Chapter 4
Mathematical Problem-Solving Using Real-World Problems

Lu Pien Cheng and Tin Lam Toh

Abstract According to the Singapore primary mathematics curriculum (2006), it is important that students tackle a variety of mathematical problems, including real-world problems, as they apply their mathematical problem-solving skills. This paper examines the challenges and affordances of using real-world problems with young children in a primary school in Singapore. Using the laboratory class cycle, the teachers in the study planned, observed and critiqued a mathematics lesson using real-world problems for primary two children. Data in this study includes the teachers' conversations during the laboratory cycle and the students' responses during the observed mathematics lessons using real-world problems. Our findings show that the real-world problem used in this study generated rich mathematical classroom discussion. The teachers' learning from using real-world problems through the laboratory cycle and the challenges they faced were discussed in this study.

Keywords Primary mathematics • Real-world problems • Mathematical processes • Problem solving

Introduction

Recent education reform efforts have been influenced by the demand of the economy for skilled workers who can apply their knowledge in flexible ways to solve novel problems (Goodman 1995). It is thus not surprising that educators measure *competence* as not only the acquisition of basic skills but also the integration of these skills in solving real-life problems (Fuchs and Fuchs 1996; Fuchs et al. 2005). In line with the above education reform, the call among the mathematics education community throughout the world to introduce mathematical tasks that are related to "real life" and the "real world" in the mathematics curriculum is a natural progression. Such a call could be traced back to as early as 1982 in the Cockcroft Report about the increased concern that adults were not able to apply the mathematics they

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had learned at school in everyday contexts (Boaler 1993). The reasons provided by the advocates of this movement can generally be classified under five main categories: (1) meeting the economic needs of the society, (2) deepening students' understanding of important issues, (3) improving students' understanding of mathematical concepts, (4) enhancing students' appreciation of mathematics and (5) improving student affect in mathematics (Beswick 2011).

The Singapore primary school mathematics curriculum has consistently emphasised the application of mathematics to solve real-world problems (MOE 2000, 2006, 2012). Students must be able to connect mathematics that they have learnt to the real world in order to enhance their understanding of key concepts and to develop mathematical competencies (MOE 2012). This matches the two main reasons proposed by Cooper and Harries (2002) for mathematical applications: (1) applications of mathematics to problems outside of the classroom and (2) improve students' understanding of mathematics concepts.

"Mathematical problem solving includes using and applying mathematics in practical tasks, in real life problems and within mathematics itself" (MOE 2000, p. 5). Central to mathematics learning is mathematical problem-solving which involves the "acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems" (MOE 2006, p. 6).

Teaching Mathematics in the Context of Real-World Problems

In this study, real-life contexts are defined "broadly to include situations that refer (directly or indirectly) to everyday activities or concern mathematical applications" (as cited in Stylianides and Stylianides 2008, p. 860). The terms real life and real world are used interchangeably in this paper. The range of practices under the umbrella of real-world connections includes simple analogies, classic word problems, analysis of real data, discussions of mathematics in society, hands-on representations of mathematics concepts and mathematically modelling real phenomena (Gainsburg 2008, p. 200).

One positive effect of using real-world problems on students is students' increased motivation in mathematics. When children make connections between the real-world and mathematics concepts, the latter becomes relevant to them, thereby motivating students to learn and get more interested in the learning process (Albert and Antos 2000). Hiebert et al. (1996, p. 18) reported that "the problems with which students will become most easily engaged are those that are taken from their everyday lives". Research also suggested that teachers can place problem-solving in real-life contexts as one way to increase their students' motivation in mathematics (Schiefele and Csikszentmihalyi 1995; Stylianides and Stylianides 2008). However, real-world activities intended "to review or enhance previously taught concepts would seem particularly expendable" when the main goal is to impart mathematical concepts and skills rather than to develop "students' ability and disposition to recognize applications and solve real problems" (Gainsburg 2008, p. 215). In our opinion, a real-life task needs to be well designed in order to "stimulate student interest and engagement and the development of a healthy, accurate view of mathematics as a useful discipline" (Trafton et al. 2001, p. 263).

Mathematical Problem-Solving in Singapore: From Story Sums to Real-World Problems

The classification scheme for the types of mathematical problems in Foong (2009) suggests the possible range of problems Singapore primary school students might be exposed. Teachers are generally comfortable with solving problems using problem-solving heuristics and thinking skills (Kaur and Dindyal 2010). In fact, teachers have received extensive preparation on the use of heuristics. Efforts were also made to associate each given type of word problems with a particular heuristics listed in the mathematics curriculum.

Students at the primary level learn most of their mathematics through *word problems* or *story problems* in general (Reusser and Stebler 1997). In Singapore classrooms, word problems can be found in typical textbook problems. These word problems are usually contextualised; students solving these problems are required to understand the context and use appropriate mathematical operations to solve these problems. It could also be seen that these contextualised problems are generally artificial with "very clean and tidy state" (Ang 2009, p. 180). The word problems are unrealistic in nature and they actually teach students to suspend real-world sense making (Greer 1997). Indeed, "many teachers consider the real contexts of word problems irrelevant distractions" (cited in Gainsburg 2008, p. 200) making it difficult to convince students about the real-life applications of mathematics. Furthermore, most of these problems are close-ended; many real-life problems are open-ended and require the solvers to engage in "interpretive activity" (Inoue 2008, p. 39). The open-endedness of such real-life problems suggests fewer constraints in problem goals, thus allowing problem-solvers more opportunities to connect their diverse everyday experiences and the problem and make sound decisions based on assumptions about the real world. Open-ended tasks encourage students to adopt divergent thinking and reasoning and therefore allow students to "respond positively and participate actively in the learning processes" (Kwon et al. 2006, p. 51). Furthermore, it has been shown that open-ended tasks focused on content-specific features are effective in promoting particular concept development and to elicit higher-order thinking (Sullivan et al. 2009).

Efforts were made to expose students to contextual open-ended mathematics problem tasks in Singapore. Foo and Fan (2007) investigated the effects of integrating authentic open-ended tasks in the Singapore mathematics classroom in a secondary school as an assessment strategy. Chan (2005) examined the teacher's and students' experiences and difficulties in using contextual open-ended mathematics problem tasks with primary six students. Results from Chan (2005) showed that the students were actively engaged at high levels of cognitive thinking through scaffolding and meaningful explanations. In the same paper, Chan reported that the authentic nature of the contextual open-ended mathematics problem tasks provided "opportunities for personal values and beliefs to be raised through the discussion". In addition, exposing students to contextual open-ended mathematics problem allowed the students to appreciate the complexity of the real world. Through the problem-solving process, contextual open-ended mathematics problem helps students connect mathematics learning to the real world. However, such tasks require much time to plan and complete.

Challenges in Teaching Using Real-World Problems

Research shows that teachers may face some difficulties in utilising mathematical tasks that are set in real-world context. For example, Rule and Hallagan (2007) noticed that the teachers in their studies had difficulties understanding algebraic generalisations set in an authentic context. This occurrence suggests some inadequacies in teachers' pedagogical content knowledge (PCK), particularly in teaching authentic problem-solving. Shulman (1986b) has highlighted the importance of PCK in order to teach the subject well. This idea was upheld by Charalambous (2008) who has shown that teachers with good PCK were able to maintain the high cognitive demands of the mathematical tasks, while teachers without good PCK generally "proceduralized even the intellectually demanding tasks [he or] she was using and placed more emphasis on students' remembering and applying rules and formulas" (p. 287). The inadequacies in PCK in teaching authentic problem-solving may be due to "teachers mainly get[ting] their ideas for real-world connections from their heads, and many feel hindered by a lack of resources, ideas, or training for making connections" (Gainsburg 2008, p. 215). Another reason could be a wider knowledge base is required to fully utilise such contextualised tasks. In a study with elementary Latina/Latino students in the use of authentic mathematical investigations, students brought "multiple and diverse funds of knowledge to the classroom" (Turner et al. 2009, p. 140). The studies suggest that teachers may require a wider knowledge base which may include a blend of students' "out-" and "in-" school experiences when using mathematics problems embedded in real-life context.

Foong et al. (1996) reported in their study that a number of teachers felt inadequately prepared to teach mathematical problem-solving when the examples had multiple possible solutions. Foong (2005) described how three primary teachers implemented the same open-ended problem-solving activities with varying degrees of success. Only one teacher implemented the tasks successfully. One of the teachers was too procedural in her instruction and the other teacher had limited understanding of the mathematical thinking embedded in the task and the kind of cognitive demands to be made of the students.

The use of contextual open-ended problems may pose challenges to collaborative group work. For example, Chan (2005) reported that "groups with quieter students made it difficult to work as a team" and students who were more vocal within the group appeared to be more engaged in the task. Bennett and Desforges (1989) cautioned that the use of such problems should be built on students' prior knowledge.

Unfamiliar contexts may cause some students to have difficulty proceeding with the problem-solving process (Rogoff and Lave 1984 as cited in Chan 2005). In the same line, Stillman reported that "cue salience and its interaction with prior knowledge are of particular importance" (2000, p. 335) in application tasks. The problem-solver will be less likely to "engage with the context of a task" in which they are unfamiliar in the "same degree" as another person who has been exposed to a similar scenario (Stillman, p. 335).

Amid the challenges faced by the teachers in using real-world problems in the mathematics classroom, much potential is yet to be explored for the problems to enhance teaching and learning of young children. This paper investigates the potential of real-world problems in mathematics by examining its benefits and challenges for teachers and young learners.

This Study

This paper continues the investigation of the potential of real-world problems in mathematics that was previously reported by Cheng (2013). The results presented in this paper were based on a subset of the data that were collected in Cheng's study (2013). The laboratory class cycle served as the platform for the teachers to plan, observe and critique the real-world mathematics problems. The observation stage is also referred to as the research lesson. In the following section, we report the teachers' engagement in one laboratory cycle in a neighbourhood primary school to develop primary two students' decision-making skills through real-world problems in mathematics. Specifically, we aim to address the following research questions:

1. What are the benefits for teachers and young children by using real-world problems?
2. What are the challenges for teachers in using real-world problems with young children?

A total of six consecutive weekly meetings were conducted with the teachers. Each meeting lasted 1 h. The first four meetings were used to plan the mathematics lesson involving the use of real-world problems. The fifth session was the research lesson and the last session was used to critique the research lesson.

Participants

Five teachers from the same school with various backgrounds, ethnicities and varying years of teaching experience participated in the study. The teachers were Mary, Mable, Ginger, Vin and Ivy. Pseudonyms were used for the teachers in this study. The research lesson was taught by Mary to the children in her class. The children were the better students in the primary two cohort in her school and the children were arranged into mixed-ability groups prior to the research lesson. For the rest of this paper, we used the term children instead of students to denote the young children involved in this study.

The Mathematics Lesson

This study utilised an approach that differs from problem-focused teaching approach. The problem-focused teaching (Riedesel et al. 1996) approached teaching mathematics in context in such a way that the real-world problems become the settings in which the mathematics are presented. That is, students were presented a word problem to solve in whatever ways that makes sense to them. The students then shared their methods for solving the problem with the class before the teacher offered a standard algorithm of solving the problem. "Skill in mathematics arises from context", rather than presenting the skills first then the context (Schwartz 2008, p. 8). The teaching approach that was applied in this study differed from problem-focused teaching in that the skills in mathematics were taught to the children first before presenting the problems embedded in real-life contexts. The purpose of the task was to further reinforce the computational skills and provide opportunities for the children to apply those skills in a more open-ended task framed in real-life context.

The task was designed by a group of teachers from a neighbourhood school for primary two children with several principles in mind (Cheng 2013): It was aligned with the 2006 Singapore mathematics curriculum, tapped and extended mastery of mathematical concepts. A pre-task of the school *Bookshop* was used to familiarise the children with the competencies before solving the actual[1] *Restaurant* problem. The pre-task required the children to spend exactly \$2 at the scenario of a school bookshop. Small numbers (10¢, 20¢, 30¢, 50¢, 80¢ and \$2) were used and the children had to choose from only nine items. The items included, for example, pencil, eraser and ruler.

The *Restaurant* problem required the children to spend close to \$30 at a restaurant scenario. Bigger numbers were used (\$18, \$5.50, \$3, \$2.50) and the children had to choose from 14 items such that each person in the group had a drink and dessert was optional. The items included, for example, fried chicken wings 1 basket of 12 for \$18, orange juice 1 cup for \$2.50 and ice cream cones 3 for \$4.50.

The *Restaurant* problem and the *Bookshop* pre-task were designed and sequenced in such a way that learning would take place at the anticipated zone of proximal development for the majority of the children in the class where the lesson was to be conducted. Polya's (1957) problem-solving steps were not formally introduced to the children but were used to guide the implementation of the tasks. Mary, one of the participants in this study, modelled the mathematical thinking, reasoning, decision-making and calculation skills to satisfy the conditions stated in the

[1] A sample of the *Bookshop* and *Restaurant* problem appeared in Cheng (2013).

Bookshop pre-task before allowing the children to investigate the *Restaurant* problem. The children worked in groups of three to record and complete the *Restaurant* problem.

Research Design and Data Analysis

The mode of inquiry we employed in this study was interpretative case study (Merriam 1988). Data collection included audio recordings of meetings, artefacts from the weekly meetings, interviews with teachers after the laboratory class cycle and the researcher's field notes. In the first phase of the data analysis, the researchers listened closely to the audiotapings of the meetings and identified issues and topics that were discussed. The issues and topics were coded (time, scaffolding questions, student grouping, etc.) and the researchers started to write memos regarding the benefits and challenges of the task for the teachers and young children in each of the codes. In the next phase, the researchers used the codes to the transcripts of the interviews and lesson plan and continued to write memos regarding the benefits and challenges of the tasks. Data from the three data sources were triangulated. The codes were organised and grouped into themes. The researchers then wrote the findings using the themes generated.

Results and Discussion

Analysis of the teachers' conversations during laboratory class revealed that the *Restaurant* problem using real-life context offered the participating teachers in this study abundant opportunities for professional development. First, we discussed the benefits of using real-world problems for the teachers and young children in this study. Next, we present three main challenges that the teachers faced when implementing the *Restaurant* problem.

Opportunities for Teachers to Engage in Deeper Discussion During Planning of Lessons

The design of mathematics problems using real-life context tasks required knowledge in several aspects, such as knowledge of curriculum, knowledge of task design and knowledge of children's thinking and their beliefs (Cheng 2013). Coming together as a group to design such tasks afforded the teachers a platform to build upon each other's expertise in creating relevant and good classroom tasks. When planning the tasks, the teachers crafted more open-ended questions of a higher-order nature, drawn away from the standard answers towards seeking "many" plausible responses by the children. In order to facilitate the children's completion of the *Restaurant* problem, the teachers had to anticipate and classify all the plausible responses and the mathematical skills required with each response. They found themselves drawing from their understanding of children's thinking to craft scaffolding questions and meaningful explanations to facilitate the task. Table 4.1 summarises how scaffolding can be used to unmask the *Restaurant* problem in the mathematics classroom (adapted from Kim and Hannafin 2011, p. 409).

Because of the diversified ethnic, cultural and family backgrounds of the children, different interpretations of the task by the children were expected. Solutions to the problems can also be vastly different as a result of individual differences, personal values and beliefs of the children. The teachers' varied backgrounds, differing beliefs and personal values have the potential to play an important role in creating appropriate context and scaffolding questions to elicit the variety of responses possible from the children. Such platforms and tasks afforded the teachers opportunities to engage in rich discussions, widen their perspectives, share and grow as a community.

The opportunity to think more deeply about the use of scaffolding questions in the *Restaurant* problem empowered the teachers to engage the children at deeper and higher levels of cognitive thinking. Ginger said:

> I really had to think more about the scaffolding questions so that the children don't just say yes or no… sitting down together [to plan the questions as a group] really helps because the scaffolding questions really help [some of] the children to articulate their thought processes [weekly meeting 6].

Mary said, "usually when I do group work, the children were able to perform the task and the discussion was not as lengthy as this lesson [Mary, weekly meeting 6]". Mary also added,

> I am now made more aware of how to question the children in order to bring out their explanation, their reasoning. So previously, the questions I asked used to be more closed. Now it's more open. It's the awareness, the conscious, because I am more conscious of that [interview].

Greater Opportunities for Teachers to Hear Children's Thinking and Understanding

During the research lesson, the social conversations that emerged during the children's group work brought a diverse range of interpretations to the problem scenario. The teachers stationed themselves with assigned groups of children and they were able to hear what the children were thinking, what they understood from the problem scenario and blockages they face in solving the problem. For example, through the questions that the children raised during group work, the teachers were more aware of the varied interpretations of key terms in the questions. One of the

Table 4.1 Scaffolding children's unmasking of the *Restaurant* problem

Problem-solving phases	Scaffolding foci	Scaffolding examples
1. Pre-task	Model the problem-solving process for an easier parallel task	Help children to acquire a sense of the problem-solving process
	Model the competencies required for the actual task	Help children develop the competencies required for the actual task
2. Actual task: hands-on experience of the problem-solving process, apply and extend competencies		
Introduction: before the problem	Identify the structure of the problem	Help children identify the "given", "to find" and assumptions in the problem
Understanding the problem	Externalise children's prior knowledge and experiences on the problems	Help children find cues and hints relevant to the problem contexts, background knowledge
		Provide resources for children to explore the problem
Launch: during problem-solving	Obtain a plan for the solution	Help children to search for and connect to similar problems that have been solved (make connections to pre-task)
Planning and doing	Pursue solution	Help children to locate the key problem concepts, data and known and unknown variables and the relationships/connections among them
	Active checking of each step of the working	Help children to identify any other information related to the context
	Handle blockages	Help children to compare solutions with assumptions of the problem
		Help children to replan when solutions do not satisfy all the assumptions or when blockages are encountered
Whole-class discussion: after problem-solving	Surface the mathematics and the mathematical processes in the tasks, e.g. compare and contrast	Help children to consolidate and reflect on the mathematical skills, mathematical processes, mathematical reasoning and problem-solving processes
Checking	Check the result/active diagnosis	Help children to verbalise solutions and explanations
		Help children to detect errors and faulty reasoning
		Help children to contemplate on potential revisions to their solutions

groups could not agree with the items that should be considered as "dessert" and this provided an opportunity to clarify what should be considered as a dessert in the given menu. In accordance with Chan (2005), the "real-world" depiction of these problem tasks provides opportunities for personal value and beliefs to be raised through the discussion. Whenever each item is removed, added or replaced with another item in the menu, ownership lies with the children to check whether all the conditions in the problem are met. Many children had different interpretations of the term “maximum to spend”. The teachers also examined the children’s solution strategies (reported in Cheng 2013) to the *Restaurant* problem during the research lesson and critique. Some of the children’s responses were not anticipated by the teachers. For example, one group of children divided $30 equally among themselves and each of the children decided what they wanted to buy with their $10. There were some instances when the teachers were surprised with what the children were able to do. Mary said (weekly meeting 6): “Some of the children surprise me … one student can understand the meaning of maximum to spend and also able to explain what this term means”. Mabel was also surprised that “some of the children are able to explain very well”.

Through the unexpected children’s solutions and responses, the teachers accommodated and assimilated their schema on their understanding of the type of solutions and explanations children would generate for such tasks. Through the expected children’s solutions and responses, the teachers reinforced their understanding of children’s thinking.

Opportunities for Young Children to Develop Mathematical Process Skills

During the research lesson, the teachers observed process skills (e.g. decision-making, comparing, reasoning, thinking, etc.), mathematical skills being reinforced, consolidated and developed through the *Restaurant* problem. The children were also engaged in more diverse and flexible thinking as the real-world problem provided them opportunities to “choose, mix and match items from the menu” and consider the appropriateness of their solution. In accordance to Kwon et al. (2006), the open-endedness of the tasks encouraged the children to adopt divergent thinking and reasoning and promoted active participation in the learning processes. The use of the *Restaurant* problem made the experience of learning mathematics more meaningful and enhanced children’s appreciation of the nature of mathematics. This result supports the findings by Albert and Antos (2000).

Ginger felt that the *Restaurant* problem afforded richer discussion of the thinking and decision processes required by the children to solve the problem. However, it is up to the individual teacher to fully utilise the affordances of the task. Ginger said:

> Some of the children’s goal was to solve the task. They are very happy when they solve the problem. They do not want to think further about the problem… Those groups, I would give feedback during the group work and challenge the children to think more deeply about the problem [paraphrase] (weekly meeting 6).

A Challenge for Teachers to Group Children

The teachers in this study grouped the children such that each group had a good mixed of high-, middle- and low-ability children. While Chan (2005) reported that more vocal students within the group appeared to be more engaged in the task, the teachers in this study faced the challenge to try to "balance the task to the high ability and low ability" (Mary, weekly meeting 6). The teachers felt that the higher-ability children could be stretched even further in the *Restaurant* problem. The low-ability children were observed to be struggling with language, computational skills, understanding the problem, identifying the known conditions and the unknowns even though a pre-task was used to familiarise the children with the actual *Restaurant* problem. One suggestion by the teachers was to group the children according to their ability groups so that the task could be differentiated for the varied groups. For the low-ability group, Mabel suggested using the same menu but reducing the number of items in the group. She also suggested planning a menu for lesser number of people. Ivy suggested reducing the categories of food to the main dish and drink. However, the challenge will be to go through the task in the whole-group discussion. Further thoughts and research are needed in this area.

A Challenge for Teachers to Complete the Task Within Curriculum Time

The second challenge was curriculum time. Such semi-open-ended real-world problems require much time to plan and complete. This is in accordance to Chan's (2005) findings. The team shared the same sentiment that more than an hour is required for rich discussion of the task for this group of children. They recommended that about three periods or 1.5 h would be more ideal for the teachers and children to fully expand and utilise the learning opportunities afforded by the task.

Dilemma as to How Much Computational Skills to Teach Before the Task

All the teachers believed that the basic and core computation skills in the curriculum should be taught first before exposing the children to such tasks. However, there were differing views about whether the "extended" computation skills required of the task should be taught first. Vin observed that the task required the children to go beyond what was normally done during the mathematics lesson, but she was unsure whether the extended skills should be taught to the children before the implementation of the tasks. Vin said:

> Some of the children were unable to add a string of numbers … normally in class, they find a total of two numbers, not a continuous [string of number] … they forget what they have added [halfway through] and they had to start to add all over again, Maybe that is a skill that needs to be taught before the task… teach them how to add 2 numbers and then how to add on and on from there (weekly meeting 6).

Conclusions and Implications

The aim of this study is to maximise the usefulness of a well-designed real-life problem by investigating the affordances of such tasks and the kind of knowledge required to facilitate the implementation the task. The study showed that there are many benefits as well as challenges for the teachers and children using real-world problems. For the teachers, such tasks have the potential to deepen the teachers' mathematical knowledge for teaching. For the children, such task afforded them opportunities to develop their mathematical processes through (1) recognition of the mathematics in real-life context and (2) application of the relevant mathematics in real-world contexts. Computational skills can also be reinforced and extended through such tasks. The real-life problem in this study provides the opportunities to enhance the twenty-first century competencies in our young learners, in particular, critical thinking and inventive thinking. One aspect of the mathematical knowledge for teaching that surfaced in this study was a deeper knowledge of context and children. Having knowledge of the context and children will assist the teachers to design appropriate task and scaffolding questions to unpack the mathematics embedded in the task.

Three main challenges were faced by the teachers in the implementation of the real-world tasks. One noticeably key challenge was to design and implement these tasks such that they are doable within the curriculum time. Real-world context is naturally appealing to many children and an excellent platform to motivate children to solve mathematics problems. However, because of the richness of the context, there is a tendency for the children and teachers to spend more time understanding and expanding the context and problem. This leaves a fraction of the curriculum time to discuss and unpack the mathematics in the problem. This may become one possible "noise" in using real-life problems in our mathematics classroom – unintentionally delaying or deviating away from the intended mathematics to be learnt through the problem. Hence, we need to be very clear when and "how fast" we want the children to get into the mathematics. Another possible "noise" is the thickness of the context. One suggestion is to vary the "thickness" of the context for different purposes of the mathematics lessons and to "dress" the context accordingly to be varied needs and abilities of the children.

The implication of this study should be treated with caution, given the fact that this study was drawn from a single laboratory class cycle. Nevertheless, what we learnt from this study is that a carefully designed real-world problem, aligned with the instructional objectives of the curriculum, provides teachers with the opportunity to learn through planning and implementation of the problem. It also provides young children "gentle small steps" to connect mathematics to their social lives. Real-world problems can be overwhelming to young children when they deal with issues they have not heard of. However, when care and caution are exercised to select context and problem situations appropriate for young children, they can appreciate and apply problem-solving processes to connect mathematics to the world they are beginning to discover. More research is required in the facilitation of such tasks especially in differentiating the tasks to cater to the varied needs and abilities of the children.

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