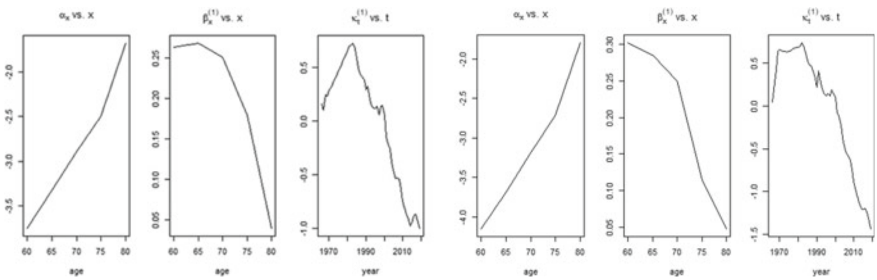

Fig. 2 Parameters of the Lee-Carter model for Malaysian males (left) and females (right) ages 60–80 over the period of 1966–1998

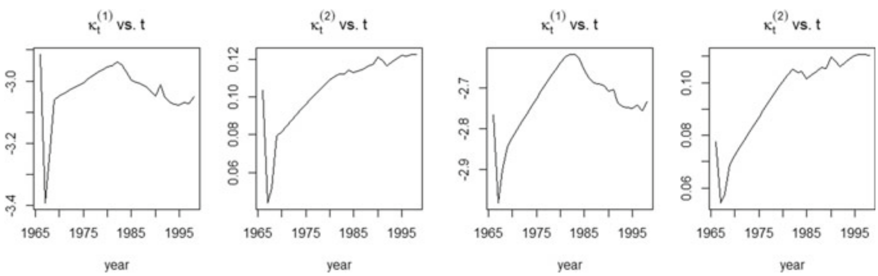

Fig. 3 Parameters of the CBD model for Malaysian males (left) and females (right) age 60–80 over the period of 1966–1998

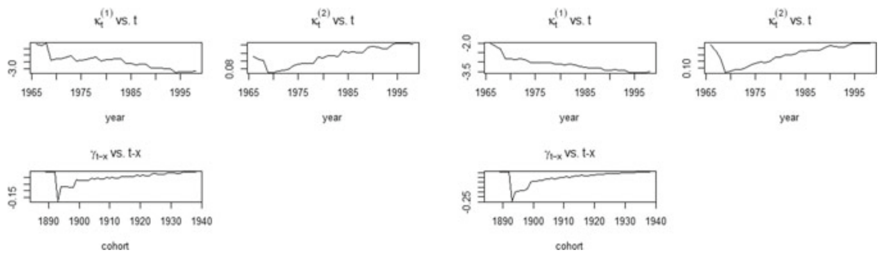

Fig. 4 Parameters of the M8 model for Malaysian males (right) and females (left) ages 60–80 over the period of 1966–1998

the mortality rates dynamically shoot up at higher ages for both genders. As for the M8 model, Fig. 4 shows that both genders experiencing quite similar trends. More information captured in the M8 model is about its cohort effect, where those who are 80 years old in 1966 data are those who were born in 1886. Hence, the cohort trend captured for both genders are showing improvements in mortality over the years. Nonetheless, both models of the CBD and M8 show the existence of mortality improvement by years.

Residual Analysis

The residuals were distributed randomly across ages and years. In order to fit the model, this study used scatter diagram to examine the model fitting. The closer the dotted to zero, the better it fits to the line. As can be seen in Fig. 5, the plot of the Lee-Carter model for both genders are scattered and most of the dotted points are far from the fitted lines. For the CBD model, both genders' show dotted lines for certain ages that is extremely too far from the fitted lines. However the patterns for both genders show a quadratic patterns which mean that this model is able to capture the cohort effects and it has been substituted by the additional quadratic effect [9]. Therefore the residuals plot for the M8 as depicted in Fig. 5 has captured the cohort effects as the presence of quadratic patterns are clearly displayed in age residuals for both genders.

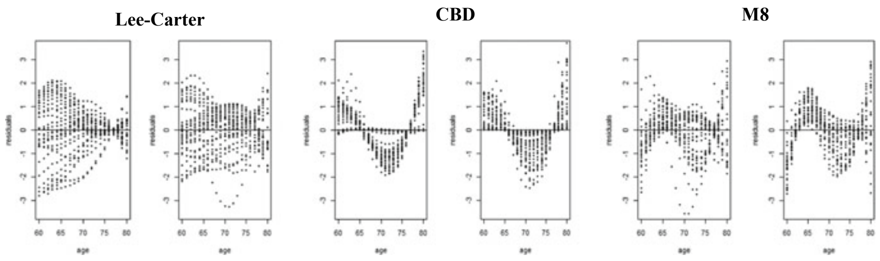

Fig. 5 The scatter residual diagrams of all the three models for Malaysian males (left) and females (right) age 60–80 over the period of 1966–1998

Forecasted Parameters for the Selected Models

Figure 6 shows the patterns of the forecasted graphs of time factors for all three models for higher age category since this study focuses on higher age (60–80). For the Lee-Carter model, it can be seen that, the $k_t^{(1)}$ values for both male and female show the declining pattern across the time. For the CBD model, the parameter $k_t^{(1)}$ shows the declining trend for both male and female, meanwhile for $k_t^{(2)}$ where the mortality is forecasted for higher age, shows the increasing trends for both genders, with narrow values of the forecasted rates. Lastly for the M8 model, the projection trend for period index of $k_t^{(1)}$ and $k_t^{(2)}$ are similar with the CBD model, however the forecasted rates for M8 model is wider as compared to the CBD model. This suggests that the M8 model is more plausible model as compared to the CBD model.

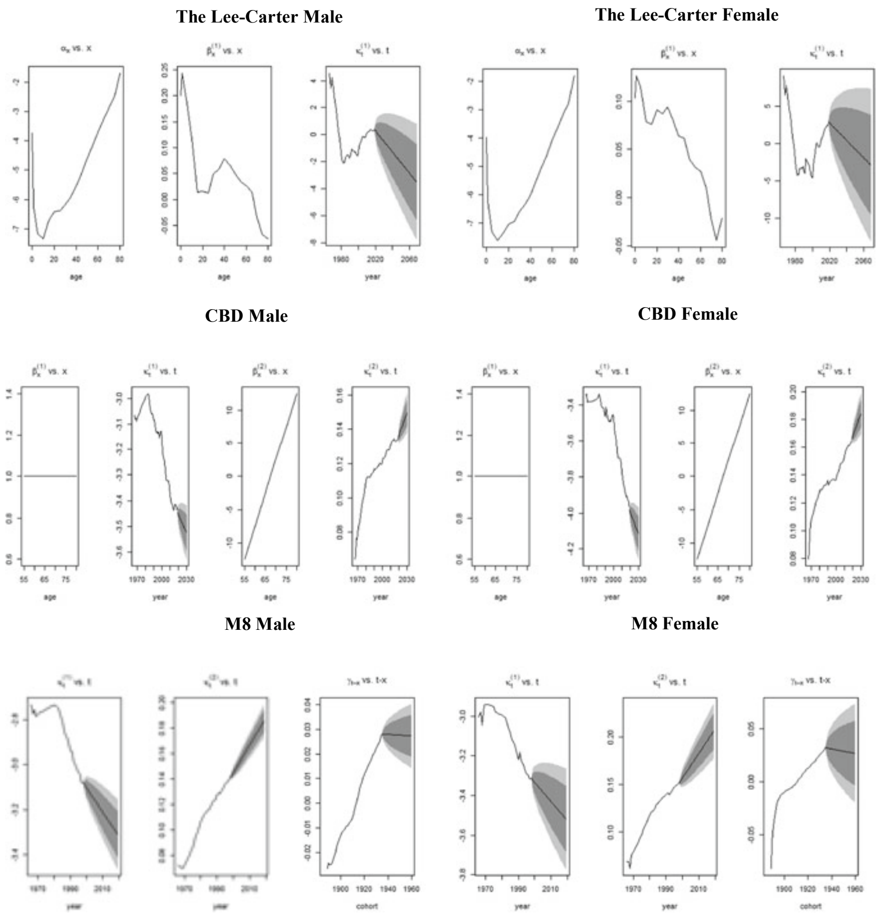

Fig. 6 The estimated age parameters and estimated and forecasted time components for the all the three models for Malaysia population age 60–80

For the cohort effects, as depicted in the M8 model, the projections for cohort male and female are declining gradually. However, for male, the declining trend is not that significant, the pattern is more towards stagnant.

3.2 The Performance of Mortality Models

Goodness-of-Fit Analysis

In this subdivision, the model evaluation for three categories namely all age (0–80), lower age (0–59) and higher age (60–80) were evaluated using AIC and BIC. The lower the AIC and BIC values, the more fit the model to the observation data [21]. Table 1 indicates that, the Lee-Carter model is the best model that fits the data best for all age and lower age categories for both males and females due to the model produced the lowest AIC and BIC. Whereas it shows that the M8 model is the best model to fit the higher age (60–80) data as it gives the lowest values of the AIC and BIC as compared to the other two models.

Out-Sample Error Analysis

Table 2 represents the RMSE and MAPE values for all the three models for all age categories. Results show that the Lee-Carter model is the most accurate model to predict mortality rate for all and lower age categories whereas the M8 model outperforms the other two models for high age category. In addition, the result is consistent with [14] that proved the CBD model is more accurate than the Lee-Carter model for older age group.

Table 1 AIC and BIC for the Lee-Carter, CBD and M8 model according to age categories from year (1966–1998)

Age range	Model	AIC		BIC	
		Male	Female	Male	Female
(0–80)	Lee-Carter	**26,783.32**	**23,750.22**	**27,948.140**	**24,915.04**
	CBD	1,306,643.00	1,083,337.00	1,307,033.00	1,083,728.00
	M8	555,230.400	375,914.900	556,271.10	376,955.60
(0–59)	Lee-Carter	**17,090.84**	**14,953.63**	**17,934.15**	**15,796.93**
	CBD	935,653.80	725,006.10	936,022.40	725,374.70
	M8	377,893.00	237,702.40	378,736.30	238,545.70
(60–80)	Lee-Carter	7079.03	7409.25	6866.68	7196.90
	CBD	7498.19	7796.75	6802.00	7100.56
	M8	**6258.75**	**6765.39**	**6151.95**	**6658.58**

Table 2 RMSE and MAPE for the Lee-Carter, CBD and M8 model according to age categories from year (1999–2019)

Age range	Model	RMSE		MAPE	
		Male	Female	Male	Female
(0–80)	Lee-Carter	**0.050**	**0.061**	**0.038**	**0.036**
	CBD	0.791	1.932	0.116	0.147
	M8	6.127	2.536	0.225	0.181
(0–59)	Lee-Carter	**0.072**	**0.085**	**0.032**	**0.032**
	CBD	0.535	1.318	0.077	0.099
	M8	14.502	14.889	0.578	0.518
(60–80)	Lee-Carter	0.242	0.0735	0.265	0.078
	CBD	0.234	0.0699	0.254	0.073
	M8	**0.194**	**0.0529**	**0.217**	**0.062**

4 Conclusions

This study compares the performance of the Lee-Carter model and its extension models namely the CBD model and M8 model for Malaysia population from the year 1966–2019. This study is focusing on higher age range of 60–80 years old population in Malaysia. In general, the performance of the M8 model outshined the other two models in every aspect of analyses particularly for higher age category. The performance of the M8 model is not only able to capture the cohort effects, but through a series of analyses in fitting the models, this model proves to fit most analyses best. As for the validation analysis, the M8 model also outperformed the other two models as this model has potential to do the forecasting more precisely. This research proved that the M8 model fit Malaysian data best as compared to other mortality models. The Malaysia data is suitable for the M8 model that include cohort effect to forecast the mortality rate especially for higher ages category. Nevertheless, based on the out-sample error for all the separate age categories namely, overall, lower and higher ages, the M8 model only significant for higher ages category but not for overall and the lower ages category.

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