Application of Queuing Theory to a Toll Plaza-A Case Study

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Abstract Queuing areas are the junctions involving vehicles waiting in lines and are characterized by an arrival pattern, a service facility arranged in a particular manner and service time. Since the maximum capacity of roads and service facility (i.e. number of booths in case of tolls) are fixed for certain period (i.e. design period), it is necessary to measure the efficiency of a facility. In case of toll booths on highways/freeways which are attractors of vehicles from different origin presumes the flow to be continuous and the vehicle inter-arrival random, adding to this is variability in demand and service (service time in toll booths), poses a problem in optimizing the service facility. In the present study, an existing toll plaza on a 4-lane divided highway having two-way movement (N–S and S–N) is evaluated based on queuing theory. Parameters like traffic volume, space-mean speed and time headway are expressed in 1 h intervals. The vehicle arrival patterns on both directions are postulated to be Poisson distributed and the observed data were fitted to the Poisson distribution. In case of N–S movement, observed frequency and theoretical frequency are found out to be equal indicating the postulated Poisson distribution to be the true population distribution. The use of chi-square test as an index of the goodness of fit for significance level 5% with 10° of freedom justifies the postulated Poisson distribution can be used for future analysis of vehicle arrivals in the respective direction. Finally, the utilization factor indicates a single booth in N–S direction to be under steady-state condition during the study period.

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T. V. Mathew et al. (eds.), *Transportation Research*, Lecture Notes in Civil Engineering 45, https://doi.org/10.1007/978-981-32-9042-6_27

Keywords Queuing theory · Observed and theoretical frequency · Chi-square test · Significance level · Poisson distribution · Utilization factor

1 Introduction

One of the major issues in traffic analysis is the analysis of delay. Delay may be seen in a traffic system where in the demand exceeds the capacity. If the delay results in formation of waiting line, such delays are called as queuing delays. Queuing delays are analysed by queuing theory. The fundamental idea of the theory is that delay in a traffic system is caused by an interruption in the flow pattern as mentioned by Drew [1]. Queuing is characterized by an arrival pattern, a queue discipline and a service facility providing service to the demand. A queuing system can be described by elements like arrival rate, queue discipline, number and arrangement of service facility and service rate. Elements like arrival patterns and service pattern are analysed based on assumptions.

The objective of the present work is as follows: (1) to choose a toll plaza on a highway facility to examine the applicability of the queuing method; (2) to collect the road inventory of the queuing system; (3) to record the traffic volume at mid-block section away from the queuing area as per Indian roads congress guidelines; (4) to present the space-mean speed of vehicles at the entry point of the queuing area and very near to the booth indicating the retardation in speed of vehicles; (5) to measure the random arrival rate of vehicles towards the toll plaza; (6) to measure the service rate of a single booth; and (7) to analyse the arrival rate follows Poisson distribution and to see goodness of fit using chi-square test and finally to check whether the system is in steady-state condition using utilization factor. All the data are collected manually. The paper does not focus on any gap in the study. Rather, the paper examines the applicability of queuing theory for a toll plaza. Young Taeson et al. [2] in their study questioned the assumption of steady-state in the analysis of the queuing system under stochastic method. They have presented a simulation-based methodology for evaluating the appropriateness of the steady-state assumption on a two-lane highway work zone. They have finally concluded that it is not always reasonable to assume steady-state tendency of a highway queuing system prior to exploiting stochastic queuing technique.Woo and Hoel [3], have worked a methodology for evaluating the capacity and level of service (LOS) for toll plazas. The LOS for the selected four toll plazas in Virginia was defined on the criterion of average density, which is highly correlated with volume to capacity ratios. Ceballos and Curtis [4], have shown that the simple analytical models can be used for an initial understanding of the queuing system but warn that good judgement must be employed while using the analytical models, as their results may differ significantly from real-life plaza operation. According to them, simulation should be used for advance planning, design, operation and management of toll and exit plaza facilities. Anokye et al [5], have shown that queuing theory can be applied in modelling the vehicular traffic flow and minimize vehicular traffic in order to reduce delays on roads of Kumasi-Ashanti region. They have suggested that the use of public transport by the government of Ghana would help in reducing congestion on the roads, which in turn boost the productivity. Vandaele et al [6], have shown that queuing models can be applied in assessing the traffic flow parameters compared to traditional empirical methods, which lack in terms of predictive power and the possibility of sensitivity analysis. They believe that speeds have a significant influence on vehicle emissions, and models can be effectively used to assess the environmental impact of road traffic. Cottrell [7] developed a three-regime model for identifying daily queuing period based on annual average daily traffic (AADT), and capacity (C) ratio (AADT/C) for Das and Levinson [8] applied queuing analytical technique to a freeway section in the city of Minneapolis, USA, to develop a new approach for validating densities and speeds on freeway sections. The selected site for the study can be traversed on Indian National Highway (NH-75) near Kirasave (chainage-119.0 km) from Bengaluru towards Mangaluru. The selected road is a divided four-lane National Highway. The site has been selected to explore the applicability of queuing theory a toll plaza.

2 Road Inventory

Road inventory data basically consist of data necessary to identify the project under evaluation. This consists of the geometric details of the project which are collected visually walking along the entire stretch. All of these data will remain constant until the pavement undergoes maintenance which is shown in Tables 1 and 2.

2.1 Traffic Volume

In the present study, the traffic count census is done on four consecutive days in both directions in accordance with Indian Roads Congress (IRC) standard on non-urban roads [9]. To take into account the randomness, the traffic volume study was carried out for duration of one hour on each day. The results are shown in Table 3.

Table 1 Details of pavement geometrics before toll plaza

SL. No	Parameters	Collected data
1	Type of pavement	Flexible pavement
2	Divided/undivided	Divided
3	Number of lanes	Four
4	Width of pavement (m)	9
5	Median width (m)	2.5
6	Shoulder width (m)	1.5
7	Type of shoulder	Earthen

Table 2 Details of queuing area

SL.NO	Parameters	Collected data
1	Type of pavement	Rigid
2	Width of pavement (one side)	30 m
3	Length of pavement on arrival side	250 m
4	Length of pavement on merging side	250 m
5	Number of toll booth on one side	5
6	Length of each toll booth	2.4 m
7	Width of each toll booth	1.9 m
8	Width between toll booths	3.6 m
9	Type of merging	Leftmost merging

Table 3 Details of traffic flow in both the directions

SL no	Direction	Time, hrs	Traffic flow, vehicles/hour	Average traffic flow, vehicles/hour
1	N–S	4:00 to 5:00	396	375
		5:00 to 6:00	518	
		9:00 to 10:00	321	
		10:00 to 11:00	262	
2	S–N	4:00 to 5:00	297	345
		5:00 to 6:00	311	
		9:00 to 10:00	354	
		10:00 to 11:00	417	

2.2 Space-mean Speed

The speed of arriving vehicles towards the toll plaza on both directions (N–S and S–N) was analysed on two designated location, one near to the booth (near speed breaker) and another location where in pavement changes from flexible to rigid (i.e. entrance of queuing area) towards the plaza. The speed data of vehicles were collected manually using the below equation. The values obtained in the field are given in Table 4.

$$\text{Space-mean speed} = \frac{\text{Distance in metres}}{\text{Average time taken in seconds}}$$

$$V_s = \frac{3.6 * d * n}{\sum ti} \tag{1}$$

Table 4 Details of space-mean speed of vehicles

Duration, hours	Direction	Average time, sec	Distance, m	Number of vehicles	Average velocity, km/h	Remark
4	N–S	0.80	10	453	20.38	Near booth
4	S–N	0.96	10	366	13.72	
4	N–S	0.63	10	598	34.17	Away from booth
4	S–N	0.76	10	556	26.33	

2.3 Time Headway

Time headway (h) or simply headway is the time counted between the passings of the fronts of two vehicles at a specified point. It is measured in seconds. Time headway is necessary to know the inter-arrival rate among the vehicles which is needed for the analysis of arrival pattern in a queuing system. In the present study, the average headway on particular days was found out manually by using a stopwatch at the designated location. The headway details collected included different class of vehicles. The arrival rate, which can be interpreted as the inverse of headway, on those respective days is presented in Table 5. The data reveal that vehicle arrival patterns are randomly distributed.

2.4 Arrival Rate

In a queuing system, it is necessary to define the distribution of arrivals. Generally, the arrival pattern has been defined around three types of distributions, namely binomial, Poisson and negative binomial. Gerlough's [10] analytical work discusses in detail the application of the Poisson distribution in highway traffic. Some of the uses open to the traffic engineer are shown to include the analysis of arrival rates at a given point by Drew [1]. In the present study, vehicle arrivals are assumed to be Poisson distributed. Field data were collected for each of 120–30 s intervals for five consecutive days. The summarized vehicle arrival rate for the study period is shown in Table 6.

Table 5 Details of headway of vehicles in both directions

Direction	Time duration (1 h)	Average headway $h_{avg} = hi/nt$ (s)	Arrival rate $q = 1/h_{avg}$ (v/h)
N–S	Day 1	20.6	174
	Day 2	21.5	167
S–N	Day 1	25.3	142
	Day 2	17.1	210

Table 6 Details of vehicle arrivals

Sl. no	Directions	Observed vehicles	Duration in hours	Average arrival rate, (v/h)
1	N–S	1991	5	398.2
2	S–N	2327	5	465.4

When the number of events becomes very large, the binomial distribution approaches the Poisson distribution as a limit and is expressed as in Eq. 2.

$$P(x) = \frac{\mathrm{e}^{-m} m^x}{x!} \quad (2)$$

The fitting of the Poisson distribution to the collected field data for each direction is presented in Tables 7 and 8. The theoretical frequency for the postulate Poisson's distribution is obtained from Eq. 3.

m = average number of vehicle arrivals per 30 s interval

$$m = \frac{1991}{600} = 3.32, \text{ (N} - \text{S)}$$

$$m = \frac{2327}{600} = 3.38, \text{ (S} - \text{N)}$$

So the Poisson distribution for the north–south direction (N–S) will be of the form

$$P(x) = \frac{\mathrm{e}^{-3.32}.(3.32)x}{x!}$$

The Poisson distribution for the south–north direction (S–N) will be of the form

$$P(x) = \frac{\mathrm{e}^{-3.88}.(3.88)x}{x!}$$

The theoretical frequency = total intervals observed $.P(x)$

$$F = 600.P(x) \quad (3)$$

2.5 Service Rate

The service rate depends upon the type of operation involved in providing service to the customers. Generally, cash collecting service takes more time than the automatic way of collection. Service rate denotes the rate at which vehicles are being served

Table 7 Details of observed frequency from north–south direction

Number of vehicles arriving for every 30 s	Observed frequency						Total number of vehicles	Probability $P(X) = e^{-3.32}.3.32^{X}/X!$	Theoretical frequency $F = 600\ P(X)$
	Day 1	Day 2	Day 3	Day 4	Day 5	Total			
0	6	7	8	2	4	27	0	0.036	21.6
1	11	10	21	13	10	65	65	0.122	72.0
2	21	22	22	32	31	128	256	0.2	120
3	19	16	23	35	34	127	381	0.22	132
4	24	20	24	20	24	112	448	0.182	109.2
5	18	19	7	9	12	65	325	0.121	72.6
6	13	15	5	6	4	43	258	0.067	40.2
7	3	5	8	3	0	19	133	0.032	19.2
8	2	2	2	0	1	7	56	0.013	7.8
9	2	2	0	0	0	4	36	0.005	3.0
10	0	1	0	0	0	1	10	0.002	1.2
11	0	1	0	0	0	1	11	0.0005	0.3
12	1	0	0	0	0	1	12	0.00013	0.08
Above 12	0	0	0	0	0	0	0	0.00013	0.82
Total	600						1991	1.000	600

Table 8 Details of observed frequency from south–north direction

Number of vehicles arriving for every 30 s	Observed frequency						Total number of vehicles	Probability $P(X) = e^{-3.88}.3.88^X/X!$	Theoretical frequency $F = 600.P(x)$
	Day 1	Day 2	Day 3	Day 4	Day 5	Total			
0	2	0	12	12	12	38	0	0.021	13.0
1	6	2	14	18	15	55	55	0.078	47.0
2	8	7	25	40	30	110	220	0.151	91.0
3	14	11	27	22	23	97	291	0.196	118.0
4	15	16	20	16	20	87	348	0.190	114.0
5	21	20	10	4	15	70	350	0.148	89.0
6	16	18	8	6	3	51	306	0.096	58.0
7	12	15	4	1	1	33	231	0.053	32.0
8	13	12	0	0	1	26	208	0.026	16.0
9	6	11	0	0	0	17	153	0.011	7.0
10	6	5	0	1	0	12	120	0.0044	3.0
11	1	2	0	0	0	3	33	0.0016	1.0
12	0	1	0	0	0	1	12	0.0005	0.3
Above 12	0	0	0	0	0	0	0	0.00016	0.7
Total	600						2327	1.000	590

in a system. But the problem with manual service facility is variation in service rate and to predict the average service rate is difficult. In the present study, the average service time of a single toll booth with first-in-first-out queue discipline was found out to be 8 s. The service rate of the booth is found out to be 450 vehicles/h.

3 Analysis of Field Data

The delay and waiting time of drivers in toll plaza depend on service time and arrival rate. The quick service time and a number of toll booths can reduce the time taken by vehicles in queue. The wasted time can be calculated and minimized by analysis of the observed data, by calculating the wasted time the performance of the servers can be analysed and also the delay in overall travel time can be found. In the present work, chi-square test is used as an index of goodness of fit. The test with a significance level of 5% was carried out to check the randomness of the data as Poisson distributed. The vehicle arrival's field data were tested for the two decisions taken from Gerlough's analytical findings [10]:

a. It is not very likely that the true distribution (of which the observed data constitute a sample) is in fact identical with the postulated distribution.
b. The true distribution (of which the observed data constitute a sample) could be identical with the postulated distribution as shown in Table 9.

The critical value for the significance test at 5% level with 8°, 9° and 10° of freedom was obtained from the chi-square table-III of appendix-A [11], and the data are presented in Table 10.

3.1 Utilization Factor

The solution to a queuing problem entails the assessment of a system's performance, which in turn is described by a set of measures of performance [12]. In the present study, a term called utilization factor is used to quantify the traffic intensity handled by the queuing system. Utilization factor is defined as the ratio of the mean arrival rate per unit time to the mean service rate per unit time. If the utilization factor of the queuing system works out to be less than unity, indicating the system to be in a steady-state condition.

$$\text{Utilization factor}(\rho) = \frac{\text{mean arrival rate } (\lambda)}{\text{mean service rate } (\mu)} < 1.0 \quad (1)$$

A single booth of the toll plaza having first-in-first-out (FIFO) queue discipline on both directions was analysed for the steady-state condition using utilization factor. The recorded value indicates that the single booth on N–S direction is under steady-

Table 9 Details of chi-square value for the vehicle arrivals for every 30 s

Vehicles arrivals for every 30 s	Observed frequency, f	Theoretical frequency, F	f^2/F	Statistical value	Chi-square test value $\chi^2 = \Sigma f^2/F - n$
0	27	21.6	33.75	Mean = 1.028	617.3 − 600 = 17.3
1	65	72.0	58.68		
2	128	120	136.5		
3	127	132	122.19		
4	112	109.2	114.87		
5	65	72.6	58.20		
6	43	40.2	46.0		
7	19	19.2	18.80	Variance = 8.932	
8	7	7.8	6.28		
9	4	3.0	5.33		
10	1	1.2	0.83		
11	1	0.3	3.33		
12	1	0.08	12.5		
Above 12	0	0.82	0.0		
Total	600	600	617.3		

Table 10 Details of significance level for both directions

Directions	Significance level	Degrees of freedom	Critical value	Computed value	Remarks
N–S	5%	8	15.51	17.3	Poisson rejected
		9	16.92		Poisson rejected
		10	18.31		Poisson accepted

Table 11 Details of traffic intensity of single booth in both directions

Direction	Mean arrival rate (λ) V/h	Mean service rate (μ) V/h	Utilization factor (ρ)
N–S	398.2	450	0.884
S–N	465.4	450	1.034

state condition, but the booth on S–N direction is not under a steady-state condition as shown in Table 11.

4 Discussions

Highway facilities providing service to the vehicles have got systems arranged in a specific manner (e.g. toll booth) at designated locations on the highway. At the entry point and exit point of those systems, vehicles have to wait and proceed in order to be served. Such systems in turn create waiting lines of vehicles called as queue length. Such queuing systems which tend to form queue of vehicles have to be analysed using queuing theory. Most of the highway systems that are analysed based on queuing methods, assume the system to be operating under the steady-state condition which is seldom in nature.

In the present study, a toll plaza located on National Highway (NH-75) was evaluated based on queuing theory over different time interval. Since it is a general practice in the traffic engineering profession to analyse traffic systems on hourly basis, the same approach was adopted for the present study. Parameters like traffic volume, space-mean speed, time headway, arrival rate and service rate are expressed in 1-hr intervals. The results of traffic volume and time headway recorded on different days ascertain that vehicle's count is highly random on hourly basis. The arrival rate was postulated to be Poisson distributed. Field data were recorded for each of 120–30 s interval for five consecutive days on both directions and the fitting of the Poisson distribution to the observed data on both directions has been presented. In case of vehicles on north–south (N–S) direction, there is a high degree of agreement between the total observed and theoretical frequencies, so the postulated theoretical (Poisson) distribution is in fact the true population distribution based on mere inspection. In case of south–north (S–N) direction, there is very less degree of agreement between the total observed and theoretical frequencies indicating that the postulated theoretical (Poisson) distribution does not reflect the true population distribution. Further, the vehicle arrival data on north–south direction were tested for goodness of fit using the chi-square(χ^2) test with a significance level of 5% (0.05) and 3° of freedom (8, 9, 10). Since the computed value ($\chi 2 = 17.3$) exceeded the critical value for degrees of freedom 8 and 9 indicating the postulated Poisson distribution to be incorrect, but for the same significance level (5%) with 10° of freedom the computed value ($\chi 2 = 17.3$) is less than the critical value indicating the postulated Poisson distribution can be used for analysis of vehicle arrival. Finally, a single booth of the toll plaza having first-in-first-out (FIFO) queue discipline on both directions was analysed for the steady-state condition using the utilization factor. The booth on the N–S direction is found to be under steady-state condition whereas the booth on S–N direction is not under steady-state condition. In brief, the findings from the study are the assumptions made in vehicle arrivals, service rate and about the steady-state system generally made in queuing methods applied to highway facilities call for attention.

5 Conclusions

The existing toll plazas located on highway systems can be analysed using queuing theory to estimate the distribution characteristics like arrival rate and service rate. But these characteristics are analysed using different probabilistic distribution to ascertain the true population from the sampled population. From the present study, the following conclusions can be drawn, and in case of arrival rate in N–S direction there is a high degree of agreement between the observed frequency and theoretical frequency, indicating the postulated Poisson distribution to be the true population distribution. Thus, the Poisson distribution can be considered for analysing the queuing systems.

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