# Advanced Wavelet Transform for Image Processing—A Survey

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**Abstract** Over the last few years, the wavelet transform has played a significant role in the field of multiresolution image analysis. The shortcomings of the wavelet transform laid the foundation of many advanced wavelets. This review paper brings together ten advanced wavelets on a common platform to discuss their importance, concept, architecture, merits and demerits in various fields of image processing. The relationships among the different advanced wavelets are also illustrated here. A comparison table serves as a catalog to know the recent trends and applications of the advanced wavelets.

**Keywords** Anisotropy · Feature extraction · Multiresolution · Sampling

## 1 Introduction

The efficient feature extraction and representation have always been a great challenge to the researchers. There has been a long way of evolution from the Fourier Transform, Discrete Fourier Transform, Fast Fourier Transform, Short-Time Fourier Transform to the Wavelet Transform. Some of the applications of the wavelet transform in image processing are coding, fusion, enhancement, compression, denoising, segmentation, content-based image retrieval and so on. Though the wavelet transform is a very popular and promising tool for sparse representation of objects with point singularities, its efficiency is greatly challenged by the objects with line singularities due to the lack of directionality and anisotropy as suggested by Do and

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Vetterli [1]. Therefore, the urgent need of higher-dimensional sparse representation of images motivates the researchers to innovate the mathematical transforms called the advanced wavelets. Here, advanced wavelet is referred as X-let.

The organization of the rest of the paper is given below. Having addressed the significance of X-let in the introductory section, ten different X-lets are presented in Sect. 2. Section 3 exhibits a compact chart for the overall comparison of the X-lets. A brief study on X-let using numerical data obtained from experimental results is mentioned in Sect. 4. The conclusion with the direction of future work is drawn in Sect. 5.

## 2 Ten Different X-lets

The ten different X-lets namely ridgelet, curvelet, contourlet, wedgelet, bandlet, grouplet, surfacelet, shearlet, Gabor wavelet and brushlet are discussed below.

### *2.1 Ridgelet Transform*

The ridgelet transform was developed by Candes and Donoho [2] in 1999 to break the limitation of the wavelet transform in the higher dimension. The ridgelet analysis is related to the theory of approximation-based superposition of the ridge functions [2]. Reviewing the fundamentals and flowgraph of the ridgelet transform presented by Starck et al. [3], it can be concluded that the ridgelet transform can resolve the problem of sparse approximation of smooth objects with straight edges. Unfortunately, in practice, the edges of the images are typically curved instead of being straight. Therefore, to study the curve singularities, one probable solution is to segregate the image into a number of sub-images and then apply the ridgelet transform individually to the sub-images as discussed by Donoho and Duncan [4, Fig. 2].

### *2.2 Curvelet Transform*

The curvelet transform was pioneered by Candes and Donoho [5] as a new multiscale directional transform. This transform in two dimensions (2D) provides nearly optimal sparse representation of objects having singularities along smooth curves. The first generation of the curvelet was developed in the continuous domain by multiscale filtering and "embedded" ridgelet transform. Unfortunately, the redundancy and the slow nature of the first generation curvelet motivated researchers to develop the second generation curvelet transform [5, 6] which is determined by the frequency partitioning technique without the use of the ridgelet transform, thereby making it a more robust and fast image analysis tool.

### 2.3 Contourlet Transform

Do and Vetterli [1] reported a double filter bank-based transform called the contourlet transform for obtaining sparse representation of images. This tool provides a variable multiresolution and directional decomposition of images. In the pyramidal filter bank [1, Fig. 7], the first stage is the multiscale decomposition by the Laplacian pyramid (LP) followed by directional decomposition by two-dimensional directional filter bank (2DDFB). The LP captures the point discontinuities and directional filter bank (DFB) links, the point discontinuities into linear structures or contour segments. The LP decomposition generates a bandpass image. This bandpass image so produced is free from "scrambled" frequencies which are associated with wavelet filter.

### 2.4 Wedgelet, Bandlet and Grouplet Transform

The role and the limitation of the X-lets in resolving the complex image boundaries with typical shapes and conditions have been discussed below with the flowchart given in Fig. 1, where $\alpha$ is the geometrical regularity determining factor. It is known that the geometrically regular functions can be described as a piecewise $C^\alpha$-regular functions ($\alpha$ times continuously differentiable) outside a set of regular edges. But the curvelet transform has the optimal image representation for only $\alpha = 2$. In practice, the images have irregular geometry with either $\alpha < 2$ or $\alpha > 2$ [7]. Therefore, when the question of regularity along the singularities of a surface arises, the failure of the curvelet transform is overcome by a novel approximate scheme called the wedgelet [8]. According to Yang et al. [7], the wedgelet divides the support of the image in dyadic adapted squares. But this approach is suitable for edges without blur and simple geometry images. Therefore, the quest for better X-let continues and one such approximation-based adaptive technique called the bandlet was introduced by Pennec and Mallat [9]. It can suitably capture the geometric regularity along the edges in an image by implementing an adaptive approximation of the image geometry when $\alpha$ is unknown and can fruitfully represent images because each bandlet atom is represented by a geometric flow showing the directions of regular variations of the gray level [7]. The geometry of the bandlet is suitable for analyzing geometrical regular images. But the key drawback of bandlet is that it cannot faithfully represent the complex geometry of textures as that of a wooden fiber. Thus, the researchers came up with another novel approach called the grouplet [10, 7] based on the Gestalt theory [11]. It suggests the recursive use of a set of grouping laws which help to model the edges of images with long range of monotonic turbulent geometry, e.g., wooden texture.

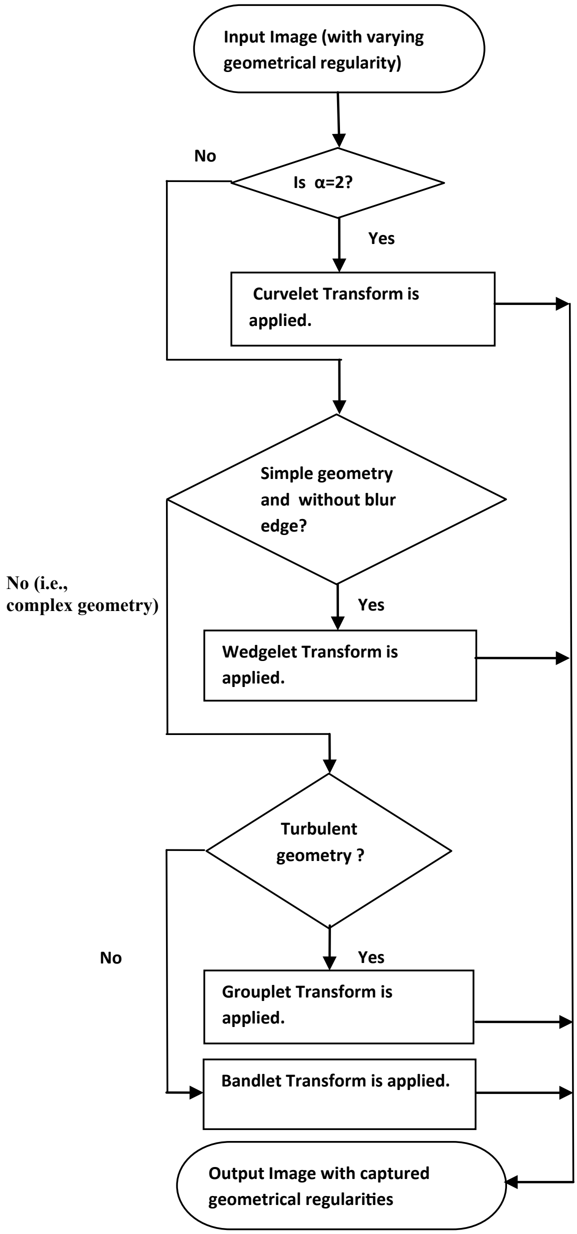


**Fig. 1** Flowchart for finding the geometrical regularity of images under different constraints

## 2.5 Surfacelet Transform

The DFB was introduced by Bamberger and Smith in the year 1992 [12]. Lu and Do [13] proposed the extension of 2DDFB to higher dimensions resulting in a new type of filter banks called three-dimensional directional filter bank (3DDFB) which retains the wonderful capacity of directional decomposition of 3D (three dimensional) signals. The 3DDFB, when combined with the LP, gives rise to the 3D surfacelet or surface transform. The block diagram representation of the surfacelet transform is presented by Lu and Do [14]. The surfacelet bears close resemblance with another transform called dual-tree wavelet transform (DTWT). The redundancy ratio in three dimensions is almost the same for both the surfacelet and DTWT. But the advantage of surfacelet over DTWT is that its angular resolution can be refined to a very high level just by raising the number of decomposition levels.

## 2.6 Shearlet Transform

Shearlet introduced by Labate et al. [15] is a new class of multidimensional image representation tool. It is popular due to its ability to represent bivariate functions sparsely. So far various directional transforms like curvelets, contourlets and surfacelets have been addressed for edge representation of images. But none of the X-lets provides a unified treatment of both the continuous and digital setting [16]. This major drawback of these multiresolution methods is overcome by the shearlet due to its uniting capacity of the continuous and digital domain [16]. Another interesting feature of the shearlet is the easy fitting in the framework of affine like systems. Shearlets are basically functions with orientation. The orientation of this function or waveform can be regulated by a parameter called shear parameter. These can be obtained by using dilation, translation and shear transformation of a given function.

## 2.7 Gabor Wavelet

In image processing, the octave-based decomposition of the Fourier plane by the wavelet transform results in a poor angular resolution. But the wavelet packets can decompose the Fourier plane optimally at the cost of the four symmetrical peaks in the frequency plane [17]. Thus, it is difficult to selectively tune and trace a unique frequency. Directionally oriented filter banks, steerable pyramid resolve the random partitioning of the Fourier plane. In this context, the role of Gabor filter needs mentioning. A Gabor filter [18] resembles a wavelet filter bank where an individual filter produces an estimate of the local frequency content. This Gabor filter is a local bandpass filter with joint localization of the spatial and frequency domain. The remarkable feature of Gabor wavelet is that the Gabor basis is not only frequency tunable but also orientation selective. The Gabor wavelets designed in close analogy to visual cortical cells of the mammalian brain helps to decompose a given image into multiple scales and multiple orientations too.

### *2.8 Brushlet Transform*

It is understood from the Gabor filter that the computation load increases abruptly due to the convolution of the original image with so many filters of the bulky Gabor filter bank. Therefore, for better angular resolution, Meyer and Coifman [17] expanded the Fourier plane into windowed Fourier bases to develop a new X-let called brushlet. This is a well-localized complex-valued function bearing a unique peak in frequency domain. Being a complex-valued function, it is associated with a phase giving the knowledge of orientation of the X-let. The size and the location of the brushlets can be adaptively selected in order to obtain the most precise representation of an image in terms of oriented features with all possible directions and low computation load as compared to Gabor filters.

## 3 Comparison of X-lets

The comparison of the X-lets is provided in Table 1.

## 4 Short Study on X-lets Using Experimental Results

There are numerous applications which experiment with X-lets. Some of the applications are (i) mammogram denoising using X-let [19, 20]; (ii) wavelet and curvelet based on soft, hard and block thresholding techniques for noise filtration of mammograms [21]. Here, a short study is presented on the numerical data obtained from [21].

The mammograms with Salt and Pepper, Speckle, Poisson, Gaussian noise are denoised by wavelet and curvelet with an intention to exploit the role of underlying soft, hard and block thresholding techniques. The thresholding technique already discussed in [21] refers to either the preservation or elimination of wavelet/curvelet coefficient generated during the implementation of wavelet/curvelet transform to the mammogram image. The average signal to noise ratio (SNR) obtained by implementing soft, hard and block thresholding techniques using wavelet and curvelet transforms to mammograms with different noises are compared in Fig. 2. When the performance of the three thresholding techniques is compared, ignoring the type of transform, block thresholding technique provides a promising result for all noises

**Table 1** Comparison of X-lets

| X-let | Significant characteristics | Merits | Demerits | Selected applications |
|---|---|---|---|---|
| Ridgelet | Optimal representation of straight-line singularities | Better than wavelet in attaining low mean square error while representing smooth functions and straight edges | Not suitable for representing images with curves and texture | Space weather monitoring, characterization of nano-structures taken by scanning tunneling microscope |
| Curvelet | Localized in position, scale and orientation | Better than wavelet and ridgelet in image compression and denoising | Higher computational cost than the wavelet transform especially in higher dimension | Fluid mechanics, Solving of partial differential equations |
| Contourlet | Direct determination of contourlet transforms on rectangular grid and compactly supported frame | Better than curvelet since immune to quantization noise and noise due to thresholding effect | Implicit oversampling by LP | CBIR, Image representation, denoising and compression |
| Wedgelet | Sub-band decomposition of the original image followed by the use of the prediction and update operator | Better than wavelet in representing the linear features of an image | Application limited to the horizon class image, i.e., binary image with single edge | Image enhancement and denoising |
| Bandlet | Specialized in capturing geometric regularity along the edges of an image | Better than curvelet and wedgelet in representing the edges of images having complex geometry like boiling water image | Unfaithful representation of the complex geometry of textures | Image compression, restoration and denoising |

(continued)

**Table 1** (continued)

| X-let | Significant characteristics | Merits | Demerits | Selected applications |
|---|---|---|---|---|
| Grouplet | The length and width of the individual atom can be modulated to match the different types of geometrical structures of natural images | Better than curvelet, bandlet and wedgelet in representing the long range regularity of fine elongated structures like hair texture | Difficult to adjust the complexity of the association fields for coding it with less number of bits for getting satisfactory compression result | Image in painting, texture analysis |
| Surfacelet | Capture and represent the signal singularities lying on smooth surfaces noticed in 3D medical and video images | Better than dual tree wavelet transform due to high degree of angular resolution caused by increasing number of decomposition levels | High redundancy of surfacelet is reduced in the higher dimensions | Processing of multidimensional volumetric data like seismic imaging and video clips, denoising of video signals |
| Shearlet | Sparse representation of bivariate functions | Better than classical wavelet because it gives information regarding the directionality within the image | Being associated with scaling and translation parameter it cannot detect directionality like conventional wavelet transform | Representation of signal at higher dimension |
| Gabor Wavelet | Multiple scale and orientation decomposition of an image | Gabor basis is frequency tunable and orientation selective | Difficult to reconstruct the Gabor wavelets by linear superposition due to the unavailability of orthonormal bases | Bio-metric application like iris identification, face recognition |
| Brushlet | Localized complex function with one peak in the frequency domain | Feature retrieval rate better than Gabor wavelets due to less computation | Though this X-let is based on adaptive representation but its construction is hard to implement | Texture segmentation, retrieval and classification |

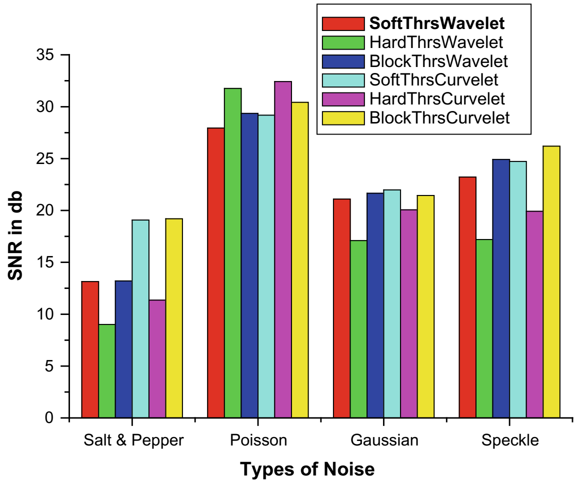


**Fig. 2** Comparison of mean SNR found using soft, hard and block thresholding techniques based on wavelet and curvelet transforms [21]

except Poisson noise. Hard thresholding gives satisfactory result for Poisson noise. When the overall denoising performance is the primary issue, curvelet outperforms wavelet.

## 5 Conclusion

The primary goal of understanding the significance and classification of X-let has been consistently presented in this paper. This paper describes the shortcomings of an X-let and its gradual development. One observation about the X-lets is that despite sharing a common structure or a similar function, they are very much application specific. And this paper gives us an idea of the applicability of the different members of the X-let family for various image processing purposes. The several less explored areas like the implementation of curvelet transform in seismic imaging or the upconversion of ordinary television image into high definition image using bandlet can be the directions of the future work.

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