# **Seismic Performance of Building Frames Considering Soil–Structure Interaction**



#### **M. G. Kalyanshetti and S. A. Halkude**

**Abstract** It is a fact that response of the soil influences the motion of the structure and the motion of the structure in turn influences the response of the soil which is known as soil–structure interaction (SSI). The present study aims to evaluate the seismic performance of building frames considering SSI effect on scale-down steel building frame model prepared corresponding to RC prototype building frames of various height which are designated with reference to H/B ratio (H and B being height and width of building, respectively) as low-rise building for H/B less than 3.5, mid-rise building for H/B in between 3.5 and 5.25, and high-rise building for H/B higher than 5.25. The building frames considered are square in plan with single bay in both directions resting on GW soil. The experimental setup for fixed and flexible base condition is developed in the laboratory. The scaled-down models are subjected to El Centro time history using uniaxial servo hydraulic shake table. The effect of SSI on various dynamic parameters, i.e. acceleration, velocity, and displacement are studied. The study reveals that SSI effect is not significant in the low-rise building frame. However, with the increasing H/B ratio, SSI effect goes on becoming significant and predominant with nonlinear response. The study further reveals that mid-rise building frames is severely affected than high-rise building frame. In high-rise building frame, lower 30% portion is not affected by SSI. The middle 30% portion is producing almost same results for both the base condition. However, upper 40% portion of the building is significantly affected by SSI. Therefore, it is inferred that, not all, but certain building frames get affected by SSI depending upon their configuration such as H/D ratio and plan geometry apart from ground input motion (El Centro) supported on soil (GW) soil. Hence, it is recommended to incorporate SSI in the analysis of mid-rise and high-rise building frames to identify its vulnerability against safety and stability.

M. G. Kalyanshetti ( $\boxtimes$ )

S. A. Halkude

P. A. H. Solapur University, Solapur, India

207

Civil Engineering Department, Walchand Institute of Technology, Solapur, India e-mail: [mgkalyanshetti@gmail.com](mailto:mgkalyanshetti@gmail.com)

<sup>©</sup> The Author(s), under exclusive license to Springer Nature Singapore Pte Ltd. 2021 T. G. Sitharam et al. (eds.), *Seismic Design and Performance*, Lecture Notes in Civil Engineering 120, [https://doi.org/10.1007/978-981-33-4005-3\\_17](https://doi.org/10.1007/978-981-33-4005-3_17)

**Keywords** Soil–structure interaction (SSI) · Fixed base · Flexible base · Shake table · Scaled-down model · Time history

### **1 Introduction**

Soil–structure interaction (SSI) includes a set of mechanisms accounting for the flexibility of the foundation support beneath a given structure resulting in altering the ground motion around the vicinity of the foundation compared to the free field. It determines the actual loading experienced by the soil–structure system resulting from the free-field seismic ground motions. The 1985 Mexico City earthquake and 1995 Kobe earthquake clearly illustrate the importance of local soil properties on the performance of structure. Many computational and analytical approaches for solving soil–structure interaction problems were identified [\[1\]](#page-12-0). Traditionally, it has been considered that SSI can be conveniently neglected for conservative design and also neglecting SSI tremendously reduces the complication in the analysis of the structures tempting designers to neglect the effect of SSI in the analysis [\[2\]](#page-12-1). Unfortunately, the assumption does not always hold true. During theoretical analysis, certain set of assumptions regarding idealization of the material, boundary conditions, etc. are required to be made to simplify the analysis to simulate the field conditions. However, these are not always true representation of what is happening on field. Therefore, in order to investigate the realistic behaviour, experimental study needs to be carried out which gives a valuable insight with respect to limitations of various assumptions made while carrying out theoretical analysis [\[3\]](#page-12-2). Halkude et al. investigated the SSI effect on various type of soil by winklers model and elastic continuum model. It is inferred that the SSI effect is greatly influenced by soft soil [\[4\]](#page-13-0). S. Hamid Reza et al. have studied an enhanced numerical soil–structure model which treats the behaviour of soil and structure with equal rigour. The proposed numerical soil– structure model has been verified and validated by performing experimental shaking table tests [\[5\]](#page-13-1). Akanshu Sharma et al. tested a 1:5 scaled reinforced concrete (RC) framed structure on shake table under dynamic loads to study effects of masonry infill panels and TLD in reducing the seismic response of structure [\[6\]](#page-13-2). Jayalekshmi B. R. evaluated the SSI effects on the seismic response of structures founded on Shedi soil of Dakshina Kannada by conducting experimental study on 1:10 scaled single-bay three-dimensional multistorey building models made of aluminium. The experimental results were presented and the modifications in dynamic characteristics due to the incorporation of soil flexibility were presented [\[7\]](#page-13-3).

Present experimental study aims to compare the seismic performance of building frames under the conventional fixed base conditions and flexible base conditions, on scale-down experimental steel model prepared using similitude laws. The buildings are assumed to rest on GW soil and are subjected El Centro time history motion.

Following are the objectives of proposed study.

- 1. To validate the experimental study and thereby study the impact of SSI on the seismic performance of building frame of various heights by studying impact on various dynamic parameters such as acceleration, velocity, displacement.
- 2. To formulate general guidelines regarding incorporation of SSI in the analysis by identifying SSI sensitivity of building frames with reference to their geometrical configuration.

# **2 Prototype RC Building Frame Considered for the Analysis**

For the present study, three RC building frames of G+3, G+5, and G+7 are considered square in plan having single bay in both directions with 'H' being the total height of building and 'B' being the width of the building. These buildings are designated as low-rise building to G+3 with H/B equal to 3.5, mid-rise building to G+5 with H/B equal to 5.25, and high-rise building to  $G+7$  with H/B equal to 7. The geometry of building frames is decided based on the feasibility of experimental study on scaledown steel model and the guidelines of I.S. codes [\[8,](#page-13-4) [9\]](#page-13-5). The details of dimensional characteristics are illustrated in Table [1.](#page-2-0)

Sr. No.	Contents	Description					
2	No. of stories	$G+3$	$G+5$	$G+7$			
3	Storey height	$3.5 \text{ m}$	3.5 <sub>m</sub>	$3.5 \text{ m}$			
$\overline{4}$	Grade of concrete	M 25	M 25	M 25			
5	Grade of steel	Fe415	Fe415	Fe415			
6	Bay width	4 m	4 m	4 m			
7	Slab thickness	$0.15$ m	$0.15 \;{\rm m}$	$0.15$ m			
8	Size of column	$0.45 \text{ m} \times 0.3 \text{ m}$	$0.45 \text{ m} \times 0.3 \text{ m}$ Top three stories $0.5 \text{ m} \times 0.3 \text{ m}$ <b>Bottom three stories</b>	$0.45 \text{ m} \times 0.3 \text{ m}$ Top four stories $0.55 \text{ m} \times 0.3 \text{ m}$ <b>Bottom</b> four stories			
9	Size of beam (DxB)	$0.4 \text{ m} \times 0.3 \text{ m}$	$0.4 \text{ m} \times 0.3 \text{ m}$	$0.4 \text{ m} \times 0.3 \text{ m}$			
10	Floor finish	$0.6 \text{ kN/m}^2$	$0.6$ kN/m <sup>2</sup>	$0.6$ kN/m <sup>2</sup>			
11	Live load	$4 \text{ kN/m}^2$	$4 \text{ kN/m}^2$	$4 \text{ kN/m}^2$			
12	Seismic zone	Ш	Ш	Ш			

<span id="page-2-0"></span>**Table 1** Properties of RC building frames

# **3 Scaled-Down Model**

The important part for experimental study is to develop an experimental model which will able to represent identical behaviour as that of prototype structure. However, it is found that every aspect of the structure could not be scaled due to limitation of pay load capacity of Shake Table. But it should not be considered as a matter of big concern, since the objective of the test is not to qualify the prototype structure by testing but to study the behaviour of structures, in general, subjected to dynamic loading. The major task in the scaling down process is to achieve 'dynamic similarity' where model and prototype experience homologous forces. For this purpose, approach suggested by Meymand [\[10\]](#page-13-6) is adopted. According to this approach, three principal test conditions establish many of the scaling parameters. The first condition is that testing is conducted in 1-g environment, which defines model and prototype accelerations to be equal. Secondly, a model with similar density to the prototype is desired, fixing another component of the scaling relations. Thirdly, the test medium is primarily composed of GW soil, whose undrained stress–strain response is independent of confining pressure, thereby simplifying the constitutive scaling requirements. In addition to the three principal test conditions, Meymand [\[10\]](#page-13-6) pointed out that the natural frequency of the prototype should be scaled by an appropriate scaling relation. By defining scaling conditions for density and acceleration, the mass, length, and time scale factors can all be expressed in terms of the geometric scaling factor (S), and a complete set of dimensionally correct scaling relations (ratio of prototype to model) can be derived for all variables being studied. Based on above discussion, following two conditions are established for preparing scaled-down model:

- (1) Natural frequency of the prototype shall be scaled by an appropriate scaling relation to that of model.
- (2) Density of the prototype and model shall be similar.

### *3.1 Scale Factor*

Adopting appropriate geometric scale factor is one of the important steps in scale modelling on shake table. Although small scale models could save cost, the precision of the results could be substantially reduced. Considering the specifications of shake table, the c/c distance between two columns is set as 0.32 m leading to a linear scale factor, of  $4.0/0.32 = 12.5$  (column spacing in prototype structure is 4 m). Therefore, employing geometric scaling factor of 1:12.5 as explained above, height, length, and width of the structural model is obtained as 1.120 m, 0.32 m, and 0.32 m, respectively. The scaling relations for the various parameter adopted in this study are shown in Table [2](#page-4-0) [\[11\]](#page-13-7).

Parameters-	$\vert$ Mass density $\vert$ Acceleration $\vert$ Frequency $\vert$		$\vert$ Time $\vert$ Length $\vert$ EI	Strain
Scale factor-		$\sigma-1/z$		

<span id="page-4-0"></span>**Table 2** Scaling relations in terms of geometric scaling factor [\[11\]](#page-13-7)

### *3.2 Scaling Down of Building Frames*

The scaled-down models of all the three building frames are prepared based on similitude laws. However, the scale-down procedure of G+3 building frame is discussed here as a representative case.

According to the first condition, the relation between natural frequency of model and prototype as per Table [2](#page-4-0) is

<span id="page-4-1"></span>
$$
f_m/f_p = S^{-1/2}
$$
  
= 3.54 (1)

Natural frequency of the G+3 prototype structure as calculated by application software (modal analysis) is,  $f_p = 1.4792$  Hz. Therefore, required frequency of the model  $(f_m)$  in accordance with Eq. [\(1\)](#page-4-1) is 5.24 Hz.

Density of the prototype structure  $(\rho_m)$  is 264.01 kg/m<sup>3</sup>. Therefore, as per the second condition, the mass of the structural model  $(M_m)$  is estimated as:

$$
M_m = \rho_m \times V_m
$$
  
= 264.01 × (1.12 × 0.32 × 0.32)  
= 30.27 Kg (2)

The dimensions of column and slab of scaled-down steel model are determined so that the weight of model nearly equals to 30.27 kg, and also, it satisfies stiffness and flexural rigidity as required by simulated laws. Considering all above, the dimensions of scaled-down steel model are worked out. The details are presented in Fig. [1a](#page-5-0). Similar calculations were done for G+5 and G+7 steel models and the details of all the three models are presented in Table [3.](#page-5-1)

#### *3.3 Validation*

The accuracy of the scaled-down models is ascertained by comparing the roof acceleration of G+3 scaled-down model with the roof acceleration of same model developed using ETAB application software subjected to El Centro ground excitation. The model developed in ETAB is shown in Fig. [1b](#page-5-0). The comparison of roof acceleration corresponding to experimental study and analytical study (using application software) is presented in Fig. [2.](#page-5-2) It is observed from Fig. [2](#page-5-2) that the roof acceleration

<span id="page-5-0"></span>





<span id="page-5-1"></span>**Table 3** Geometric and material properties of steel scaled-down model



<span id="page-5-2"></span>



produced in experimental study is almost 10 to 15% higher than that of analytical study. This variation is obvious due to limitation in reproducing the replica model. In the analytical study, analysis is carried out by applying uniaxial excitation and acceleration is obtained in the same direction by restraining the movement of structure in other direction. However, in the laboratory even though the excitation is uniaxial but

the model is not restrained to respond in only one direction; hence, the acceleration produced in the laboratory is higher than that produced by analytical study. Therefore, variation in the acceleration is considered to be reasonably within accepted limit, and thus, the scale-down model is validated. The same ideology is adopted to validate G+5 and G+7 model.

### <span id="page-6-0"></span>**4 Experimental Setup on Shake Table**

The shake table at the Civil Engineering Department, Walchand Institute of Technology, is uniaxially driven. The table size is  $2 \text{ m} \times 2 \text{ m}$  with maximum payload capacity of 30 kN with an operating frequency range of 0.01–50 Hz. Objective of the present study is to evaluate the change in the various responses of structure such as acceleration, velocity, displacement, etc. for fixed base and flexible base condition. Accordingly, experimental setups are developed in the laboratory to produce fixed base condition and flexible base condition. These are described in following sections.

# *4.1 Fixed Base Condition*

The fixed base response of the scaled-down model is investigated by securing the foundation of the model directly to the platform of the shaking table. Four accelerometers are used to acquire the acceleration data. Accelerometer no. 1 is placed at bottom of shake table (actuator) to verify the preciseness of simulations of the ground acceleration. Accelerometer no. 2, 3, and 4 are used to record accelerations of various floor levels. The fixed base setup is shown in Fig. [3.](#page-7-0)

# *4.2 Flexible Base Condition*

Flexible base conditions refer to the case, wherein the foundation along with the sub soil is considered. To simulate this condition in the laboratory, a laterally confined soil mass beneath the footing is required to be used. Therefore, a container made of steel plates is used to produce this lateral confinement. The experimental scaleddown model is square in plan; therefore, in order to have equal quantity/mass of soil all around the model on all sides, the square container is used. The plan dimensions of this container are kept to the maximum possible extent considering the size limitation of shake table  $(2 \times 2 \text{ m}^2)$  and adequate space required for the fixtures for mounting the container on shake table. The depth of the container is decided based on the requirement to account for embedment depth [\[12\]](#page-13-8). Payload capacity (30 kN) of shake table is also considered while developing the setup. Thus, in view of all these, the steel container of size 1.5 m  $\times$  1.5 m  $\times$  0.7 m is used. The soil is filled in and



<span id="page-7-0"></span>**Fig. 3** Experimental setup of fixed base condition

compacted in layers of 10 cm thickness with 95% of MDD. Then the experimental model is kept on the soil mass. The complete setup of the flexible base condition is shown in Fig. [4.](#page-7-1)

The locally available soil is used for the study. The various properties of soil used are determined in the laboratory in accordance with respective IS codes [\[13](#page-13-9)[–16\]](#page-13-10). These properties are presented in Table [4.](#page-8-0)

<span id="page-7-1"></span>

(a) Size details (b) Laboratory set up



<span id="page-8-0"></span>

#### *4.3 Input Time History*

For the seismic analysis of structure, one of the most important tasks is to select the proper real ground motion. Among the many existing ground motions, the severest ground motion should be selected that can drive the structure to its critical response and thereby result in the highest damage potential. El Centro 1940 (E-W) record obtained during the Imperial Valley, California earthquake of 18 May 1940 is one of such severest ground motion record that is used extensively in research studies. Therefore, in the present study also, the El Centro time history used. The acceleration spectrum is presented in Fig. [5.](#page-9-0)

### **5 Results and Discussion**

The scaled-down models are used in the laboratory to evaluate their responses for fixed and flexible base condition to investigate SSI effect. The laboratory setup as described in Sect. [4](#page-6-0) is developed. All the models are subjected to El Centro time history. The acceleration is recorded by accelerometers, velocity is calculated by integrating acceleration, and further displacement is calculated by integrating velocity. Thus, these parameters at each floor level of all the building frames with fixed and



<span id="page-9-0"></span>Fig. 5 El Centro time history [\[17\]](#page-13-11)

flexible base conditions are calculated and plots are developed to represent the variation of these parameters with respect to time. Typical plots of G+3 building only are shown below for fixed base condition and flexible base condition in Figs. [6](#page-9-1) and [7,](#page-10-0) respectively. The plots are of responses at roof level.

It is observed from Fig. [6](#page-9-1) that it is observed that fixed base condition is producing maximum acceleration of 2.10 g, maximum velocity of 0.4 m/s, and maximum displacement of 9.2 mm, whereas from Fig. [7,](#page-10-0) the same responses for flexible base condition are  $1.23$  g,  $0.25$  m/s and 5 mm, respectively. This shows that flexible base is producing almost 55–65% lower responses in comparison with fixed base condition. However, this response is of roof level only. Such plots of all floor levels are obtained to identify the effect of flexible base all over the building frame. Such kind of plots for G+5 and G+7 building frames are also obtained at all floor levels. The maximum acceleration, velocity, and displacement corresponding to fixed and flexible base of all building frame are compared to identify SSI effect. These plots are presented in next section.



<span id="page-9-1"></span>**Fig. 6** Fixed base response of G+3 building frame



<span id="page-10-0"></span>**Fig. 7** Flexible base response of G+3 building frame

### *5.1 Comparison of Fixed and Flexible Base Response*

The comparison of dynamic response corresponding to fixed base and flexible base of G+3 (low-rise), G+5 (mid-rise), and G+7 (high-rise) building frame is presented in Figs. [8,](#page-10-1) [9](#page-10-2) and [10,](#page-11-0) respectively.

#### (a) **G+3 (low-rise) building frame**:

It is observed from Fig. [8a](#page-10-1), b that the acceleration and velocity corresponding to fixed base condition goes on increasing nonlinearly from first floor to fourth



<span id="page-10-1"></span>**Fig. 8** Comparison of dynamic parameters for G+3 (low-rise) building frame



<span id="page-10-2"></span>**Fig. 9** Comparison of dynamic parameters for G+5 (mid-rise) building frame



<span id="page-11-0"></span>**Fig. 10** Comparison of dynamic parameters for G+7 (high-rise) building frame

floor. The rate of increment is observed to be in the range of 20% to 30% up to third floor level, which further increases to in the range of 45% to 50% from third to fourth storey. From Fig. [8c](#page-10-1), it is observed that the displacement is increasing almost linearly at all floor levels in the range of 50% to 55% in comparison with previous floor.

The comparison between fixed and flexible base conditions presented in Fig. [8a](#page-10-1)– c reveals that flexible base condition is producing lower responses than fixed base. However, this difference is not the same in all floors. The acceleration and velocity are almost 10–15% lower than fixed base up to third floor, and suddenly, this increase becomes steeper up to 30% from third to fourth floor. However, displacement corresponding to flexible base is constantly 30–35% lower than fixed base at all floor levels. It is to be noted here that since the flexible base conditions are producing lower response than fixed base condition, SSI is not significant in low-rise building.

#### (b) **G + 5 (mid-rise) building frame**:

Figure [9a](#page-10-2)–c shows that nature of variation in dynamic response with respect to no. of storey for both fixed and flexible conditions are of alike and nonlinear in nature.

The comparison between fixed and flexible base conditions presented in Fig. [9a](#page-10-2)– c reveals that the response is reversed as compared to G+3 building. Flexible base condition is producing higher response than fixed base. Flexible base is producing almost 30–40% higher response in the bottom (upto 3rd floor) and top floors (floor 6–8), whereas in the middle floors (floor 3–6), the response is 20–25% higher than the fixed base condition. Thus, middle floors are least affected by SSI. However, since at all floor, flexible base is producing higher response than fixed base, SSI is significant in mid-rise building frame.

#### (c) **G + 7 (high-rise) building frame**:

It is observed from Fig.  $10$  that the reversal of trend is observed  $\omega$  5th floor. Flexible base is producing lower response than fixed base up to @ 5th floor, and beyond this, flexible base is producing higher response than fixed base.

The comparison between fixed and flexible base conditions presented in Fig. [10a](#page-11-0)– c reveals that unlike G+5 building frame, SSI effect is not significant at all floors. In the lower 30% of total height, flexible base condition is producing 20–30% lower response than fixed base condition. In the mid-height from 30 to 60% of total height, the fixed and flexible base conditions are producing almost similar response with a marginal variation. In the remaining upper 40% of total height, flexible base condition is producing 30–40% higher response than fixed base condition. This indicates that SSI is significant in top 40% height of building frame.

## **6 Conclusion and Recommendation**

In the present study, SSI effect is investigated experimentally using G+3, G+5, and G+7 scaled-down steel building frames which are designated as low-rise building (H/B equal to 3.5), mid-rise building (H/B equal to 5.25), and high-rise building (H/B equal to 7) respectively. It is well understood that SSI is influenced by structural configuration, supporting soil and the ground excitation. However, in the preset study supporting soil (GW soil) and the ground excitation (El Centro) being constant, the study is focused on assessment of performance of building frame only. Based on the obtained data, the following conclusions are drawn.

- 1. SSI effect is not significant for low-rise building frames. It is due to the fact that these buildings are not influenced by seismic forces. Hence, the conventional fixed base analysis is recommended for such buildings.
- 2. The study reveals that mid-rise and high-rise buildings are influenced by SSI. In mid-rise building, SSI effect is prominently observed as compared to high-rise buildings. In high-rise building, SSI effect is not significant in almost lower 60% of height, whereas only upper 40% of height is affected by SSI. This reveals that the structural configuration plays an important role. Due to this fact, in the present study, it is observed that the mid-rise buildings are greatly affected than high-rise building frame.
- 3. In case of any variation in the parameters, the performance of building frame is likely to vary.
- 4. The above discussions reveal that conventional analysis excluding SSI may not be adequate to ensure the structural safety. Therefore, it is recommended to incorporate SSI in the analysis of at least mid-rise and high-rise building frames.

# **References**

- <span id="page-12-0"></span>1. Gazetas G (1991) Formulas and charts for impedances of surface and embedded foundations. J Geotech Eng 117(9):1363–1381
- <span id="page-12-1"></span>2. Dutta SC, Bhattacharya K, Roy R (2004) Response of low-rise buildings under seismic ground excitation incorporating soil–structure interaction. Soil Dyn Earthq Eng 24:893–914
- <span id="page-12-2"></span>3. Hirave V, Kalyanshetti M (2018) Seismic response of steel braced building frame considering soil structure interaction (SSI): an experimental study. J Inst Eng India, Ser A 99(1):113–122
- <span id="page-13-0"></span>4. Halkude SA, Kalyanshetti MG (2014) Soil structure interaction effect on seismic response of R.C. frames with isolated footing. Int J Eng Res Technol 03(1):2767–2775
- <span id="page-13-1"></span>5. Tabatabaiefar SHR, Fatahi B, Samali B (2014) Numerical and experimental investigations on seismic response of building frames under influence of soil-structure interaction. Advan Struct Eng 17(1):109–130
- <span id="page-13-2"></span>6. Sharma A, Reddy GR, Vaze KK (2012) Shake table tests on a non-seismically detailed RC frame structure. Struct Eng Mech 41(1):1–24
- <span id="page-13-3"></span>7. Jayalekshmi BR, Lohith K, Shivashankar R, Venkataramana K (2008) Experimental investigation on dynamic characteristics of structures founded on dispersive soil. The 12th international conference of international association for computer methods and advances in geomechanics (IACMAG) Goa, India, pp 2717–2724
- <span id="page-13-4"></span>8. IS 1893 (part 1) (2002) Indian standard criteria for earthquake resistant design of structures, Bureau of Indian Standards, New Delhi
- <span id="page-13-5"></span>9. IS 456 (2000) Plain and reinforced concrete—Code of practice, Bureau of Indian Standards, New Delhi
- <span id="page-13-6"></span>10. Meymand PJ (1998) Shaking table scale model tests of nonlinear soil-pile-superstructure interaction in soft clay. Ph.D. thesis in civil engineering. University of California, Berkeley
- <span id="page-13-7"></span>11. Sabnis GM, Harris HG, White RN, Mirza MS (1983) Structural modeling and experimental techniques, prentice hall Inc., Engelwood Cliff, New Jersey
- <span id="page-13-8"></span>12. Bowles JE (1996) Foundation analysis and design. Tata McGraw-Hill, New Delhi
- <span id="page-13-9"></span>13. IS-2720 (Part 3) (1980) "Methods of test for soil—determination of specific gravity" Bureau of Indian Standards, New Delhi
- 14. IS-2720 (Part 7) (1980) Methods of test for soils—determination of water content-dry density relation using light compaction. Bureau of Indian Standards, New Delhi
- 15. IS-2720 (Part 13) (1986) Methods of test for soils—direct shear test. Bureau of Indian Standards, New Delhi
- <span id="page-13-10"></span>16. IS-2720 (Part 5) (1985) Methods of test for soils—determination of liquid and plastic limit. Bureau of Indian Standards, New Delhi
- <span id="page-13-11"></span>17. Chopra AK (2003) Dynamics of structures, theory and application to earthquake engineering. Prentice Hall, New Delhi