The Moisture Evaporation Rate of Walls Revisited and Predicting Temperature Profile of Moisture Containing Walls Exposed to Fire

Tensei Mizukami and Takeyoshi Tanaka

Abstract Most building materials more or less contain moisture. It is often observed at temperature measurements at fire resistance tests and also at numerical calculations of heat conduction in walls containing moisture that the temperature rises stagnate around 100 °C for some time periods. This temperature stagnation significantly contributes to the high fire resistance performance of some types of wall, e.g., gypsum board. In this paper, the existing moisture evaporation rate formula has been revisited and re-examined the heat continuity equation from the standpoint of the existence of heat loss not only for moisture evaporation but also by raising the temperature of the wall. It reduces the number of uncertainties in the heat continuity equation and allows us to skip the step of regression fits to numerical simulation. The new formula is compared to the numerical calculation, and the range of application is validated.

Keywords Moisture evaporation rate · Moisture containing wall · Semi-infinite body theory

Nomenclature

C_0	Specific heat of wall material (kJ/kg K)
C_w	Specific heat of water (kJ/kg K)
F	Fourier number (–)
F_ϕ	Fourier number corresponding to drying front (–)
C	Proportionality in ISO 834-11 (–)
D	Proportionality constant (–)
D_p	Temperature stagnation time in ISO 834-11 (min)
Δt_v	Temperature stagnation time (s)

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L_v	Latent heat of water evaporation (kJ/kg)
$\dot{q}''$	Conductive heat flux to drying front (kW/m^2 s)
T	Temperature (°C)
ΔT	Temperature rise (K)
t	Time (s)
d_p	Fire protection material thickness in ISO 834 (mm)
x	Distance from exposed surface (m)
erfc()	Complementary error function

Greek symbols

α	Thermal diffusivity of wall material (m^2/s)
ϕ	Mass ratio of moisture content to wall material (kg/kg)
λ	Thermal conductivity of wall material (kW/m K)
ρ	Density of wall material (kg/m^3)

Subscripts

c	Insulation criterion
ϕ	At drying front
(x, t)	At a given point in time and place
f	Fire
0	Initial
v	Water evaporation

1 Introduction

Most of the building materials more or less contain moisture. It is often observed at temperature measurements in fire resistance tests and also at numerical calculations of heat conduction in walls containing moisture that the temperature rises stagnate at around 100 °C for a certain time period, as shown in Fig. 1.

$$D_p = C d_p^3 \tag{1}$$

$$C = \left(\sum_{i=1}^{n} d_p^3 \times D_p \right) \Bigg/ \sum_{i=1}^{n} d_p^6 \tag{2}$$

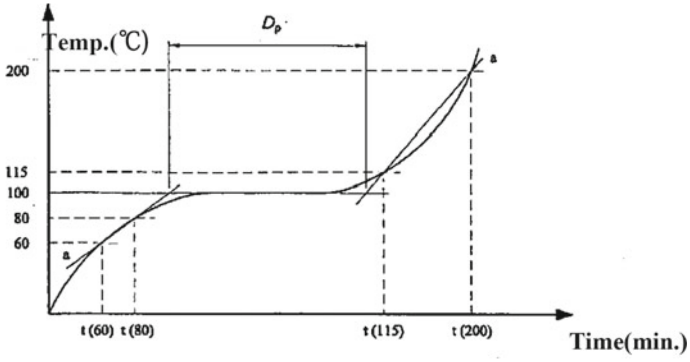

Fig. 1 Evaluation of temperature stagnation period of general moisture containing walls in ISO 834-11

This temperature stagnation significantly contributes to high fire resistance performance. Moreover, some types of wall material, e.g., gypsum board contains crystal water. ISO 834-11 [1] provides a method to estimate the moisture plateau length for the assessment of fire protection to structural steel elements, but it is a method empirically obtained from fire resistance test data, where C is a constant, n is the number of specimens, D_p is the temperature stagnation period for each specimen tested as specified in Fig. 1, and d_p is the fire protection material thickness of each specimen tested. According to Eq. 1, the temperature stagnation period is proportional to the cube of fire protection material thickness.

On the other hand, Mizukami [2], based on theoretical consideration, proposed a simple analytical formula for temperature stagnation period as follows:

$$\Delta t_v \approx \frac{L_v \rho \phi}{\lambda (T_f - T_v)} \cdot x \cdot \Delta x \tag{3}$$

where T_f and T_v are the heating temperature and the water evaporation temperature, respectively, and L_v is the latent heat of water evaporation, ρ is the density of wall material (dry base), ϕ is the mass fraction of moisture to wall mass (dry base), x is the distance from the exposed surface, Δx is the thickness of the slice (Δt_v and x are corresponding to D_p and d_p in Eq. (1), respectively). This formula explains that stagnation period, and Δt_v becomes longer as the distance from exposed surface, x, increases, or more specifically that the Δt_v is proportional to the distance x. Hence, the stagnation period (Δt_v, D_p) is proportional to the cube of the thickness (x, d_p) in ISO 834-11, while it is predicted to be proportional to the thickness in Eq. (3). Also, the moisture content is assumed to be constant, and the correction factor, C, is derived empirically by experimental data set in ISO 834-11, while the stagnation period is predicted to be proportional to the moisture content, ϕ, in Eq. (3).

Also Eq. 3 implies that the stagnation time, Δt_v, should disappear if Δx is taken to be very small, although Δt_v will be observed in reality since thermocouple probes have a certain inspection volume. This was verified by numerical simulations as shown in Fig. 2, which compared the temperatures between the cases with $\Delta x = 10\,\text{mm}$ and 1.25 mm. This implies that the temperature stagnant region does not actually exist, but there are dried and wet parts and the interface between the two parts proceeds inward continuously. A formula is proposed to predict the time at which the evaporation front to reach an arbitrary depth from the exposed surface is as follows:

$$t_{\mathrm{v}} = \left(\frac{1}{D}\right) \cdot \frac{1}{2} \cdot \left(\frac{L_{\mathrm{v}}\phi}{C_{\mathrm{w}}(T_{\mathrm{f}} - T_{\mathrm{v}})}\right) \cdot \left(\frac{x}{2\sqrt{\alpha}}\right)^2 \tag{4}$$

where C_{w} is the specific heat of the wall material, α is the thermal diffusivity of the wall, i.e., $\alpha = \lambda / \rho C_{\mathrm{w}}$.

However, the coefficient, D in Eq. 4 had to be obtained by the regression fits to the results of the numerically computed heat conduction equations under the calculation condition in Table 1. Then, Eq. 4 is rewritten as follows:

$$t_{\mathrm{v}} = \frac{1}{D} \cdot 5.4 \times 10^6 \cdot \phi x^2 \tag{5}$$

Figure 3 shows the numerically calculated dried-up times for different moisture contents, ϕ, and Fig. 4 shows the estimated coefficient number in terms of ϕ to fit

Fig. 2 Comparison of the temperature development with different segment size

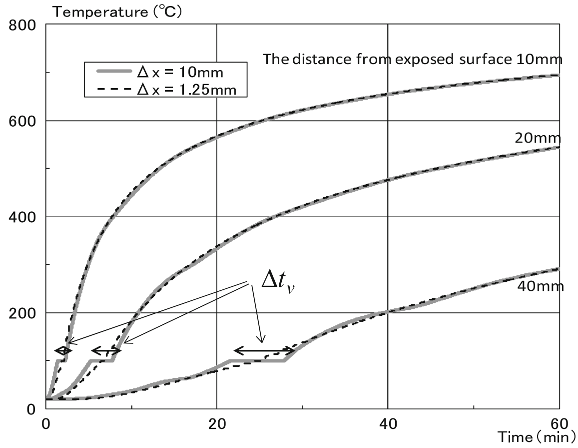

Table 1 Calculation condition

C_{w} [kJ/kg K]	0.88	T_{f} [°C]	875
T_{v} [°C]	100	T_0 [°C]	20
L_{v} [kJ/kg]	2590	ϕ [−]	0.0375–0.3
α [m^2/s]	0.334×10^{-6}	Δx [mm]	1
		Δt [s]	1

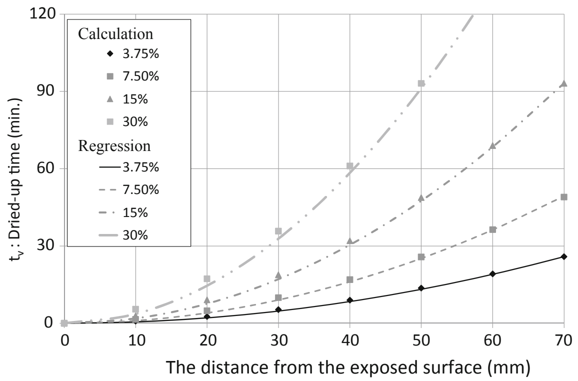

Fig. 3 Numerically calculated dried-up times for different moisture contents

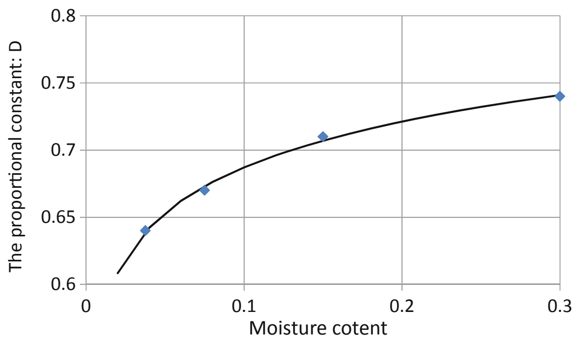

Fig. 4 Estimated coefficient number in terms of moisture contents

Eq. 5 to the regression lines in Fig. 3, which goes to

$$D = 0.049 \log_e \phi + 0.8 \tag{6}$$

In developing the simple formula, Eq. (4), it was approximated that the temperature gradient from exposed surface to drying front is linear and the conductive heat flux is proportional to the approximated temperature gradient. However, the actual temperature gradient is not linear, but becomes gradually shallower as the distance gets closer to the drying front. This is the reason why the regression by Eq. (6) was introduced.

In this research, instead of introducing the coefficient adjustment, the heat continuity equation is re-examined considering not only the heat for moisture evaporation, but also the heat of raising the temperature of the wall. It reduces the number of uncertainties in the heat continuity equation and allows us to skip the step of regression fits to numerical simulation to estimate the drying front. In the latter part of this research, an extended application method is proposed to estimate the temperature rise of arbitrary distance by using a ratio of Fourier number. The new formula is compared to the numerical calculation, and the range of application is validated.

2 Revised Heat Continuity Equation

The fact that the temperature gradient in the dried region is steep near the exposed surface and shallow near the drying front means that a certain amount of conductive heat is accumulated in the dried region to raise its temperature. Considering a slice element of the wall with an area ΔA and thickness Δx at the boundary of the dried and wet regions, as shown in Fig. 5, and letting the distance of the boundary from the exposed surface be x. When the evaporation front proceeds by Δx, the $\dot{q}''$ needs to provide the heat for evaporating water contained in the slice element and raising the temperature of the wall as shown in Fig. 5. Hence, letting Δt be the time for the evaporation front to take to proceed by Δx, we have the heat continuity equation as follows:

$$\dot{q}''\Delta A\Delta t = L_{\mathrm{v}}\phi(\rho\Delta A\Delta x) + C_0\rho(T_{\mathrm{f}} - T_{\mathrm{v}})\frac{1}{2}\Delta A\Delta x + \{C_0 + C_{\mathrm{w}}\phi\}\rho(T_{\mathrm{v}} - T_0)\frac{1}{2}\Delta A\Delta x \tag{7}$$

where L_{v} is the latent heat of evaporation of water, C_0 and C_{w} are the specific heat of the wall material (dry base) and the water liquid phase, respectively, ρ is the density of the wall, and ϕ is the mass fraction of moisture to wall mass (dry base), and the temperature difference profiles between before and after the evaporation of the slice element as shown in Fig. 5, i.e., between t and $t + \Delta t$, is approximated by triangles approximating the conductive heat flux, $\dot{q}''$, as follows:

$$\dot{q}'' \approx \lambda\frac{T_{\mathrm{f}} - T_{\mathrm{v}}}{x} \tag{8}$$

From Eqs. 7 and 8, we have

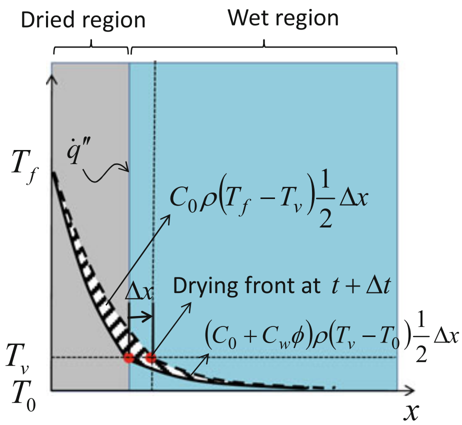

Fig. 5 Schematic illustration of conservation of energy for moisture containing wall

$$\Delta t = \frac{\left\{L_v\phi + \frac{1}{2}C_0(T_f - T_0) + \frac{1}{2}C_w\phi(T_v - T_0)\right\}\rho}{\lambda(T_f - T_v)} x\Delta x \tag{9}$$

Hence, it follows that Δt, which is the time that the drying front makes Δx, is supposed to increase in proportion to the distance x. This appears to be consistent with the observation that the stagnation time at a location in a wall with moisture increases with its distance from the exposed surface, as exemplified in Fig. 2. However, Eq. 9 implies another important fact, i.e., the stagnation time Δt will depend upon, or more specifically in proportion to, the thickness of the slice, Δx, in the numerical calculation as already mentioned by Mizukami [2]. Hence, it is more appropriate to deal with the velocity of the evaporation front than the stagnation period. Rewriting Eq. 9 for the velocity of evaporation front, we have

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda(T_f - T_v)}{\left\{L_v\phi + \frac{1}{2}C_0(T_f - T_0) + \frac{1}{2}C_w\phi(T_v - T_0)\right\}\rho x} \tag{10}$$

3 Simple Calculation Formula for Drying Front

In the above discussion, it is assumed that temperature stagnation does not exist in the moisture containing wall exposed to fire heating, but the wall is simply dried up from the exposed surface and the drying front, i.e., the boundary between dried and wet layers, moves toward the unexposed surface continuously. The time, t, that the drying front reaches an arbitrary distance, x, can be obtained by changing Eq. 10 to the differential equation shown below

$$\mathrm{d}t = \frac{\left\{L_v\phi + \frac{1}{2}C_0(T_f - T_0) + \frac{1}{2}C_w\phi(T_v - T_0)\right\}\rho}{\lambda(T_f - T_v)} x\,\mathrm{d}x \tag{11}$$

and integrating Eq. 11 as

$$\begin{aligned} t &= \frac{\rho C_0}{2\lambda} x^2 \frac{\left\{L_v\phi + \frac{1}{2}C_0(T_f - T_0) + \frac{1}{2}C_w\phi(T_v - T_0)\right\}}{C_0(T_f - T_v)} \\ &= \frac{x^2}{4\alpha}\left\{\frac{2L_v\phi + C_0(T_f - T_0) + C_w\phi(T_v - T_0)}{C_0(T_f - T_v)}\right\} \end{aligned} \tag{12}$$

Hence the dried-up time, t, that the drying front reaches at a certain distance, x, increases proportionally to the square of the distance, x. Figure 6 shows the results of stagnation time, Δt_v, and the dried-up time, t_v, calculated using numerical predictions under the calculation condition in Table 1. Finite difference method [2] is used to simulate the heat conduction and evaporation in a wall, assuming that the incident heat flux is exhausted by the latent heat of vaporization at the evaporation temperature

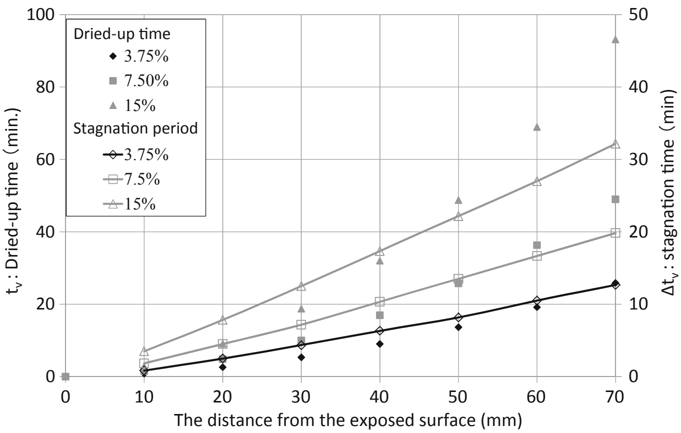

Fig. 6 Dried-up time and stagnation time by numerical calculation

until the mass of water is dried off. Temperature dependence of thermal properties and moisture migration does not take into account. As can be seen in Fig. 6, Δt_v is nearly proportional to x, and t_v is nearly proportional to the square of x that predicted in Eqs. (11) and (12).

3.1 *Validation with Numerical Simulation*

The time also increases with the increase of moisture fraction. Noting that

$$\frac{2L_v\phi + C_0(T_f - T_0) + C_w\phi(T_v - T_0)}{C_0(T_f - T_v)} = \frac{2L_v\phi + (C_0 + C_w\phi)(T_v - T_0)}{C_0(T_f - T_v)} + 1 \tag{13}$$

It is known that the decrease of fire temperature causes the delay of the drying front to move to a certain distance.

Also, Eq. 12 can be regarded as the formula to obtain the distance of the drying front, x, at an arbitrary time, t. Therefore, it may be convenient to generalize Eq. 13 using Fourier number defined as

$$F_\phi = \left.\frac{x}{2\sqrt{\alpha t}}\right|_\phi = \left\{\frac{C_0(T_f - T_v)}{2L_v\phi + C_0(T_f - T_0) + C_w\phi(T_v - T_0)}\right\}^{1/2} \tag{14}$$

where subscript ϕ denotes the point where moisture fraction, ϕ, becomes zero, i.e., the point of drying front.

The right-hand side of Eq. 14 is a function of fire temperature, T_f, initial temperature, T_0 and moisture content, ϕ, so becomes constant when these values are

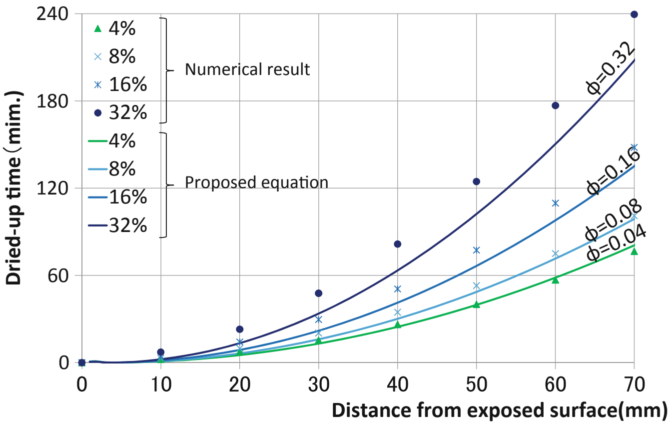

Fig. 7 Comparison of dried-up time with different moisture content

Table 2 Calculation condition

C_w [kJ/kg K]	0.88	T_f [°C]	780
T_v [°C]	100	T_0 [°C]	20
L_v [kJ/kg]	2590	ϕ [−]	0–0.32
α [m^2/s]	0.334×10^{-6}	Δx [mm]	1
		Δt [s]	1

specified, since the others, i.e., specific heat of wall material, C_w, specific heat of liquid water, $C_0 = 4187$ J/kg K, evaporation temperature of water, T_v, and latent heat of water evaporation, $L_v = 2257$ kJ/kg, are basically constants as material properties.

Figure 7 compares the time of drying front in a wall with different moisture contents to reach different distances from the exposed surface predicted by Eq. 12 and the results of numerical computation of heat conduction equation under the following conditions in Table 2. Equation 12 agrees well with numerical results. The gap seems to be a little larger as the moisture content increases, but it seems to be rather the same taking the ratio of the gap over the dried-up time. If that is the case, the multiple data in Fig. 7 is bundled together by non-dimensioned.

3.2 *Error Analysis and Compensation*

The dependence of the drying front in terms of the generalized drying front, F_ϕ, on moisture content, ϕ, is calculated by Eq. 14 and compared with numerically calculated results as shown in Fig. 8. It is confirmed by the results in Fig. 8 that the gap between Eq. 14 and the numerical result becomes a little larger as the moisture

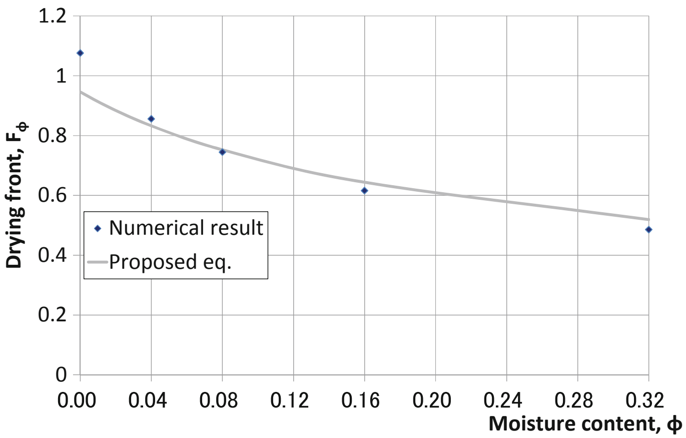

Fig. 8 Relation between drying front and moisture content

content decreases, but that the agreement is fairly good for a significantly wide range. For just checking the extreme case for the simple formula, let us assume $\phi \to 0$, i.e., for dry materials. Then Eq. 14 becomes as

$$F_{\phi=0} = \left.\frac{x}{2\sqrt{\alpha t}}\right|_{\phi=0} = \left(\frac{T_f - T_v}{T_f - T_0}\right)^{1/2} = \left(1 - \frac{\Delta T_v}{\Delta T_f}\right)^{1/2} \tag{15}$$

Equation 15 may be regarded as the simple formula to predict the Fourier number corresponding to ΔT_v for dry wall. On the other hand, the theory of heat conduction of the semi-infinite body in dry materials is well known and available in general heat transfer textbook such as Carslaw [3]. From this, we have

$$F_{\phi=0} = \left.\frac{x}{2\sqrt{\alpha t}}\right|_{\phi=0} = \mathrm{erfc}^{-1}\left(\frac{T_v - T_0}{T_f - T_0}\right)^{1/2} = \mathrm{erfc}^{-1}\left(\frac{\Delta T_v}{\Delta T_f}\right)^{1/2} \tag{16}$$

Therefore, the gap comes from the simplification of the heat conduction in the wall; however, it gives reasonable value over 4% moisture content. And it is safer side treatment to assume materials containing moisture less than 4% as dry material.

4 Temperature in the Dried Region of Materials Containing Moisture

As long as fire-resistant performance is concerned, the building elements containing moisture usually stay sound until the moisture is lost by evaporation. So, it is the dried condition where fire resistance can be issued. Therefore, the target wall is assumed as a semi-infinite body which has a virtual drying front beyond the back surface of the real wall thickness in this paper.

4.1 Simple Estimation of the Temperature in the Dried Region

In order to obtain the temperature in the dried region, it is necessary to know the temperature profile in the region. However, Eq. 14 only gives the position of drying front, but does not give the temperature profile in the dried region. The simplest method is to assume the linear temperature profile from the surface to the drying front, which is consistent with the assumption for heat conduction in this region by Eq. 8. The advantage of the linear approximation of the temperature profile in the dried region is the simplicity in predicting the time, location or temperature in terms of Fourier number. The schematic of the calculation of the temperature assuming the linear temperature profile in the dried region is shown in Fig. 9.

Although the process of the approximation can be applicable for arbitrary temperatures as well, a temperature with specific interest will be a temperature that has some critical meaning for fire resistance performance. Let T_c be such a critical temperature and F_c be the corresponding Fourier number here. Then, it is easy to find the following relationship:

$$\left(1 - \frac{\Delta T_c}{\Delta T_f}\right) \Big/ \left(1 - \frac{\Delta T_v}{\Delta T_f}\right) = \frac{T_f - T_c}{T_f - T_v} = \frac{F_c}{F_\phi} \tag{17}$$

Hence, the Fourier number, F_c, corresponding to the temperature criterion, T_c, is given by

$$F_c = \left\{ \left(1 - \frac{\Delta T_c}{\Delta T_f}\right) \Big/ \left(1 - \frac{\Delta T_v}{\Delta T_f}\right) \right\} \cdot F_\phi \tag{18}$$

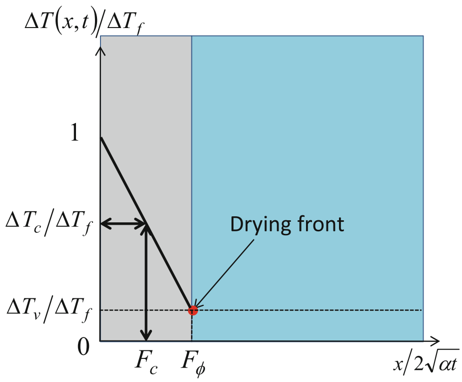

Fig. 9 Linear approximation for temperature profile in the dried region

Note that letting t_c and x_c be the time and the distance from the exposed surface, respectively, the above F_c is

$$F_c = \frac{x_c}{2\sqrt{\alpha t_c}} \tag{19}$$

Equation 18 gives the time and the location in the dried region in the wall at which the temperature becomes critical temperature.

Incidentally, if we chose such F_c as $F_c < F_\varphi$, we can predict the temperature, T_c, at arbitrary time and location in the dried region from Eq. 17 as

$$\frac{\Delta T_c}{\Delta T_f} = 1 - \left(1 - \frac{\Delta T_v}{\Delta T_f}\right)\frac{F_c}{F_\phi} \tag{20}$$

4.2 Comparison of the Temperature Between Simple Estimation and Numerical Calculation

Figure 10 compares the linear temperature profiles calculated by Eq. 20 and the results of the numerical computation of the heat conduction equation under the conditions in Table 2 with different moisture contents. The temperature profile by the numerical result in Fig. 10 represents the one at a fixed time, 30 min. Note that the x-axis is non-dimensioned in terms of Fourier number, which consists of the material thickness divided by the square root of the time expressed as Eq. 19.

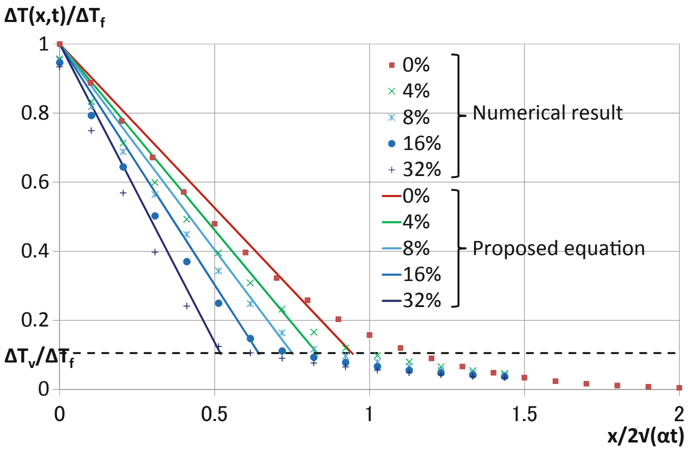

Fig. 10 Comparison of temperature profiles of numerically computed and the linear approximation in the dried region

As can be seen in Fig. 10, while the temperature profile by numerical computation for dry wall starts steeper than the linear profile at low Fourier number, in other words, near the fire exposed surface and becomes gradually gentle, the profiles for high moisture contents are nearly straight in the dried area and matches well with the linear profile. Even in the profiles for low moisture contents, the gap seems to be reduced reasonably for higher temperature, e.g., higher than $0.2\Delta T_{\mathrm{f}}$. In most cases of practical fire safety design, the critical temperature for fire resistance performance is larger than $0.2\Delta T_{\mathrm{f}}$.

4.3 Practical Use of the Simple Temperature Estimation

In practical fire safety design, the relationship between temperature profile and Fourier number can be used for both of the two purposes: (1) To estimate the spatio-temporal position (x, t) in terms of Fourier number when the ratio of critical temperature rise to fire temperature rise, $\Delta T_{\mathrm{c}}/\Delta T_{\mathrm{f}}$ is given, as shown in Fig. 11 and (2) To estimate temperature rise in terms of $\Delta T(x, t)/\Delta T_{\mathrm{f}}$ when the position, (x, t), is given, as shown in Fig. 12. Equations 18 and 20 are available for the first and second purpose, respectively. And its procedures are shown as the arrows in Figs. 11 and 12, respectively. The symbols and lines in Figs. 11 and 12 represent numerical calculations and proposed equations, respectively.

Figure 13 takes the numerical results of Fourier number corresponding to the critical temperature rise in the x-axis and the calculated Fourier number by Eq. 18 on the same condition in y-axis to search the range of application. The symbols represent moisture content. Then, the gap from the diagonal line shows the error, and a safe

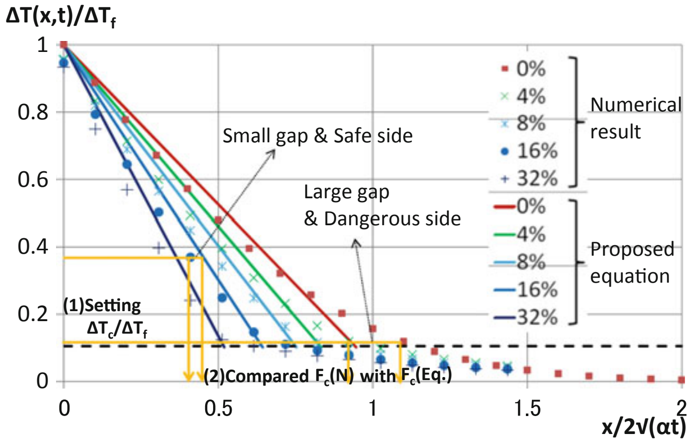

Fig. 11 Flow chart of obtaining the gap of Fourier number between numerical result and proposed Eq. 18

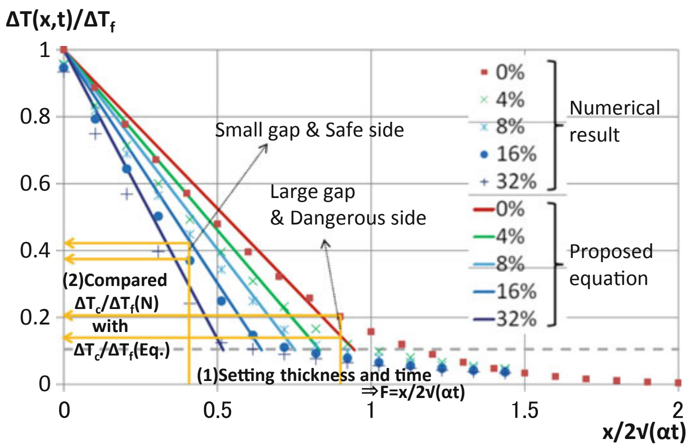

Fig. 12 Flow chart of obtaining the gap of non-dimensional temperature rise between numerical result and proposed Eq. 20

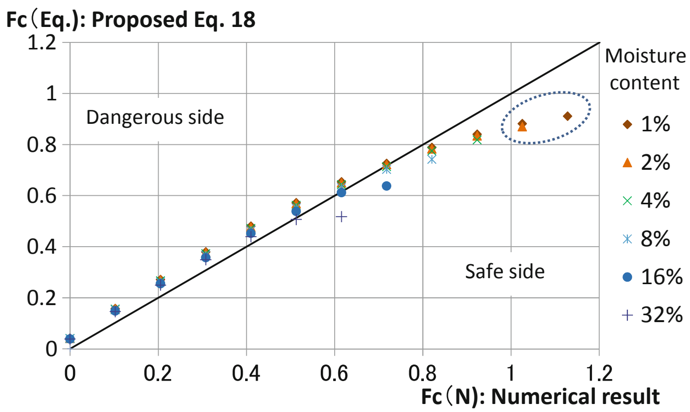

Fig. 13 Comparison of Fourier number between numerical result and proposed Eq. 18 with different moisture content

side is bottom-right-hand side. It is shown that the Fourier number corresponding to the critical temperature rise by Eq. 18 matches well with numerical results in most of the range. As the Fourier number becomes large, Eq. 18 tends to underestimate it. Note that there is an upper limit of the Fourier number, F_{ϕ}, in each moisture content since the proposed equation is only applied to the dried-up region. And the gap seems to be large close to the upper limit in small moisture content.

In another way, the gap in the non-dimensional temperature rise between proposed Eq. 20 and numerical results is shown in Fig. 14. Note that a safe side is up-left-hand side in this case. It shows that the non-dimensional temperature rise is over-estimated a little in a wide range which is the safe side. However, it becomes underestimated

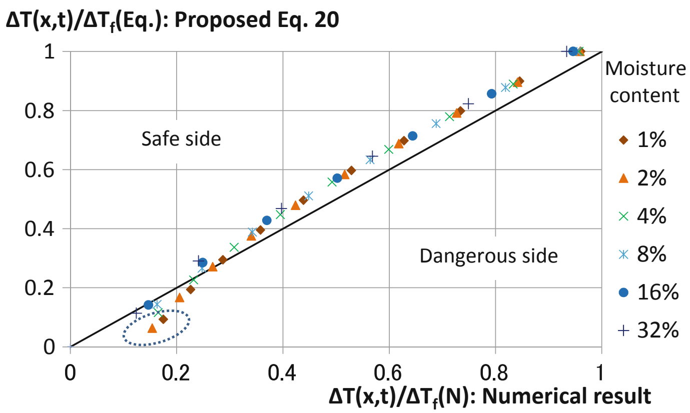

Fig. 14 Comparison of the non-dimensional temperature rise between numerical result and proposed Eq. 20 with different moisture content

and the gap seems to be large with less than 2% moisture content, $\phi \leq 0.02$, and less than 0.1 temperature rise, $\Delta T_c / \Delta T_f < 0.1$. As an example, $\Delta T_c / \Delta T_f$ is about to 0.23 assuming insulation criterion in ISO 834 as $\Delta T_c = 180$ K and assuming the average fire temperature rise at 60 min in ISO 834 as $\Delta T_f = 780$ K. The fire exposed temperature and insulation criterion are set arbitrarily in the performance-based fire safety design; however, it seems to be covered by the range of application in practice. And it is safer side treatment to assume the materials containing smaller moisture content than 2% as dry material.

5 Conclusion

The existing moisture evaporation rate formula has been revisited and re-examined considering not only the heat for moisture evaporation, but also the heat of raising the temperature of the wall. It reduces the number of uncertainties in the heat continuity equation and allows us to skip the step of regression fits to numerical simulation to estimate the drying front.

The new formula is further developed to the simple formula for estimating temperature profile in the dried region of materials containing moisture. It compared to the numerical calculation with good agreement in large moisture content. For small moisture content cases, the range of application is presented to be $\phi > 0.02 \cup \Delta T_c / \Delta T_f > 0.1$.

The advantage of this method is that there is no need to employ numerical simulation to estimate the incident heat flux to the drying front. Without it, the location of the drying front and the temperature profile in the dried region are obtained by the simple equation.

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