New LMI Criteria to the Global Asymptotic Stability of Uncertain Discrete-Time Systems with Time Delay and Generalized Overflow Nonlinearities

Pushpendra Kumar Gupta and V. Krishna Rao Kandanvli

Abstract This paper investigates the problem of stability analysis of discrete-time systems under the effect of generalized overflow nonlinearities, parameter uncertainties, and time delay. The systems under assumption involve norm-bounded parameter uncertainties. Two stability criteria based on Linear Matrix Inequality (LMI) approach are presented. The usefulness of the presented criteria is numerically proved.

Keywords Generalized overflow nonlinearity · Global asymptotic stability · Time delay · Parameter uncertainty · Lyapunov method · Linear matrix inequality

1 Introduction

Several practical engineering systems such as Markovian jump systems [1], network control systems [2], neural networks [3], sensor networks [4], etc. can be transformed as discrete systems which can be represented as state-space model.

In these discrete-time systems, the delay can be occurred due to channel, for e.g., transportation delay or some other reason, which may tend the system to be unstable. So, stability analysis of systems having delay is important. Delay may be constant, time-varying, and random in nature. Several studies based on the concept of delay are previously reported [1, 3, 5–13].

Parameter uncertainty is also an important factor for the instability of the discrete systems. Parameter uncertainty arises due to various factors such as modeling errors, finite resolution of the measuring equipment, variation in system parameters, and some other ignored factors. [14–16].

P. K. Gupta (✉) · V. K. R. Kandanvli
Department of Electronics and Communication Engineering, Motilal Nehru National Institute of Technology Allahabad, Prayagraj 211004, India
e-mail: pushpendranit30@gmail.com

V. K. R. Kandanvli
e-mail: rao@mnnit.ac.in

D. Dutta et al. (eds.), *Advances in VLSI, Communication, and Signal Processing*, Lecture Notes in Electrical Engineering 587,
https://doi.org/10.1007/978-981-32-9775-3_79

While implementing linear discrete-time systems in digital computer with finite wordlength processors, nonlinearities may occur in the systems. Owing to the presence of such nonlinearities, there may be a chance that systems exhibit unstable nature. So, it is very important to find the system parameters range for which the given system is stable. In the present work, the effects of overflow nonlinearities are only taken into account and the quantization effects are assumed as negligible [5, 17–23].

The stability analysis problem of a class of discrete-time uncertain systems having generalized overflow nonlinearities and time delay is significant and more realistic. Very little contribution has been done in the literature [5, 17] so far. In [5], a delay-independent stability criterion is presented and in [17], delay-dependent stability result is provided by employing free-weighting matrix method. Generally, delay-dependent stability analysis provides less conservative results than delay-independent one. A tighter bound inequality, i.e., Wirtinger-based inequality is used in [24, 25] for getting better stability results. Motivated by these concerns and inspired by the work presented in [5, 7, 17, 19, 24–27], we revisit the problem under consideration.

The main aim of the paper is to establish improved delay-dependent criteria for the stability of the systems under consideration. The key involvements of the paper are (1) The system presented covers a wider class of discrete-time systems employing norm-bounded parameter uncertainties, time delay, and generalized overflow nonlinearities. (2) Wirtinger-based inequality [24, 25] is used to deal the sum and cross terms present in the forward difference of Lyapunov function for deriving the delay-dependent stability criteria of the present system which may provide better results. (3) The presented criteria are computationally less complex as they are Linear Matrix Inequality (LMI) based.

The rest of the paper and its organization are given as follows: In Sect. 2, descriptions of the considered systems and some required lemmas are given. Delay-dependent criteria for the stability analysis of the discrete-time systems are presented in Sect. 3. Section 4 shows the effectiveness of the proposed criteria with numerical examples. Finally, concluding remarks are provided in Sect. 5.

In this paper, notations are considered as follows: $\mathbb{R}^k$ represents k-dimensional Euclidean space, $\mathbb{R}^{\alpha \times \beta}$ denotes set of $\alpha \times \beta$ matrices with real elements, $\mathbf{0}$ is a matrix or vector with all elements are zero, $\boldsymbol{I}$ refers identity matrix of compatible dimension, $\boldsymbol{P}^{\mathrm{T}}$ stands transpose of the matrix $\boldsymbol{P}$, $\boldsymbol{P} > \mathbf{0} (< \mathbf{0})$ shows $\boldsymbol{P}$ is positive (negative) definite real symmetric matrix, symbol $*$ stands symmetric terms in a symmetric matrix. $diag(a, b, c)$ means diagonal matrix with diagonal elements a,b,c.

2 System Description

The description of the system under consideration is given by

$$\begin{aligned}\hat{\boldsymbol{x}}(r+1) &= \hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) \\ &= [\hat{f}_1(\hat{y}_1(r))\ \hat{f}_2(\hat{y}_2(r))\ \cdots\ \hat{f}_n(\hat{y}_n(r))]^{\mathrm{T}}\end{aligned} \tag{1a}$$

$$\begin{aligned}\hat{\boldsymbol{y}}(r) &= \overline{\boldsymbol{A}}\,\hat{x}(r) + \overline{\boldsymbol{A}}_d\ \hat{\boldsymbol{x}}(r-d) \\ &= \left[\hat{y}_1(r)\ \hat{y}_2(r)\ \cdots\ \hat{y}_n(r)\right]^{\mathrm{T}}\end{aligned} \tag{1b}$$

$$\hat{\boldsymbol{x}}(r) = \boldsymbol{\varphi}(r),\ \forall r\ \in [-d,\ 0] \tag{1c}$$

$$\bar{\boldsymbol{A}} = \boldsymbol{A} + \Delta\boldsymbol{A},\ \ \bar{\boldsymbol{A}}_{\boldsymbol{d}} = \boldsymbol{A}_{\boldsymbol{d}} + \Delta\boldsymbol{A}_{\boldsymbol{d}} \tag{1d}$$

where $\hat{\boldsymbol{x}}(r) \in \mathbb{R}^n$ is the state variable; $\boldsymbol{A},\ \boldsymbol{A}_{\boldsymbol{d}} \in \mathbb{R}^{n\times n}$ are matrices (known constant); the matrices (unknown) $\Delta\boldsymbol{A},\ \Delta\boldsymbol{A}_{\boldsymbol{d}} \in \mathbb{R}^{n\times n}$ having uncertainties in $\boldsymbol{A},\ \boldsymbol{A}_{\boldsymbol{d}}$, respectively; at time r, the initial state value is $\boldsymbol{\varphi}(r) \in \mathbb{R}^n$; d is the positive integer for time delay.

The characteristic of generalized overflow nonlinearities $\hat{f}_i(\hat{y}_i(r))$ is specified by

$$\left.\begin{array}{ll} L \le \hat{f}_i(\hat{y}_i(r)) \le 1, & \hat{y}_i(r) > 1 \\ \hat{f}_i(\hat{y}_i(r)) = \hat{y}_i(r), & -1 \le \hat{y}_i(r) \le 1 \\ -1 \le \hat{f}_i(\hat{y}_i(r)) \le -L, & \hat{y}_i(r) < -1 \end{array}\right\} i = 1, 2, \ldots n \tag{2a}$$

where

$$-1 \le L \le 1. \tag{2b}$$

With proper choice of L, (2) covers different types of overflow arithmetics, for e.g., saturation ($L = 1$), zeroing ($L = 0$), two's complement and triangular ($L = -1$), etc. In the state matrices, the parameter uncertainties are assumed as

$$\Delta\boldsymbol{A} = \boldsymbol{B}_0\,\boldsymbol{F}_0\,\boldsymbol{C}_0 \tag{3a}$$

$$\Delta\boldsymbol{A}_{\boldsymbol{d}} = \boldsymbol{B}_1\,\boldsymbol{F}_1\,\boldsymbol{C}_1 \tag{3b}$$

where $\boldsymbol{B}_{\boldsymbol{i}} \in \mathbb{R}^{n\times p_i}$, $\boldsymbol{C}_{\boldsymbol{i}} \in \mathbb{R}^{q_i\times n}$ $(i = 0,\ 1)$ are matrices (known constant) and $\boldsymbol{F}_{\boldsymbol{i}} \in \mathbb{R}^{p_i\times q_i}(i = 0,\ 1)$ is matrix (unknown) which satisfies

$$\boldsymbol{F}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{F}_{\boldsymbol{i}} \le \boldsymbol{I},\ i = 0, 1. \tag{3c}$$

For the proof of main results, we present the following lemmas.

Lemma 1 [5, 17–19] *A positive definite matrix* $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} = [\hat{h}_{ij}] \in \boldsymbol{R}^{n\times n}$ *satisfies*

$$\hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{M}\hat{\boldsymbol{y}}(r) - \hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{M}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) \geq 0 \tag{4}$$

provided that the matrix $\boldsymbol{M}$ *is characterized by*

$$\hat{h}_{ii} = s_i + \sum_{j=1,\, j\neq i}^{n} (\hat{o}_{ij} + \hat{\rho}_{ij}),\ i = 1, 2, \ldots n \tag{5a}$$

$$\hat{h}_{ij} = \hat{h}_{ji} = \left(\frac{1+L}{2}\right)(\hat{o}_{ij} - \hat{\rho}_{ij}), \quad i, j = 1, 2, \ldots n\,(i \neq j) \tag{5b}$$

$$\hat{o}_{ij} = \hat{o}_{ji} > 0,\ \hat{\rho}_{ij} = \hat{\rho}_{ji} > 0 \quad i, j = 1, 2, \ldots n\,(i \neq j) \tag{5c}$$

$$s_i > 0, \quad i = 1, 2, \ldots n \tag{5d}$$

$$-1 \leq L \leq 1 \tag{5e}$$

where $\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r))$*is the nonlinearities described by (2) and it is very easy to understand that, for the case where* $n = 1$, $\boldsymbol{M}$ *corresponds to a positive scalar '* γ*'.*

Lemma 2 [24, 25] *For a matrix* $\boldsymbol{R} > \boldsymbol{0}$ *and three nonnegative integers* a_1, a_2, r, *as* $a_1 \leq a_2 \leq r$, *if*

$$\begin{aligned}\boldsymbol{\xi}(r, a_1, a_2) &= \frac{1}{a_2 - a_1}[(2\sum_{s=r-a_2}^{r-a_1-1} \hat{\boldsymbol{x}}(s)) + \hat{\boldsymbol{x}}(r-a_1) - \hat{\boldsymbol{x}}(r-a_2)],\ a_1 < a_2 \\ &= 2\hat{\boldsymbol{x}}(r-a_1),\ a_1 = a_2\end{aligned} \tag{6}$$

then

$$-(a_2 - a_1)\sum_{s=r-a_2}^{r-a_1-1} \boldsymbol{\eta}^{\mathrm{T}}(s)\boldsymbol{R}\boldsymbol{\eta}(s) \leq -\begin{bmatrix}\boldsymbol{\theta}_0 \\ \boldsymbol{\theta}_1\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}\boldsymbol{R} & \boldsymbol{0} \\ \boldsymbol{0} & 3\boldsymbol{R}\end{bmatrix} \begin{bmatrix}\boldsymbol{\theta}_0 \\ \boldsymbol{\theta}_1\end{bmatrix} \tag{7}$$

where

$$\boldsymbol{\theta}_0 = \hat{\boldsymbol{x}}(r-a_1) - \hat{\boldsymbol{x}}(r-a_2), \tag{8}$$

$$\boldsymbol{\theta}_1 = \hat{\boldsymbol{x}}(r-a_1) + \hat{\boldsymbol{x}}(r-a_2) - \boldsymbol{\xi}(r, a_1, a_2) \tag{9}$$

$$\boldsymbol{\eta}(s) = \hat{\boldsymbol{x}}(s+1) - \hat{\boldsymbol{x}}(s) \tag{10}$$

Lemma 3 [14, 15] *Let* $\boldsymbol{\Xi}$, Θ, $\boldsymbol{F}$ and $\boldsymbol{U}$ *be matrices (real) of suitable dimensions with* $\boldsymbol{U} = \boldsymbol{U}^{\mathrm{T}}$, *then*

$$\boldsymbol{U} + \boldsymbol{\Xi}\,\boldsymbol{F}\,\Theta + \Theta^{\mathrm{T}}\,\boldsymbol{F}^{\mathrm{T}}\,\boldsymbol{\Xi}^{T} < \boldsymbol{0} \tag{11}$$

for all $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} \le \boldsymbol{I}$, *iff there exists a positive scalar* ε *such that*

$$\boldsymbol{U} + \varepsilon^{-1}\,\boldsymbol{\Xi}\,\boldsymbol{\Xi}^{\mathrm{T}} + \varepsilon\,\Theta^{\mathrm{T}}\,\Theta < \boldsymbol{0} \tag{12}$$

Lemma 4 [19–22, 27] *Consider* $\boldsymbol{B} = [\hat{c}_{uv}] \in \boldsymbol{R}^{n\times n}$ is *given by*

$$\hat{c}_{uu} = \sum_{v=1,\, v\neq u}^{n} (\chi_{uv} + \delta_{uv}),\ u = 1, 2 \ldots n \tag{13a}$$

$$\hat{c}_{uv} = L(\chi_{uv} - \delta_{uv}),\ u, v = 1, 2, \ldots n\,(u \neq v) \tag{13b}$$

$$\chi_{uv} > 0,\ \delta_{uv} > 0,\ u, v = 1, 2, \ldots n\,(u \neq v) \tag{13c}$$

$$0 \le L \le 1, \tag{13d}$$

(*for* $n = 1$, $\boldsymbol{B}$ *becomes a positive scalar*)**,** *then*

$$\sum_{u=1}^{n} 2[\hat{y}_u(r) - \hat{f}_u(\hat{y}_u(r))]\left[\sum_{v=1,\, v\neq u}^{n} (\chi_{uv} + \delta_{uv})\hat{f}_u(\hat{y}_u(r)) + L(\chi_{uv} - \delta_{uv})\hat{f}_v(\hat{y}_v(r))\right]$$
$$= \hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{B}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) + \hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{B}^{\mathrm{T}}\hat{\boldsymbol{y}}(r) - \hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))(\boldsymbol{B} + \boldsymbol{B}^{\mathrm{T}})\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) \ge 0, \tag{14}$$

where $\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r))$ *is given by* (2a) *and* (13d).

Lemma 5 [19, 20, 23, 27] *Consider* $\boldsymbol{Z} = diag\,(d_1, d_2,\ldots, d_n) > \boldsymbol{0}$. *Then, pertaining to* (2a) *and* $-1 \le L < 0$, *the following relation is satisfied*

$$\begin{aligned}&\sum_{k=1}^{n} 2d_k[\hat{y}_k(r) - \hat{f}_k(\hat{y}_k(r))]\,[-L\hat{y}_k(r) + \hat{f}_k(\hat{y}_k(r))]\\ &= (1 + L)\hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{Z}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) + (1 + L)\hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{Z}\hat{\boldsymbol{y}}(r)\\ &\quad - 2\hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{Z}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) - 2L\hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{Z}\hat{\boldsymbol{y}}(r) \ge 0.\end{aligned} \tag{15}$$

Remark 1 In this paper, we have considered two ranges of L, i.e., $0 \le L \le 1$ and $-1 \le L < 0$ which together cover (2b).

3 Main Results

In Sect. 3, two criteria are presented for the global asymptotic stability of the system (1a), (1b), (1c), (1d)–(3a), (3b), (3c). The first criterion (Theorem 1) is applicable to the situation where $0 \le L \le 1$ and the second one (Theorem 2) is valid for $-1 \le L < 0$.

Theorem 1 *The system* (1a), (1b), (1c), (1d)–(3a), (3b), (3c) *with* $0 \le L \le 1$ *is globally asymptotically stable if there exit suitable dimensioned matrices* $\boldsymbol{P} = \begin{bmatrix} \boldsymbol{p}_1 & \boldsymbol{p}_2 \\ * & \boldsymbol{p}_3 \end{bmatrix} > \boldsymbol{0}$, $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > \boldsymbol{0}$, $\boldsymbol{R} = \boldsymbol{R}^{\mathrm{T}} > \boldsymbol{0}$ *and positive scalars* ε_0, ε_1

$$\begin{bmatrix} -\boldsymbol{p}_1 + ((\boldsymbol{p}_2 + \boldsymbol{p}_2^{\mathrm{T}})/2) + \boldsymbol{Q} + d^2\boldsymbol{R} - 4\boldsymbol{R} + \varepsilon_0 \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{C}_0 & -(\boldsymbol{p}_2/2) - 2\boldsymbol{R} & -d((\boldsymbol{p}_2 - \boldsymbol{p}_3)/2) + 3\boldsymbol{R} & (\boldsymbol{p}_2^{\mathrm{T}}/2) - d^2\boldsymbol{R} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M} & \boldsymbol{0} & \boldsymbol{0} \\ * & -\boldsymbol{Q} - 4\boldsymbol{R} + \varepsilon_1 \boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{C}_1 & -d(\boldsymbol{p}_3/2) + 3\boldsymbol{R} & -(\boldsymbol{p}_2^{\mathrm{T}}/2) + \boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{B} & \boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{M} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & -3\boldsymbol{R} & d\,\boldsymbol{p}_2^{\mathrm{T}}/2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & \boldsymbol{p}_1 + (d^2\boldsymbol{R}) - \boldsymbol{M} - (\boldsymbol{B} + \boldsymbol{B}^{\mathrm{T}}) & \boldsymbol{0} & \boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}_0 & \boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}_1 \\ * & * & * & * & -\boldsymbol{M} & \boldsymbol{M}\boldsymbol{B}_0 & \boldsymbol{M}\boldsymbol{B}_1 \\ * & * & * & * & * & -\varepsilon_0 \boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & * & * & -\varepsilon_1 \boldsymbol{I} \end{bmatrix} < \boldsymbol{0} \tag{16}$$

where the matrices $\boldsymbol{M}$ *and* $\boldsymbol{B}$ *are given by (5) and (13), respectively.*

Proof Consider a quadratic Lyapunov function

$$\boldsymbol{V}(\hat{\boldsymbol{x}}(r)) = \boldsymbol{\Gamma}^{\mathrm{T}}(r)\boldsymbol{P}\,\boldsymbol{\Gamma}(r) + \sum_{s=r-d}^{r-1} \hat{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}\hat{\boldsymbol{x}}(s) + d \sum_{\theta=-d+1}^{0} \sum_{s=r+\theta-1}^{r-1} \boldsymbol{\eta}^{\mathrm{T}}(s)\boldsymbol{R}\boldsymbol{\eta}(s) \tag{17}$$

where

$$\boldsymbol{\Gamma}^{\mathrm{T}}(r) = [\hat{\boldsymbol{x}}^{\mathrm{T}}(r) \; \sum_{s=r-d}^{r-1} \hat{\boldsymbol{x}}^{\mathrm{T}}(s)\,]$$

and

$$\boldsymbol{\eta}(r) = \hat{\boldsymbol{x}}(r+1) - \hat{\boldsymbol{x}}(r) = \hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) - \hat{\boldsymbol{x}}(r).$$

Defining

$$\begin{aligned}\Delta V(\hat{\boldsymbol{x}}(r)) &= V(\hat{\boldsymbol{x}}(r+1)) - V(\hat{\boldsymbol{x}}(r)) \\ &= \boldsymbol{\chi}^{\mathrm{T}}(r)\boldsymbol{\Psi}(d)\boldsymbol{\chi}(r) + \hat{\boldsymbol{x}}^{\mathrm{T}}(r)\boldsymbol{Q}\hat{\boldsymbol{x}}(r) - \hat{\boldsymbol{x}}^{\mathrm{T}}(r-d)\boldsymbol{Q}\hat{\boldsymbol{x}}(r-d) \\ &\quad + d^2\boldsymbol{\eta}^{\mathrm{T}}(r)\boldsymbol{R}\boldsymbol{\eta}(r) - d\sum_{s=r-d}^{r-1}\boldsymbol{\eta}^{\mathrm{T}}(s)\boldsymbol{R}\boldsymbol{\eta}(s)\end{aligned} \tag{18}$$

where

$$\boldsymbol{\chi}^{\mathrm{T}}(r) = \left[\hat{\boldsymbol{x}}^{\mathrm{T}}(r)\ \hat{\boldsymbol{x}}^{\mathrm{T}}(r-d)\ \boldsymbol{\xi}^{\mathrm{T}}(r,0,d)\ \hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\right],$$

$$\boldsymbol{\Psi}(d) = \begin{bmatrix} -\boldsymbol{p}_1 + (\boldsymbol{p}_2 + \boldsymbol{p}_2^{\mathrm{T}})/2 & -\boldsymbol{p}_2/2 & -d((\boldsymbol{p}_2 - \boldsymbol{p}_3)/2) & \boldsymbol{p}_2^{\mathrm{T}}/2 \\ * & \boldsymbol{0} & -d\,\boldsymbol{p}_3/2 & -\boldsymbol{p}_2^{\mathrm{T}}/2 \\ * & * & \boldsymbol{0} & d\,\boldsymbol{p}_2^{\mathrm{T}}/2 \\ * & * & * & \boldsymbol{p}_1 \end{bmatrix} \tag{19}$$

and $\boldsymbol{\xi}(r,0,d)$ is given by (6).

Next, by the use of Lemma 2, the last term of $\Delta V(\hat{\boldsymbol{x}}(r))$ is expressed as

$$\begin{aligned}&-d\sum_{s=r-d}^{r-1}\boldsymbol{\eta}^{\mathrm{T}}(s)\boldsymbol{R}\eta(s) \\ &\le -d\begin{bmatrix}\hat{\boldsymbol{x}}(r) - \hat{\boldsymbol{x}}(r-d) \\ \hat{\boldsymbol{x}}(r) + \hat{\boldsymbol{x}}(r-d) - \boldsymbol{\xi}(r,0,d)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{R} & \boldsymbol{0} \\ \boldsymbol{0} & 3\boldsymbol{R}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{x}}(r) - \hat{\boldsymbol{x}}(r-d) \\ \hat{\boldsymbol{x}}(r) + \hat{\boldsymbol{x}}(r-d) - \boldsymbol{\xi}(r,0,d)\end{bmatrix}\end{aligned} \tag{20}$$

Now, from Lemmas 1 and 4, (18) can be rearranged as

$$\Delta V(\hat{\boldsymbol{x}}(r)) = \boldsymbol{\chi}^{\mathrm{T}}(r)\boldsymbol{\phi}(d)\boldsymbol{\chi}(r) - \beta \tag{21a}$$

where

$$\begin{aligned}\beta &= \hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{M}\hat{\boldsymbol{y}}(r) - \hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{M}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) + \hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{B}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) \\ &\quad + \hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{B}^{\mathrm{T}}\hat{\boldsymbol{y}}(r) - \hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))(\boldsymbol{B} + \boldsymbol{B}^{\mathrm{T}})\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) \ge 0.\end{aligned} \tag{21b}$$

$$\boldsymbol{\phi}(d) = \begin{bmatrix} -\boldsymbol{p}_1 + ((\boldsymbol{p}_2 + \boldsymbol{p}_2^{\mathrm{T}})/2) + \boldsymbol{Q} + d^2\boldsymbol{R} - 4\boldsymbol{R} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{A}} & -(\boldsymbol{p}_2/2) - 2\boldsymbol{R} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{A}}_d & -d((\boldsymbol{p}_2 - \boldsymbol{p}_3)/2) + 3\boldsymbol{R} & (\boldsymbol{p}_2^{\mathrm{T}}/2) - d^2\boldsymbol{R} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{B} \\ * & -\boldsymbol{Q} - 4\boldsymbol{R} + \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{A}}_d & -d\,\boldsymbol{p}_3/2 + 3\boldsymbol{R} & -(\boldsymbol{p}_2^{\mathrm{T}}/2) + \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{B} \\ * & * & -3\boldsymbol{R} & d\,\boldsymbol{p}_2^{\mathrm{T}}/2 \\ * & * & * & \boldsymbol{P}_1 + (d^2\boldsymbol{R}) - \boldsymbol{M} - (\boldsymbol{B} + \boldsymbol{B}^{\mathrm{T}}) \end{bmatrix} \tag{21c}$$

From (21a), (21b), (21c), it is implied that $\Delta V(\hat{\boldsymbol{x}}(r)) < 0$ if $\boldsymbol{\phi}(d) < \mathbf{0}$. By the aid of Schur's complement, $\boldsymbol{\phi}(d) < \mathbf{0}$ is written as

$$\boldsymbol{\phi}(d) = \begin{bmatrix} -\boldsymbol{p}_1 + ((\boldsymbol{p}_2 + \boldsymbol{p}_2^{\mathrm{T}})/2) + \boldsymbol{Q} + d^2\boldsymbol{R} - 4\boldsymbol{R} & -(\boldsymbol{p}_2/2) - 2\boldsymbol{R} & -d((\boldsymbol{p}_2 - \boldsymbol{p}_3)/2) + 3\boldsymbol{R} & (\boldsymbol{p}_2^{\mathrm{T}}/2) - d^2\boldsymbol{R} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{B} & \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{M} \\ * & -\boldsymbol{Q} - 4\boldsymbol{R} & -d\,\boldsymbol{p}_3/2 + 3\boldsymbol{R} & -(\boldsymbol{p}_2^{\mathrm{T}}/2) + \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{B} & \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{M} \\ * & * & -3\boldsymbol{R} & d\,\boldsymbol{p}_2^{\mathrm{T}}/2 & \mathbf{0} \\ * & * & * & \boldsymbol{p}_1 + d^2\boldsymbol{R} - \boldsymbol{M} - (\boldsymbol{B} + \boldsymbol{B}^{\mathrm{T}}) & \mathbf{0} \\ * & * & * & * & -\boldsymbol{M} \end{bmatrix} < \mathbf{0} \tag{22}$$

Using (3a), the inequality (22) can be expressed as

$$\boldsymbol{\phi}_0(d) + \bar{\boldsymbol{B}}_0\boldsymbol{F}_0\bar{\boldsymbol{C}}_0 + \bar{\boldsymbol{C}}_0^{\mathrm{T}}\boldsymbol{F}_0^{\mathrm{T}}\bar{\boldsymbol{B}}_0^{\mathrm{T}} < \mathbf{0} \tag{23}$$

where

$$\bar{\boldsymbol{B}}_0^{\mathrm{T}} = \left[\mathbf{0}\ \mathbf{0}\ \mathbf{0}\ \boldsymbol{B}_0^{\mathrm{T}}\boldsymbol{B}\ \ \boldsymbol{B}_0^{\mathrm{T}}\boldsymbol{M}\right], \bar{\boldsymbol{C}}_0 = [\boldsymbol{C}_0\ \mathbf{0}\ \mathbf{0}\ \mathbf{0}\ \mathbf{0}] \tag{24}$$

$$\boldsymbol{\phi}_0(d) = \begin{bmatrix} -\boldsymbol{p}_1 + (\boldsymbol{p}_2 + \boldsymbol{p}_2^{\mathrm{T}})/2 + \boldsymbol{Q} + d^2\boldsymbol{R} - 4\boldsymbol{R} & -(\boldsymbol{p}_2/2) - 2\boldsymbol{R} & -d((\boldsymbol{p}_2 - \boldsymbol{p}_3)/2) + 3\boldsymbol{R} & (\boldsymbol{p}_2^{\mathrm{T}}/2) - d^2\boldsymbol{R} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M} \\ * & -\boldsymbol{Q} - 4\boldsymbol{R} & -d\,\boldsymbol{p}_3/2 + 3\boldsymbol{R} & -(\boldsymbol{p}_2^{\mathrm{T}}/2) + \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{B} & \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{M} \\ * & * & -3\boldsymbol{R} & d\,\boldsymbol{p}_2^{\mathrm{T}}/2 & \mathbf{0} \\ * & * & * & \boldsymbol{p}_1 + d^2\boldsymbol{R} - \boldsymbol{M} - (\boldsymbol{B} + \boldsymbol{B}^{\mathrm{T}}) & \mathbf{0} \\ * & * & * & * & -\boldsymbol{M} \end{bmatrix} \tag{25}$$

Employing Lemma 3, (23) can be written as

$$\boldsymbol{\phi}_0(d) + \varepsilon_0^{-1}\bar{\boldsymbol{B}}_0\bar{\boldsymbol{B}}_0^{\mathrm{T}} + \varepsilon_0\bar{\boldsymbol{C}}_0^{\mathrm{T}}\bar{\boldsymbol{C}}_0 < \mathbf{0} \tag{26}$$

where $\varepsilon_0 > 0$. Next, with the aid of Schur's complement, (26) yields

$$\begin{bmatrix} -\boldsymbol{p}_1 + ((\boldsymbol{p}_2 + \boldsymbol{p}_2^{\mathrm{T}})/2) + \boldsymbol{Q} + d^2\boldsymbol{R} - 4\boldsymbol{R} + \varepsilon_0 \boldsymbol{C}_0^T \boldsymbol{C}_0 & -(\boldsymbol{p}_2/2) - 2\boldsymbol{R} & -d((\boldsymbol{p}_2 - \boldsymbol{p}_3)/2) + 3\boldsymbol{R} & (\boldsymbol{p}_2^{\mathrm{T}}/2) - d^2\boldsymbol{R} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M} & \mathbf{0} \\ * & -\boldsymbol{Q} - 4\boldsymbol{R} & -d(\boldsymbol{p}_3/2) + 3\boldsymbol{R} & -(\boldsymbol{p}_2^{\mathrm{T}}/2) + \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{B} & \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{M} & \mathbf{0} \\ * & * & -3\boldsymbol{R} & d\boldsymbol{p}_2^{\mathrm{T}}/2 & \mathbf{0} & \mathbf{0} \\ * & * & * & \boldsymbol{p}_1 + d^2\boldsymbol{R} - \boldsymbol{M} - (\boldsymbol{B} + \boldsymbol{B}^{\mathrm{T}}) & \mathbf{0} & \boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}_0 \\ * & * & * & * & -\boldsymbol{M} & \boldsymbol{M}\boldsymbol{B}_0 \\ * & * & * & * & * & -\varepsilon_0\boldsymbol{I} \end{bmatrix} < \mathbf{0} \tag{27}$$

Next, following the analysis similar to (23)–(27), one can easily get (16) from (27).

This ends the proof.

Next, we present the following result.

Theorem 2 *The system* (1a), (1b), (1c), (1d)–(3a), (3b), (3c)*along with* $-1 \le L < 0$ *is globally asymptotically stable if there exit suitable dimensioned matrices* $\boldsymbol{P} = \begin{bmatrix} \boldsymbol{p}_1 & \boldsymbol{p}_2 \\ * & \boldsymbol{p}_3 \end{bmatrix} > \mathbf{0}$, $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > \mathbf{0}$, $\boldsymbol{R} = \boldsymbol{R}^{\mathrm{T}} > \mathbf{0}$, *and positive scalars* $\varepsilon_0, \varepsilon_1$ *satisfying*

$$\begin{bmatrix} -\boldsymbol{p}_1 + (\boldsymbol{p}_2 + \boldsymbol{p}_2^{\mathrm{T}})/2 + \boldsymbol{Q} + d^2\boldsymbol{R} - 4\boldsymbol{R} + \varepsilon_0 \boldsymbol{C}_0^T \boldsymbol{C}_0 & -(\boldsymbol{p}_2/2) - 2\boldsymbol{R} & -d\,(\boldsymbol{p}_2 - \boldsymbol{p}_3)/2 + 3\boldsymbol{R} & (\boldsymbol{p}_2^{\mathrm{T}}/2) - d^2\boldsymbol{R} + (1+L)\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Z} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M} & (-2L)^{1/2}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Z} & \mathbf{0} & \mathbf{0} \\ * & -\boldsymbol{Q} - 4\boldsymbol{R} + \varepsilon_1 \boldsymbol{C}_1^T \boldsymbol{C}_1 & -d(\boldsymbol{p}_3/2) + 3\boldsymbol{R} & -(\boldsymbol{p}_2^{\mathrm{T}}/2) + (1+L)\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{Z} & \boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{M} & (-2L)^{1/2}\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{Z} & \mathbf{0} & \mathbf{0} \\ * & * & -3\boldsymbol{R} & d(\boldsymbol{p}_2^{\mathrm{T}}/2) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & \boldsymbol{p}_1 + d^2\boldsymbol{R} - \boldsymbol{M} - 2\boldsymbol{Z} & \mathbf{0} & \mathbf{0} & (1+L)\boldsymbol{Z}\boldsymbol{B}_0 & (1+L)\boldsymbol{Z}\boldsymbol{B}_1 \\ * & * & * & * & -\boldsymbol{M} & \mathbf{0} & \boldsymbol{M}\boldsymbol{B}_0 & \boldsymbol{M}\boldsymbol{B}_1 \\ * & * & * & * & \mathbf{0} & -\boldsymbol{Z} & (-2L)^{1/2}\boldsymbol{Z}\boldsymbol{B}_0 & (-2L)^{1/2}\boldsymbol{Z}\boldsymbol{B}_1 \\ * & * & * & * & * & * & -\varepsilon_0\boldsymbol{I} & \mathbf{0} \\ * & * & * & * & * & * & * & -\varepsilon_1\boldsymbol{I} \end{bmatrix} < \mathbf{0} \tag{28}$$

where $\boldsymbol{M}$*is given by (5) and* $\boldsymbol{Z}$*is a positive definite diagonal matrix of appropriate dimension.*

Proof Pertaining to the case where $-1 \le L < 0$ and in view of Lemmas 1 and 5, Eq. (18) (obtained in the proof of Theorem 1) can be mathematically re-expressed as

$$\Delta V(\hat{\boldsymbol{x}}(r)) = \boldsymbol{\chi}^{\mathrm{T}}(r)\boldsymbol{\phi}_1(d)\boldsymbol{\chi}(r) - \beta_1 \tag{29a}$$

where

$$\begin{aligned}\beta_1 = {} & \hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{M}\hat{\boldsymbol{y}}(r) - \hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{M}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) + (1+L)\hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{Z}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) \\ & + (1+L)\hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{Z}\hat{\boldsymbol{y}}(r) - 2\hat{\boldsymbol{f}}^{\mathrm{T}}(\hat{\boldsymbol{y}}(r))\boldsymbol{Z}\hat{\boldsymbol{f}}(\hat{\boldsymbol{y}}(r)) - 2L\hat{\boldsymbol{y}}^{\mathrm{T}}(r)\boldsymbol{Z}\hat{\boldsymbol{y}}(r) \geq 0,\end{aligned} \tag{29b}$$

$$\boldsymbol{\phi}_1(d) = \begin{bmatrix} \begin{matrix} -\boldsymbol{p}_1 + ((\boldsymbol{p}_2+\boldsymbol{p}_2^{\mathrm{T}})/2) + \boldsymbol{Q} + d^2\boldsymbol{R} - 4\boldsymbol{R} + \\ \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{A}} - 2L\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Z}\bar{\boldsymbol{A}} \end{matrix} & -(\boldsymbol{p}_2/2) - 2\boldsymbol{R} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{A}}_d - 2L\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Z}\bar{\boldsymbol{A}}_d & -d((\boldsymbol{p}_2-\boldsymbol{p}_3)/2) + 3\boldsymbol{R} & (\boldsymbol{p}_2^{\mathrm{T}}/2) - d^2\boldsymbol{R} + (1+L)\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Z} \\ * & -\boldsymbol{Q} - 4\boldsymbol{R} + \bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{M}\bar{\boldsymbol{A}}_d - 2L\bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{Z}\bar{\boldsymbol{A}}_d & -d(\boldsymbol{p}_3/2) + 3\boldsymbol{R} & -(\boldsymbol{p}_2^{\mathrm{T}}/2) + (1+L)\bar{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{Z} \\ * & * & -3\boldsymbol{R} & d\,\boldsymbol{p}_2^{\mathrm{T}}/2 \\ * & * & * & \boldsymbol{p}_1 + d^2\boldsymbol{R} - \boldsymbol{M} - 2\boldsymbol{Z} \end{bmatrix} \tag{30}$$

From (29a), (29b), it follows that $\Delta V(\hat{\boldsymbol{x}}(r)) < 0$ if $\boldsymbol{\phi}_1(d) < \mathbf{0}$.

Next, following the similar steps as shown in the proof of Theorem 1, the condition $\boldsymbol{\phi}_1(d) < \mathbf{0}$ leads to (28). This ends the proof.

Remark 2 The given criteria are in LMI setting and can be mathematically tractable by MATLAB with YALMIP 3.0 parser [28].

Remark 3 Note that it is worth to compare the present results with Corollary 9 of [17]. To handle the sum and cross terms of the forward difference of the Lyapunov function, Wirtinger-based inequality (see Lemma 2) is considered in this paper while in [17], free-weighting matrix approach is used for the same. Consequently, the present approach requires less number of decision variables as compared to [17]. Hence, the present approach gains improvement in terms of computational complexity relative to [17].

Remark 4 As future work, the idea of the present paper can be extended to the stability of discrete-time uncertain systems with external disturbances and/or time-varying delay using generalized overflow nonlinearities, $\boldsymbol{H}_\infty$ stability of discrete-time uncertain systems with generalized overflow nonlinearities and time delay, etc. Further, one can also apply the present idea for two dimensional (2-D) and multidimensional ($n > 2$) systems.

Remark 5 The time delay considered in the existing literature may be constant and/or time-varying. In the modeling of the practical engineering/industrial systems, i.e., microgrid systems with constant communication delay [29] and networked control systems with constant feedback delay [30] where constant time delay has also been taken into account, the main objective of all these concerns including the present paper is to find the allowable time delay that assures the system stability.

4 Numerical Examples

The usefulness of the presented criteria is given by the following numerical examples.

Example 1 Consider the system (1a), (1b), (1c), (1d)–(3a), (3b), (3c), where

$$\boldsymbol{A} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.91 \end{bmatrix}, \ \boldsymbol{A}_d = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}$$

$$\boldsymbol{B}_0 = \boldsymbol{B}_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \ \boldsymbol{C}_0 = [0.01 \quad 0], \ \boldsymbol{C}_1 = [0 \quad 0.01] \tag{31}$$

Pertaining to $L = 1$, it is found that (16) is feasible for maximum delay $d = 18$. Hence, according to Theorem 1, global asymptotically stability of the system under consideration is achieved whereas Corollary 9 of [17] fails to obtain the same.

Example 2 Consider the discrete-time system described by (1a), (1b), (1c), (1d)–(3a), (3b), (3c) with (31) and $L = -1/3$. It is verified easily that Theorem 2 assures the global asymptotic stability of the system over a maximum delay $d = 13$. By contrast, Corollary 9 of [17] fails to this end.

Examples 1 and 2 support the fact that the presented approach may yield stability results which are not covered by [17]. In summary, the present results may establish improved stability conditions (less conservative and computationally efficient) which are uncovered by existing results.

5 Conclusions

In this paper, two delay-dependent criteria for stability analysis of discrete-time uncertain systems with time delay and generalized overflow nonlinearities have been established. The presented criteria are in the form of LMIs and, hence, computationally tractable. The effectiveness of the proposed criteria is proved with the numerical examples.

Acknowledgements The corresponding author wishes to thank the TEQIP-III grant, MNNIT Allahabad for providing scholarship to pursue his research work. The authors of the paper wish to thank the reviewers for their comments and suggestions.

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