Chapter 11
Reflections of a Korean Middle School Mathematics Teacher on Improving the Teaching of Mathematics

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Abstract Reflection with appropriate feedback plays an important role in enhancing professional development of teachers. This chapter reports how Teacher-K, an experienced middle school mathematics teacher, used the affordances of grounded images of five videoed lessons and the feedback offered by a mathematics educator to improve her teaching of mathematics that necessitates the incorporation of new curricular initiatives. Initial reflection of Teacher-K was limited in scope, but quantity and quality of reflection improved with each subsequent reflection session. The teacher reconsidered the critical teaching actions of each lesson, reorganises them and sought ways to improve her lesson. She demonstrated aspects of being a reflective practitioner by her willingness to apply these ideas to her teaching. This study showed this experienced teacher was not only concerned with teaching of mathematics in general but also the specific ideas of mathematics ignored by others as trivial. Mathematics educators have much to learn from such experienced teachers.

Introduction

Since the publication of Shulman's (1986) seminal paper, pedagogical content knowledge (PCK) of teachers has generated much interest among researchers, practitioners and policymakers. The reason for this is that content-related category of teacher knowledge symbolises the 'blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction' (Shulman, 1987, p. 8). PCK brought attention to the fact that the knowledge required for teaching is not a simple matter of knowing the subject contents

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and pedagogy. This led to various studies that attempted to identify exactly what characterises the knowledge required for teaching of specific subject matters.

Science, Technology, Engineering, Arts, and Mathematics (STEAM) is Korea's response to US STEM Education. It seeks to apply technological and engineering design-based pedagogical approaches to teach science and mathematics. Teaching of mathematics in tandem with other disciplines such as science and the arts can enhance the learning of mathematics (Sanders, 2009).

The Korea Ministry of Education, Science and Technology (MEST) adopted STEAM as the official government programme to develop Creative and Convergent Talents in primary and secondary schools (MEST, reference). The aim of STEAM education is to help improve Korea's science and technology competitiveness (Ko et al., 2013). Although Korea is a high-performing country in the Programme for International Students Assessment (PISA) for science and mathematics, students' appreciation of science and mathematics is poorly ranked, being 43rd out of 49 countries. To bridge this divide and bolster Korea's push for greater innovation to face the unknown challenges of the twenty-first century, STEAM education is perceived as an important process to facilitate students' interest for and understanding of science and technology and develop their integrated thinking and problem-solving ability (Maes, 2010).

To promote STEAM education policy of MEST, the Korea Foundation for the Advancement of Science and Creativity (KOFAC) is spearheading research to prepare teachers at all levels to use STEAM in their teaching of mathematics and science. Teachers at all levels need well-structured professional development courses to prepare them to adopt STEAM policies (Sanders et al., 2011). Teachers' teaching performance is closely related to their professional development. 'Professional development' here includes not only professional knowledge and domain skills but also high-level cognitive skills that can help teachers solve problems and deal effectively with new situations (Park et al., 2008).

Various studies evaluate the effectiveness of teaching through the ability to engage teachers in reflective thinking and deepen their reflective activity (Choi, 2003; Dinkelman, 1997; Hamrick, 1995). Most of the existing studies on teachers' reflective thinking are based on their reflective writings (Jo, 2011; Lee & Kim, 2013). Self-reflection through writing allows teachers to ponder deeply over their practices; it is, however, limited as teachers still have to rely on their own memory and awareness of classroom events and developments. On the other hand, when teachers' self-reflections are supported by video technology, they are more likely to assess their teaching, the meaning of events or situations more accurately. There is therefore a pressing need to conduct an empirical study on the nature of teachers' reflections on teaching and identify the ways in which to improve this reflective thinking using grounded images collected using video technology.

This chapter reports on the nature of a middle school mathematics teacher's reflections on her teaching of mathematics. Given this opportunity to review a number of lessons, what aspects of her teaching would she select to improve? What reasons did she provide for wanting to improve these areas of her teaching?

Theoretical Framework: Teacher's Knowledge

The knowledge base of teachers should, as a minimum, incorporate the following seven categories: (1) content knowledge, (2) general pedagogical knowledge, (3) pedagogical content knowledge, (4) curriculum knowledge, (5) knowledge of learners and their characteristics, (6) knowledge of educational contexts and (7) knowledge of educational ends, purposes and values and their philosophical and historical grounds (Shulman, 1987)

Content knowledge comprises, inter alia, knowledge about the facts, concepts, principles, definitions, theorems, rules and structures, which are crucial elements for teacher effectiveness. It concerns the amount of knowledge and how that knowledge is organised, which may determine the effective development of mathematical concepts. It also incorporates disciplinary knowledge, such as understanding the rules of evidence and proof, the methods of enquiry into the domain, what warrants a particular proposition, the importance of knowing the proposition and its associations with others. In essence, the 'teacher need not only understand that something is so; the teacher must further understand why it is so' (Shulman, 1986, p. 9). Ball further delineated Shulman's characterisations according to three criteria: knowledge of concepts and procedures, understanding the underlying principles and meanings and an appreciation and understanding of the connections between mathematical ideas. In addition to content knowledge, there is the knowledge about mathematics, which includes:

> understandings about the nature of mathematical knowledge and of mathematics as a field. What counts as an "answer" in mathematics? What establishes the validity of an answer? What is involved in doing mathematics? In other words, what do mathematicians do? Mathematical knowledge is based on both convention and logic. Which ideas are arbitrary or conventional and which are logical? What is the origin of some of the mathematics we use today and how does mathematics change? (Ball, 1990, p. 458).

General pedagogical knowledge transcends subject matter, referring to the broad principles and strategies of classroom management and organisation that appear to transcend subject matter. Effective teachers of most disciplines would recognise the applications of these principles and strategies.

PCK differs from general pedagogical knowledge as it is discipline specific. This knowledge differentiates a content specialist from a pedagogue (Shulman, 1987, p 8). It enables teachers to transform content knowledge into forms that are comprehensible and accessible to a specific audience. This includes knowing the most appropriate examples to introduce a concept, using and connecting appropriate representations for the particular concept and using appropriate analogies to deepen understanding of it. PCK is the 'special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding' (Shulman, p. 8).

Within PCK, we find the knowledge of content and students (Ball, Thames, & Phelps, 2008). The effective delivery of a lesson is contingent upon teachers' knowledge of their students. What preconceptions do students have towards mathematics? What misconceptions or difficulties do they have towards a specific mathematical concept?

Curriculum knowledge informs teachers how a mathematical programme is structured and what and when to deliver specific mathematical topics. Teachers need to know the variety of available instructional materials that will support their delivery of a particular mathematical programme. Such knowledge 'serve[s] as both the indications and contraindications for the use of particular curriculum or programme materials in particular circumstances' (Shulman, 1987, p. 10). This knowledge is crucial, particularly with the introduction of new initiatives. Teachers need examples of how to teach mathematics when the focus of the curriculum changes. For example, with the incorporation of information and communication technology (ICT) into the teaching of mathematics, teachers need examples of how to conduct specific mathematical topics. What mathematical concepts are best developed using ICT and how may its use deepen students' understanding of specific concept?

Knowledge of learners and their characteristics combines knowing about both students and mathematics. Examples of this useful knowledge for teachers are the knowledge of what motivates students, the ways in which they conceive mathematics, what aspects they find interesting or confusing and how they choose to solve mathematical problems. Armed with such knowledge, teachers are better prepared in terms of the mathematical tasks that they choose to engage students in and the solutions that they anticipate from different students. Teachers who know their students well may be able to engage with them using language appropriate to their level and extend on these students' responses to particular mathematics tasks (Ball, Thames, & Phelps, 2008).

Knowledge of educational contexts includes understanding the dynamics of the group or classroom with regard to 'the governance and financing of school districts, to the character of communities and cultures' (Shulman, 1987, p. 8)

Knowledge of educational ends, purposes and values and their philosophical and historical grounds is rarely discussed in the literature. A plausible reason for its omission could be that it would appear more suitable for policymakers and curriculum designers.

The above discussion shows that the knowledge needed by mathematics teachers for good teaching has many dimensions. Knowledge of mathematics content is clearly very important, but it is not sufficient to ensure quality teaching (Hill, Schilling, & Ball, 2004). Knowledge of general pedagogical techniques is also a reasonable prerequisite to good teaching. However, teaching proves to be more complex than a simple application of pedagogical techniques to convey mathematical content (Ball, Thames, & Phelps, 2008).

Reflection and Its Relationship to Professional Development

Reflection is a critical and creative activity that describes, analyses, interprets and evaluates a class by considering the phenomenon of the class comprised of teachers and students. Dewey's work on reflection undergirds studies on teachers' reflective thinking. Reflective thinking results from personal mental activity, such as detailed analysis based on observation, data collection and evidence (Dewey, 1910). Dewey

(1944) argued that the purpose of reflection was to identify the relation between actions and their consequence. Reflection is the process in which teachers make continuous decisions to improve their teaching (Van Manen, 1977), apply self-criticism when looking back on their teaching and considering multiple viewpoints and decide how to change their practices to address specific issues (Hatton & Smith, 1995). Park et al. (2008) defined reflection on teaching to be a process of self-reflection to achieve better decision-making by rethinking and evaluating teaching behaviour to gain a sense of the logical rationale behind it. When teachers become more reflective, they develop valuable theories about their own practice in their own class settings (Dewey, 1904).

When a person acts and reflects, they become a researcher in a practical setting (Schön, 1983). As reflective practitioners, teachers do not only rely on previously established theories and technology but also on their own theories based on their own unique experiences. For this to happen, classes should be viewed as a space where teachers' reflections affect their solutions of authentic classroom problems.

Using Video Technology to Increase Teacher's Reflective Thinking

Teachers' reflection of teaching needs to be based on grounded images and not only on their subjective thinking and memories. Video technology can provide graphic and grounded images about the language and behaviour of teachers and students in the classroom. Teachers see, hear and reflect based on clear evidence of what happened during the class, which allows a much more accurate reflection. Moreover, data using video technology allows teachers to observe the atmosphere in the classroom in a more comprehensive manner. It helps them to obtain various opinions and experiences by looking for the meaning of experience inside the context of what is occurring in the classroom.

The goal of the reflection using videos is not to evaluate, but rather to understand the proceedings of the class. Further, through reflection on teaching, teachers are not just given more knowledge, skills or abilities relevant to teaching, but rather they come to know about self-teaching through a psychological equilibrium: acquiring the necessary practical knowledge by discovering 'how' and 'why' and not just on the superficial level of their teaching subject. Through video observation, teachers see their class from a different perspective, and through the process of interpreting and reinterpreting the meaning of what they observe, they are able to experience a growth in their own awareness. In this manner, they define their own awareness about educational circumstances, obtain alternative points of view and construct and reconstruct knowledge about what constitutes excellent teaching. Accordingly, the goal of such reflection needs to promote reflective thinking as an important approach in the professional development of teachers. The ultimate goal of reflection is to enable teachers to become independent evaluators and critics of their own practice, thus enabling them to use and apply their knowledge rationally, and so bring about a qualitative improvement in their classrooms (Dewey, 1904).

The Study

Teacher-K had videoed herself teaching mathematics to a class of middle school students. She watched the lesson to analyse the effectiveness of her teaching. However, she realised that without the feedback of an expert, the process of self-reflection and hence her growth as a teacher were limited. Her growth would improve if she were able to receive critical inputs from a mathematics educator. Teacher-K approached me for support.

I agree to be part of Teacher-K's learning experience. To be of value to her, it was necessary to determine what she had been unable to perceive from watching herself at work and then work with her over a series of videoed mathematics lessons in order to examine how her subjective evaluation evolved over the course of a number of lessons. Hence, the objectives of this study were twofold. Firstly, it examined how the qualities of reflection changed as Teacher-K observed herself at work over a series of five mathematics lessons. Secondly, it investigated the aspects of professional knowledge that were the focus for Teacher-K.

Method

Among the five videoed lessons, the first and second lessons were on the teaching of graphs of quadratic functions, the third and fourth on the application of quadratic functions and the fifth on a STEAM-based material entitled 'Star in the Night Sky'. Each lesson lasted for approximately one hour. A single camera was placed centrally at the back of the classroom, irrespective of whether it was an activity-based lesson or a lecture.

Teacher-K reviewed the recorded lesson with the researcher during the week. Firstly, she reflected on her own teaching methods using the 'think-aloud approach' (Brown & Rogers, 2002) before being asked to expand on her self-reflections. So as not to disturb her self-reflection, the researcher used unstructured interviews to gain greater insights into Teacher-K's values and beliefs on teaching, choices of teaching methods and actions. She gave me permission to record her self-evaluation of the taught lessons and interviews for further analysis.

Participants

Teacher-K was a middle school mathematics teacher with 6 years of teaching experience. She is also the author of mathematics textbooks and teacher guidebooks. Furthermore, she was interested in conducting research in content mathematics and mathematical activities, which were of interest to middle school students, and pedagogy to address their interests and aptitudes.

Nevertheless, Teacher-K felt she had limited knowledge of good teaching practices. Other than the theories of mathematics education acquired as a pre-service teacher, she had not received other forms of mentoring on how to improve her practices. She wanted to change her current teaching style, which she described as expository in nature. She welcomed the opportunity to participate in this research as this gave her a chance to learn about alternative ways of teaching mathematics. Furthermore, Teacher-K was aware that she had only a superficial understanding of the STEAM education system that was introduced in 2013. She did not know how to conduct mathematics lessons based on STEAM; hence, she wanted to evaluate this new STEAM initiative. By articulating her thoughts on STEAM, she would be encouraged to reflect on how to improve the teaching of STEAM-based mathematics. She explained:

> I only know that STEAM education is the fusion of disciplines. I can give a lesson, but I don't know how to apply it to mathematics. I do it as I think it is only centred on activity… It will be important to let students have ideas or design it. Thus, I need to organise my next class on 'creative design' (5th reflection session).

Data Analysis

The primary data were re-examined by Teacher-K and the researcher. The researcher used 'stimulated recall' (Nunan, 1989) with Teacher-K, which enabled both parties to clarify any unclear portions of the transcriptions.

Teacher-K's reflections were analysed using a constant comparative analysis of open coding, categorisation and category verification (Ezzy, 2002) using the following three steps. Firstly, three mathematics educators, including the researcher, labelled and categorised each piece of data using the open-coding process. Secondly, the three educators double-checked the classified and coded data, classified them into similar subjects and labelled the coding names. Thirdly, all data were compared to similar data in a constant and repetitive manner. The three educators referred to Shulman (1987) to create categories that best expressed the characteristics of the data set.

Analysis and Findings

The analyses of Teacher-K's reflections were grouped into two sections. Section one discussed how the video helped Teacher-K to evaluate her teaching. Section two analysed the aspects of her teaching that Teacher-K considered important, which were divided into six categories: (1) pedagogical knowledge, (2) teaching strategy, (3) teaching skills, (4) understanding students, (5) teaching environment and (6) overall class.

Usefulness of the Video in Helping Teacher-K Evaluate Her Teaching

Teacher-K found the grounded images captured by the videos extremely helpful because she used these concrete examples to anchor her discussion with respect to the way she responded to students' questions and responses. Figure 11.1 shows that without the aid of the grounded images, what she remembered was based on vague and subjective images, and these seemed more positive than what happened in the class. Thus, the grounded images provided evidence-based data that challenged what she remembered from her lessons. The use of grounded images thus helped Teacher-K construct clear and objective recollections of her lessons.

Although Teacher-K thought she had communicated clearly and that the students were cognitively engaged in her mathematics class, the grounded images showed otherwise. She did not ask suitable questions to help the students think about the mathematics. Furthermore, she did not address the misconceptions found in the students' answers to solutions:

> I usually thought that I communicated well with students and helped them understand the contents taught in my class. I thought the knowledge that students should know was constructed in my class through a shared understanding with students. However, through the video observation, I realised that I didn't consider posing appropriate questions to address students' misconceptions.

Contrary to her perception that students had understood the lesson very well, the grounded images proved her wrong as the students' understanding was questionable. This misplaced perception arose because she had based her perceptions on interactions with only a small group of students:

> I thought that the students' understanding was pretty high in my class. However, when I looked at my class through the video observation, I realised that students' responses and understanding as I had imagined them were quite different from the reality. I found that the reason was that I tended to focus on only a few students who pay attention in my class.

Fig. 11.1 Role of video technology

Teacher-K was rewarded for her efforts when she made concerted efforts to improve her explanations. She could pinpoint exactly the portion of the lesson on which the improvement was focused:

> When watching the video last time, I think there were some problems… I contemplated what to do better in this class. That is why I did this or that this time… I think the process of explanation in the middle was better than in the previous class.

Teacher-K also became more sensitive about time management. Because of poor time management, she was always in a hurry to complete the lesson:

> I did not know it, but there seemed to be a problem with time management in my class. I noticed that I would start the class slowly, but be in a hurry by the end in order to finish the class… I realised through video observation that I talked faster at the end of the class. Due to this, I realised that the contents at the end of class were not being properly delivered. I realised that this was serious when I watched the video.

Furthermore, with the aid of the grounded images, Teacher-K realised that she should put more thought into designing better lessons so that she would not end up giving a lecture when she intended to give a more interactive lesson:

> First of all, I thought about the contents to be taught, but I gave a lecture without an overall design in my class. When I looked at my class through the video observation, I criticised and analysed about whether my class was generally well designed or not.

After watching the videos, Teacher-K became concerned with issues of equity. She realised that not all students in her class received her attention:

> I think a teacher should show an interest equally by looking at all students in the classroom. So far, I thought I looked at all students equally when I gave and received feedback from many students in my class. However, I realised that my expected behaviour was different from what I saw through the video observation. Through this, I saw where I didn't look and whom I was more concerned about.

Teacher-K was so focused on delivering the planned lesson that she ignored other factors that could also affect her teaching and students' learning:

> In addition, I did not pay attention to other internal factors, aside from the contents to be taught, and I did not realise the need for various external factors, such as my expressions, gestures, and writing on the board.

Teacher-K explained how the videoed lessons helped to improve her delivery of future lessons. For example, having reviewed a particular lesson, she realised that she had not addressed particular aspects of a concept. The evidence reminded her of these omissions. In the follow-up lesson, not only did she improve her time management, but she also addressed these specific omissions from the previous lesson. The videoed lessons meant that the data were available on demand. More importantly, she was able to analyse her actions in her own time and without having to ask others for help. This meant that she was saved the embarrassment of others critiquing her teaching, something that is not easy to accept:

> I easily forgot ideas that I felt were lacking, things I shouldn't have done or should have done better. However, while I observed my behaviour through a video, I was able to concentrate on

> my behaviour and it helped remind me of the aspects that I wanted to work on. For example, when I had a class on the same content after watching the recorded video, not only was I was able to manage my time more efficiently, but I was able to work on parts that were lacking. Likewise, I was not only aware of the problems that were previously unknown, but I was also highly motivated to improve my behaviour. The biggest advantage was to have time to look back on my classes by examining at my behaviour in the class without help from others.

In summary, the grounded images of the videos challenged a very important assumption held by Teacher-K, notably that a teacher cannot assume that the students shared the teacher's understanding of the taught concepts. To assess students' understanding of a taught lesson, she needed to give greater consideration to students' responses to questions posed. It was important for her to identify why students lost interest and hence why they were not focused in the lesson. She made it her responsibility to improve her teaching by addressing factors, such as the mathematics to be taught, the nature of the activity used to deliver the content and the choice of teaching methods:

> Since I realised that students' response and understanding were different from what I considered in the class, I observed students' responses to the overall class contents and teaching method. After observing the video, I was motivated to make attempts to change my class in various ways, such as class design, teaching methods, and teaching contents. After broadly analysing the causes of and responses to students who did not concentrate in class, I was able to put greater effort into general fields, such as the understanding and concentration of students in the class.

Changes of K-teacher's Reflection on Teaching Practice

In the previous section, Teacher-K provided an overview of how the videos helped her to reconsider her teaching of mathematics. This section, however, shows how her reflections evolved over the five reflection sessions. Although she was less forthcoming in the first session, she became more articulate about her observations with each subsequent session. Her reflections showed that she gave greater considerations to the needs of the students, particularly those who failed to follow or lost interest in the lesson. Teacher-K's reflections on her teaching were evaluated based on six categories that she identified to be important. Table 11.1 provides a summary of these six categories and compares how the different components changed between the first and fifth reflection sessions.

Curriculum Knowledge Two constructs, awareness of mathematical content and reorganisation of teaching instruction, are subsumed under this category. Awareness of mathematical content refers to a teacher's appreciation of the academic and practical value of mathematics as a discipline. The introduction of STEAM into the Korean mathematics curriculum renders this knowledge particularly important. Although Korean students perform very well in international comparative studies such as Trends in International Mathematics and Science Study (TIMSS) and PISA, they do not have a high appreciation of mathematics. This may inadvertently affect the innovativeness of Korean students in the twenty-first century.

Table 11.1 Evolution of Teacher-K's reflections from the first to fifth session across the six categories

Category	First reflection session	Fifth reflection session
Curriculum knowledge	Reorganisation of teaching practice (2[a])	Reorganisation of teaching practice (7)
		Awareness of content (2)
Pedagogical content knowledge (PCK)	Strategy (1)	Strategy (3)
	Organisation (4)	Organisation (1)
	Form (4)	Form (5)
		Utilisation (7)
		Planning (8)
		Evaluation (1)
Knowledge of learners and their characteristics	Interaction (1)	Interaction (7)
	Level of learners (3)	Level of learners (5)
		Feedback (4)
General pedagogical practice	Skill of teaching practice (6)	Skill of teaching practice (3)
Knowledge of educational contexts		Teaching environment (1)
		Teaching atmosphere (9)
Class overall	Holistic approach (1)	Holistic approach (8)

[a]Frequencies of occurrences

In the first session, Teacher-K's reflections did not include both categories. She focused specifically on one component, namely, how to reorganise her teaching, and she was limited in the number of reflections. However, by the fifth reflection, she was more articulate, and the number of reflections on how best to reorganise her teaching had increased from two to seven (Fig. 11.2). By the fifth session, Teacher-K reflected on how she chose to integrate mathematics into other unrelated areas of learning:

> I was contemplating what could be associated with the topic or what mathematical element was related to the beautiful night sky. I was thinking about what should be drawn and be the outcome. Fortunately, I could create polygons that perfectly matched to the topic. (5th reflection session)

A reorganisation of the teaching instruction emphasised the importance of a sound knowledge of the mathematics curriculum as well as how the contents were organised in mathematics textbooks. Mathematics teachers should be cognisant of the changes in the curriculum, the mathematics to be taught in a specific year and the best way to teach this content. Without such knowledge, teachers may not be able to prepare effective lessons to cultivate the interest of Korean students. For example, Teacher-K knew that she lacked an understanding of the concept of the STEAM education system, which was a new curricular initiative. She was concerned about how to introduce this new initiative into her teaching. Her knowledge of STEAM was limited to what was discussed among her colleagues, but she wanted to learn more about it, either by discussing the new initiative with her colleagues or

Fig. 11.2 The theme of the class for STEAM is 'Beautiful Night Sky'. The screen at *left* shows a night sky. The figure at *right* shows the students constructing polygons

by learning about the alternative teaching strategies advocated in STEAM by observing the practice of her colleagues. As she explained:

> I only know that STEAM education is the fusion of discipline contents. I can give a lesson, but I don't know how I to apply it to the mathematics. I do it as I think it is only centred on activity… It will be important to let students have ideas or design it. Thus, I need to organise my next class focused on 'creative design' (5th reflection session).

Teacher-K thus recognised the need to consider the individual differences in students and provide them with the meaningful learning opportunities. For example, in implementing the STEAM-based curricular materials, she was able to summarise why the teaching outcomes may vary despite teachers sharing the same instructional materials. She may deliver the same content differently to her colleagues, while her students may have a different set of prior knowledge and experiences, thus interacting differently to the same curricular materials:

> Of course, [the material] should be discussed in detail with other colleagues. Even if the same materials are shared, students may learn about them differently. My direction may be different from that of other teachers. The most important thing in STEAM education is to configure the situation and have an idea about why this situation comes about based on the problems that students may have (5th reflection session).

Pedagogical Content Knowledge This construct examines the planning and organisation of instruction to achieve the goals of teaching and learning mathematics. Teacher-K focused a large part of her reflection on this construct. An analysis of her reflections identified the following six components in this construct:

1. Organisation
2. Form of instruction (macroscopic strategy with regard to planning teaching in general)

3. Type of teaching practice (group activities, individual teaching, descriptive teaching, etc.)
4. Planning class instruction (planning with regard to procedures and progress of teaching practice, such as introduction-development-summary-evaluation)
5. Evaluation (preparation of evaluation methods and procedures, development of evaluation tools)
6. Utilisation of tools and materials (various teaching-learning tools or materials that a teacher creates, prepares and uses for practice)

Although PCK comprised these six components, Teacher-K focused on only two in the first session, organisation and form of instruction. However, by the fifth session, Teacher-K had included the remaining four components.

For example, in the first reflection session, Teacher-K was aware of students' difficulties in understanding the concept of functions and wanted to invest more class time to help students identify the characteristics of the graph of a given function, but she did not have alternative methods to improve her teaching of this concept:

> Since there are many students who do not know the section of a function or how to draw a graph, I would like to invest more time in drawing a function graph. However, the conditions do not allow this. (1st reflection session).

In one lesson on function graphs, Teacher-K used a computer to show various graphs to the class. In this case, the instruction was teacher oriented, but she wanted the teaching and learning process to be learner oriented. One alternative strategy was to ensure that as many students as possible were engaged in constructing their own graphs (Fig. 11.3). She was thus aware of the need for the effective utilisation of computers:

> If students go to the computer room and do it by themselves in the next class, they would explore and understand them while saying, 'Ah, it moves up' or 'The formula changed like

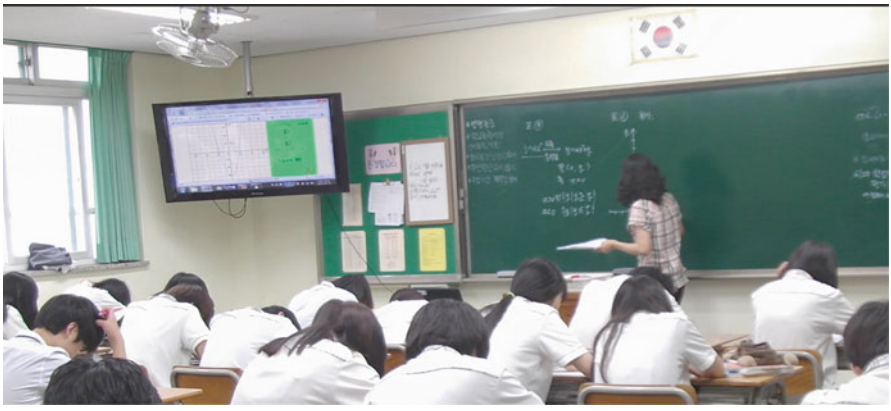

Fig. 11.3 Constructing graphs on the computers provided students the opportunity to manipulate and explore the structure of graphs as the 'formula' changed

> this'. It would be good for students. Because we cannot draw it every time [on paper], it is nice to find more things in common (4th reflection session).

Such a reflection is partially related to the notion 'how can I handle it in a different way' and the movement of 'reorganisation' (Hole & McEntee, 1999; Smyth, 1989). It also seems to show the aspect of reflection at a 'comparative level' in terms of finding other explanations or alternative points of view.

In addition, Teacher-K knew that she lacked an understanding of the STEAM education system, and she tried to look for other types of teaching practice with her colleagues and talked about her thoughts and judgements with regard to STEAM education:

> I only know that STEAM education is the fusion of contents. I give a lesson, but I don't know how to apply it to mathematics. I do it as I think it is only centred on activity… It will be important to let students have ideas or to design it. Thus, I need to organise my next class with a focus on 'creative design' (5th reflection session).

Teacher-K's reflection showed that she was aware that good instruction should integrate various modes of teaching strategies, such as student-oriented group work, teacher-oriented teaching or descriptive teaching. Furthermore, good instruction should incorporate an evaluation of students learning. Teacher-K critically reflected on the aspects of 'evaluation'; in other words, she clearly recognised that teaching practice should be organised as a complete structure to achieve the teaching goal, and she thus displayed the aspect of critical reflection on teaching practice with regard to the organisation of teaching at a later stage. She reflected on these components only in the fifth reflection section.

Knowledge of Students Teacher-K paid attention to students' participation in the mathematics lessons. An analysis of her observations showed that she paid attention to the following three components of learning:

1. Level of learners (considering the prior knowledge, interests and emotions of learners)
2. Interaction (facilitation of students' responses, facilitation and induction of interaction)
3. Feedback based on students' response (identification and treatment of misconceptions)

In the first reflection session, her reflections were limited to the level of learners and the interactions between them. However, the quality and quantity of her reflections increased in each subsequent session. In the fifth session, Teacher-K's reflection focused on her communication with students and their cognitive and affective behaviour.

Teacher-K was concerned for students who did not participate in the mathematics lessons, even when she used group work to encourage their participation. She realised that the weak students were not able to participate in the current lesson because they did not have the prerequisite knowledge to engage with the concept discussed that very day. She saw how unproductive this lesson was for such stu-

dents. She noticed that grouping weak students together further compounded their lack of participation. Reteaching the lesson would not benefit such students. Teacher-K felt responsible for the progress of this group of students who possessed other good attributes, such as good leadership qualities and kindness. She wondered how best to help such middle grade students, but she did not even have the knowledge about the elementary school curriculum. Teacher-K wondered what changes could be made to the middle school curriculum to address the needs of such students:

> There is already a student sleeping in class. He is almost no good. He is totally no good. He is lacking, even as an elementary school student. Even if I repeatedly explain it to him, he does not understand. Those who are not good in a group [small group configuration] spend an hour doing nothing. I think that we need activities for students who are not good at studying (4th reflection session).
>
> He is a class president. He is not good at mathematics, but he is kind. Thus, he does not ask [for scissors to create a polygon]. I think I may have to assign roles in a class with small groups next time. I also may need to distribute materials (5th reflection session).

Nevertheless, Teacher-K saw that group work benefitted others who were not usually receptive to the mathematics presented in the conventional transmission method, that is, when they had to listen to the teacher teaching:

> I believe that these two have become a lot better. He used to be a student that would always sleep in class. Even though he is not concentrating, he looks happy. They look happy when they do group activities together (5th reflection session).

General pedagogical knowledge: In the first reflection session, Teacher-K placed greater emphasis on pronunciation, intonation, facial expression, movement, writing on the board and time allocation:

> I was generally satisfied. I wrote the goal of the class (on the board). I also gave students some time for activities and I initiated cooperative learning. Since it was my first attempt, I didn't know what to do. So I felt a bit awkward. (1st reflection session (1st reflection session).

Here Teacher-K's reflection showed she was satisfied with her teaching because she had introduced cooperative learning, a mode of teaching that she had never previously used in her mathematics class (Fig. 11.4). She had written the objective of the lesson on the board and given time to students to participate in cooperative learning. However, her reflection showed that she was more concerned with her performance than the students' learning from the activity. Nevertheless, in her second reflection, she questioned the effectiveness of such collaborative teaching, as once students completed the activity, they lost interest in the task. Teacher-K thus saw the need to bring the students together through whole class teaching in which the teacher conducted a class discussion. However, she did not express the effectiveness of the class discussion after the process of cooperative learning activity:

> Yes, moreover… students' concentration and cooperation between them decreased without group teaching. Once they found the answers to the questions, they lost concentration (2nd reflection session).

Fig. 11.4 The class that she was satisfied with

Knowledge of Educational Contexts Teaching environment and teaching atmosphere come under this category. The former includes the various materials and engineering tools available in the classroom for teaching and learning mathematics. The latter focused on the number of students, their concentration and the permissive or repressive atmosphere of a mathematics classroom. Initially, Teacher-K did not reflect on the importance of the teaching environment and atmosphere. However, in the later reflection sessions she became more aware of the environment and teaching atmosphere and how these could influence students' learning.

Professional Development of the Mathematics Educator

Teacher-K provided me with greater insights into how teachers grow over the course of their careers and the nature of the support appreciated by them. The quality of a teacher's reflection is not determined by their length of service (Ko, Maeng, & Nam, 2013). The act of reflecting is not a natural process when there is no necessity to justify one's choice of pedagogical actions. Thus, the experience of watching a video of one's own teaching may not be meaningful when the objective of the experience is unclear. Unless learners consciously search for a deeper meaning, not all experiences are educational (Dewey, 1938). In this section, I discuss two important concerns that teachers have with regard to their work. First, established teachers need opportunities to talk about their work with a critical friend; in this study, the mathematics educator was the critical friend. Second, established teachers need to talk seriously about the mathematics content that they teach.

Engaging in Their Work with a Critical Friend Although her peers considered her an effective mathematics teacher, Teacher-K nevertheless wished to improve her practice. Her time spent on watching the videos of her mathematics teaching would not have had a significant impact if she had not worked with the mathematics educa-

tor who chose not to remain as a neutral observer. The mathematics educator was not only willing to listen to her talk about her lessons, her justifications about her choice of pedagogical actions and her concerns about the students in her class but also prepared to question her about these decisions. Cognitively demanding questions were asked, such as the following: 'Why do you feel that way?' 'How did you determine that?' 'What did your students think about it?' The following dialogue shows how Teacher-K, when provided with the opportunities to share her teaching concerns, openly evaluated what was effective in her practice and what needed improving. Furthermore, she considered alternatives to improve the learning of her students. Such deep reflections would most probably not have occurred if there were no opportunities to share her thoughts with someone who was willing to listen.

Educator: You barely said a word an hour ago, but now you are telling many different stories

Teacher-K: I guess you're right… Speaking [with you] seems to have made me more talkative. I've realised many things [about my class] that I hadn't noticed before. Now I say to myself, 'This is me' and think, 'Oh, I need to change this [thing]'or 'That [idea] was pretty good!' (3rd reflection session)

Educator: These students are responding to the solving process now, aren't they?

Teacher-K: Yes

Educator: Last time, were they asked to give the answer only?

Teacher-K: Yes, at the time, I gave students more time to set up an equation

Educator: Why do you ask them to do so?

Teacher-K: I was concerned about the students who didn't solve the problems last time. I wanted to examine whether they understood them. However, I didn't. So, I wanted to see whether they could set up an equation, which was the intermediate process (reflection session)

In-Depth Conversations About Mathematics Content Among Teachers The mathematics that could be problematic to teachers need not be very challenging. In fact they could be rather trivial. Although such matters were trivial, they were a cause of frustration for Teacher-K because she was a careful teacher and she was concerned for the success of her students in the written examinations. Teacher-K was not confident whether a comma would suffice in presenting the solution of a quadratic equation or whether the conjunction 'or' could be used instead. Although such matter seemed trivial, it merited serious discussion at a level that reflected a deeper understanding of the mathematics by Teacher-K. Such a matter would not perturb teachers with less mathematical knowledge:

> In quadratic equations, I teach that $x = 3$ or $x = 5$ after doing factorisation. Many students asked whether 'or' must be written or whether writing the comma is wrong. I'm also very confused. Thus, I can't give a clear answer and I'm frustrated.
>
> In the test, even if one student was not wrong mathematically, it was graded as a wrong answer, because it was not shown in the textbook and it was beyond the scope of middle school. However, I didn't know what to do. Since I didn't teach it, it was graded as a wrong.

Working with Teacher-K was an enriching experience. The grounded images of the videos provided us with powerful tools to deepen the learning of both parties and identify the needs of established teachers.

Conclusions

This study showed that the process of reflection improved when teachers and mathematical educators work with grounded images taken from videos. In addition, challenging questions encouraged the teacher and mathematics educator to think more deeply about the teaching and learning of mathematics. The teacher actively and continually identified the problems faced by students and considered alternatives and detailed plans to solve problems that arose during the class. Over the process of five reflections, Teacher-K became more concerned with those who failed to engage with the mathematics, and she showed a genuine desire to help those challenged by mathematics. She was particularly interested in finding alternative methods to treat the issues. A reflective teacher should therefore analyse and criticise the process of solving educational problems, which results in gaining feedback and improving problem-solving skills (Zeichner, 1983). If this viewpoint is accepted, Teacher-K exhibited a qualitative growth in terms of her reflections on teaching practice.

Mathematics educator's deep interest in a teacher's narratives and the related classroom experiences could prompt even greater self-reflection on the part of the teacher. When mathematics educator asks good questions, the teacher is encouraged to reflect, restructure and reorganise her practical knowledge based on her shared ideas and stories of her personal experiences. It is important to provide feedback that valued the teacher's varied cognitive, psychological and emotional needs. For this process of learning to take hold, the teacher and educator should use grounded images objectively as a practical case material to learn from each other.

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