Calculation and Experimental Study on Comprehensive Stiffness of Angular Contact Ball Bearings

Peng Sun, Weifang Chen[(✉)], Chuan Su, and Yusu Shen

College of Mechanical and Electrical Engineering,
Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
1826193818l@l63.com, meewfchen@nuaa.edu.edu.cn,
1275ll062l@qq.com, shenyususue@sina.com

Abstract. The stiffness of the rolling bearing has a great influence on the dynamic characteristics of motorized spindle. In order to obtain the stiffness of angular contact ball bearing under oil-jet lubrication, the theoretical calculation model and experiment method are proposed in this paper. Based on hertz contact theory and elastohydrodynamic lubrication theory, the quasi-static analysis model of angular contact ball bearing is established. The factors including ball centrifugal force, gyro moment, preload and oil film thickness are taken into consideration and theoretically analyzed. The test platform on comprehensive stiffness is designed and the stiffness is experimentally verified under different factors. The results show that the comprehensive stiffness increases with the increase of axial, radial load and preload but decreases with the increase of rotational speed. Therefore, the adjustment of the working condition of angular ball bearing can make it work in a better mechanical state and thus improve the bearing life.

Keywords: Angular contact ball bearings · Comprehensive stiffness ·
Oil film stiffness · Contact stiffness

1 Introduction

Angular contact ball bearings are important driving and supporting parts in numerical control machines. In the process of supporting loads, the boundary conditions of bearings are constantly changing, so the description of the stiffness matrix is more complicated. Besides, as an elastomer, the mechanical properties of bearings play a key role in the dynamic characteristics of a motorized spindle system and can significantly influence the dynamic performance of the motorized spindle system.

Many scholars had done some research on this. Chen [1] regarded angular contact ball bearing as the research object and derived five-DOF expression of stiffness without considering factors such as oil film or centrifugal force. Yu [2] established a calculation model of angular contact ball bearings according to the principle of elastohydrodynamic

lubrication and the result showed that the stiffness of bearing tends to decrease when considering the lubricating oil. Tang [3] built a quasi-dynamics analysis model with centrifugal force, gyroscopic moment and lubricating oil film, which demonstrated that the contact stiffness and oil film stiffness are not much different when the rotational speed of bearings reaches a certain level. Xiong [4] developed a calculation model of bearing considering lubricating oil, the deformation of ring, the effects of temperature and the centrifugal force. Other factors were also studied under different preload and speed condition. Shi [5] set up the test platform on axial stiffness of bearing, concluding that the hertz analytical solution is rather consistent with the measured value when the load value is small. However, as the load increases, the error will increase. Li [6] obtained the dynamic stiffness of the angular contact ball bearing by making use of the synchronous rotational radial load with unbalanced mass. However, the rotational speed and the radial load have an inter-related relationship. Besides, the test platform has certain limitations and can't comprehensively analyzes the influencing factors. Fang [7] established the deformation expression of angular contact ball bearing with the coordinate transformation method and designed the relevant test platform. However, what the test platform measured is the absolute displacement of the bearing and can't fully meet the exact requirement. Ali [8] and Lambert [9] obtained the radial stiffness of bearing by measuring the mass, acceleration and relative displacement of the inner and outer ring, which indirectly expresses the contact force between the shaft and the inner ring by the product of acceleration and mass of shaft and provides a new idea for solving the stiffness of bearing through the reasonable matching of sensor position and data processing method.

At present, the existing mechanical model of bearings can realize the approximate calculation of stiffness, but the comprehensive factors have not been fully analyzed. It has not yet reached the precise level in the test part of stiffness. Due to the poor repeatability of the test results, it can not play a good guiding significance. There are not many related references about the test part of the rolling bearing comprehensive stiffness. Thus, a perfect test plan has not yet been formed. Based on above investigation and survey, angular contact ball bearing will be carefully studied with an improved quasi-static model. Furthermore, a complete experimental scheme is proposed to verify the theoretical analysis.

2 Theoretical Calculation Model

According to the hertz elastic contact theory [10] and elastohydrodynamic lubrication theory, analysis of the stress condition of angular contact ball bearings has been conducted. As shown in Fig. 1, the position change of inner and outer ring curvature center, before and after the angular contact ball bearing is loaded, has been showed in the schematic diagram. Since the outer ring and the housing are fixed, point E and E′ coincide.
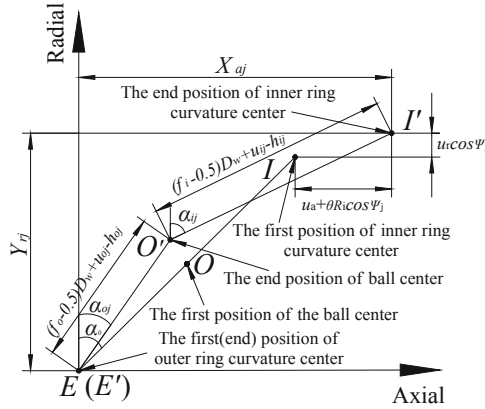
Fig. 1. The position change of inner and outer ring curvature center before and after loaded

2.1 Geometric Equation

According to Fig. 1, for the ball in any position j, the following formula holds:

$$\begin{cases} X_{aj} = L_{IE} \sin \alpha_0 + u_a + \theta R_i \cos \psi_j \\ Y_{rj} = L_{IE} \cos \alpha_0 + u_r \cos \psi_j \\ R_i = 0.5 D_m + (f_i - 0.5) D_w \cos \alpha_0 \end{cases} \quad (1)$$

where α_0 is the initial contact angle; L_{IE} is the distance between inner and outer ring curvature center when the bearing is not loaded; u_a, u_r and θ are respectively axial and radial deformation and rotation angle after the bearing is loaded; R_i is the radius of the circle where the inner ring curvature center is located; D_m is the diameter of the bearing pitch; D_w is the diameter of the ball; $f_{i(o)}$ is the curvature of inner (outer) groove.

Considering the axial and radial load, the centrifugal force of balls, the gyro moment and the lubricating oil film, the contact angle of the inner ring (outer ring is the same) can be expressed as:

$$\begin{cases} \sin \alpha_{ij} = \frac{X_{aj} - [(f_i - 0.5)D_w + u_{oj} - h_{oj}] \sin \alpha_{oj}}{(f_i - 0.5)D_w + u_{ij} - h_{ij}} \\ \cos \alpha_{ij} = \frac{Y_{rj} - [(f_i - 0.5)D_w + u_{oj} - h_{oj}] \cos \alpha_{oj}}{(f_e - 0.5)D_w + u_{ij} - h_{ij}} \end{cases} \quad (2)$$

where u_{ij} and u_{oj} are the hertz deformation between the jth ball and the contact point of inner and outer ring; h_{ij} and h_{oj} are the oil film thickness between the jth ball and the contact point of inner and outer ring.

2.2 Force Balance Equation Between the Ball and the Inner and Outer Ring

When the angular contact ball bearing rotates in high speed, in addition to the contact force generated by external load, the combined effect of centrifugal force and gyro

moment are also applied on the ball. The force on the ball and the inner ring is shown in Fig. 2
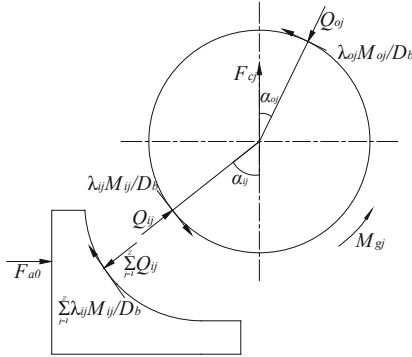

Fig. 2. The force on the ball and the inner ring

The force balance relationship is determined by the following formulas:

$$\begin{cases} Q_{ij} \sin \alpha_{ij} - Q_{oj} \sin \alpha_{oj} - \lambda_{ij} \frac{M_{gj}}{D_b} \cos \alpha_{ij} + \lambda_{oj} \frac{M_{gj}}{D_b} \cos \alpha_{oj} = 0 \\ Q_{ij} \cos \alpha_{ij} - Q_{oj} \cos \alpha_{oj} + \lambda_{ij} \frac{M_{gj}}{D_b} \sin \alpha_{ij} - \lambda_{oj} \frac{M_{gj}}{D_b} \sin \alpha_{oj} + F_{cj} = 0 \end{cases} \quad (3)$$

where Q_{ij} and Q_{oj} are the forces between the ball and the inner and outer ring, they could be expressed as:

$$\begin{cases} Q_{ij} = K_{ij} u_{ij}^{1.5} \\ Q_{oj} = K_{oj} u_{oj}^{1.5} \end{cases} \quad (4)$$

When the system is statically balanced, the radial load and bearing load on the inner ring of the bearing can be expressed respectively as:

$$\begin{cases} F_a + F_{a0} - \sum_{j=1}^{Z} (Q_{ij} \sin \alpha_{ij} + \lambda_{ij} \frac{M_{gj}}{D_b} \cos \alpha_{ij}) = 0 \\ F_r - \sum_{j=1}^{Z} (Q_{ij} \cos \alpha_{ij} - \lambda_{ij} \frac{M_{gj}}{D_b} \sin \alpha_{ij}) \cos \psi_j = 0 \\ M - \sum_{j=1}^{Z} [(Q_{ij} \sin \alpha_{ij} + \lambda_{ij} \frac{M_{gj}}{D_b} \cos \alpha_{ij}) R_i - r_i \lambda_{ij} \frac{M_{gj}}{D_b}] \cos \psi_j = 0 \end{cases} \quad (5)$$

where Z is the number of balls; F_{a0} is the initial preload; r_i is the radius inner ring curvature.

Equations (1), (2), (3), (4) can be combined as the first group of equations, and (5) as the second group of equations, then they can be solved jointly by Newton-Raphson

iterative method. Under the condition, the axial and radial deformation and the angle of the bearing could be solved and then the comprehensive stiffness of the bearing with these parameters can be obtained.

3   Test and Analysis

3.1   Test Platform

In order to verify the correctness of the theoretical model, a test platform of bearings on comprehensive stiffness is designed in this paper. As shown in Fig. 3, the test platform consists of five parts: power output system, spindle transmission system, axial and radial loading system, lubrication system and data acquisition system.
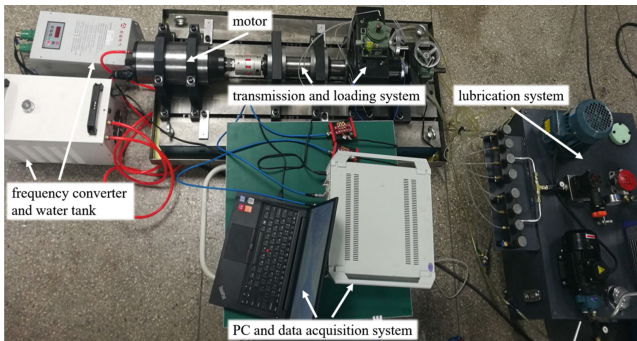
Fig. 3.   The test platform of bearing on comprehensive stiffness

The axial loading and testing part includes a loader, a piezoelectric force sensor and a displacement sensor. The axial force is applied on the outer ring of the bearing through a cylinder by the axial loader and can be measured by the force sensor located between the loader and the cylinder. The displacement sensor is mounted in the center hole of the cylinder for measuring the axial displacement of the end face of the shaft. The value measured by the axial displacement sensor represents the relative displacement of the inner and outer ring since the inner ring and the shaft, the outer ring and the cylinder have the same displacement in the axial direction.

The radial loading and testing part includes a loader, a piezoelectric force sensor, a displacement sensor and the housing. Similar to the principle of axial testing, the inner ring and the shaft, the outer ring and the housing have the same displacement in the radial direction and the radial value measured by the sensor which is embedded in the housing represents the relative displacement of the inner and outer ring in the radial direction.

The supporting assembly fixed on the baseboard is used to support for the transmission shaft. Several oil and gas holes and rotating shaft seals are provided on the outer surface of the housing for bearing lubrication.

3.2 Principle

The test principle of comprehensive stiffness is shown in Fig. 4. The slowly increasing load in the axial and radial direction will be applied on the bearing, and then the physical output signal of each sensor is converted into an identifiable analog signal through the transmitter. After filtering and other calculation, the actual force-displacement data can be obtained. Finally the axial and radial stiffness of bearings are obtained by deriving the force-displacement curve which is obtained by data fitting.
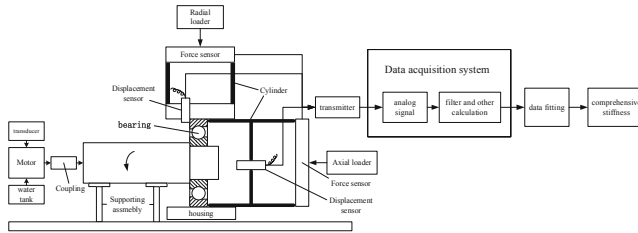

**Fig. 4.** The test principle of comprehensive stiffness

3.3 Result and Analysis

The type of the angular contact ball bearing selected in this paper is S7212AC. The basic parameters are shown in Table 1.

Table 1. The basic parameters of S7212AC

Parameter	Symbol	Value
Outer diameter	D	110 mm
Inner diameter	d	60 mm
Width	B	22 mm
Ball number	Z	16
Elasticity modulus	E	2.07×10^{11} Pa
Density	P	7800 kg/m^3
Ball diameter	D_w	12.5 mm
Initial contact angle	α	25°
Curvature of outer groove	f_e	0.53
Curvature of inner groove	f_i	0.52
Poisson's ratio	μ	0.3
material	l	9Cr18

The Utel dynamic signal acquisition device is used for data acquisition. The acquired displacement and force data are polynomial fitted to obtain a fitting curve. The slope of the curve represents the stiffness in a certain working condition. Figure 5 shows the stiffness comparison of the test processing result and the theoretical result.
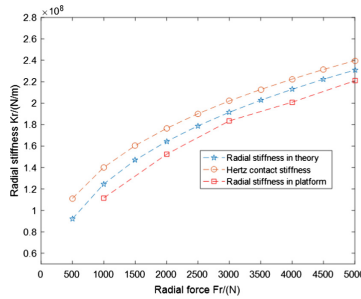
(a)   The stiffness comparison in radial direction
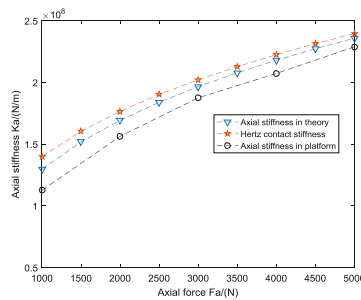
(b)   The stiffness comparison in axial direction

Fig. 5. The stiffness comparison of the test processing result and the theoretical result

When the radial load changes from 0 to 5000 N, the test of comprehensive stiffness on the angular contact ball bearing under operating conditions is carried out. When the bearing is running, the displacement data is periodically fluctuated. The mean value of the displacement data corresponding to the related load is fitted to obtain the force-displacement relationship of the angular contact ball bearing and then the bearing stiffness is obtained.

As shown in Fig. 6, comparative tests have been done under different working conditions. It can be seen that the stiffness generally decreases with the increase of the rotational speed. And the larger the radial load at the same speed is, the larger the radial comprehensive stiffness becomes.
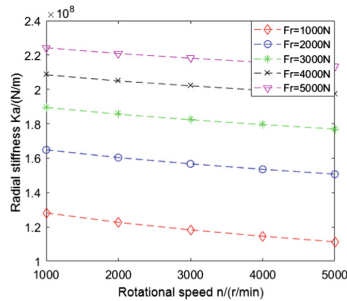
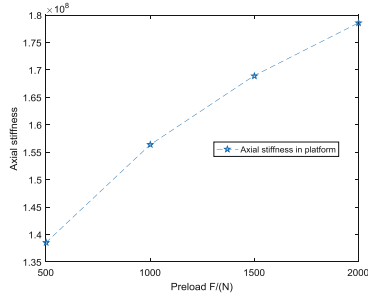(a) Radial stiffness under different working conditions
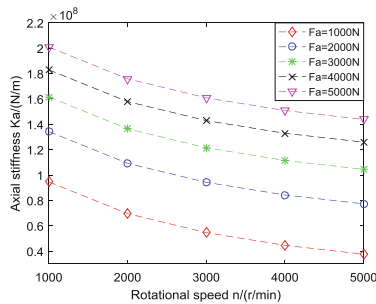
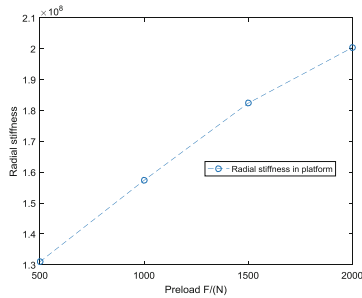(b) Axial stiffness under different working conditions

Fig. 6. The comparative test under different working condition

The comparative test of the comprehensive stiffness of angular contact ball bearings subjected to different preload has been carried out. As the preload increases, the contact load between the ball and the inner and outer ring increases when other factors remain unchanged, which will lead to the increase of the contact stiffness and account for why the axial and radial comprehensive stiffness increase in Fig. 7.

(a) Radial stiffness under different preload

(b) Axial stiffness under different preload

Fig. 7. The stiffness under different preload

4   Conclusions

In this paper, the calculation model of the comprehensive stiffness of angular contact ball bearings was established. The effects of external load, centrifugal force, rotational speed, lubricating oil film, preload and gyro moment were fully considered. The test platform of comprehensive stiffness was built and the influence under different working conditions was analyzed. The main conclusions are as follows:

(1) Under the premise that other factors remain unchanged, the axial and radial comprehensive stiffness of angular contact ball bearings increase with the increase of axial and radial loads. The maximum error between the theoretical radial comprehensive stiffness and the measured radial stiffness is 6.5%, while the maximum error between the theoretical axial stiffness and the measured axial stiffness is 13% and the error decreases with the increase of the load;

(2) The comprehensive stiffness of the angular contact ball bearing will decrease when the elastohydrodynamic lubrication effect is considered;

(3) With the increase of the rotational speed, the axial comprehensive stiffness of the angular contact ball bearing will decrease, for the oil film thickness of the lubricating oil is reduced, which leads to the increase of the oil film stiffness. Different from the axial comprehensive stiffness, the radial comprehensive stiffness is almost unchanged;

(4) The increase of the preload will make the axial and radial comprehensive stiffness of the angular contact ball bearing increase.

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