Mathematics Teacher Education 10

Swee Fong Ng *Editor*

Cases of Mathematics Professional Development in East Asian Countries

Using Video to Support Grounded Analysis

Springer

Mathematics Teacher Education

Volume 10

Scope

The Mathematics Teacher Education book series presents relevant research and innovative international developments with respect to the preparation and professional development of mathematics teachers. A better understanding of teachers' cognitions as well as knowledge about effective models for preservice and inservice teacher education is fundamental for mathematics education at the primary, secondary and tertiary level in the various contexts and cultures across the world. Therefore, considerable research is needed to understand what facilitates and impedes mathematics teachers' professional learning. The series aims to provide a significant resource for teachers, teacher educators and graduate students by introducing and critically reflecting new ideas, concepts and findings of research in teacher education.

More information about this series at http://www.springer.com/series/6327

Swee Fong Ng
Editor

Cases of Mathematics Professional Development in East Asian Countries

Using Video to Support Grounded Analysis

Springer

Editor
Swee Fong Ng
National Institute of Education
Singapore

Mathematics Teacher Education
ISBN 978-981-287-404-7 ISBN 978-981-287-405-4 (eBook)
DOI 10.1007/978-981-287-405-4

Library of Congress Control Number: 2015938104

Springer Singapore Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer Science+Business Media Singapore Pte Ltd. is part of Springer Science+Business Media (www.springer.com)

Contents

Chapter 1
How Researchers and Teachers Could Use Videos: The Role of Grounded Images

Swee Fong Ng

Abstract Before the availability of video technology, the teachers formed images of classroom scenes based on the narratives provided by researchers. The availability of video technology has changed the ways researchers engage teachers in professional development courses. Because both parties are able to observe the lesson from the same perspective, they form grounded images of the classroom scene in question. This chapter discusses the theory of grounded images and provides the reader a summary of the seven chapters in Section I.

Brophy's (2004) work *Using Video in Teacher Education* provided various examples how videos could be used to enhance teacher education in different subject domains. Mathematics has always been a challenging subject to learn and to teach. Mathematics educators and teachers are continually searching for ways to improve the teaching and learning of mathematics. Furthermore, findings from international comparative studies such as TIMSS and PISA have put the spotlight on mathematics education. Policymakers, researchers, mathematics educators and teachers are curious why some countries are performing better than others. Although teaching is culture specific (Stigler & Hiebert, 2009), nevertheless, researchers, mathematics educators and teachers wish to learn from their peers. The affordances of video have meant that it is possible to learn by observing others at their practice. Videoed materials can be stored, retrieved, transmitted with a minimal of fuss and used repeatedly without any damage to the original content. The use of videos in the professional development of teachers is now an established methodology. However, researchers, mathematics educators and teachers continue to find ways to use videos to improve their practice. This book introduces the theory of grounded image to discuss how mathematics educators from Hong Kong, Indonesia, Korea, Malaysia and Singapore capitalise on the affordances of videos to design professional development courses for teachers.

S.F. Ng (✉)
National Institute of Education, Singapore
e-mail: sweefong.ng@nie.edu.sg

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_1

Although videos have their drawbacks, nevertheless, their affordances make video a useful tool to support teacher education. Sherin (2004) identified three affordances. (1) Once captured the video images are lasting records of the events and these images can be accessed on demand. Digitised materials have resulted in even greater and speedier access to specific episodes. (2) Although the original video is a record of the chronological progression of a lesson, the original contents can be edited, grouped and reorganised into a format that is very different from its original. In fact different episodes from different lessons can be grouped together as a specific example of a particular construct. (3) Videos provide researchers, mathematics educators and teachers the opportunity to engage with a new set of pedagogical practices “that are very different from typical pedagogical practices” (Sherin, p. 14).

Although videos can be a useful tool to enhance teacher education, it does not mean that teachers would naturally learn from watching videos of classroom practices. There must be a clear objective why videos are used and great care must be exercised to select the appropriate content to be viewed (Brophy, 2004). It is not enough to present the content to the practitioners. Individuals learn best with proper feedback (Klein, Nir-Gal, & Darom, 2000). In their work, Klein and Darom found that integrating adult feedback in pre-school computer learning environments facilitated informed use of computer technologies and has positive effects on children’s performance. Extrapolating from this study, it is reasonable to hypothesise that with proper feedback provided by facilitators of professional development courses, teachers participating in professional development courses are likely to benefit from watching the videos of specific lessons.

The content could consist of a complete or selected episodes of a lesson conducted by a teacher other than the participants involved in the professional development or by a teacher involved in the professional development course itself. The sociocultural context of the lesson could be one which the participants are familiar with or one with a foreign context. In this case the language of the content may be foreign to the participants and dubbing or translation of the contents may be required to help participants engage with the proceedings of the lesson presented.

The participants could be an individual who has identified a specific pedagogical practice which he or she wishes to improve upon. Participants could also be groups of individuals coming together to improve on a specific curricular area. This could be teachers forming video clubs to address specific curricular needs such as how to improve the teaching of problem-solving in their classrooms. Participants could also be a group of teachers who wishes to learn how teachers in other countries conduct their mathematics lessons with the specific aim of learning how to develop repertoires which may be foreign to them. Japanese lesson study is a good example of how practitioners from one culturally different group learn how others teach and the efficacy of their methods.

Facilitators could comprise researchers whose sole interest may be to use videoed content to help them study classrooms and teaching across cultures. Stigler and Hiebert’s (2009) work is one such example. Mathematics educators may choose to use videos of lessons in the delivery of the mathematics methods courses.

Lampert and Ball are pioneers in the field. Their unique work in the Space for Learning and Teaching Exploration (SLATE) project provided a rich and detailed corpus of materials for American educators (Lampert & Ball, 1998). Star and Strickland's (2008) work with pre-service secondary mathematics teachers helped them categorise what pre-service teachers noticed in classrooms captured on videos. Borko, Jacobs, Eiteljorg and Pittman's (2008) work with the Supporting the Transition from Arithmetic to Algebraic Reasoning (STARR) project with middle-school mathematics teachers showed how the participating teachers' conversations around videos of lessons from one another's classrooms became increasingly reflective and productive.

The Use of Grounded Images in Mathematics Teachers' Professional Development: The Theoretical Framework

The introduction of electronic recording devices has changed the way researchers work. The drawings in Charles Darwin's book *The Origin of Species* (1998) were valuable records of plants and animals he encountered in their natural habitat. In the 1920s, cultural anthropologists such as Margaret Mead kept copious handwritten field notes, and photos were used to capture the participants of her study. Her work *Coming of Age in Samoa* (2001) became a bestseller. The images in these classics were stills and much was left to the imagination of the reader. Compare and contrast these works with those of the naturalist Sir David Attenborough. The images and the accompanying commentary have meant that the audience could engage with the wonders of the natural world which are presented in their minutest detail. For example, the flight of the flying lizard could be examined as a complete sequence or paused for the audience to examine the alignment of the webbed feet. At the same time the commentary provides the audience with the researchers' interpretations of the lizard's flight. The researchers and the audience share the same image as they have a common perspective of the event. Furthermore once captured these images can be recalled on demand for continual analysis.

In a similar vein the availability of electronic recording devices has added another dimension to professional development of mathematics teachers. In the days of handwritten notes, the researchers observe a teacher or group of teachers at work. In post-lesson interviews, the researcher and the teacher as participant discuss the contents of the lesson. Where they have difficulties understanding particular actions of the teacher, the researchers describe proceedings of that section in question. The teacher tries to recall that particular classroom scene and tries to answer the researcher's queries. Whilst the image of the classroom scene in the researcher's mind is formed from the perspective of the researcher observing the lesson, that formed by the teacher is based on the teacher listening to the description offered by the researcher. This is because the teacher did not have the opportunity to observe herself or himself at work. Because they come from different perspectives,

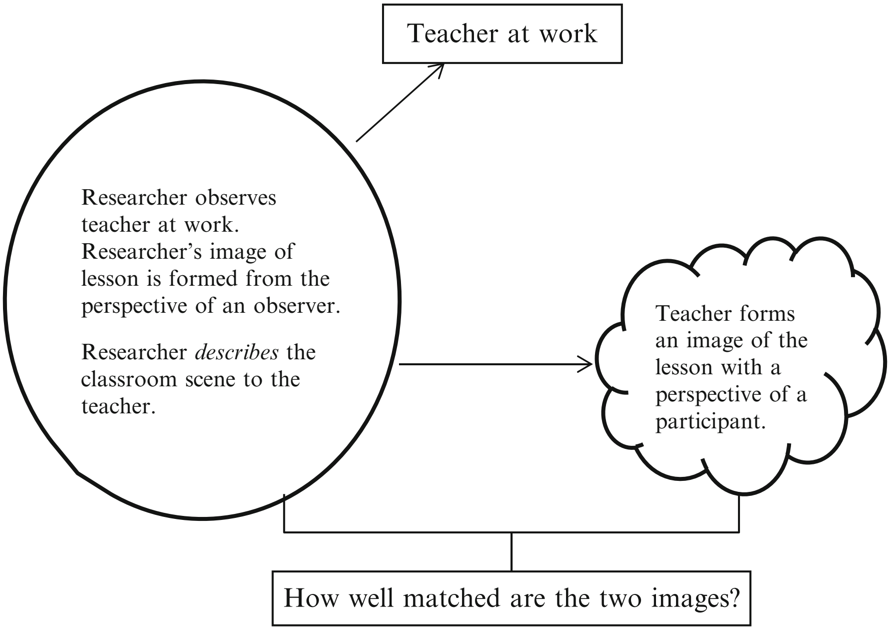


Fig. 1.1 Prior to the availability of video technology. Different perspective taking means that the researcher and teachers form different images of the same classroom scene

it is unlikely that the researcher and the teacher share the same image of the lesson. As illustrated in Fig. 1.1, the researcher's perspective is that of the teacher at work where the teacher is the object in the researcher's perspective. Because the two images are not grounded from the same perspective, the answers provided by the teacher are the teacher's best attempt to address the issues raised by the researcher.

Video technology has changed all this. Figure 1.2 shows how the availability of data captured on videos means that teachers are now able to reposition themselves so that they have the added advantage where they are simultaneously the actor and the evaluators of a specific lesson. When the researcher and the teachers look at the videoed lesson from the same perspective, they share the same grounded image of the lesson. Thus, with video technology, teachers see themselves as others see them. Although it is not necessary for the researcher and the teachers to have the same interpretation of the shared grounded image, there is synchrony in their discussion. There is no doubt in their minds that they are discussing the same teaching sequence. Where the researcher has doubts, the relevant sections of the lesson could be played, paused and replayed for further analysis and discussion.

Teachers can choose to study their actions in their own time or they could choose to be even more critical of their work by inviting others in the field to explore ways they can improve their practice. With video images the minutiae of each action could be analysed for further discussion. Teachers who choose to video their lessons

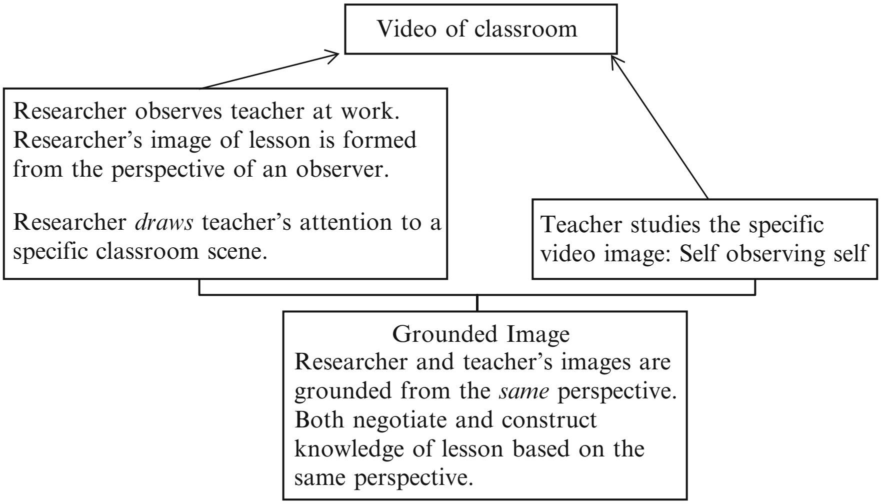


Fig. 1.2 With the availability of video technology, researchers and teachers construct grounded images

for self-improvement could decide for themselves the specific impact a mathematical task has on students' behaviours and the outcome of a particular pedagogy on the behaviours of groups of students. The teachers learn from the grounded images whether they were audible to all those present, the effects of their facial expressions and the inflections and intonations of their voice on specific students. Were their hand gestures useful in drawing students' attention to the work on the board? How did they respond to those students who were less forthcoming in classroom discussions? Primarily teachers could learn from the grounded images the impact their teaching actions have on the students. Furthermore researchers, with the permission of the participating teachers, could use video clips to improve the pedagogy of those who seek to learn. This is particularly true in situations where teachers have no knowledge of the proposed pedagogy. Observing other teachers applying a particular pedagogy and the resulting outcomes in the learning of the students may provide teachers participating in professional development courses some guidelines how to conduct specific mathematics tasks and the possible outcomes of such lesson on students. These grounded images help participating teachers form a clearer picture of their own classroom should they wish to adopt alternative pedagogies. Guided by such grounded images, the participating teachers may be less apprehensive about how to conduct such activities and may likely encourage them to try some of these activities which could benefit students.

Researchers also benefit from the affordances of videos. Although researchers' recollections of the classroom are supported by well-kept field notes, reviewing video clips of lessons may provide greater insights and may even challenge their

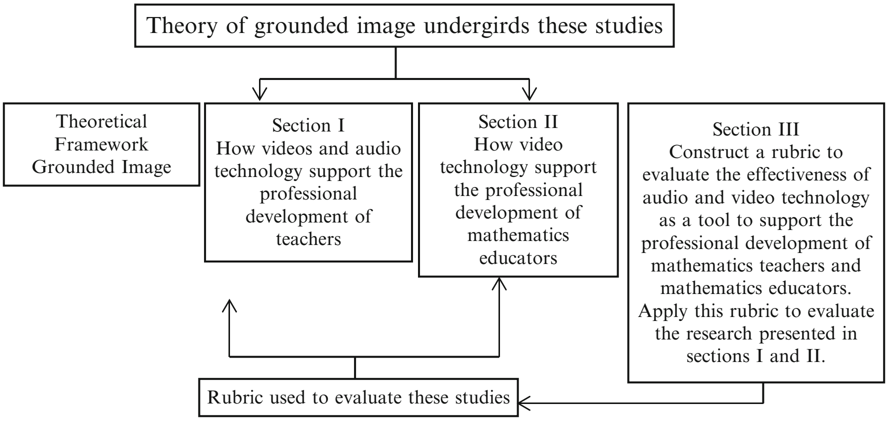


Fig. 1.3 The organisation of the book

initial reactions to an observed lesson. For example, the researcher questioned the efficacy of Teacher T's (see Chapter 5) practice of getting the children to repeat after her the part-part-whole relationship captured by the model drawing. However, after repeated reviewing of the videoed lesson, she had to change her initial reaction to this practice when she saw how the children benefitted from that practice.

Professional development of teachers is paramount in ensuring the availability of effective teachers who are capable to deliver the best possible instruction to every child (Barber & Mourshed, 2009). Whether teachers engage with a new teaching methodology will depend on their state of preparedness. This book reports on how video and audio technology are used to promote the professional development of teachers. Figure 1.3 presents the organisation of the book. Section I discusses how videos and audio technology support the professional development of teachers, and Section II provides evidence how professional development of mathematics educators can benefit from work with and by teachers. However, the various pieces of research reported in Sections I and II are eclectic in nature, yet each piece of work shows some measure of success and learning by the teachers and also by the mathematics educators/researchers. Then what defines effective use of video and audio technology? Are there some common features of professional development courses which utilise video and audio technology effectively? The chapter in Section III of this book summarises research conducted using videos in the western literature, presents a rubric to evaluate the effective use of video and audio technology and then uses this rubric to evaluate the research conducted in the six Asian countries, Hong Kong, Indonesia, Korea, Malaysia, the Philippines and Singapore.

References

Barber, M., & Mourshed, M. (2009, July 7). *How good education systems can become great in decade ahead*. Report on the International Education Roundtable, Singapore. Retrieved from http://www.mckinsey.com/locations/southeastasia/knowledge/Education_Roundtable.pdf

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24*, 417–436.

Brophy, J. E. (Ed.). (2004). *Using video in teacher education*. Bingly, UK: JAI Press.

Darwin, C. (1998). *The origin of species*. London, UK: Wordsworth Editions Ltd.

Klein, P. S., Nir-Gal, O., & Darom, E. (2000). The use of computers in kindergarten, with and without adult mediation, effects on children's cognitive performance and behaviour. *Computers in Human Behavior, 16*, 591–608.

Lampert, M., & Ball, D. (1998). *Teaching, multimedia and mathematics: Investigations of real practice*. New York: Teacher's College Press.

Mead, M. (2001). *Coming of age in Samoa. A psychological study of primitive youth for western civilisation*. New York: Harper Collins Publishers.

Sherin, M. G. (2004). New perspective on the role of video in teacher education. In J. Brophy (Ed.), *Using video in teacher education* (pp. 1–27). Bingly, UK: JAI Press.

Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to observe. *Journal of Mathematics Teacher Education, 11*, 107–125.

Stigler, J. W., & Hiebert, J. (2009). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Part I

Chapter 2
Into Part I: How Videos and Audio Technology Support the Professional Development of Teachers

Part I comprises five chapters. The first three chapters report the work conducted in Singapore. The diverse contents of each of the chapter illustrate how video technology is used to address the different needs of some of the Singapore teachers. The penultimate chapter of Part I illustrates how a group of Philippine teachers used video clubs to help them engage with the extension component of problem-solving. The last chapter provides empirical evidence of the challenges faced by Malaysian mathematics educators as they try to introduce lesson study methodology to teachers from a multiethnic society.

With the introduction of new initiatives, finding new ways to deliver the intended curriculum is necessary. In Singapore the 2012 *Primary Mathematics Teaching and Learning Syllabus* defines clearly the processes of applications and mathematical modelling and the nature of the mathematics teachers are expected to engage their students.

> Students should have opportunities to apply mathematical problem-solving and reasoning skills to tackle a variety of problems, including open-ended and real-world problems. Mathematical modelling is the process of formulating and improving a mathematical model to represent and solve real-world problems. Through mathematical modelling, students learn to deal with ambiguity, make connections, select and apply appropriate mathematics concepts and skills, identify assumptions and reflect on the solutions to real-world problems, and make informed decisions based on given or collected data. (Ministry of Education, 2012, p. 17–18)

Although the processes and the mathematics may be well defined, mathematical modelling is a very novel way to engage learners with mathematics they know. Often teachers are unsure how to conduct lessons on mathematical modelling because they are challenged how best to facilitate for rich student mathematisation processes during such tasks. Kit Ee Dawn Ng et al.'s chapter reports how a multitiered teaching experiment using design research methodology was conducted to build teachers' capacity in designing, facilitating, and evaluating student mathematisation during mathematical modelling tasks with an intact class of Primary 5 students (aged 10–11). The use of

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_2

videos was critical because grounded images helped capture the dynamics and complexity of authentic classroom interactions. Their chapter highlights how video recordings of teacher-student interactions during a modelling task were harnessed to activate critical moments of learning for the teacher towards developing her competencies in facilitating students' mathematisation processes.

While Kit Ee Dawn Ng et al.'s work dealt with supporting teachers with novel approaches to learning mathematics, Lu Pien Cheng's chapter, however, addresses the very fundamental needs of seven primary teachers who found teaching of fractions and conversion between different units of measurement challenging. Although the teachers were keen to improve their pedagogies, they were, however, apprehensive about being videotaped conducting such lessons. Cheng adopted a two-phase approach to encourage teachers to consider the value of videotaping lessons for continual evaluation and professional development. In the first phase the lesson on fractions was audiotaped. Cheng then used the contents of this lesson to construct a questioning framework which she used to help the participating teachers improve the quality of their reflections, moving from Level 1, the lowest to Level 3, the highest. During this phase, the teachers had difficulties recalling the exact details of the observed lesson, and this prompted them to embrace video technology. But because the teachers were still reluctant to be videotaped, Cheng volunteered to be videotaped conducting a lesson on conversion between different units of measurement. The teachers used the grounded images of Cheng at work as catalysts to promote richer reflections.

Although the part-part-whole concept and its related pictorial representation conceptualised as the Singapore model drawing may seem beguilingly simple, it is not. Swee Fong Ng's study showed that the teacher in the study was unable to help early learners of the model method make the connections between the pictorial representation and the accompanying mathematical sentences. Comparing and contrasting the data from the two videoed lessons showed that the very prescriptive mediational processes of the teacher in the second lesson helped the children to discern and identify the critical components of the part-part-whole representation. Repetition of the related mathematical language and the specific relationships until these stuck in their consciousness enabled the children to engage with the lesson. The children could state the desired rule. When a child failed to understand the related concepts, repeated engagement of the specific names meant that she could volunteer her lack of understanding to the teacher who was then able to address her query.

Romina Ann Yap and Yew Hoong Leong's study was situated within a 7-month professional development programme aimed at supporting Secondary 1 mathematics teachers in the teaching of mathematical problem-solving in the Philippines. This chapter examined whether and how video clubs influenced teachers' mathematical problem-solving classroom instruction particularly in the area of teaching extensions, the final phase of Polya's four problem-solving phases. Evaluations at the end of the programme reveal that teachers considered the video club as one of the components of the programme that had the most impact on them.

Chap Sam Lim and Liew Kee Kor's chapter discusses how edited videos were used as a medium to introduce the concepts and processes of lesson study to 54 multiethnic Malaysian teachers as part of their professional development. These teachers were teaching in one of the three different types of vernacular primary schools which were underperforming in mathematics and science. In the introductory workshop Lim and Kor showed the participating teachers edited clips of the Japanese model of lesson study. English subtitles were provided to help teachers engage with the contents of the video. However, this was not sufficient. Because the teachers were most proficient with their mother tongue, Lim and Kor supplemented the professional development sessions with the language most appropriate to the teachers. The chapter concludes that although video can be an effective medium for professional development of teachers, teachers' commitment to and senior management's support for professional development of teachers remain the most important factor for the success of any professional development programme.

Reference

Ministry of Education. (2012). *Primary mathematics teaching and learning syllabus*. Singapore: Author.

Chapter 3
Developing Teacher Competencies Through Videos for Facilitation of Mathematical Modelling in Singapore Primary Schools

Kit Ee Dawn Ng, Wanty Widjaja, Chun Ming Eric Chan, and Cynthia Seto

Abstract Mathematical modelling tasks which are situated in real-world contexts encourage students to draw connections between school-based mathematics and the real world, enhancing their engagement in learning. Such tasks often require varied interpretations of the real-world problem context resulting in multiple pathways of solutions. Although mathematical modelling has been introduced in the Singapore mathematics curriculum since 2007, its incorporation in schools has been limited. One reason for this could be that teachers are challenged by how best to facilitate for rich student mathematisation processes during such tasks. This chapter reports how a multitiered teaching experiment using design research methodology was conducted to build teachers' capacity in designing, facilitating, and evaluating student mathematisation during mathematical modelling tasks with an intact class of Primary 5 students (aged 10–11). The use of videos was critical because grounded images helped capture the dynamics and complexity of authentic classroom interactions. This chapter highlights how video recordings of teacher-student interactions during a modelling task were harnessed during design methodology cycles, particularly during the retrospective analysis phase, to activate critical moments of learning for the teacher towards developing her competencies in facilitating students' mathematisation processes.

K.E.D. Ng (✉) • C.M.E. Chan
National Institute of Education, Nanyang Technological University, Singapore
e-mail: dawn.ng@nie.edu.sg; eric.chan@nie.edu.sg

W. Widjaja
Faculty of Arts and Education, School of Education, Deakin University, Burwood, Victoria, Australia
e-mail: w.widjaja@deakin.edu.au

C. Seto
Academy of Singapore Teachers, Ministry of Education, Singapore
e-mail: cynthia_seto@moe.gov.sg

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_3

Introduction

The Organisation for Economic Co-operation and Development (OECD) defines *mathematical literacy* for the twenty-first century as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments, and to engage with mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen" (OECD, 2010, p. 4). Students are thus expected to be "active problem solvers" (OECD, p. 6) demonstrating the *capacities* to "analyse, reason, and communicate ideas effectively as they pose, formulate, solve and interpret mathematics in a variety of situations" (de Lange, 2006, p. 15), particularly in real-world problems. The Singapore mathematics curriculum also emphasises the importance of equipping students with *competencies* to meet the challenges of the twenty-first century. These competencies refer to students' abilities to use appropriate tools (e.g. ICT), knowledge, and skills to solve real-world problems in collaboration with others (Curriculum Planning and Development Division [CPDD], 2012, p. 1).

Incorporated into the Singapore mathematics curriculum framework since 2007 (CPDD, 2006), mathematical modelling is defined as the "process of formulating and improving a mathematical model to represent and solve real-world problems" (CPDD, 2012, p. 2). The intent is to encourage Singapore primary and secondary schools to embrace a pedagogy which will equip students with modelling competencies for real-world problem solving. Mathematical modelling in Singapore schools involves students working with ambiguity and authentic data so that they learn to "make connections, apply appropriate mathematics concepts, skills, identify assumptions and reflect on the solutions to real-world problems and make informed decisions" (CPDD, 2012, p. 3). In addition, students' cognitive reasoning can be fostered when they manage complex systems within the real-world context during model development (English & Sriraman, 2010), highlighting connections between school mathematics and the real world. The purpose of mathematical modelling in Singapore schools can be said to draw upon the theoretical perspective of "contextual modelling" (Kaiser & Sriraman, 2006, p. 306) which promotes decision making within real-world constraints along with the integration of socially constructed knowledge during the modelling process.

Primary school students are capable of modelling situations when presented with model-eliciting activities (MEAs) (Chan, 2010; Chan, Ng, Wijaja, & Seto 2012; English, 2010). Such activities allow learners to engage in mathematisation processes (de Lange, 2006) where the problem is interpreted, represented, and solved in multiple ways through the development of representations that are continually tested and revised until they reach an appropriate solution. However, the implementation of modelling activities in Singapore schools has been limited and sporadic. Although the National Institute of Education and the Ministry of Education have made concerted efforts to encourage the use of such activities in mathematics classes (see Balakrishnan, 2011; Ng & Lee, 2012), mathematical modelling has yet to gain a firm foothold in Singapore schools (Ang, 2010). Studies conducted (e.g. Ng, 2010; Stillman, 2010) have identified teacher competencies in facilitating modelling as a major factor for its implementation.

This chapter seeks to answer this research question: What teacher competencies were crucial in the facilitation of model-eliciting activities (MEAs) in primary mathematics classrooms? In other words, for teachers who have undergone some teacher development to implement their first MEAs, we would like to investigate what teacher competencies would surface as pertinent during their facilitation and evaluation of students' mathematisation processes. Teachers have varying entry positions and comfort levels for the implementation of mathematical modelling in their classrooms. One main goal of the research is to help teachers new to mathematical modelling identify their learning needs for the successful implementation of MEAs in their primary classrooms and in the process prompt teachers to make *conscious efforts* to build upon their existing competencies or foster new ones. Another goal of the research is for teacher educators to explore *non-prescriptive* ways to help teachers develop their own competencies in incorporating MEAs in their primary classrooms. This means that the researchers of the study who are teacher educators would adopt a methodology that allows for the co-construction of knowledge through the negotiation of meaning within the established community of participants.

The chapter is organised into various sections. It begins with a section outlining the modelling cycle and desired modelling competencies. After all, teacher competencies should be aligned with the fostering of modelling competencies in students. The next section summarises key teacher competencies for facilitating mathematical modelling and how a lack of these competencies may impede the successful implementation of mathematical modelling in Singapore schools. The following section describes the methodology adopted in the study and explains the rationale of choice of methodology in view of the two goals of the study mentioned above. Findings are presented in the section after. The chapter then closes with the final section of concluding remarks and implications for future incorporation of mathematical modelling in Singapore schools, proposing directions of teacher education.

Modelling Cycle and Modelling Competencies

Mathematisation, the crux of mathematical literacy, is the process of formulating a mathematical problem from a real-world situation and solving the problem in view of real-world constraints (de Lange, 2006, p. 17). During mathematisation, mathematical lenses are used to represent, model, and solve the problem. Some aspects of reality may be set aside in the process. The mathematical solution is then examined on whether it makes sense within the real-world situation the problem started with. Modelling competencies are often manifested during mathematisation. Although there are various perspectives on where mathematisation becomes explicit within

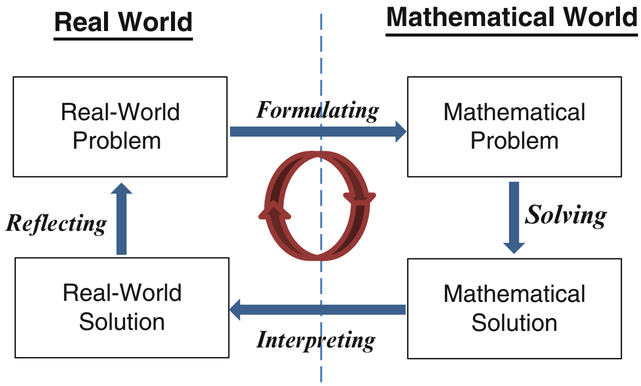


Fig. 3.1 A simplified diagram of the modelling cycle adapted from the Singapore curriculum documents (CPDD, 2012, p. 11)

the modelling process (e.g. Maaß, 2006), de Lange suggests that the process of mathematisation weighs heavily *throughout* the modelling cycle. Figure 3.1 shows a simplified diagram of the modelling cycle as used in this study. The modelling cycle comprises four stages: (1) real-world problem, (2) mathematical problem, (3) mathematical solution, and (4) real-world solution. A modeller engages with formulating, solving, interpreting, and reflecting as they move between stages (1) to (2), (2) to (3), (3) to (4), and (4) to (1), respectively (CPDD, 2012, pp. 9–18).

Formulating

Using mathematical lenses, modellers formulate a mathematical problem from the real-world problem by identifying the key variables, forming relationships between variables, and making assumptions about the problem situation. Reality may be trimmed away in an attempt to represent the problem mathematically when the modellers quantify the measurement of the variables and possible relationships between variables. Modellers also determine which conditions or parameters need to be further consolidated for the construction of a preliminary mathematical model.

Solving

Various mathematical problems can be formulated from the same real-world problem situation because modellers may perceive differing variables associated with the situation. Subsequently, a range of mathematical models can be developed, promoting a myriad of solution pathways. Mathematical models can take geometric, graphical, algebraic, or statistical forms along with appropriate mathematical arguments in support of a claim made in response to the problem posed.

Interpreting

Once a mathematical solution is in view, the modeller proceeds to interpret the solution within the original real-world problem context. This involves the modellers making sense of their mathematical solution by re-examining the variables involved, assumptions, and parameters so that a real-world solution could be developed to answer the original problem more appropriately and reasonably.

Reflecting

This occurs when the newly developed real-world solution is evaluated for its adequacy and limitations within real-world constraints. Proposed improvements to the mathematical model or further investigations into other variables may arise from this reflection, prompting the modeller to backtrack to any previous stage of the modelling cycle. Ideally, modellers should engage in reflection throughout the cyclical modelling process (Galbraith, 2013) as they move between the stages reviewing their preliminary models periodically towards the development of more sophisticated models. This constant reflection is represented by the central circular arrows in Fig. 3.1. In addition, a main feature of MEAs is to review the model for its applicability or generalisability to other contexts.

Houston (2007) classified modelling competencies generally as *holistic* (i.e. focusing on modellers' overall sense making of the real-world situation and their efforts at generating mathematical models) or *microscopic* (i.e. skill based such as goal clarification, stating assumptions, selecting variables, and formulating mathematical statements). However, Maaß (2006) provided an alternative classification framework for modelling competencies involving five competency clusters (pp. 116–117) which are predominantly skill based: (a) interpreting the real-world problem and formulating a real-world model based on reality, (b) developing a mathematical model from the real-world model, (c) solving mathematical questions within the mathematical model, (d) interpreting the mathematical results in the real-world problem, and (e) validating the solution. A number of studies have identified the central role *metacognition* plays in the modelling process (e.g. Kaiser, Schwarz, & Tiedemann, 2010; Stillman, 2011; Stillman, Brown, & Galbraith, 2010) and called for teachers to facilitate for students' metacognition so that they can monitor and remove blockages during the modelling cycle.

Teacher Competencies in Facilitating Modelling

The role of the teacher during facilitation of mathematical modelling activities is complex and multifaceted. Existing research on teacher competencies in modelling centres on two aspects: (a) teacher knowledge about modelling and the teaching of modelling (e.g. Doerr, 2007; Kaiser et al., 2010, Lesh & Doerr, 2003) and (b) teacher intervention strategies during students' modelling process (e.g. Borromeo Ferri, & Blum, 2010; Warner, Schorr, Arias, & Sanchez, 2010). The first aspect refers to the knowledge of the teacher regarding what modelling is, the stages of the modelling cycle, and modelling competencies associated with the mathematisation process. These have been discussed in detail in the section above. The second aspect on teacher intervention involves moment-to-moment facilitation decisions of the teacher during real-time implementation of modelling tasks. This can include the nature of questioning and how the teacher supports, refutes, and even redirects the proposed mathematical representations and conclusions of students. Meta-metacognition of teachers (Borromeo Ferri & Blum, 2010; Stillman, 2009) could be another crucial aspect of teacher competencies in the facilitation of mathematical modelling. It involves the teacher constantly monitoring and regulating their own thinking and pedagogical decisions during facilitation as well as fostering the metacognition of their students. The quality of teacher meta-metacognition may well orchestrate a delicate balance between student independence in mathematisation and teacher guidance.

Limited preliminary Singapore research revealed that *teacher readiness* is a key factor in determining the types of teacher competencies in mathematical modelling. We will elaborate briefly on this in order to provide a rationale for the identified goals of the research presented above and our chosen methodology in the next section. Teacher readiness can be viewed from two grounds: mindset and facilitation expectations.

Mindset

This refers to teachers' interpretation of what mathematical modelling is, their beliefs about the nature of mathematics and problem solving, as well as whether they had enriching initial experiences being modellers themselves. Many Singapore primary school teachers interpret mathematical modelling as the model-drawing approach often used to solve structurally complex word problems (Chan, 2008). In her exploratory study with 48 experienced primary school teachers involving the use of MEAs for primary level implementation, Ng (2010) found that many teachers in the sample believed that mathematical representations should mainly involve algorithms and formulae. Their perceived problem solving pathway was rather linear, starting from understanding the problem before moving on to identifying the necessary algorithm or formula to solve the problem and lastly working towards one correct answer. The perceptions of teachers on mathematical modelling and the nature of mathematics from Chan's (2008) and Ng's (2010) research contrasted sharply with the essence of mathematical modelling discussed in this chapter. In addition, Ng (2013) found that some teachers may not be able to appreciate the potentials of modelling activities because of their less than enriching prior experiences as modellers themselves. At the same time, these teachers may lack the competence in facilitating for higher quality mathematical outcomes from students' modelling activities.

Facilitation Expectations

Singapore teachers may feel challenged with MEAs which have open structures embedded in rich contexts due to fear of moving out of their comfort zone of prescriptive teacher-centred lessons (Chan, 2008). Doerr (2007) emphasised the use of more student-centred delivery mode during modelling facilitation. However, this will depend on whether the teachers are ready to adopt such mode. Indeed, this concern is not unique to Singapore teachers. Borromeo Ferri and Blum (2010) found that the interventions of German teachers during the modelling attempts of their students are often intuitive, not strategic towards preserving the independent thought processes of students. The teachers are more concerned with the application of fixed mathematical content and classroom management. This may account for them trying to facilitate students towards their favoured solutions to the modelling task.

Teacher competency is viewed as the crux for successful and sustainable incorporation of mathematical modelling in Singapore schools. The current lack of Singapore research to date in this area specifically for primary schools provided a strong impetus for the study reported in this chapter. Implications from the findings presented here may serve as a springboard for charting future directions in teacher professional development for mathematical modelling. Nonetheless, the mindset of Singapore teachers and their perceived facilitation expectations from modelling tasks imply that research on teacher competencies would have to take cautious considerations of the prerequisite perceptions of teachers and their comfort levels to mathematical modelling at entry point. Hence, the fostering of teacher competencies by the teacher educators would have to be non-prescriptive. Moreover, crucial competencies to be identified for further development within each teacher should be co-constructed by the teacher and teacher educators, supported by a shared understanding of what was enacted in actual classroom conditions. In other words, the teacher makes a conscious choice of the nature of development of teacher competencies. Design research methodology was selected in view of the purposes and constraints of this research. The role of video is vital in this research because video provides a grounded image which serves as a common platform for seeking consensus in identifying critical teacher competencies as espoused in the research question.

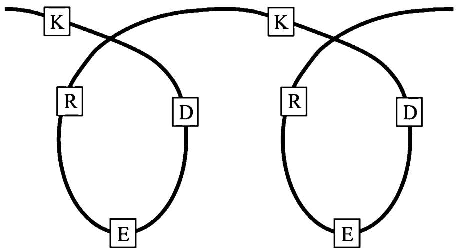


Fig. 3.2 The cyclical process of knowledge (*K*), designing (*D*), experiment (*E*), and retrospective analysis (*R*) phases (Dolk et al., 2010, p. 175)

This Study

This chapter came from part of a larger study which used Dolk, Widjaja, Zonneveld, and Fauzan's (2010) version of design research methodology framework (Fig. 3.2) involving four phases conducted in iterative cycles. The word limit of this chapter only allows us to discuss our findings for the first cycle. This cycle was situated on a researcher-designed MEA entitled the "Bus Route Task" (Appendix), and it focused on acquainting the teacher with the features of modelling tasks along with scaffolding strategies for mathematisation.

Knowledge Phase

This phase took into consideration teacher's knowledge of the following prior to the design and implementation of the modelling task: (a) mathematical content knowledge, (b) pedagogical content knowledge, (c) curriculum content knowledge, (d) knowledge of mathematical modelling, particularly MEAs, and (e) knowledge of teacher scaffolding emphasis in support of mathematisation. Teacher-S, the participating teacher reported here, had 30 years of teaching experience. She had been with her Primary 5 (aged 11) mixed ability class from a typical Singapore government primary school for at least 8 months before the research was conducted. Since Teacher-S was new to mathematical modelling, our foci on teacher competency development were on (d) and (e). We facilitated Teacher-S' first modelling attempt using the Bus Route Task. Teacher-S worked together with her colleagues as modellers to experience all four stages of the modelling cycle illustrated in Fig. 3.1. Next, we prompted Teacher-S to reflect on her own mathematisation process during model development so that she could articulate her mathematical thinking and the difficulties she faced in solving the problem. After which, our facilitation procedure was summarised, and she was guided to identify how we helped navigate her through the modelling stages.

Design Phase

Teacher-S drew upon her knowledge to examine the design of the Bus Route Task for feasibility and flow of implementation specifically for her Primary 5 class. We subsequently revised the task based on comments from Teacher-S.

Teaching Experiment Phase

Teacher-S utilised three consecutive lessons of 1 h each to carry out the Bus Route Task with her class of 33 students. The students worked in groups during the task. Student-grouping was decided by Teacher-S based on her experience with the class. At the end of each session, she reflected about the lesson with the researchers, highlighting facilitation issues and posing questions for the researchers.

Retrospective Analysis Phase

The researchers conducted a post-task reflection focusing on helping Teacher-S identify and reflect on her competencies in incorporating modelling tasks.

Method

Data Collection and Analysis

Video recording was conducted throughout all sessions. Videos were used to capture the discussion between researchers and Teacher-S during the design phase. In the teaching experiment phase, videos were used to capture the dynamics of teacher-student interactions as well as student-student interactions. Three concurrent video recordings were done in this phase: a roving camera on Teacher-S as she moved about in the classroom and two stationary cameras with two case-study student groups. Carefully selected video segments from the teaching experiment phase were used as a means to engage Teacher-S during retrospective analysis whereby she was prompted using video-stimulated recall (Lyle, 2003) to reflect upon her competencies in navigating her students through the potentials and challenges of the modelling task towards advancing students' mathematisation. The researchers used a non-prescriptive stance highlighting instances of teacher-student interactions as shown in the video excerpts to elicit her awareness of the aspects in focus. Subsequently, Teacher-S consciously articulated her discovery of critical teacher competencies she would like to develop during her discussions with the researchers.

Observations from the Teaching Experiment Phase

Prior to learning about mathematical modelling, Teacher-S admitted that she has been using a predominantly prescriptive teacher-centred mode of teaching. She made a conscious effort to switch to a more student-centred facilitation approach during the teaching experiment phase. The students were surprised by the change in her teaching style and the open-ended nature of the Bus Route Task. Initially, a few students were reluctant to engage with the task because they were not familiar with its open-ended format. Those who worked with the task required substantial and continual reassurance from Teacher-S to test and further develop their ideas within their groups.

In the first of three teaching experiment sessions, Teacher-S set the stage for the modelling task by generating a whole class discussion on the goal of the problem and identifying the variables affecting bus route efficiency. These key points such as cost, distance, speed, and convenience were recorded on the board, and Teacher-S continually drew students' attention to them (Fig. 3.3, left) as they went about working in their groups. Video footages captured the teacher diligently attending to the eight student groups, prompting them intensely to work with the identified variables. After direct instructions from Teacher-S, most of the student groups began to use the resources provided (e.g. strings, rulers, markers, etc.) to assist in their initial formulation of the model. Figure 3.3 at the right shows two students using a string to measure the distance covered by one of the three proposed bus routes marked out on a real-world street map provided along with the task sheet. The teacher later discovered that the students were not progressing as she had expected in response to the five questions on the task sheet. She called the class to attention to set goals for their progress and discussed the assumptions they could make about the problem situation.

The groups continued their discussions in the second session. Three dominant preliminary mathematical models of the problem emerged: At the left of Fig. 3.4 is

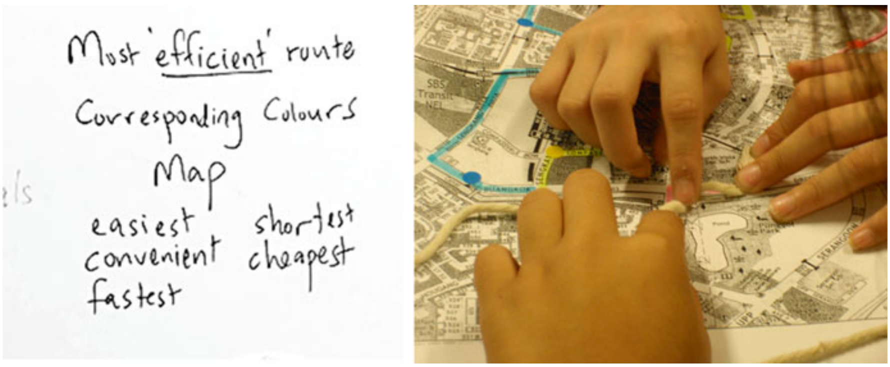


Fig. 3.3 At the *left*, a screen grab of teacher notes on the board generated from whole class discussion about the variables affecting the most efficient bus route. At the *right*, a screen grab of two students using a string to help measure the distance covered by one of the given bus routes

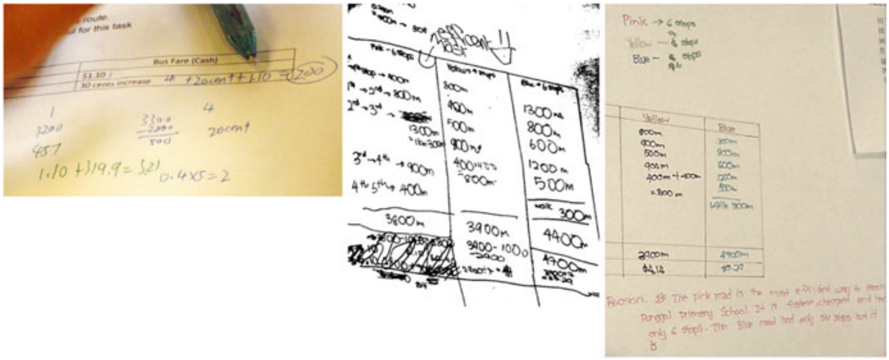


Fig. 3.4 Screen grab of a cost-based model, at the *left*; distance-based model, in the *centre*; and distance-cost model, at the far *right*

the cost-based model, at the centre is the distance-based model, and at the far right the distance-cost model. In refining the model, Teacher-S encouraged student groups to look into other variables and the relationships between them as part of a more holistic consideration of the most efficient bus route. However, it could be observed from the video footages that many groups seemed to have concluded their work once they worked out their model, thinking that they already had a solution to the problem. The students were perplexed by Teacher-S' request to review their models and were reluctant to do so. Such reactions on the part of the students could be due to their lack of experience with open-ended real-world problems.

In the third and final session, students were given time to revise their models before they gave an oral presentation of their work. There was a question-and-answer session after each presentation. Most of the questions centred on the assumptions made by the groups during calculations (e.g. the traffic conditions, speed of the bus). The teacher concluded the presentation sessions with a summary of the modelling processes undertaken by the students, a review of the models developed, and the mathematical outcomes of the task.

In the next section, we present findings on crucial teacher competencies identified from video data of teacher-student interactions during the teaching experiment phase and audio data from video-stimulated recall conducted in the retrospective analysis phase.

Finding 1: Competencies Associated with Teacher Knowledge of Modelling Cycle

Video data showed that Teacher-S displayed some competencies in facilitating students' movements from the real-world problem stage to the mathematical problem stage of the modelling cycle (see Fig. 3.1). She prompted students to offer their interpretations of the most efficient bus route, eliciting the variables affecting bus

Table 3.1 Excerpts taken from video transcripts showing teacher competencies being displayed during the first session

Speaker	Line	Content
Teacher:	1	[Referring to the task sheet and the street map]
	2	Right. Group 1 says what stands out for them is this most efficient route. Okay. Anyone has
	3	something different that stood out for you as most important?
Teacher:	4	Okay, good. Put your hands down. Can we look at this word, efficient? We are going to find the
	5	most efficient route. So what does efficient mean then? Yes?
Teacher:	6	Shortest, cheapest. Do I hear anything else? Any other words that come to mind when you
	7	talk about the word "efficient"? So this is what you think efficient means? Okay just to
	8	define the word "efficient", is actually to achieve something, a desired result. Something that
	9	you want to achieve in the best possible way. You don't want to have any wastage in any … ***(defining most efficient bus route, eliciting variables)*** [Speaking to a student group]
Teacher:	10	So you have to write down your thoughts. Why you have decided to look at the bus stops. Think
	11	through. Is there any other considerations you have to take into account? Things like that. Okay
	12	This is just the initial part. See if there's anything else that's important as well ***(prompting students to work out a preliminary model)***

route efficiency and helping students formulate their first mathematical model based on their selected variables (Table 3.1, lines 2–3). All student groups had no difficulty in working out the solutions from the preliminary models. Hence, minimal teacher facilitation was required to move students to the mathematical solution stage of the modelling cycle. Although the students' assumptions about the problem displayed sense making within some real-world constraints, Teacher-S faced challenges in prompting students to move on to the real-world solution stage of the modelling cycle. Quite a few groups were unable to reflect critically about the model in terms of its adequacy and limitations to address the original real-world problem. Only a few groups revised their initial models.

Finding 2: Competencies Associated with Teacher Intervention

This component involves moment-to-moment facilitation decisions of Teacher-S during real-time implementation of modelling tasks. The video footages showed that she displayed a wide repertoire of questioning skills during her facilitation (Tables 3.2 and 3.3), prompting students to think and justify their models. She occasionally supported the ideas of students and built upon them when she

Table 3.2 Group 8 teacher-student discourse

Speaker	Line	Content
Teacher:	1	[Pointing at the table of values which recorded the map distance of each bus route]
	2	This is based on?
Student:	3	We measure how long {the distance of the bus route is}
Teacher:	4	The length of the route, okay the distance. ***What else are you going to do? Is that enough you***
	5	***think?***
Student:	6	No
Teacher:	7	How else would you do?
Student:	8	Find the bus fare
Teacher:	9	You're going to work out the bus fare. Is there anything else that you hope ***to make your***
	10	***argument stronger***? Any other factor? ***Have you looked at the map more closely***? Is there ***any***
	11	***other factor*** you want to include? It is up to you; you may want to look at other factors; you
	12	may want to choose some; you may want to leave out some; it's up to you. You ***decide and***
	13	***record, record the discussion*** that you've had, how you decided to select distance and cost, is
	14	it? ***Why***? Why distance and cost and not some of the other things. Okay? Any question?
Student:	15	[No response]
Teacher:	16	[Moves to another group]

Table 3.3 Group 1 teacher-student discourse

Speaker	Line	Content
Teacher:	1	[Referring to one of the members in the group]
	2	She is doing the recording; the rest of you what are you doing?
Student:	3	We are just giving her ideas
Teacher:	4	**How many factors {determining the choice of the most efficient route} have you considered so**
	5	**far?**
Student:	6	Bus fare, distance, etc.
Teacher:	7	Okay, so **which have you decided on? Which route?**
Student:	8	The pink route
Teacher:	9	Okay, you've got two factors; **how did you decide the pink was better?**
Student:	10	We measure{d} three different routes, and we marked it with markers
Teacher:	11	[Distracted by students who are off tasks]
	12	Sorry, just give me a minute
	13	[She returned after 10 min]
Student:	14	[Repeating] We use markers to mark it and measure

(continued)

Table 3.3 (continued)

Speaker	Line	Content
Teacher:	15	Okay, that is how you did, how you arrived at it. But **why {have you decided} between distance**
	16	**and cost? You said the pink route is better based on…**
Student:	17	The cost
Teacher:	18	[Referring to students' calculations on the bus fares for the respective routes]
	19	How about this part of it, **does it affect the distance? Did you link these two? What is the link**
	20	**between these two?** {What is the relationship between bus fares and bus route distance?}
Student:	21	This is the {calculations for the} pink line {route} and this is for the yellow line {route}; we
	22	find out the difference between the two
Teacher:	23	You mean in distance then…
Student:	24	Yeah and the amount of fares, meaning we are trying to find out which line {route} is the
	25	cheapest by the distance
Teacher:	26	Okay
	27	**So did you find some kind of a link between the two?**
Student:	28	Yes
Teacher:	29	**What did you find out?**
Student:	30	We should know that {a} shorter {route} is cheaper
Teacher:	31	**The shorter, the cheaper so you decided on pink route based on that**
Student:	32	Yeah
Teacher:	33	Ok. [Teacher leaves the group]

sensed the groups making headway in the task. However, there was little evidence of her refuting the claims of the students explicitly although she tried to question the assumptions underlying the claims. She was hoping that the students would come to a conclusion about their error and redirect their own model development.

Three competencies were highlighted for further work by the teacher during the retrospective analysis phase of the research. Ng, Widjaja, Chan, and Seto (2012) and Ng, Chan, Widjaja, and Seto (2013) provided preliminary discussions of these competencies.

Finding 3: Teacher Competency in Striking a Balance between Questioning and Listening

Video played a central role in helping Teacher-S identify the need to attend to this crucial competency during her facilitation. Two contrasting video excerpts of teacher-student discourse from Groups 8 and 1 were purposefully selected to frame the scenario for activating Teacher-S' critical moment of learning during the retrospective analysis phase. When triangulated with the transcripts of the excerpts (Tables 3.2 and 3.3), the video-stimulated recall segment served as a platform for constructing a grounded image of Teacher-S' approaches with the two groups. Subsequently, through reflection prompted by the researchers in view of the video excerpts, Teacher-S recognised and identified her first crucial competency for her self-development in modelling facilitation: the importance of striking a balance between questioning and listening when probing students' thinking. Table 3.4 shows excerpts of researcher-teacher discourse during the video-stimulated recall. The portions in bold are highlighted for discussion of the teacher's questioning approaches. Square brackets recorded actions of the teacher or students. Curly brackets contained additional notes added to the transcript for a more complete interpretation. Cross references between Tables 3.2, 3.3, and 3.4 serve for a better understanding of the retrospective analysis phase.

Table 3.4 Researcher-teacher discourse during video-stimulated recall

Speaker	Line	Content
Teacher	1	Ok. I thought that … with Group 8, I was able to take them a bit further, whereas for
	2	Group 1, I didn't really go so deep. {For Group 1} I didn't really go into the real-world
	3	constraints I think as much as with Group 8. Because Group 8 seemed more settled and
	4	more…they seemed to have arrived at a conclusion, whereas Group 1, I felt was
	5	still…trying to…finish the task. They hadn't quite completed. That was my feel so I
	6	thought I would let them just carry on. So they could make some more observations on
	7	their own
Researcher:		[From another portion of the discourse]
	8	…So we notice that in the first group {Group 8} the interaction that they have, you
	9	actually asked lots of good prompts. Do you find that at that time you also listen to the
	10	students I mean…what are they thinking after that particular after you posed the
	11	questions…?
Teacher:	12	Not really
Researcher:	13	Not really. Ok. And then what did you find {what happened} when you didn't listen to
	14	the students?

(continued)

Table 3.4 (continued)

Speaker	Line	Content
Teacher:	15	I couldn't actually eh, take them through their thinking process. I wasn't really listening
	16	to what they were {talking about}, so we didn't really move {on} in that direction {that
	17	I'd wanted them to}
Researcher:	18	Yes, exactly. So that's why we found that sometimes eh, even with good prompts, when
	19	you actually asked them lots of good prompts at once, at one go you see,…, but actually
	20	it was not really connected to what they have said to you or what they have
	21	done…maybe the students' voice hasn't come in at that time {wasn't heard}
Teacher:	22	Yeah, it's easier to…actually guide them along eh, their thinking when I follow up with
	23	their responses…rather than just asking them questions because I want them to think in
	24	that way
Teacher:	25	[After some other discussion]
	26	I think you've covered the main issue for, with me, with the questioning part and the
	27	listening, I think more working towards that. That will help to bring about better
	28	outcomes for the next, next time

The intention behind Teacher-S' questioning approach for Groups 8 and 1 was similar: the need to expand on the students' preliminary mathematical models (Table 3.2, lines 1–7; Table 3.3, lines 15–28). However, there were contrasting reactions from both groups. In the case of Group 8, Teacher-S had intended to help them "go deeper" (Table 3.4, lines 1–4) into the model by moving them from the mathematical solution stage to the real-world solution stage of the modelling cycle. The question prompts she used were mainly hinting for the presence of other factors from the street map without explicitly listing them out. Unfortunately, Group 8 was unable to make sense of their model by interpreting it further within the real-world constraints, nor did they consider other factors affecting bus travel (e.g. travelling time, traffic conditions, the number of bus stops, and how crowded the bus stops were). Group 8 did not revise their original distance-cost model (Fig. 3.5) even after Teacher-S' intervention (Table 3.2, lines 9–14). They had already made their decision on the most efficient bus route based on the shortest distance travelled and relating it to the cost of travelling.

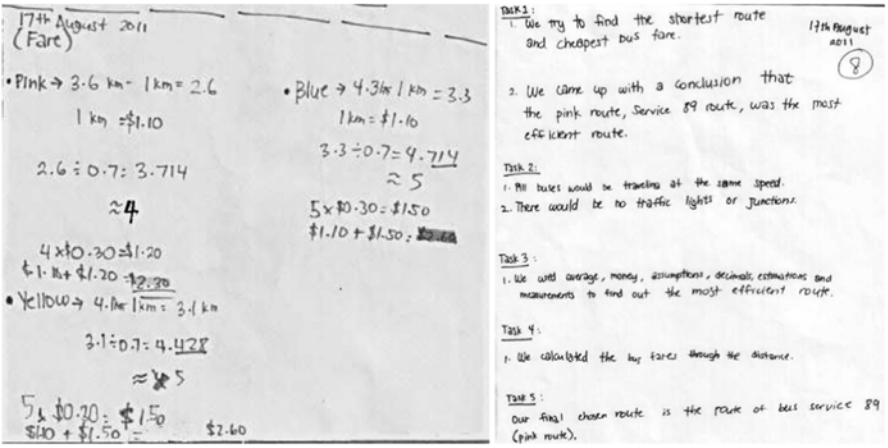


Fig. 3.5 Distance-cost model from Group 8

In contrast, the preliminary model of Group 1 consisted of simple measurements of the different routes given in the street map using strings. They had initially considered working with only one factor when deciding on the most efficient bus route, that is, distance. However, Teacher-S observed that the model they produced was too superficial, and she wanted to move them on (Table 3.4, lines 4–7). Her questioning approach was more specific towards drawing links between the factors of distance and cost. She paused occasionally during her flow of questioning to listen and redirect the mathematical reasoning of the group. The excerpt in Table 3.3 (lines 24–25 and 30) showed that Group 1 was receptive towards Teacher-S' intervention. In their final model, they managed to draw connections between cost and distance travelled for the selection of the most efficient bus route (Fig. 3.6).

The difference in outcomes from Teacher-S' questioning approaches between Groups 8 and 1 could be attributed to whether she achieved the delicate balance of questioning and listening during the facilitation process (Table 3.4, lines 15–17). With group 1, the time allocated to listening to their responses allowed Teacher-S to extend the group's model development process. However, with Group 8, she could be perceived as "interrogating" the students with a series of questions, leaving little time for them to reflect on her queries (Table 3.2, lines 9–14). The video footages discussed here were powerful stimuli to create in Teacher-S the awareness that she needed to strike a balance, between offering questions to the students and taking a step back to pause and listen for students to provide a considered response. Indeed, the interview excerpt below showed that Teacher-S had successfully identified that a balance of questioning and listening was crucial in steering the students towards more sophisticated mathematical reasoning (Table 3.4, lines 13–24). The video was instrumental in creating this grounded image between the teacher and the researcher.

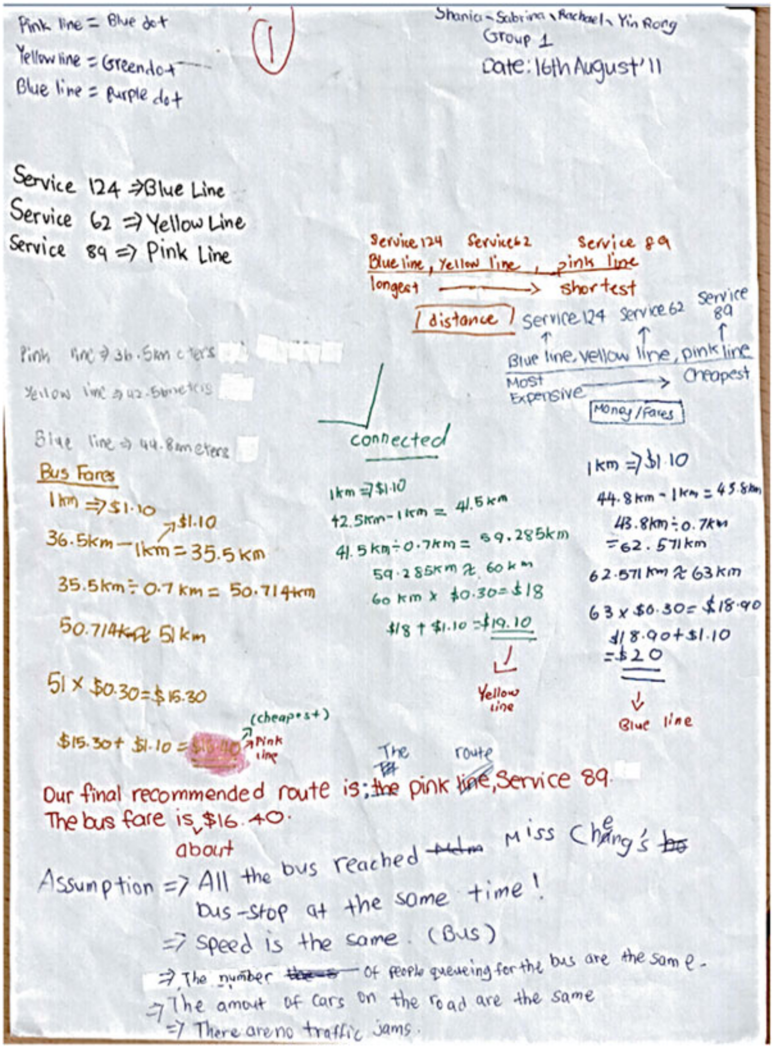


Fig. 3.6 Final model from Group 1

Finding 4: Teacher Competency in Fostering Metacognition in Students

The use of metacognitive strategies can be perceived in two areas in this research: teacher-S' own metacognition and her efforts at activating her students' metacognitive behaviours. Throughout her facilitation of the Bus Route Task, Teacher-S had consciously monitored and regulated her own pedagogical decisions in view of the current progress of each group. At times, to encourage more student-directed inquiry into the task, Teacher-S had deliberately moved away from her routine prescriptive approach:

> Teacher-S: … my natural instinct, my natural self would be to tell, which is what most of the time I do when I teach, which is faster that way. Sometimes they don't quite understand what I'm trying to put across so I usually tell them. So here I was actually trying to get them to tell me, that was my intention. I wanted it to come from them, rather than me actually giving them the answers.

Whilst Teacher-S had consciously kept her "natural self" at bay, she also had in mind to pose certain questions to help students along. Teacher-S seemed to be under the impression that *she* had to pose questions to students in order to avoid "telling" them what to do:

> Teacher-S: …so there must be a kind of a balance. You don't let out so much, but you know you have to kind of help them along, by giving them a bit more information … maybe I selected certain questions which I thought would be helpful. So these questions were at the back of my mind. So I was actually focussing more on certain types of questions.

However, her overemphasis on selected questions coupled with a lack of listening time might have impeded students' progress instead, as shown in Table 3.2. Whilst Teacher-S could identify the need to foster her own metacognition, she had neglected the need to foster students' metacognition. She might not have realised these two sets of needs until she was presented with the two video excerpts of contrasting cases between Groups 8 and 1 that she had to allow for student reflection to occur. Just as she herself had monitored and regulated her own facilitation foci, she did not create platforms for her students to monitor and regulate their mathematical thinking in reaction to her queries during the task. This is an innate sense of flexibility required in Teacher-S' current use of metacognitive strategies. She now has to consciously level herself towards a more macroscopic plane – that of fostering the metacognitive behaviours of others within the fluidity of discourse dynamics, not just microscopically doing so during her own pedagogical decision making.

Discussion and Implications

This chapter aimed to discuss teacher competencies during the facilitation of modelling tasks. It outlined some teacher competencies displayed by the teacher involved in the case study research. These are mainly associated with teacher knowledge of the modelling cycle and teacher intervention. For example, it was observed that the teacher's facilitation prompted the students to move from one stage of the modelling cycle to the next, focusing on certain elements of the modelling process such as formulating and interpreting. However, results also revealed that the same teacher faced challenges in two competencies which were considered crucial in her professional development: (a) striking a balance between questioning and listening and (b) fostering metacognition in students. Instrumental to the identification of these competencies was the use of videos which provided a strong avenue for forming grounding images between the researchers and the teacher.

The use of videos was instrumental towards achieving the two goals of this study mentioned above. Using video as a tool for teacher learning has been widely recognised in teacher education (Alsawaie & Alghazo, 2010) and teacher professional development (Sherin & Han, 2004). Video clips provide authentic and reliable classroom teaching situations as credible evidence that is suitable for promoting reflection of the teachers' own practices (Merseth & Lacey, 1993). The focus from limited research on mathematical modelling in Singapore primary schools has largely been on student modelling outcomes and their capabilities (e.g. see Chan, 2008, 2010). No study to date has harnessed the affordances of video analysis to advance teachers' knowledge and facilitation skills in mathematical modelling at Singapore primary schools. Particularly for (a), the use of contrasting cases in video-stimulated recall was sufficiently powerful to elicit teacher reflection resulting in her conscious move to identify crucial competencies to focus upon for self-development. This sets the methodology in this research apart from others on mathematical modelling at the primary level in Singapore schools. Video is fundamental in assisting the researchers to establish a grounded image of the competencies the teacher displayed in real-time classroom implementation of MEAs because it allows for playback references in the negotiation of meaning towards sense making. Consensus can be formed between the teacher and the researchers who interpreted the teacher competencies observed or the lack of them in some instances. This can help identify specific teacher competencies required by the teacher in a non-prescriptive manner.

The importance of listening during teacher intervention was highlighted by Doerr and English (2006). They proposed that teachers should adopt a pattern of listening-observing-questioning right at the beginning of the facilitation process so as to glean an understanding of students' thinking and note the different ways in which the real-world problem was interpreted and represented. This could be related to what Borromeo Ferri and Blum (2010) proposed as having a balanced teacher intervention where students' independence is preserved during teacher guidance. Indeed, when the teacher appears too forceful in bringing across her questions leaving little time for student deliberations, students may be deterred from justifying their own mathematical thinking which they might not have had the chance to present clearly to the teacher. However, the issue of when and how to seek out this question-and-listen balance towards more enriching modelling experiences for both the teacher and students still remains as many teacher interventions are more intuitive than strategic. In addition, different types of teacher questioning during modelling tasks could lead to varying degrees of success in eliciting students' mathematisation process (Warner et al. 2010). Video footages could be a bank of resources for teacher educators to use for the explication of desirable teacher intervention techniques in relation to question and listen.

Stillman et al. (2010) identified the importance of the use of metacognitive strategies in overcoming blockages of lower intensity during the modelling cycle and emphasised the need for teachers to be sensitive to the blockages faced by students.

The findings in this chapter extend these ideas by suggesting a focus of teacher competency on two fronts during teacher development: addressing teacher's use of metacognitive strategies during facilitation and enhancing student metacognition. Nonetheless, more could still be done with the use of videos to help teachers activate critical moments of learning with respect to teacher meta-metacognition. This is because video footages allow teachers to analyse their teaching approaches and the impact of these approaches, empowering teachers to chart their professional development in incorporating open-ended contextualised activities such as modelling tasks.

Lastly, the development of teacher competencies in mathematical modelling has to exist in the midst of teacher tensions during facilitation. There is a need for a change in mindset towards more teacher-centred facilitation in modelling (Doerr, 2007; Ng 2010). de Oliveira and Barbosa (2010) highlighted several other tensions which may strike a chord with the researchers and teachers in this field: the tension of deciding how to intervene, the tension of student engagement, and the tension of delaying dealing with the wrong mathematics that students use sometimes for the pedagogical purpose. Perhaps, this tension has to be video-recorded to provide positive professional development exemplars for teacher competencies in modelling.

On a final note, this chapter has illustrated the use of grounded images from video recordings as an integral part of teacher professional development during case studies involving teacher reflective practice towards the incorporation of mathematical modelling at Singapore primary schools. However, the use of videos has broader applications and implications on the scalability of teacher education on mathematical modelling or other open-ended contextualised tasks. This is because archived videos can be powerful means of demonstrating good practices within real classroom constraints for representing the perspectives of students, teachers, researchers, and teacher educators, drawing meaningful links between theory and practice.

Appendix

Determining the Most Efficient Bus Route

Ms. Chang recently moved to Block 297C Punggol Road. She is going to start teaching at Punggol Primary School next week and needs to know how to travel to the school. However, the MRT is always too crowded for her to take, and it also requires her to take a feeder bus which results in inconvenience. Ms. Chang realises that there are three bus services that ply different routes to her school. Help her to find the most efficient route to travel by bus from her home to the school. The location of her home is marked in the map. Currently, the three bus services that are available for Ms. Chang to choose are Service 124, Service 62, and Service 89. The routes for Service 124, Service 62, and Service 89 are marked as blue, yellow, and pink lines, respectively, on the map. The bus stops along each bus route are marked with stickers with corresponding colours.

Your task is to give Ms. Chang a proposal consisting of the following:

1. How your group determines what is meant by the "most efficient" bus route
2. *Assumptions* about the problem your group made in order to help Ms. Chang
3. The *mathematics* used to decide which route is the most efficient
4. How your group *justifies* that the selected route is the most efficient
5. The final *recommended* route for Ms. Chang

For us to better understand your work, you can attach the following to your proposal:

(a) A map containing the chosen bus route
(b) The information you found useful for this task

Bus Fares

Distance range	Bus fare (cash)
First 1 km	$1.10
Up to every 0.7 km increase	30 cents increase

Note: The student groups were given a map in which the three different routes for selection were marked using pink, blue, and yellow colours. Bus stops were indicated along the routes using coloured stickers. In Singapore, bus fares are calculated according to the distance travelled.

References

Alsawaie, O. N., & Alghazo, I. M. (2010). The effect of video-based approach on prospective teachers' ability to analyze mathematics teaching. *Journal of Mathematics Teacher Education, 13*(3), 223–241.

Ang, K. C. (2010). Mathematical modelling in the Singapore curriculum: Opportunities and challenges. In A. Araújo, A. Fernandes, A. Azevedo, & J. F. Rodrigues (Eds.), *Educational interfaces between mathematics and industry: Proceedings of the EIMI 2010 conference* (pp. 53–62). New York: Springer.

Balakrishnan, G. (2011). *Mathematical modelling: Insights from Singapore*. Paper presented at the connecting to practice – Teaching practice and the practice of applied mathematicians: The 15th international conference on the teaching of mathematical modelling and applications, Australian Catholic University (St. Patrick), Melbourne, Australia.

Borromeo Ferri, R., & Blum, W. (2010). Insights into teachers' unconscious behaviour in modeling contexts. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: Proceedings of the 13th international conference for the teaching of modelling and applications* (pp. 423–432). New York: Springer.

Chan, C. M. E. (2008). Using model-eliciting activities for primary mathematics classrooms. *The Mathematics Educator, 11*(1/2), 47–66.

Chan, C. M. E. (2010). Tracing primary 6 pupils' model development within the mathematical modelling process. *Journal of Mathematical Modelling and Application, 1*(3), 40–57.

Chan, C. M. E., Ng, K. E. D., Wijaja, W., & Seto, C. (2012). Assessment of primary 5 students' mathematical modelling competencies. *Journal of Science and Mathematics Education in Southeast Asia, 35*(2), 146–178.

Curriculum Planning and Development Division [CPDD]. (2006). *Mathematics syllabus (Primary)*. Singapore: Ministry of Education.

Curriculum Planning and Development Division [CPDD]. (2012). *Mathematical modelling resource kit*. Singapore: Ministry of Education.

de Lange, J. (2006). Mathematical literacy for living from OECD-PISA perspective. *Tsukubu Journal of Educational Study in Mathematics, 25*, 13–35.

de Oliveira, A. M. P., & Barbosa, J. C. (2010). Mathematical modeling and the teachers' tensions. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: Proceedings of the 13th international conference for the teaching of modelling and applications* (pp. 511–517). New York: Springer.

Doerr, H. M. (2007). What knowledge do teachers need for teaching mathematics through applications and modelling? In W. Blum, P. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 69–78). New York: Springer.

Doerr, H., & English, L. D. (2006). Middle grade teachers' learning through students' engagement with modeling tasks. *Journal of Mathematics Teacher Education, 9*(1), 5–32.

Dolk, M., Widjaja, W., Zonneveld, E., & Fauzan, A. (2010). Examining teacher's role in relation to their beliefs and expectations about students' thinking in design research. In R. K. Sembiring, K. Hoogland, & M. Dolk (Eds.), *A decade of PMRI in Indonesia* (pp. 175–187). Bandung: APS International.

English, L. D. (2010). Young children's early modelling with data. *Mathematics Education Research Journal, 22*(2), 24–47.

English, L. D., & Sriraman, B. (2010). Problem solving for the 21st century. In L. D. English & B. Sriraman (Eds.), *Theories of mathematics education: Seeking new frontiers* (pp. 263–285). Berlin: Springer.

Galbraith, P. (2013). *Discussant notes for symposium on mathematical modelling*. Paper presented at the thinking: Time for a rethink? The 5th international redesigning pedagogy conference National Institute of Education, Singapore.

Houston, K. (2007). Assessing the "phases" of mathematical modelling. In W. Blum, P. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 249–256). New York: Springer.

Kaiser, G., Schwarz, B., & Tiedemann, S. (2010). Future teachers' professional knowledge on modeling. In R. Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 433–444). New York: Springer.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM – The International Journal of Mathematics Education, 38*(3), 302–310.

Lesh, R. A., & Doerr, H. M. (2003). Foundations of a models and modeling perspective in mathematics teaching and learning. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 3–34). Mahwah: Lawrence Erlbaum Associates.

Lyle, J. (2003). Stimulated recall: A report on its use in naturalistic research. *British Educational Research Journal, 29*(6), 861–878.

Maaß, K. (2006). What are modelling competencies? *ZDM – The International Journal of Mathematics Education, 38*(2), 113–142.

Merseth, K., & Lacey, C. (1993). Weaving stronger fabric: The pedagogical promise of hypermedia and case methods in teacher education. *Teacher & Teacher Education., 9*(3), 283–299.

Ng, K. E. D. (2010). Initial experiences of primary school teachers with mathematical modelling. In B. Kaur & J. Dindyal (Eds.), *Mathematical modelling and applications: Yearbook of association of mathematics educators* (pp. 129–144). Singapore: World Scientific.

Ng, K. E. D. (2013). Teacher readiness in mathematical modelling: Are there differences between pre-service and experienced teachers? In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Connecting to practice: Teaching practice and the practice of applied mathematicians* (pp. 339–348). Dordrecht: Springer.

Ng, K. E. D., Chan, C. M. E., Widjaja, W., & Seto, C. (2013). Fostering teacher competencies in incorporating mathematical modeling in Singapore primary mathematics classrooms. In M. Inprasitha (Ed.), *Innovations and exemplary practices in mathematics education: Proceedings of the 6th East Asia regional conference on mathematics education (EARCOME 6)* (Vol. 3, pp. 219–228). Phuket, Thailand: Center for Research in Mathematics Education (CRME), Khon Kaen University.

Ng, K. E. D., & Lee, N. H. (2012). *Mathematical modelling: A collection of classroom tasks*. Singapore: Alston Publishing House Private Limited.

Ng, K. E. D., Widjaja, W., Chan, C. M. E., & Seto, C. (2012). Activating teacher critical moments through reflection on mathematical modelling facilitation. . In J. Brown & T. Ikeda (Eds.), The 12th international congress on mathematical education (ICME-12) electronic pre-conference proceedings TSG17: Mathematical applications and modelling in the teaching and learning of mathematics (pp. 3347–3356). Korea/Seoul: ICME12.

Organisation for Economic Co-Operation and Development [OECD]. (2010, November 30). *Draft PISA 2012 mathematics framework*. Retrieved February 2, 2012, from http://www.oecd.org/document/53/0,3746,en_32252351_32236130_46963701_1_1_1_1,00.html

Sherin, M. G., & Han, S. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education, 20*, 163–183.

Stillman, G. (2009, June 5). *Fostering metacognitive competencies during beginning mathematical modelling tasks.* Paper presented at the Seminar for mathematics and mathematics education academic group, National Institute of Education, Singapore.

Stillman, G. (2011). Applying metacognitive knowledge and strategies in applications and modelling tasks at secondary school. In G. Kaiser, G. W. Blum, R. B. Ferri, & G. Stillman (Eds.), *International perspectives on the teaching and learning of mathematical modelling 1: Trends in teaching and learning of mathematical modelling – ICTMA14* (pp. 165–180). Dordrecht: Springer.

Stillman, G., Brown, J., & Galbraith, P. (2010). Identifying challenges within transition phases of mathematical modeling activities at year 9. In R. Lesh, C. R. Haines, P. L. Galbraith, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 385–398). New York: Springer.

Stillman, G. A. (2010). Implementing applications and modelling in secondary school: Issues for teaching and learning. In B. Kaur & J. Dindyal (Eds.), *Mathematical applications and modelling: Yearbook 2010 of the association of mathematics educators* (pp. 300–322). Singapore: World Scientific.

Warner, L. B., Schorr, R. Y., Arias, C. A., & Sanchez, L. (2010). A case study of two teachers: Teacher questions and student explanations. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: Proceedings of the 13th international conference for the teaching of modelling and applications* (pp. 519–529). New York: Springer.

Chapter 4
Developing Critical Reflection Through Audio and Video Technology for Some Singapore Primary School Mathematics Teachers

Lu Pien Cheng

Abstract Audiotaping and videotaping are useful but constantly evolving technological tools. This chapter examined how a critical commentator (CC, the author of this paper) used the affordances of audio and video technology to facilitate reflection amongst seven Singapore primary mathematics teachers. With the aid of a questioning framework, the CC was able to help these teachers improved in the quality of their reflections, moving from level 1, the lowest, to level 3, the highest. Although the participating teachers were initially resistant to the use of video technology, difficulties in recalling exact details of the observed lesson prompted them to embrace video technology. Grounded images were used as catalysts that promote richer reflections. This study showed that informed leadership offered by the head of department could be a way to ensure research continues even when the CC withdraws from the study.

Introduction

Teachers' development is contingent upon their ability to reflect on their own practice (Clarke, 2000; Shulman, 1986). Knowledge of their own practice delineates for them areas for improvement and personal effectiveness (Lederman & Gess-Newsome, 1999; Morine-Dershimer & Kent, 1999; Muir & Beswick, 2007). Teacher educators are motivated to help teachers to be more reflective about their practice (Power, Clarke & Hine, 2002) with the aim of helping teachers become lifelong critically reflective practitioners (Martinez & Mackay, 2002). Dewey (1933, 1981/1933) and Schön (1983, 1987), two pivotal theorists on reflection, saw reflection as a mental progression where ideas are refined through action. Many

L.P. Cheng (✉)
National Institute of Education, Nanyang Technological University, Singapore
e-mail: lupien.cheng@nie.edu.sg

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_4

educational professionals have described reflection as discreet events (Day, 1999; Power et al., 2002). In fact, Beswick (2007) framed teachers' understanding of their reflection into three increasingly sophisticated levels of reflection: technical description, deliberate reflection and critical reflection.

Level 1 Technical Description

A teacher describes general accounts of classroom practice, often with a focus on technical aspects, with no consideration of the value of the experiences. For example, in a lesson on division of fraction by another fraction, where the divisor is smaller than the dividend, such as $1/2 \div 1/8$, a teacher would describe the materials used to teach the concept without any consideration of the consequences on the choice of materials on the learning of the students.

Level 2 Deliberate Reflection

The teacher identifies critical incidents and offers a rationale or explanation for the action or behaviour. The teacher could explain how by superimposing the 1/8 piece of the fraction disc on the 1/2 piece of the fraction discs may help the students see why four 1/8 pieces are needed to fill 1/2 of the fraction discs.

Level 3 Critical Reflection

The teacher moves beyond identifying critical incidents and provides explanations for considering others' perspectives and offers alternatives on how the lesson could be improved. The teacher could suggest that it is important for students to relate the actions of superimposing the 1/8 pieces onto the 1/2 pieces to its written symbolic form such as $1/2 \div 1/8 = 4$ and why the multiplication sign could replace the division sign and the need to invert the denominator.

However, when teachers reflect on their own, without the support from others, there is limit to what they can learn from their reflections and the objectivity of their reflections (Day, 1999; Muir & Beswick, 2007). The help of a 'mentor' or critical friend inside or outside the school can be beneficial in helping teachers systematically investigate the outcomes of the reflective process (Clarke, 1997; Day, 1998, 1999; Muir & Beswick, 2007).

Video has become increasingly popular as an artefact in teacher professional development because of its unique ability to capture the richness and complexity of classrooms for later analysis (Brophy, 2004; Brouwer, 2011; Fadde & Rich, 2010;

Fuller & Manning, 1973; Tochon, 2008; Fadde & Rich, 2010; Brouwer, 2011; Tochon, 2008). Professional development leaders can select video excerpts to address particular features of teaching and learning that *they* want to examine; and the video can be stopped, replayed or otherwise manipulated to focus the teachers' conversations on those features. Used in these ways, video can support collaborative learning focused on reflection, analysis and consideration of alternative pedagogical strategies in the context of a shared common experience (Brophy, 2004; Muir, Beswick, & Williamson, 2010, p. 129). These participants have 'the opportunity to develop a different kind of knowledge for teaching—knowledge not of 'what to do next,' but rather, knowledge of how to interpret and reflect on classroom practices' (Sherin, 2004, p. 14).

Teachers, however, do not necessarily gain new insights about their practice from watching classroom videos (Brophy, 2004). To be an effective tool for teacher learning, a video must be viewed with a clear purpose in mind. When used within a professional development programme, clips should be purposefully selected to address specific programme goals and be embedded within activities that are carefully planned to scaffold teachers' progress towards those goals (Brophy, 2004; Seidel et al., 2005). When used judiciously videos become a resource that can enhance learning (LeFevre, 2004) [cited in Borko, Jacobs, Eiteljorg & Pittman, 2008 p. 418–419].

In summary, teachers are more likely to grow in their profession when they are more reflective about their practice. Such growth is contingent upon on appropriate mentoring offered by critical others. Teachers and professional teachers have found videos to be useful resources as they could view and review selected episodes, the choice of which were more often than not determined by professional developers.

In the TIMSS Video Study (Stigler & Hiebert, 1999), the teachers featured therein were willing partners of the study and were not related to whoever chooses to view the videos. Teachers also could video their own lesson and use these lessons as aids for reflection. Being featured in the video and to allow others to observe the recorded lessons may not be an issue to these teachers. However, the problem arises when teachers are not comfortable to be videoed *and* for others to use their recorded lessons as objects of discussion. The research in this chapter documents such a situation. A group of seven teachers in the upper primary grades wanted to improve the effectiveness of their teaching by learning from their peers. They approached the author of this paper, herein identified as the critical commentator (CC) for support. Although these teachers knew that the affordances of videoed lessons (such as availability of the resource on demand, selecting episodes for discussion) were invaluable for their learning, nevertheless, they were 'self-conscious about being videotaped' (Sherin & Han, 2004, p. 166).

This chapter discusses how the CC customised a professional development course which simultaneously addressed the professional needs of these teachers and their sense of discomfort. The course made eclectic use of audio recording of lessons, students' work, photos of students' work and videos in the evaluation of their work.

Table 4.1 The framework of nine questions used to facilitate reflection

Questioning examples	Questioning foci
What did the teacher do?	Recall the pedagogical movements
	Understand the pedagogical movements in the research lesson
	Examine the difference between the research lesson and the planned lesson
Why did the teacher do that?	Understand the rationale behind the pedagogical movements
	Use what the teacher knows about the context and theories to explain and justify the movements
How did the teacher do that?	Understand how learning outcomes were achieved
	Understand how students' errors and misconceptions were addressed
What did the teacher not do?	Understand why the teacher did not do what was planned
What the teacher should not do?	Examine further the teacher's pedagogical movements
	What should not have been done during the lesson?
	What evidence did the teacher have that students did not achieve the learning goals? (adapted from Santagata, 2011, p. 154)
What the teacher could have done?	Critical reflection and extend beyond the lesson
	What alternative strategies could the teacher use?
	How did the teacher anticipate these strategies would impact learning outcomes?
What could the teacher have done differently?	Critical reflection and extend beyond the lesson
	What alternative teaching strategies could the teacher use for a different class profile, for infusing curriculum initiatives and reforms?
What would I do?	Accommodate and assimilate any new understanding formed
	Internalise new understanding by thinking about what and how they would approach the situations that arose
	Operationalise the reflections
What were the students' responses?	Assess the students' learning
	Find evidence of students' learning and misconceptions
	Understand students' thinking

The CC discussed how she constructed and used a framework of nine questions in Table 4.1 to guide her analysis of data. The aims of the study were:

(1) To examine how the use of audio technology and the framework assisted the CC to plan for the reflection of a mathematics lesson that encouraged reflective practice amongst the teachers
(2) To examine how the use of digital technology encourages reflective practice amongst the teachers

Figure 4.1 shows how the CC used audio technology and digital images as a tool for professional development in the research lesson.

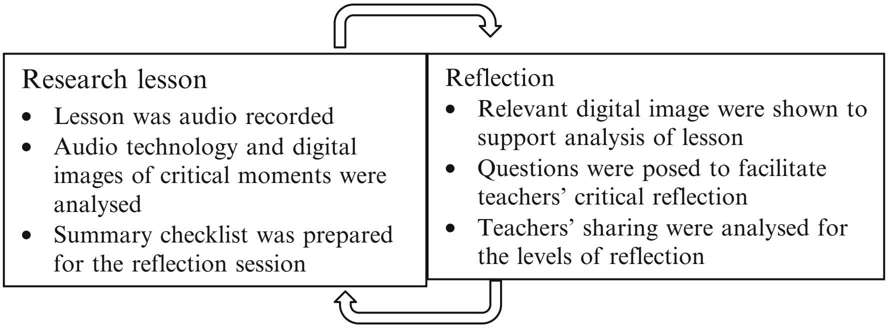


Fig. 4.1 Audio technology and digital images as a tool for the CC

The Study

Setting and Participants

In Singapore, some schools practised flexible scheduling of curriculum time such that teachers could use this 'time-tabled time' for professional development purposes. The school administrator of the school for this research project nominated a group of seven upper primary mathematics teachers from diverse backgrounds, ethnicities and with varying years of teaching experience to participate in a professional development course. Its objective was for the teachers to discuss, more systematically, problems they encountered in teaching primary school mathematics. I was approached by these teachers to help them address pedagogical issues related to the teaching of these two topics: (1) the division of a proper fraction by a proper fraction, a concept taught at Primary 6, and (2) conversion of units used in measurement, a topic taught at Primary 5. The teachers were apprehensive about teaching division of a fraction by a fraction as it is a new topic introduced in the 2007 syllabus. For the unit on conversion, the students, when asked to convert 1 *cm* to 1 *m*, would apply the procedural strategy of moving the decimal point two places to the left. The teachers were unsure how they could help the students understand the reason for this strategy. The course comprised two cycles. The first cycle focused on the teaching of fraction by teacher John. This lesson is identified as the fraction lesson. The teaching of the conversion lesson by the CC was the focus of the second cycle. Each cycle consisted of six 1 h sessions. Four sessions were spent on planning the lesson, one session was spent on teaching the lesson, and in the last session the teachers recollected specific moments of learning and teaching, and they critiqued what they had observed of the research lesson. The fraction lesson was audiotaped and conversion lesson videotaped. The recordings from all sessions were transcribed for further analysis. The reflection session for the fraction lesson was about 53 min and 38 min for the conversion lesson. Table 4.2 shows the structure of the two teaching cycles.

Table 4.2 Structure of research cycle

Meeting	Purpose	Data collected	Duration
Cycle 1			
1–4	Teachers and the CC *planned* the fraction lesson	Lesson plan	4 h
		Audio recording	
5	Teachers and CC observed lesson taught by John	Field notes	1 h
		Students' work	
		Photographs	
		Audio recording	
	To prepare for the reflection session, CC used the framework to analyse artefacts from John's lesson		
6	Reflection session with teachers	Audio recording	1 h (53 min)
	The CC used the framework to guide teachers' reflections and to try to improve the quality of their reflections		
Cycle 2			
7–10	Teachers and the CC *planned* the conversion lesson	Lesson plan	4 h
		Audio recording	
11	Teachers observed CC teach the conversion lesson	Video recording	1 h
		Students' work	
12	Reflection	Audio recording	1 h (38 min)

The objective of cycle 1 was to collect data which the CC then used, with the aid of the framework, to select critical moments to facilitate the teachers' reflections conducted in the 6th meeting. In cycle 2, the teachers were given the opportunity to apply the skills of reflection acquired from cycle 1 to the conversion lesson. The professional development course adopted the same structure of investigation (planning, research lesson and reflection) as the lesson study cycle (Lewis, Perry & Hurd, 2009) except that the cycles were not repeated.

Planning Stage In both cycles, the teachers and the CC worked together to plan the lessons. Figures 4.2 and 4.3 show the lesson ideas developed by the teachers during the planning stage for cycles 1 and 2 respectively.

Observation Stage In both cycles, the teachers were assigned to observe the learning activities of specific groups of students. They took photographs of students' work which they felt reflected students' learning and thinking. They were also encouraged to write down any incidents that captured their interest. The teachers could use these artefacts in the reflection sessions.

Preparing for the First Reflection Session After the fraction lesson, I listened repeatedly to the audiotaped lesson and used the nine questions to select appropriate critical moments to help teachers improve the quality of their reflections in the first cycle.

Fraction Lesson

- Ask students "How many one-eighths are there in half?"
- Teacher demonstrates $\frac{1}{2} \div \frac{1}{8}$ using either manipulatives or model diagrams
- Introduce a possible story sum for $\frac{1}{2} \div \frac{1}{8}$. "John had ½ of a pizza. He cut it into a number of pieces. Each piece was $\frac{1}{8}$ of the whole pizza. How many pieces did John cut into?" Teacher explains why the story sum is appropriate for $\frac{1}{2} \div \frac{1}{8}$

Problem Posing Activity

- Students work in small groups to tackle either activity 1 or 2 (Teacher to group students according to their abilities)

Activity 1:

Students create 2 story sums for $\frac{2}{3} \div \frac{1}{6}$ using different context and/or different mathematics topics (e.g. whole numbers, mass, measures / mensuration). Explain why your story sum is appropriate for $\frac{2}{3} \div \frac{1}{6}$.

Activity 2:

Create a story sum for fraction divided by fraction. Choose your own fractions. Explain why and how you choose the two fractions. Explain why your story sum is appropriate.

- Whole class discussion: Selected groups are asked to present their story sums for activity 1 followed by activity 2. During the presentations, students are asked to explain why their story sums are appropriate

Fig. 4.2 Fraction division lesson idea developed by the teachers

Conversion Lesson

- Benchmarking using the height of one of the students
- Convert the height from cm to m
- Predict the length of the items before measuring and recording their answers on the worksheet provided
- Whole class discussion on reasonableness of measurement and accuracy of the conversion from *cm* to *m*

Fig. 4.3 Conversion lesson idea developed by the teachers

Reflection Stage Because of the tight teaching schedule, the reflection sessions were held one week after the taught lesson. Teachers' reflections were recorded and transcribed.

The First Reflection Session Based on the Fraction Lesson There was no prescribed structure to the reflection session. The teachers were free to choose any critical moments or issues they wish to discuss. I used the framework of nine questions to facilitate the teachers' reflections. These questions or their variations were posed to trigger teachers' in-depth reflection of the research lesson, to help teachers construct knowledge about a concept with which they may be unfamiliar or to deepen knowledge of a construct of which they may have vague notions. This was necessary because the teachers were not used to sharing their reflections with their peers. Teachers could use any relevant digital images to support their reflections, to clarify their intentions, to discuss a teachable moment or critical incident, to elicit further information about the teaching practices observed and the reasons behind them and to challenge the teachers to suggest alternatives to improve the lesson. The digital images were important resources which teachers could use to support their reflections on the fraction lesson. I had no control over these exchanges. Instead I had to be flexible and recognise suitable moments which could be used to further teachers' reflections.

The Second Reflection Session Based on the Conversion Lesson I assumed the role of a researcher during the second reflection lesson. Data from this session would enable me to understand how video technological tool supported the teachers in their reflections. Emily, the head of department, was the facilitator of the reflection session. The teachers had access to these artefacts: the videotape, samples of students' work and photographs of the conversion lesson before and during the reflection session. The second reflection session served as a platform for the teachers to practice and apply the skills of 'critical reflection' acquired from cycle 1. The second reflection session was also intended to prepare the teachers to engage in critical reflection of mathematics lessons on their own when they conducted their own research independent of external critical commentators.

Analysis

I analysed the proceedings of each reflection session to identify those critical moments identified by the teachers. These were then compared and contrasted against the three levels of reflection: (1) level 1 (L1), technical description; (2) level 2 (L2), deliberate reflection; and (3) level 3 (L3), critical reflection. I then examined which aspect of the questioning framework may have caused the reflections or enhanced their levels of engagement. Finally the findings from each session were compared and contrasted against each other to identify whether there was a change in the quality of the reflections.

The First Reflection

Because there were no video images, the teachers and I used the photographs of students' work to facilitate our discussion. The data showed that the teachers reflected at level 1 when they were required to engage with mathematical concepts with which they were unfamiliar, they had difficulties recollecting details of the lesson or they were examining strategies with which they were very familiar:

- Engaging with unfamiliar mathematical concepts (23:13 to 52:00). For example, when I asked to pose a word problem for $1/4 \div 1/2$, they struggled with the activity. Rina said, 'I could not visualise $1/4 \div 1/2$'. The rest of the teachers also found it challenging to reflect on word problems involving division by fractions, particularly when the divisor is bigger than the dividend.
- Difficulties recollecting details of the lesson. For example, Winnie had to recall from her memory when she said in one instance, 'From what I remember…'. John taught the first research lesson, and he was reflecting mostly at level 1 in the first critique. John said he could not remember in two occasions when he was asked to explain what he did and why he did certain things during the research lesson (15:23–19:00). The teachers could only reflect on the technical details because these incidents were not captured in their field notes nor in students' work or in photographs.
- Examined strategies with which they were very familiar (21:55 to 23:12). For example, when I shared my observation about the use of helping words in the first problem-posing activity, John said, 'those were the helping words'. The rest of the teachers showed no interest in discussing the strategy of using helping words to facilitate the problem-posing activity because it was already a common and useful strategy adopted by them.

Further analysis of the data showed that unplanned action in the research lesson afforded many opportunities for reflective practice amongst the teachers. I found the questioning framework a useful tool to support the teachers' progress in their reflections. The following critical incidents illustrate how the use of audio technology and the questioning framework helped teachers improved the quality of their reflections.

Critical Incident 1: Analysis of Students' Errors Seemed to Promote Deeper Reflections by Some Teachers

Although the teachers worked as a group to plan the fraction lesson, John, the teacher, did not follow closely the scripted lesson. Instead of re-presenting $(1/2 \div 1/8)$ as $(1/2 \times 8/1)$, the student, Sean, had written $(2/1 \times 1/8)$. However, John did not tell the student he was wrong. Instead with John's guidance, the student concluded that his answer was wrong: 'I shouldn't have converted the first one' (referring to the dividend). Emily noticed this particular *unexpected moment* and brought it to the group's attention. This moment was unexpected because in the planning stage the teachers did not plan to address situations where students provided erroneous solutions. In order to help the student understand the error in his solution, John had ventured away from the planned lesson. However, because no

Table 4.3 Reflecting on the error $1/2 \div 1/8$ as $(2/1 \times 1/8)$

Speaker	Line	Content
Emily:	1	Initially Sean computed it incorrectly. Instead of ½ times 8 over one, he
	2	puts as 2 over 1 × 1/8. Later as you [John] guide him, Sean realise, 'I
	3	shouldn't have converted the first one'.
CC:	4	I remember John did not tell Sean he was wrong. John did something and
	5	Sean realise for himself that he should not invert the first fraction.
	6	Do you remember what John did?
John:	7	I asked the rest of the class what their answers were. [L1]
Nina:	8	John asked for more answers and wrote the answers, 4 options, on the
	9	board. Then Sean realised he made a mistake. [L2]
CC:	10	What do you think about this approach?
	11	Not telling Sean the correct answer, instead elicit more responses from the
	12	class and put the 4 options on the board for discussion?
Emily:	13	Very often we will say. Nope wrong. It is good that we delay the answer. [L2]
CC:	14	The focus of this lesson was not to correct the students' algorithm, so I am
	15	very curious to see how John helped the children overcome this error and
	16	what makes John pursue in this direction because this was not in our lesson
	17	plan.
John:	18	It was that moment …. It just happened. I didn't plan for this either. I
	19	wasn't expecting this answer. [L1]
CC:	20	Think about the approach (directing the question to the group).
	21	Would you do what John had done if students come up with the wrong
	22	answer in computation?
John:	23	It depends on the situation and the topic…especially those pupils who
	24	consistently got it wrong …. [L2]

responses were forthcoming from the team, I *prompted the teachers to recall what John* did and *what guided John's decision* to help Sean identify and rectify his own errors.

Table 4.3 shows an excerpt of the reflection. At the beginning of the excerpt, Emily seemed to recognise the value of these experiences but did not elaborate why [lines 1–3]. I tried to generate more discussion by *sharing one observation* [line 4] which the teachers probably had forgotten or felt unimportant. By *rephrasing* Emily's descriptions of John's pedagogical movements [line 5], and prompting the teachers to *recall* what John did during the fraction lesson [line 6], John was able to recall what he did describing mainly the incident [line 7] but he was still reflecting at level 1. Nina built on John's recollection [lines 8–9] and showed the photograph in Fig. 4.4 to support her reflection.

I tried to move the teachers to level 2 reflection by having them *examine* the rationale for not 'correcting' the students' erroneous responses immediately. I wanted the teachers to investigate why the students were given the time and opportunities to think, reflect, discuss and correct their own errors [line 10].

Fig. 4.4 Photograph of board work during the first research lesson

Table 4.4 Addressing the errors for $1/2 \div 1/8$

Speaker	Line	Content
CC:	1	Other observations?
Emily:	2	I look here for $1/2 \div 1/8$. Shauna says at first how many eights make 1/2 [paraphrase].
	3	I think she means 1/2 divided by 8. That's the wrong concept, 1/2 divided by 8
	4	parts …. You [John] showed the fraction manipulatives [to help Shauna sees
	5	that he is wrong]. [L2]
CC:	6	By listening to the student's misconception and picking this up
	7	for discussion, in this case with manipulatives to show how many 1/8 are
	8	there in 1/2, might be helpful to rectify some of these misconceptions.
Emily:	9	How about next time, when children give this type of responses again, we
	10	write down on the board 1/2 divided by 8 and ask them 'what is the
	11	difference between 1/2 divided by 8 and 1/2 divided by 1/8'. [L3]

I then *summarised* what had been discussed so far [lines 11–12]. Emily then reflected and recognised the value of delaying 'giving the correct answer instantly to students' [line 13]. With appropriate prompts, the teachers were encouraged to examine how John helped the students overcame those errors [lines 14–15], why John deviated from the planned lesson and the theories that guided John's actions [line 16]. Despite the prompting, John continued to reflect at level 1 [lines 18–19]. In a further attempt to move the teachers' reflections on the same issue to level 2, I posed another question from the framework *Would you do what John had done?* [lines 21–22]. With this question, John was more willing to share more deeply about his actions [line 23]. The question in lines 21–22 appeared to help the teachers *translate* the whole process of reflection [lines 1–24] into actions that the teachers may possibly use in their mathematics classrooms. This drew the reflections closer to the teachers' personal experiences in the mathematics classroom and made the reflections more relevant to the teachers' practice.

With further prompts Emily continued to share her observations of the *unexpected moment* [Table 4.4, line 1]. She noticed and then offered an explanation for another student, Shauna's misconception [line 3]. She was reflecting at L2 then. I then shared what John did with the fraction circles to help Shauna overcome this difficulty [Table 4.4, lines 6–8]. Besides posing questions to encourage the teachers' reflection, I contributed to the reflections and provided suggestions to rectify students' misconceptions. My contribution and suggestion may have prompted Emily to think more deeply about helping students learn from their misconceptions. Emily was seen to move to L3 reflection in Table 4.4, lines 9–11.

The above excerpts show that teachers could be encouraged to move to another higher level of reflection if they were given appropriate prompts to focus on specific errors made by students. Knowledge of what prompts and questions to use is vital for the richness of the discussion and quality of reflections. Without the use of video recordings, the facilitators of the reflection session need to be persistent and patient in helping the teachers recollect the taught lesson and use appropriate photographs and field notes to support them in their reflections.

Critical Incident 2: Analysis of Students' Responses in the Problem-Posing Activity Promoted Deeper Reflections by Some Teachers

In the problem-posing activity, students were asked to create a story sum (Fig. 4.2). The teachers were assigned to observe pairs of students during the lesson. I facilitated the analysis of students' responses in the problem-posing activity. During the reflection, teachers were invited to *share, in turn,* their observations of the students' responses during the problem-posing activity [Table 4.5, lines 16, 24 and 33]. In the sharing, I posed only one question [Table 4.5, line 11] from the framework to facilitate teachers' reflections. The teachers mostly volunteered their observations. Ann and Winnie chose to focus on how students engaged with the problem-posing task. The discussion offered by these two teachers suggested that they were reflecting at level 2 [lines 1–10]. Ann noticed that the students wrote down the fraction division statement and its solution first before constructing the story sum [lines 1–4]. This was in contrast with Winnie's who noticed that the students did the reverse [lines 5–10]. They constructed the story sum before constructing the fraction division statement and its solution. Winnie reflected that some story sums required a whole number solution and the students investigated this conjecture and manipulated the dividend and divisor.

When challenged to recall how the students decided on the fractions during the problem-posing activity (*What were the students' responses?*) [line 11], Winnie moved to level 3 when she explained that the students used trial and error to figure out the fractions to use for the solution to be a whole number [lines 13–15]. The CC supported Winnie's reflection by showing examples of how the students played with numbers in Fig. 4.5.

Rina was very specific in her reflection. She began her reflection at L1 by describing her observations. She was at L2 in lines 40–43 when she noticed the important role of the word one whole and the importance of units. Rina used the example in Fig. 4.6 to support her observation. Her conversation changed from the use of region to measurement model in fractions, and she started to reflect more deeply about

Table 4.5 Reflection on how students choose the fractions for problem-posing activity

Speaker	Line	Content
Ann:	1	My 2 pupils were given the 2nd task; they think of 2 numbers and
	2	divide … do. I get a whole number or fractions. Because whole numbers
	3	can represent some objects … then they think about the context to make it
	4	realistic. [L2]
Winnie:	5	They crafted the question first and then the algorithms; then they realise the
	6	answers do not work out to a whole number and one of the students panic
	7	He was like … how to have 1/2 a person; then they paused and they wonder
	8	how to change and which one to change? It took them a bit of time and
	9	somehow they managed to work it out again and they erased the
	10	wrong …. [L2]
CC:	11	Do you remember how the 2 students work it out?
Winnie	12	From what I remember A was asking B which one to change. Then A says
	13	anyone. He also does not know. Then they just plug any other fractions
	14	and try again until they get a whole number answer. It was like trial and
	15	error. That's their way of learning. [L3]
CC:	16	How about Nina?
Nina:	17	C and D were quite systematic. Also it just happened that the fractions
	18	they chose work out to give a whole number answer. What happened was
	19	C wrote the question and B worked out the answer. Then C wrote another
	20	question and B worked out the answer. There was no discussion. It was
	21	like I ask the question, and if something is wrong, then we think about
	22	what to do next. It just happened that what they chose came out to be
	23	whole numbers. [L2]
CC:	24	How about Emily?
Emily:	25	E and F, the first time was no problem, but they had problem with the
	26	second problem-posing activity because they chose very big fractions like
	27	4/15 and $1/30\left[4/15 \div 1/30\right]$. Got the algorithm correct. Then don't know why
	28	they drew a model. When they drew a model, they drew 4/15. Instead of
	29	dividing by 1/30… instead of dividing the whole into 30 parts, they divided
	30	the 4/15 into 30 parts and they got into trouble and [they] don't know what
	31	to do and they gave up and they changed the fractions. After the lesson I
	32	went and tell them what was wrong with their diagram [paraphrased] [L1]
CC	33	What about Rina?
Rina	34	For the first question they were very specific with their language … [L1]
	35	they take this amount cut into … the second fraction is specifically stated
	36	that it was [1/6] of the whole. So no errors there. In the second problem-
	37	posing activity, they started originally with dividing the present. Then
	38	they worked out and realised [that a] present cannot [be] cut into so many
	39	pieces. So they changed into a ribbon because a ribbon can [be] cut.
	40	But the ribbon they didn't specify how long. So I don't know that will
	41	affect. They just say given 1/2 a ribbon, cut this ribbon into 1/10 of the
	42	whole ribbon. So my question was 'what was 1/2 of a ribbon'? What was
	43	the whole ribbon? [L2]
	44	If I was just dealing with fractions without the units of measurement ….
	45	Still can get the answer [L3].

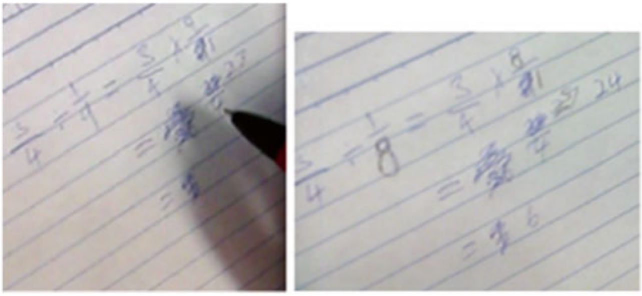


Fig. 4.5 Students' trial and error in finding whole number solutions

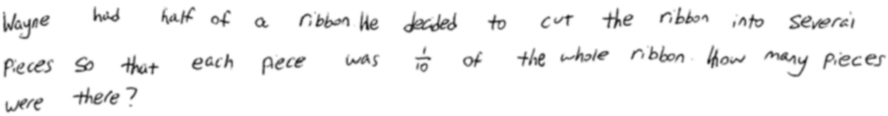


Fig. 4.6 Sample of students' work from problem-posing activity

measurement concepts in fractions. Rina was at L3 when she concluded that the units may not play a major role in finding the solution to measurement the fraction division problem [lines 44–45].

The above examples illustrated how the teachers were identifying, analysing and understanding the thinking processes of the students. Teachers used samples of students' work and photographs of their work to support their reflections. Although only one question from the framework (*Do you remember how the 2 students work it out?*) was used to facilitate the process of reflection, the teachers were able to reflect at levels 2 and 3. The teachers were more willing to share during critical incident 2 because they were not asked to reflect on a fellow colleague's teaching. In critical incident 2, they were more forthcoming because they were reflecting on students' work, not on their colleagues' teaching. Critical incident 2 suggests that reflections that focused on students promoted deeper reflections for some teachers as compared to reflections that focused on the teachers' pedagogical movements. Critical incident 2 also suggests that besides knowing what questions to scaffold teachers' reflections, a deliberate effort to invite *all the teachers to share* their observations and reflections in a nonthreatening environment is vital in assisting the teachers to progress in their reflections.

The Need for Grounded Image

The teachers sensed that the students struggled more with problem-posing activity 2 as compared to problem-posing activity 1 (see Table 4.5, lines 34–36). Their reflections focused mainly on problem-posing activity 2. I observed this lack of

Table 4.6 Teacher John could not recall his teaching action

Speaker	Line	Content
CC:	1	You were saying in the first activity they were very specific in the question
	2	and the second activity they had more problems because they had to
	3	create fractions and a story sum along with it. It struck me that at one point
	4	before moving onto the second activity, John was deep in thoughts walking
	5	around and thinking about something. Do you remember what John
	6	did? I think it was about 1/2 and 1/8. Where do the 2 fractions come
	7	from? Did 1/8 come from 1/2 or did 1/8 comes from the whole? I was
	8	wondering if you notice that some of the children were having
	9	difficulties when the children try to show how many 1/8 in 1/2 [first part
	10	of the lesson, critical incident 2]. I thought John deliberately used the
	11	manipulatives. Did I interpret you correctly?
John:	12	I cannot remember after 1 week.

discussion on problem-posing activity 1. To understand the lack of discussion on problem-posing activity 1, I recounted a specific teaching action to John [Table 4.6, lines 3–5]. Unfortunately, John was unable to recall anything [line 12] about the incident.

None of the teachers pursued this matter. Emily explained that she was unaware of this particular learning difficulty. Furthermore because she had only vague recollections of the incident, she did not think deeply into John's actions. Emily felt that that particular concept of fraction brought up by the CC was important, and thus, it was important for teachers to develop deeper understanding of the concept of fraction division. However, the group was unable to pursue the incident because they had no grounded images to help them recollect the incident. Without objective images of the incident, the groups were unable to engage in any reflection. Although photographs were available and were used to help the group recall 'what John' did, the photographs were insufficient to examine more deeply into why the actions were performed. Emily realised that without grounded images provided by videos, the quality of teachers' reflections was limited and hence compromising the efficacy of the professional development of the teachers. Hence she was open to the idea of videotaping the conversion lesson for the next reflection cycle. However, she felt that the teachers were still not ready nor comfortable to be videoed. After much deliberation, the group welcomed my offer to be videotaped while they were conducting the conversion lesson.

The Second Reflection

The Conversion Lesson: Taught by the CC and Facilitated by the Head of Department

Emily facilitated the second reflection session. The teachers had in their possession the video-recorded lesson, but they were not required to view the recorded lesson. During the reflection session the teachers used memory-stimulated recall to reflect on a number of key teaching points, and these are summarised in Table 4.7. The analysis showed that while most teachers were reflecting at levels 1, 2 and 3, they were mainly at L2 level.

Missed Opportunity to Investigate Acceptable Mathematical Language

The discussion in Table 4.8 showed how the teachers identified a very important conceptual matter related to decimals but failed to see that the students' inappropriate use of the mathematical language reflected their lack of knowledge of place value. If the students understood place value, 1.60 should be read as one point six, not one point 60. This was because the digit six is in the tenths position, not the tens position. In Singapore, with technology widely available, it is not uncommon to hear information presented in decimals without giving due attention to the position of the digits in a rational number. The teachers observed a phenomenon, described the phenomenon, identified a possible reason for the phenomenon and left it at that. They did not explain why it was not appropriate to read decimals without attending to the place value of the respective digits. Emily, who facilitated this reflection session, did not challenge the teachers to question whether this verbal representation of decimals was acceptable and if it was not, what pedagogical actions were needed to address this misconception. The teachers did not use the videotaped materials to further their discussion. If they did, they could have paused the video and

Table 4.7 Summary of critical moments and levels of reflections identified for conversion lesson

Audio segment of reflection	Conversion lesson	Level of reflection identified
	Key ideas discussed during reflection session	
0000–0910	Introduction: guess my height	L1, L2, L3
0910–1650	Measurement activity	L2
1650–2033	Conversion from smaller to larger unit	L1, L2
2033–3129	View video (selected segment) of research lesson on conversion of units	
3129–3800	Measuring objects more than 1 m long	L3 (only Emily)

Table 4.8 Reading decimals

Speaker	Line	Content
Emily:	1	Let's recap on the introduction of the lesson. The class had to guess Han
	2	Ching's height using Peter's height as a reference point [L1]
John:	3	Joseph said one point 60 m. He didn't read it as one point six [L1]
Ann:	4	He didn't read the decimal correctly. After the decimal point, he should be
	5	reading out all the digits one by one. [L1]
Winnie:	6	Is it the American way of reading? [L2]
John:	7	Possible. A lot of children even my Primary 6 students read like that.
	8	But [for this lesson, they seem to know the concept, so it does not matter
	9	how they read it]. [That's because] a lot of them are familiar with their
	10	height so they were able to guess the height correctly. [L2]

emphasised the concept of place value by pointing out that six was in the tenths position and not the tens position. Although the teachers had possession of the video, they did not see it as a valuable resource to support them in their professional development. Perhaps first, teachers need to learn to value the use of videos, and then perhaps they would be self-motivated to want to learn how to use videos for their professional development. Some parts of the transcript from the reflection are presented in Table 4.8 to illustrate the levels of reflection identified in the reflection before the teachers decided to use the videos to support their reflections.

The Use of Video for Reflection

The teachers recognised that powerful mathematical ideas were underpinning the concepts of decimals, and they wanted to learn how to unpack those mathematical ideas by examining the pedagogy employed in the teaching of conversion. The teachers also wanted to examine more deeply the students' responses to the scaffolding questions for conversion of units in measurement. However, during the reflection, the teachers were unable to recall specific details of the lesson which depicted how scaffolding questions were used to unpack and connect mathematical concepts (segments 16:50–20:33 of reflection). There were also mixed views about whether all of the students were able to connect fractions to measurement conversion. These two reasons triggered the teachers to turn to the video recording of the lesson. The teachers viewed 20:33–31:29 segments of the video to investigate those questions because they believed that the video would stimulate their recall and 'provide an unbiased account of their teaching for other teachers to examine' (Zhang, Lundeberg, Koehler & Eberhardt, 2011, p. 459). That is, the video would provide more objective and reliable images to describe the taught lesson as compared to their memory-based images. In the next section, I discuss how the use of video technology supports the teachers' reflection.

Ideas Unpacked Through the Use of Video Technology: The Case of Emily

Emily was observed to be the only teacher in the group sharing her reflections using the video replay. The rest of the teachers appeared to be unable to discuss what they have watched from the video. One reason could be the rich mathematical ideas embedded in the conversion lesson. Another reason could be the large segment of the video being replayed 'all at once'. The teachers needed more time to *chew on* the video replay. Winnie explained:

> some parts of the [research] lesson were "*very dense*" and I thought that this lesson was difficult … The CC was the one who taught the lesson. There was [a lot of concepts to help the students to connect] and I was thinking how to phrase it such that the students can understand. That's the difficult part for me [if I were to] teach the lesson.

Although Emily was the facilitator, analysis showed she was using the video replay for her own reflections rather than facilitating the teachers' reflections. Her reflections were at level 3. Table 4.9 illustrates how Emily, after viewing the video, was able to unpack for herself the important concepts related to the teaching of conversion of units. She was able to re-examine the students' difficulty in *converting smaller to larger unit* and suggest ways to help students overcome this learning difficulty. She (1) drew personal inferences from the video by considering learning from her students' perspectives [lines 1–3]; (2) realised the danger of overgeneralising students' learning [lines 4–5]; (3) offered alternative suggestions to improve the teaching, e.g. connecting current concept (conversion) to prerequisites (fractions) [lines 6–8]; and (4) extended current concept to later mathematics topics [lines 9–11].

Table 4.10 [lines 1–14] shows how Emily used the video replay to help her recall and discuss more deeply how the students build connections between mathematical concepts. For example, in lines 1–4, Emily pointed out that developing students' sense of one metre as the benchmark helped students view measurement as iteration

Table 4.9 Emily unpacked the measurement lesson after viewing the video

Speaker	Line	Content
Emily:	1	For my weak pupils I am just afraid some of them may not know ¾ is 75
	2	over 100. They may not be able to visualise this…or cutting the ruler
	3	into 4 equal parts … and that 73 is roughly 3 parts.
	4	We cannot take for granted that the weak pupils are able to see this. For
	5	the good pupils, they should be able to see this.
	6	It is important for us to emphasise [and recap concepts of fractions using
	7	denominators 100] to the pupils [concepts of fractions] 73 out of 100 parts
	8	and that is why we divide by 100.
	9	So later when we are teaching conversion km to m and m to km, [it is
	10	easier for them to understand and visualise] why we divide by 1,000 [when
	11	we convert m to km] because it is so many out of 1,000 parts. [L3]

Table 4.10 Emily reflected and facilitated the reflection after viewing the video

Speaker	Line	Content
Emily:	1	Somebody pointed out to use a 2 m ruler [for measurements between 1 *m*
	2	and 2 *m*]. [The video] shows that you can spilt the height to 1 ruler plus 22
	3	*cm*. Those [students] who can catch this part will be able to see that it is
	4	1.22.
	5	But I am just worried for my weak pupils because this [was done very
	6	quickly due to time constraint during the research lesson], and the
	7	children [in the video were supposed to explain the difference between
	8	1.20 *m* and 1.02 *m*].
	9	The Primary 5 teachers should follow this up with their classes. Hopefully
	10	the students can [explain the difference by using their prerequisite in
	11	Primary 3 to spilt the readings into 1 *m* 2 *cm* and 1 *m* 20 *cm*.
	12	Is there anything that we can improve in this lesson? For me I would like
	13	to try this lesson out with them again. Because they are a weak class. [I want
	14	to see how the lesson works with LA pupils]. [L3]

of 1 m, that is, $122\,cm = 1\,m + 22\,cm$. The remaining length $22\,cm$ is less than a metre, hence a fraction of a metre, 22/100, which is 0.22. This realisation of the importance of helping students build connections across mathematical topics by tapping on students' prerequisite knowledge spurred Emily to reflect more deeply for the low-ability students in her mathematics class [lines 5–8]. The dialogue also showed Emily giving instructions to the Primary 5 teachers to follow up on the problem that she identified from the video. In line 9, Emily told the teachers what to do rather than providing the teachers the opportunities to reflect and suggest on what to do.

In line 12, Emily invited the teachers to suggest ways to improve the teaching of conversion. However, before the teachers could suggest anything, she continued to share her reflections. This observation fits what Mason (2003) described as teacher lust, that is, "the desire to enact change within teachers… desire to impose his [the facilitator's] values and ideas upon others by replicating his experience for them"(cited in Tyminski, 2010, p. 297). The facilitator "simply wants the teachers to have rewarding experiences and thought that urging his experiences on others would accomplish this goal" (Tyminski, p. 297). Or, Emily took her leadership role seriously as the head of department, and she felt responsible for the performance of her teachers and improving teachers' practice. Emily knew the purpose of the second reflection, and she was preparing herself to take up the responsibility of facilitating future in-house professional development.

Although one of the main aims of viewing the video was to investigate how the scaffolding questions could enhance teachers' reflections, nowhere was this seen in the discussion. The video recording was meant to provide teachers the opportunities to revisit the taught lesson to examine further different aspects of the research lesson that enhanced students' learning. However, Emily was the only teacher who appeared to benefit from the use of video recording during the second reflection.

Conclusions

In this study, the reflection took place 1 week after the research lesson due to constraints of the teachers' timetable. In the first research cycle, the CC used audio recording, photographs and the framework to prepare for the first reflection. The audio recording enabled the CC to recognise important patterns in teachers' practice to enhance the professional development. During the first reflection session, the teachers were at level 1 reflection when they engaged in mathematical concepts that they were unfamiliar, had difficulties recollecting details of the lesson and examined strategies common to the teachers' practice. At level 1, even when participants commented on the issues they remembered from their lesson, their comments tend to be very general. The findings suggest that teachers could be encouraged to move to another higher level of reflection if they were given prompts on what to focus. The teachers were more forthcoming in their reflection when examining students' responses and when the CC participated in the reflection. However, instances when the teachers were engaged in level 3 reflection were limited without grounded images provided by videos. The second research lesson, the conversion lesson, was thus videotaped.

In the second research cycle, the head of department facilitated the reflection without the support of a mentor or critical friend. Video recording was used during the second reflection session to help teachers recall specific details of issues they were interested in examining and find evidence of students' learning. Although the clips were carefully selected and embedded within the reflection, only the head of department appeared to benefit from the use of video technology. Perhaps if the teachers were given more time to reflect, they would view and review selected episodes and make better use of the affordances of video.

The framework of nine questions and video of the conversion lesson are examples of tools used in this professional development. The framework of nine questions is a useful tool for teachers to reflect on their mathematics lessons in the absence of a 'mentor' or critical friend. However, most of the questions in the framework focus on the teacher's pedagogical movements. Only one question focuses on the students. As such, when using the framework to reflect lessons taught by their fellow colleague, the reflection requires critical colleagueship, that is, 'an atmosphere in which members trust each other but at the same time participate in a professional discourse that includes and does not avoid critique' (Lord, 1994, p. 195). The facilitator needs to balance between eliciting reflections from all the teachers and participating in critical colleagueship. Through a safe and friendly environment to reflect, and the opportunities for everyone to 'air' their views, the teachers would be more willing to share, learn and build from each other's reflections. Such meaningful activities when conducted in a nurturing environment have the potential of assisting the teachers to develop and apply 'sharper' eyes for 'noticing and examining students' work' in their everyday practices. In this study, the CC assumed that the teachers were able to see the *value of the video* and the teachers were able to use the video to enhance their learning during the reflection.

This study suggests such assumptions are unfounded. When teachers are not comfortable to be videoed, the CC must be willing to take on additional responsibilities. The CC has to conduct the lesson, be videoed in the process and let the teachers analyse the lesson.

Just like any other tools in mathematics teacher education, having powerful tools available for the teachers' in-house professional development is insufficient to promote critical reflection and teacher growth. The teachers need to see the value of the tools and know how to use the tools before the tools become resources that can enhance learning.

References

Beswick, K. (2007). Teachers' beliefs that matter in secondary mathematics classrooms. *Educational Studies in Mathematics, 65*(1), 95–120.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24*(2), 417–436.

Brophy, J. (2004). *Using video in teacher education*. Amsterdam: Elsevier.

Brouwer, N. (2011, April). *Imaging teacher learning: a literature review on the use of digital video for preservice teacher education and professional development*. In Paper presented at the annual meeting of the American educational research association, New Orleans.

Clarke, D. M. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education, 28*(3), 278–301.

Clarke, D. (2000). Time to reflect. *Journal of Mathematics Teacher Education, 3*, 201–203.

Day, C. (1998). Working with the different selves of teachers: Beyond comfortable collaboration. *Educational Action Research, 6*(2), 255–275.

Day, C. (1999). Professional development and reflective practice: Purposes, processes and partnerships. *Pedagogy, Culture & Society, 7*(2), 221–233.

Dewey, J. (1933). *How we think: A restatement of the relation of reflective thinking to the educative process*. Boston: Heath.

Dewey, J. (1981/1933). How we think. In J. A. Boydston (Ed.), *John Dewey: The later works, 1925–1953* (Vol. 8, pp. 105–352). Carbondale, IL: Southern Illinois University Press.

Fadde, P., & Rich, P. (2010). Guerrilla video: A new protocol for producing classroom video. *Educational Technology Magazine: The Magazine for Managers of Change in Education, 50*(1), 4–7.

Fuller, F. F., & Manning, B. A. (1973). Self-confrontation reviewed: A conceptualization for video playback in teacher education. *Review of Educational Research, 43*, 469–528.

Lederman, N. G., & Gess-Newsome, J. (1999). Reconceptualizing secondary science teacher education. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge* (pp. 199–213). Dordrecht, Netherlands: Kluwer.

LeFevre, D. M. (2004). Designing for teacher learning: Video based curriculum design. In J. Brophy (Ed.), *Advances in research on teaching* (Using video in teacher education, Vol. 10, pp. 235–258). Oxford, UK: Elsevier.

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education, 12*(4), 285–304.

Lord, B. (1994). Teachers' professional development: Critical colleagueship and the role of professional communities. In N. Cobb (Ed.), *The future of education: Perspectives on national standards in education* (pp. 175–204). New York: College Entrance Examination Board.

Martinez, K., & Mackay, G. (2002, December 1–5). *Structuring critical reflection in professional experience*. Paper presented at the 2002 AARE annual conference, Brisbane.
Mason, J. (2003). Seeing worthwhile things: A response to Alan Schoenfeld's review of researching your own practice in JTME 6.1. *Journal of Mathematics Teacher Education, 6*(3), 281–292.
Morine-Dershimer, G., & Kent, T. (1999). The complex nature and sources of teachers' pedagogical knowledge. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge* (pp. 21–50). Dordrecht, Netherlands: Kluwer.
Muir, T., & Beswick, K. (2007). Stimulating reflection on practice: Using the supportive classroom reflection process. *Mathematics Teacher Education and Development, 8*(Special issue), 74–93.
Muir, T., Beswick, K., & Williamson, J. (2010). Up, close and personal: Teachers' responses to an individualised professional learning opportunity. *Asia-Pacific Journal of Teacher Education, 38*(2), 129–146.
Power, A., Clarke, M., & Hine, A. (2002). *Internship learning connects the dots: The theory and practice of reflection*. Retrieved November 29, 2013 from http://www.aare.edu.au/publications-database.php/4550/internship-learning-connects-the-dots-the-theory-and-practice-of-reflection
Santagata, R. (2011). From teacher noticing to a framework for analysing and improving classroom lessons. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: seeing through teachers' eyes. Studies in mathematical thinking and learning*. New York/London: Routledge.
Schön, D. A. (1983). *The reflective practitioner*. New York: Basic Books.
Schön, D. A. (1987). *Educating the reflective practitioner. Towards a new design for teaching and learning in the professions*. London: Jossey-Bass Publishers.
Seidel, T., Prenzel, M., Rimmele, R., Schwindt, K., Kobarg, M., Meyer, L., et al. (2005). *Do videos really matter? The experimental study LUV on the use of videos in teacher's professional development.* Paper presented at the eleventh conference of the European association for research on learning and instruction, Nicosia, Cyprus.
Sherin, M. G. (2004). New perspectives on the role of video in teacher education. In J. Brophy (Ed.), *Advances in research on teaching: Using video in teacher education* (Vol. 10, pp. 1–27). Oxford, UK: Elsevier.
Sherin, M. G., & Han, S. Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education, 20*(2), 163–183.
Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14. *Participant observation*. New York: Holt, Rinehart and Winston.
Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*. New York: Free Press.
Tochon, F. (2008). A brief history of video feedback and its role in foreign language education. *CALICO Journal, 25*(3), 420–435.
Tyminski, A. M. (2010). Teacher lust: Reconstructing the construct for mathematics instruction. *Journal of Mathematics Teacher Education, 13*(4), 295–311.
Zhang, M., Lundeberg, M. A., Koehler, M. J., & Eberhardt, J. (2011). Understanding affordances and challenges of three types of video for teacher professional development. *Teaching and Teacher Education, 27*(2), 454–262.

Chapter 5
How a Singapore Teacher Used Videos to Help Improve Her Teaching of the Part-Whole Concept of Numbers and the Model Method

Swee Fong Ng

Abstract Although the part-part-whole concept and its related pictorial representation conceptualised as the Singapore model drawing may seem beguilingly simple, it is not. Comparing the data from the two videoed lessons showed that the very prescriptive mediational processes of the teacher in the second lesson helped the children to discern and identify the critical components of the part-part-whole representation. Repetition of the related mathematical language and the specific relationships until these stuck in the consciousness enabled the children to engage with the lesson. The children could state the desired rule. More importantly facility with the mathematical terms gave children a means to request for further clarifications about the concept and to extend the concept. The MISC framework gave the participating teacher greater structure to her teaching actions. Watching videoed episodes of herself at work helped improved her practice in two ways. She designed meaningful instructional materials to address the intellectual needs of the children. The entire process challenged her assumptions on how children process information and the difficulties they may have with making sense of what seems to be a simple representation of part-part-whole nature of numbers.

The singular factor that impacts learning is the quality of feedback teachers provide to learners (Hattie, 1999; Klein, Nil-Gar, & Darom, 2000). This chapter discusses how a grade 2 teacher (identified as T) used feedback provided by videos of herself at work helped improved her teaching of the part-part-whole concept of numbers and its attending representation known as the model drawing to 12 grade 2 children (8+) who were identified as having difficulties learning mathematics. Feedback in the form of teacher-learner mathematical communication would be based on the amount of mathematics-related talk the teacher engages the learners. Teacher-learner mathematical communication was identified as a potentially useful way of measuring mathematical input in mathematical classrooms (Klibanoff, Levine,

S.F. Ng (✉)
National Institute of Education, Singapore
e-mail: sweefong.ng@nie.edu.sc

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_5

Huttenlocher, Vasilyeva, & Hedges, 2006). Their work was used to guide T's work. This is discussed in section one.

Feedback provided by teachers to those in their care is contingent upon the kinds of tasks presented to the learners as well as the questions teachers ask and the subsequent responses to the answers they elicited from the learners. Provision of good feedback is a highly skilled task, and teachers benefit from more support, not less (Ball, 2009). The Mediational Intervention for Sensitizing Caregivers (MISC) approach provides teachers with a useful guide to enriching the quality of teacher-learner interactions (Klein, 1992, 2003). The section 'The Part-Part-Whole Nature of Numbers and Its Attending Representations' discusses the MISC approach.

Various modes of representations can be used to represent a mathematical concept. It is necessary to know that all these different modes of representation convey the same concept. It is through the use of examples that teachers help children abstract the concepts embedded in them. In Singapore, it is crucial for children to have a sound understanding of the part-part-whole concept of numbers if they are to be able solve structurally complex word problems, including algebraic type word problems. The section 'This Study' discusses the part-part-whole concept, its attending representations, viz. the model drawing and how the Singapore curriculum develops this concept and its representations for the lower grades.

The section 'Method' discusses the study and explains why videography is the method of choice. This is followed by analyses of the data. The findings showed that the MISC framework and the lessons learnt from reviewing the first video enabled T to design instructional materials that increased the amount of mathematical inputs into the lesson. In the second lesson the children had opportunities to share their doubts about their own understanding and also what they understood of the part-part-whole concept of numbers. These findings are reported in the penultimate section. The concluding section discusses the effectiveness of the MISC framework in helping T provide children in her class with effective feedback and the efficacy of videos as a professional development tool.

Teacher-Learner Mathematical Communication

Teachers' mathematical input propels the growth of children's mathematical knowledge (Klibanoff et al., 2006). Mathematical inputs that had the potential to promote children's mathematical knowledge occurred in different contexts. The contexts could range from planned mathematical instruction in a specific mathematical content to contexts that are not related to any specific mathematics lesson to incidental comments to learners. But the most significant finding from their work showed that there was an association between the early mathematical development of young children and the amount of their exposure to mathematically relevant language. Their study demonstrated that the amount of mathematically relevant input preschool teachers provided in their speech was related to the growth of children's mathematical knowledge over the 4-year-old nursery school year. Although some inputs were more instructive than others, all these inputs had the potential to promote the acquisition of mathematical language and concepts. The most noteworthy finding was that the amount of teachers' mathematics-related talk was significantly related to the growth of preschoolers' conventional mathematical knowledge over the school year but was not related to their mathematics knowledge at the start of the school year.

Although preschoolers were the participants of the study, nevertheless Klibanoff et al.'s study was pertinent to this study. The children in the current study consistently failed to meet the demands of the mathematics curriculum. Therefore, if the teacher could increase the amount of mathematically relevant input into the classroom talk, perhaps these grade 2 children's mathematics knowledge could also be increased, independent of their existing knowledge.

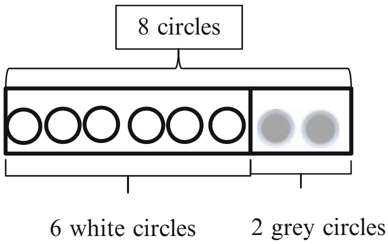
Mediational Intervention for Sensitizing Caregivers (MISC): A Framework to Enhance Teacher-Learner Interactions

Although Klibanoff et al.'s work with preschoolers showed that teacher-learner interactions helped children acquire mathematical concepts and to use them in problem-solving situations, it did not, however, provide teachers with a framework on how to increase the quantity and quality of the mathematical input. It would be useful if teachers were provided with some specific framework which could help them improve the quantity and quality of the teacher-learner interactions. This is particularly important for work with slightly older children where they would be engaging with more mathematical content and thus a need to make connections between content areas and the various modes of representations that are so essential in the teaching and learning of mathematics beyond preschool years. Although it is not customary for preschool children to record their written work or to make sense of mathematical symbols, this is not so for children in grades 1 and beyond.

This study proposes to use the Mediational Intervention for Sensitizing Caregivers approach (Klein, 1992, 2003) as a tool for enriching the quality of interaction between the participating mathematics teacher and the children under her care. The objective of the MISC (Klein, 1988, 2003) is to help sensitise teachers to the needs of children and to relate to them in a way that will enhance their cognitive, emotional and behavioural functioning. The framework in Table 5.1 lists the five mediational processes (Klein, 1992) and how each process focuses on the actions of the teacher and how these actions address specific intellectual needs of the children. Hypothetical examples based on discerning the parts from the whole are used to illustrate how each of the five mediational processes and the related intellectual needs are operationalised and addressed respectively.

The MISC is a useful framework which teachers can use to guide them in their teaching. It is important for teachers to be clear and precise in the delivery of their lessons. Without a framework to guide them, teachers' interactions with the children may be less focused. With the MISC framework, each of the mediational process is identified with a specific name. Naming each of the process provides teachers a clear structure, providing them with specific reasons for carrying out that process and how that process addresses particular intellectual needs of the children.

Table 5.1 Definition of each mediational process of the MISC framework (Klein, 1996) and hypothetical examples of how each process can be operationalised and the nature of the intellectual needs addressed



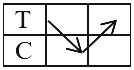
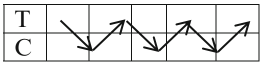
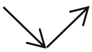
Mediation processes	Examples of the process	Intellectual needs of the child
1. Focusing (intention and reciprocity) The teacher focuses the children's attention on the mathematical problem and how the different representations relate to each other. The teacher's intention is to help the children focus, see, hear and feel clearly. Such specificity ensures that there is reciprocity between the teacher and the children	The teacher uses her hands to discern the parts from the whole. First, the teacher uses her left and right hands to capture the rectangle with six circles. This is one part. It has six circles. The same action is used to identify the rectangle with two circles. This is another part. It has two circles. This is the whole. It has eight circles	Children need visual information to be presented and expressed precisely. Children are not at a loss as to how to make sense of the different representations
2. Mediation of meaning The teacher draws from the children the meaning of the mathematical process. The teacher expresses pleasure, either vocally or with a smile, when a child can identify and name the parts correctly	The teacher could ask: Which is the whole rectangle? How many parts are there in the whole rectangle?	Children need to test what they think they have learnt. They can use their hands to capture the whole or use language to describe the whole
3. Expanding on responses The teacher could then raise the children's awareness to metacognitive aspects of thinking by asking children to explain how they knew. They could be encouraged to expand their explanations by associating the current experiences to some past experience or to purely logical rules	The whole rectangle has two parts. Can the whole rectangle be partitioned into three parts? How can this be done?	Children need to go beyond what they already know. Or if doubts persist, they can seek for more information or clarifications *Going beyond:* The whole rectangle is divided into two parts, one part with 6 circles and another part with 2 circles. I can divide the whole rectangle into three parts, one part with 5 circles, another part with 2 circles and a third part with 1 circle *Seeking further clarification:* I don't know how to find the parts

(continued)

Table 5.1 (continued)

Mediation processes	Examples of the process	Intellectual needs of the child
4. Encouraging feelings of competence The teacher provides words of encouragement or praise in a way that is meaningful to the children. Such words of affirmation are specific to activities that are well conducted or to well-written answers or solutions that are clearly presented. Such informative feedback is well timed in relation to the event	That is a good and clear answer. You have explained how to partition the whole into three parts. You also explain how many circles there are in each part	Children need to experience success and to be able to identify and summarise activities that led to success
5. Organising and planning This process refers to regulation of behaviours by showing children how to organise and plan their activities	This whole rectangle has eight circles. We can partition the whole into two parts, one part with 6 circles and another part with 2 circles. We take a ruler and draw a line after the sixth circle	Children need to learn how to plan before acting, clarify goals to meet subgoals, pace one's activities and also how much energy to invest in an activity

Table 5.2 Interactions of different lengths of ping pongs

There is only one 'ping pong'[a] in this exchange	There are three 'ping pongs' in this interaction
T: How many parts are there in the whole? (I)	T: How many parts are there in the whole?
P: There are two parts in the whole? (R)	P: There are two parts in the whole?
T: That is the correct answer (E)	T: What are these two parts?
	P: One part has 6 circles white circles; another part has 2 grey circles
	T: How do you know that is the correct answer?
	P: 6 white circles plus 2 grey circles. Altogether there are 8 circles. The whole has 8 circles
	T: Good. You checked your answer 
	This is considered as one ping pong exchange

In conventional classrooms, teacher-learner dyads are often characterised by monosyllabic responses. The teacher poses a question and the child responds and the teacher evaluates the accuracy of the response. This ‘initiate-response-sequence’ (IRE) ends when the teacher confirms the accuracy or otherwise of the answer (Cazden, 1988). In contrast, the MISC framework with its five named processes serves as a useful guide for teachers to increase their mathematical inputs in the teacher-learner interactions. For example, ‘expanding on responses’ encourages a longer and more meaningful interaction between a teacher and the children. Klein describes the length of the action-reaction exchanges metaphorically as the length of the ‘ping pong’ (Jaegermann & Klein, 2010) which could be quantified. Table 5.2 illustrates two interactions with different lengths of the ‘ping pong’; the interaction on the left is of one ping pong length. The hypothetical dialogue on the right has a prolonged exchange. The child providing the correct response does not signal the end of the teacher-child exchange. Instead the teacher encourages the child to expand on the answer. The exchange is concluded only when the child could identify all the relevant parts and the whole and with the teacher praising the child for having checked the answer.

Naming each of the five mediational processes helps to facilitate the interchange between the researcher and the teacher. When the mediational process is not named, it would be necessary to describe the process to draw attention to it, a time-consuming activity. However, naming the mediational processes puts a structure to each of the process. Hence, the discussion between the different parties becomes more focused. For example, the hypothetical question ‘What gestures can be used to focus children’s attention on the whole?’ concentrates the two parties’ attention on the focusing process of the MISC.

The Part-Part-Whole Nature of Numbers and Its Attending Representations

To conceptualise a number as comprising two or more parts is the most important relationship that can be developed about numbers (Van de Walle, 2001; Neuman, 1987). Resnick emphasised that:

> Probably the major conceptual achievement of the early school years is the interpretation of numbers in terms of part and whole relationships. With the application of a Part-Whole schema to quantity, it becomes possible for children to think about numbers as compositions of other numbers. This enrichment of number understanding permits forms of mathematical problem solving and interpretation that are not available to younger children. (1983, p. 114)

For example, 8 can be partitioned into two smaller subsets comprising solely of whole numbers, (1,7), (2, 6), (3, 5) and (4, 4), or comprising 3 smaller subsets (1, 1, 6) and (4, 2, 2) or 4 smaller subsets (1, 2, 2, 3) or 2 subsets of rational numbers (3.5, 4.5), etc. Although 8 is a whole, it can be part of a bigger whole. For example, 8 is be part of 9: (8, 1), etc. This conceptualisation of numbers does not depend on children’s knowledge of operations. In Fig. 5.1 a set of 8 rectangles is perceived as two smaller subsets of 3 rectangles and 5 rectangles.

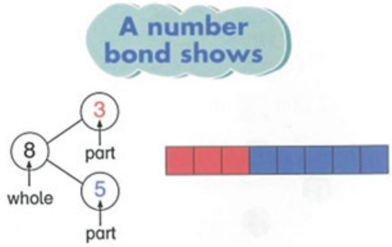


Fig. 5.1 The two different representations of part-part-whole for the number 8; the number bond representation (at *left*) and the model drawing where absolute number of rectangles are used to specify the parts and the related whole (at *right*)

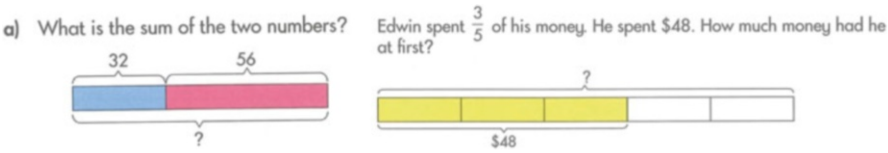
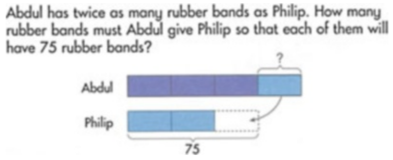


Fig. 5.2 Lengths of the rectangles are relative to one another; longer rectangle is used to represent bigger number, shorter rectangle for smaller number (at *left*), rectangles of equal lengths represent information involving proportional reasoning (at *right*)

The Singapore primary mathematics curriculum places great importance on the development of the part-part-whole concept. Figure 5.1 (Collars, Koay, Lee, Ong, & Tan, 2007, p. 18,) provides a common representation of the part-part-whole concept of numbers. The development of this concept is developed in tandem, as number bonds (8: 3, 5) and also pictorially as one whole comprising eight rectangles comprising three rectangles and five rectangles. This pictorial method of representing part-part-whole nature of numbers has such great currency that it has its own nomenclature: the model method. In the early primary grades, e.g. grades 1–3, the product, known as the model drawing, is a schematic representation used to capture all the information presented in simple arithmetic word problems. In upper primary years, children are taught to use the model method to solve algebraic type word problems. Thus, it is of utmost importance that lower primary children are taught the part-part-whole concept of numbers and how to construct appropriate model drawings to represent this relationship (Ministry of Education, 2009).

In the early grades, the actual number of rectangles is used to specify the different parts and the related whole. In the upper primary grades when the numbers are larger and when it is impossible to represent the exact number of rectangles, the lengths of the rectangles are relative to one another. The example at the left of Fig. 5.2 (Collars, Koay, Lee, & Tan, 2007a, b, p. 30) shows that when it is no longer possible to draw 32 smaller rectangles, the lengths of the rectangles are dictated by the size of the numbers and are relative to each other. The representation at the right of Fig. 5.2 (Collars, Koay, Lee, Ong, & Tan, 2007, p. 65) shows that rectangles of equal lengths are used to represent information involving proportional reasoning. Although the problem involves knowledge of fractions, the solution of the problem dispenses with the need to operate with fractions.



3 units = 75 rubber bands
1 unit = 25 rubber band.
Abdul must give 25 rubber bands to Philip.

Using letter-symbolic algebra
Let a represent the number of rubber bands held by Abdul, p the number of rubber bands by Philip.
x represents the number of rubber bands given away by Abdul.
Before Abdul gave away the rubber bands.

$$a = 2p$$

After Abdul gave away the rubber bands.

$$a - x = 75$$
$$p + x = 75$$
$$x = 25$$

Fig. 5.3 Efficient use of the model method (at *left*) and algebraic method (at *right*) to solve algebraic word problems

Children without access to letter-symbolic algebra may choose to use the model method to solve structurally complex word problems. Figure 5.3 presents a grade 5 (Collars, Koay, Lee, Ong & Tan, 2008, p. 32) example where the model method is used to solve algebraic type word problems. In the model drawing each rectangle is identified as a unit. The model drawing demonstrates an elegant solution to the problem. The algebraic method taught only in the secondary years shows how the letter-symbolic method is used to solve the same problem.

This Study

Participants

T has been teaching for 7 years. Although parental consent was sought from all the 12 children, not all parents agreed to have their children participate in the video section of the study. Permission was given to video the teacher and to record interactions between T and the children and to use the children's oral responses to T's questions. However, the children's work could not be used as a measure of the effectiveness of T's teaching as parents did not provide permission to assess their children's work. Pseudonyms are used to identify the children. The letter P was used when the exact identity of the child was unknown.

The 12 children who participated in this study were underperforming in mathematics. They were not part of an intact class. Because they failed to achieve a pass grade in the various continual mathematics assessments, they were given extra attention to help them improve their mathematics. They came together for the extra mathematics support given by T. They rejoined the mainstream classes for the rest of the curriculum.

Method

T wanted to improve her teaching on the part-part-whole concept of numbers. Her ultimate objective was for the children to construct appropriate model drawings to solve word problems. She had taught one lesson on the model method, but the children were unable to construct appropriate model drawings to represent the numerical information. Given the import of the part-part-whole concept and the attending representations have on problem-solving in the Singapore primary curriculum, it was essential that children have a sound knowledge of this concept.

The two lessons discussed in this chapter were part of a bigger study involving the use of the MISC approach in the teaching of mathematics where I was the researcher. Because of her involvement in the bigger study, T was familiar with the MISC approach. Before we could consider how T could improve the teaching of the part-part-whole concept, it was necessary to have some documentary evidence of what was going on in the classroom.

The Use of Video Technology in Professional Development of Teacher-T

Rather than relying on questionnaires or checklists to assess inputs, it was agreed that lessons were videotaped which meant that it was not necessary to rely on T's memory to recall what happened in the lessons. Viewing the recorded lessons meant that T was watching herself re-enacting the taught lesson. By repositioning herself, T took on two roles, that of an observer and as a teacher. T as the observer could study the outcomes of her actions on the children's learning. An added benefit of using videos meant that the contents of the entire lesson were available on demand. T could review the entire lesson in her own time and reconsider which parts of the lesson could be improved.

Videography supports the work of the researcher in a number of ways. By reviewing the lesson over and over again, the researcher would have a better sense of the structure of the entire lesson. How effective were the examples used to develop a concept? Did the children find the examples engaging in that they encouraged children to ask questions? The researcher could review the teacher-children interactions. What were the teacher's facial expressions when she posed a question and when she answered a response from the class? How did the children respond to those facial expressions? Furthermore, when the researcher has doubts about certain actions or decisions of the teacher, the relevant section could be reviewed together with the teacher. First, there would be consensus that they were referring to the same set of actions. The ensuing discussion was grounded on the same image or episode. The researcher could seek clarifications from the teacher. Thus, by investigating the grounded images afforded by the videos, the discussion between the teacher-researcher would be more focused and less contentious. The researcher's questions could serve as a stimulus, encouraging the teacher to reflect on her own actions or nonactions.

Data Collection

There were two lesson observations on the teaching of the part-part-whole concept and model drawing. The first lesson was conducted just before the first term test which was followed by a midterm break. Hence, there was a 3-week gap between the first and second lesson. A research assistant videoed the lesson and the researcher kept field notes. Each observed lesson was 1 h long. At the end of the lesson observation, T and the children continued with the rest of the curricular activities. After evaluating the children's homework, T concluded that the delivery of the first lesson was ineffective because the 'models were not well drawn'. T wanted to know how to improve on the first lesson.

I used the five MISC processes as a framework to prepare a summary of the lesson which was emailed for T's consideration. We met over the term break and discussed the proceedings of the first lesson. We used the lesson summary in tandem with the selected video episodes of the MISC processes to discuss T's mediation for children's learning for each process. We spent 2 h discussing the first lesson. T designed the next set of instructional materials for Lesson 2.

Lesson 1

The first lesson did not achieve the specified objectives because the example used to develop the concept was inappropriate. Furthermore, an abstract representation was used to construct the model.

Inappropriate Choice of Example For concept formation those critical features of the concept 'must be discerned and held in our awareness simultaneously' (Runesson & Mok, 2004, p. 63). What are the critical features in model representation that must be attended to and be held simultaneously? Model drawing is related to the part-part-whole concept of numbers. To draw a particular model to represent a number and its related bonds, for example, (8: 6, 2), it is necessary to identify and name the whole (8) and relate that to the longest rectangle, identify and name the related number bonds (6, 2) to the relevant parts of the whole rectangle. The critical features of a model representation are that the whole rectangle can be partitioned into two or more parts and that each rectangle represents a specific number in the number bond triad.

To develop mathematical concepts it is important to select appropriate examples such that it is not intellectually challenging to discern these critical features. It is prudent to select examples that use small numbers and contexts that are simple (Collis, 1973) because children can make better sense of a problem when numbers used are small. Also children can make sense of problems that are structurally simple. Children may be deflected from engaging with the critical features of the mathematical concept when the context used to convey the concept is structurally too convoluted (Collis, 1973). The soldier example was used to start the first lesson:

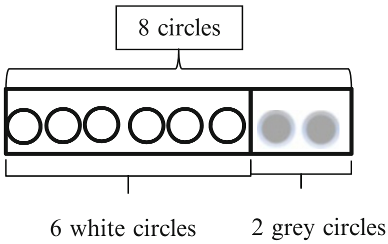
> Soldier example: There are 8 soldiers in an army camp. Six of them are officers. How many of them are not officers. (5:20–5:50)

Mathematically, this example has three variables, soldiers, officers and not officers. The concept of mutually exclusive subsets is conceptually demanding for young children (Piaget, 1952; Inhelder & Piaget, 1964). Although the numbers used are small, the structure is not. It was possible that the children in this study were challenged by the concept of disjoint sets: officers and not officers. Furthermore, these two disjoint sets form a whole. Because there was no discussion about ranks of soldiers, it was not clear whether the children were able to discern the ranks of the soldiers. For children to engage with the mathematics and to be able to construct an appropriate model drawing, they must have a clear idea of the critical features of the concept and to relate the critical features to the problem: Soldiers form the whole, officers and not officers are parts of the whole, and together they make up the whole.

To introduce model drawings to children, examples with simple context and which can be represented would be most helpful. The affordances in the representations used should enable the children to hold all the information simultaneously. The whole is partitioned into individual parts and how these representations relate to the number bonds. It would be better for beginning learners to learn to use discrete objects to represent the whole and how these set of discrete objects can be partitioned into their respective parts.

The soldier problem could be replaced with a simple and structurally direct problem:

> There are 8 circles. Six circles are white. 2 circles are grey. Draw a model to represent this information.

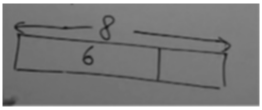


With the above representation, hand gestures and horizontal braces could be used to focus children's attention on the different sets of circles, to identify and name the different parts and to relate the parts to the whole. The length of each rectangle is absolute. The length of the rectangle representing white circles contains exactly six white circles. There are exactly two grey circles in that part of the rectangle representing grey circles. The rectangle representing the whole is represented by the two shorter rectangles with the two contiguous edges of adjacent rectangles represented by a single line.

An Abstract Representation Was Used to Construct the Model Instead of using absolute units to represent eight soldiers, a relative representation of 8 was used instead. A whole rectangle with the number eight written at its top was used to represent the total number of soldiers. The whole rectangle was partitioned into two parts, the longer of the two rectangles to represent 6, the bigger of the two subsets. This relative representation of the whole meant that the length of the rectangle representing the whole is a variable. The arbitrariness of the length of the rectangle may have made it difficult for the children to discern that as a whole. Could a longer rectangle be used instead? Why was that a whole? However, if absolute units were used instead such that a rectangle containing 8 discrete objects represented the whole, children may be able to discern the whole and the parts of the model drawing to the total of the set.

Rather than using mathematical language to help children discern the parts from the whole, and the parts made a whole, T relied on hand gestures instead. The hand gestures may have been useful if she had used it simultaneously to capture the entire length of the whole rectangle drawn on the whiteboard. However, this was not the case. With the board behind her, T gestured with her hands by drawing apart the left and right hands to coincide with the word ‘whole’. The episode in Table 5.3 showed that even when it was clear from children’s responses that they were not able to discern which of the two numbers, 8 or 6, represented the whole, she repeatedly used hand gestures to try to focus children’s on the meaning of whole.

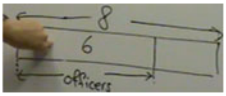
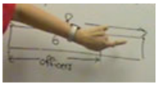
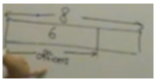
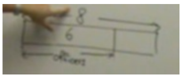
Table 5.3 Dialogue and actions recorded by the video showing that the relative representation of the model drawing was ineffective in helping children discern the parts from the whole

	Dialogue	Action by T
		
T	My whole is 8. So if I draw my model, this is my whole and there are 8.	T drew a long rectangle (10:33).
T	My arrows point to the beginning and to the end of it.	T drew the arrows wrote 8 in between the two arrows (10:35).
T	This is my whole.	T indicated the extreme right end of the rectangle.
T	Ok let us look at the second part of the question. Second part says that six of them are officers.	T pointed to every word.
T	If six of them are officers, that means the rest are not officers. Either you are an officer or not an officer.	T waved her left hand to show that the remaining set did not belong to the officer class.
T	Tell me which number here represents the whole. That means everything. The eight or the six?	Again the hands were used to drawn apart to gesture ‘everything’.
P1	Six	
T	P1 said six. Anyone else has a different answer?	
T	P2 says six.	Teacher nodded her head towards P2.
T	Let me have a show of hands. Who says eight is the whole? Everything.	Again the teacher uses the same gesture to indicate the whole (when both her hands were farthest apart from each other).
T	Who says 6 is the whole?	Some children raised their hands. Teacher used her eyes to scan the class from left to right.

The children had difficulties discerning which part of the model drawing represented the subset 'not officers'. Although T used her index finger to point to the shorter of the two rectangles, the children used the teacher's facial reaction such as nods and puzzled look to provide T the response she expected to hear from the class. The children failed to discern the parts from the parts and the parts from the whole. More importantly they failed to see the relationship between the parts and the whole.

The episode in Table 5.4 shows that opportunities where children could practise the words part and whole were rare. Therefore, children did not acquire the mathematical language used in the lesson. Furthermore, the objective of the lesson was not clear to the children. Why was it so important to identify the part that represented the subset not officers? If there was a rule, what was the rule? Children needed to test whether they have learnt, but the opportunities to test what they have learnt were not there. Because T asked and answered most of her questions, the children did not have the opportunity to request information from T. Since the

Table 5.4 Opportunities where children could practise the words part and whole were rare

	Dialogue	Action by T
T		
	So when I ask you how many are not officers, do I want this part?	T pointed to the rectangle representing officers (17:48).
T	I don't want this part right?	T shook her head.
T		
	Because the question is asking for how many are not officers.	She pointed to the shorter rectangle for emphasis (17:54).
T		
	So where should I put my question mark? Over here?	T pointed to the officer part of the rectangle (17:56).
T		
	Or over here?	T pointing to the shorter rectangle (17:59).
Ps*	No.	
T		
	Here?	(18:01)
Ps	No.	T was puzzled and her look of puzzlement was very obvious to the class (18:01).
Ps	Yes, yes, yes.	

[a]Refers to pupils giving chorus answers

questions T asked did not require the children to explain or expand upon their answers, the opportunities to encourage feelings of competence amongst the children were almost non-existent. Furthermore, the children were not shown how to organise and plan for the drawing of models.

Thinking About the Next Lesson During the feedback session we discussed how to improve the focusing process so that the children could discern the whole from the parts. Furthermore, T would have to consider ways to increase the mathematical inputs into her lesson. She needed to use language to show the relationship between the parts and the whole. The two examples showed how the whole was partitioned into parts. This introduction was not robust enough as a whole can be partitioned into more than two parts. The repertoire of examples has to be extended so children were introduced to a more robust knowledge of part-part-whole. Could there be more ways to encourage feelings of competence amongst children? Perhaps children could be asked to offer their own examples which required them to suggest how to draw models and to relate the parts with the whole. In Lesson 1 the ping pong in the teacher-children dyads were short, made up mostly of children's monosyllabic responses to T's questions. How could the lengths of the ping pongs be increased? How could T regulate mathematical behaviours? What instructions would she give to help children construct models?

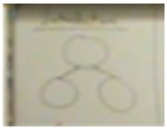
Lesson 2

T ensured that children were able to discern the terms part from whole and to relate these critical features to the parts and the whole in the model drawing by constant repetition of these terms and constantly highlighting how these were related to different representations: number bond representation, number sentences, model drawings and abstract representations of the relationship, part plus part equals whole written as $P+P=W$. She introduced the terms part and whole and the relationship between the parts with the whole by activating the children's prior knowledge of number bonds. She used mathematical contexts that were familiar to the children. To assess whether the children were getting a sense of the concept, she checked for learning by asking children to provide examples of their own. To preempt the possibility that children acquire the misconception that a whole can only be partitioned into two parts, and to help children abstract the part-part-whole concept, different examples were used, some where the whole was partitioned into two parts and another where the whole was partitioned into three.

Activating the Children's Prior Knowledge of Number Bonds These children's experience of the part-part-whole concept began in Primary 1 by way of introduction of number bonds. Their experiences with number bonds may serve as a powerful entry route to the pictorial representation of the part-part-whole concept as explained by Marton, Runesson and Tsui:

> Powerful ways of acting derive from powerful ways of seeing, and the way something is seen or experienced is a fundamental feature of learning. If we want learners to develop certain capabilities, we must make it possible for them to develop a certain way of seeing

Table 5.5 Activating children's prior knowledge of number bonds

Ping pong		Dialogue	Action by T
	Teacher-pupil		
	T	Do you know the number bond for 5? Mary?	Children raised their hands.
	Mary:	3 and 2.	
	T:	Bee is Mary correct?	
	B:	Yes.	
	T:	Shanti? Is Mary correct?	
	S:	Yes.	
	T:	Shanti tell me how do you get 3 and 2?	
	S:	Plus.	
	T:	3 plus 2 equals 5.	
	T:	Can someone tell me the names we call these?	(Pointing to the individual circles).
	P:	Mother.	
	T:	Mother? Do you remember why you call this mother? (pointing to the representation)	
	P:	Story. Mrs. K told the story. Mother and child.	

P refers to the same child. Dialogue captured during 4.28–5.59

> or experiencing. Consequently, arranging for learning implies arranging for developing learners' ways of seeing or experiencing, that is developing through which the world is perceived. (2004, p. 8,)

The episode in Table 5.5 showed how T's decision to tap on children's prior knowledge resulted in more meaningful dyadic interactions between teacher and children and the ping pongs were longer than those in Lesson 1. When the number bond representation was presented on the board, the children immediately called out aloud 'part-whole' (4:28). T asked more questions of the children, and she also asked children to check whether their peers were correct. By opening up the discussion, some children in the class recalled how their Primary 1 mathematics teacher used stories to help them to remember the meaning of the part-part-whole representations. Metaphors such as mother and child were used to help relate the whole to the mother and child to the parts.

T spent another five minutes reinforcing the part-part-whole concept with the number operations. She repeatedly asked the children to identify and name the whole and the part of the number bond relationship which the children articulated as $3+2=5$ and $2+3=5$ and finally as part plus part equals whole which T recorded as $P+P=W$.

Using Mathematical Contexts Familiar to Children T dispensed with the idea of using a context to introduce part-part-whole concept of numbers (see Fig. 5.4.). A

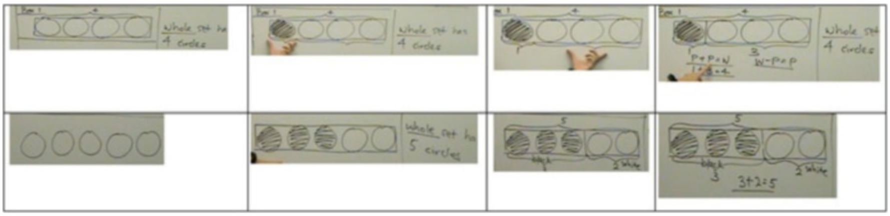


Fig. 5.4 Using examples which are very visual and obvious. Screen grab during 19.50–34.45, starting from top extreme *left*

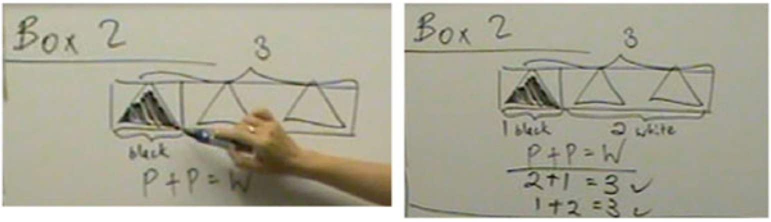


Fig. 5.5 Example presented by children. Ticks used to affirm children's conclusion of part plus part equals whole. Screen grab during 39.17–42.45

context was only necessary when she wanted the children to use the model method to solve word problems. Instead she used examples which were very visual and obvious. In the first example, she partitioned a set of four circles into two parts, one part with a black circle and the other part with three white circles. She further reinforced this idea of partitioning a whole into parts with another example of five circles. This example gave her the opportunity to link the number bond representation (5: 3, 2) with the model drawing.

Checking for Learning Children's ability to provide examples of their own is evidence that they have begun to make sense of what the teacher has been teaching (Watson & Mason, 2002, 2006). To assess whether the children were able to make sense of the part-part-whole concept, T asked the class to provide an example of the 'part plus part equals whole' relationship. As she articulated this relationship, simultaneously she wrote $P+P=W$ on the board. To prevent the children from misconstruing that relationships are based on circles, she asked the children what shapes they would use instead. She picked a child who chose to start with a set of three triangles and partitioning the set of three triangles into one black triangle and two white triangles. T recorded the articulation of the child. One child gave the number equation $2+1=3$ and another child, $1+2=3$. See Fig. 5.5.

For Discernment to Happen There Is a Need for Variation To help the children discern the parts from the whole, critical features of the part-part-whole concept, T selected examples where the whole was partitioned into two parts and another where the whole was partitioned into three parts. In Lesson 1, both examples were

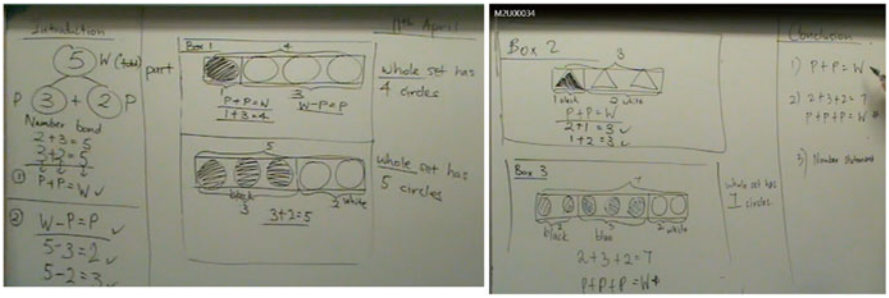


Fig. 5.6 Whiteboard presenting all the examples used for Lesson 2

similar in that the whole was divided into two parts. Children may misconstrue that any whole could only be divided into two parts. Or the children may not consider representations where the whole is partitioned into three or more parts as representation of the part-part-whole concept. If there were examples where the whole was partitioned into more than two parts, children may be better able to discern parts from whole and abstract the concept of part-part-whole and that it was possible for the whole to be partitioned into many more parts. Furthermore, if the whole was partitioned into more than two parts, children may be better able to discern the concepts of parts. In Lesson 2, three examples were used. Two examples showed how a whole was partitioned into two parts: (4: 1, 3) and (5: 3, 2). The third, the whole was partitioned into three parts, (7: 2, 3, 2). Different colours were used to differentiate each of the parts: 2 black circles, three blue circles, and 2 white circles. See Fig. 5.6.

Repetition as a Learning Process Compared to Lesson 1, Lesson 2 showed that T played an important role in helping the children acquire the appropriate mathematical language of part and whole. After she had introduced the terms part and whole to the class, she kept using them meaningfully. One could almost say that she followed an algorithm to the way she used these terms. After drawing the whole rectangle, she identified and named the rectangle as the whole. Next, the children were asked to count the number of objects in the whole. After the whole was partitioned into the respective parts, she used her thumb and index finger to identify the part and then named that as a part (e.g. one part with 1 black circle. This part has 3 white circles). This was followed by the numerical relationship (e.g. $3+1=4$). This process was repeated until all the parts were identified and named. Next, she summarised the relationship between the different parts and the whole as part plus part equals whole. The summary was presented verbally as well as symbolically by writing the equation $P+P=W$. She then related the numbers to the corresponding letters; 3 and 1 referred to the parts represented by the letter P and 4 to the whole represented by W. The children were asked to state the relationship by repeating after her: part plus part equals whole. As the children repeated this statement, she pointed to each specific part of the model drawing. Repetition of the part-part-whole relationship provided the children an opportunity to practise using these words. The repetition process meant that the children could remember the words and were able

Table 5.6 A whole can be partitioned into more than two parts

Ping pong	Teacher-pupil	Dialogue	Action by T
	T	How come there are three parts this time?	T pointed to the example in box 3.
	T	whereas there are two parts here	T pointed to the triangle example.
	P1:	Because there are 2 black circles.	
	T:	One part.	(T pointed to the 2 black circles).
	P1:	3 blue circles.	
	T:	One part.	(T pointed to the 3 blue circles).
	P1	And 2 white circles.	(T pointed to the 2 white circles).

Exchange conducted during 01.01–1.37, second file

to use the words to communicate their answer and their doubts to the teacher. The willingness of these children to volunteer their doubts to the teacher could happen only when they have acquired the necessary mathematical language such that they could draw upon these mathematical terms to engage the teacher:

Shanti: I don't know how to see the parts (25:25).
P: Why got three parts? (4:14)

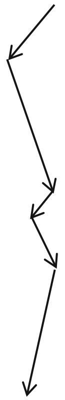
Not only did T repeat the part-part-whole relationship to a specific example. She assessed whether the class has acquired the meaning of part-part-whole relationship by comparing two different examples. The interaction in Table 5.6 showed how the child, albeit hesitatingly, was able to explain why some examples comprised two parts and others three.

Furthermore, in her summing up of the lesson (Table 5.7), T asked the children to name one thing they have learnt from that day's lesson.

The above mathematical inputs by children can only happen when they have acquired the use of relevant mathematical terminologies and relationships. Unless and until these mathematical terminologies and relationship stuck in the minds of the children, they may not be able to use them or lack the confidence to use them. For those who have yet to achieve a meaningful grasp of these terminologies and their attending relationships, but because they have access to them, these children could use them to seek for further clarifications. For those who have grasped these meanings, their confidence was enhanced when they could use them to sum up what they have learnt. Thus, access to these terminologies addressed their intellectual needs. This learning through repetition is a good example of the Stickiness Factor discussed by Gladwell (2000).

T had more opportunities to encourage competence amongst the children. Whenever the children were able to state the rule part plus part equals whole, T drew a star next to the rule $P+P=W$ and affirmed the children's response. Such affirmation may have helped children abstract for themselves the point of the lesson: the part-part-whole relationship of numbers. T gave the children specific guidance on how to partition the whole into parts. Thus, T showed the children how to organise and plan the construction of the part-part-whole representation.

Table 5.7 Teacher summing up the lesson

Ping pong	Teacher-pupil	Dialogue	Action
	T	Can anyone tell me what we have learnt so far? Just one thing you have learnt.	
	P1	Part plus part equals whole.	
			(T recorded the child's response by writing $P+P=W$ on the board).
	T	Anybody else learnt something else?	
	P2	Two plus three plus two equals seven.	T recorded the P2's response. $2+3+2=7$.
	T	You want to elaborate more about this part? You just learnt the number two plus three plus 2 equals 7? What can you say about these numbers?	T directing her question to P2.
	P2	Part plus part plus part equals whole.	
			T wrote $P+P+P=W$ beneath the number equation $2+3+2=7$.

Exchange conducted during 2.18–3.21, second file

Conclusions

This study shows that although the part-part-whole concept and its attending external representations may look beguilingly simple, it is not. It is very likely that adults, including many teachers, may perceive that the decomposing of a whole number into its constituent parts to be self-evident, without realising the intellectual challenge children face with this concept. Given the pictorial representations, children have to be able to discern part from parts and parts from the whole and name the whole, the size of the whole, the constituent parts and the size of each part, the number of parts and the relationship of the parts to the whole. Comparisons of the two lessons show that the MISC framework did help sensitise the teacher to the intellectual needs of the children. In the second lesson the teacher placed much thought and care in designing instructional materials, particularly in the selection of appropriate examples, to help children hold various pieces of information of the part-part-whole concept and its pictorial representation simultaneously. T found that it was inappropriate to use relative representations of the whole, whereas absolute representations of the part and whole helped the children discerned the parts from the whole. Precise hand gestures played an important role in focusing children's attention on the appropriate aspects of the part-part-whole representation. Precise focusing helps to address children's intellectual needs to make sense of the different aspects of the pictorial representation. Furthermore, repetition of critical mathematical terms, part and whole, enabled children to make use of these terms to seek further clarification from the teacher. Although children may not have understood the part-part-whole concept, nevertheless, they could use these words as currency to seek for more information or to expand on their responses. Appropriate words of praise encouraged feelings of competence amongst the children addressing their intellectual need to experience success and to summarise activities that led to success. The most noteworthy finding of this study is that repetition enables key ideas to stick in children's minds. When these ideas stick in their mind, they can act on it.

Reflections of the Teacher T reported she found the videos useful in a number of ways. She could use the videos on demand. Because of the review function, she could recall selected episodes of the lesson, pause at critical junctures which meant that she could analyse the consequences of her actions on children's learning. The images meant that she could interrogate children's actions to ascertain the nature of their learning difficulties. She saw the usefulness of the whiteboard and how the clear organisation of the contents on the whiteboard facilitated learning. She learnt to ask more useful questions, questions that require pupils to think more about the concept.

By repositioning herself as an observer, T found herself questioning her own pedagogy and the children's epistemology. Why was it that the children were able to respond to her oral questions but were unable to work independently? Could it be that children need more verbalisation of concepts before they could write down these concepts? Do they have difficulty connecting their thoughts with writing? Does this mean that they need more examples where the teacher and the children solve problems collaboratively with the children telling the teacher what to do and the teacher serves as a recorder of their thoughts on the whiteboard? She concluded that she 'cannot assume that pupils know' just because she had taught the lesson.

For this teacher the professional development did not end with watching the videos of her lessons. Rather once sensitised, the grounded images captured by the videos provoked the teacher to think and question deeply how children learn mathematics.

Reflections of the Researcher When working with grounded images, it was important for the researcher and the teacher to be guided with a suitable framework. Once the latter was familiar with the framework, the teacher could identify which specific pedagogical practices she wanted to improve and what form the improvements could take. The teacher was responsible for her learning trajectory. More importantly, the images allowed the researcher to challenge her own initial reactions to certain pedagogical practices which she thought were rather ineffective. For example, the researcher questioned the value of the teacher's practice of getting children to repeat the relationship part plus part equals whole. However, continual review of the teacher and the children's dialogue showed that children's repetition of the phrase enabled them to communicate effectively with the teacher. The children used the same phrase to inform the teacher the status of their learning. Hence, the video images enabled the researcher to check her subjective reactions to certain pedagogical practices which actually had positive learning outcomes for the children.

References

Ball, D. L. (2009, June 1). *Making mathematics learnable in school: What is the work of teaching mathematics*. Keynote address at the 3rd redesigning pedagogy international conference. Singapore: National Institute of Education.

Cazden, C. B. (1988). *Classroom discourse – The language of teaching and learning*. Portsmouth, NH: Heinemann.

Collars, C., Koay, P. L., Lee, N. H., Ong, B. L., & Tan, C. S. (2007). *Shaping maths coursebook 4A* (2nd ed.). Singapore: Marshall Cavendish Education.

Collars, C., Koay, P. L., Lee, N. H., Ong, B. L., & Tan, C. S. (2008). *Shaping maths coursebook 5A* (2nd ed.). Singapore: Marshall Cavendish Education.

Collars, C., Koay, P. L., Lee, N. H., & Tan, C. S. (2007a). *Shaping maths coursebook 1A* (2nd ed.). Singapore: Marshall Cavendish Education.

Collars, C., Koay, P. L., Lee, N. H., & Tan, C. S. (2007b). *Shaping maths coursebook 3A* (2nd ed.). Singapore: Marshall Cavendish Education.

Collis, K. F. (1973). A study of children's ability to work with elementary mathematical systems. *Australian Journal of Psychology, 25*(2), 121–130.

Gladwell, M. (2000). *The tipping point: How little things can make a big difference*. London: Abacus.

Hattie, J. (1999, August 2). *Influences on student learning*. John Hattie Inaugural Lecture: Professor of Education, University of Auckland.

Inhelder, B., & Piaget, J. (1964). *The early growth of logic in the child: Classification and seriation*. London: Routledge & Kegan Paul.

Jaegermann, N., & Klein, P. (2010). Enhancing mothers' interactions with toddlers who have sensory processing disorders. *Infant Mental Health Journal, 31*(3), 291–311.

Klein, P. S. (1988). Stability and change in interaction of Israeli mothers and infants. *Infant Behaviour and Development, 11*, 55–70.

Klein, P. S. (1992). Assessing cognitive modifiability of infants and toddlers: Observations based on mediated learning experience. In H. C. Haywood & D. Tzuriel (Eds.), *Interactive assessment* (pp. 223–250). New York: Springer.

Klein, P. S. (1996). *Early intervention: Cross –cultural experiences with a mediational approach*. New York: Garland.

Klein, P. S. (2003). Early intervention: Mediated intervention for sensitizing caregivers. In A. S. H. Seng, L. K. H. Pon, & O. S. Tan (Eds.), *Mediated learning experience with children: Applications across countries*. Singapore: McGraw-Hill Education.

Klein, P. S., Nil-Gar, O., & Darom, E. (2000). The use of computers in kindergarten, with or without adult mediation; effects on children's cognitive performance and behavior. *Computers in Human Behavior, 16*, 591–608.

Klibanoff, R., Levine, S., Huttenlocher, J., Vasilyeva, M., & Hedges, L. (2006). Preschool children's mathematical knowledge: The effect of teacher's "math talk". *Developmental Psychology, 42*, 59–69.

Marton, F., Runesson, U., & Tsui, A. B. (2004). The space of learning. In F. Marton & A. B. M. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 3–40). Mahwah, NJ: Lawrence Erlbaum Associates.

Ministry of Education. (2009). *The Singapore model method for learning mathematics*. Singapore: EPB Pan Pacific.

Neuman, D. (1987). *The origin of arithmetic skills*. Goteborg, Sweden: Acta Universitalis Gothenburgensis.

Piaget, J. (1952). *The child's conception of number*. London: Routledge & Kegan Paul.

Resnick, L. B. (1983). A developmental theory of number understanding. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 109–151). Orlando, FL: Academic.

Runesson, U., & Mok, I. A. C. (2004). Discernment and the question, "What can be learned". In F. Marton & A. B. M. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 63–87). Mahwah, NJ: Lawrence Erlbaum Associates.

Van de Walle, J. (2001). *Elementary and middle school mathematics. Teaching developmentally* (4th ed.). New York: Addison Wesley Longman.

Watson, A., & Mason, J. (2002). Extending examples spaces as a learning/teaching strategy in mathematics. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th conference of the international group for the psychology of mathematics education* (Vol. 4, pp. 378–385). Norwich, UK: PME.

Watson, A., & Mason, J. (2006). *Mathematics as a constructive activity. Learners generating examples*. Mahwah, NJ: Lawrence Erlbaum Associates.

Chapter 6
Using Video Clubs to Learn for Mathematical Problem-Solving Instruction in the Philippines: The Case of Teaching Extensions

Romina Ann S. Yap and Yew Hoong Leong

Abstract The study reported here is situated within a 7-month professional development programme aimed at supporting Secondary 1 mathematics teachers in the teaching of mathematical problem-solving in the Philippines. A video club was part of the programme. In a video club, the teachers view video clips of their own or their peer's classroom teaching and discuss certain aspects of teaching. Evaluations at the end of the programme revealed that teachers considered the video club as one of the components of the programme that had the most impact on them. This chapter examined whether and how the video club influenced teachers' mathematical problem-solving classroom instruction particularly in the area of teaching extensions. The findings offer a provisional theory for the trajectory of teacher learning in video clubs.

Introduction

Rivendell[1] High School is a private secondary school in the Philippines. Reputed to be among the premier secondary schools in the country, Rivendell commits itself to the mission of forming and nurturing students who can positively contribute to the development of the nation and the global community. In line with this mission, part of the vision of Rivendell's mathematics department is to contribute to nation-building by moulding students to become problem solvers through its different programmes. They believe that when students are equipped with problem-solving skills, they will be better prepared not only to face the uncertainties that lie ahead but also to lead the country in much-needed social, economic and political transformations.

[1] The name of the school, the teacher participants and the students appearing in this chapter are pseudonyms.

R.A.S. Yap (✉) • Y.H. Leong
National Institute of Education, Nanyang Technological University, Singapore
e-mail: rominayap@yahoo.com; yewhoong.leong@nie.edu.sg

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_6

Among the principal ways by which the mathematics department directly attended to developing the students' problem-solving skills was through designating sessions specifically for mathematical problem-solving (MPS). They recognised, however, that teachers would require better support to help them realise the vision of developing problem solvers through their programmes. Thus, the mathematics department of Rivendell High School welcomed the opportunity to participate in a professional development (PD) programme aimed at supporting MPS instruction.

The PD programme conducted with a group of mathematics teachers in Rivendell spanned seven months in the year 2012. It had multiple components, among which was a video club. In brief, a video club (Sherin, 2004) is a group of teachers who meet regularly to view video clips of their own classroom teaching and discuss certain aspects of teaching that the video may have shown. As a teacher development undertaking, video clubs possess some of the characteristics of what are purported to be effective PD (e.g. Hawley & Valli, 1999; Smith, 2001), it being sustained, collaborative and practice based. In addition, because of the medium used, Sherin and Han (2004) suggested that video clubs "offer teachers the opportunity to examine teaching and learning in new ways and have the potential to foster the learning called for by reform" (p. 163).

At the end of the PD programme, teachers identified the video club as one of the components of the PD that had the most impact on them. In this chapter, we report on whether and how teachers' participation in the video club influenced their instruction of MPS. In particular, we focus on teachers' instruction of the extension aspect of problem-solving. Extending is an important but often ignored element in problem-solving. In fact it was an instructional feature of MPS that was new to all participating teachers. By focusing our study on the instruction of extensions, we focus on teachers' learning of new instructional ideas.

We conducted this study particularly to inform us on how to conduct the PD programme better in the future in light of its intention to support Filipino teachers in their instruction of MPS. In general, however, reporting the results of this study also contributes to the literature that aims to examine how teachers translate their learning from PD to their respective classrooms. While the practice of PD is common and is conducted with the end of benefitting teachers' instruction, whether and how they actually influence classroom practice is often unclear (Wilson & Berne, 1999).

Before proceeding to describe the study and report the findings, we first provide a brief background of mathematical problem-solving, with particular interest in the role of extensions and the instruction of MPS in general.

Mathematical Problem-Solving: Focus on Extensions and MPS Instruction

Mathematical problem-solving began gaining international attention in the 1980s partly due to the publication of an *Agenda for Action: Recommendations for School Mathematics of the 1980s* which advocated making problem-solving the "focus of school mathematics" (NCTM, 1980, p. 1). Since then, various mathematics curricula in both Western and Eastern countries have clearly acknowledged the importance of problem-solving as reflected by its increasing prominence in curriculum documents (Wu & Zhang, 2007). This is also the case in the Philippines where problem-solving and critical thinking were identified as the central goals of the mathematics curriculum in basic education (Department of Education [DepEd], 2012).

Despite the prominence of MPS in curriculum documents, however, there is no standard definition of MPS (Schoenfeld, 1992). We take the view that it is the process of engaging in seeking a solution to mathematical tasks for which the solver has no direct knowledge or means for solving it (see, e.g. NCTM, 2000; Van de Walle, 2003). We also add that MPS involves the process of extending the problem and/or the obtained solution(s) to the problem (Pólya, 1945/1973). The definition of MPS in the latest Philippine curriculum document (DepEd, 2012) is consistent with the first part of our definition. But like many other documents or programmes, it does not make any explicit reference to the process of extending as part of problem-solving.

By extending the problem, we mean the act of making and considering new problems related to the original problem including generalisations. We placed an emphasis on extending because it is a significant but often overlooked part of MPS. This conception of extension finds its roots in the last stage of Pólya's (1945/1973) four-stage model for problem-solving. The first three stages – Understanding the Problem, Devising a Plan and Carrying Out the Plan – are almost self-explanatory. The last stage, Looking Back, has been usually reduced to mean mere checking of the correctness of one's answer. However, Pólya's conception of Looking Back actually goes beyond this, as he also exhorts the solver to reflect on questions such as: "Can you check the argument? Can you derive the result differently? Can you see it in a glance? Can you use the result or the method for some other problem?" (Pólya, 1945/1973, p. xvii).

In an effort to clarify and better operationalise the concept of Looking Back, Cai and Brook (2006) specify the activities of generating, analysing and comparing alternative solutions, posing new problems and making generalisations as part of Looking Back. Although this conception of Looking Back may seem superfluous, it is sometimes considered to be one of the most important parts of MPS (J. W. Wilson, Fernandez, & Hadaway, 1993) as it is where solvers can be more deeply engaged with the underlying mathematical connections and structures that a problem has to offer.

Because extending in the process of Looking Back is seen as work *in addition* to solving the original problem, as previously mentioned, it is often overlooked (Kantowski, 1977; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005). Most are content with being able to arrive at a certain answer. To address this tendency of neglecting the role and importance of extensions in MPS, Toh et al. (2011) developed a pedagogical tool called the Practical Worksheet. This worksheet was designed to support a solver's MPS through a series of guiding questions and explicitly builds in the process of extending. Using Pólya's model as its organising structure, solvers are encouraged to go through the stages according to Pólya's original intention. Thus, under the section for Looking Back, it does not only prompt solvers to check their answers, but it also prompts them to look for alternative solutions

("Write down your level of satisfaction with your solution. Write down a sketch of any alternative solution(s) that you can think of") and extend the problem ("Give one or two adaptations, extensions or generalisations of the problem. Explain succinctly whether your solution structure will work on them").

The Practical Worksheet was developed together with a problem-solving module and a corresponding teacher development programme as part of a larger design experiment in Singapore that sought to bring out the centrality of problem-solving in the mathematics classroom. In one of their studies, they found that the Practical Worksheet can be a facilitative tool for teachers in making extensions a significant part of MPS instruction (Leong, Tay, Toh, Quek, & Dindyal, 2011).

It should be noted, however, that conducting MPS classrooms is generally challenging for teachers (Anderson, Sullivan, & White, 2004; Foong, Yap, & Koay, 1996) even as descriptions of various classroom portraits of instructional practices that support an MPS curriculum have been widely written about (e.g. Charles, 1989; Charles & Lester, 1984; Curcio & Artzt, 2003; Grouws, 2003; Van de Walle, 2007). One reason for this is that a teacher's instruction is highly influenced by the image of teaching he or she formed as a student watching his or her own teachers (Lortie, 1975). For most mathematics teachers, this image is characterised by straight lectures, answering exercises and applying formula (Romberg & Kaput, 1999). Although these modes of instruction are not altogether unproductive, they need to be modified in a way that is consistent with MPS instruction – where students play a far more active role in determining how instruction proceeds through their genuine engagement in problem-solving. Another reason why problem-solving instruction is difficult for teachers, however, is that teachers themselves may not have experienced what it means to be problem solvers (Blanco, 2004). Clearly, it would be difficult to teach something that one has little experience with.

These challenges to and opportunities for MPS instruction informed the design of the PD programme implemented in this study. This design is described in the next section.

The Study

We provide the backdrop within which the study was undertaken in this section. We begin with describing the overall design of the conducted PD programme. We then provide more details on the video club component of the PD. Finally, we report on the findings about teachers' response to the conducted PD.

The Overall PD Programme for MPS Instruction

The PD programme within which this study was situated was conceptualised with the primary goal of enabling mathematics teachers to integrate MPS in regular classroom instruction. It integrated in its design the conduct of regular collaboration

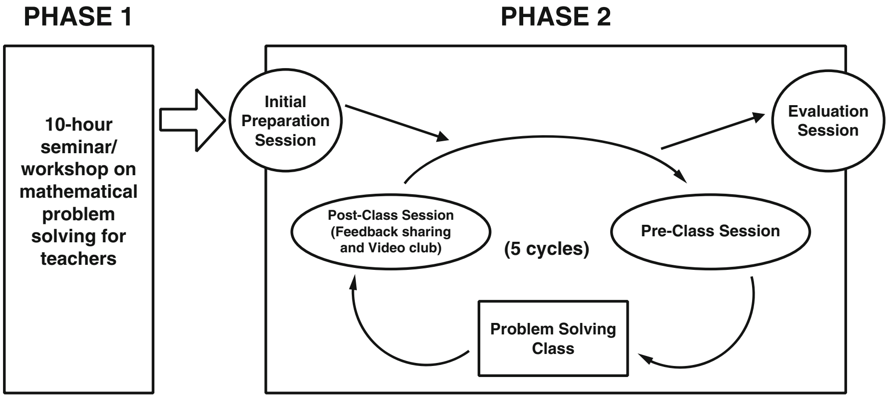


Fig. 6.1 Schematic diagram of the PD phases

sessions among the teachers to serve as the venue for collective sharing and learning. Through conducting regular collaboration sessions, the PD also aimed to build a culture of professional learning in community which in turn was intended to support teachers' individual and collective learning better (Vescio, Ross, & Adams, 2008).

The PD also utilised Toh et al.'s (2011) Practical Worksheet as a pedagogical tool. The text of the Practical Worksheet mainly used in this PD programme (slightly modified from the original to suit the context of the teacher participants) can be found in Appendix A. The worksheet required solvers to give at least one extension and to describe a possible solution for it.

The PD programme spanned 7 months and consisted of two main phases, the basic components of which are illustrated in Fig. 6.1. The first phase comprised a 10-h seminar/workshop conducted before the beginning of the school year.[2] This seminar was aimed at providing teachers a better understanding of MPS through presenting issues surrounding MPS and giving teachers the experience of doing MPS using the Practical Worksheet. A 1-h module was devoted to extending which allowed teachers to experience how extensions could bring to surface the underlying mathematical concepts behind a problem in a more enriching manner. In this module, the following were given as general suggestions for making extensions: change the numbers, change the conditions or number of conditions, reverse the given and the required information, change the approach while keeping certain features intact, state the problem in a generalised form or a combination of the said suggestions could also be made.

Teachers were not expected to be expert problem solvers after completing Phase 1. More MPS experience would be required for any individual to achieve proficiency. Phase 1, however, enabled teachers to discuss MPS and its instruction based on a common experience which facilitated the collective process of forming a shared understanding.

[2] The academic school year in the Philippines typically commences in June.

In the second phase of the PD, we held regular collaboration sessions. Aimed at supporting teachers' instruction of MPS, these sessions took place during the school year as the teachers conducted problem-solving classes. A total of 12 sessions were conducted where the first served to initiate the teachers in the activities planned for succeeding sessions while the last served to evaluate the process undertaken in the PD. The ten sessions in the middle can be generally classified either as a pre-class session or a post-class session.

A pre-class session allowed teachers to discuss the solutions and instructional considerations for the mathematical problem before using it in their scheduled problem-solving class. Teachers were given the problem, usually chosen by the facilitator in consultation with some of the participating teachers,[3] a week before the scheduled session. They were then expected to come to the session with their own answers on the Practical Worksheet to facilitate the discussion.

The teachers' problem-solving class that followed was videotaped, and a post-class session was held a few days after. A post-class session served both to gather feedback about teachers' implemented MPS lessons and to gather learning points for succeeding MPS classes. To address these two aims, two activities were held in a post-class session. The first was a sharing of teachers' feedback, and the second was a video club. The specific use and conduct of video clubs in the PD are detailed in the next subsection.

From Fig. 6.1, it is evident that pre-class sessions and post-class sessions were intended to be held alternately. However, a number of unexpected disruptions in the school calendar were experienced which did not make this always possible. Figure 6.2 provides a general picture of the actual schedule that the PD programme followed from the first pre-class session to the last post-class session.

The typical duration of each collaboration session and problem-solving class was 50 min. The two problem-solving classes that are close together (represented by C1a and C1b in Fig. 6.2) were sessions in which MPS and the Practical Worksheet were introduced to the students over two consecutive days.

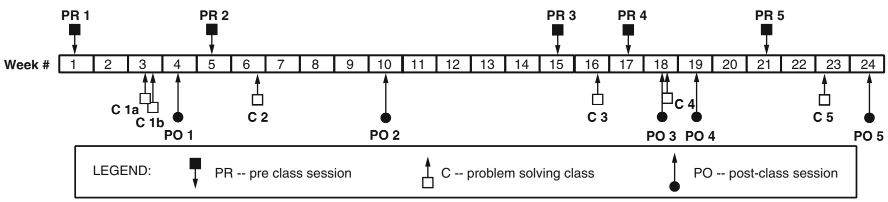


Fig. 6.2 Overview of the actual schedule of the problem-solving classes and the pre- and post-class sessions

[3] Some of the criteria for choosing the problem included its alignment with the topics in the regular mathematics curriculum, its level of difficulty considering the students' abilities and its suitability to highlight certain MPS processes.

The Video Club

In the first chapter of this volume, Ng highlighted the potential of video technology to provide a common grounded image of instruction from which knowledge can be constructed. The PD programme described in this chapter also capitalised on this technological affordance by incorporating a video club in its design. It is significant to mention that the video club also addressed a more practical logistical concern. As mentioned earlier, one of the means that the PD utilised was building a teacher community of professional learners. Such successful communities have been characterised, among other things, by the practice of conducting observations of each other's classrooms and providing each other feedback and support for the primary purpose of teacher development (Hord, 2004). In practice, finding the opportunity and the time to observe the classroom of peers is difficult. But participating in a video club makes this possible albeit in a limited way.

The main focus of the clips in a video club may vary depending on the purpose for conducting the video club. But in terms of process, Sherin (2004) provided a picture of its typical activities. It begins with capturing a teacher's classroom instruction on video. A facilitator and/or concerned teachers will then choose the clip that will be shown during a session based on the goals upon which the club was established. During the sessions, the video to be shown is first contextualised either by the facilitator or the featured teacher by describing the events that led up to the point where the video clip begins. During the viewing, teachers are free to comment or interrupt, although facilitated discussions will usually commence only after the viewing. Discussions often take off from what teachers notice and their comments and reflections as a result of these observations.

The group of Sherin and van Es has done considerable work in video club use in mathematics teacher development. In their studies, video club meetings took place over a period between 1 and 2 years and were used primarily to elicit teacher observations and reflections on students' mathematical thinking. Through analysing teachers' discourse during the meetings, they have shown how teachers' participation in video clubs positively developed over time in various ways (e.g. Sherin & Han, 2004; van Es, 2009, 2012). They have also reported on observing noticeable shifts in teachers' instruction in relation to their participation in the video club (Sherin & van Es, 2009; van Es & Sherin, 2010). Sherin and van Es, however, acknowledged that too little work on video clubs has been conducted to generalise its benefits or to specify its enabling mechanisms.

Unlike the works of Sherin and van Es, the video club in the present PD was primarily intended to focus on teacher instruction rather than on student thinking. This was done with a view of helping teachers improve on their instruction in MPS classes. As previously mentioned, the video club was conducted within post-class sessions where teachers also engaged in sharing feedback on the previous class. As such, the time exclusively devoted for the video club was only about 20–30 min.

A maximum of two video clips were shown in every session, and the clips were 2–7 min long in length. The facilitator – the first author of this chapter, henceforth referred to in the first person singular pronoun – picked and prepared the video clips that were shown during the video clubs. My primary consideration for choosing a clip was its perceived potential to elicit among teachers reflection and discussion that would contribute to enriching their instruction of MPS. It was also influenced by what I perceived were the needs of the teachers based on what I observed from the classes. For example, in one lesson I noticed that some teachers had markedly different ways of overseeing students during their allotted time for problem seatwork. One teacher was very involved with the students' processes to the extent that she was leading them to a solution. Another was very minimally involved, only reminding students of the remaining time. Given this imbalance, I chose a clip from Calvin's class which I thought was more representative of what a problem-solving class should look like.

I had other secondary, but nonetheless important, considerations in the choice of the video clips. First, since the teachers were new to the video club process, and some of them expressed some apprehension at the prospect of other teachers viewing a clip from their class, I made an effort to avoid choosing clips that would portray teachers negatively or clips that might lead to unfair comparisons between two different teachers. Second, in the spirit of fairness, I also made an effort to feature a clip from every teacher by the end of the PD. And finally, to avoid redundancy, variety in terms of focus of the video was considered. A short description of the video clips shown in the sessions can be found in Appendix B.

Teachers' Evaluation of the PD

At the end of the PD programme, teachers' feedback and evaluation were obtained through evaluation forms, a focused group discussion and individual interviews. All of the teachers expressed that they benefitted from the PD programme. A review of the teachers' responses and their participation reveals that the video club helped the teachers in at least four ways. First, watching their peers in the classroom enabled them to learn teaching strategies that they could also try out in their own classrooms. Second, video clubs encouraged teachers to reflect on their own instruction whether as a result of being able to compare their own classroom actions to that of their peers or of being able to view their own instruction more impartially and listening to peer comments on them. Third, video clubs provided a platform for teachers to discuss collaboratively about issues related to MPS instruction. Fourth, some teachers whose video clips were featured in a video club received affirmation about their practice.

In the portion of the evaluation form that inquired as to which of the different components of the PD had the most impact on them, five of the seven participating teachers had the video club as their answer or part of their answer. Thus, one can perhaps conclude that the video club contributed significantly to the professional development of the teachers in terms of MPS instruction. Despite assertions of benefitting from the video club, it is not always clear whether and how teachers' instruction was influenced. Determining this influence, however, is important especially in light of the fact that the ultimate objective of conducting the PD for teachers was to support teachers' MPS instruction in the classroom. This study therefore investigated how the video club influenced teachers' MPS instruction. We focus particularly on the instruction of *extensions*, as enacting this aspect of MPS instruction was a new undertaking for all participating teachers. Thus, this chapter addresses the question of how teachers' participation in a video club affected their instruction of the extension aspect of mathematical problem-solving.

Method

The Participating School and Teachers

As mentioned at the beginning of this chapter, the mathematics department of Rivendell High School envisioned their students to become problem solvers. To carry this out, they deliberately incorporated a problem-solving component in their instruction which evolved with their understanding of MPS and their students' needs. For instance, the early MPS programme consisted only of giving homework to students. By 2005, they began to set aside classroom sessions for MPS to highlight certain MPS heuristics through carefully chosen problems. By 2011, prior to their participation in the present PD, they used Toh et al.'s (2011) Practical Worksheet in their instruction,[4] modified in such a way that Stage 4 in Pólya's model involved only checking the solution but not extending the problem. A former head of the mathematics department explained that the exclusion of extending was due in part to the fact that teachers felt that students were already burdened enough with solving the problem, but also in part to the fact that teachers did not understand the concept of extensions fully nor grasp the importance of its role in problem-solving.

Rivendell recognised there was much room for improvement in their MPS programme. Among their concerns, the most critical was that most of their mathematics teachers had not actually experienced authentic problem-solving. Thus, teachers were not likely to teach MPS in an authentic manner and also not likely to carry out problem-solving approaches to instruction outside of the MPS classes. It was these circumstances that led the mathematics department of Rivendell High School to participate in the PD as reported in this study.

Phase 1 of the PD was conducted as an in-service session for all of Rivendell's mathematics teachers who were available at the start of the school calendar. A total of twenty teachers attended. For Phase 2, it was decided with the department head that the PD focus on building a programme for the Secondary 1 students who were following the regular curriculum.[5] This group of students was chosen partly because

[4] The Rivendell mathematics department became acquainted with the Practical Worksheet through a faculty member who encountered it in her postgraduate studies in Singapore.

[5] To a certain degree, Rivendell implements a system of student streaming. Most students follow the regular curriculum, but a smaller fixed fraction of students are placed in an accelerated track.

Table 6.1 Summary of participating teachers' profile

	Teacher's name						
	Alice	Bessie	Calvin	Derrick	Elsa	Frank	Yvonne
Gender	Female	Female	Male	Male	Female	Male	Female
Total no. of years teaching	25	13	2	2	2	3	17
Total no. of years teaching in Rivendell	25	13	2	2	2	1	16

their calendar of activities was more flexible than the other groups and because we wanted to make the curricular adjustments as early as possible. Thus, the teachers who taught at least one of these classes participated in Phase 2.

Table 6.1 provides a general profile of the participating teachers. While there are seven teachers reflected in Table 6.1, Alice actually left the group soon after post-class session 4 because she was seconded to another arm of the school. On the other hand, Yvonne only began participating in the PD during pre-class session 5 as she was the one tasked to take over Alice's teaching responsibilities.

Data Collection and Data Analysis

The primary raw data consisted of video recordings of all the post-class sessions which included the video club and video recordings of most[6] of the MPS classes of the participating teachers. Supplementary data included video recordings of all the pre-class sessions, the researcher's journal and field notes, teachers' written reflections and evaluations as well as audio recordings of teachers' interviews during and after the PD implementation. Transcripts of all the video and audio recordings were made to form part of the data for analysis.

As the study reported here only focused on the teacher's instruction of extensions as influenced by the video club, the primary data were first sifted such that only data relevant to this focus were included. Portions of the post-class sessions where teachers discussed general feedback from their classes were not reviewed. Furthermore, video club discussions that did not make any reference to extensions were also excluded. Similarly, only segments of instruction pertaining to extensions were retained for the data on MPS classes.

The resulting segments of video with their corresponding transcripts were then reviewed and provided with thick descriptions along with supporting commentaries to highlight both the observed events in the segment and the context surrounding the event in relation to the research objective (Powell, Francisco, & Maher, 2003). Lengthy video and transcript segments were parsed into smaller episodes to make the descriptions and commentaries more focused. This process of parsing was guided by changes in the initiator and object or goal of the discussions (Schoenfeld, Minstrell, & van Zee, 2000).

[6] A few classes were only audio-recorded or not recorded altogether due to logistical limitations.

We made use of narratives to weave together the teachers' video club participation and their classroom teaching to bring to surface the probable influence of the former to the latter setting. The approach adopted is akin to what Derry et al. (2010) described as a "play-by-play" analyses of video data where "interpretations of episodes that follow each other in time are presented sequentially. Play-by-play analyses are particularly effective at showing how the sequentially developing context relates to what happens next" (p. 22).

In constructing a narrative, a chronological review of the relevant annotated data segments was first made for each teacher. A narrative chronology of how the episodes unfolded in relation to one another was then made. These resulting narratives were further refined through another round of review; this time, relevant supplementary data were also consulted to support the developing narratives.

During narrative writing, it was found that specifying teachers' actions in their instruction of extensions revealed interesting findings. This was therefore undertaken for each teacher to substantiate the narratives. It was the final narratives that we primarily used to determine whether and how the video club influenced the teachers' MPS classroom instruction.

Findings

We derived a provisional theory of how the video club influenced the participating teachers' instruction of extensions based on the narrative accounts. We present this through expounding on a trajectory of teacher learning that was exhibited in this study. In reporting, we intersperse portions of narratives taken from Bessie and Elsa's chronological trajectories to illustrate the points. We chose to use Bessie and Elsa's narratives because the data collected from these teachers were the most complete. In addition, their learning experiences offered an illustrative contrast.

Before reporting the findings, we first share how we characterised the teachers' actions during their instruction of extensions as these characterisations figure prominently in the discussion. We also first describe Bessie and Elsa's early instruction of extensions to contextualise the findings that we report better.

Characterisation of the Teacher Actions in Teachers' Classroom Practice

Six distinct teacher actions corresponding to student responses during the instruction of extensions were identified. These are summarised and described as follows:

- Defining extending and/or explaining its value – students listening
- Giving general suggestions for extending – students listening
- Giving specific examples of extensions – students listening

- Acquiring students' inputs for extensions – students giving general suggestions
- Acquiring students' inputs for extensions – students giving specific examples
- Posing extensions as new problems to solve – students solving

"General suggestions" refer to suggestions of extensions that do not provide enough information to be solved directly. For instance, in a problem that includes in its given conditions the perimeter of a figure, a teacher suggesting to change the perimeter without providing a specific value was considered to have given a general suggestion. We find that it is relevant to differentiate general suggestions from specific examples since specific examples seem to promote further thinking into the problem as they pointed to further problem-solving.

Identifying these teacher actions allowed us to trace their development over time. To provide an indicative picture of this progression in tabular form, the total amount of time teachers spent on extensions in every class was obtained, and the time they spent for each of the teacher actions above was estimated. Tables 6.2 and 6.3, respectively, provide summaries of how Bessie and Elsa utilised the time they spent in the instruction of extensions in the six MPS classes, where each class was 50 min long. While examining the utilisation of the time spent does not give the full picture of how instruction of extensions was conducted, it allowed a comparison in terms of teachers' commitment – in the use of this precious resource of time – to this important aspect of problem-solving.

Extensions were discussed only during the second and third video club sessions, that is, after Class 2 and Class 3, respectively. The original narratives were therefore

Table 6.2 Teacher Bessie's instruction of extensions in the problem-solving lessons

	Classroom activity on extensions in Bessie's class	Class #					
		1a	1b	2	3	4	5
Number of minutes spent	Defining extending and/or explaining its value – students listening	0.5	1.5	0	1.5	0	0.5
	Giving general suggestions for extending – students listening	0.5	1	0.5	0.5	0.5	0.5
	Giving specific examples of extensions – students listening	0.5	0	0	3.5	4	0.5
	Acquiring students' inputs for extensions – students giving general suggestions	0	0.5	0.5	0	0	0
	Acquiring students' inputs for extensions – students giving specific examples	0	1	2.5	0	2[a]	0
	Posing extensions as new problems to solve – students solving	0	0	0	0	0.5	7.5
	Others	0	0	0	0	0	0
	Total time allotment (minutes)	*1.5*	*4*	*3.5*	*5.5*	*7*	*9*

[a]Bessie chose students to share their extensions based on what she had read after quickly going over the submitted papers

Table 6.3 Teacher Elsa's instruction of extensions in the problem-solving lessons

	Classroom activity on extensions in Elsa's class	Class #					
		1a	1b	2	3	4	5
Number of minutes spent	Defining extending and/or explaining its value – students listening	0.5	1	0	0	0.5	1
	Giving general suggestions for extending – students listening	1	0	0.5	0.5	1	0
	Giving specific examples of extensions – students listening	1	0	0	1.5	2	1
	Acquiring students' inputs for extensions – students giving general suggestions	1.5	2	0	0.5	3	0
	Acquiring students' inputs for extensions – students giving specific examples	2	1[b]	0.5	0	1.5	0
	Posing extensions as new problems to solve – students solving	0	0	0	1	3	4.5
	Others	2[a]	0	0	0	1[c]	0
	Total time allotment (minutes)	*8*	*4*	*1*	*3.5*	*12*	*6.5*

[a]Elsa asked the students to answer the part of the worksheet under extensions after she gave some general suggestions
[b]Elsa did not actually ask for students' inputs, but she read from the worksheets that students submitted
[c]Elsa asked students to give an example of a generalisation, but no response was obtained

organised around these two events, and thus only these two video club sessions are included in the discussion in later sections.

Instruction of Extensions Before the Video Club Discussions on Extensions

Although teachers recognised the value of extensions, teachers' instruction did not always reflect their appreciation of extensions. This was more evident in the teachers' first three problem-solving classes. For instance, both Bessie and Elsa articulated in interviews conducted between Class 2 and Class 3 that they valued extensions in MPS instruction. For Bessie, she believed that it made one understand the problem better. On the other hand, Elsa considered extending as opportunities for students to exercise their creativity in a subject known more for its systematic rigour. Bessie and Elsa explicitly articulated these sentiments in their respective classes as well. The utilisation of and the time spent on extensions in Bessie and Elsa's MPS instruction in the first three classes, however, were not always evocative of what they say they valued about extending.

As the data from Tables 6.2 and 6.3 indicate, Bessie and Elsa allocated very little time to the instruction of extensions in Classes 1a, 1b and 2, that is, except for Elsa's

Class 1a. One contributing factor was the sentiment that there was too little time to carry out all the instructional goals specified in the classes. And since extensions were often scheduled for the latter part of the class, they were the ones sacrificed. This was particularly true in Bessie's case as she always found herself still lecturing minutes after the end-of-class bell rang. In Elsa's case, however, lacking time did not seem to be an issue. Of all the teachers, she was the only one who was able to carry out all the instructional goals the team set for Class 1a with time to spare, and she had 8 min to devote for extensions. In Class 1b and Class 2, she also did not have to race against time, and yet she only spent 4 min and 1 min, respectively, on discussing extensions. This seemed to indicate that extensions were not her priority then.

Another key concern identified in the teacher's instruction of extensions was that teachers spent very little time sharing their own specific examples of extensions. It seemed that teachers simply expected students to know what it was they were to produce for extensions instead of modelling what extensions could look like. In Elsa's first class, for example, while students were given considerable time (i.e. 2 min) to make attempts to provide examples of extensions, most of these attempts were poorly articulated and did not always reflect a clear understanding of extensions. In Bessie's case, she would often give an assessment of an example given by a student on whether it was a "worthwhile" problem, without really defining or giving examples of what she meant.

A third concern in teachers' instruction was that teachers spent no time in genuinely posing questions related to extensions.

A Trajectory of Teacher Learning from a Video Club

Discussions about the instruction of extensions surfaced only in the second and third video clubs. Here we present three events that we observed about the trajectory of teacher learning from a video club. In the interest of space, we focus more on the video club and classroom episodes surrounding the third video club.

During the video club, *teachers were presented with ideas and possibilities for their own classrooms*. Perhaps similar to most PD programmes that focus on teacher pedagogy, the video club provided a venue where teachers were able to gather and create ideas for future instruction. In the present study, discussions related to the instruction of extensions were drawn either directly or indirectly from the video clips that were shown in the video club. The second video club was an example of the latter since the video clips viewed did not actually feature instruction on extensions. However, as the facilitator, I used the discussion ensuing from the second clip to segue into the discussion of extensions. I particularly pointed out how teachers did not seem to give enough time and attention to extensions and alternative solutions in their lessons.

In the third video club, teachers viewed a clip from Calvin's class featuring how he went about discussing extensions with his students. In the clip, Calvin posed

The perimeter of the identified usable space in a certain room is found to be 108 meters. The space is roughly represented by the shaded region in the figure on the right. Given the indicated dimensions, find its area. (All intersecting lines are perpendicular to each other.)

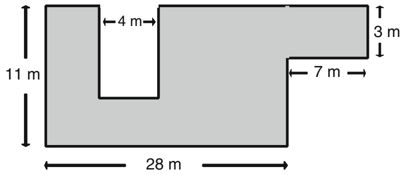


Fig. 6.3 Main problem for MPS Class 3

extensions as new problems to solve during class, and students were seen to be actively engaged in answering these new problems. The main problem for the said class is given in Fig. 6.3. And the transcript[7] of a portion of the video clip (together with snapshots from the clip which display some of Calvin's actions directed at the prepared slides at certain instances) that was shown during the video club is given in Fig. 6.4.

I chose this clip because Calvin's goals in discussing extensions appeared to differ from the rest of the teachers. The other teachers were mostly confined to showing students examples of extensions such that their goal for instruction seemed to be focused on how students can think about *making* extensions. On the other hand, by posing extensions as new problems to solve, Calvin was focused on how students can *think through* extensions.

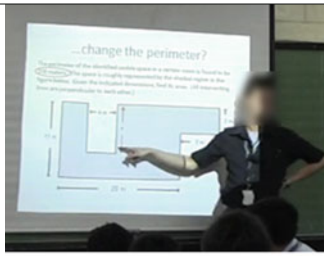
During the video club, Bessie said that she liked how "it wasn't just a reporting of the extensions, but the students were made to actually think of how that extension would work. They were thinking. At least when they do it on their own, they can feel more confident that they can do it". Elsa in her reflection said that she appreciated how Calvin "concentrated on the students' thought processes" which she said she usually forgot.

In this third video club, the ideas for instruction of extensions were directly derived from the video clip shown, with one teacher's *actualities*, as represented by Calvin's instruction featured in the clip, becoming *possibilities* for other teachers. We further conjecture that the perceived feasibility of the viewed instructional ideas was strengthened by the fact that Calvin was a peer teaching the same lesson and bounded by the same constraints they were bound to. It was, so to say, a grounded image of instruction that was accessible to Bessie and Elsa.

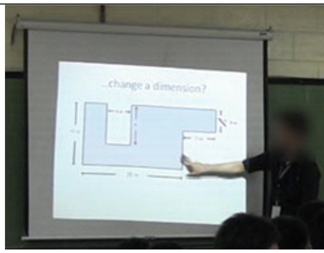
Before the next MPS class, ***the conceived ideas and possibilities in the video club discussions were translated into a shared goal which was embodied into a collective action plan.*** Pre-class sessions were secondary sources of data in this particular study. But in writing the narratives to weave together teacher's encounters

[7] In Rivendell, classroom instruction is carried out in English, although it is common for some Filipino words to be injected in discourse. On the other hand, the collaboration sessions and interviews were conducted using a mix of English and Filipino words as was common in this Philippine setting. For convenience, all the excerpts appearing in this paper are the English translations. Efforts were made to keep the contextual meanings accurate.

CALVIN	If we change the numbers, if we change the perimeter, what would change? Remember this is 108 (REFERRING TO THE GIVEN PERIMETER). If I made this to 110? What's the value here?
C*	9.
CALVIN	9. If I made this 112?
C	Ten.
CALVIN	If I made it 111?
C	11...oh, no, no. 9.5.
CALVIN	What?
C	9.5.
CALVIN	There will be a decimal that's half. Okay.
CALVIN	If I change this to six, would it change the perimeter?
C	Yes.
CALVIN	If I change this (REFERRING TO THE LENGTH ORIGINALLY LABELED "3 m") to six, what will this be? [INDISCERNIBLE RESPONSE] What was that?
C	Five.
CALVIN	This is going to be five. Will our perimeter change?
C	No. No.... Sir, isn't it...
CALVIN	Wouldn't. These are just possible extensions; I would be interested to see your extensions. Are there other possible extensions?



"What's the value here?"



"..., what will this be?"

Fig. 6.4 Transcript of a portion of the video clip shown in the video club from Calvin's class, accompanied by snapshots from the video clip which display Calvin's actions at certain instances. *Calvin was posing the questions to the class in general and multiple students were answering from their seats, "C" refers to the responses from the class that cannot be attributed to only one person

with extensions in the video club and in their instruction, the role of intermediary activities could not be ignored. Pre-class sessions provided an opportunity for video club discussions to be revisited such that ideas and possibilities raised during the video club could be translated into shared goals and plans. In this way, the video club discussions on extensions impacted on the general design of the next lesson.

For instance, after the second video club, it was agreed that discussion on Pólya's first stage be limited to make more time for the discussion of extensions. And to aid teachers' instruction, common presentation slides of the problem, its solution and some extensions were prepared for the first time.[8] Furthermore, to help teachers manage the available classroom time better, parts of the planned lesson were labelled

[8] While the presentation slides were made for teachers to use, it was emphasised that teachers were free to modify or even disregard the slides depending on what they saw was appropriate for their class.

as a "NEED" if it was found that it was necessary to go through it given the lesson goals or simply a "WANT" if it was found to be relevant but not crucial for the execution of the lesson goals.

Because of previously explained schedule adjustments, the third video club took place after the fourth pre-class session (see Fig. 6.2) instead of it taking place before it. Nevertheless, the ideas and possibilities during this video club did influence the collective plans made for the succeeding classes (both the fourth and fifth class). In particular, the presentation slides in succeeding lessons incorporated relevant cues that teachers could take advantage of when posing extensions as new problems for students to genuinely think about. These opportunities for teachers to enrich their instruction of extensions were discussed during the pre-class sessions to help them prepare their own lessons.[9]

Before proceeding to the next point, it is relevant to mention that teacher's instruction of extensions were noticeably more purposive after the second video club where the apparent lack of attention to extensions was pointed out. Indicative of how Bessie and Elsa's attention to extensions increased was the increase in the time they spent on extensions in class as reflected in Tables 6.2 and 6.3. The nature of their actions also changed. For instance, both Bessie and Elsa spent more time to model what "worthwhile" extensions were by sharing more specific examples. Their instructional repertoire for extensions also developed to become more varied. This increased attention to extensions made their instruction more consistent with their avowed beliefs about the importance of extensions in MPS which were reported earlier.

Therefore, to an extent, the shared goals and collective action plans that grew out of the video club discussions were implemented in Bessie and Elsa's classrooms. The next point, however, highlights what seemed to be a crucial element for defining how teachers bring the possibilities presented to them from the video club into their classrooms.

During the MPS class, ***how instruction played out was shaped by a teacher's ownership and personalisation of the collective goals and action plans.*** A shared goal and a shared action plan were not sufficient for a teacher's instruction to resemble what was viewed and discussed during the video club or what was laid out in the action plan. Rather, we found that it also depended on how much a teacher owned and personalised these collective goals and action plans as his or her own. To illustrate, we make a comparison between Bessie's and Elsa's instruction of extensions in Class 4 which followed the third video club.

The main problem in Class 4 is given in Fig. 6.5. Presentation slides were prepared for the discussion of the problem with this class. In the segment allotted for extensions, the slides prepared essentially provided different specific examples of extensions for the problem, and it also included slides which teachers can use to support further engagement of students with the extensions. Cues for encouraging such engagement were also discussed during the pre-class session. While both Elsa

[9] During the fourth pre-class session, I initiated these discussions having previewed the clips that were used in the video club that took place a week after.

You are interested in buying an Xbox from an online store which offered it at 20% discount. However, the buyer must also pay a 7% delivery charge. Out of curiosity, you inquired if they applied the discount first or the delivery charge first when computing the final price for the Xbox. However, instead of a straight answer, the customer service representative simply replied by saying that they always had the best interest of their customer in mind. Which do you think would be in your best interest – that they apply the discount first, or the delivery charge first? Is this always the case? Adequately support your answer.

Fig. 6.5 Main problem posed for MPS Class 4

and Bessie decided to make use of these slides in their class, they differed in their utilisation of these resources.

Elsa spent 12 min on extensions, and she spent almost an equal amount of time on giving examples, hearing out students' own examples and posing extension questions to students. Often, she would first ask students to share their extensions, and then she would comment on the appropriateness of the suggested problem. She would then add to the students' examples by going through the ones prepared in the slides. The cues incorporated in the slides designed to probe more into students' understanding of the problem through extensions were well utilised. When a student answered, Elsa would follow up with a "why" question that would oblige students to articulate how they had thought about it.

On the other hand, Bessie started the discussion on extensions by asking two students to share their extensions. These students were pre-picked based on what Bessie read from their papers. After the students shared, Bessie then pointed out what she liked or found interesting in the extensions that they gave. From Table 6.2, one can see that Bessie spent most of the remaining time sharing extensions with the class. She did this using the prepared slides. However, the slides that had cues meant to support students' further engagement with the problem were skipped without commenting about them.

To illustrate the contrast in Elsa's and Bessie's instruction, Fig. 6.6 juxtaposes parts of the transcript from Elsa's and Bessie's class for the same segment of instruction on extensions represented by the corresponding slides that they were referring to. Considering the prior events, Bessie's class particularly followed a less likely script.

It can be said that Bessie might have not really valued extensions in the way that Calvin used them as seen in the video club. But an earlier incident in the same class made it plain that something else was amiss. In that incident, a student was sharing his solution to the class. The solution contained a common misconception[10] that the team had discussed during the pre-class session. The presentation slides even included a

[10] The student applied the discount and delivery charge in the following additive manner:

Case 1: $x - 0.20x + 0.70x = 0.87x$
Case 2: $x + 0.70x - 0.20x = 0.87x$

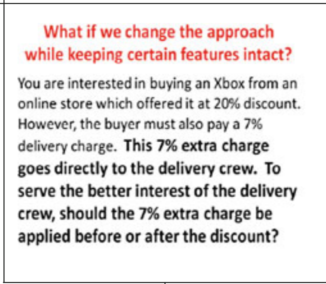
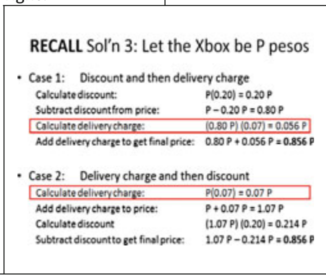
ELSA'S CLASS	SLIDE REFERRED TO	BESSIE'S CLASS
ELSA: Okay. Now, this one, you have the 20% discount and the 7% delivery charge. The question would be - this 7% charge goes to the delivery crew. Now, the question is which would serve the better interest of the delivery crew, if the 7% extra charge would be applied before or after the discount? What do you think, before or after the discount? Sean. SEAN: Before. ELSA: Why before the discount? SEAN: 'Cause after the discount, it would be 7% of 80%. ELSA: So, that is,- that is lower already than the original price. So, that's correct, right?		BESSIE: What if you change the approach by keeping certain features intact, so your other ideas for extensions? So, you're interested in buying an Xbox from an online store which offered it at 20% discount; however, the buyer must also pay a 7% delivery charge. This 7% extra charge goes directly to the delivery crew. To serve the better interest of the delivery crew, should the 7% extra charge be applied before or after the discount? So, we are changing the approach while keeping certain features intact. Okay? Okay.
ELSA: If you apply the discount first – to calculate the delivery charge, you have 0.056 of P; but, if you apply the delivery charge first, you have 0.07 off P. So, this [REFERRING TO CASE 2 ON THE SLIDE] would be greater than if the discount was applied first.		[BESSIE THEN SKIPS THIS SLIDE AND THE TWO OTHER SLIDES THAT FOLLOW.]

Fig. 6.6 A portion of the transcript from Elsa's and Bessie's instruction of extensions in their respective classes, accompanied by the supporting slides the teachers referred to

slide that illustrated this misconception. Bessie, however, overlooked this and certified the correctness of the solution and even pointed out some features that she liked about it. Bessie only realised her mistake after I pointed it out to her after the class. She then said that she will correct this mistake in her next meeting with the class.

In an interview conducted after the PD implementation, Bessie owned up to her shortcomings.

> …I'd like to say … it was a personal mistake to sometimes be too confident with our meetings that I sometimes don't look at the slides right before a class, and then, you know, get surprised when something comes up… So I feel like I also have a mistake with the collaborative sessions because I became too confident… My promise to myself in the future is that should not make me prepare less.

It seemed that the shared goal and plan provided Bessie with a *false sense of security*. Confident that her participation in collective planning and the prepared instructional materials were enough to conduct the class, she did not take the time to *own* or *personalise* the lesson. But she also failed to own the problem in the sense that she failed to recognise the incorrect solution. This rendered her lesson lacking in connectedness to the goals it was intended to carry out.

We claim that the three events outlined in this trajectory of teacher learning are all essential for teachers' instruction to be positively influenced by the video club. Without the initial contact with the possibilities during sessions whether as a result of watching videos or participating in the ensuing discussions, teachers would not have been able to develop the necessary amount of interest or buy-in for new instructional approaches. Left on their own after the video club, we doubt that every teacher would have sufficient resources and resolve to reform or reorganise one's instruction according to the possibilities presented. Thus, a shared goal translated into a collective action plan was necessary to formalise and concretise the recognised possibilities. Finally, as was already expounded on, ownership and personalisation of this action plan is needed to tailor it more appropriately to one's class. Such a personalisation should, however, continue to be rooted on the intended goals that the possibilities featured.

Conclusion

As mentioned at the beginning of this chapter, the teachers considered the video club as among the components of the PD that had the most impact on them. An examination of their classroom practices vis-à-vis their video club participation – illustrated in this chapter by zooming in on teachers' instruction of extensions – revealed that the video club did indeed influence teachers' instruction as well, though the regularity, extent and quality of observed influence varied. Considering that the time devoted for the video club was relatively short, the teachers' receptiveness to the process and their efforts to bring their video club learning to their classrooms were nonetheless encouraging.

Viewing the results in the context of the larger PD, we could say that the video club was able to contribute to the PD programme's objectives of supporting teachers' MPS instruction. But we also acknowledge that it could still be better conducted for future undertakings. For instance, more time to conduct the video club could be negotiated to allow for more substantive discussions. In terms of choice of video clips, as the teachers become more comfortable with the group and the process, perhaps it need not be limited to featuring commendable instruction as there is also merit in viewing not-so-ideal images of teaching. Finally, drawing on the results of this study, to raise the efficacy of video clubs further in influencing teachers' classroom instruction, more opportunities for teachers to reflect on and articulate their resolve can be created. This can support their process of owning or personalising the collective goals or action plans that may arise from the video club process.

This chapter contributed to literature that endorses video clubs for teacher professional development (e.g. Sherin & van Es 2009) by offering a provisional theory on the trajectory of teacher learning from a video club. It also hinted at possible considerations when designing a programme which includes a video club. For instance, the choice of video clip is important as it defines the starting point of conversation. But while video recordings of actual classrooms can be powerful conveyors of reform ideas, reflection on video alone does not guarantee influence on practice. How the conversation points are followed up or linked to instruction also needs to be attended to. This includes considering how teachers own and personalise their instruction for the actual lesson.

It must be emphasised, however, that the provisional theory offered above was made based on the accounts of teachers participating in a video club which particularly focused on teacher instruction which was different from the focus in Sherin and van Es's works. Furthermore, they were made from the perspective of one who participated by watching a peer's video and not as the one being watched on video. A different conceptualisation of learning from video clubs might be required for the case when the video club focuses on other aspects of teaching or if it involves one whose classroom video clip was featured during the video club. We also consider this provisional theory of teacher learning in a video club as a conjecture open for further testing.

Appendices

Appendix A – Practical Worksheet Utilised in the PD

Problem

Instructions

- Work on the problem above by completing the worksheet doing stages I–IV.
- It may be necessary to return to certain stages.
- Articulate your thinking processes clearly.

1. *Understand the problem.*

   (You may have to return to this section a few times. Number each attempt to understand the problem accordingly as Attempt 1, Attempt 2, etc.)

   (a) Write down your feelings about the problem. Does it bore you? Scare you? Challenge you? Why?
   (b) Write down the parts you do not understand now or that you misunderstood in your previous attempt.
   (c) Write down your attempt(s) to understand the problem, and state the strategies you used to understand the problem.

2. *Devise a plan.*

   (You may have to return to this section a few times. Number each new plan accordingly as Plan 1, Plan 2, etc.)

   (a) Write down the key Math skills and ideas that might be involved in solving the problem.
   (b) Do you think you have the required knowledge to implement the plan?
   (c) Write out each plan concisely and clearly. State the heuristics to be used for solving.

3. *Carry out the plan.*

   (You may have to return to this section a few times. Number each implementation accordingly as Plan 1, Plan 2, etc., or even Plan 1.1, Plan 1.2, etc., if there are two or more attempts using Plan 1.)

   Write out each implementation in detail. Write down the conjectures that might arise from your attempts and provide justifications when necessary.

4. *Check and extend.*

   (a) Write down how you checked your solution.
   (b) Are you happy and confident with your solution? Write down an outline of at least one alternative solution that you can think of.
   (c) Give at least one extension of the problem. Briefly, give a possible solution method or strategy for at least one of the extensions you gave.

Appendix B – Short Description of the Video Clips Featured in the Video Club

	Video club 1	Video club 2	Video club 3	Video club 4	Video club 5
Clip #1: Teacher featured	Derrick	Bessie	Calvin	Elsa	Derrick
Short description	*Teacher asking student to articulate his solution better*	*Teacher preparing the class for problem-solving*	*Teacher monitoring students as they engage in problem-solving*	*Teacher asking class to review a peer's work*	*Teacher asking class what they considered important in problem-solving*
Clip #2: Teacher featured	– NONE –	Bessie	Calvin	Alice	Yvonne
Short description	Only one clip was shown in this video club	*Teacher focusing on the heuristics the students were using*	*Teacher giving students time to answer extensions*	*Teacher deconstructing a common misconception*	*Teacher asking class to explain reason behind a solution step*

Note: Frank expressed discomfort at being videotaped in the classroom. Thus, none of his classes were featured in the video clubs.

References

Anderson, J., Sullivan, P., & White, P. (2004). *The influence of perceived constraints on teachers' problem-solving beliefs and practices.* Paper presented at the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville.

Blanco, L. (2004). Problem solving and the initial practical and theoretical education of teachers in Spain. *Mathematics Teacher Education and Development, 6*, 31–42.

Cai, J., & Brook, M. (2006). Looking back in problem solving. *Mathematics Teaching, 196*, 42–45.

Charles, R. I. (1989). Teacher education and mathematical problem solving: Some issues and directions. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Charles, R. I., & Lester, F. K. (1984). *Teaching problem solving*. London: Edward Arnold.

Curcio, F. R., & Artzt, A. F. (2003). Reflecting on teaching mathematics through problem solving. In F. K. Lester (Ed.), *Teaching mathematics through problem solving: Prekindergarten – Grade 6* (pp. 127–141). Reston, VA: National Council of Teachers of Mathematics.

Department of Education [DepEd]. (2012). *K to 12 curriculum guide: Mathematics*. Pasig: DepEd Philippines. Retrieved from http://multilingualphilippines.com/wp-content/uploads/2012/06/MATHEMATICS-K-12-Curriculum-Guide.pdf

Derry, S. J., Pea, R. D., Barron, B., Engle, R. A., Erickson, F., Goldman, R., et al. (2010). Conducting video research in the learning sciences: Guidance on selection, analysis, technology and ethics. *Journal of the Learning Sciences, 19*(1), 3–53.

Foong, P. Y., Yap, S. F., & Koay, P. L. (1996). Teachers' concerns about the revised mathematics curriculum. *The Mathematics Educator, 1*(1), 99–110.

Grouws, D. A. (2003). The teacher's role in teaching mathematics through problem solving. In H. L. Schoen (Ed.), *Teaching mathematics through problem solving: Grades 6–12* (pp. 129–141). Reston, VA: National Council of Teachers of Mathematics.

Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development: A new consensus. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 127–150). San Francisco, CA: Jossey-Bass.

Hord, S. M. (2004). *Learning together, leading together: Changing schools through professional learning communities*. New York: Teachers College Press.

Kantowski, M. G. (1977). Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education, 8*(3), 163–180.

Leong, Y. H., Tay, E. G., Toh, T. L., Quek, K. S., & Dindyal, J. (2011). Reviving Pólya's "Look Back" in a Singapore school. *Journal of Mathematical Behavior, 30*, 181–193.

Lortie, D. C. (1975). *Schoolteacher: A sociological study*. Chicago: University of Chicago Press.

National Council of Teachers of Mathematics [NCTM]. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s*. Reston, VA: NCTM.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Pólya, G. (1945/1973). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton University Press.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *Journal of Mathematical Behavior, 22*, 405–435.

Romberg, T. A., & Kaput, J. A. (1999). Mathematics worth teaching, mathematics worth understanding. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 3–17). Mahwah, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Schoenfeld, A. H., Minstrell, J., & van Zee, E. (2000). The detailed analysis of an established teacher's non-traditional lesson. *Journal of Mathematical Behavior, 18*(3), 281–325.

Sherin, M. G. (2004). New perspective on the role of video in teacher education. In J. Brophy (Ed.), *Using video in teacher education* (pp. 1–27). Oxford: Elsevier Ltd.

Sherin, M. G., & Han, S. Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education, 20*, 163–183.

Sherin, M. G., & van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education, 60*(1), 20–37.

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. T. F. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in mathematics classroom. *Journal of Mathematical Behavior, 24*(3–4), 287–301.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Toh, T. L., Quek, K. S., Leong, Y. H., Dindyal, J., & Tay, E. G. (2011). *Making mathematics practical: An approach to problem solving*. Singapore: World Scientific Publishing Co. Pte. Ltd.

Van de Walle, J. A. (2003). Designing and selecting problem-based tasks. In F. K. Lester (Ed.), *Teaching mathematics through problem solving: Prekindergarten – Grade 6* (pp. 67–80). Reston, VA: National Council of Teachers of Mathematics.

Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.). Boston: Pearson/Allyn and Bacon.

van Es, E. A. (2009). Participants' roles in the context of a video club. *The Journal of the Learning Sciences, 18*, 100–137.

van Es, E. A. (2012). Examining the development of a teacher learning community: The case of a video club. *Teaching and Teacher Education, 28*(2), 182–192.

van Es, E. A., & Sherin, M. G. (2010). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education, 13*(2), 155–176.

Vescio, V., Ross, D., & Adams, A. (2008). A review of research on the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education, 24*, 80–91.

Wilson, J. W., Fernandez, M. L., & Hadaway, N. (1993). Mathematical problem solving. In P. S. Wilson (Ed.), *Research ideas for the classroom: High school mathematics* (pp. 57–78). New York: Macmillan Publishing Company.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. *Review of Research in Education, 24*, 173–209.

Wu, M., & Zhang, D. (2007). An overview of the mathematics curricula in the West and East. In F. K. S. Leung, K. Graf, & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions – A comparative study of East Asia and the West* (pp. 181–193). New York: Springer.

Chapter 7
Video as a Medium for Introducing Lesson Study to Multi-ethnic Malaysian Mathematics Teachers

Chap Sam Lim and Liew Kee Kor

Abstract This chapter discusses how edited videos were used as a medium to introduce the concepts and processes of Lesson Study to 54 multi-ethnic Malaysian teachers as part of their professional development. These teachers were teaching in one of the three different types of vernacular primary schools which were underperforming in mathematics and science. The chapter begins with a brief introduction of the Malaysian school system, the recent decline of the Malaysian student performance in international studies and thus the need to upgrade the teacher quality. The Japanese model of Lesson Study was chosen, but to address the language proficiency issues of the participating teachers, various strategies were used to improve teachers' engagement with the contents of the edited videos during the introductory workshop. The chapter concludes that video can be an effective medium; however, teacher commitment and administrators support remain the most important factor for the success of any professional development programme.

Introduction

Malaysia is a multi-ethnic country with three major ethnic groups: Malays (60 %), Chinese (26 %) and Indians (7 %). The Malaysian school system is made up of four levels: primary (Grade 1–6), lower secondary (Grade 7–9), upper secondary (Grade 10–11) and form six or matriculation (Grade 12–13). To cater to the linguistic needs of the various multi-ethnic groups, parents can choose to enrol their children in any of the three types of primary schools. These are (a) Malay medium national schools

C.S. Lim (✉)
School of Educational Studies, Universiti Sains Malaysia, Pulau Pinang, Malaysia
e-mail: cslim@usm.my

L.K. Kor
Department of Computer & Mathematical Sciences, Universiti Teknologi MARA, Shah Alam, Malaysia
e-mail: korlk564@kedah.uitm.edu.my

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_7

(SK) where the medium of instruction is the Malay language, (b) Chinese medium national-type schools (SJKC) where the medium of instruction is Mandarin and (c) Tamil medium national-type schools (SJKT), Tamil language being the medium of instruction. Although the medium of instruction may be different, all primary schools share the same curriculum. However, at the secondary level, all schools deliver a common curriculum using the Malay language, the lingua franca of the land.

Although all teachers have to be fluent in the Malay language, most vernacular school teachers tend to be more fluent in their mother tongue. While teachers are expected to be able to communicate in English, they are challenged to do so. For many, the English language would be their third language. Their lack of command of the English language poses a challenge particularly when teachers are introduced with new ideas and practices. They find it difficult to engage with the literature, particularly when English is used to communicate these new ideas.

The continuous decline of the Malaysian student performance in the international comparative studies such as the Trends in International Mathematics and Science Study [TIMSS] has also caused great concerns among the Malaysian society at large. The disconcerting fact is that the percentage of those who failed to meet the minimum proficiency level has increased from 18 % in the TIMSS-2007 to 35 % in TIMSS-2011. Students who performed below the low benchmark were identified as possessing only limited mastery of basic mathematical concepts and mathematics achievement. Table 7.1 shows the level of mathematics performance among the Malaysian Grade 8 students in TIMSS 2003–2011.

Various factors could be offered to account for these students' poor performance in the examination, for example, low-quality teaching, lack of interest and motivation among teachers and students, as well as school culture. The key findings from the TIMSS 2011 revealed that higher average achievement was associated with schools that execute rigorous curricular goals and teacher factor (such as an effective teacher). Indeed past studies on classroom and school factors have suggested that teacher effects account for a large part of variation in mathematics achievement. For example, Rice (2003) alleges that teacher quality exerts the most influence on student achievement. Akiba, LeTendre and Scribner (2007) conducted a cross-national study using TIMSS 2003 data from 46 countries on teacher quality based

Table 7.1 The Malaysian 8th graders at the international benchmarks of mathematical achievement from 2003 to 2011

Year		Advanced benchmark	High benchmark	Intermediate benchmark	Low benchmark
	Range of total scores	Above 625	550	475	400
2003		6 % (1.0)	30 % (2.4)	66 % (2.1)	93 % (0.9)
2007		2 % (0.5)	18 % (2.1)	50 % (2.7)	82 % (1.9)
2011		2 % (0.4)	12 % (1.5)	36 % (2.4)	65 % (2.5)

Source: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS), 2003, 2007 and 2011
Note: ( ) standard errors are reported in parentheses

on four characteristic indicators: (a) full certification, (b) mathematics major, (c) mathematics education major and (d) teaching experience of 3 or more years. They found that countries with better teacher quality produced higher student mathematics achievement. Owings, Kaplan, Nunnery, Marzano and Myran (2006) attest that teacher quality is the most important in predicting student achievement outcomes. The McKinsey's report (Barber & Mourshed, 2009) that documented the International Education Roundtable discussion from seven school systems around the world (Singapore, Canada, Hong Kong, the People's Republic of China, Sweden, the United States and Australia) recognises the impact of high-performing teachers on student learning. It pointed out empirically that learning with a high-performing teacher can make a 53-percentile difference for two students who start at the same achievement level in a period of 3 years (see Fig. 7.1).

Sensing the importance of teacher quality and student achievement, the Malaysian government has taken great efforts to stem the decline in students' performance by addressing the need to improve the quality of mathematic teaching. In the Malaysian Education Blueprint (MEB) (2013–2025), the Ministry has identified 11 shifts to transform the current education system. These shifts may represent a change in strategy and direction or operational changes of the implemented existing policies. It is stated in the blueprint that "Each shift will address at least one of the five system outcomes of access, quality, equity, unity, and efficiency, with quality as the common underlying focus across all shifts due to the fact that this is the dimension

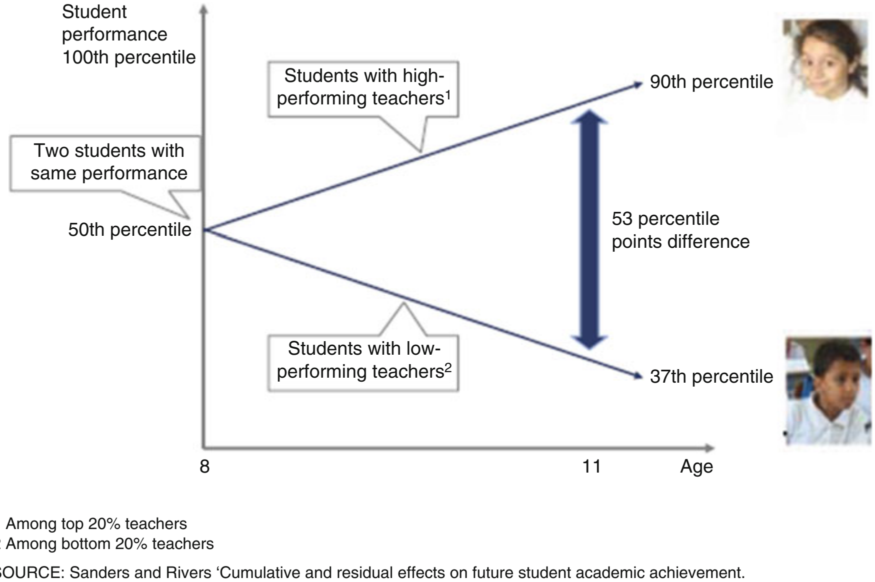


Fig. 7.1 Teacher quality: the determinant of student outcomes (Source: cited in Barber and Mourshed (2009). *Shaping the future: How good education systems can become great in the decade ahead*. p. 27)

which requires the most urgent attention" (Ministry of Education, 2012, p. E19). The 11 shifts are:

Shift 1: Provide equal access to quality education of an international standard
Shift 2: Ensure every child is proficient in Bahasa Malaysia and English language
Shift 3: Develop values-driven Malaysians
Shift 4: Transform teaching into the profession of choice
Shift 5: Ensure high-performing school leaders in every school
Shift 6: Empower State Education Departments, District Education Departments, and schools to customize solutions based on need
Shift 7: Leverage ICT to scale up quality learning across Malaysia
Shift 8: Transform Ministry delivery capabilities and capacity
Shift 9: Partner with parents, community, and private sector at scale
Shift 10: Maximize student outcomes for every ringgit
Shift 11: Increase transparency for direct public accountability

The fourth shift in the latest MEB recognised the crucial role played by the teachers and the need to upgrade their quality by continuous professional development (CPD). The plan is "to upgrade the quality and personalization of CPD from 2013 with greater emphasis on school based training" (Ministry of Education, 2012, p. E15). This fourth shift recognised not only the important role of continuous professional development but also the significant advantage of school-based professional development.

In Malaysia there is a research that focused on the challenges faced by teacher educators (Goh, 2012), student teachers' efficacy (Bakar, Mohamed, & Zakaria, 2012) and teacher quality and performance (Abd Hamid, Syed Hassan, & Ismail, 2012). Goh (2012) deliberated the subject of the use of the Malaysian Teacher Standard (MTS) to improve teacher quality. She highlighted three challenges faced by the Malaysian teacher educators in aligning their practice with the standards. The first is transforming ingrained beliefs, values and biased perceptions of the student teachers. The second is the guiding principle of applying the MTS standards to teaching, while the third is the actual training of teachers. Bakar et al.'s (2012) study on 675 final-year student teachers found that in general the respondents scored high in teaching efficacy and they were also highly efficacious in student engagement, instructional strategies and classroom management. Abd Hamid et al. (2012) investigated the validity of a structural model of teacher's cognitive abilities and personalities in predicting teacher's performances through the use of structural equation model (SEM). Based on a large random sample of 1366 teachers from different types of schools, they obtained a model fit with both cognitive abilities and personality predicting classroom management. However, research investigating the relationship between teacher quality and pupil achievement is still scarce.

Unlike other professions such as law, accounting or medicine, teaching has been "a profession without a practice" (Barber & Mourshed, 2009, p. 30). The McKinsey report (Barber & Mourshed, 2009) reveals that successful education systems are those that create more opportunities for teachers to "work together in sharing practice and research, developing lessons plans, and building consensus on what constitute good teaching practice" (p. 30). It highlighted specifically the importance and function of the Professional Learning Communities (PLCs), a means of collaborative professional development, which provides a reference with which teachers work towards achieving the consensus about what constitutes good teaching practice. An example is the practice of teachers working together to analyse and develop model lessons in Japan.

Teacher education is a lifelong affair. As such to produce good quality teachers, teacher education must not cease at pre-service. All in-service and practising teachers need to engage with continuous professional development where they are kept abreast with best practices as well as how to address the needs of the younger generation and to prepare the younger generation to face the needs of the twenty-first century. However, the teacher professional development programme in Malaysia was usually provided and sponsored by the Ministry of Education (MOE), yet conducted in a *one-shot* and *top-down* manner. Short courses or workshops which lasted from 1 day to 2 weeks were usually conducted by the Curriculum Development Department, Teacher Education Division, State or District Education Department, under the auspices of MOE. One teacher from each selected schools was then instructed by the school administrator to attend these courses or workshops. Very rarely do teachers volunteer themselves to attend these courses as most of them do not see this as an opportunity for them to improve their professionalism, but rather as a burden or extra workload. Most of these courses are given as “recipes” to be brought home to be shared with their peers. A typical professional development in Malaysia is dominant with the facilitator presenting the theory/ideas/information through Power Point presentation. For example, in a week-long Geometer’s Sketchpad (GSP) workshop, teachers were shown the power of GSP and were taught the basic skills of using GSP. These teachers were usually excited with the new software and keen to bring back the new ideas to share with their colleagues. Upon completing the courses/workshops, the participating teachers are expected to conduct an in-house training usually of 2 h duration (on Saturday or in the afternoon) for their colleagues. However, most of the time, these teachers are not shown how to deliver an innovative lesson based on what they have learnt. These teachers have no grounded images of how an authentic lesson looks like. They have to struggle to integrate what they have learnt into their daily lessons. Very rarely do they receive continuous support from the facilitators or organisers. Neither is there peer collaboration from their colleagues because usually only one teacher is sent for the course. Thus, without any form of support, the teacher becomes disillusioned and the knowledge gained from the workshop remains as knowledge held by that teacher.

Thus, there is an urgent need to search for a suitable model of teacher professional development that can attend to the aforementioned two issues: providing examples that how a lesson can be carried out and that encourage teachers to work together to improve their lessons. A literature search shows that one of the key contributing factors behind the success of the Japanese students in TIMSS and PISA is the Japanese teacher professional development practice of Lesson Study (LS). We anticipate that LS is an effective means that provides a platform for in-service or practising teachers to discuss and to enhance their pedagogical knowledge and teaching practice and hence improve their teaching quality. Videos of the processes of how to conduct LS are likely to provide grounded images which could give participating teachers an idea how to conduct innovative practices.

This chapter discusses how videos were used to introduce the concept and processes of Lesson Study to seven primary schools which were underperforming in mathematics and science as part of their professional development. This project comprised two major components: developmental and research. The developmental component aimed to introduce LS as a teacher professional development model to some low-performing schools so as to improve the teaching quality of science and mathematics teachers. The research component attempted to investigate the effect of LS on teachers' teaching quality. Hence, in the following sections we provide a brief history of LS and the background to the LS project and the challenges faced in introducing LS to Malaysian teachers. In the Research Design section, we discussed how videos were used to introduce the concept of LS to multi-ethnic Malaysian teachers and details of the participating schools. Strategies to improve teachers' engagement with the contents of the edited videos are discussed in the section "The Use of Video for Teacher Professional Development". The learning acquired by the teachers is discussed in the penultimate section. The concluding section summarises the findings of this project.

Lesson Study (LS)

Lesson Study has gained increasing attention from mathematics educators since the release of the TIMSS Video Study (Stigler, Gonzalez, Kawanaka, Knoll and Serrano, 1999 cited in Doig & Groves, 2011). This form of professional development has spread beyond Japan to countries such as the United States, the United Kingdom, South America, South Africa, Australia as well as the Southeast Asian countries such as Thailand, Indonesia and Malaysia (Lewis, Perry, & Hurd, 2009).

LS is a primary form of professional development in the Japanese elementary schools. The Japanese LS "jugyokenkyuu" (授业研究) is favourably known as LS (Yoshida, 1999) or Research Lesson (Lewis and Tsuchida 1997). Makinae's (2010) research on the origin of LS (LS) recorded that in the early 1900s, a number of local boards of education in Japan held conferences called "Jugyo-hihyo-kai (criticism lesson conference)" or "Jugyo-kenkyu-kai (LS conference)". Since then, the Japanese teachers have been conducting LS by researching their own practice in school-based in-service training called "konaikenshu" 校内研修 to encourage teachers to develop, to share as well as to critic new teaching methods. Unlike most of the teacher training development activities, LS is neither funded nor mandatory and is organised by teachers themselves in their own school with the purpose to create changes in teachers' knowledge and beliefs, enhancing professional community, and to enrich teaching–learning resources (Doig & Groves, 2011).

Unperturbed by differences in cultural setting, a LS cycle is always conducted around four phases, that is, investigation, planning, research lesson and reflection (Fernandez & Yoshida 2004; Lewis, 2002). Lewis et al. (2009) also rationalised that because LS is a locally designed process, there are some variations in the features emphasised in a LS cycle. Drawing on various LS research and studies on

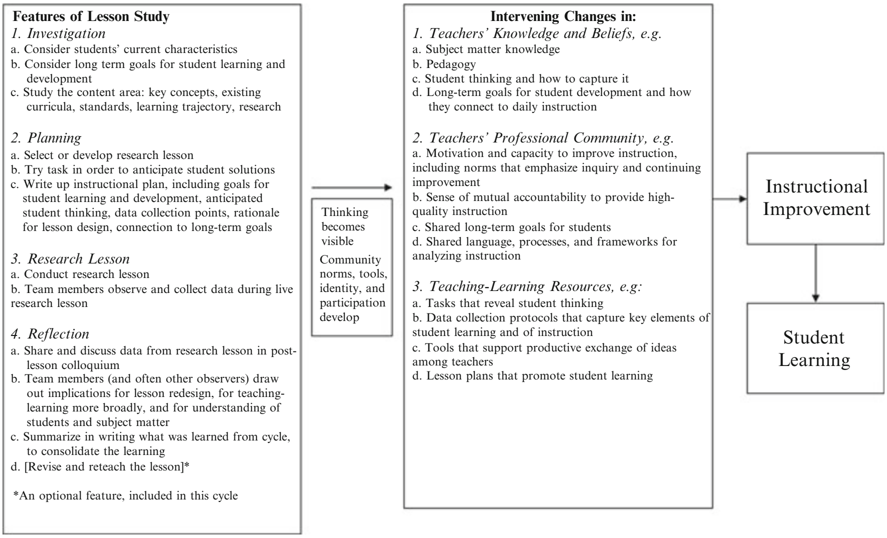


Fig. 7.2 A theoretical model of LS (Source: Lewis, Perry, & Hurd (2009). *Improving mathematics instruction through Lesson Study: a theoretical model and North American case*, p. 287)

mathematics teachers' professional learning, Lewis et al. have developed a theoretical model of LS and used it to examine the features and impact of a North American LS case (see Fig. 7.2).

Lewis et al. (2009) used the above theoretical model to investigate whether and how LS impacts teachers' knowledge and beliefs, professional community and teaching–learning resources. The model was used to examine data from a US lesson to prove the effectiveness of LS outside Japan. Data for the study included videotapes of all participants from five different schools in one LS group as they planned, taught, observed, revised and retaught a research lesson. Videotapes of all group meetings and both research lessons and written lesson plans were also collected. Lewis et al. also used the naturally occurring artefacts of LS, the publicly available video with provided transcripts as a source of reference about LS. Their findings strongly concurred with Zawojewski et al. (2008) that videotaping is useful in teaching individuals about "LS that is not yet well understood and where individual teachers are expected to differ in what is learned" (Lewis et al., 2009, p. 289).

The LS Project: Introducing LS to Mathematics Teachers in Malaysia

After much review from the literature, LS offers to be a good option for teacher professional development model for Malaysian teachers since it is school based and could provide continuous support to allow teachers to grow professionally. However, LS is a new concept to many Malaysian teachers. In particular, LS encourages collaborative culture of planning the lesson together, observing peer's teaching and reflecting for professional improvement. Most Malaysian teachers tended to plan their own lessons and teach their own classes. They seldom observe other teachers' teaching except for administrative and evaluation purposes such as for promotion exercises or annual appraisal. Therefore, introducing LS is like inculcating a new culture among teachers. It was going to be challenging to explain this new concept to the teachers. Without observing how the process of LS was developed among a group of teachers, teachers would be unclear about the processes of LS. If they were unclear about the processes, it was highly unlikely they would attempt to try it out in their schools as part of their professional development, much less implement the process. Despite these challenges we believe that it is plausible to engage Lewis et al.'s (2009) work on using video to introduce LS to the Malaysian teachers who are from a different cultural background of the Japanese and of diverse ethnicity.

In the past, some countries who wanted to learn about LS would send their teacher educators or teachers to Japan to learn about LS. Or else, they invited exponents of LS to their country to share their experiences with the local teachers what is LS and to conduct LS together with the local teachers. Although these approaches are useful, they are too costly and do not reach a wider section of teachers.

In Malaysia, there are a number of obstacles that hinder the use of such approaches. The prohibitive costs of sending teachers and teacher educators to Japan rule out that option. Furthermore such a methodology benefits only a few. The issue of selecting the right participants to attend such courses further complicates the problem. However, the most deterring factor is the issue of language. Most Malaysian primary mathematics teachers are conversant in Malay and their mother tongue. English language is seldom their medium of communication at work and daily life. Thus, sending teachers to Japan defeats the purpose as most of these teachers are neither proficient in Japanese nor English. It would not be effective to invite proponents of LS to Malaysia as most of these experts can only speak Japanese and English. Therefore, the more practical alternative is to show videos of Japanese teachers conducting LS and to have explanatory notes in either Malay or Mandarin (depending on the type of primary schools) language accompanying these videos. Showing videos of the LS process in this way would clearly help to reach many more of the local teachers.

Method

Participating Schools, Teachers and Students

In 2011, under the Tenth Malaysia Plan (2011–2015), all schools were to be evaluated and ranked according to performance band using a standard quality instrument known as Standard Quality Education Malaysia (SQEM) (EPU, 2010). In the ranking process, it is stated that "the performance of all public schools will be assessed and ranked annually according to a composite score. The composite score comprises the Grade Point Average based on the school's performance in public examinations and the Standard for Quality Education in Malaysia, which measures the quality of teaching and learning, organizational management, educational program management and student accomplishment" (EPU, 2010, p. 204). Figure 7.3 displays the summary approach for ranking schools.

MOE has classified schools into high performing (Band 1 and 2), average performing (Band 3, 4 and 5) and low performing (Band 6, 7 and 8). Low-performing schools were schools whose students attained below the 40 % level of achievement in the sixth-grade Primary School Assessment Test (Mohd Sofi Ali, 2003). The classification was intended to motivate schools to do better and to allow the MOE to target resources to schools that need the most support. The head teachers and principals of high-performing schools (HPS) will receive a reward for achieving significant improvements in the performance of their schools, while the 10 % bottom-performing head teachers and principals will be receiving remedial and developmental training to help them improve their school performance.

In this project, low-performing primary schools were chosen as participating schools. Initially a total of nine primary low-performing schools in three northern states of Malaysia – Kedah, Penang and Perak – were chosen to participate in this project. The nine schools were made up of three types of primary schools: (a) Malay medium national schools (SK), (b) Chinese medium national-type schools (SJKC) and (c) Tamil medium national-type schools (SJKT). However, after the introductory workshop, two of the three SK schools withdrew from the project due to lack of administrator support or teacher commitment and time constraint.

Although one of the SK school principals has agreed to participate in our project, she failed to get the full commitment of her teachers. She was newly appointed as a principal

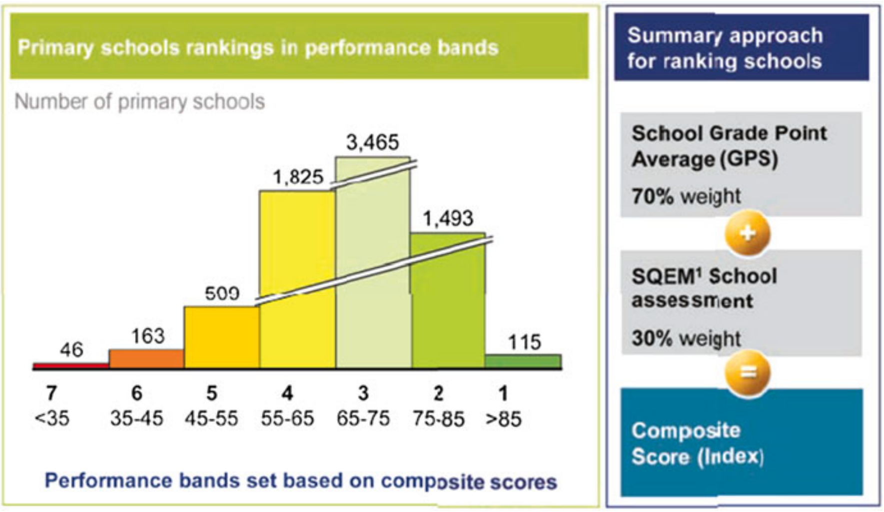


Fig. 7.3 Summary approach for ranking schools (Source: *10th Malaysia Plan Chapter 5: Developing and retaining a first-world talent base*, p. 204). [1]Standard Quality Education Malaysia (SQEM) is the standard quality instrument for evaluating schools based on four main dimensions (vision and mission, organisational management, educational programme management and pupil management)

to that school. When we first approached her to participate in the project, she was just 3 months into her tenure as principal of the school. All the 24 mathematics and science teachers of the school participated in the introductory workshop and all of them responded positively to the idea of LS. However, when the administrator identified the three mathematics and three science teachers who were to form the LS group, they protested against their selection. They questioned why they were chosen and why they were not informed about this project. In essence they were unhappy as their participation in the project meant that they had to stay back in the afternoons for the discussion meetings and lesson planning. They could not see how they could have time to participate in this LS project when they were already loaded with too much teaching and administrative responsibilities. However, we had to relinquish this SK school as the administrator and the project team failed to convince these teachers that their participation would benefit.

As for the second SK school which withdrew from the project, the issue was slightly different. Both the administrators (particularly the senior assistant) and the participating teachers responded positively to our project initially. After the introductory workshop, we planned out the dates for lesson planning and discussion. However, before the date, the senior assistant asked to postpone the date as the school was busy with other programmes. After two postponements, the senior assistant suggested that their school would begin the LS cycle in the following year. Unfortunately when we called to make an appointment with the senior assistant in the following year, we were informed that both the principal and the senior assistant had been transferred to another school. We approached the new principal and he said he would ask his teachers to see if they would like to continue with the project. Sadly, after a month, when we called him again, he confirmed that his teachers could not join the project as they were all heavy loaded with teaching tasks and lack of time.

Thus, the project ended with seven schools. Each school has two LS groups: one for mathematics and one for science. However, the only participating SK school has only one science teacher; thus, that school has only one mathematics LS group. Altogether the project had 13 LS groups, each group made up of three to five mathematics or science teachers. So, a total of 54 mathematics and science teachers as well as 39 classes of students were involved in the project.

Research Design

Because LS is a new concept, the initial plan was to conduct an introductory workshop on LS to be attended by all the teachers and school administrators of the participating schools at one centre. However, because it was difficult to arrange for one session where all participants could meet due to logistics and time constraint, we changed our plan to have the introductory LS workshop in each of the participating schools. In total we conducted nine LS workshops to the nine schools at different times during the months of January to May of 2011. This approach demanded much more of our time and as a result we had to extend the data collection for this project from 1 to 3 years. Nevertheless, the repeated workshops provided us the chance to revise our approach of giving the introductory workshop. Further details will be discussed in the next section.

After the introductory workshop, each LS group carried out three LS cycles for the stipulated time. Each cycle consisted of the following steps:

(a) Identifying the goal of the lesson.
(b) Planning the lesson collaboratively.
(c) Refine the lesson plan together.
(d) One of the LS members teaches, while the other members observe the lesson.
(e) Immediately after the lesson, all members reflect and revise on the taught lesson.
(f) Revise the lesson and reteach the lesson to another class if there is a need or possibility.

Step (f) was optional because, except one SJKT, all the participating schools have only one class in one grade; therefore, no reteaching was possible. For the one SJKT, reteaching was carried out for all the three cycles. Each step was carried out in one visit/meeting, but step (d) and (e) was carried out in one visit as reflection on the lesson was carried out immediately after the lesson observation.

During the first LS cycle, the researchers were involved in all the first five steps stated above. But the involvement was slowly reduced in the second cycle and further reduced in the third cycle. In the second cycle, the participating LS groups were expected to identify the goal of the lesson and planned a draft lesson plan before the researchers came in. The researchers were involved in refining the lesson plan, observing and reflecting on the lesson. For the third cycle, the researchers participated only in the lesson observation and reflection. Reducing the involvement of the researchers was strategic as this encouraged the teachers to gain confidence in the project, have more ownership over the project and hopefully be able to conduct the LS by themselves.

The Use of Video for Teacher Professional Development

This section discusses how videos were used to introduce the concept and processes of LS to the participating teachers.

During the LS Introductory Workshop

The concepts and implementation of LS are new to many Malaysian teachers. To provide the participating teachers a better idea of what is LS and how to carry out LS, we showed the participants an edited Japanese video of LS which was prepared and edited by Yoshida & Fernandez (2002). The video lasted 20 min and 54 s. It comprises three main parts:

Part I: What is LS? The video explained briefly the concepts of LS and the three main activities involved in LS. These include (1) creating LS goal, (2) working on "Study Lesson" and (3) producing a LS report.

Part II: Taking a closer look at the three main activities. The video explained in detail each of the three main activities. For example, in Activity One, teachers in the LS group attempted to set the goal of their LS by identifying the gap between what the missions or goals of the school and what the students already have achieved. Three exemplary goals were given to illustrate. In Activity Two, the process of how a LS group was formed from a small group of teachers, ideal size of 4–6 teachers, from the process of collaboratively planning the lesson, to defining the lesson plan, to teaching and observing the lesson till the reflection on the taught lesson was described and elaborated step by step. The optional step of reteaching the lesson by different teachers to different groups of students and the involvement of external adviser were also explained. Activity Three shows a LS report was compiled and produced from the collection of teachers' detailed lesson plan, minutes of the discussion, some examples of students' work as well as teachers' reflection about the LS.

Part III: Sharing the LS idea. This part explains how the idea of LS can be shared through LS Open House and publication of reports. During the LS Open House, principals and teachers from other schools were invited to observe a teaching lesson and then an open discussion after the observation. Lesson plan and teaching materials were provided to the guests and feedbacks from these guest teachers were highly invited. In Japan, the LS reports were commonly published and distributed widely in the market. It seems that Japanese teachers published more than the researchers.

Subtitles and Commentaries in Appropriate Languages

The commentary accompanying the video was in American English without subtitles. Because the Malaysian participants have limited command of the English language, we edited the video clips by providing English subtitles (see Fig. 7.4). The subtitles were meant to aid the participants' understanding of the spoken language as they were not used to the American accent. However, many of them still could not understand the English subtitles. To enhance their comprehension, we provided further explanation using the language most familiar to the teachers. For example, in

Fig. 7.4 Video clips with added English subtitles to aid the participants' oral understanding

the SK schools, we used Malay language to provide further commentaries to the videos; Mandarin was used in SJKC, and both English and Malay mixed with some Tamil language were used in SJKT. It might be noteworthy to acknowledge that we did not provide the video clip with subtitles in the different languages, as it was prohibitive to do so. The English subtitles were provided in printout when we bought the LS introductory video by Yoshida and Fernandez (2002). We believe most of the teachers could understand the video although some of them were not so proficient in English. We believe the added English subtitles and explanation in different languages should be sufficient to help the teachers.

To provide further explanations, we used a Power Point presentation which was presented in various languages, such as Malay or Mandarin or English depending on the type of schools. The Power Point slides displayed key points of our discussion. The slides can be divided into three major parts. The first part discussed the need to have teacher professional development. The second part explained the concepts of LS and the third part provided examples in the form of photographs of LS projects conducted in Malaysia and some APEC countries, such as Thailand, Singapore and Japan.

Providing More Structure Helped Focus Participants' Attention

This section discusses how providing greater structure to the video watching session helped participating teachers focus their attention on the contents of the video.

We used the video (Yoshida & Fernandez, 2002), to explain to the participants, step by step, the process of LS. During the introductory workshop to the first school, we showed the whole video in its entirety. However, we noticed that some participants lost their concentration after 3–4 min into the video. After the whole video was shown, we still have to replay some parts of it to help teachers recall the contents of the video. Many of the teachers could not grasp the full meaning of the video, either due to failure to engage with the English language or lack of focus. Therefore, we adopted a different strategy when we conducted the workshop to the second school. We paused the video at certain strategic points and discussed with the participants the major points in the next video clip. For example, a Power Point slide with this question prompted the participants to focus on the contents of the next video clip: "What is Activity Two?"After showing the Activity Two, the video was paused again and the main points about Activity Two were discussed as shown in the following slide (see Fig. 7.5). Our experience showed that pausing the video at strategic points and then explaining or discussing the contents was effective in not only focusing participants' attention to the specific contents of the video but also aided in their understanding through highlighting specific aspects/issues. In fact, one of the participants commented at the end of the introductory workshop that if merely watching the video, he had only a superficial understanding of its contents. However, the explanation and highlighted key points helped him acquire a deeper understanding of the contents.

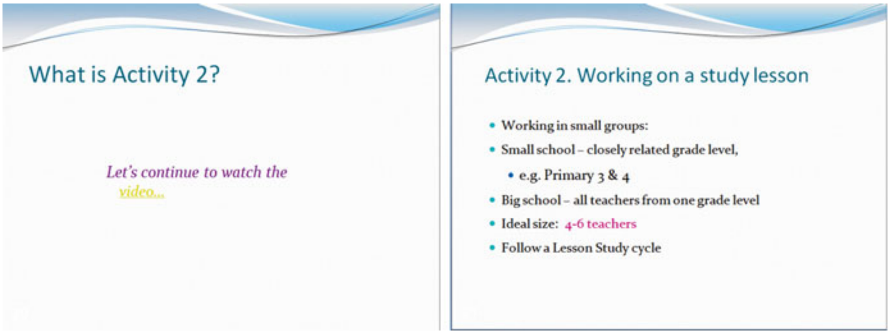
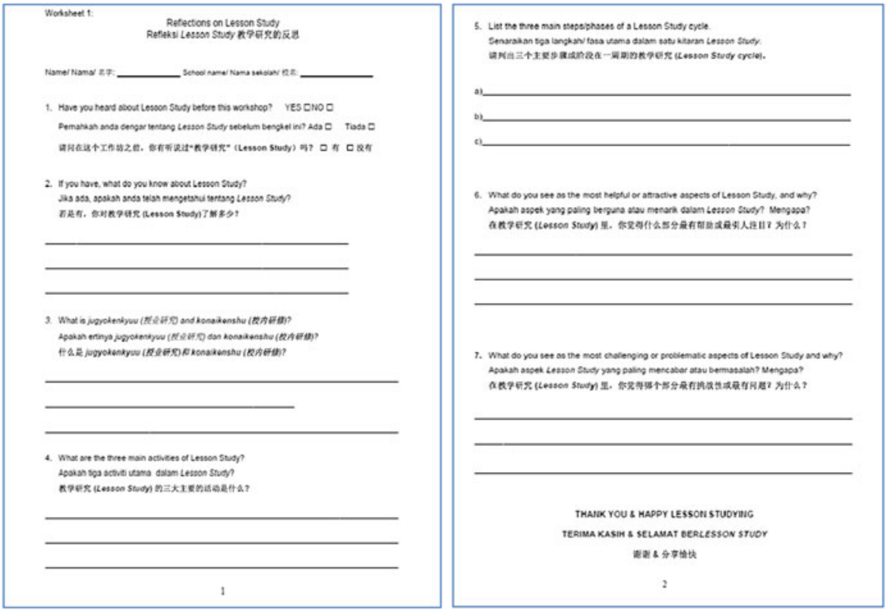


Fig. 7.5 Power Point slides providing prompts which helped to focus participants' attention on the contents of the videos



Worksheet 1:

Reflections on Lesson Study
Refleksi *Lesson Study* 教学研究的反思

Name/ Nama/ 名字: ____________ School name/ Nama sekolah/ 校名: ____________

1. Have you heard about Lesson Study before this workshop? YES ☐ NO ☐
   Pernahkah anda dengar tentang *Lesson Study* sebelum bengkel ini? Ada ☐ Tiada ☐
   请问在这个工作坊之前，你有听说过"教学研究"（Lesson Study）吗？ ☐ 有 ☐ 没有

2. If you have, what do you know about Lesson Study?
   Jika ada, apakah anda telah mengetahui tentang *Lesson Study*?
   若是有，你对教学研究 (Lesson Study)了解多少？

3. What is *jugyokenkyuu* (授业研究) and *konaikenshu* (校内研修)?
   Apakah ertinya *jugyokenkyuu* (授业研究) dan *konaikenshu* (校内研修)?
   什么是 *jugyokenkyuu* (授业研究)和 *konaikenshu* (校内研修)?

4. What are the three main activities of Lesson Study?
   Apakah tiga activiti utama dalam *Lesson Study*?
   教学研究 (Lesson Study) 的三大主要的活动是什么？

1

5. List the three main steps/phases of a Lesson Study cycle.
   Senaraikan tiga langkah/ fasa utama dalam satu kitaran *Lesson Study*.
   请列出三个主要步骤或阶段在一周期的教学研究 (*Lesson Study cycle*)。

a)
b)
c)

6. What do you see as the most helpful or attractive aspects of Lesson Study, and why?
   Apakah aspek yang paling berguna atau menarik dalam *Lesson Study*? Mengapa?
   在教学研究 (*Lesson Study*) 里，你觉得什么部分最有帮助或最引人注目？为什么？

7. What do you see as the most challenging or problematic aspects of Lesson Study and why?
   Apakah aspek *Lesson Study* yang paling mencabar atau bermasalah? Mengapa?
   在教学研究 (*Lesson Study*) 里，你觉得哪个部分最有挑战性或最有问题？为什么？

THANK YOU & HAPPY LESSON STUDYING
TERIMA KASIH & SELAMAT BER*LESSON STUDY*
谢谢 & 分享愉快

2

Fig. 7.6 Response sheet on LS (Worksheet 1). There are seven prompting questions written in three languages: English, Malay and Mandarin

Using Response Sheet (Worksheet 1)

Next, we also realised that it was more effective when participants were given a response sheet which they have to complete after they watched the video. Figure 7.6 shows an example of a response sheet (Worksheet 1) to the video "Introduction to LS". The response sheet contains seven questions. Questions 3, 4 and 5 directed the participants' attention to the content of the video. If the participants paid attention to the contents of the video, they would be able to answer these questions. More noteworthy was the strategy where before watching the video, the participants were given the response sheet which required them to find answers from the video. This strategy encouraged the participants to pay more attention and became more focused while watching the video than if they were just asked to watch a video. Questions 6 and 7 aimed to obtain their feedback about the attractive aspects of LS (if there is any) as well as the issues and challenges foreseen by these participants.

Again, to address the language needs of the participants from different ethnic groups, the response sheet was printed with three languages: English, Malay and Mandarin. The participants were also encouraged to write in whatever languages that they were most competent with.

Using Local Example of LS

Besides using the Japanese example, the participants were also shown a video of a Malaysian model of LS. The choice of example was very important as the contents could influence the participants differently. The video with Japanese content provided the participants an authentic example of how LS was carried out in Japan but a video with Malaysian content would suggest to the participants that it was possible to try innovative pedagogies in a Malaysian classroom and in fact, some Malaysian teachers have carried out LS in Malaysia. Videos with local content are more convincing than those examples with foreign content (e.g. video with Japanese content) because they offer empirical evidence that LS is not culture specific. When participants were provided evidence that LS was conducted successfully in local context, the participants may be more confident to try it out in their own schools. The argument that it would only work for Japanese teachers was no longer valid.

Using Edited Mathematics Teaching Video for Promoting Reflection During Teacher Professional Development Programme

For most Malaysian teachers, the act of self-reflection on one's teaching is not a common practice of their teaching experience. Tee's (2007) study showed that Malaysian secondary mathematics teachers practised reflective thinking moderately and there was no significant differences of reflective thinking practices based on teachers' background factors. The findings also revealed that time constraint was a factor which significantly affected reflective thinking practices. In a qualitative study by Siti Mistima and Effandi Zakaria (2010), their finding shows that the mathematics teachers managed to reflect on their classroom teaching but failed to identify the right course of action to overcome the confronted problem.

Because it was difficult to ask participants to reflect on their own teaching during the LS introductory workshops as it was not possible to discuss their own teaching, we took the opportunity to encourage the participating teachers to observe the practice of other teachers and to use these examples as stimuli to encourage them to reflect on their teaching. Also it is easier to comment on the work of others than to engage in self-critique. Therefore, the LS project provided us with the opportunity to encourage participants to reflect on the teaching of others. After understanding the general concepts and process of conducting LS, another important feature of the LS project was to encourage teachers to reflect on the observed lesson. To demonstrate to the LS participants how to reflect and to experience reflection, we showed an edited video of a primary mathematics lesson. To cater for the language diversity of each type of primary school, different video clips were used for each type of school. For example, we used an edited video lesson from SK for the SK schools and likewise for the other schools. There were six edited videos available. They were collected from another project on characteristics of effective mathematics teaching (see Lim & Kor, 2012 for details). Each edited video clip was about 12 min in length. To enhance the reflective practices, we also provided the participating teachers with the lesson plan (Fig. 7.7 shows one example of a lesson plan on the topic mass) and a list of guiding questions (as in Fig. 7.8: Worksheet 2).

Therefore, the guiding questions provided the participants with some reference points or basis for reflection. For instance, Question 1 “What do you think about this lesson?” sought the participants’ general impression or first reaction after observing the lesson. This open question encouraged the participants to express their opinions about anything that they considered as significant. Questions 2 and 3 focused participants’ attention on students’ learning. By asking the participants to offer examples

Mathematics Lesson Plan (Standard 5 students)

Topic: Mass

Unit: Compare mass of objects

Date:
Time:

No. of Students: 34

Lesson Objectives:
1. Pupils are able to measure, read and record masses of objects in kilogram and gram using the weighing scale.
2. Pupils are able to dettermine how many times the mass of an object as compared to another

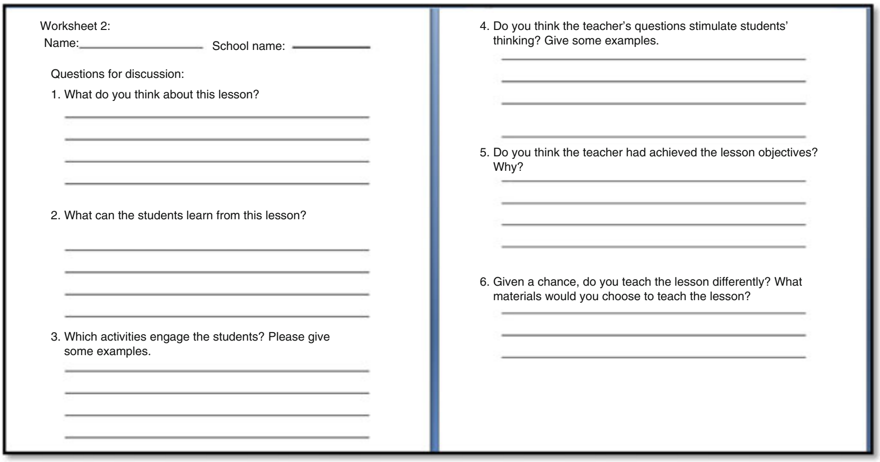
Instructional Materials:
1. Weighing scale
2. Packet food
3. worksheet

Focus of This Lesson:
1. Measure and record mass of objects in kilogram and gram.
2. Compare the masses of 2 objects using kg and g, stating the comparison in multiples or fractions.

Sequence of Teaching Activities:
1. Teacher displays packet foods on the table. Then she asks the students what characteristics (type of food, price, and quantity) they will consider of when buying things while shopping. Teacher discusses with students and brings their awareness to mass.
2. Students are divided into 6 groups. Each group of students are given 5 packets of food. They are asked to measure, read and record the masses of packet foods on the worksheet.
3. Each group of students is asked to present their worksheet on the blackboard. Teacher compares and checks their answers.
4. Teacher asks the students to compare the masses of two packet foods by asking question: “which packet food is heavier?”, “how many times the mass of sugar as compare to the mass of rice?” and“what is the fraction of mass of red beans as compared to the mass of green peas?”.
5. Teacher encourages the students to compare mass of any of the two packet food freely and find the comparative mass answers.
6. Then students will demonstrate their answer on the blackboard. This step will ensure the students’ understanding on the concept of comparative mass, further assess and correct students’ answer.
7. Teacher explains in detail sbout the strategies of finding the correct answer to strengthen and enrich their skills of comparing mass.

Possible Questions for discussion:
1. What do you think about this lesson?
2. What can the students learn from this lesson?
3. Which activities engage the students? Please give some examples.
4. Do you think the teacher’s questions stimulate students’ thinking? Give some examples.
5. Do you think the teacher had achieved the lesson objectives? Why?
6. Given a chance, do you teach the lesson differently? What materials would you choose to teach the lesson?

Fig. 7.7 An example of a lesson plan on the teaching of mass taught to a class of Primary 5 of the observed lesson for SJKC



Worksheet 2:
Name: ______ School name: ______

Questions for discussion:

1. What do you think about this lesson?
2. What can the students learn from this lesson?
3. Which activities engage the students? Please give some examples.
4. Do you think the teacher's questions stimulate students' thinking? Give some examples.
5. Do you think the teacher had achieved the lesson objectives? Why?
6. Given a chance, do you teach the lesson differently? What materials would you choose to teach the lesson?

Fig. 7.8 Worksheet 2 to guide participants with the reflection on the observed lesson

encouraged them to pay attention to students' activities. Some of the participants' response to Question 4 provided some interesting insights which will be discussed in greater detail in the next section.

Question 5 asked participants to evaluate if the lesson has achieved the planned objectives and to justify their comments, whereas Question 6 which asked that "Given a chance, do you teach the lesson differently? What materials would you choose to teach the lesson?" stimulated various responses from the participants. This open question provided the participants more space and opportunities to reflect on their own practices. If given a chance, some of them may like to teach it differently, whereas other teachers agreed with the way that the teacher taught in the video lesson. More detailed discussion on the responses will be discussed in the next section.

Analysis

Teachers' Reflections on the Two Worksheets

In this section, we discussed the reflections gathered from the worksheets and provided by the participants of the study according to school types.

Findings from Response Sheet (Worksheet 1)

For Worksheet 1, we mainly analysed the participating teachers' feedback on Question 6 and 7. This is because as discussed in the earlier section, Questions 1–5 were given with the aim of directing the participants' attention to the content of the video. If the participants paid attention to the contents of the video, they would be able to answer these questions. Tables 7.2 and 7.3 show the results of analysis of Questions 6 and 7, respectively.

As shown in Table 7.2, comparisons were made among the types of school to the question "What do you see as the most helpful or attractive aspects of LS, and why?" In general, 14 aspects were extracted from the three different types of schools and classified under four common themes: "knowledge sharing", "student focused", "teacher improvement" and "teamwork".

In the "knowledge sharing" category, all the three types of schools communicated the same opinion that LS promotes better teaching approaches through brainstorming among the teachers. Participants from the three types of schools agreed that lessons conducted in the LS were more "student focused". For example, a SJKC teacher responded that LS has changed her pedagogical belief from "what I want to do" to become "what do the pupils need" (从 "我要做什么" 改变成 "小朋友需要什么"). LS also encourages "teamwork" as concurred by the participants from all the schools. In particular, participants from SJKC and SJKT believed that through the reflection and discussion with their colleagues, they can learn from others' mistakes and be able to improve their weaknesses. For example, another SJKC teacher wrote that "We can learn from other teachers and can take the opportunities to find out our weaknesses", while two participants from SJKT remarked "Critics

Table 7.2 Attractive features of the LS perceived by three different types of primary schools

Common theme	Perceived feature	SK $N=4$	SJKC $N=17$	SJKT $N=20$
Knowledge sharing	Sharing of ideas, knowledge and teaching approaches	2	4	6
	Generating more ideas for teaching	2	2	4
	Learning from practice	0	5	2
	Open class	0	1	2
	Opportunity to revise the lesson plan together	0	0	2
Student focused	Lesson is more student focused	1	4	2
	Promote student's thinking	0	1	1
Teacher improvement	Improve quality of teaching by observing other teachers	0	3	2
	Develop confidence among teachers	0	0	1
	Reflection brings self-improvement	0	0	1
	Improve teachers' knowledge	0	0	2
Teamwork	There are discussions among teachers	0	0	3
	Collaboration among teachers	0	1	2
	Planning better student activities	1	1	1
	Planning and creating a lesson together	0	2	2

Table 7.3 Challenging aspects of the LS perceived by three different types of primary schools

Common theme	Perceived problem	SK $N=4$	SJKC $N=17$	SJKT $N=20$
School	Shortage of time	0	8	10
	Facility	2	0	0
	Common time for group discussion	0	1	5
	Heavy school workload	0	3	2
	Shortage of teachers	0	1	1
Teacher	Lack of knowledge of LS implementation	1	1	2
	Teachers' attitudes	0	1	0
	Collaboration/cooperation among teachers	0	0	6
Student	Managing problem students	0	5	0
	Student attendance	0	0	1

from a LS. This part will drive a teacher to improve his or her own self and quality" and "It helps the teacher to reflect on the teaching and learning …and improve the quality of teaching".

Table 7.3 reports on the analysis of participants' reflection on the question "What do you see as the most challenging or problematic aspects of LS, and why?" Three common themes were extracted from participants' written feedback: "school", "teacher" and "student".

The SK participants were more concerned with the school facility and the teachers' knowledge in implementing the LS. The participants from SJKC and SJKT, however, expressed their concern about the time factor. Detailed analysis showed that these teachers were concerned with the problem of teacher shortage and the resulting heavy workload on them. This is a perennial problem confronting the vernacular schools in Malaysia and is constantly receiving attention from the Ministry of Education as well as the society. In addition, SJKC participants highlighted the issue of teacher attitudes and managing problem students, whereas the SJKT participants identified student's attendance and the difficulty in getting teachers to cooperate as factors that could affect the implementation of LS in their workplace. Although the issue of student's attendance was an isolated case in this study as it was raised by only one of the SJKT participants, nonetheless the subject is significant enough to draw serious attention from the researchers. TIMSS 2011 reported that students attending school with discipline or safety problems had much lower achievement than those in safe and orderly schools. Further work is still needed to attend to the root cause in this problem.

Tables 7.2 and 7.3 show that SJKT teachers were more forthcoming than the SJKC teachers in their feedbacks. One possible reason for this difference in feedback could be that the SJKT teachers may have a better command of languages, English and Malay, than their SJKC counterparts and therefore may be more willing to share their opinions.

Findings from Response Sheet (Worksheet 2)

This section discusses participants' responses to Question 4 and Question 6 of Worksheet 2. These two questions were chosen because the responses given yield some interesting results, whereas the response to the other four questions (Question 1–3 and Question 5) were rather general. As different video-recorded lessons focusing on different topics and content were used with the different type of schools, it was not possible to compare the participants' response as in Worksheet 1.

The analysis of participants' response to Question 4 indicates that most of these teachers were not critical and reflective enough. However, Question 4 did alert them to pay more attention to the teacher's questions. Many of them still could not state specific examples. Perhaps some of them were not paying full attention to the video clips.

Nine SJKC participants observed the mathematics lesson on "mass and ratio". This Year 5 lesson comprised two main parts. In the first part of the lesson, students were divided into six groups. Each group of students was asked to measure, read and record the masses of five packets of foods on the worksheet. In the second part of the lesson, the students were asked to compare the masses of two packet foods by questions such as: "Which packet food is heavier?", "How many times the mass of sugar as compared to the mass of rice?" and "What is the fraction of mass of red beans as compared to the mass of green peas?"All the nine participants gave positive responses that the teacher's questions had stimulated students' thinking. However, only three of them managed to cite specific example of question, such as "The mass of XX is how many times the mass of YY? What fraction is the mass of XX compare to mass of YY? (XX 的质量是 YY质量的几倍？XX的质量是 YY的质量几分之几). Other participants did not quote any specific example of question".

For the seven SJKT participants, they were given to watch a video clip on the topic "Perimeter", taught by an Indian teacher in a Tamil school. This Year 5 mathematics lesson had this specific objective: at the end of the lesson, students would be able to calculate the perimeter of two dimensional composite shapes which may make up of square, rectangle and/or triangle. The lesson began with a revision of the method to calculate the perimeter of basic shapes such as square, rectangle and triangle. Then the teacher demonstrated how to calculate the perimeter of a composite shape which is made up of two rectangles. The class were then divided into groups of 5–6 students and each group was given a different problem to solve. Each group was then asked to explain their solutions in front of the class. The teacher facilitated the presentation by asking some questions or adding some explanation. At the end of the observation, three of the SJKT teacher participants commented that the teacher's questions were "normal questions" or "He only asked simple questions (Beliau meminta soalan-soalan yang mudah sahaja)", while three of them opined that some questions that the teacher asked were challenging as they said, "Yes, because the teacher asked the students for the missing length as in the given problem".

For the four participants from SK, they were given an edited video Year 4 lesson on "length" to watch. This lesson was taught by a female Malay teacher. At the start of the lesson, the teacher asked four pupils to come to the front to paste on the board four cards which have written on it the four different units to measure length: kilometre, metre, centimetre and millimetre. A student was then asked to state the relationship/connection between the four different units. The teacher revised the conversion formulae with the pupils. She then introduced Polya's four-step problem-solving strategies. Based on a given word problem, the teacher discussed with the students how to solve the problem using Polya's four-step strategies. The students were asked, "What kind of information was given?", "What is asked for?" and "What kind of operation to use?" The pupils were then divided into small groups of four to five pupils and given another word problem to solve. After that, each group leader was asked to explain their solutions in front of the class. The teacher concluded the lesson by giving them a worksheet to practise what they were taught. After the observation, all the four teacher participants shared a similar opinion that questions such as "Yes, teacher asks the students what kind of operation was required by the question (Ya, guru menyoal murid operasi apa yang dikehendaki dalam soalan)" was challenging to pupils.

Comparatively, the analysis of responses to Question 6 yielded much more interesting results. Four out of nine SJKC participants agreed with the method used in the observed video lesson. The other five participants gave various suggestions which focused on the materials used to convey the concept such as "use objects in the class or students' belongings to measure such as pencil, box, books, shoes etc. (利用班上或学生的物品来进行测量。如, 铅笔, 盒, 书本, 鞋子等)" and "must give more examples of multiplier to ensure that all students understand (叫倍数是要多给例子以确保所有学生懂)".

All the SJKT and SK participants gave different ideas and suggestions of materials that could help to enhance the lesson, for example, "Yes, use ICT to display the map shown by the teacher just now (Ya, menggunakan ICT bagi memaparkan peta yang ditunjukkan oleh guru tersebut)"; "Yes, video showing a farmer fencing his garden or ask students to curtain a table, then move or do pictures"; and "Yes, I can teach the lesson differently. I will bring the students around the school compound and related with the nature perimeter. I think that can make students understand the perimeter much better".

The participants' responses to Question 6 showed that it was effective in provoking the participating teachers to reflect and to generate new ideas from their own teaching experiences. Sharing these ideas among the teachers could clearly enhance their pedagogical content knowledge.

Conclusions

This chapter discusses how edited videos were used to introduce the concepts and processes of LS to the multi-ethnic Malaysian teachers who could be teaching in one of the three different types of vernacular primary schools. The first set of video clips contained contents from Japan. To help Malaysian teachers engage with the Japanese content, English subtitles were provided. However, because of the teachers' poor command of the English language, the researchers provided commentaries in Bahasa Melayu. The second set of edited videos provided participants with authentic examples of an "Excellent Teacher" at work. This second set of videos provided exemplary models of instructional methods and classroom management. The longitudinal design of the project showed that it was necessary to provide structure to the viewing of the video clips. When participants were asked to video the selected video clips, they were unsure what to focus on. However, providing participants with specially designed worksheets has helped to alert them to pay attention to the contents and hence the quality of their reflections. Their responses to Worksheet 1 showed the participants had a clearer picture of LS as they were able to identify the significant features of LS and the potential challenges faced should they adopt LS as a method of professional development.

Comparatively, the second worksheet has yet to succeed in making the participants reflect critically. Nevertheless, Question 6 of Worksheet 2 which asked the participants whether they would teach differently if they were the teacher in the "Excellent Teacher" video was more successful in encouraging participants to reflect further as they were able to provide some alternative examples in their responses. In conclusion the findings showed that it is important to provide teachers with very specific guidelines to help them engage more productively with the contents of the videos,

At this juncture, we have managed to implement LS in the seven primary schools and each school has conducted three cycles of LS. During this 3-year project, we have to use both "push" and "pull" strategies to motivate teachers to engage in LS. The preliminary analysis of the pedagogical flow of the mathematics lessons before and after the implementation of LS by one SJKC teacher in the project (see more detailed discussion in Lim, Kor and Chia 2013) has shown that there were observed changes in the teacher's lesson from a merely procedural and teacher-dominant lesson (in pre-LS) to more pupil involvement (group work) and pupil-oriented lessons (pupils pose questions).

In conclusion this study showed that the single most important factor that can contribute to the success of any professional development programme is the commitment of the senior management and the teachers involved. If the senior management (principals and vice-principals) and the teachers are not committed to professional development, there is no chance for any form of professional development to proceed. On the contrary when the senior management is supportive of professional development but the teachers are resistant to invest time in their own growth, no amount of resources and good intention would bear fruit. However when the senior management and the teachers are committed to professional development, such as the case of the SJKT teachers who struggled with heavy and demanding workload, they managed to learn from the help proffered by researchers. Given the importance of teacher quality on students' learning, teachers and leaders of the schools are responsible to ensure that school-based professional development courses are effective. Otherwise the decline of Malaysian students in international comparative studies will continue with its downward trend,

Our Reflection

The main aim of this study was to improve mathematics teachers' teaching practices through the introduction of LS as a teacher professional development [TPD] model for Malaysian teachers. We were glad that, albeit taxing and time consuming, LS has appeared to be effective in changing some of the participating mathematics teachers' teaching practices as well as in their perceptions towards the importance of TPD. The two key features of LS – collaboration and reflection – have shown to be more effective for the participating teachers than the usual short and one-shot workshop model experienced by most Malaysian teachers. The evidence suggests LS provided the in-service teachers a reliable and sustainable platform whereby they could work collaboratively to discuss the challenges they faced in their teaching and how best to resolve these obstacles. This platform was particularly important for novice teachers who lacked field experiences. We suggest that LS be extended to be used in pre-service teacher education.

During the study, we attempted to use video as a medium to provide grounded images for the participants about the process of carrying out LS. We were very happy that the video worked particularly after the various adaption of using multi-languages and guided worksheets. We also noticed that inviting the participants in a TPD workshop to observe and critique a video featuring a lesson taught by an "Excellent Teacher" from the same teacher community and teaching the topic in their curriculum can bring about unexpected outcomes. This opportunity not only enhanced the participants' observation skills but also provided the participants an avenue to provide feedback on how to improve teaching that particular lesson objectively.

We have extended the use of video clips of the practices of "Excellent Teachers" to the pre-service as well as in-service teacher courses. We found that the videos provided vivid images of what was happening and how particular strategies could be applied in actual classroom context. Seeing is believing and a picture is worth a thousand words. We believe that video technology has helped transform the traditional mode of delivering professional development of teachers.

Acknowledgement The study reported in this paper was made possible by the generous support from the Research Grant of the Universiti Sains Malaysia, Penang, Malaysia. Highest appreciation is also granted to Dr Ng Swee Fong for her insightful comments and suggestions for the early draft of this paper.

References

Abd Hamid, S. R., Syed Hassan, S. S., & Ismail, N. H. (2012). Teaching quality and performance among experienced teachers in Malaysia. *Australian Journal of Teacher Education, 37*(11). Retrieved from http://dx.doi.org/10.14221/ajte.2012v37n11.2

Akiba, M., LeTendre, G. K., & Scribner, J. P. (2007). Teacher quality, opportunity gap and national achievement in 46 Countries. *Educational Researcher, 36*(7), 369–387. doi:10.3102/0013189X07308739.

Bakar, A. R., Mohamed, S., & Zakaria, N. S. (2012).They are trained to teach, but how confident are they? A study of student teachers' sense of efficacy. *Journal of Social Sciences*, *8*(4), 497–504. doi: 10.3844/jsssp.2012.497.504. Retrieved from http://www.thescipub.com/jss.toc

Barber, M., & Mourshed, M. (2009, July 7). *How good education systems can become great in decade ahead*. Report on the International Education Roundtable, Singapore. Retrieved from http://www.mckinsey.com/locations/southeastasia/knowledge/Education_Roundtable.pdf

Doig, B., & Groves, S. (2011). Japanese Lesson Study: Teacher professional development through communities of inquiry. *Mathematics Teacher Education and Development, 13*(1), 77–93.

EPU (2010). *Tenth Malaysia plan (2011–2015)*. The Economic Planning Unit, Prime Minister's Department, Putrajaya. Retrieved from http://www.pmo.gov.my/dokumenattached/RMK/RMK10_Eds.pdf

Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Goh, P. S. C. (2012). The Malaysian teacher standards: A look at the challenges and implications for teacher educators. *Educational Research for Policy and Practice, 11*, 73–87. doi:10.1007/s10671-011-9107-8.

Lewis, C. (2002). Does lesson study have a future in the United States? *Nagoya Journal of Education and Human Development, 1*, 1–23.

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through Lesson Study: A theoretical model and North American case. *Journal of Mathematics Teacher Education, 12*, 285–304. doi:10.1007/s10857-009-9102-7.

Lewis, C., & Tsuchida, I. (1997). Planned educational change in Japan: The shift to student-centered elementary science. *Journal of Educational Policy, 12*(5), 313–331.

Lim, C. S., & Kor, L. K. (2012). 'Excellent' primary mathematics teachers' espoused and enacted values of effective lessons. *ZDM Mathematics Education, 44*(1), 59–70. doi:10.1007/s11858-012-0390-5.

Lim, C. S., Kor, L. K., & Chia, H. M. (2013, November 11–14). *One mathematics teacher's evolving pedagogical practice through Lesson Study*. Paper presented at the 5th international conference on science and mathematics education [CoSMED 2013], "Empowering the future generation through science and mathematics education", at SEAMEO RECSAM, Penang.

Maat, S. M., & Zakaria, E. (2010). An exploration of mathematics teachers' reflection on their teaching practices. *Asian Social Science, 6*(5), 147–152.

Makinae, N. (2010). *The origin of lesson study in Japan*. Paper presented at the 5th East Asia regional conference on mathematics education: In search of excellence in mathematics education, Tokyo. Retrieved from http://www.lessonstudygroup.net/lg/reading_table.php

Ministry of Education. (2012). *Malaysia education blueprint 2013–2015: Preliminary report-executive summary*. Retrieved from http://www4.unescobkk.org/nespap/sites/default/files/Preliminary-Blueprint-ExecSummary-Eng_0.pdf

Mohd Sofi Ali. (2003). English language teaching in primary schools: Policy and implementation concerns. *IPBA E-Journal*. Retrieved from http://kajianberasaskansekolah.files.wordpress.com/2008/04/article70.pdf

Owings, W. A., Kaplan, L. S., Nunnery, J., Marzano, R., & Myran, S. (2006). Teacher quality and troops to teachers: A national study with implications for principals. *NASSP Bulletin, 90*, 102–131. doi:10.1177/0192636506289023.

Rice, J. K. (2003). *Teacher quality: Understanding the effectiveness of teacher attributes*. Washington, DC: Economic Policy Institute.

Tee, Y. J. (2007). *Reflective thinking practices among secondary mathematics teachers*. Master thesis, Universiti Putra Malaysia. Retrieved from http://psasir.upm.edu.my/4824

Yoshida, M. (1999) *Lesson study: An ethnographic investigation of school-based teacher development in Japan*. Unpublished doctoral dissertation, University of Chicago.

Yoshida, M., & Fernandez, C. (2002). *Lesson study: An introduction [CD-ROM]*. Madison: Global Education Resources.

Zawojewski, J., Chamberlin, M., Hjalmarson, M., & Lewis, C. (2008). Developing design studies in mathematics education professional development: Studying teachers' interpretive systems. In A. Kelly, R. Lesh, & J. Baek (Eds.), *Handbook of innovative design research in science, technology, engineering, mathematics (STEM) education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Chapter 8
Summary of Part I and Introduction to Part II

The findings reported in the five chapters in Part I broadly support the use of videos in professional development of teachers. One message emerging from the work of the various authors suggests that effective leadership provided by senior management is crucial to the professional growth of teachers. Senior management must value and hence be willing to invest in the professional development of teachers. When professional growth of teachers is seen as an important contributing factor to how students learn, senior management would be willing to invest time and space for teachers to meet and discuss how best to conduct their professional growth. Furthermore they are open to listen to what mathematics educators have to share with teachers in such professional development courses. The role senior management plays in the professional development of teachers is shown clearly in Lim and Kor's work with the Malaysian teachers. When senior management offered their support to the mathematics educators, those teachers who attended the in-service courses learnt about the nature of lesson study and also the good practices of their peers. By participating in the in-service courses, the teachers have some grounded images of how lesson study could be conducted. However when such support was not forthcoming, it was not possible for teachers to have any form of grounded images of lesson study.

Teachers' receptiveness to the methodology is crucial to the successful use of videos as a tool for professional development. When participating teachers overcame their apprehension about being videotaped, they were able to collect videos of themselves at work. By comparing one set of grounded images against another, teachers were able to plan their professional development. Teacher-T in Swee Fong Ng's chapter crafted her lesson and the appropriate pedagogy so that the Primary 2 children in her class could summarise for themselves the part-part-whole relationship of numbers. Teacher Bessie in Romina Yap et al.'s chapter identified her shortcoming and made a decision to be mindful not to assume that she had the knowledge to deal with the extension phase of the problem-solving task. Teacher-S in Kit Ee Dawn Ng's chapter realised that while it was necessary to monitor her own decision-

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_8

making process, it was equally important of her to create space for the students to monitor and regulate their mathematical thinking in reaction to her queries during the task.

The provision of specific guiding questions helped focused the participating teachers' attention on specific pedagogical actions. Their viewing of the videos became meaningful as this meant that there was synchrony of their watching of the video and the intent of the mathematics educators. The guiding questions also helped the participating teachers select the relevant grounded images, and this helped in the discussion following from watching the video. Although it was possible to replay the selected video clip to refresh teachers' recollection of a specific episode, the guiding questions helped teachers to reflect on the outcome of the actions.

Into Part II

Besides reporting on the learning outcomes of the participants of the study, the four chapters in this part highlighted what the mathematics educators learnt what was valued by the participants of the study.

Factorisation is a fundamental but a challenging topic for teachers to teach and students to learn. Two chapters in this part, Ida Mok's work with a competent Hong Kong teacher, Teacher-HK, and Weng Kin Ho et al.'s work with 82 pre-service Singapore secondary mathematics teachers, provide contrasting approaches to the teaching of factorisation of linear and quadratic expressions. Teacher-HK wrote seven increasingly complex algebraic expressions on the blackboard and through a technique of question and answer taught factorisation of linear and quadratic expressions to a class of Secondary 2 students. Teacher-HK was confident that this technique of compare and contrast of suitably selected mathematical tasks would help the students see that factorisation is the reverse of expansion and thus help prepare these students with related work on factorisation of linear and quadratic expressions.

In contrast, in the Singapore study 82 pre-service secondary mathematics teachers were shown an online video suite, available on demand, which showed how a secondary mathematics teacher used concrete materials known locally as Algecards to teach factorisation of linear and quadratic expressions to a group of low-ability secondary 2 (14+) mathematics students. The Head-of-Department, teachers and students were recorded discussing the merits of such an innovative approach to the teaching and learning of factorisation. Reflecting on the feedback of the Head-of-Department, teachers and students, the pre-service teachers were then asked whether they would 'buy in' to this concrete approach of teaching factorisation.

The penultimate chapter of this part reports on the work of Teacher-K, an experienced middle school mathematics teacher in Korea who used the affordances of grounded images of five videoed lessons and the feedback of a mathematics educator who served as a critical commentator to improve her teaching of mathematics that necessitates the incorporation of new curricular initiatives. Teacher-K became more reflective with each subsequent encounter with the critical commentator. Her reflections showed her becoming increasingly concern about those students who seemed to be marginalised from the learning of mathematics and her desire to engage these students.

Widjaja and Dolk's work with Indonesian teachers provides useful information to teachers, researchers and mathematics educators alike. The chapter showed teachers struggling and succeeding to adopt Realistic Mathematics Education approaches to teaching and learning mathematics to Grade 2 children who had used different methods to solve mathematical tasks involving big numbers.

Chapter 9
What Is Important in a Lesson on Factorisation? The Reflection of an Experienced Teacher in Hong Kong

Ida Ah Chee Mok

Abstract Factorisation is a fundamental topic in school algebra but also a challenging topic for teachers to teach and students to learn. This chapter highlights what an experienced teacher valued in the teaching of factorisation. A competent Hong Kong teacher, Teacher-HK, was invited to comment on his lesson by the video-stimulated recall technique. The analysis was based on the interviews with Teacher-HK unfolding his strategy for helping his secondary 2 (14+) students learn about the complexities associated with the learning of factorisation of linear and quadratic expressions and also the rationalisation for his actions. Finally, how the example may benefit teacher professional development is discussed.

Introduction

The Hong Kong school mathematics curriculum introduces algebra as 'generalised arithmetic' (Usiskin, 1988), where students use letters as variables to replace numbers in arithmetic statements. Based on their experiences with arithmetic, students extend their experiences with arithmetic to generalised cases in algebra (Mok, 2010a). Factorisation of algebraic expressions is an integral part of secondary school algebra. The concepts and skills of factorisation are fundamental aspects of algebra because they are necessary to the simplifying and solving of algebraic expressions and equations, a key activity of mathematics. However, many students find this topic challenging when the algebraic expressions are of a complex nature, in particular those algebraic expressions involving negative signs. Furthermore although there are many possible factors to a given algebraic expression, it is necessary to find the unique set of factors that best represents a given algebraic expression. Often students do not know why a particular set of factors is the best solution. Teachers are often perplexed how best to teach this topic to help the students discern

I.A.C. Mok (✉)
Faculty of Education, University of Hong Kong, Hong Kong, China
e-mail: iacmok@hku.hk

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_9

why one particular set of factors is the best solution to a given algebraic expression.

Teaching is a complex activity, and there may be various approaches to develop complex concepts and skills. The enterprise of teaching involves planning of a lesson, implementation of that lesson, assessment of students' learning and evaluation of the teacher's own lessons. A teacher's belief, knowledge and goals form an overarching and intertwining basis for the enactment of these processes. Expert teachers have been found to conduct such processes better than novices. In the last three decades, a number of studies have sought to highlight the differences between the practices of novices and those of the experts. It is important for mathematics educators and those responsible for conducting professional development courses to be cognisant of such differences in practice. Experts in the field agree that those conducting professional development courses would be well served if they are armed with such knowledge (Artzt & Armour-Thomas, 1998; Ball et al., 2005; Berliner, 1986; Lampert et al., 2010; Leinhardt & Greeno, 1986).

Research shows that within the teaching profession, there is a need for teachers to develop a deeper understanding of their own practice and to learn from experts in the field how they carry out their own practice (Berliner, 1986; Borko et al., 2008; Lampert et al., 2010; Loughran, 2002). Such understanding is important in informing teachers about alternative paradigms within the limitation of the curriculum and the setting of schools and classes. Watching videos of experts teach sometimes may help novices and less experienced teachers see problems from a different perspective. Hence with new-found perspectives, less experienced teachers and novices may reframe existing problems in different ways and to seek for more feasible solutions. Without the opportunity to view the problems in other ways, teachers may tend to rationalise that students failed to learn because they were not willing to invest themselves with a set piece of work. For example, novices may perceive a class not engaging themselves in the work as having poor attitude towards their work (Loughran, 2002) when the cause could be one of ineffective teaching. However, interplay between experience and reflection may bring about changes in perceptions of the teaching and learning in context. Expert teachers very often have different insightfulness, sensitivity, wide knowledge base and deep interpretation of problems in contrast to a novice teacher (Yang & Leung, 2013). When such differences in practice are highlighted, novices may learn how to conduct good mathematics lessons. Mathematics educators and those responsible for professional development could share such processes with the novices, thus sensitising them to what makes an effective teacher. Novices are only convinced that such effective processes are possible when they are provided with credible evidence from the field. Thus, it is important that professional development courses are embedded with work of expert teachers. Although description of practices of effective teachers may suffice to offer novices examples of effective practices, professional development courses could be even more effective if such practices were supported by images of actual lessons.

This chapter highlights what an experienced teacher valued in the teaching of factorisation. The data presented in this chapter were taken from the international project Learner's Perspective Study (LPS) which had its genesis in 1999. A set of consecutive 8th Grade mathematics lessons conducted by a competent teacher was recorded, and three post-lesson interviews were carried out. The Learner's Perspective Study (LPS), represents an innovative venture in classroom video study, complementing emergent national norms of student achievements and teaching practices with an in-depth analysis of classroom learning from the perspective of the learners as well as the teachers. With respect to design and methodology, it provides the opportunities for researchers to study participating teachers' reconstructions of classroom events of 'well-taught' 8th Grade mathematics lessons (Clarke et al. 2006). The teacher was invited to comment on his own lesson in a post-lesson interview with the method of video-stimulated recall.

In the next sections, a description of the methods of data collection and analysis will be given, followed with the results of the analysis of the teacher's commentary of his own lessons. First I discuss Artzt and Armour-Thomas problem-solving metaphor which is used to frame this study with the teacher. The teacher metacognition framework (TMF) sheds light on the relationships between cognitions and teaching practice in mathematics and suggests that teachers' knowledge, beliefs and goals directly influence thinking across three stages of teaching, namely, before, during and after the lesson. The framework is adapted to analyse the transcript of the post-lesson teacher interview. The challenges students faced with the structural aspects of factorisation will be discussed next. This is followed by a discussion how the use of videos enhanced the contribution of the teacher. The selection of the participating teacher, the protocols used with the participating teacher and the framework used to analyse the data are discussed next. This is followed by a discussion of the results and the findings. Screenshots of the selected sections are used to support the discussion. Such screenshots provide examples of grounded images of the lesson selected by Teacher-HK. The penultimate section discusses the learning points that are of value to prospective teachers. The final section illustrates how the work discussed in the chapter serves to benefit me as a mathematics educator who is actively involved in professional development of teachers, novices and those who wish to improve their teaching of factorisation.

Theoretical Perspectives

The Teacher Metacognition Framework (TMF)

In a metaphorical sense, teachers are always solving problems. For teachers the perennial problem is 'how to teach a lesson that will promote student learning with understanding' (Artzt & Armour-Thomas, 1998, p. 6). Artzt and Armour-Thomas referred to teacher's commentaries about a lesson and about the processes associated with teaching that lesson as the metacognitive components of the process of problem-solving that consist of goals, beliefs, knowledge, planning, monitoring, regulating, assessing and revising. Artzt and Armour-Thomas (1998) proposed the teacher metacognition framework (TMF) to understand the teacher's thoughts associated with their instructional practice. Taking the problem-solver's perspective and other researchers' (Fennema & Franke, 1992; Jackson, 1968; Leinhardt, Putnam, Stein, & Baxter, 1991; Peterson, 1988; Shulman, 1986), Artzt and Armour-Thomas defined teacher knowledge as 'an integrated system of internalised information acquired about pupils, content and pedagogy' (p.7). In the TMF, knowledge of the pupils, content and pedagogy; beliefs of the roles of the student and the teacher; and goals of the lessons were the overarching metacognition. In three structured post-lesson interviews, they asked the teachers to explain their lesson plans and describe their thoughts as they developed the lesson for the class, explaining their specific decision-making during the lessons and reflecting upon their lessons. Applying TMF, they studied the metacognition that teachers use in making decisions and judgments before (planning), during (monitoring and regulating) and after (assessing and revising) a lesson. The results of their study showed that TMF helped to discern the components of the teacher's metacognition. The patterns of metacognition appeared to be related to the pattern of the nature of instructional practice.

In this study, the focus was on lesson episodes which the teacher said were important from his own perspective. In addition to delineation of what the teacher said in the interviews, the teacher metacognition framework (TMF) developed by Artzt and Armour-Thomas (1998, 1999) was modified to provide an alternative perspective of effective teaching. The work of Artzt and Armour-Thomas focused on teacher's decision-making episodes in the lesson, whereas the study in this chapter described what the teacher valued. With respect to the metacognition aspect, the analysis of the transcript tapped the teachers' knowledge and beliefs of pedagogy, content and students. The TMF describes three lesson dimensions, each with several attributes, namely, the tasks dimension, the learning environment dimension and discourse dimensions, which cover a range of lessons by different groups of teachers. The dimension indicators help highlighting the instructional practice and the associated teacher's cognition. Not all dimension indicators were relevant to this lesson. However, the task dimension and the discourse dimension match what Teacher-HK valued, therefore, helping describe the important qualities for effective teaching in the lesson. The relevant dimension attributes and indicators are listed in Table 9.1 (Artzt & Armour-Thomas, 1999, p. 217).

Students' Potential Difficulties with the Topic of Factorisation in the Context of the Hong Kong School Curriculum

The Hong Kong mathematics curriculum introduces the topic factorisation at secondary 2 (equivalent to Grade 8). Students are expected to learn factorisation as a reverse process of expansion: factorise polynomials by using common factors and grouping of terms; factorise polynomials by using identities including difference of two squares and perfect square expressions; and factorise by the cross method (The Education Department, Hong Kong, 1999).

Table 9.1 The task dimension and discourse dimension that are relevant to the HK lesson

The task dimension: sequencing/difficulty level	Sequences tasks such that students can progress in their cumulative understanding of a particular content area and can make connections among ideas learned in the past to those they will learn in the future. Uses tasks that are suitable to what the students already know and can do and what they need to learn or improve on
The discourse dimension: teacher-student interaction	Encourages the participation of each student. Requires students to give explanations and justifications or demonstrations orally and/or in writing. Listens carefully to students' ideas and makes appropriate decisions regarding when to offer information and when to provide clarification
Questioning	Poses a variety of levels and types of questions using appropriate wait times that elicit, engage and challenge students' thinking

Adapted from Artzt and Armour-Thomas framework (1999)

Hong Kong students find factorising specific algebraic expressions particularly challenging. These include algebraic expressions containing the negative signs and higher-degree variables. This section focuses on the difficulties students have with three types of algebraic expressions:

Uniqueness of factors: Students often have difficulties deciding which of these equivalent expressions $n(2a+2b)$, $2n(a+b)$ and $2(na+nb)$ are the most suitable set of factors when factorising the expression $2na+2nb$. Based on the simplified interpretation that factorisation is the reverse of expansion (or multiplication), all three can be answers. Nonetheless, embedded in advanced mathematics, the answer demands the representation by a unique set of irreducible factors over the field of integers. Therefore, the algorithm of finding factors does not stop at $n(2a+2b)$ and $2(na+nb)$. These details cannot be explained clearly till students work in the area of abstract algebra in the university. At school algebra, the case is often handled intuitively by seeking for the set of factors where each factor is of the 'simplest' form, hence, developing an algorithm to keep taking out the common factors till the simplest form is obtained. This can be concluded by 'taking the highest common factor (HCF)' for $2na$ and $2nb$ for the expression $2na+2nb$.

Difficulties with the negative sign: However, the situation becomes more complex and ambiguous when the expression contains the negative signs. Both these solutions $2n(-a-b)$ and $-2n(a+b)$ are correct solutions to the expression $-2na-2nb$. But the choice may make a subtle difference for later calculation. The difference may not be a reason in the mathematical context but simply a phenomenon that students tend to make more careless mistakes when factorising expressions with negative signs. For example, some may leave out one negative sign resulting in such erroneous factors: $2n(-a+b)$ or $-2n(a-b)$.

Difficulties with variables containing higher indices: Similarly, the level of difficulty increases when the expressions contain variables of a higher degree, e.g. $2na+2n^2b$ and $2na+2n^2b^2$. For example, some students may not know the real meaning of the indices and see them as an arbitrary symbol attached to the letter.

Therefore, they may ignore the indices and work with what they know and provide erroneous solutions such as $2n(a+b)$ or $2(na+n^2b)$; or simply give up and make no attempt to find the factors.

In the lesson discussed in this chapter, the teacher used students' difficulties factorising such expressions to form the backbone of his lessons and guide him in the choice of examples he used in addressing the difficulties students have factorising such algebraic expressions.

The Efficacy of Video Technology in Teacher Professional Development

The use of videos for teacher education has grown in importance because technology provides a unique capacity to capture the richness and complexity of classroom (e.g. Borko et al., 2008; Calandra et al., 2008; Mok, 2010b; Zhang et al., 2011). The effective use of videos in the professional development of teachers in a variety of contexts is well documented in the literature. There are many different ways for using videos, e.g. student teachers watching their own videos for self-evaluation (Calandra et al., 2008; Mok, 2010b) or for discussion in learning programmes (Borko et al., 2008) and in-service teachers watching lesson videos either individually for reflection or in group setting for collaborative work (Zhang et al., 2011). It is generally agreed that videos can promote reflective practice, which is essential for a teacher's professional growth. However, simply watching the video superficially is not equivalent to reflective practice (Loughran, 2002). Effective reflective practice involves careful consideration of both watching the video and enacting in authentic contexts with a purpose of promoting sharing and learning through experience. In the study described in this chapter, Teacher-HK was a very experienced teacher, and he was aware that he was in a research project and prepared to explain his pedagogical ideas in detail. Although what he said in the post-lesson interview might be on hindsight, he chose episodes from the videoed lessons to support what he valued in his retrospection.

The Study

The Participating Teacher and the Students

At the time of this study, the participating teacher, Teacher-HK, who has more than 20 years of teaching experience, was active in curriculum development and research activities. Members of the LPS research team in consultation with the school principal and members of the local mathematics education community identified Teacher-HK as a competent mathematics teacher. This recognition was further endorsed by his colleagues and his students.

The students were secondary 2 students, aged 13–14. The school was of average standard in Hong Kong using Chinese as medium of instruction which was the medium of instruction used by the majority of the schools in Hong Kong. Pseudonyms are used to identify students.

Data Collection

Teacher-HK was filmed for 18 consecutive lessons. Three cameras (a teacher camera, a student camera and a whole-class camera) were used for the video recording of the lessons. An on-site mixing of the images from two video cameras was carried out to provide a split-screen record of both teacher and student actions, and the video was used for stimulated recall for the post-lesson interviews for the focused students. Teacher-HK participated in three video-stimulated interviews and completed two substantial questionnaires before and after videotaping, as well as a shorter questionnaire after each videotaped lesson. In addition to the videos and interviews, copies of student-written materials, textbook pages and worksheets used in class were made. The rich data set made detailed reconstructed accounts of the lessons possible, hence, allowing analysis to be carried out to address classroom issues from a variety of perspectives.

The Teacher's Interviews

In each interview, Teacher-HK chose a lesson video to give his comments. The following protocol was used in the interview:

- What are the objectives of the lesson?
- Why does he think that the objectives are important for the students?
- What is important during the lesson? The teacher was asked to play the video and stop the video at episodes that he saw important and gave his commentary for the episode by explaining why he thought that was important.

The Method of Analysis

The following questions were used to guide the analysis of the interview transcripts:

1. What were seen as important in the teacher's perspective?
2. What were the teacher's knowledge and beliefs that may guide his planning and actions in the lesson?

The interview protocol was designed to invite Teacher-HK to explain what was important based on his own personal pedagogical views in order to capture the rationale and interpretation for the events in the lessons. The analysis aimed to capture Teacher-HK's perspective as close to the teacher's view as possible. The interview transcript was read several times and divided into segments based on where Teacher-HK stopped the lesson video. The corresponding lesson video was viewed to supplement the understanding of Teacher-HK's commentary. Keywords and phrases in the transcript were highlighted (e.g. 'the student asked a question', 'minus sign', 'common factors'), and remarks (e.g. belief, goals, student, knowledge, pedagogy) were marked in the margin of the transcript to remind the main ideas in his conversation and to provide a triangulation from the perspectives of TMF.

The interviews were carried out in Cantonese. The audio was first transcribed in Cantonese, then translated into English. The translation was carried out by a student helper and checked by the research assistant. Queries that arose from the translation were referred to the researcher (the author) for clarification.

Findings

What an Experienced Teacher Valued in a Lesson on Factorisation

Overview of the lesson: Teacher-HK explained that he chose the lesson because the topic of factorisation was fundamental and important for the learning of algebra and other advanced mathematical topics in later stage. The first lesson was on the topic 'factorisation of polynomials'. Before this lesson, the students were taught the topic 'multiplication of polynomials'. Teacher-HK began the lesson by writing the topic 'factorisation of polynomials' and asked what the students might associate with this topic. Then the meaning of factorisation of the polynomial was first explained as the reverse action of multiplication. Teacher-HK worked out seven examples with the class (Fig. 9.1). The lesson was carried out in a teacher-led whole-class discussion till the end of the seven tasks. Then Teacher-HK assigned some exercises from the textbook for the students to carry out as individual seatwork. During the individual seatwork, Teacher-HK moved about to provide support to individual students. This continued till the end of the lesson.

During the interview, Teacher-HK identified five episodes as important. In brief, he found the following important: (1) helping students build the correct habit, (2) his choice of examples, (3) giving feedback to students, (4) helping students reflect upon their work and (5) monitoring and regulating the pace of teaching and the students' progress. Analysis of the interview suggested that Teacher-HK's knowledge of the mathematics content, beliefs of how students learn and the goals of a specific lesson were the intertwining factors that underpinned the teacher's pedagogy and decision-making and the conduct of his lesson.

Teacher-HK had very clear ideas of the difficulties students have with factorisation of polynomials of degree one, degree two and those involving factors with

Fig. 9.1 The seven tasks of increasing complexity presented in the first lesson

negative signs. Thus Teacher-HK's knowledge of what students found challenging influenced the tasks he chose to engage the students in his class. Thus he selected polynomials that were of increasing complexity. In terms of pedagogical design of the content, he put in effort to the sequence and design of the examples. He made use of the comparison method and built in the complexity and gradual change in the form of the algebraic expressions in the seven examples (episode 3).

Teacher-HK's goal for the lesson was to let the students know clearly what constitutes factorisation. A good introduction to the concept of factorisation would help the development of the other part of the lesson. This is discussed under the section Using Pre-existing Knowledge to Construct New Knowledge. He was aware of the conventions and requirements in expressing mathematics in a certain form, and he called these as 'habits' such as reading mathematics from left to right and the minus sign in the final answer to a question (episode 1, episode 3). He also planned specific questions for his class discussion to help students reflect upon the answers. In episode 2, he used three examples to help students see the limitations of defining factorisation of polynomials as the reverse of multiplication and the need to use HCF. In episode 4, he asked a special question to guide the students to compare two examples and reflect more deeply for the reasons supporting the answers.

His planning and enactment of the lesson were based on his knowledge of what the students had already learned and the teacher's understanding of the difficult part for the topic. Therefore, he chose to introduce factorisation of polynomials as a reverse of multiplication (episode 1). By doing this, the new knowledge (factorisation) was built upon prior knowledge of multiplication. Teacher-HK made special effort to evaluate students' progress so that he could monitor and regulate the pace of his teaching. For example, he asked students to raise their hands to indicate whether they could understand the content (episode 5). He moved between desks and helped students complete their work after he had assigned exercises (seatwork) for the students to complete individually. During the seatwork period in other lessons, he checked the progress of the students and made use of students' contribution for further development of the instruction, for example, inviting some students to work on the board, sharing the correction of mistakes and giving the feedback to students' questions in public. All these formed practical and effective routines in class to ensure student engagement in the mathematics.

Overall, he made use of his knowledge of the content and the knowledge of students to plan the demonstration examples and the teacher-led discussion. He built in the habits for mathematics, critical aspects of the content, comparison method and reflection in his pedagogical design and implementation of the actual lesson.

Design and Sequence of the Examples

Often the choice of examples does not feature strongly when teachers plan and design a lesson. Sometimes an example is selected as long as it requires the application of the taught concept. However, this is not the case with Teacher-HK. Figure 9.1 shows the screenshot of the seven examples in the lesson. Table 9.2 lists the seven examples.

Teacher-HK chose his teaching examples very carefully. The seven examples were of increasing complexity. Examples 1 and 3 formed a triad where it was possible to have different solutions, albeit correct, but the goal was for students to know that there should be a unique set of factors. Examples 1 and 2 were very similar replications of the introductory example, and the students were expected to give the answers without difficulty, but they also served the purpose of giving Teacher-HK a chance to see whether the students had followed the earlier explanation. Example 3 which built upon the earlier two examples showed a possibility of more than one possible answer, hence, providing a platform to explore the meaning of a unique set of factors. Examples 4 to 7 provided opportunities for students to explore the difficulties with negative signs and quadratics. Teacher-HK's choice of examples demonstrates the importance of selecting good examples to enhance learning. The sequence and design of the tasks in this lesson demonstrate the possibility for creating an experience of factorisation with increasing complexity while overcoming difficulties in handling complex algebraic expressions.

Table 9.2 Seven examples of increasing complexity presented in the first lesson

Example	Expressions
Example 1	$na+nb$
Example 2	$2a+2b$
Example 3	$2na+2nb$
Example 4	$-2na-2nb$
Example 5	$-2na+2nb$
Example 6	$2na+2nb^2$
Example 7	$2na+2n^2b^2$

New Knowledge Is Taught Based on Prior Knowledge

Episode 1 (at 08:29, about 1 min): The teacher had just explained the meaning of factorisation as a contrast of multiplication by referring to reading the equation $m(a+b)=ma+mb$ in two possible directions. Therefore, doing multiplication in the reverse way, in his words 'making the answer into the question', is called factorisation. Teacher-HK asked the class to give an example of factorisation. A student, Mark, first suggested $2(m+s)=2m+2s$. Teacher-HK wrote Mark's answer on the board and continued to ask Mark to tell the direction of reading the line when he chose for factorisation while telling him that this answer was only an example of expansion and could not be counted as factorisation. Attempting to explain his answer, Mark said, 'Two is their common factor'. As Mark did not answer the teacher's question directly, Teacher-HK rephrased his question and asked Mark again whether he read from left to right or from right to left. Mark could then answer, 'From right to left'. Teacher-HK then corrected Mark's answer on the board by replacing it with $2m+2s=2(m+s)$, and spoke to the whole class to emphasise this point, 'From left to right. We simply call this factorisation' (Fig. 9.2).

In the interview, the teacher explained why he thought this was important for several reasons. Firstly, he used this (the opposite for multiplication of polynomial) as the meaning of factorisation. He said, 'Because this is actually what factorisation is, opposite to multiplication of polynomial'. Secondly, his teaching method for teaching concepts was to introduce the new by making use of students' prior knowledge, and he believed that this is easier for the students. He said, 'Because the new knowledge, especially the concepts, if it is taught based on the prior concepts, it is easier for students to accept'. Thirdly, in this episode he explained the opposite relationship between factorisation and multiplication; he said, 'Doing this step could point out the relationship between factorisation and multiplication. Specially, their aspects are opposite'. Fourthly, this was important because this was a student's example. He wanted the student to contribute an example in order to show what they knew about factorisation after listening to the introductory explanation. The student's initial answer was not correct. Teacher-HK guided the student to realise the correct answer in a whole-class discussion. In the process, the teacher removed the student's suggestion of $2(m+s)=2m+2s$ and replaced it with $2m+2s=2(m+s)$. He further explained this action in terms of

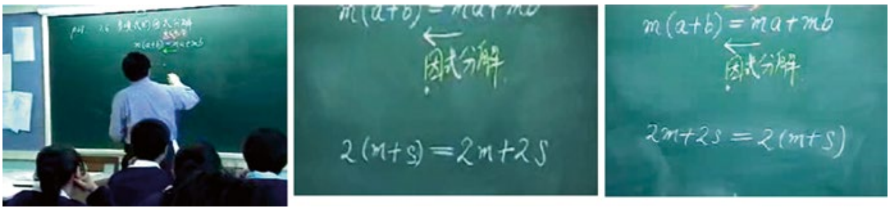


Fig. 9.2 Factorisation as the reverse of multiplication

his understanding of the student's knowledge and writing habits, and these aspects were what he wanted to emphasise for the students. He said:

> But this new knowledge was totally reversed from this one. The topic of my lesson was factorization. I had to emphasize that the transformation was actually from right to the left, but because of our writing habits, which were writing from the left to the right, so, the students were writing from the left to the right naturally.

What May Be Acceptable, What May Be Most Acceptable: Highlighting Mathematical Conventions

Episode 2 (at 10:35, about 8 min): In the lesson, the teacher had given seven questions for the students to do as examples and exercises. He referred to the three examples that he worked with the whole class interactively in a whole-class discussion as important. Examples 1, 2 and 3 in Table 9.1 were of increasing complexity:

For example, for the expression $2na+2nb$, Teacher-HK recorded the three equivalent sets of factors offered by students. $n(2a+2b)$, $2n(a+b)$ and $2(na+nb)$ (as in Figure 9.3). Then he led the class to examine the situation of three possible answers; and for the need of one answer, he led the students into a discussion for the need of using the idea of highest common factor.

First the teacher asked the class to inspect the three answers, 'Are they the same as the original expression?' The class replied, 'The same', but the class also agreed that the three answers could be all correct. Then the teacher asked the students to discuss in pair about this phenomenon and tried to work out a conclusion. The students discussed in pair for about 30 s. Then the teacher resumed the attention of the whole class. A student chose the first one $n(2a+2b)$ with the reason, 'It seems to be'. Many students laughed at this answer. The teacher then asked the students to recall how they did factorisation of numbers in primary school. He wrote $12=4\times 3$ on the board and rewrote it further to '$2\times 2\times 3$'" to explain the idea of 'factorise

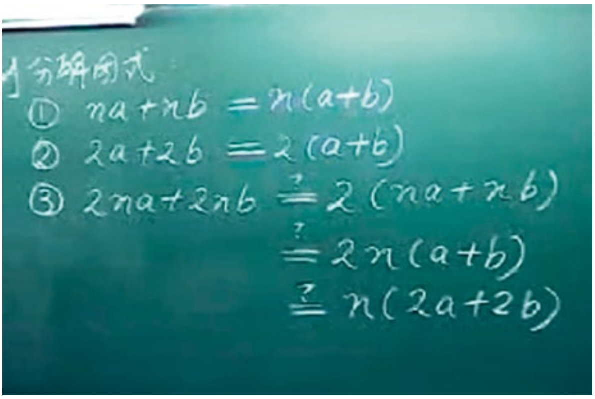


Fig. 9.3 Teacher-HK recorded the three equivalent sets of factors for Example 3

further and further'. Using this example as an analogy, the teacher pointed to $2n(a+b)$ and asked the class whether this was also an example of 'factorise further and further'. The class replied, 'Yes'. Then the teacher called upon a student, Leo, to answer the question again. Although Leo chose the correct answer $2n(a+b)$, he failed to give the correct explanation. When the teacher asked for a reason, Leo's first reason was 'the simplest'. When the teacher gave a hint of 'what factor', Leo mixed up LCM and HCF and said 'LCM'. The teacher corrected his answer and made a conclusion for the discussion: 'If you want to have a complete factorisation, you must have the HCF'.

In the interview, the teacher explained that this example was very important because the results revealed the limitation of the earlier established meaning of factorisation 'the reverse transformation of multiplication'. Based on this meaning, all three answers were acceptable. This led to a phenomenon contradicting the fact that there should be a unique factorisation. And this phenomenon required further investigation. He explained:

> It is impossible for the three answers to be correct …Because the answer of factorization is unique …Then, then we have to study… why a question is so important and there would be some logical problem in the students' mind…This occurred because, yeh, if I only emphasized on the reverse transformation by multiplication, the three answers would be correct as the truth was the multiplication of the three answers were equal to the question.

Episode 3 (at 18:00, about 2 min): The fourth algebraic expression $-2na-2nb$ – was a variation of the first three examples, but the presence of the negative sign increased the complexity of the task (Fig. 9.4). A student, Robert, first gave the answer $2n(-a-b)$. Although the factors were extracted correctly, the common factor of negative one was yet to be extracted from $(-a-b)$. Therefore, Teacher-HK gave a positive feedback to the student and called on another student, Mark, to give another answer. Then he led the class to arrive at the complete answer of $-2n(a+b)$ where all the factors, including negative one, were extracted.

T (to Robert): Firstly, I have to show my appreciation on your performance, you all have done a good job! However, our habit is that we avoid the presence of negative signs. It is in fact correct, but how can we deal with the negative sign? Mark.

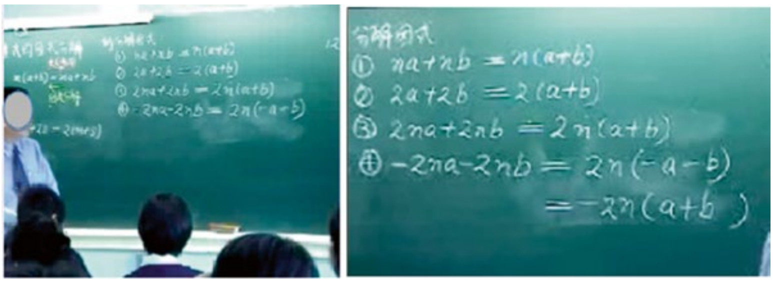


Fig. 9.4 Working with the negative sign

Mark: Minus two n brackets a plus b.

T (to the class): Right! That suits our habit! Robert is also correct! Okay! Now, this habit is a bit weird! Have a look at this! The habit is that if we find the negative sign in the first term, we should single it out, and turn it into one without a negative sign.

In the interview, the teacher elaborated his feedback to Robert's answer. He explained that Robert's answer $2n(-a-b)$ was not wrong because Robert's answer was an equivalent of the original $-2na-2nb$. His feedback was to help the class to learn the habit of reducing the number of negative signs from the answers, i.e. the form of the answers required by convention. He explained:

> His answer was not wrong as it could be changed back into the question. The problem is that there are lots of rules on factorization. Because, em, our habit is to reduce the negative sign. If our equation, which is minus a minus b, we take out the subtract sign. This is the normal factorization, habit, the habit of the requirement of the answer.

Using Examples with Almost Similar Structure to Deepen Reflection on the Answers

Episode 4 Teacher-HK used the following question to focus the students' attention on Example 6 $2na+2n^2b$ and Example 7 $2na+2n^2b^2$ (Fig. 9.5).

> Why do you single out the n where there's an $nsquare$ in question six? But in question seven, where there's a $bsquare$, why don't you single the b out?

The specific objective of this question was to encourage the students to compare the answers of the two questions and thus to engage with a deeper reflection of the answers. With his questioning, the student's answer changed from 'it is very simple'

Fig. 9.5 Inspecting the factors closely

to ‘we can single the b out only when b exists on both sides!’ Finally, Teacher-HK rounded up the discussion by recapitulating the idea of common factor.

T (to the class) Common Factor. Since it is a common factor, they both need to have ‘b’. If it does not contain a ‘b’, we will not single ‘b’ out. And you need to know ‘b’ is not the common factor of those two terms.

In the interview, Teacher-HK explained that he intentionally guided the students to compare the two questions to experience a deeper reflection and better understanding of the idea of taking out the highest common factor (HCF). He explained:

> I would like to make a comparison between question seven and question six. I asked question was to strengthen the concept of the consideration of taking out the H.C.F. I asked a question which was specially designed. I asked why the n in the n square in question six could be taken out, while b in the b square of question seven could not.

Looking After the Cognitive and Emotional Needs of the Students: Getting Feedback for Regulating the Pace and Progress

Episode 5: After discussing the seven examples, at 28:30 Teacher-HK checked whether students could follow the discussion (Fig. 9.6). He explained that he wanted some form of feedback from the students so that he could adjust the pace of his teaching according to the students’ needs.

> T: I’ll stop here today. If it is clear to you so far, raise your hands!

Fig. 9.6 Asking students to raise their hands

He explained that he would ‘ask them to put up their hands when they know the answer. If there are few people putting up their hands, I would adjust my teaching progress. If many people put up their hands, I would teach faster’. Furthermore after discussing the seven tasks, the students were asked to spend the remaining 7 min of the lesson on assigned exercises as individual seatwork. Teacher-HK then provided between-desk support for individual students as they completed the set work. Teacher-HK then assigned the students the homework task.

Discussion

The basic assumption for studying the practice of expert teacher is that the findings can inform the professional development of prospective or novice teachers. Some well-designed routines of expert teachers may help novice teachers carry out some activities efficiently without diverting significant mental resources for more substantive activities (Lampert, et al., 2010). Analysis of Teacher-HK’s commentaries provided in juxtaposition with the episodes captured from the videoed lessons helped mathematics teachers and mathematics educators understand his pedagogical beliefs and knowledge and how these determine his teaching actions, his choice of examples and the questions he posed to the students. For example, if a teacher has a class of students who are not ready to speak up, then simple exchange routines such as call-on, revise and asking for clarification may help the teacher to accomplish explanatory work that needs students to articulate their ideas. These routines were well demonstrated by Teacher-HK. The videos showed that the students were fully engaged with the mathematical tasks throughout the whole-class question-and-answer session and the individual seatwork. Throughout the lesson, the students were given opportunities to answer questions, to explain their answers, to realise that their answers were wrong and to express their ideas and queries.

The teacher metacognition framework (TMF) (Artzt & Armour-Thomas, 1998) appears to play a useful role to help understand the teacher’s rationale for the lesson events and how these events might be conducive for the students’ learning of factorisation. There are apparently two aspects. The rationale for specific design of the teaching content or skills providing an experience of a particular facet of the concept is a blended result of the teacher’s knowledge, belief and goals. With respect to providing an effective instructional environment, Teacher-HK had demonstrated his expert knowledge in the task dimension and discourse dimension. The seven teacher-designed examples paved the way such that students could progress in their cumulative understanding of the meaning of factorisation as a reverse of expansion and taking out a unique set of factors. By the constructive teacher-student interaction in the lesson, the teacher had guided the students to make connection and comparison between consecutive tasks. The enactment of the design was driven by the teacher’s belief of effective teaching, his goals that he would like the students to achieve and his knowledge of the content and his students.

In the interviews, his elaborations unfolded his interpretations for the meaning of the factorisation concept in the context of the school curriculum. More noteworthy were the teacher's understanding of his own students, his awareness of their difficulties with the concept of factorisation and how he planned and executed his plan to help them overcome the obstacles that they might encounter in the learning of factorisation. The expert teacher's elaborations provide valuable insight to an expert teacher's philosophy of teaching. While it is important to have a sound understanding and deep knowledge of the subject matter, it is equally important to have deep knowledge of the students and his rationales for his action with reference to the authentic and complex context of a mathematics lesson.

It is difficult to learn these by imitation, but the analysis contributes to an understanding of the topic of factorisation in a pedagogical context. Nonetheless, the knowledge of expert teachers is profound (Berliner, 1986). Bearing in mind that the teachers' belief and conceptions are influenced by culture and experience, the purpose for studying the practice of expert is not necessary for direct transmission of teaching models. The pursuit of reflective practice using video technology in fact helps understand the feasibility of alternative practices in a deep sense, therefore, equipping teachers and teacher educators to face challenges for the teacher's professional development.

This chapter helps mathematics educators in several capacities. The study unfolded the teacher's instructional practice and associated cognitions in an integrated manner, hence, providing an opportunity for understanding an example of effective teaching. Specific to the topic of factorisation, the findings demonstrate some features that are pertinent to effective teaching, namely, the designing of good questions in a lesson based on an understanding of student's potential difficulties for the topic and the appropriate sequencing of tasks from simple to complex that required students to construct new knowledge based on their prior knowledge. These practices may well be applicable to the teaching of other topics. The teacher's reflection of his own lesson also highlighted these two aspects which are consistent with the framework offered by Artzt and Armour-Thomas (1999). Finally, the chapter demonstrates how the interplay between task dimension and discourse dimension may serve as a way for facilitating prospective and in-service teachers to reflect on lesson videos with reference to these two dimensions while designing their own instructional plans or reflecting upon their own lessons for understanding and improving their own teaching.

References

Artzt, A. F., & Armour-Thomas, E. (1999). A cognitive model for examining teachers' instructional practice in mathematics: A guide for facilitating teacher reflection. *Educational Studies in Mathematics, 40*(3), 211–235.

Artzt, A. F., & Armour-Thomas, E. (1998). Mathematics teaching as problem solving: A framework for studying teacher metacognition underlying instructional practice in mathematics. *Instructional Science, 26*, 5–25.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(3), 14–22, 43–46.

Berliner, D. C. (1986). In pursuit of the expert pedagogue. *Educational Researcher, 15*(7), 5–13.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24*(2), 417e436.

Calandra, B., Gurvitch, R., & Lund, J. (2008). An exploratory study of digital video editing as a tool for teacher preparation. *Journal of Technology and Teacher Education, 16*(2), 137e153.

Clarke, D., Emanuelsson, J., Jablonka, E., & Mok, I. A. C. (Eds.). (2006). *Making connections: Comparing mathematics classrooms around the world*. Rotterdam: Sense Publishers B.V.

Education Department, Hong Kong. (1999). *Syllabuses for secondary schools. Mathematics (Secondary 1 to 5)*. Access at http://www.edb.gov.hk/en/curriculum-development/kla/ma/curr/sec-math-1999.html

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164). New York: Macmillan Publishing Company.

Jackson, P. W. (1968). *Life in classrooms*. New York: Holt, Rinehart & Winston.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In *Instructional explanations in the disciplines* (pp. 129–141). New York: Springer.

Leinhardt, G., & Greeno, J. G. (1986). The cognitive skill of teaching. *Journal of Educational Psychology, 78*(2), 75.

Leinhardt, G., Putnam, R. T., Stein, M. K., & Baxter, J. (1991). Where subject knowledge matters. In J. E. W. Brophy (Ed.), *Advances in research on teaching: Teachers' subject matter knowledge and classroom instruction* (Vol. 2, pp. 87–113). Greenwich, CT: JAI Press.

Loughran, J. J. (2002). Effective reflective practice: In search of meaning in learning about teaching. *Journal of Teacher Education, 53*(1), 33–43.

Mok, I. A. (2010a). Students' algebra sense via their understanding of the distributive law. *Pedagogies: An International Journal, 5*(3), 251–263.

Mok, I. A. C. (2010b). Towards a reflective practice: The case of a prospective teacher in Hong Kong. *Journal of Mathematics Education, 3*(2), 25–39.

Peterson, P. L. (1988). Teachers' and students' cognitional knowledge for classroom teaching and learning. *Educational Researcher, 17*(5), 5–14.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. Coxford (Ed.), *The ideas of algebra, K–12* (1988 Yearbook, pp. 8–19). Reston, VA: National Council of Teachers of Mathematics.

Yang, X., & Leung, F. K. (2013). Conception of expert mathematics teacher: A comparative study between Hong Kong and Chongqing. *ZDM, 45*(1), 121–132.

Zhang, M., Lundeberg, M., Koehler, M. J., & Eberhardt, J. (2011). Understanding affordances and challenges of three types of video for teacher professional development. *Teaching and Teacher Education, 27*(2), 454–462.

Part II

Chapter 10
The Impact of Online Video Suite on the Singapore Pre-service Teachers' Buying-In to Innovative Teaching of Factorisation via AlgeCards

Weng Kin Ho, Yew Hoong Leong, and Foo Him Ho

Abstract A group of pre-service teachers at one institute of education was assigned to record their reflections as they viewed an online video suite which recorded (1) a secondary mathematics teacher employing a Concrete-Pictorial-Abstract (CPA)-based approach to teaching quadratic expansion and factorisation to a group of low-ability secondary 2 (14+) mathematics students; (2) students using concrete manipulative, AlgeCards, and pictorial representations, Rectangle Diagram, to perform quadratic expansion and factorisation; and (3) teachers, including a Head of Department (HOD), sharing their teaching experience in the use of the teaching package based on the abovementioned CPA approach. This chapter studied the impact of the online video suite on the pre-service teachers' willingness to *buy-in* to innovative teaching of factorisation via AlgeCards through inspecting the written responses of these teachers to preset Milestone Tasks placed at different points along the progression of the video watching. The term 'buy-in' refers to the evidential shift of teacher's belief in the applicability of the lesson innovation. We identified the salient features of the video suite which were responsible for bringing about pre-service teachers' buying-in to the use of AlgeCards in teaching factorisation. Based on these findings, this chapter makes some recommendations on the design of teacher preparation method course using video technology.

W.K. Ho (✉) • Y.H. Leong
National Institute of Education, Nanyang Technological University, Singapore
e-mail: wengkin.ho@nie.edu.sg; yewhoong.leong@nie.edu.sg

F.H. Ho
Curriculum Planning and Development Division, Ministry of Education, Singapore
e-mail: Foo_Him_Ho@moe.edu.sg

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_10

Introduction

One of the persistent problems in pre-service teacher education is the theory-practice gap that is purportedly experienced by pre-service teachers (henceforth, PT for short) as they move from methods courses offered by institutes of teacher education (perceived as focusing primarily on 'theory') to field placements in school (which they perceive as the arena for 'practice'). This sense of disjunction is not helped by the traditional structure of pre-service programmes: frontloading theoretical ideas taught usually in university classroom settings with scarce opportunities to interact with authentic practice and then positioning field placement as a follow-up course where these theoretical ideas are *applied* in actual practice (Darling-Hammond, 2010; Zeichner, 2010). We think that one way to smoothen this perceived theory-practice divide is to restructure methods courses in such a way that authentic practices are featured alongside the introduction of theoretical ideas. In so doing, methods courses are injected with a certain degree of authenticity that can serve to increase the sense of direct relevance of these courses and to heighten anticipation for the realities of practice during field placements.

It is, however, not always easy to provide opportunities to observe real-time classroom instructions within the duration of the methods courses. In the case of the consistently large pre-service cohorts of over 150 pre-service teachers at the research site, henceforth known as the Institute, the logistical cost of doing so arguably outweighs its benefits. Video records of authentic classroom instruction thus provide a potential avenue to realise the goal of bridging the theory-practice gap.

Videos for Teacher Development

Video has the potential of capturing the richness and complexity of classroom discourse. It allows one to enter the world of the classroom without being physically present in the teaching-in-the-moment (Sherin, 2004). Videos afford repeated analyses of images and audio that take place within the classroom discourse—the detailed interactions between teacher and students and between groups of students—all at times and locations to the convenience of the user. Unlike live observation of a lesson, video records are available on demand. Digital video clips can be processed and enhanced easily by embedding notes and instructions to aid learning and facilitate understanding. Also, video excerpts addressing a particular feature of teaching and learning can be isolated for analysis and discussion. Used in this way, video is a good tool to direct viewers' attention to focus on the desired learning objectives (van Es & Sherin, 2008, 2010). The problem pre-service teachers had with factorisation was that they were unable to connect factorisation of quadratic expressions to a concrete meaning that was visible to beginners.

We, the authors of this chapter, using our experiences in teaching the mathematics methods course in the Institute to several cohorts of PTs, agreed that most PTs who taught factorisation of quadratic expressions in their micro-teaching sessions (where PTs practised teaching certain topics to their peers) continued to use the usual 'cross-method' of factorisation. This was despite the fact that PTs were introduced the use of AlgeCards (or algetiles—a similar manipulative without algebraic labels) to teach factorisation of quadratic expressions. It therefore prompted us to investigate if the use of video suites for this particular topic has any positive effect in changing the beliefs of PTs. In particular, we wanted to ascertain whether the PTs would be convinced of the efficacy of the use of AlgeCards in the teaching of factorisation if they observed established teachers teaching using such tools to the extent that they express their willingness to try this teaching approach in their own lesson in the future. In addition, since video technology is most suitable for capturing the real-life use of the AlgeCards, the choice of this topic amongst others seems compelling.

Design Considerations of a Video Suite

Although there is growing literature on task design for teacher education in recent years (Thompson, Carlson & Silverman 2007), there is a lack of discussion on the following domains which are the foci (and constraints) of our work as mathematics teacher educators: (1) use of videos (2) on authentic classroom instructional work (3) for the purpose of teacher development, (4) based on tasks that are in-built with features which exemplify or lead to an awareness of sound pedagogical ideas in the teaching of mathematics and (5) that are suitable for use by a large cohort of PTs.

For Domain (1), the use of videos is a technological means. Domain (2) demands the use of genuine classroom teaching to show sound pedagogical practices. In pre-service teacher training, relevant theoretical framework and assumptions of several learning theories were taught to the pre-service teachers. Usually, these theories were supported by some case studies documented in the literature. *However*, pre-service teachers did not have the opportunity to see a live application of these theories in an authentic classroom situation. The contents of the video necessarily address this specific need that was not addressed by the theoretical phase of their training. Domain (3) refers to teacher training. For Domain (4), task design principles must be present to guide PTs in their training as teachers. Finally, Domain (5) demands that whatever the method of pre-service teacher training, it must be applicable to cater for large numbers of PTs who were undergoing the methods course at one time. In addition, Domain (5) demands that the means of instruction must also be available on demand for training of the PTs. Notice that the current methodology of using video for pre-service teacher training presents itself as a natural intersection of all these five domains.

The models of video use for professional development of teachers attend to some but not all of the requirements listed above.

The following six principles guided us in designing the video suite:

(a) The videos capture authentic classroom practices. In the Singapore context, local classroom conditions typically include (1) one teacher giving instructions to a class of size not exceeding 40, (2) grouping of students of roughly the same overall academic ability and (3) requirement for the teacher to complete a fixed set of topics listed in the mathematics syllabus via a commonly shared (amongst other teachers) scheme of work.
(b) The classroom practices exemplify instructional innovation that is based on sound pedagogical principles.
(c) The materials contain sufficient evidence that directs the learner to assess the applicability of the innovation and to infer the pedagogical principles underpinning the innovation.
(d) Within the overarching anticipated learning trajectory embodied in a sequence of tasks, there is room for individual learner's preferences and interests.
(e) There are opportunities for learners to reflect on their learning.
(f) The suite is fully accessible online.

The motivation for (a) and (b) was to show PTs how teaching approaches and learning theories taught in the methods course could be applied to a typical classroom in a local school. The emphasis here was about applicability. One common concern of teachers is that the innovation may be time consuming. They may choose not to use the innovation if the trade-off is not so favourable. More precisely, they have to decide if the innovation is too time consuming and the learning outcome is not obvious or too insignificant for them. We now give elaborations on (c)–(f). Video is only a tool for learning (LeFevre, 2004). So, for (c), the tasks offered for learning and the nature of facilitation with video need to be coordinated in such a way as to capitalise on what video has to offer. Thus, directing the learners to assess an instructional innovation and to look for the theoretical underpinnings provides this needed focus to their work on the video-based tasks. At the same time, we wanted to retain a sense of flexibility within the task sequences so as to allow elements of self-directedness in the learning. This balance was provided for in (d).

Reflection is an integral component for teacher thinking and development (Dewey, 1933; Schön, 1983, 1987). Hence, (e) is an important component of this study, and the use of reflections will be elaborated in a later section of this chapter. Through purposeful reflections, learners were given the opportunity to compare and contrast what they have observed against their existing beliefs (Alsawaie & Alghazo, 2010; Stockero, 2008). This pairing could result in coherence, confrontation or changes. In any case, the task of reflection was intended to provoke some of these outcomes that can facilitate deeper thinking about mathematics teaching. As for (f), given the current technology and infrastructure available, online was a mode that was appropriate to allow learners access to the videos. It allowed for multiple entries at different time junctures (throughout the day) to the video suite and thus offers flexibility that could help circumvent problems associated to time scheduling and diversities in learning styles. We were also motivated by the ease in importing this video suite as a stand-alone unit across relevant methods courses that were offered concurrently.

Constructing Contents for the Video Suite

The second and third authors of this chapter had just completed a *Lesson Study* project with some mathematics teachers of a local secondary school. The teachers were implementing an instructional innovation that was based on the Concrete-Pictorial-Abstract (CPA) approach to the teaching of secondary 2 (to students of 14+) mathematics. CPA is adapted from Bruner's (Bruner, 1966) enactive-iconic-symbolic theory and is an approach commonly advocated (Leong, Ho, Cheng, & Ho,, 2013) for mathematics teaching in Singapore schools. Readers may refer to Leong et al. (2010) for full details of the study.

The intervention in the *Lesson Study* project involved the use of a manipulative—known to the teachers in the project school as AlgeCards—consisting of three types of 'tiles': a square tile with dimensions of x units and its area duly labelled as 'x^2', a rectangle tile with dimensions of 1 unit by x unit and its area duly labelled as 'x' and a square tile with dimension of 1 unit and its area duly labelled as '1'. Students were given these tiles, and they worked on them to form a rectangle with combined areas represented by appropriate quadratic expressions. Figure 10.1 provides one example of a rectangle where the area is represented by the quadratic expression $x^2 + 3x + 2$. The length and breadth of the rectangle are the factors of the trinomial. The basic objective of the task was for the students to see the association between the area of the rectangle and the product of the two linear factors. The higher objective was for students to achieve the level of abstraction, i.e. the product of linear factors (dimensions of the rectangles) gives the value of the area of the rectangle, namely, the quadratic expression. After substantial experimenting with the AlgeCards, the students then proceeded to represent the rectangles pictorially by drawing an abstracted form using paper and pencil. Finally, they expressed the

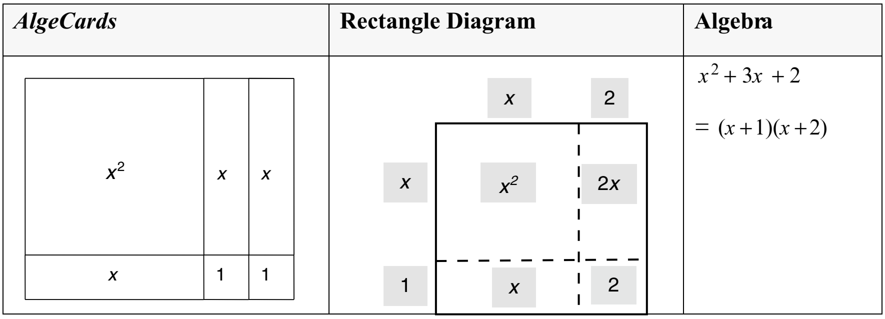


Fig. 10.1 Linking *AlgeCards* to rectangle diagram and to the algebraic factorisation

factorisation algebraically. The three modes described correspond roughly to the concrete, pictorial and abstract stages of CPA, and they are illustrated in Fig. 10.1. The resident teacher taught the three lessons, each lesson of 45 min duration.

The secondary mathematics teachers had chosen to apply the innovative approach to the teaching of factorisation of quadratic trinomials because the students found factorisation of a quadratic trinomial challenging. In particular, they failed to see the relationship between the two linear factors and the trinomial. The lessons were innovative because they showed how a teacher in a typical Singapore classroom was able to employ the CPA approach, using AlgeCards, to help students establish the aforementioned relationship. Hence, the use of AlgeCards as an instructional innovation made the lessons more meaningful to the students than in a conventional classroom where the teacher would show the algorithmic procedure for finding the factors of a quadratic trinomial.

Furthermore, the work of this teacher was carried out in real time, under the usual local classroom conditions (e.g. 35 students), and exemplified a lesson implementation that was designed and supported by sound pedagogical principles (i.e. CPA approach) and thus fulfilled the design principles (a) and (b) for selecting materials for the videos.

All the relevant data from the project school—videos of classroom happenings, videos of teachers' and students' reflections, lesson plans and worksheets—we collected from the *Lesson Study* enterprise were processed for the purpose of packaging them in a way that would fulfil design principle (c), that is, through accessing the videos, the learner will not only see a limited view of the lessons; rather, they will be able to take different angles (such as students' learning, teacher-observers' points-of-view) to enable them to follow the entire flow of the lessons and thus assess whether it is feasible to deploy the CPA approach in their own teaching of factorisation.

The contents were packaged into an e-learning suite which consists of five sequential sections. The structure and contents are summarised in Table 10.1. Videos were cut into snippets of no more than 4 min each. In particular, the video snippets in section 'Videos for Teacher Development' demonstrated in a step-by-step fashion how the teacher exemplified the use of the AlgeCards over a large number of trinomials. In line with design principle (d), the PTs have the flexibility to view as many of these demonstrations as was needed to gain an understanding of how the AlgeCards helped students learn about factorisation and how they can be deployed in a typical classroom situation with 35 students. Similarly, in section 'The Study', a number of teachers who had used the same teaching innovation with their students were interviewed; the PTs may choose to view all or some of these interviews to get as much information as was needed to appreciate the feasibility of this teaching innovation through the lens of the teachers themselves.

The PTs were required to proceed with each section sequentially. To ensure that the PTs spent sufficient time viewing a number of video snippets so as to have a more holistic idea of the actual innovation as it was carried out in the project class, an administrative device was set in the video suite to prevent PTs to proceed to the next section at will. Also, the exact duration of each section was not made known to

Table 10.1 Structure of the e-learning suite that pre-service teachers accessed

Section	Main content	Data from Lesson Study project	Post-section Milestone Task for student teachers
1	Overview of instructional innovation	Lesson plans and worksheets used, article by Leong et al. (2010)	Milestone Task 1: tasks that check PTs' proficiency in the use of the Rectangle Diagram in expansion and factorisation
2	Zoom-in to Lesson 2	Selected video snippets of Lesson 2 that highlight the key moves of the lesson, including selected snippets of students' seatwork and discussions	Milestone Task 2: a series of questions about Lesson 2
3	Zoom-out to before and after Lesson 2: Lessons 1 and 3	Selected video snippets of Lessons 1 and 3 that highlight the key moves of the lesson, including selected snippets of students' seatwork and discussions	Milestone Task 3: a question about PTs' overall response to the module
4	Views of teachers and students	Video snippets of interview sessions conducted with selected teachers in the Lesson Study team. Video snippets of interview sessions with two students selected by the teachers as amongst the most mathematically challenged	Milestone Task 4: a review of PTs' response in Milestone Task 3 in the light of feedback from teachers and students
5	Overall reflections	None	Overall response to the package in relation to ideas about teaching mathematics covered in the mathematics methods course at NIE

the PTs; otherwise, such knowledge of these durations may result in some PTs skipping the entire segment of the video within a section and only returning to the end of it for transit to the next section. Moreover, the Milestone Tasks at the end of each section were intended to provide focus and goal to what the PTs would notice from the videos. The Milestone Tasks were also built in for the purpose of providing opportunities for their reflections (design principle (d)). In line with design principle (e), the PTs accessed all the videos and attempted the Milestone Tasks solely through electronic means over a recommended total time—not necessarily continuous in one go—of 6 h. Prior to the implementation of this video suite reported in this chapter, a pilot version was implemented to an earlier cohort of PTs for the same methods course. From this preliminary implementation, we gathered sufficient feedback from the PTs concerning both the time spent in each section and the total time spent in working through the entire video suite. A majority of those PTs indicated that both these durations were long enough for them to work comfortably towards the completion of the Milestone Tasks. Reassured by this feedback, we decided that the timing set was about right and made no changes to these durations.

The Study

The focus of this study is to examine how video technology could be used to influence pre-service teachers' belief so that they were more likely to buy-in to innovative teaching of factorisation via AlgeCards. In the research literature on teachers' belief, it has been well established that a teacher's set of core beliefs are very resistant to changes (Ball & Cohen, 1999; Schoenfeld, 1998, 2003; Törner, 2002; Törner, Rolka, Rösken, & Schoenfeld,, 2006; Törner, Rolka, Rösken, & Sriraman, 2010). Such resistance could be partly explained by Lortie's notion of 'apprenticeship of observation' (Lortie, 1975, p. 61), i.e. PTs wanted to teach the way they were taught. Once PTs have acquired a set of beliefs of what works in their class, they are very resistant to changes. Because beliefs are very difficult to change, it is important to present to PTs alternative pedagogies which are powerful and produce results that are significant. In this study, we propose that video technology can be harnessed to influence the PTs' belief in the usefulness of the teaching innovation of using AlgeCards in teaching factorisation. Using this video suite, together with the Milestone Tasks, we want to identify which aspects of the suite have brought about observable changes, if any, in the PTs' belief about the usefulness of the teaching innovation of the AlgeCards. The Milestone Tasks were designed with the specific aim of eliciting the PTs' views about the CPA approach (via AlgeCards) as they progressed through the different sections of the suite. We hypothesised that the PTs' responses to those questions in the Milestone Tasks would provide evidence for shifts, if any, of the PT's beliefs.

This chapter focuses on how, with the aid of AlgeCards and the CPA approach, teachers were able to help students acquire a more meaningful understanding of factorisation and to make sense of the three representations, concrete, pictorial and algebraic (in this case, the CPA approach).

Analysis

We adapted Manouchehri's (2002) five levels of reflection, *describing*, *explaining*, *theorising*, *confronting* and *restructuring*, to classify the levels of reflection manifested in the PTs' responses. Reflection at the *describing* level was a surface description of the happenings in the classroom. Reflection at the *explaining* level was an attempt to relate PTs' actions to a teacher's approach to teaching; in other words, a cause-and-effect explanation was dominant at this level. At the *theorising* level, a PT's reflection on certain trajectory of teaching or teaching approaches was supported by research findings or learning theories. Reflections in the c*onfronting* category referred to PTs' ability to provide deeper analysis whereby they expressed different opinions about certain teaching approaches and teachers' actions. In the confronting stage, the PT questions about the existence of alternative theories or explanations that justify what has been observed in a certain learning-teaching moment. In the original formulation of 'confronting', Manouchehri, 2002 restricted its scope to those activities that involved the PT challenging one's own views about one's teaching or a peer's teaching. To be relevant to our present context in which the PT views the teaching practice of the videoed teacher, we have widened the scope of interpretation for 'confronting' to include those reflective processes of challenging one's own views about the teaching carried out by the videoed teacher. For example, those PTs who reflected by asking 'Did she do it right?' or 'Is there another way to explain what she was doing?' were classified as engaging at the confronting level. At the fifth level, *restructuring* referred to PTs' reflection that focused on 'how an experience (either their own or another teacher's) can be redesigned to avoid potential problems or better support learning' (p. 377, Stockero, 2008). In this study, restructuring occurred when a PT undergoes a change of beliefs that fit with modified conceptions of teaching. For example, some PTs, after watching how the teacher in the video snippet used the AlgeCards, began thinking of ways to modify the approach used by the teacher filmed in the video in order to suit to their own students' needs (e.g. PT's students may be of a different profile from the one shown in the video) because they might think that the teaching style demonstrated in the video was more suitable for a lower-ability class. A typical question such PTs would ask at this level of reflection is 'I think her method of teaching applies better to lower-ability students, but my students are higher-ability ones, and so what can I do differently from this teacher that would work better for my students?' Then such PTs would be considered to have experienced 'restructuring' in their own reflective stance.

Method

The participants in this study were 82 pre-service secondary mathematics teachers who were undergoing a 1-year Postgraduate Diploma in Education programme in one institute of teacher education. A mandatory component of this programme was a 66-h methods course in which pedagogical theories related to teaching and learning of mathematics at the secondary school level are taught. In particular, the Concrete-Pictorial-Abstract approach underpinned by Bruner's 'enactive-iconic-symbolic' modes of representation was a feature of this course. We use these modes of representation as a convenient subject of discussion upon which the PTs' reflection can be anchored. To familiarise the PTs with the CPA approach to teaching mathematics, the PTs attended an introductory lecture that illustrated how this approach could be adopted in conventional classrooms.

It was unreasonable to assume that the PTs would have acquired a clear idea of the CPA approach from one mass lecture. As a follow-up activity, the PTs were required to access online learning suite that contained video recordings of lessons on quadratic expansion and factorisation using the CPA approach that was implemented in the lesson study described previously.

Formulation of the Milestone Tasks

Because the raw data for this study were the PTs' online responses to the tasks which were placed at different time junctures of their learning journey (see Table 10.1), the way in which the questions were crafted and phrased in these tasks was of significance. We used the task design principles listed above to design the Milestone Tasks. Not only should the questions in each Milestone Task meet the learning objective of reinforcing the PTs' learning of how CPA approach could be applied in the teaching of factorisation of quadratic expressions and expansion of two linear factors but also elicit the responses that would adequately address our research questions stated in the purpose of our present study. Our present formulation was guided by the existing preliminary study (Leong et al., 2010) conducted with an earlier cohort of PTs who underwent the same programme and the same contents in the online suite, with this Milestone Task 2:

1. Discuss the role of AlgeCards in the instructional process in this lesson.
2. How is the smooth transition from AlgeCards to Rectangle Diagram critical to students' learning about quadratic factorisation in this lesson? Make reference(s) to the videos in your answer to this question.
3. Is there evidence to suggest that students in the video benefitted from this mode of learning? If so, what do you think are the main ingredients that contributed to its success?

In that study, although they were not asked to discuss the CPA approach, most of the responses focused on its applications. These PTs' responses were evidence that the online suite had the potential to be used as an example of how schools applied the CPA approach. Since the online suite was intended to deepen the PTs understanding of the CPA approach and how it can be applied in the actual classroom, the task design principles invited the PTs into a deeper state of reflection about the use of AlgeCards. To ensure that the PTs provided more in-depth reflections of the CPA approach, we revised the questions in Milestone Task 2. The revisions should leave no doubts in the minds of the PTs that their reflections should focus on CPA approach but still provided them with ample opportunities to express their thoughts and observations of the applications of the CPA approach to teach factorisation and expansion of quadratic expressions in the project school. To achieve this end, Questions 1 and 3 were retained, but Question 2 was amended to the following set of instructions.

2. View the lesson through the Concrete-Pictorial-Abstract (CPA) sequence; identify (1) the 'concrete' element(s), (2) the 'pictorial' element(s), (3) the 'abstract' element(s) and (4) the connections amongst these modes.

Furthermore, to guide the PTs to provide higher-order reflections, Milestone Task 3 provided PTs with a hypothetical situation where they were challenged to consider how they would implement the CPA approach to their own teaching.

Milestone Task 3: Imagine you are a team member in this teaching innovation project and you are about to carry out the module of lessons for your resident Sec 2NA Mathematics class. What are some things you buy-in to the innovation? What are some things you will change/add? In each case, provide supporting reasons by way of appealing to sound practices or theories you have learnt in the last semester.

With these amendments, it was hypothesised that, as the PTs progressed from Milestone Task 2 to 3, the emphasis of their reflections would begin to shift from *theorising* with the application of the CPA approach to *confronting* their own beliefs on how to teach factorisation and expansion of quadratic expressions. The intent of the question 'What are some things you buy-in to the innovation?' was an attempt to capture changes, if any, in the teacher's belief system, which may come in the form of reinforcement or a complete change of viewpoint. Section IV provides the final video suite, where the PTs view the feedback provided by the teachers who taught these lessons, the Head of the Mathematics Department of the participating school and selected students who participated in these lessons.

Milestone Task 4: Have the feedback of the teachers and students about their experiences in the project altered or reinforced your views about this teaching approach as expressed in your comments above? Explicate your thoughts/observations below.

The objective of Milestone Task 4 was to capture possible shifts in PTs' beliefs and to guide the student teacher into experiencing *restructuring* in his or her reflective stance. Before exiting from the entire learning suite, the participating PTs were required to write down their overall reflections of the innovations featured in the video suite. The PTs' reflections were crucial part of this study as they provided evidence which parts of the video suite heightened the PTs' reflective noticing and which parts were responsible for the PTs' buy-in to the innovation.

This research design would allow us to use the their feedback as evidence of (1) the PTs' reflective stance and noticing ability and (2) PTs' beliefs about the application of the CPA approach to teaching based on these grounded video images. Alongside this qualitative analysis of the students' reflection notes, we concurrently identify those salient features of the video suite, be it the structure of the suite or the way in which the Milestone Tasks were phrased, that are connected to those significant reflective activities observed in the PTs' comments.

Analysis

Each PT was identified using a numerical code, 1–82. PTs' responses to the questions from Sections II to IV were extracted from the online web-based platform specially designed for the implementation and monitoring of this study. Each PT's responses were analysed section by section against the five reflective noticing levels: (1) *describing*, (2) *explaining*, (3) *theorising*, (4) *confronting* and (5) *restructuring* (Manouchehri, 2002).

Coding was done by two of the authors of this chapter. Before proceeding with the actual coding process, the two coders classified and coded some responses. This was to ensure that there was consistency between the two coders. Those ambiguous responses that were difficult to code were singled out for discussion and recoding by both coders. Table 10.2 shows some example of responses belonging to different levels of reflective noticing.

Findings

Reflection Levels

Table 10.3 shows the proportion of PTs reflecting the various levels of reflection. Since PTs may reflect at some or all of the levels of reflection, the total responses *need not* be 100 %. The reflections in the 'describing', 'explaining' or 'theorising' level came mainly from Milestone Task 2 questions, whereas Milestone Task 3 and 4 questions elicited majority of the responses from the next two reflection levels, viz. 'confronting' and 'restructuring'. On average, about 15 % more PTs provided high-level (confronting and restructuring) than low-level reflections. It is interesting that the PTs in this study provided more at higher-level than lower-level reflections. This finding is in stark contrast to the findings reported by Stockero (2008) where PTs' reflections about others and their own teaching were predominantly at low levels. One possible reason for this difference could be that the sequential nature of the tasks and the way the Milestone Task questions were crafted and worded may have helped to channel and direct the PTs' attention on noticing the essential components of the teaching approach.

'Buy-in' referred to PTs who believed that the CPA approach was effective for teaching a class of academically challenged students the factorisation and expansion process. Table 10.4 shows that approximately 72 % (59 out of 82) of the PTs indicated 'buy-in' to the teaching approach. Table 10.4 shows the proportion of those who buy-in to the approach and their reflections at each of these levels.

The 'buy-in' PTs were sufficiently engaged at each reflection level, with an average proportion of 73 % engagement across the five levels. The highest frequency of reflection occurred at the 'confronting' level, followed by the 'restructuring'. One would have expected that for the non-'buy-in' group, the higher levels of reflection

Table 10.2 Some sample responses classified into the five levels of reflection

Level	Examples of reflective noticing by PT
Describing	PT 9: The students seemed to have enjoyed using the manipulatives more than I had imagined
	PT 23: I saw some students were still hesitant in arranging AlgeCards even though they had been taught the systematic way of arranging it
	PT 41: The students in the videos generally look engaged in the lesson, and most seemed to follow the lesson well. Some were even able to quickly skip the AlgeCards and move on to the tile diagram or rectangle diagram
Explaining	PT 25: The AlgeCards also helps the teacher to relate the area of a rectangle to the concept of expansion and similarly the length and breadth of a rectangle to the concept of factorisation
	PT 41: I think one of the reasons for the success is because it has concretised an abstract algebraic procedure and made it into a more tangible process for students. For students, it seems easier to move pieces around to form a rectangle than to work out different combinations of possible factors to find the correct factorisation
	PT 44: The students firstly experiment with the concrete element 'hands-on', developing some understanding through placing the AlgeCards at different positions to form a rectangle for each problem. Subsequently, with the help of the pictorial elements, students are able to visualise the concrete element without physically manipulating the cards and thereby begin to use the abstract elements to solve the mathematical problems related to expansion and factorisation
Theorising	PT 40: This innovation has tapped on a multimodal representation, i.e. according to Bruner's cognitive modes of enactive (concrete), iconic (visual) and symbolic (algebraic)
	PT 44: According to Piaget's cognitive process of assimilation and accommodation, this approach uses previously learnt mathematical concepts which students are fluent and familiar with to integrate new perspectives and create new learning outcomes
	PT 82: According to the paper 'Concretising factorisation of quadratic equations', it was mentioned that the building of the rectangle has to begin with placing the constant term (term without an 'x') at the corner of the x^2 term
Confronting	PT 18: will consider grouping the students into pairs such that every pair will work on one set of AlgeCards to promote collaborative work
	PT 44: Perhaps, students should be encouraged to discuss more with their desk partners as peer learning has been proven to be relatively effective in strengthening learning outcomes for student
	PT 61: The Rectangle Diagram is somewhat similar to the 'cross-method', but I am afraid the students will be stuck upon finding areas and fail to understand the way to bring out common terms or grouping of like terms

(continued)

Table 10.2 (continued)

Level	Examples of reflective noticing by PT
Restructuring	PT 23: Personally, I think it would be nice if at the end of these lessons, teachers can compare the 'cross-method' for factorisation with the Rectangle Diagram. As such, whatever they had been taught would not be left as different pieces of mathematics
	PT 25: However, I will not buy-in the use of AlgeCards and Rectangular Diagram throughout all sessions. Students should wean off the use of such representations; otherwise, they will be overreliant on it, which is not beneficial for exam purposes
	PT 62: The teacher could make the lesson more interesting by asking students to go to the board to do some examples and correct them if there is mistake. Teachers can bring students to computer lab to use an interactive Algebra disc programme to let students explore and learn algebra and virtual manipulatives

Table 10.3 Proportion of pre-service teachers reflecting at different levels of reflection ($n=82$)

Reflection level	Describing	Explaining	Theorising	Confronting	Restructuring
Percentage	59 %	67 %	57 %	80 %	73 %

Table 10.4 Proportion of pre-service teachers who buy-in to the teaching approach and the different levels of reflection ($n=59$)

Reflection level	Describing	Explaining	Theorising	Confronting	Restructuring
Percentage	66 %	75 %	68 %	80 %	80 %

Table 10.5 Proportion of pre-service teachers who do not buy-in to the teaching approach and the different levels of reflection ($n=23$)

Reflection level	Describing	Explaining	Theorising	Confronting	Restructuring
Percentage	39 %	44 %	30 %	83 %	57 %

were not as frequently engaged in contrast to the lower levels. But this is not so (see Table 10.5).

Tables 10.4 and 10.5 indicate that (1) the non-'buy-in' group has a generally much shallower engagement in the lower levels of reflection and (2) the 'buy-in' group was more reflective at the theorising level than the non-'buy-in' group.

It is important to examine carefully the underlying assumptions made in Manouchehri's 5-level model concerning the transitions from explaining to theorising and from theorising to restructuring. At the explaining level, the PT 'identifies and discusses the causal factors in the course of an interaction', but the PT's acknowledgement of these factors 'is not supplemented by a description of how one can analyze the event or further study the problem' (Manouchehri, 2002, p. 721). At the third level of reflection, theorising, the PT attempts to substantiate his/her explanation by reasoning from data and invokes known theories from research or experience from previous coursework to justify his/her explanation. At the fourth level of reflection, confronting, the PT uses the reasoning obtained at the level of theorising to revisit his/her explanation made in the explaining stage and checks its validity. During the stage of confronting, the PT questions the alternative ways of assessing the situation. Thus, confronting can manifest itself as a challenge to one's own views or a challenge to the views and practices of others.

For the buy-in group, there was evidence that before transiting to the next level of reflection, a relatively high level of engagement had already occurred in the present (and preceding) level(s). This explains why, on the average, the buy-in group exercised more reflection at each level as compared to the non-'buy-in' group (i.e. 73 % versus 51 %). This somehow suggests that the buy-in group has been more conscientious in completing each Milestone Task and progressing through the different levels of reflection, i.e. paying adequate attention to each of the levels, and caring not to skip to the next level until sufficient thinking has been carried out at the present level.

Although the non-'buy-in' group seemed to exercise a high frequency in confronting, a careful look into the comments given by PTs belonging to this group revealed that they were merely challenging the teaching practice of the teacher in the video basing their arguments mostly on their own views or experiences, rather than on established learning theories. Evidently, 70 % of the non-'buy-in' group did not engage theorising at all.

Reasons for Buy-In

The 59 PTs who were willing to 'buy-in' to the CPA approach wrote about their 'support' at either Milestone Task 3 or 4. One of the reasons for the 'buy-in' was the presence of the 'insider's opinion' or an 'internal reflection'. All the PTs entered their comments based on their observation of the lesson. It was only at Section IV that they heard the 'other side of the story', i.e. how the teachers who taught the lessons felt, how the students thought about their learning experience, etc. For instance, PT7's belief in this teaching innovation was reaffirmed when his/her opinions resonated with students' and teachers' views.

> I think the feedback of the teachers and students have reinforced my views about this teaching approach, … is more meaningful to the students to make sense of what they are learning.

More importantly, it would appear the opinions of those in authority may influence PTs' beliefs. Many PTs' beliefs in the efficacy of the CPA approach were reinforced by the comments made by the Head of Department (HOD) and students. More than 85 % of the PTs made some comments on what they thought based on the HOD's comments. The following are examples of responses from four PTs:

> After hearing the HOD, I think that the rectangle method helps promote understanding and this helps students learn better as they are able to make sense out of it. (PT13)
>
> It is insightful to learn from the HOD of Maths that the stronger students should also be exposed to this method so that they can make sense of what they are doing and not just apply methods mechanically. It has altered my view on whom to expose this method to and now I felt that it would be good to expose every student to this method. (PT14)

> However, what struck me the most is the words of the HOD. She did acknowledge the fact that this method might not appeal to the better students, but the bottom line is that the rectangular method can give meaning to the mathematics that the students do. (PT51)

In fact, unlike many preceding studies on the use of video technology in preservice teacher education, the video suite incorporates the concept of 'internal reflection'. This 'internal reflection' came in the form of the video clips in Section IV, where the teachers, the students and the HOD talked about their experience in the teaching package. While the remarks made by the teachers and the students had some impact on the reflective stance of the PTs, the data seem to point heavily towards the presence of the HOD in Section IV. In the Singapore school system, the HOD is seen to be a school leader who not only is well versed in both the content and pedagogical knowledge but also has an influential role in shaping the curriculum and approaches to teaching in the school. Thus, it was not surprising to see that PTs' beliefs were, to a great extent, influenced by the viewpoints of the HOD; and in certain cases, such an impact results in a shift in the PT's attitudes and opinions about the teaching approach. While we must be careful not to claim that 'buy-in' equates to a more desirable outcome, it is perhaps wise to realise that the presence of this 'internal reflection' somewhat widens the PTs' perspective, reduces the possibility of biasness and encourages an open mindset that we educators have always been advocating.

Implications and Conclusions

Implications

Contrary to the findings by Stockero (2008) which showed that PTs' reflections about others and their own teaching were usually at the low levels, the PTs in this study showed more engagement at higher-level than lower-level reflections. This difference in findings could be a consequence of the nature of the tasks which were designed created scaffolds for the PTs to focus on their reflective thinking.

The structure of the e-learning suite may have aided the PTs by encouraging them to think and reflect more deeply about teaching, students' learning and the use of the AlgeCards in the CPA approach. The design and the structure of the video were guided by six principles. Because the online suite did engage the PTs in deeper reflection, it is then instructive to examine more closely the usefulness of the guiding task design principles (a)–(f) in ensuring effectiveness of the online learning suite. Thus, it is useful to discuss the impact of the six design principles individually.

(a) *The videos capture authentic classroom practices.* If a teaching innovation is shown to be effective in a typical local classroom, the PTs are more likely to be convinced about its applicability. The following PT's view supports this point:

I do believe that these ingredients can be incorporated into typical lesson in Singapore schools consisting of mathematically challenged students. (PT22)

(b) *The classroom practices exemplify instructional innovation that is based on sound pedagogical principles.* With Bruner's CPA as a well-founded theory of instruction, the teacher in the video showed how lessons can be conducted in the usual Singapore classroom context that made used of AlgeCards.

(c) *The materials contain sufficient evidence that directs the PTs to access the applicability of the innovation and to infer the pedagogical principles underpinning the innovation.* The video clips capture the essential parts of the teaching package, recording how the teacher used the AlgeCards in a step-by-step manner and how the students engaged in using this manipulative to learn factorisation of quadratic trinomials. This claim is supported by evidence from the PTs' description of what they observed took place in the classroom captured in the video, for instance:

The role of the AlgeCards is to concretise the concept of expansion and factorisation. Students are able to *have hands-on experience and opportunities to arrange the cards in order to obtain the 'best' rectangle.* The introduction of AlgeCards helps the students to raise questions on 'what is the best rectangle?' and it helps the students to better visualise the process of factorisation and expansion.

Because the video snippets were arranged in such a way that the PTs could generate a continuum of what, in their mind, would be a very close approximation to the entire implementation, the PTs were able to judge for themselves what would be, in their opinion, the effective features of the teaching innovation and to make connections with a relevant learning theory associated to the teaching innovation. One PT's reflection notes supported this claim:

The concrete element (AlgeCards) and pictorial element serve as strategies to concretise and scaffold the abstract algebraic representations of quadratic expressions which most students find it hard to understand. The concrete element in particular brings in the abstract concepts into forms that allow the students to 'do' and 'see' for themselves (i.e. help the students to achieve another perspective of the abstract concepts). The use of the concrete representations will allow the students to better learn and understand the basics using tools that are more developmentally appropriate for students who are concrete operational.

(d) *Within the overarching anticipated learning trajectory embodied in a sequence of tasks, there is room for individual learners' preferences and interests.* The Milestone Tasks focus the PT on some aspects of the teaching innovation shown in the video but allow the PT to freely choose those observations that were of interest to the PT and to comment on them.

(e) *There are opportunities for learners to reflect on their learning.* The Milestone Tasks scaffold the reflection process throughout the video suite. The questions were phrased in such a way to invite the various levels of reflection to occur. Milestone Task 1 leads PTs into describing and explaining, Milestone Task 2 leads them to theorising, and Milestone Tasks 3 and 4 lead them to confronting and restructuring.

(f) *The suite is fully accessible online.* This feature allows users to access the video suite at any time they felt convenient—an exploitation of modern technology to circumvent problems of time scheduling and to promote e-learning.

Our statistics indicated that amongst the 'buy-in' group, not only were the highest occurrences of the reflection taking place at the higher level, there was sufficiently high average engagement in the lower levels of reflections. The situation was different for the non-'buy-in' group in that the average engagement in the lower levels of reflections was substantially lower than that observed in the 'buy-in group', while there still was a high proportion of engagement in confronting' and 'restructuring'. This interesting observation somewhat suggests that 'buy-in' is more likely to take place when a PT experiences a more holistic range of (and hence more authentic) reflection levels, beginning from the basic levels before proceeding to the advanced levels. From the responses supplied by the PTs in the 'buy-in' group, one can see that these participants took more effort to give detailed descriptions of what they reflected upon in each section. In short, these PTs were conscientious about the entire reflection process as they continued to stay focused on what was asked of them in all the Milestone Tasks.

Stockero (2008) raised an important open question of 'whether it is desirable, or even possible, to move entirely away from this type of reflection [i.e., at the describing and explaining levels]' and further cautioned that it may be the case that some amount of lower-level reflection (i.e. describing and explaining) is a *necessary* part of reflective discourse, as 'it sets the stage for higher level reflections' (p. 389, Stockero, 2008). To a certain extent, our present finding answers negatively to Stockero's question. Indeed, more than just being necessary, lower levels of reflection should occur at a substantially high amount before a PT gains enough 'energy' to jump to the higher levels of reflection. The upshot of this conclusion is that in designing tasks which are intended to develop and deepen PTs' reflecting stance during their pre-service teacher education, the instructor should help PTs build a sufficiently rich experience in the lower levels of reflections by giving them appropriate scaffolding prior to exercising their reflection at the higher levels.

It is important to realise that PTs in the 'buy-in' group were stronger in theorising than those in the non-'buy-in' group. PTs who 'buy-in' are likely to support their explanations of what they have observed by referring to a well-known learning theory that was taught to them earlier in the methods course. Based on this theory, such PTs would use this theory they considered to confront the explanation or views given earlier on. Then reassessing other possible approaches, factors and theories, such PTs would then move on to suggest what improvement or modification could be made if they were the one teaching the lesson. Here is a sample of a 'buy-in' PT who passed to the levels of confronting and restructuring via theorising:

E: It (CPA approach) is also easier for students to construct their own links to how length and breadth of the rectangles actually has more purpose than they already know.

T: Using the approach of perceiving rectangle area and its side with expansion and factorisation made it easier for the students to relate to. It is similar to engaging the students' prior knowledge and linking what they already know with something new. According to *Piaget's cognitive processes*, this approach falls under assimilation where the teaching of new knowledge (expansion and factorisation) is done by expanding existing schemata (area of rectangle).

C: Although the students found it easier to solve their factorisation problems using the new method, they were unable to give a clearer description to it. In fact, there is still a risk this approach would be similar to cross-method approach where students know how to use but do not really understand what and why they are doing it. From Zaki, it was evident that he did not catch the relationship of the area of a rectangle to expansion, while Hariss felt that he found it easier to get his answers using the method.

R: When introducing the individual AlgeCards, it might have been better if students are given some time to think how the card area can be x and let them identify by themselves that if the area is x, ($x \times 1 = x$), the sides of it are x and 1.

Our findings suggest that deeper levels of reflection can only take place when PTs engage sufficiently in the lower levels and must make meaningful connections of what has been observed (describing) and explained (explaining) with sound pedagogical theories that they have learnt in the methods course (theorising). Without going through theorising rigorously, some PTs might appear to be engaging in confronting, but what they may be merely doing is a baseless 'fighting' against the views and practices of others rather than that of their own. Therefore, it is important for PTs, during their pre-service teacher training, to acquire a sound knowledge of learning theories so that they can begin to think of how their teaching can be restructured after having observed how others teach.

Theorising is therefore a crucial piece in this jigsaw puzzle of teacher's reflective stance and teacher's belief system. Our present findings suggest that a change of teacher's belief towards 'buying-in' a new teaching innovation or approach is most likely to occur if the teacher can invoke relevant learning theories to challenge his or her own explanation of what was being observed and then make creative modifications to the new teaching innovation or approach. This realisation then brings us back to begin where we talked about the theory-practice divide that we singled out as a common deficiency in most methods courses. The implication is therefore to integrate an essential exercise of developing PTs' reflective abilities taking advantage of video technology and the six task design principles into the methods course so that PTs are given a natural setting to invoke the learning theories they acquired earlier in the methods course and translate them into new teaching approaches and overall improvement in the quality of their own teaching.

In conclusion, video technology could support the learning of various key players in teacher development. As mathematics educators, we are encouraged by the affordances provided by video technology. We found that grounded images have deepened the connection between pedagogical theories and classroom practices. This connection lays the very foundation for authentic contextual learning in a pre-service teacher preparation course: sound pedagogical theories account for the processes that take place in classrooms, and teaching practices are effective because they are informed by relevant theories. Thus, we would like to reiterate the importance for PTs to have sound theories of learning supported by research and to provide examples which modelled good pedagogical practices *and* for significant others to support the innovation.

For mathematics teachers, video technology has the potential to enhance their reflective stance. Based on grounded images, teachers have the opportunity to recapitulate accurately the processes that had taken place while they (or their peers) were teaching. Only when critical and constructive reflection, supported by sound pedagogical theories, takes place can concrete and well-informed improvements be made in their teaching practice.

For school leaders and that in authority, in particular the Heads of Department, they are role models for professional growth. They themselves must be highly engaged in every level of reflection so that they can also buy-in to the most appropriate innovative practices.

References

Alsawaie, O. N., & Alghazo, J. M. (2010). The effect of video-based approach on prospective teachers' ability to analyze mathematics teaching. *Journal of Mathematics Teacher Education, 13*, 223–241.

Ball, D., & Cohen, D. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Skyes (Eds.), *Teaching as a learning profession* (pp. 3–32). San Francisco, CA: Jossey-Bass.

Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge, MA: Harvard University Press.

Darling-Hammond, L. (2010). Teacher education and the American future. *Journal of Teacher Education, 61*(1–2), 35–47.

Dewey, J. (1933). *How we think*. Boston, MA: D. C. Heath and Company.

LeFevre, D. M. (2004). Designing for teaching learning: Video-based curriculum design. In J. Brophy (Ed.), *Using video in teacher education* (Vol. 10, pp. 235–258). Amsterdam: Elsevier.

Leong, Y. H., Ho, W. K., Cheng L. P. & Ho, F. H. (2013). Concrete-pictorial-abstract approach in Singapore mathematics instruction: Surveying its origins and charting its future. *Journal of Mathematics Teacher Education, article under review.*

Leong, Y. H., Yap, S. F., Teo, M. L., Thilagam, S., Karen, I., Quek, E. C., et al. (2010). Concrete factorization of quadratic expressions. *The Australian Mathematics Teacher, 66*(3), 19–25.

Lortie, D. (1975). *Schoolteacher: A sociological study*. Chicago, IL: The University of Chicago Press.

Manouchehri, A. (2002). Developing teaching knowledge through peer discourse. *Teaching and Teacher Education, 18*, 715–737.

Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education, 4*(1), 1–94.

Schoenfeld, A. H. (2003, April 19–23). *Dilemmas/decisions: Can we model teachers' on-line decision-making?* Paper presented at the annual meeting of the American educational research association, Montreal, Quebec, Canada.

Schön, D. (1983). *The reflective practitioner: How professionals think in action*. New York: Basic Books.

Schön, D. (1987). *Educating the reflective practitioner*. San Francisco, CA: Jossey-Bass Higher Education Series.

Sherin, M. G. (2004). New perspectives on the role of video in teacher education. In J. Brophy (Ed.), *Using video in teacher education* (pp. 1–27). Bingly, UK: JAI Press.

Stockero, S. L. (2008). Using a video-based curriculum to develop a reflective stance in prospective mathematics teachers. *Journal of Mathematics Teacher Education, 11*, 373–394.

Thompson, P. W., Carlson, M. P., & Silverman, J. (2007). The design of tasks in support of teachers' development of coherent mathematical meanings. *Journal of Mathematics Teacher Education, 10*, 415–432.

Törner, G. (2002). Mathematical beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 73–94). Dordrecht: Kluwer Academic Publishers.

Törner, G., Rolka, K., Rösken, B., & Schoenfeld, A. (2006, April 7–11). *Teacher monologue as a safety net: Examining a German mathematics classroom situation through the lens of Schoenfeld's theory of teaching in context.* Paper presented at the 2006 annual meeting of the American educational research association, San Francisco, USA.

Törner, G., Rolka, K., Rösken, B., & Sriraman, B. (2010). Understanding a teacher's actions in the classroom by applying Schoenfeld's theory *Teaching-In-Context*: Reflecting on goals and beliefs. Theories of Mathematics Education, Advances in Mathematics Education (pp. 401–420). Berlin/Heidelberg: Springer.

van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers "learning to notice" in the context of a video club. *Teaching and Teacher Education, 24*, 244–276.

van Es, E. A., & Sherin, M. G. (2010). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education, 13*, 155–176.

Zeichner, K. (2010). Rethinking the connections between campus courses and field experiences in college- and university-based teacher education. *Journal of Teacher Education, 61*(1–2), 89–99.

Chapter 11
Reflections of a Korean Middle School Mathematics Teacher on Improving the Teaching of Mathematics

Ho Kyoung Ko

Abstract Reflection with appropriate feedback plays an important role in enhancing professional development of teachers. This chapter reports how Teacher-K, an experienced middle school mathematics teacher, used the affordances of grounded images of five videoed lessons and the feedback offered by a mathematics educator to improve her teaching of mathematics that necessitates the incorporation of new curricular initiatives. Initial reflection of Teacher-K was limited in scope, but quantity and quality of reflection improved with each subsequent reflection session. The teacher reconsidered the critical teaching actions of each lesson, reorganises them and sought ways to improve her lesson. She demonstrated aspects of being a reflective practitioner by her willingness to apply these ideas to her teaching. This study showed this experienced teacher was not only concerned with teaching of mathematics in general but also the specific ideas of mathematics ignored by others as trivial. Mathematics educators have much to learn from such experienced teachers.

Introduction

Since the publication of Shulman's (1986) seminal paper, pedagogical content knowledge (PCK) of teachers has generated much interest among researchers, practitioners and policymakers. The reason for this is that content-related category of teacher knowledge symbolises the 'blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction' (Shulman, 1987, p. 8). PCK brought attention to the fact that the knowledge required for teaching is not a simple matter of knowing the subject contents

H.K. Ko (✉)
Graduate School of Education, Ajou University, Suwon, South Korea
e-mail: kohoh@ajou.ac.kr

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_11

and pedagogy. This led to various studies that attempted to identify exactly what characterises the knowledge required for teaching of specific subject matters.

Science, Technology, Engineering, Arts, and Mathematics (STEAM) is Korea's response to US STEM Education. It seeks to apply technological and engineering design-based pedagogical approaches to teach science and mathematics. Teaching of mathematics in tandem with other disciplines such as science and the arts can enhance the learning of mathematics (Sanders, 2009).

The Korea Ministry of Education, Science and Technology (MEST) adopted STEAM as the official government programme to develop Creative and Convergent Talents in primary and secondary schools (MEST, reference). The aim of STEAM education is to help improve Korea's science and technology competitiveness (Ko et al., 2013). Although Korea is a high-performing country in the Programme for International Students Assessment (PISA) for science and mathematics, students' appreciation of science and mathematics is poorly ranked, being 43rd out of 49 countries. To bridge this divide and bolster Korea's push for greater innovation to face the unknown challenges of the twenty-first century, STEAM education is perceived as an important process to facilitate students' interest for and understanding of science and technology and develop their integrated thinking and problem-solving ability (Maes, 2010).

To promote STEAM education policy of MEST, the Korea Foundation for the Advancement of Science and Creativity (KOFAC) is spearheading research to prepare teachers at all levels to use STEAM in their teaching of mathematics and science. Teachers at all levels need well-structured professional development courses to prepare them to adopt STEAM policies (Sanders et al., 2011). Teachers' teaching performance is closely related to their professional development. 'Professional development' here includes not only professional knowledge and domain skills but also high-level cognitive skills that can help teachers solve problems and deal effectively with new situations (Park et al., 2008).

Various studies evaluate the effectiveness of teaching through the ability to engage teachers in reflective thinking and deepen their reflective activity (Choi, 2003; Dinkelman, 1997; Hamrick, 1995). Most of the existing studies on teachers' reflective thinking are based on their reflective writings (Jo, 2011; Lee & Kim, 2013). Self-reflection through writing allows teachers to ponder deeply over their practices; it is, however, limited as teachers still have to rely on their own memory and awareness of classroom events and developments. On the other hand, when teachers' self-reflections are supported by video technology, they are more likely to assess their teaching, the meaning of events or situations more accurately. There is therefore a pressing need to conduct an empirical study on the nature of teachers' reflections on teaching and identify the ways in which to improve this reflective thinking using grounded images collected using video technology.

This chapter reports on the nature of a middle school mathematics teacher's reflections on her teaching of mathematics. Given this opportunity to review a number of lessons, what aspects of her teaching would she select to improve? What reasons did she provide for wanting to improve these areas of her teaching?

Theoretical Framework: Teacher's Knowledge

The knowledge base of teachers should, as a minimum, incorporate the following seven categories: (1) content knowledge, (2) general pedagogical knowledge, (3) pedagogical content knowledge, (4) curriculum knowledge, (5) knowledge of learners and their characteristics, (6) knowledge of educational contexts and (7) knowledge of educational ends, purposes and values and their philosophical and historical grounds (Shulman, 1987)

Content knowledge comprises, inter alia, knowledge about the facts, concepts, principles, definitions, theorems, rules and structures, which are crucial elements for teacher effectiveness. It concerns the amount of knowledge and how that knowledge is organised, which may determine the effective development of mathematical concepts. It also incorporates disciplinary knowledge, such as understanding the rules of evidence and proof, the methods of enquiry into the domain, what warrants a particular proposition, the importance of knowing the proposition and its associations with others. In essence, the 'teacher need not only understand that something is so; the teacher must further understand why it is so' (Shulman, 1986, p. 9). Ball further delineated Shulman's characterisations according to three criteria: knowledge of concepts and procedures, understanding the underlying principles and meanings and an appreciation and understanding of the connections between mathematical ideas. In addition to content knowledge, there is the knowledge about mathematics, which includes:

> understandings about the nature of mathematical knowledge and of mathematics as a field. What counts as an "answer" in mathematics? What establishes the validity of an answer? What is involved in doing mathematics? In other words, what do mathematicians do? Mathematical knowledge is based on both convention and logic. Which ideas are arbitrary or conventional and which are logical? What is the origin of some of the mathematics we use today and how does mathematics change? (Ball, 1990, p. 458).

General pedagogical knowledge transcends subject matter, referring to the broad principles and strategies of classroom management and organisation that appear to transcend subject matter. Effective teachers of most disciplines would recognise the applications of these principles and strategies.

PCK differs from general pedagogical knowledge as it is discipline specific. This knowledge differentiates a content specialist from a pedagogue (Shulman, 1987, p 8). It enables teachers to transform content knowledge into forms that are comprehensible and accessible to a specific audience. This includes knowing the most appropriate examples to introduce a concept, using and connecting appropriate representations for the particular concept and using appropriate analogies to deepen understanding of it. PCK is the 'special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding' (Shulman, p. 8).

Within PCK, we find the knowledge of content and students (Ball, Thames, & Phelps, 2008). The effective delivery of a lesson is contingent upon teachers' knowledge of their students. What preconceptions do students have towards mathematics? What misconceptions or difficulties do they have towards a specific mathematical concept?

Curriculum knowledge informs teachers how a mathematical programme is structured and what and when to deliver specific mathematical topics. Teachers need to know the variety of available instructional materials that will support their delivery of a particular mathematical programme. Such knowledge 'serve[s] as both the indications and contraindications for the use of particular curriculum or programme materials in particular circumstances' (Shulman, 1987, p. 10). This knowledge is crucial, particularly with the introduction of new initiatives. Teachers need examples of how to teach mathematics when the focus of the curriculum changes. For example, with the incorporation of information and communication technology (ICT) into the teaching of mathematics, teachers need examples of how to conduct specific mathematical topics. What mathematical concepts are best developed using ICT and how may its use deepen students' understanding of specific concept?

Knowledge of learners and their characteristics combines knowing about both students and mathematics. Examples of this useful knowledge for teachers are the knowledge of what motivates students, the ways in which they conceive mathematics, what aspects they find interesting or confusing and how they choose to solve mathematical problems. Armed with such knowledge, teachers are better prepared in terms of the mathematical tasks that they choose to engage students in and the solutions that they anticipate from different students. Teachers who know their students well may be able to engage with them using language appropriate to their level and extend on these students' responses to particular mathematics tasks (Ball, Thames, & Phelps, 2008).

Knowledge of educational contexts includes understanding the dynamics of the group or classroom with regard to 'the governance and financing of school districts, to the character of communities and cultures' (Shulman, 1987, p. 8)

Knowledge of educational ends, purposes and values and their philosophical and historical grounds is rarely discussed in the literature. A plausible reason for its omission could be that it would appear more suitable for policymakers and curriculum designers.

The above discussion shows that the knowledge needed by mathematics teachers for good teaching has many dimensions. Knowledge of mathematics content is clearly very important, but it is not sufficient to ensure quality teaching (Hill, Schilling, & Ball, 2004). Knowledge of general pedagogical techniques is also a reasonable prerequisite to good teaching. However, teaching proves to be more complex than a simple application of pedagogical techniques to convey mathematical content (Ball, Thames, & Phelps, 2008).

Reflection and Its Relationship to Professional Development

Reflection is a critical and creative activity that describes, analyses, interprets and evaluates a class by considering the phenomenon of the class comprised of teachers and students. Dewey's work on reflection undergirds studies on teachers' reflective thinking. Reflective thinking results from personal mental activity, such as detailed analysis based on observation, data collection and evidence (Dewey, 1910). Dewey

(1944) argued that the purpose of reflection was to identify the relation between actions and their consequence. Reflection is the process in which teachers make continuous decisions to improve their teaching (Van Manen, 1977), apply self-criticism when looking back on their teaching and considering multiple viewpoints and decide how to change their practices to address specific issues (Hatton & Smith, 1995). Park et al. (2008) defined reflection on teaching to be a process of self-reflection to achieve better decision-making by rethinking and evaluating teaching behaviour to gain a sense of the logical rationale behind it. When teachers become more reflective, they develop valuable theories about their own practice in their own class settings (Dewey, 1904).

When a person acts and reflects, they become a researcher in a practical setting (Schön, 1983). As reflective practitioners, teachers do not only rely on previously established theories and technology but also on their own theories based on their own unique experiences. For this to happen, classes should be viewed as a space where teachers' reflections affect their solutions of authentic classroom problems.

Using Video Technology to Increase Teacher's Reflective Thinking

Teachers' reflection of teaching needs to be based on grounded images and not only on their subjective thinking and memories. Video technology can provide graphic and grounded images about the language and behaviour of teachers and students in the classroom. Teachers see, hear and reflect based on clear evidence of what happened during the class, which allows a much more accurate reflection. Moreover, data using video technology allows teachers to observe the atmosphere in the classroom in a more comprehensive manner. It helps them to obtain various opinions and experiences by looking for the meaning of experience inside the context of what is occurring in the classroom.

The goal of the reflection using videos is not to evaluate, but rather to understand the proceedings of the class. Further, through reflection on teaching, teachers are not just given more knowledge, skills or abilities relevant to teaching, but rather they come to know about self-teaching through a psychological equilibrium: acquiring the necessary practical knowledge by discovering 'how' and 'why' and not just on the superficial level of their teaching subject. Through video observation, teachers see their class from a different perspective, and through the process of interpreting and reinterpreting the meaning of what they observe, they are able to experience a growth in their own awareness. In this manner, they define their own awareness about educational circumstances, obtain alternative points of view and construct and reconstruct knowledge about what constitutes excellent teaching. Accordingly, the goal of such reflection needs to promote reflective thinking as an important approach in the professional development of teachers. The ultimate goal of reflection is to enable teachers to become independent evaluators and critics of their own practice, thus enabling them to use and apply their knowledge rationally, and so bring about a qualitative improvement in their classrooms (Dewey, 1904).

The Study

Teacher-K had videoed herself teaching mathematics to a class of middle school students. She watched the lesson to analyse the effectiveness of her teaching. However, she realised that without the feedback of an expert, the process of self-reflection and hence her growth as a teacher were limited. Her growth would improve if she were able to receive critical inputs from a mathematics educator. Teacher-K approached me for support.

I agree to be part of Teacher-K's learning experience. To be of value to her, it was necessary to determine what she had been unable to perceive from watching herself at work and then work with her over a series of videoed mathematics lessons in order to examine how her subjective evaluation evolved over the course of a number of lessons. Hence, the objectives of this study were twofold. Firstly, it examined how the qualities of reflection changed as Teacher-K observed herself at work over a series of five mathematics lessons. Secondly, it investigated the aspects of professional knowledge that were the focus for Teacher-K.

Method

Among the five videoed lessons, the first and second lessons were on the teaching of graphs of quadratic functions, the third and fourth on the application of quadratic functions and the fifth on a STEAM-based material entitled 'Star in the Night Sky'. Each lesson lasted for approximately one hour. A single camera was placed centrally at the back of the classroom, irrespective of whether it was an activity-based lesson or a lecture.

Teacher-K reviewed the recorded lesson with the researcher during the week. Firstly, she reflected on her own teaching methods using the 'think-aloud approach' (Brown & Rogers, 2002) before being asked to expand on her self-reflections. So as not to disturb her self-reflection, the researcher used unstructured interviews to gain greater insights into Teacher-K's values and beliefs on teaching, choices of teaching methods and actions. She gave me permission to record her self-evaluation of the taught lessons and interviews for further analysis.

Participants

Teacher-K was a middle school mathematics teacher with 6 years of teaching experience. She is also the author of mathematics textbooks and teacher guidebooks. Furthermore, she was interested in conducting research in content mathematics and mathematical activities, which were of interest to middle school students, and pedagogy to address their interests and aptitudes.

Nevertheless, Teacher-K felt she had limited knowledge of good teaching practices. Other than the theories of mathematics education acquired as a pre-service teacher, she had not received other forms of mentoring on how to improve her practices. She wanted to change her current teaching style, which she described as expository in nature. She welcomed the opportunity to participate in this research as this gave her a chance to learn about alternative ways of teaching mathematics. Furthermore, Teacher-K was aware that she had only a superficial understanding of the STEAM education system that was introduced in 2013. She did not know how to conduct mathematics lessons based on STEAM; hence, she wanted to evaluate this new STEAM initiative. By articulating her thoughts on STEAM, she would be encouraged to reflect on how to improve the teaching of STEAM-based mathematics. She explained:

> I only know that STEAM education is the fusion of disciplines. I can give a lesson, but I don't know how to apply it to mathematics. I do it as I think it is only centred on activity… It will be important to let students have ideas or design it. Thus, I need to organise my next class on 'creative design' (5th reflection session).

Data Analysis

The primary data were re-examined by Teacher-K and the researcher. The researcher used 'stimulated recall' (Nunan, 1989) with Teacher-K, which enabled both parties to clarify any unclear portions of the transcriptions.

Teacher-K's reflections were analysed using a constant comparative analysis of open coding, categorisation and category verification (Ezzy, 2002) using the following three steps. Firstly, three mathematics educators, including the researcher, labelled and categorised each piece of data using the open-coding process. Secondly, the three educators double-checked the classified and coded data, classified them into similar subjects and labelled the coding names. Thirdly, all data were compared to similar data in a constant and repetitive manner. The three educators referred to Shulman (1987) to create categories that best expressed the characteristics of the data set.

Analysis and Findings

The analyses of Teacher-K's reflections were grouped into two sections. Section one discussed how the video helped Teacher-K to evaluate her teaching. Section two analysed the aspects of her teaching that Teacher-K considered important, which were divided into six categories: (1) pedagogical knowledge, (2) teaching strategy, (3) teaching skills, (4) understanding students, (5) teaching environment and (6) overall class.

Usefulness of the Video in Helping Teacher-K Evaluate Her Teaching

Teacher-K found the grounded images captured by the videos extremely helpful because she used these concrete examples to anchor her discussion with respect to the way she responded to students' questions and responses. Figure 11.1 shows that without the aid of the grounded images, what she remembered was based on vague and subjective images, and these seemed more positive than what happened in the class. Thus, the grounded images provided evidence-based data that challenged what she remembered from her lessons. The use of grounded images thus helped Teacher-K construct clear and objective recollections of her lessons.

Although Teacher-K thought she had communicated clearly and that the students were cognitively engaged in her mathematics class, the grounded images showed otherwise. She did not ask suitable questions to help the students think about the mathematics. Furthermore, she did not address the misconceptions found in the students' answers to solutions:

> I usually thought that I communicated well with students and helped them understand the contents taught in my class. I thought the knowledge that students should know was constructed in my class through a shared understanding with students. However, through the video observation, I realised that I didn't consider posing appropriate questions to address students' misconceptions.

Contrary to her perception that students had understood the lesson very well, the grounded images proved her wrong as the students' understanding was questionable. This misplaced perception arose because she had based her perceptions on interactions with only a small group of students:

> I thought that the students' understanding was pretty high in my class. However, when I looked at my class through the video observation, I realised that students' responses and understanding as I had imagined them were quite different from the reality. I found that the reason was that I tended to focus on only a few students who pay attention in my class.

Fig. 11.1 Role of video technology

Teacher-K was rewarded for her efforts when she made concerted efforts to improve her explanations. She could pinpoint exactly the portion of the lesson on which the improvement was focused:

> When watching the video last time, I think there were some problems… I contemplated what to do better in this class. That is why I did this or that this time… I think the process of explanation in the middle was better than in the previous class.

Teacher-K also became more sensitive about time management. Because of poor time management, she was always in a hurry to complete the lesson:

> I did not know it, but there seemed to be a problem with time management in my class. I noticed that I would start the class slowly, but be in a hurry by the end in order to finish the class… I realised through video observation that I talked faster at the end of the class. Due to this, I realised that the contents at the end of class were not being properly delivered. I realised that this was serious when I watched the video.

Furthermore, with the aid of the grounded images, Teacher-K realised that she should put more thought into designing better lessons so that she would not end up giving a lecture when she intended to give a more interactive lesson:

> First of all, I thought about the contents to be taught, but I gave a lecture without an overall design in my class. When I looked at my class through the video observation, I criticised and analysed about whether my class was generally well designed or not.

After watching the videos, Teacher-K became concerned with issues of equity. She realised that not all students in her class received her attention:

> I think a teacher should show an interest equally by looking at all students in the classroom. So far, I thought I looked at all students equally when I gave and received feedback from many students in my class. However, I realised that my expected behaviour was different from what I saw through the video observation. Through this, I saw where I didn't look and whom I was more concerned about.

Teacher-K was so focused on delivering the planned lesson that she ignored other factors that could also affect her teaching and students' learning:

> In addition, I did not pay attention to other internal factors, aside from the contents to be taught, and I did not realise the need for various external factors, such as my expressions, gestures, and writing on the board.

Teacher-K explained how the videoed lessons helped to improve her delivery of future lessons. For example, having reviewed a particular lesson, she realised that she had not addressed particular aspects of a concept. The evidence reminded her of these omissions. In the follow-up lesson, not only did she improve her time management, but she also addressed these specific omissions from the previous lesson. The videoed lessons meant that the data were available on demand. More importantly, she was able to analyse her actions in her own time and without having to ask others for help. This meant that she was saved the embarrassment of others critiquing her teaching, something that is not easy to accept:

> I easily forgot ideas that I felt were lacking, things I shouldn't have done or should have done better. However, while I observed my behaviour through a video, I was able to concentrate on

> my behaviour and it helped remind me of the aspects that I wanted to work on. For example, when I had a class on the same content after watching the recorded video, not only was I was able to manage my time more efficiently, but I was able to work on parts that were lacking. Likewise, I was not only aware of the problems that were previously unknown, but I was also highly motivated to improve my behaviour. The biggest advantage was to have time to look back on my classes by examining at my behaviour in the class without help from others.

In summary, the grounded images of the videos challenged a very important assumption held by Teacher-K, notably that a teacher cannot assume that the students shared the teacher's understanding of the taught concepts. To assess students' understanding of a taught lesson, she needed to give greater consideration to students' responses to questions posed. It was important for her to identify why students lost interest and hence why they were not focused in the lesson. She made it her responsibility to improve her teaching by addressing factors, such as the mathematics to be taught, the nature of the activity used to deliver the content and the choice of teaching methods:

> Since I realised that students' response and understanding were different from what I considered in the class, I observed students' responses to the overall class contents and teaching method. After observing the video, I was motivated to make attempts to change my class in various ways, such as class design, teaching methods, and teaching contents. After broadly analysing the causes of and responses to students who did not concentrate in class, I was able to put greater effort into general fields, such as the understanding and concentration of students in the class.

Changes of K-teacher's Reflection on Teaching Practice

In the previous section, Teacher-K provided an overview of how the videos helped her to reconsider her teaching of mathematics. This section, however, shows how her reflections evolved over the five reflection sessions. Although she was less forthcoming in the first session, she became more articulate about her observations with each subsequent session. Her reflections showed that she gave greater considerations to the needs of the students, particularly those who failed to follow or lost interest in the lesson. Teacher-K's reflections on her teaching were evaluated based on six categories that she identified to be important. Table 11.1 provides a summary of these six categories and compares how the different components changed between the first and fifth reflection sessions.

Curriculum Knowledge Two constructs, awareness of mathematical content and reorganisation of teaching instruction, are subsumed under this category. Awareness of mathematical content refers to a teacher's appreciation of the academic and practical value of mathematics as a discipline. The introduction of STEAM into the Korean mathematics curriculum renders this knowledge particularly important. Although Korean students perform very well in international comparative studies such as Trends in International Mathematics and Science Study (TIMSS) and PISA, they do not have a high appreciation of mathematics. This may inadvertently affect the innovativeness of Korean students in the twenty-first century.

Table 11.1 Evolution of Teacher-K's reflections from the first to fifth session across the six categories

Category	First reflection session	Fifth reflection session
Curriculum knowledge	Reorganisation of teaching practice (2[a])	Reorganisation of teaching practice (7)
		Awareness of content (2)
Pedagogical content knowledge (PCK)	Strategy (1)	Strategy (3)
	Organisation (4)	Organisation (1)
	Form (4)	Form (5)
		Utilisation (7)
		Planning (8)
		Evaluation (1)
Knowledge of learners and their characteristics	Interaction (1)	Interaction (7)
	Level of learners (3)	Level of learners (5)
		Feedback (4)
General pedagogical practice	Skill of teaching practice (6)	Skill of teaching practice (3)
Knowledge of educational contexts		Teaching environment (1)
		Teaching atmosphere (9)
Class overall	Holistic approach (1)	Holistic approach (8)

[a]Frequencies of occurrences

In the first session, Teacher-K's reflections did not include both categories. She focused specifically on one component, namely, how to reorganise her teaching, and she was limited in the number of reflections. However, by the fifth reflection, she was more articulate, and the number of reflections on how best to reorganise her teaching had increased from two to seven (Fig. 11.2). By the fifth session, Teacher-K reflected on how she chose to integrate mathematics into other unrelated areas of learning:

> I was contemplating what could be associated with the topic or what mathematical element was related to the beautiful night sky. I was thinking about what should be drawn and be the outcome. Fortunately, I could create polygons that perfectly matched to the topic. (5th reflection session)

A reorganisation of the teaching instruction emphasised the importance of a sound knowledge of the mathematics curriculum as well as how the contents were organised in mathematics textbooks. Mathematics teachers should be cognisant of the changes in the curriculum, the mathematics to be taught in a specific year and the best way to teach this content. Without such knowledge, teachers may not be able to prepare effective lessons to cultivate the interest of Korean students. For example, Teacher-K knew that she lacked an understanding of the concept of the STEAM education system, which was a new curricular initiative. She was concerned about how to introduce this new initiative into her teaching. Her knowledge of STEAM was limited to what was discussed among her colleagues, but she wanted to learn more about it, either by discussing the new initiative with her colleagues or

Fig. 11.2 The theme of the class for STEAM is 'Beautiful Night Sky'. The screen at *left* shows a night sky. The figure at *right* shows the students constructing polygons

by learning about the alternative teaching strategies advocated in STEAM by observing the practice of her colleagues. As she explained:

> I only know that STEAM education is the fusion of discipline contents. I can give a lesson, but I don't know how I to apply it to the mathematics. I do it as I think it is only centred on activity… It will be important to let students have ideas or design it. Thus, I need to organise my next class focused on 'creative design' (5th reflection session).

Teacher-K thus recognised the need to consider the individual differences in students and provide them with the meaningful learning opportunities. For example, in implementing the STEAM-based curricular materials, she was able to summarise why the teaching outcomes may vary despite teachers sharing the same instructional materials. She may deliver the same content differently to her colleagues, while her students may have a different set of prior knowledge and experiences, thus interacting differently to the same curricular materials:

> Of course, [the material] should be discussed in detail with other colleagues. Even if the same materials are shared, students may learn about them differently. My direction may be different from that of other teachers. The most important thing in STEAM education is to configure the situation and have an idea about why this situation comes about based on the problems that students may have (5th reflection session).

Pedagogical Content Knowledge This construct examines the planning and organisation of instruction to achieve the goals of teaching and learning mathematics. Teacher-K focused a large part of her reflection on this construct. An analysis of her reflections identified the following six components in this construct:

1. Organisation
2. Form of instruction (macroscopic strategy with regard to planning teaching in general)

3. Type of teaching practice (group activities, individual teaching, descriptive teaching, etc.)
4. Planning class instruction (planning with regard to procedures and progress of teaching practice, such as introduction-development-summary-evaluation)
5. Evaluation (preparation of evaluation methods and procedures, development of evaluation tools)
6. Utilisation of tools and materials (various teaching-learning tools or materials that a teacher creates, prepares and uses for practice)

Although PCK comprised these six components, Teacher-K focused on only two in the first session, organisation and form of instruction. However, by the fifth session, Teacher-K had included the remaining four components.

For example, in the first reflection session, Teacher-K was aware of students' difficulties in understanding the concept of functions and wanted to invest more class time to help students identify the characteristics of the graph of a given function, but she did not have alternative methods to improve her teaching of this concept:

> Since there are many students who do not know the section of a function or how to draw a graph, I would like to invest more time in drawing a function graph. However, the conditions do not allow this. (1st reflection session).

In one lesson on function graphs, Teacher-K used a computer to show various graphs to the class. In this case, the instruction was teacher oriented, but she wanted the teaching and learning process to be learner oriented. One alternative strategy was to ensure that as many students as possible were engaged in constructing their own graphs (Fig. 11.3). She was thus aware of the need for the effective utilisation of computers:

> If students go to the computer room and do it by themselves in the next class, they would explore and understand them while saying, 'Ah, it moves up' or 'The formula changed like

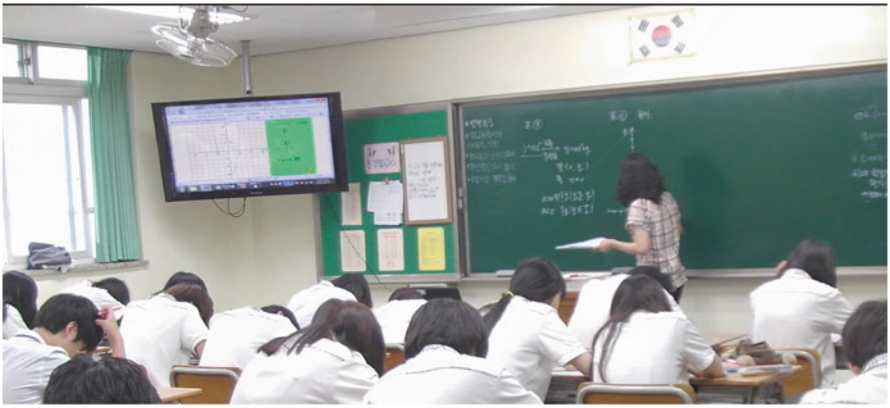


Fig. 11.3 Constructing graphs on the computers provided students the opportunity to manipulate and explore the structure of graphs as the 'formula' changed

> this'. It would be good for students. Because we cannot draw it every time [on paper], it is nice to find more things in common (4th reflection session).

Such a reflection is partially related to the notion 'how can I handle it in a different way' and the movement of 'reorganisation' (Hole & McEntee, 1999; Smyth, 1989). It also seems to show the aspect of reflection at a 'comparative level' in terms of finding other explanations or alternative points of view.

In addition, Teacher-K knew that she lacked an understanding of the STEAM education system, and she tried to look for other types of teaching practice with her colleagues and talked about her thoughts and judgements with regard to STEAM education:

> I only know that STEAM education is the fusion of contents. I give a lesson, but I don't know how to apply it to mathematics. I do it as I think it is only centred on activity… It will be important to let students have ideas or to design it. Thus, I need to organise my next class with a focus on 'creative design' (5th reflection session).

Teacher-K's reflection showed that she was aware that good instruction should integrate various modes of teaching strategies, such as student-oriented group work, teacher-oriented teaching or descriptive teaching. Furthermore, good instruction should incorporate an evaluation of students learning. Teacher-K critically reflected on the aspects of 'evaluation'; in other words, she clearly recognised that teaching practice should be organised as a complete structure to achieve the teaching goal, and she thus displayed the aspect of critical reflection on teaching practice with regard to the organisation of teaching at a later stage. She reflected on these components only in the fifth reflection section.

Knowledge of Students Teacher-K paid attention to students' participation in the mathematics lessons. An analysis of her observations showed that she paid attention to the following three components of learning:

1. Level of learners (considering the prior knowledge, interests and emotions of learners)
2. Interaction (facilitation of students' responses, facilitation and induction of interaction)
3. Feedback based on students' response (identification and treatment of misconceptions)

In the first reflection session, her reflections were limited to the level of learners and the interactions between them. However, the quality and quantity of her reflections increased in each subsequent session. In the fifth session, Teacher-K's reflection focused on her communication with students and their cognitive and affective behaviour.

Teacher-K was concerned for students who did not participate in the mathematics lessons, even when she used group work to encourage their participation. She realised that the weak students were not able to participate in the current lesson because they did not have the prerequisite knowledge to engage with the concept discussed that very day. She saw how unproductive this lesson was for such students. She noticed that grouping weak students together further compounded their lack of participation. Reteaching the lesson would not benefit such students. Teacher-K felt responsible for the progress of this group of students who possessed other good attributes, such as good leadership qualities and kindness. She wondered how best to help such middle grade students, but she did not even have the knowledge about the elementary school curriculum. Teacher-K wondered what changes could be made to the middle school curriculum to address the needs of such students:

> There is already a student sleeping in class. He is almost no good. He is totally no good. He is lacking, even as an elementary school student. Even if I repeatedly explain it to him, he does not understand. Those who are not good in a group [small group configuration] spend an hour doing nothing. I think that we need activities for students who are not good at studying (4th reflection session).
>
> He is a class president. He is not good at mathematics, but he is kind. Thus, he does not ask [for scissors to create a polygon]. I think I may have to assign roles in a class with small groups next time. I also may need to distribute materials (5th reflection session).

Nevertheless, Teacher-K saw that group work benefitted others who were not usually receptive to the mathematics presented in the conventional transmission method, that is, when they had to listen to the teacher teaching:

> I believe that these two have become a lot better. He used to be a student that would always sleep in class. Even though he is not concentrating, he looks happy. They look happy when they do group activities together (5th reflection session).

General pedagogical knowledge: In the first reflection session, Teacher-K placed greater emphasis on pronunciation, intonation, facial expression, movement, writing on the board and time allocation:

> I was generally satisfied. I wrote the goal of the class (on the board). I also gave students some time for activities and I initiated cooperative learning. Since it was my first attempt, I didn't know what to do. So I felt a bit awkward. (1st reflection session (1st reflection session).

Here Teacher-K's reflection showed she was satisfied with her teaching because she had introduced cooperative learning, a mode of teaching that she had never previously used in her mathematics class (Fig. 11.4). She had written the objective of the lesson on the board and given time to students to participate in cooperative learning. However, her reflection showed that she was more concerned with her performance than the students' learning from the activity. Nevertheless, in her second reflection, she questioned the effectiveness of such collaborative teaching, as once students completed the activity, they lost interest in the task. Teacher-K thus saw the need to bring the students together through whole class teaching in which the teacher conducted a class discussion. However, she did not express the effectiveness of the class discussion after the process of cooperative learning activity:

> Yes, moreover… students' concentration and cooperation between them decreased without group teaching. Once they found the answers to the questions, they lost concentration (2nd reflection session).

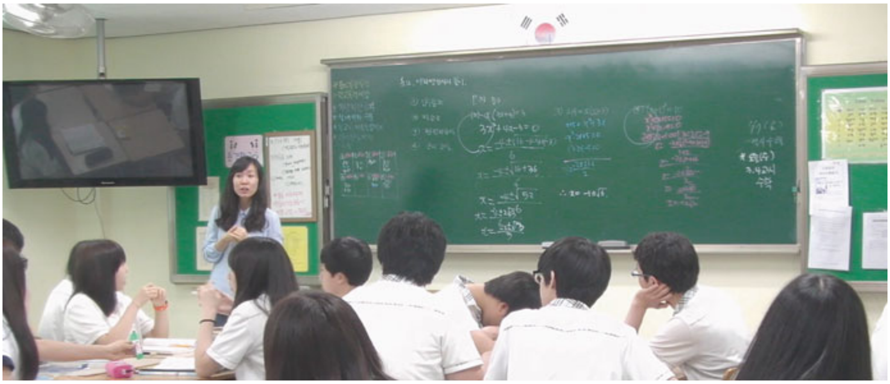


Fig. 11.4 The class that she was satisfied with

Knowledge of Educational Contexts Teaching environment and teaching atmosphere come under this category. The former includes the various materials and engineering tools available in the classroom for teaching and learning mathematics. The latter focused on the number of students, their concentration and the permissive or repressive atmosphere of a mathematics classroom. Initially, Teacher-K did not reflect on the importance of the teaching environment and atmosphere. However, in the later reflection sessions she became more aware of the environment and teaching atmosphere and how these could influence students' learning.

Professional Development of the Mathematics Educator

Teacher-K provided me with greater insights into how teachers grow over the course of their careers and the nature of the support appreciated by them. The quality of a teacher's reflection is not determined by their length of service (Ko, Maeng, & Nam, 2013). The act of reflecting is not a natural process when there is no necessity to justify one's choice of pedagogical actions. Thus, the experience of watching a video of one's own teaching may not be meaningful when the objective of the experience is unclear. Unless learners consciously search for a deeper meaning, not all experiences are educational (Dewey, 1938). In this section, I discuss two important concerns that teachers have with regard to their work. First, established teachers need opportunities to talk about their work with a critical friend; in this study, the mathematics educator was the critical friend. Second, established teachers need to talk seriously about the mathematics content that they teach.

Engaging in Their Work with a Critical Friend Although her peers considered her an effective mathematics teacher, Teacher-K nevertheless wished to improve her practice. Her time spent on watching the videos of her mathematics teaching would not have had a significant impact if she had not worked with the mathematics educator who chose not to remain as a neutral observer. The mathematics educator was not only willing to listen to her talk about her lessons, her justifications about her choice of pedagogical actions and her concerns about the students in her class but also prepared to question her about these decisions. Cognitively demanding questions were asked, such as the following: 'Why do you feel that way?' 'How did you determine that?' 'What did your students think about it?' The following dialogue shows how Teacher-K, when provided with the opportunities to share her teaching concerns, openly evaluated what was effective in her practice and what needed improving. Furthermore, she considered alternatives to improve the learning of her students. Such deep reflections would most probably not have occurred if there were no opportunities to share her thoughts with someone who was willing to listen.

Educator: You barely said a word an hour ago, but now you are telling many different stories

Teacher-K: I guess you're right… Speaking [with you] seems to have made me more talkative. I've realised many things [about my class] that I hadn't noticed before. Now I say to myself, 'This is me' and think, 'Oh, I need to change this [thing]'or 'That [idea] was pretty good!' (3rd reflection session)

Educator: These students are responding to the solving process now, aren't they?

Teacher-K: Yes

Educator: Last time, were they asked to give the answer only?

Teacher-K: Yes, at the time, I gave students more time to set up an equation

Educator: Why do you ask them to do so?

Teacher-K: I was concerned about the students who didn't solve the problems last time. I wanted to examine whether they understood them. However, I didn't. So, I wanted to see whether they could set up an equation, which was the intermediate process (reflection session)

In-Depth Conversations About Mathematics Content Among Teachers The mathematics that could be problematic to teachers need not be very challenging. In fact they could be rather trivial. Although such matters were trivial, they were a cause of frustration for Teacher-K because she was a careful teacher and she was concerned for the success of her students in the written examinations. Teacher-K was not confident whether a comma would suffice in presenting the solution of a quadratic equation or whether the conjunction 'or' could be used instead. Although such matter seemed trivial, it merited serious discussion at a level that reflected a deeper understanding of the mathematics by Teacher-K. Such a matter would not perturb teachers with less mathematical knowledge:

> In quadratic equations, I teach that $x = 3$ or $x = 5$ after doing factorisation. Many students asked whether 'or' must be written or whether writing the comma is wrong. I'm also very confused. Thus, I can't give a clear answer and I'm frustrated.
>
> In the test, even if one student was not wrong mathematically, it was graded as a wrong answer, because it was not shown in the textbook and it was beyond the scope of middle school. However, I didn't know what to do. Since I didn't teach it, it was graded as a wrong.

Working with Teacher-K was an enriching experience. The grounded images of the videos provided us with powerful tools to deepen the learning of both parties and identify the needs of established teachers.

Conclusions

This study showed that the process of reflection improved when teachers and mathematical educators work with grounded images taken from videos. In addition, challenging questions encouraged the teacher and mathematics educator to think more deeply about the teaching and learning of mathematics. The teacher actively and continually identified the problems faced by students and considered alternatives and detailed plans to solve problems that arose during the class. Over the process of five reflections, Teacher-K became more concerned with those who failed to engage with the mathematics, and she showed a genuine desire to help those challenged by mathematics. She was particularly interested in finding alternative methods to treat the issues. A reflective teacher should therefore analyse and criticise the process of solving educational problems, which results in gaining feedback and improving problem-solving skills (Zeichner, 1983). If this viewpoint is accepted, Teacher-K exhibited a qualitative growth in terms of her reflections on teaching practice.

Mathematics educator's deep interest in a teacher's narratives and the related classroom experiences could prompt even greater self-reflection on the part of the teacher. When mathematics educator asks good questions, the teacher is encouraged to reflect, restructure and reorganise her practical knowledge based on her shared ideas and stories of her personal experiences. It is important to provide feedback that valued the teacher's varied cognitive, psychological and emotional needs. For this process of learning to take hold, the teacher and educator should use grounded images objectively as a practical case material to learn from each other.

References

Ball, D. L. (1990). The mathematical understanding that prospective teachers bring to teacher education. *Elementary School Journal, 90*(4), 449–466.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Brown, J. D., & Rogers, T. S. (2002). *Doing second language research*. Oxford: Oxford University Press.

Choi. O. G. (2003). *The effects of reflective teaching on teacher's question level*. Unpublished doctoral dissertation, Korea National University of Education, Seoul.

Dewey, J. (1904). The relation of theory to practice in the education of teachers. In C. A. Mcmurry (Ed.), *The third yearbook of the national society for the scientific study of education*. Chicago: The University of Chicago Press.

Dewey, J. (1910). *How we think*. Mineola: Dover Publications, Inc.

Dewey, J. (1938). *Experience and education*. New York: Touchstone.

Dewey, J. (1944). *Democracy and education*. New York: The Macmillan Company (Original work published 1916).

Dinkelman, T. D. (1997). The promise of action research for critically reflective teacher education. *The Teacher Educator, 32*(4), 250–274.

Ezzy, D. (2002). *Qualitative analysis: Practice and innovation*. London: Routledge.

Hamrick, J. C. (1995). Elementary teachers' reflection on major influences impacting their decision in reading and language arts instruction during their first two years of teaching. *Dissertation Abstracts International, 57*(1), 87.

Hatton, N., & Smith, D. (1995). Reflection in teacher: Education: Towards definition and implementation. *Teaching & Teacher Education, 11*(1), 33–49.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105*(1), 11.

Hole, S., & McEntee, G. H. (1999). Reflection is at the heart of practice. *Educational Leadership, 56*(8), 34–37.

Jo, D. J. (2011). A study of pre-service program focusing on student teachers' reflective thinking. *The Journal of Yeolin Education, 19*(4), 139–167.

Ko, H., Choi, S., Yoo, M., Oh, W., Kim, J., & Lee, K. (2013). *Development of model to improve contents, methods and evaluation of STEAM education for mathematics and other subjects in middle school*. Korea Education Development Institute Research Report PR-2013.

Ko, H., Maeng, E., & Nam, G. A. (2013). Case study on factors and levels of teachers' reflective thinking on their instructions. *Research of Curriculum Instruction, 17*(3), 603–627.

Lee, S. A., & Kim, H. J. (2013). The characteristics of American pre-service teachers' reflective thinking and their relationships with epistemological beliefs. *The Journal of Yeolin Education, 21*(1), 233–256.

Maes, B. (2010). *Stop talking about "STEM" education! "TEAMS" is way cooler*. http://bertmaes.wordpress.com. Accessed 22 June 2012.

Nunan, D. (1989). *Understanding language classroom*. London: Prentice-Hall.

Park, S. R., Han, S. H., & Lee, M. J. (2008). Reconsideration of the research trend for the reflection on teaching. *Journal of Learner-Centered Curriculum and Instruction, 8*(2), 403–422.

Sanders, M. (2009). STEM, STEM education, STEM mania. *The Technology Teacher, 68*(4), 20–26.

Sanders, M., Kwon, H. S., Park, K. S., & Lee, H. N. (2011). Integrative STEM (Science, Technology, Engineering, and Mathematics) education: Contemporary trends and issues. *The Secondary Education Research, 59*(3), 729–762.

Schön, D. A. (1983). *The reflective practitioner: How professionals think in action*. New York: Basic Books (Reprinted in 1995).

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–22.

Smyth, J. (1989). Developing and sustaining critical reflection in teacher education. *Journal of Teacher Education, 40*(2), 2–9.

Van Manen, M. (1977). Linking ways of knowing with ways of being practical. *Curriculum Inquiry, 6*, 205–228.

Zeichner, K. (1983). Alternative paradigms of teacher education. *Journal of Teacher Education, 34*(3), 3–9.

Chapter 12
The Use of Videos and Classroom Artefacts in Professional Development of Teachers and Teacher Educators in Indonesia

Wanty Widjaja and Maarten Dolk

Abstract This study is grounded on adaptations of Realistic Mathematics Education, Lesson Study and design-based research in Indonesian classroom contexts. Design-based research has gained currency in educational research over the past decade due to its strength to bridge the divide between theoretical research and educational practice in naturalistic settings. Design-based approaches involve a process of designing mathematical tasks, observing the enacted design in classrooms and reflecting on the process from analysing the classroom artefacts. Video plays a central role in supporting teachers and teacher educators to study and reflect on students' mathematical thinking and in capturing the dynamic of classroom teaching and learning process. This chapter will examine and analyse practitioners' lenses in capturing the dynamic and complexity of classroom mathematical learning using video vignettes and classroom artefacts including digital photos of classroom moments and students' work. Practitioners' lenses are taken as a window to capture key teaching and learning moments from the lessons. Analysis of this selection of these video vignettes along with other classroom artefacts based on practitioners' lenses provides insights into practitioners' views on key teaching and learning moments in mathematics lessons.

W. Widjaja (✉)
Faculty of Arts and Education, School of Education, Deakin University, Burwood, VIC, Australia
e-mail: w.widjaja@deakin.edu.au

M. Dolk
Freudenthal Institute, Utretch, The Netherlands
e-mail: m.dolk@uu.nl

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_12

Introduction

In Indonesia, there has been a concerted effort to improve the quality of mathematics teaching and learning in classrooms. The large number of under-certified teachers and the rural nature of a large part of the country ask for professional development programmes that empower teachers through active collaborative peer involvement. In an era with increasingly more powerful mobile phones with better video- and photo-taking capabilities, collecting artefacts of teaching and learning in a classroom becomes more affordable and feasible. A critical issue is to provide a set-up that allows teachers to create self-supporting research groups.

The adaptation of Realistic Mathematics Education (RME) in Indonesia was driven by the intention to make mathematics more accessible and meaningful for students and improve students' attitude towards mathematics and quality of mathematics teaching in schools (Gijse, 2010). Indonesia is a large and culturally diverse country; an adaptation of RME – called *Pendidikan Matematika Realistik Indonesia* (PMRI) – needs to fit the diverse local contexts in Indonesia. PMRI places critical importance in learning mathematics through students' active engagement in communicating their thinking during the learning process. In contrast to the Indonesian educational tradition in which teachers are seen as the sole authority of knowledge and students are expected to learn standard algorithms and use these to solve simple mathematical applications, PMRI calls for a pedagogical approach with an emphasis on problem solving and communicating the thinking process in learning. Sembiring – one of the founding fathers of PMRI – likens the PMRI pedagogy with 'democratic teaching through mathematics' (Gijse, 2010, p. 20) as PMRI encourages students and teachers to listen to each other, defend and justify their thinking, value different opinions and discuss various mathematical approaches.

Realistic Mathematics Education started in the Netherlands at the end of the 1960s to renew Dutch mathematics education. It also formed the Dutch 'response' to the New Math movement in the United States. Freudenthal believed that mathematics should be thought of as a human activity of mathematising. To him, mathematics is a process of structuring, schematising and modelling, not a discipline of structures that could be transmitted (see, for instance, Freudenthal, 1978, 1991).

In RME, students are not seen as passive recipients of existing mathematics, but they should be guided to reinvent mathematical knowledge. The idea of guided reinvention 'is to allow students to come to regard the knowledge that they acquire as their own private knowledge, knowledge for which they themselves are responsible' (Gravemeijer & Doorman, 1999, p. 116). The learning starts with situations or problems that are 'experientially real' to the students (hence the name Realistic Mathematics Education). These are situations where the students realise what they can do and what makes sense to them in those situations. These can be 'real-world' situations but also situations from the history of mathematics, from applications of mathematics, from a fantasy world or from the world of numbers that are real to the students. These starting points for learning should not only be 'experientially real', they should also be justifiable in terms of the potential ending points of the sequence (Cobb, 2000). While working on context problems, the students develop strategies, knowledge, representations and models closely related to the situation. By offering different contexts and by generalising aspects of the contexts, the students develop more formal mathematical knowledge, and the models emerge from models of the situations into models to think with (Gravemeijer, 2004).

Gathering systematic evidence from classroom practice by research to inform teaching is perceived as a critical aspect in any reform initiative in education (Cobb, 2000; Jacobs, Lamb, & Phillip, 2010; Mason, 2002). Analysing classroom artefacts has been recognised to contribute to teachers' growth of knowledge, skills, beliefs and dispositions to improve teaching (Borko, 2004; Putnam & Borko, 2000). Mason (2002) claims that sharpening skills to notice relevant things in classrooms are vital for teachers as they constantly have to act and make pedagogical decisions in the moment. In similar vein, Jacobs, Lamb and Philipp (2010) underscore the significant role of professional noticing children's mathematical thinking as a foundation for productive learning experience for teachers. They conceived professional noticing of children's mathematical thinking as an interrelated skill comprising skills to: (a) attend to children's strategies, (b) interpret children's understanding and (c) use children's understandings as a springboard for instruction.

Extensive studies have documented the use of video as an artefact of practice to support teachers in developing their ability to notice and reflect on classroom practices (Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008; Sherin, 2004; van Es & Sherin, 2008). Video affords opportunities to capture the dynamics and complexity of classroom interactions and provides grounded images to hone in particular aspects of teaching (Ng, this volume). Video provides teachers and researchers the opportunity to engage in fine-grained analysis of classroom practice using multiple perspectives. Video vignettes can be examined several times with different foci and levels of insight in analysis to foster productive professional discussions for teachers' professional development (Borko, Jacobs, Eiteljorg, & Pittman, 2008). Lesh and Lehrer (2000) highlight the significance of complementing and triangulating video data in the light of the restricted lens of the camera with other evidence such as students' works and photos of the teaching and learning process.

This study was situated in the context of an intensive 5-day professional development programme for primary school teachers and mathematics teacher educators to learn design research approach for designing and enacting classroom investigations and examining evidence of mathematical learning in classroom practice. It was part of the larger project of *Pendidikan Matematika Realistik Indonesia (PMRI)* which has its genesis in 2001 (Sembiring, Hoogland, & Dolk, 2010; Zulkardi, 2013). The role of video and classroom artefacts to support teachers and teacher educators in noticing the key teaching and learning moments in mathematics classrooms will be examined. In this set-up, practitioners (primary school teachers and teacher educators) exercised their professional noticing by capturing and analysing what they considered as key teaching and learning moments in the classroom through video camera, digital still cameras and classroom artefacts such as students' works over 3 days. We contend that this set-up requires practitioners to revisit their own practice and beliefs on what constitutes critical elements of mathematics teaching and learning. Furthermore, we argue that video technology allows practitioners to ground their discussions based on collected evidence of what works in the classroom. This chapter will illuminate our learning points as teacher educators from working together with teachers and practitioners in this programme. The potentials and limitations of the set-up with respect to the role of video will be discussed.

Theoretical Framework

Lesson Study has been widely adapted in many countries as a platform for professional development of teachers (see, e.g. Doig & Groves, 2011; Lewis, Perry, & Hurd, 2009). Key elements of Lesson Study such as collaborative planning, observations of public research lessons and reflections during post-lesson discussions are recognised as valuable ways to deepen teachers' professional knowledge. Teachers who are able to orchestrate a productive whole class discussion carefully plan detailed lessons paying explicit attention to key mathematical ideas, students' anticipated solutions and students' learning trajectories (Watanabe, Takahashi, & Yoshida, 2008).

Lesson Study engages teachers as 'investigators of their own classroom practices' and 'researchers of teaching and learning in the classroom' (Takahashi & Yoshida, 2004, p. 438). Collecting specific and detailed data of students' learning including their misconceptions or difficulties, in order to have a better grasp of students' learning, is vital during the observation. Practitioners play a key role by collecting specific evidence on student learning and the teacher pedagogical move that might not be noticed by the teacher during the research lesson (Lewis & Tsuchida, 1998). Various tools including video camera, digital camera and observation record sheets are frequently utilised to document evidence of students learning in detail. With a growing concern to improve teacher professional competencies and to incorporate real classroom practice as the basis of in-service teacher training, Lesson Study has gained increased acceptance as a promising approach in Indonesia.

Aware of the common practice of evaluating teacher's performance during observations, our study draws on the Lesson Study model by predicting anticipated students' solutions as an integral part of the planning stage. In our set-up, anticipating students' solutions was made explicit to support practitioners' retrospective analysis on students' mathematical thinking and development rather than on teachers' or students' behaviours. The cyclical process of planning-doing-reflecting and the collaborative nature of capturing students' learning through observing the classroom practice in Lesson Study approach were integrated in this study.

The Use of Video and Classroom Artefacts to Facilitate Professional Noticing

It has been well documented that engaging teachers in professional development that focuses on student thinking is valuable and powerful. Skills to notice, attend and respond to students' mathematical thinking in classroom settings need to be developed. Video and classroom artefacts such as students' work are valuable tools in providing opportunities for teachers to cultivate skills to notice students' mathematical thinking.

Video has been used for a long time in education community and become increasingly popular as an artefact of practice in teacher professional development because of its unique ability to capture the richness and complexity of classrooms for later analysis (Brophy, 2004; Dolk, Faes, Goffree, Hermsen, & Oonk, 1996; Lin, 2002). Seago (2004) contends that video cases afford teachers the opportunity to develop a more complex view of teaching, new norms of professional discourse and mathematical knowledge needed for teaching by honing in particular aspect of teaching. Video also affords the luxury of time and opens up opportunities for teachers to engage in fine-grained analysis and reflection of classroom practice with varying lenses (Brophy, 2004; Sherin, 2004). Rosaen, Lundeberg, Cooper, Fritzen and Terpstra (2008) find that video affords a more tangible view of teachers' teaching that is more specific, more complex and more focused on instruction and students than respective memory-based reflections. Similarly, van Es (2012) reveals that video affords teachers to become more student centred and evidence based in their analysis and reflection of classroom interactions as they engage in collaborative inquiry of their classroom practice.

Lesh and Lehrer (2000) point out that video enables teachers and teacher educators using both theoretical and practical perspectives to work collaboratively in analysing data. Furthermore, they underscore the importance of using methods for triangulation by using other artefacts such as students' works to overcome the restricted lens of camera when analysing videotape data. Roschelle (2000) concurs with Lesh and Lehrer's point on the methodological challenges of video data and the need to devise complementary technique to attend to the potential biases of video data. He highlights the benefits of direct field observations, interviews, field notes and students' work to provide an alternative view of the capturing data and to fill some of the details missing on the video.

Hall (2000) challenges claims that videotape provides 'objective' or 'realistic' records of human action (p. 658). According to him, video records of human activity systematically miss the experience of participants but at the same time provide access to events that practitioners might miss. Brophy (2004) contends that watching classroom video does not necessarily lead to new insights for teachers. It is critical to select video vignettes with clear goals and embed these in activities that are planned carefully to help teachers in meeting those goals within a professional development programme.

In this 5-day professional development programme, video vignettes were an add-on to direct observations and field notes by the practitioners and one of the windows into the practitioners' lenses used to capture key teaching and learning moments. Therefore, the video vignette allows for analysis of the moment itself and offers insight into practitioners' observation of the classroom teaching.

Design-Based Research Approach

Design-Based Research

Design-based research approach is grounded on a model of collaborative practitioner inquiry, which is most likely to embed pedagogical reform in practice (Design-Based Research Collective, 2003). Practitioners and researchers work closely together to implement innovative forms of learning and to study the means that are designed to support them in the contexts of practice (Cobb, 2000; Gravemeijer & van Eerde, 2009).

Design research entails a cyclical process between development stage and research stage (Gravemeijer, 1994, 2004). Theory and evidence from prior research inform and guide the development of mathematical tasks or activities, and the enactment of these tasks is evaluated based on evidence collected during research stage and fed back into a new cycle of envisioning and action. During the development stage, researchers and practitioners engage in an iterative exchange between thought and practical experiments. The iterative process of design, implementation and retrospective analysis corresponds well with the Japanese Lesson Study process of planning, observing and reflecting. Bannan-Ritland (2008) highlights the value of having first-hand experience of designing, implementing and reflecting in design-based research to teacher professional growth as adaptive experts:

> Teacher design research (TDR), whose goal is to promote the growth of teachers as adaptive experts … the instructional aspects of TDR comes not from outside experts, but, rather from the teachers' cognitive dissonance experiences as designers in design cycles. (p. 247)

Enacting Design-Based Research in a Professional Development Programme

The Study

A 5-day professional development programme took place in a private primary school in Surabaya, Indonesia. Teachers and teacher educators from PMRI centres and schools participated in the programme. The aim of this professional development was to equip participants (teachers and teacher educators) with first-hand experience of the design-based research 'mini' cycles of knowledge-designing-experimenting-retrospective analysis (Dolk, Widjaja, Zonneveld, & Fauzan, 2010) as depicted in Figs. 12.1 and 12.2.

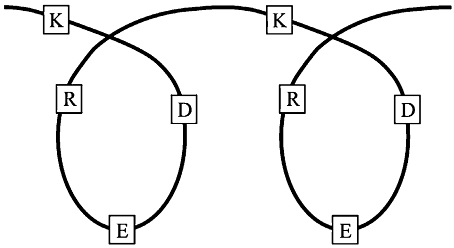


Fig. 12.1 The cyclical process of knowledge, designing, experimenting and retrospective analysis

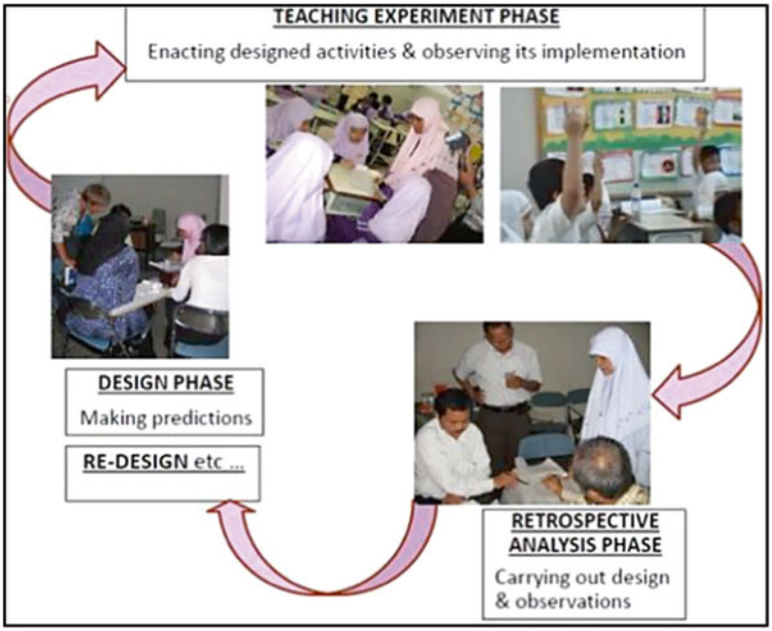


Fig. 12.2 The enactment of design-based research phases

The Participants

Eighteen teachers and teacher educators from nine PMRI centres and schools participated in this programme facilitated by the authors and two other teacher educators. Three grade 2 teachers and two grade 5 teachers from the host primary school participated in the whole programme and enacted the lessons with their respective students. In this chapter, we will refer to teachers and teacher educators who were participants in the programme as practitioners to differentiate them from the resident teachers who enacted the lessons. The resident teachers who enacted the lessons will be referred to as teachers. The authors and the other two teacher educators will be referred to as facilitators.

Structure of Groups

The practitioners worked in five small groups with two to three members in each group. Each group worked with one resident teacher and observed the same teacher and students throughout the programme. Because there were only three grade levels available, practitioners were encouraged to work with the year level that was most relevant for their own teaching post. Each member of the group was responsible for collecting specific sets of data. The member could be responsible for taking photos, videotaping lesson vignettes or writing field notes of the lessons. As practitioners collected evidence of teaching and learning in classrooms, they were requested to refrain themselves from teaching the students or talking to the teacher.

Method

Design and Planning Phase

During the design process and the planning of the lessons, practitioners, teachers and facilitators discussed the role of the context and the investigation, the role of the teacher, the role of the students, the development of mathematics by the students and the social norms in the class.

The Mathematical Investigation

The mathematical investigation was adapted from *The Young Mathematician at Work* series (Dolk & Fosnot, 2005). The facilitator chose numbers carefully to allow students to approach the problem using addition strategies involving 12 and 25 which offer potentials for students to extend their additive thinking to multiplicative thinking (see Fig. 12.3). It was also anticipated that some students might approach the problem by thinking of 25,000 as 25 thousand. Therefore, the task might be solved as 12×25 instead of $12 \times 25{,}000$. It should be pointed out that in adapting the context of the investigation, we took into account the fact that a kilogram of chicken costs about 25,000 rupiah. In the Indonesian language, this is commonly stated as '25 ribu rupiah' or 25 thousand. Formally, this is written as 'Rp. 25.000' as Indonesian used a decimal point instead of comma to mark a thousand.

The Indonesian mathematics curriculum introduces and develops multiplication by two digit numbers in Grade 2. The facilitators engaged practitioners and teachers to adapt a context 'The Turkey Investigation' (Dolk & Fosnot, 2005) for the Indonesian context (Fig. 12.3). Adaptation of this context of investigation includes the choice of local context and numbers that are relevant and appropriate for students. It is vital that the numbers were chosen carefully to offer multiple entry points for students and allow students to come up with alternative strategies to approach the investigation. The choice of numbers should make the investigation 'realistic' and 'enticing' for students to solve. The adaptation process also involved discussion

Turkey investigation

I need your help! My entire family is coming over for Thanksgiving dinner.

Here is the sign I saw in the supermarket: “Turkeys for sale. $1.25 per pound.”

The largest Turkey I could find was 24 pounds. With your partner, discuss how you could figure out how much will the turkey cost? How much will the turkey cost?

Adaptation of Turkey investigation

Ustadzhah wants to buy 12 kilograms of chicken. Each kilogram of chicken costs 25 thousand rupiah. How much money will Ustadzah need to bring?

Follow-up Turkey investigation

Last year when I went to my sister for Thanksgiving at her house, we waited for hours for the Turkey to be done because it was such a huge Turkey it needs to be cooked for a long long time. So last night, I was thinking I don’t want everybody to have to wait and I want to make sure that my Turkey gets cooked on time so I got out my old favourite cookbook and here marked on this page. Listen very carefully because I need your help one more time...

It says here “If you are not using a thermometer allow up to 20 minutes to the pound for a bird up to 6 pounds… For a larger bird, which my Turkey is, allow up to 15 minutes per pound”.

So it says it here on the book I have to cook it for 15 minutes per pound. What I’d like you to do now is to work out how long you think I need to cook this Turkey. With your partner, think about how you are going to figure that out.

Adaptation of follow-up the turkey investigation

Ustadzhah want to ask your help one more time.

Ustadzhah is going to cook 12 kilograms of chicken for the whole family to celebrate the Ied.

It turns out that cooking one kilogram of chicken takes about 15 minutes.

How long do you think it will take Ustadzhah to cook 12 kilograms of chicken?

Fig. 12.3 The mathematical investigations and their adaptions

about expectations of what students could do given the investigation, what approach and solution to the investigation could be and how that might support the development of students’ mathematical knowledge and insights. Because the lessons to be conducted were different compared to a traditional Indonesian classroom, it was necessary to provide the teachers some examples of the nontraditional lessons.

Short video vignettes from *Turkey investigations* (Dolk & Fosnot, 2005) were shown to the teachers and practitioners on the first day during the planning phase. In the video, the teacher presented the investigation as her own personal problem in cooking Thanksgiving dinner and invited students to help her in solving some of her problems. The video was not dubbed into *Bahasa Indonesia*, but the first author provided the translation into *Bahasa Indonesia*.

The teachers and practitioners were concerned that the Grade 2 students may find the adapted Turkey investigation too challenging because it involved solving a multiplication of 25,000 by 12. Multiplication and division involving two digit numbers (tens) are taught in the second semester of Grade 2. The three Grade 2 teachers were clearly nervous to ask their students to solve this problem. At that time, the facilitators negotiated with the teachers that the facilitators would be responsible should the students fail to engage with the investigation. The practitioners were encouraged to collect evidences that support or contradict these two questions: *what proof do we have that this context is too difficult for the children* versus *will the students be able to solve this problem using their common sense*.

Responsibilities of Facilitators The facilitators designed and supported the teachers during the teaching experiment. After each of the teaching experiments (day 2, day 3, day 4), the facilitators met with the five teachers. While the meeting was brief (10–15 min), it allowed teachers to reflect and express their thoughts and concerns. Hence, the facilitators could gather insights into teachers' personal take of how the lesson went and what support they would need for the next day. As teachers were new to the pedagogy, they might not be comfortable in sharing their concerns in public space. The facilitators also offered support for the teachers during the teaching if they were uncertain about the mathematical content or the selection of students' work to be discussed.

Responsibilities of Practitioners After each classroom teaching experiment, each group of practitioners met to discuss and analyse the evidence they collected during the teaching experiments. Based on the analysis of evidence collected during observations, each group prepared a poster containing a selection of twelve snapshots of key teaching and learning moments, a 5-min video vignette and observation notes to share their evidence and insights to other groups.

Teaching Experiment Phase

Day 1: Practitioners and teachers watched the video vignettes of Turkey investigation and solved the mathematical problem followed by discussion of adaptation of the problem into the local context. Groups of teacher-practitioners-facilitator were formed for observations on day 2, day 3 and day 4. The role of teachers, the role of practitioners, and the research questions were discussed.

Day 2: The adaption of Turkey investigation was presented to the students. Students worked on the adapted investigation in small groups and prepared their posters that contained various strategies to solve the investigation. Each group of practitioners worked on collected evidence to prepare a group poster reporting on their classroom observation from day 2.

Day 3: Students presented and shared their strategies during a whole class discussion orchestrated by the teacher. Group of practitioners worked on collected evidence to prepare a group poster reporting on their classroom observation from day 3.

Day 4: Students worked on an investigation to figure out the length of time required to cook 12 kg of chicken if it takes 15 min to cook 1 kg of chicken. Whole class discussion of students' strategies took place. Group of practitioners worked on collected evidence to prepare a group poster reporting on their classroom observation from day 4.

Day 5: Reflecting on the 4-day experience as an exercise of retrospective analysis phase. The sessions were run concurrently for teachers and teacher educators. Both authors facilitated the sessions for teacher educators with the focus on reflecting over the 4-day experiences and linking those experiences with the design research cycle. The other two facilitators ran a parallel reflection session with the teachers.

Retrospective Analysis Phase

The analysis was based on ten selected key moments captured through practitioners' camera lens, a 5-min video vignette of the lesson and classroom artefacts from three Grade 2 classes.

Practitioners' Insights in Capturing the Classroom Practice

In this section, we discuss the evidence gathered by practitioners and teachers. We start by articulating the roles of the videos before presenting evidence of practitioners' insights captured through their video vignettes, a collection of 12 photos and field notes of students' works over the 3-day classroom teaching experiments. The final section discusses what the authors have learnt from working with the teachers.

The Roles of Videos in This Professional Development Programme

The videos play three important roles:

1. To cue teachers into the mathematical investigation different from conventional Indonesian classrooms

The video presented a model for teachers in this study to engage students in mathematical investigations in a real-world situation. While the choice of numbers was open for discussion, the phrasing of the investigation was left to individual teacher's personal and professional judgement. It was evident that the video vignettes of the Turkey investigation sparked discussion among teachers and practitioners.

2. To provide teachers with models of how socio-mathematical norms are being enacted and negotiated in the classroom by the teacher and students at various junctures of the lesson

   The social norms guide the class on the normative expectations of interactions such as the need not only to provide an answer but also explanations that lead to the answer. In mathematical learning, 'socio-mathematical norms' signify what counts as mathematically different or acceptable mathematical explanation (Yackel & Cobb, 1996; Yackel, Cobb, & Wood, 1998). In Indonesian classrooms, teachers are the arbiters of knowledge. They decide on whether students' answers are right or wrong. Showing video vignettes of socio-mathematical norms being enacted was vital to engage practitioners in thinking and planning ways to establish classrooms culture and socio-mathematical norms that encourage students in articulating their ideas and negotiating their interpretations. The video offers participating teachers with socio-mathematical norms that were not consistent with conventional mathematics classroom in Indonesia.

3. To gather insights about what teachers and practitioners value in mathematics lessons

   In this professional development, teachers and practitioners collected evidence from the classroom using video, a collection of 12 photos and field notes of students' works which provided insights about what teachers and practitioners valued in mathematics lessons.

Evidence Captured Through Practitioners' Lens

The collected artefacts depicted what happened in the classroom seen through lens coloured by practitioners' expectations before the lesson and by the suggested research question. All artefacts collected by the participants – video, photographs and observation reports – pointed into the same direction showing a prominent attention towards students' mathematical thinking and their work, shifts in the role of the teacher and development of socio-mathematical norms in the classroom.

Noticing Students' Mathematical Thinking and Students' Works

During the classroom experiment, practitioners could focus on what happened in class based on their expectations, and during the selection of the artefacts, they could compare what happened in class with their expectations. Given the critical

Fig. 12.4 Students' engagement captured through the practitioners' lens

debate during the design phase, we are not surprised that the practitioners pay much attention to the students' work and to students' explanations of their thinking. All groups of practitioners captured students as their focal points documenting students' working in groups and samples of students' work. The fact that practitioners noticed students' mathematical thinking as central was apparent in the proportion of snapshots that were centred on students' work and students' working on the investigation. Various strategies including some of students' misinterpretation of the task were documented. Observing these artefacts enabled group of teachers, practitioners and facilitators to make sense of students' mathematical thinking. It allowed us to notice practitioners' shift of attention to students' mathematical thinking and their expectations about students' mathematical capacity (Fig. 12.4).

The practitioners' lens focused on the students' solution of this difficult problem and balances between a focus on traditional strategies and on alternative strategies. The practitioners also balance between selecting moments in class that showed understanding by the students and moments where the students were following a standard procedure. An example of the latter was an instance when one student decomposed 63 into 0+60+3 (0 hundreds, 60 units in the tens and 3 units in the ones). Although the practitioners' artefacts still paid attention on classroom management, the focus of most artefacts shifted to students' mathematical thinking. An explanation of these shifts can be found in the earlier discussion about the suitability of the problem and the choice of the large numbers.

Students' multiple strategies in solving the problem were documented on observation notes from different teams. Contents of day 2 and day 3 posters showed that there was a strong focus on analysing students' strategies and students' works to reflect their prediction of what strategies students might use. One team recorded that students demonstrated good mathematical thinking. The team identified and classified them into three main strategies: repeated addition using doubles, direct proportion and decomposition. The team observed that students were able to record the total sum of money using appropriate notation *Rp. 300.000* and to associate this with '300 thousand' correctly. The team pointed out that they found evidence of two out of eight groups applied the anticipated strategy of using 'a table form'. Finally, the team noted that some students did not pay attention during the lesson.

Repeated Addition

1 chicken = 25 thousand.
2 chickens = 25 thousand + 25 thousand → 1 chicken + 1 chicken = 2.
4 chickens = 50 thousand + 50 thousand → 2 chicken + 2 chicken = 4.
8 chickens = 100 thousand + 100 thousand → 4 chicken + 4 chicken = 8.

Direct Proportion

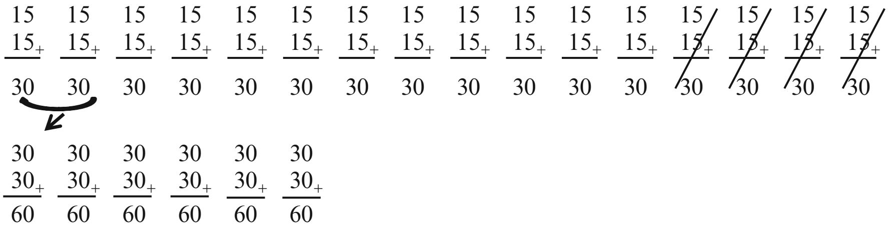
1 chicken = 25 thousand.
2 chickens = 50 thousand.
4 chickens = 100 thousand.
8 chickens = 200 thousand.
12 chickens = 100 + 200 = 300 thousand.

Decomposition

$2 = 1 + 1 \rightarrow 25 + 25 = 50.$
$4 = 2 + 2 \rightarrow 50 + 50 = 100.$
$8 = 4 + 4 \rightarrow 100 + 100 = 200.$
$12 = 8 + 4 \rightarrow 200 + 100 = 300.$

Observation record from day 3 continued to have a strong focus on students' strategies but also highlighted the fact that students were getting used to working in groups and to sharing ideas among group members. Multiple strategies including an incorrect strategy that indicated lack of place value knowledge in working out the multiplication algorithm were documented below:

Repeated addition and doubling



One to one correspondence and counting on by 15 s

1 chicken: 15 min
2 chickens: 15 + 15 = 30 min
3 chickens: 15 + 15 + 15 = 45 min
⋮
24 chickens: $\underbrace{15+15+15+\ldots+15}_{24\text{ times}} = 360$ minutes

Multiplication and direct proportion

$24 \times 15 = 300$.
1 chicken requires 15 min.
2 chickens require 30 min.
⋮
24 chickens requires 360 min.

Confusion about place value in multiplication algorithm

$$\begin{array}{r} 25 \\ \underline{12}\ \times \\ 39 \\ \underline{15}\ + \\ 54 \end{array}$$

The practitioners' focal lens was not directed on the teacher but mainly on the students and their work. The video vignettes captured the practitioners' lens of key teaching and learning moments on day 2 of the teaching experiment whereby a significant portion of the lesson was devoted to group presentations. More than 5 min of the video was focused on the process of students presenting their solution strategies documented in the posters in front of the classrooms. Students' multiple strategies were evident, and the teacher invited some groups to explain their strategies to the teacher, the practitioners and their peers.

The video vignette captured the teacher asking a student to work on decomposition strategy in adding two whole numbers at the end of the lesson. The student showed some understanding of decomposing a number 63 into 60 and 3 and addition strategy. At this point, the practitioners chose to focus on students' written work on the board (Fig. 12.5) which documented the use of addition sign instead of an equal sign. Capturing this moment on the video provided evidence of students' knowledge of place value as shown in the decomposition of $63 = 0 + 60 + 3$ and $6 = 0 + 0 + 6$. The teacher ended the lesson by posing some problems (Fig. 12.5) to help students notice the relationships between $63 + 20 = 83$ and $63 + 19 = 82$. This action was intended to consolidate students' learning on place value and addition and to extend their addition strategy by looking at the relationships between the addends (i.e. 20 and 19) and the sums (i.e. 83 and 82).

Shifts in the Role of the Teacher and the Students

Both videos and photos captured the teacher at work mainly at the beginning of the lesson when she introduced the problem, set up groups and managed the classroom. Hence, the teacher was not out of the picture, but only one or two snapshots were centred on the teachers, and these were mainly recorded on day 1 and day 3. Other moments that featured the teacher at the centre were when the teacher practised

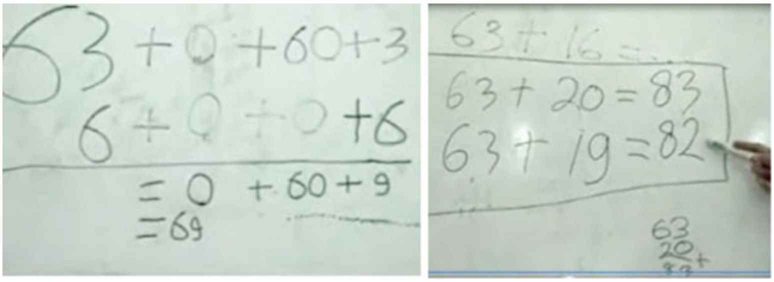


Fig. 12.5 Revisiting addition strategy

classroom norms and socio-mathematical norms such as dealing with disengaged students, getting students to contribute their explanations and posing questions to set a new norm. There was a clear shift in the teacher's role with more attention to encourage students to articulate their thinking and ideas to the teacher and the rest of the class. Rather than telling students the main learning point from yesterday's lesson, the teacher invited students to identify those learning points. Students listed addition, multiplication, story problem, division (that the teacher crossed out from the list) and addition with carrying over as their learning points.

Students had a more active role by working together in groups to explore multiple solutions and preparing a poster to document their solution strategies and presenting them to their peers. As students were engaged in the investigation, the video captured the teacher spending time to observe students' strategies and listening to their discussion. The teacher dealt with classroom management issues such as calling students' attention back to the lesson. Although the practitioners' artefacts paid attention to classroom management, the focus of most artefacts shifted to students' mathematical thinking.

The teacher was – given the design process – able to create a classroom that was different from the traditional Indonesian classroom. She encouraged students to explain and justify their strategies and spent time on different strategies including 'incorrect' solutions and negotiated the 'new' social norms with the students in the classroom. Schoenfeld (2002) underscores the significant role of the teacher in shaping students' understanding of mathematics through a carefully crafted design and plan. He argued that productive exchanges among students were not a spontaneous act but rather a reflection of a consistent practice of a classroom discourse community. While the evidence indicated that the classroom discourse might not yet been part of a consistent practice in these classrooms, the artefacts reflected attempts that mathematical discursive habits were being negotiated by the teacher and the students (Sfard, 2000; Yackel & Cobb, 1996). Based on the collected artefacts, we noticed a growing appreciation and attention to other aspects of teaching such as active role of the students and how students develop mathematical knowledge and reasoning, and the use of alternative strategies.

Fig. 12.6 The teacher at work captured through the practitioner's lens

Fig. 12.7 Dynamic interaction of students at work (screen grabs extreme *left* and *centre*, and the teacher checking herself, extreme *right*)

Development of Social Norms and Socio-mathematical Norms

The practitioners chose the moments that indicated the progress of development of social norms in the classroom. Evidence of the development of socio-mathematical norms in the classroom being practised and negotiated by the teacher and the students was consistent in both the videos and the collection of 12 photos by the practitioners. The video vignettes captured in a Grade 2 class revealed that the teacher initiated the norms to explain and justify students' strategies by saying 'Please explain to *Ustadzah*, to *Uztadzah's* friends and to all your friends how do you solve the problem using those strategies' (Fig. 12.6).

Establishing new classroom norms and socio-mathematical norms is neither easy nor straightforward. In fact, the process might create tensions for both the teacher and the students when these norms are not yet accepted as part of the classroom culture (Yackel et al., 1998). The grounded images from the video captured the tension experienced by the teacher and the students as the new norms were being negotiated and enacted in the classroom. After inviting a group whose work indicated the use of repeated addition and multiplication to explain their work, the teacher asked one student to demonstrate the use of multiplication algorithm to multiply 12 and 25. When the student finished his work, the teacher posed a question to the student but quickly checked herself by asking '*Oh why Ustadzah was the one who ask questions*?' (min 3:20, Fig. 12.7). Here, the moment when the teacher noticed the need to practise a new norm to invite other students in posing questions to other students was captured in the practitioner's lens.

The development of socio-mathematical norms was reflected as the main focus of the whole class discussion. It was clear that the goal of the whole class discussion was not to find out whether the answer was correct or incorrect but to understand students' strategies and the thinking behind those strategies. The video vignettes documented the teacher's attempt to probe into student thinking albeit students' responses did not reveal much.

Teacher: Oh according to '*mbak*' Ani (pseudonym) this is incorrect.
Ani: That is incorrect.
Teacher: How about this one? Is this correct or incorrect? Why did you hide behind? Why does a beautiful girl hide behind? Okay so why is this incorrect?
Ani: Because the answer is only 37.
Teacher: Because the money is only 37. Why is 37 not acceptable?
Ani: Because the money is not enough.
Teacher: Oh according to Ani, 37 is not enough money. Now, please help explain why there are so many twenty fives; Ustadzah is still confused. Please explain your thinking behind one and two.

The grounded images brought to light the importance of looking out for the socio-emotional needs of children who were overtly shy and hid behind the poster during her group presentation. In contrast, there were students who dominated the presentation or made a scene in front of the class during the group presentations. These artefacts were indicative of challenges students faced when new social norms and socio-mathematical norms were enacted and being negotiated. We anticipated that having all members of the group share their work would alleviate the pressure for shy students. The evidence suggests the new classroom social norms take time to develop and teachers need to be prepared to deal with such group dynamics.

The development of socio-mathematical norms was evident when the teacher invited another student to explain another group's strategy whose work was shown in Fig. 12.8. Furthermore, the teacher underlined the importance of understanding other students' ideas by saying 'It was *a must* to pose questions if you don't understand'. The student explained the second strategy from group 2 by articulating '25 plus 25 is 50, 25 plus 25 is 50 so the sum of these two is 100'. She continued with repeated addition of 25s until she reached 300.

The collected artefacts captured some students struggling with having to explain and justify their thinking during presentations (see, e.g. Fig. 12.8). Students were not used to speaking up during the presentation, so a microphone was used to ensure that their peers could follow other students' explanations and engaged in the discussion after the presentation. It was not surprising that the practitioners focused on the development of social norms in the classroom. What was remarkable was that the practitioners' lens was on the students when they were explaining their strategy and did not shift towards the teacher even when she asked a question. The focal point on the student was indicative that students were the dominant source of attention for the practitioners.

Min 3:57 video | Min 4:36 video | Min 4:40

Fig. 12.8 Dynamic collaboration among students (Group 2) as they worked on the board

Learning Points from Working with Teachers and Practitioners

In this chapter, we describe a programme where a group of teachers and practitioners became design researchers. This programme was based on a hybrid of Lesson Study, design research and Realistic Mathematics Education. We had the teachers and the practitioners codesign an investigation and anticipate what they thought would happen in class. Furthermore, the teachers were asked to formulate one question they liked to be able to answer after the class observation and to find evidence that would help to understand that question better and to answer the question in this particular case. Specific roles and tasks were assigned to every team member in collecting grounded images from the classroom using video, cameras or observation notes of students' works. The expectations to use these grounded images to explain students' written work, their discussions and their thinking to the other teams were made very clear throughout the programme. As the participants were active codesigners of the investigation, they had insights what to expect in these lessons as discussed in Section 4.2. These classroom artefacts informed our next design, teaching experiment and retrospective analysis.

The lessons carried out in this programme were not an average mathematics lesson in the Indonesian context. The majority of mathematics lessons in Indonesia are still teacher centred. Traditionally, in Indonesia, the teacher explains the mathematics and sets the students work practising and applying the new knowledge. During this time, the teacher walks around to support students, to check upon their work and to indicate if the answers are correct (Dolk et al., 2010; Widjaja, Dolk, & Fauzan, 2010). Artefacts from such a lesson would focus on the teacher, on the correct explanation of the mathematics at hand, on the support the teacher gives to the students and on the correct answers by the students. The artefacts collected during this lesson are different. The artefacts not only focused on the teacher but also showed *a balance* between the teacher and her work and the students and their work. They signal practitioners' redefining the role of teacher and students' work in classrooms and what teachers and practitioner value in their classrooms. We contend that three simultaneous processes involved in the design phase are working towards this:

- Design of the lesson emphasising students' mathematical reasoning, the use of alternative strategies and not correct or incorrect answers
- The emphasis during the design phase to 'other' aspects to teaching (e.g. active role of the students, how students develop mathematical knowledge, social norms in a classroom)
- Aspects of the educational theory and knowledge that the facilitators emphasised during the design process and the way the facilitators supported the teacher in her preparation of the lesson

The teachers and the practitioners were triggered by the discussion, by the facilitators' confidence that bigger numbers will serve as a challenge and not as an obstacle and by the arguments by the facilitators that the chosen numbers allowed the students to use their common sense to solve this problem in multiple ways. Furthermore, practitioners were triggered by an explicit call to focus on evidence in the classroom. Their attempt to understand if the problem was within reach of the students implied that they had to focus on students thinking, actions and work. The students and their work supported our claim that the large numbers were not an obstacle and that the careful choice of numbers enabled students to use common sense in developing alternative strategies rather than rely only on the formal algorithm. The fact that the practitioners focused on the students' thinking and work had two effects. Firstly, they focused less on behavioural aspects of the teacher's role, aspects like classroom management. Secondly, they focused more on the development of the mathematics by the students. This was the change in the practitioners' lens that the facilitators anticipated with this programme. In particular, there was a clear shift on the use of student works as a springboard for analysing teaching and revisiting the plan for the subsequent task.

The set-up of this professional development proved to be rich. During the course of this professional development programme, the teacher was supported by a teacher educator who facilitated the programme on a day-to-day basis. The collaborative effort between the teacher and the teacher educator in carrying out this classroom practice was critical in creating an environment that supports the learning of mathematics. It is important that teachers feel that they are being supported and not 'judged' by the presence of other teachers and teacher educators in the classrooms. To create this, the practitioners were not allowed to talk with the teacher or to teach the students. If they felt the teacher had to do something different or if they had a suggestion for the teacher, they could inform the co-teacher (one of the teacher educators). It was up to the co-teacher's discretion how best to proceed. Furthermore, shifting the attention from the teacher to the students' mathematical thinking is really critical, particularly in Indonesian context where there is a strong tendency to focus on teachers' actions during classroom observations (Saito, Hawe, Hadiprawiroc, & Empedhe, 2008).

We argue that this set-up is powerful for three reasons. Firstly, the division of several observation tools and detailed observation 'rules' created a wide range of artefacts of the lesson. The different tools supported and complemented each other. For instance, the limitations of video (video footage does not show the whole situation and is not as ‘wide’ as real-time observation) were compensated by the other observation tools. Secondly, the discussion about their expectations and the focus on evidence towards a research question the practitioners formulated before the lesson created a backdrop against what the practitioners could reflect on the lesson. Thirdly, the nature of the set-up provides opportunities for teachers and teacher educators to engage in collaborative learning to examine and reflect on their classroom practice as members of a community of inquiry (Jaworski, 2008). Such opportunities are vital for sustained professional learning of teachers and teacher educators.

Future Directions

Analysis of practitioners’ video vignettes along with photos of key teaching and learning moments and students’ works suggests that teachers and teacher educators in this study paid an increased attention to students’ works and their mathematical thinking and less on the action of the teachers. This concurs with findings reported by van Es and Sherin (2008) that teachers increase their focus on interpreting students’ mathematical thinking in detailed ways after sharing and discussing vignettes of their video-recorded classroom practices. While realising the affordances of video in capturing key teaching and learning moments and supporting teachers’ and teacher educators’ reflections, it is critical to complement the use of videos with photos and students’ work. This is in line with points raised by Lesh and Lehrer (2000) and Roschelle (2000) on the importance of complementing classroom video data with students’ works and field notes from the classroom observation. The nature of this study did not allow us to trace the impact of this programme when the participants returned to their regular classroom practice. However, our work with some teachers in Indonesia following a similar set-up documented teachers’ pedagogical growth in noticing and engaging students in developing their mathematical thinking (Widjaja, 2012; Widjaja, Dolk, & Fauzan, 2010).

This set-up situated in a classroom setting empowered practitioners through active involvement in designing, observing and analysing data using video, camera and field notes. Participation in this professional development programme provided opportunities for teachers and teacher educators to further their collaborations in order to improve classroom practices in their PMRI centres and schools. Through collaborative work, teachers and teacher educators construct educational knowledge on how to design a learning trajectory for a mathematics topic, to establish classroom norms that support learning and to create a situation in which students can construct mathematical knowledge. This study shows that the input of the teacher educator was still crucial. Further research and development is needed to design a system that allows teachers in rural settings to establish self-supporting design groups that utilise classroom video and photo artefacts to empower their knowledge about children’s learning and to improve their noticing of mathematical moments in class.

The use of grounded images employing a combination of video vignettes of lessons, a collection of 12 photos and observation notes allowed the teachers and practitioners to work together in researching students' learning, teachers' learning and classroom practices. The videos enabled all participants to analyse students' thinking and teachers' actions in depth. Sharing the short video vignettes and other classroom artefacts with other teachers and teacher educators who observed different classrooms offered opportunities and challenges for the practitioners to 'tell a story' grounded on evidence rather than relying on their personal views or beliefs.

The process of designing, observing, selecting classroom video vignettes and photos and analysing offers participants a high level of commitment to and ownership of the professionalisation. The in-service discourse was to a high extent about their thinking and their professional noticing (Mason, 2002, 2011). It has to be acknowledged that technical issues of selecting and creating a 5-min video segment on a daily basis for 3 days were quite challenging for some participants. In upcoming courses of intensive professional development with limited time, we would provide more support in both the process of selecting and in the technical aspect of the selection.

Acknowledgements The authors would like to thank participating teachers and mathematics teacher educators for sharing their insight and learning journey and a primary school in Surabaya for their generous support. We would like to acknowledge the contribution of Ellen Zonneveld, Ahmad Fauzan and the core team of PMRI for their support.

References

Bannan-Ritland, B. (2008). Teacher design research: An emerging paradigm for teachers' professional development. In A. E. Kelly, R. A. Lesh, & J. Y. Baek (Eds.), *Handbook of design research methods in education: Innovations in science, technology, engineering, and mathematics learning and teaching* (pp. 246–262). New York: Routledge.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3–15.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24*(2), 417–436.

Brophy, J. (2004). Discussion. *Advances in Research on Teaching: Using Video in Teacher Education, 10*, 287–304.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307–333). Mahwah, NJ: Lawrence Erlbaum.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5–8.

Doig, B., & Groves, S. (2011). Japanese lesson study: Teacher professional development through communities of inquiry. *Mathematics Teacher Education and Development, 13*(1), 77–93.

Dolk, M., Faes, W., Goffree, F., Hermsen, H., & Oonk, W. (1996). *A multimedia interactive learning environment for future primary school teachers*. Utrecht, The Netherlands: Freudenthal Institute: NVORWO.

Dolk, M., & Fosnot, C. T. (2005). *Turkey investigations, grades 3–5: A context for multiplication*. Portsmouth, UK: Heinemann.

Dolk, M., Widjaja, W., Zonneveld, E., & Fauzan, A. (2010). Examining teacher's role in relation to their beliefs and expectations about students' thinking in design research. In R. K. Sembiring, K. Hoogland, & M. Dolk (Eds.), *A decade of PMRI in Indonesia* (pp. 175–187). Utrecht, The Netherlands: APS International.

Freudenthal, H. (1978). *Weeding and sowing: Preface to a science of mathematics education*. Dordrecht, The Netherlands: D. Reidel Publishing Company.

Freudenthal, H. (1991). *Revisiting mathematics education, China lectures*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Gijse, A. (2010). Towards a democratic future. In R. K. Sembiring, K. Hoogland, & M. Dolk (Eds.), *A decade of PMRI in Indonesia* (pp. 13–27). Utrecht, The Netherlands: APS.

Gravemeijer, K. (1994). *Developing realistic mathematics education*. Utrecht, The Netherlands: Freudenthal Institute.

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105–128.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39*, 111–129.

Gravemeijer, K., & van Eerde, D. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *The Elementary School Journal, 109*(5), 510–524.

Hall, R. (2000). Videorecording as theory. In A. E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 647–664). Mahwah, NJ: Lawrence Erlbaum.

Jacobs, V. R., Lamb, L. L. C., & Phillip, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal of Research in Mathematics Education, 41*(2), 169–202.

Jaworski, B. (2008). Building and sustaining inquiry communities in mathematics teaching development. In K. Krainer & T. Wood (Eds.), *Participants in mathematics teacher education* (pp. 309–330). Rotterdam, The Netherlands: Sense Publishers.

Lesh, R., & Lehrer, R. (2000). Iterative refinement cycles for videotape analyses of conceptual change. In A. E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 665–708). Mahwah, NJ: Lawrence Erlbaum.

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education, 12*(4), 285–304.

Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator, 12*(Winter), 12–17. 50–52.

Lin, P.-J. (2002). On enhancing teachers' knowledge by constructing cases in classrooms. *Journal of Mathematics Teacher Education, 5*, 317–349.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London: Routledge Falmer.

Mason, J. (2011). Noticing: Roots and branches. In M. G. Sherin, V. R. Jacobs, & R. A. Phillip (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 35–50). New York: Routledge.

Putnam, R., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4–15.

Rosaen, C. L., Lundeberg, M., Cooper, M., Fritzen, A., & Terpstra, M. (2008). Noticing noticing: How does investigation of video records change how teachers reflect on their experiences? *Journal of Teacher Education, 59*(4), 347–360.

Roschelle, J. (2000). Choosing and using video equipment for data collection. In A. E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 709–729). Mahwah, NJ: Lawrence Erlbaum.

Saito, E., Hawe, P., Hadiprawiroc, S., & Empedhe, S. (2008). Initiating education reform through lesson study at a university in Indonesia. *Educational Action Research, 16*(3), 391–406.

Schoenfeld, A. (2002). A highly interactive discourse structure. *Social Constructivist Teaching, 9*, 131–169.

Seago, N. (2004). Using video as an object of inquiry for mathematics teaching and learning. In J. Brophy (Ed.), *Advances in research on teaching* (Vol. 10, pp. 259–286). Oxford, UK: Elsevier.

Sembiring, R. K., Hoogland, K., & Dolk, M. (2010). *A decade of PMRI in Indonesia*. Utrecht, The Netherlands: APS International.

Sfard, A. (2000). On reform movement and the limits of mathematical discourse. *Mathematical Thinking and Learning, 2*(3), 157–189.

Sherin, M. G. (2004). New perspectives on the role of video in teacher education. *Advances in Research on Teaching, 0*, 1–27.

Takahashi, A., & Yoshida, M. (2004). Ideas for establishing lesson-study communities. *Teaching Children Mathematics, 10*(9), 436–443.

van Es, E. A. (2012). Using video to collaborate around problems of practice. *Teacher Education Quarterly, 39*(2), 103–116.

van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education, 24*, 244–276.

Watanabe, T., Takahashi, A., & Yoshida, M. (2008). Kyozaikenkyu: A critical step for conducting effective lesson study and beyond. *AMTE Monograph, 5*, 131–142.

Widjaja, W. (2012). Exercising sociomathematical norms in classroom discourse about data representation: Insights from one case study of a grade 6 lesson in Indonesia. *The Mathematics Educator, 13*(2), 21–38.

Widjaja, W., Dolk, M., & Fauzan, A. (2010). The role of contexts and teacher's questioning to enhance students' thinking. *Journal of Science and Mathematics Education in Southeast Asia, 33*(2), 168–186.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477.

Yackel, E., Cobb, P., & Wood, E. (1998). The interactive constitution of mathematical meaning in one second grade classroom: An illustrative example. *Journal of Mathematical Behavior, 17*(4), 469–488.

Zulkardi, Z. (2013). *Future challenges and educational responses: Curriculum 2013 and PMRI as an innovation in Indonesian mathematics education*. Paper presented at the EARCOME 6 Phuket.

Chapter 13
Summary of Findings of Chapters in Part II and Introduction to Part III

The two studies, the Singapore and the Hong Kong study, provide contrasting evidence of how two groups of teachers choose to teach factorisation. Teacher-HK in the Hong Kong study placed great emphasis on the choice of tasks used to teach factorization. By selecting suitable tasks which are related but with increasing complexity, Teacher-HK presented factorization as a reverse of expansion and how a set of unique factors could be extracted from a given expression. In fact Teacher-HK wanted to ensure that the students in the class understood why each expression had a unique set of factors and why it was necessary to extract all the factors. He used specific mathematical concepts to highlight why a given set of factors is correct. For example, although $n(2a+2b), 2n(a+b)$ and $2(na+nb)$ are three equivalent sets of factors for the expression $2na+2nb$, only $2n(a+b)$ is the acceptable form. Teacher-HK helped students relate factorization of algebraic expressions to the factorization of whole numbers. Although it is correct to express twelve as the product of two factors 4×3, it was possible to factorise 12 completely to '$2 \times 2 \times 3$'. Teacher-HK tried to ensure that the students knew when they had the complete set of factors.

In comparison, the Singapore study had a different focus. The content of the online video suite showed the pre-service teachers how the concrete-pictorial-abstract approach could be used to teach factorization to a group of low-ability students. The objective was to relate the algebraic form of the algebraic expression to its geometrical representation and how the factors represented the dimensions of the resulting rectangle. The objective of the contents was to show the participating teachers how the teaching of factorization could be made more meaningful for low-ability students. The objective of showing the online video suite was to ascertain whether the pre-service teachers were willing to engage in innovative ways of teaching. Whether the set of factors was complete was not an issue discussed by the teachers in the online video suite. However some of the participating teachers who watched the online video suite were willing to question the efficacy of the method as noted by one participant, 'When introducing the individual AlgeCards, it might

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_13

have been better if students are given some time to think how the card area can be x and let them identify by themselves that if the area is , $(x \times 1 = x)$, the sides of it are x and 1'. Here it could be this participant may be wondering how students in the video come to know that x can be completely further factorised to x and 1.

In conclusion, the Hong Kong study was focused on the concept of factorization, namely, to find all the factors of a given algebraic expression, while the focus of the Singapore study was the process of factorization.

Again the influence of those in position of authority came across very clearly in the Ho et al.'s study. Two factors influenced whether the pre-service teachers were willing to consider buying in to the innovative practices. Pre-service teachers who had good knowledge of learning theories were willing to consider alternative pedagogical practices. However they were further convinced when the Head-of-Department supported such innovations.

Comparing and contrasting the reflections of Teacher-K from Korea and Teacher-HK showed that these two teachers were also concerned with fundamental concepts of mathematics, in their case how best to present solutions to problems. Teacher-K was frustrated that she could not provide an answer she was confident of when questioned by her students whether a comma or the conjunction 'or' should be used to identify the roots of a given quadratic equation:

> In quadratic equations, I teach that x = 3 or x = 5 after doing factorisation. Many students asked whether 'or' must be written or whether writing the comma is wrong. I'm also very confused. Thus, I can't give a clear answer and I'm frustrated.

Mathematics educators could benefit much from the reflections of teachers such as Teacher-HK and Teacher-K. The reflections of these teachers, identified by their peers as good and effective teachers, suggest that teaching of mathematics should not only focus on the processes but also to very fundamental concepts of mathematics, which many would deem as trivial. Thus teacher preparation courses should not only be about how to teach mathematics but what of mathematics to emphasise.

The mathematics educators and researchers did not dictate what the participants (teachers and pre-service teachers) in these four chapters should focus on. The participants were free to identify what they valued. Teacher-HK chose clips which emphasised the importance he placed on well-structured tasks and clear explanations why some answers were more acceptable than others. All the teachers in the four chapters were concerned how to engage slower learners and also the thinking processes of the students. Although initial recordings were focused on classroom management, the subsequent video recordings of the Indonesian teachers and their choice of artefacts showed that they were more concerned about the thinking strategies of children when they worked with large numbers. All the teachers placed great importance that as many students as possible in their classes were engaged with the mathematics. Thus mathematics educators need to place greater importance in their work on how best to help teachers engage as many students as possible. Strategies must not only be innovative, they need to be effective in helping as many students as possible learn mathematics.

Into Part III

Whether teachers engage with a new teaching methodology will depend on their state of preparedness. The works reported in Parts I and II provide evidence how video and audio technology are used to promote the professional growth of teachers. However the various pieces of research are eclectic in nature, yet each piece of work shows some measure of success and learning by teachers and also by the mathematics educators/researchers. Then what defines effective use of video and audio technology? Are there some common features of professional development courses which utilise video and audio technology effectively? The final part of this book presents a rubric to evaluate the effective use of video and audio technology.

Part III

Chapter 14
The Use of Video Technology in Pre-service Teacher Education and In-service Teacher Professional Development

Kim Koh

Abstract This chapter provided a review of the western literature on the use of video technology in teacher education and professional development. The importance of using videos in building pre-service and in-service mathematics teachers was discussed in the context of digital literacy and current movements in teacher education and professional development. The role of video in teacher education and professional development has evolved since the 1960s based on both prevailing learning theories and technological innovations. Following the review, a rubric was constructed to evaluate the effectiveness of video technology used in pre-service and in-service mathematics teacher education and professional development in six Asian countries: Hong Kong, Indonesia, Korea, Malaysia, Singapore, and the Philippines.

Introduction

In this second decade of the twenty-first century, with the advent of technology, digital literacy has become one of the essential twenty-first century learning outcomes that are desired in the curriculum frameworks of many education systems. Likewise, teachers' mastery of digital knowledge and skills will benefit them in their quest for lifelong professional development. The Partnership for 21st Century Skills (2002) has touted professional development as one of the key mechanisms to help education systems to achieve the vision of the twenty-first century learning. The literature on teacher learning and professional development has long pointed out that ongoing, sustained professional development is more effective than ad-hoc, one-shot workshops to improve teachers' instructional and assessment practices (Garet, Porter, Desimone, Birman, & Yoon, 2001; Wiliam & Thompson, 2008). Using video as a medium to provide a shared experience and to serve as a focal

K. Koh (✉)
Werklund School of Education, University of Calgary, Calgary, AB, Canada
e-mail: khkoh@ucalgary.ca

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_14

point for teachers' collaborative exploration in mathematics teacher professional development has been advocated by researchers who embrace practice-based professional development (e.g., Borko, Jacobs, Eiteljorg, & Pittman, 2006; Borko, Koellner, Jacobs, & Seago, 2011; Hall & Wright, 2007; Santagata, 2009; Sherin & Han, 2004; Sherin & van Es, 2009).

The use of effective videos to help situate teaching activity by capturing practice in context and relating these with associated evidence of student learning is deemed to enable pre-service and in-service teachers to reflect on their instructional and assessment practices and to have productive conversations centred around student learning and thinking. Pre-service teachers can be effectively immersed in school culture by watching and analysing teaching and learning activities prior to, during, and following formative field experience. Similarly, in-service teachers can engage in meaningful professional dialogue in a safe and collective learning environment whereby they are empowered to analyse and refine their instructional practices. According to Borko (2004), many teacher professional development programmes in mathematics are not of high quality, offering "fragmented, intellectually superficial" (p. 3) seminars or workshops. Additionally, these programmes are unable to provide ongoing support for teachers to sustain their implementation of new curricula or pedagogies.

A better approach to building teachers' capacity in instructional and assessment practices is school-based professional learning community because it can be localized and contextualized in a teacher's classroom (Wiliam & Thompson, 2008). However, many school-based professional learning communities are found to be haphazard and fail to produce deep change in school cultures. This is because of their tendency to "stick to the technicalities of specifying narrow performance goals, defining a focus, examining data, and establishing teams" (Hargreaves & Fullan, 2012, p. 129). Hence the use of videos enables a better focus of contents and the selected videotaped lessons can also serve as a focal point for teachers' active learning and collective participation in school-based professional learning communities.

A Situative Lens of Professional Development

Since the seminal work of Lee Shulman's (1986) on pedagogical content knowledge, mathematics researchers and educators have placed greater emphasis on designing professional development that substantially increases teachers' knowledge of mathematics for teaching, supports the improvements of their instructional practices, and promote student learning and achievement. Using a situative perspective, Borko et al. (2008) pointed out three major design principles that are essential for mathematics teacher development programmes: (1) establishing a professional learning community, (2) using artefacts of practice, and (3) establishing community around video. Using Lave and Wenger's (1991) idea of community of practice, Borko and Koellner (2008, p. 1) argued that:

> From a situative perspective, learning is both an individual process of coming to understand how to participate in the discourse and practices of a particular community, and a community process of redefining norms and practices through the ideas and ways of thinking that individual members bring to the discourse. In educational settings, a situative perspective suggests that strong professional learning communities can foster the enhancement of teachers' professional knowledge and improvement of practice.

Strong professional learning communities create opportunities for teachers to collectively engage in productive conversations about ways to improve their teaching and assessment practices as well as to support each other as they work to transform their classroom practices. These opportunities are often localized and contextualized in teachers' everyday classroom activities (Wiliam & Thompson, 2008).

Situative theorists posit that teachers' own classrooms are powerful contexts for professional development, as Sherin (2004) has aptly explained, "Video allows one to enter the world of the classroom without having to be in the position of teaching in-the-moment" (p. 13). Artefacts of classroom practice such as videotapes of lessons and student work samples produced during a lesson can be used to anchor teachers' professional conversations about one another's instructional practices and student thinking, which in turn lead to collective ideas for improvement. Extant research has also shown that teachers who watched video from their own classrooms found the professional learning experience to be more meaningful than those who watched video from someone else's classroom. Because video clearly exposes actual teaching practices, sharing one's video in professional development settings is likely to be seemed as threatening if a teacher does not feel confident that his or her video clips will provide valuable learning opportunities and create opportunities for productive conversations for him or herself and his or her colleagues. Hence, establishing and maintaining a strong community is of paramount importance when teachers are asked to share video clips from their own classrooms with colleagues.

A Brief History of Video Use in Teacher Education and Professional Development

A chronological history of the use of video in teacher education has been reviewed by Hall and Wright (2007) and Sherin (2004). According to them, the use of video recordings for the purposes of teacher education and professional development began in the 1960s when Allan and Ryan used videos for microteaching. The focus in microteaching was on student teachers' mastery of selected skills using videotaped lessons for their instructors' feedback. In the 1970s, interaction or lesson analysis was introduced to teacher education wherein student teachers use an observation instrument in the analysis of teaching. Based on the behaviourist learning approach, student teachers learned how to identify particular student and teacher behaviours for analysis. In 1971, Greenburg used videos in conjunction with instructor supervision to improve the teaching of physical education teacher whereas videotaping and telephone conversations were used by Koorland et al. in 1985.

In the early 1980s, behaviourism was replaced by cognitive psychology. Hence, researchers and teacher educators began to focus more on teacher cognition rather than teacher behaviours (Sherin, 2004). There was an increasing use of videos in teacher education programmes. For example, videos of expert teaching and videotaped commentaries were used to create opportunities for pre-service teachers to observe and analyse the teaching practices of expert teachers. Typically, expert teaching was often modelled for novice teachers through videotapes of actual instruction. In some cases, novice teachers were shown excerpts of video along with videotaped commentaries on the lessons by expert teachers. As such, novice teachers had the opportunities to observe how expert teachers make instructional decisions and to understand the knowledge that served as the basis for those decisions (Sherin, 2004). A mastery of such pedagogical content knowledge is important for novice teachers.

Video recordings have also been used by expert teachers as part of their own professional development. For example, teachers in the United States will have to submit videotapes of their teaching along with written commentaries in order to meet the National Board for Professional Teaching Standards (Silver, Mesa, Morris, Star, & Benken, 2009). For expert teachers, video can be used in two ways: excerpts of video are shared with a researcher who provides critical feedback and with their peers in order to generate conversations on their teaching practices in professional development settings.

By the late 1980s, teacher education programmes had shifted to the use of video-based cases, which provided novice teachers with rich examples of pedagogical dilemmas. Further, such platforms enabled novice teachers to adopt reflective practices in their teaching but also to develop pedagogical content knowledge (Sherin, 2004).

In the early 1990s, the invention of hypermedia programmes had allowed for good linkages of videos to text and graphics. It also enabled a selection of video segments for teacher professional development. In most cases, videos were presented along with a framework to guide teachers' reflections and discussions. One of the good examples was the Teaching Mathematics Methods Using Interactive Videodisc (TMMUIV) (Hatfield & Bitter as cited in Sherin, 2004), in which mathematics teachers were trained to use manipulatives in their teaching of elementary school mathematics. In addition, one of the notable international studies, i.e., the Trends of International Mathematics and Science Study (TIMSS) had included video studies of teachers' instructional practices in 1995 and 1999. Following this, a group of Australian researchers led by David Clarke had carried out a large-scale video study to examine students' perspective in learning (Janik, Seidel, & Najvar, 2009). In the United States, the use of videos in practice-based professional development programmes in mathematics was originated by Borko and her colleagues.

The invention of video technology also facilitated the recording of field observations, which allowed a substitution of live observations with videotaped supervision in pre-service teacher education (Sherin, 2004). Such recordings not only save supervisors' time, but also enable them to use segments of video to give live feedback to pre-service teachers. The video can also be used for group reflections and peer feedback.

With the advent of digital technology in the 2000s, the use of videos had increased exponentially in teacher education and professional development programmes. For instance, the development of web-based video libraries of lessons was made possible by the availability of internet. One of the significant technological innovations that took place in the second half of the twentieth century was videography. Videography is ethnography that uses video technology to analyse people acting in social settings by video. For example, it enables the recording and analysis of particular structures and patterns of interaction in classrooms. The availability of qualitative data analysis programmes such as NVivo and StudioCode also enables meaningful analysis of videos for empirical studies conducted in the area of teacher learning and professional development.

According to Sherin (2004), the use of video in teacher education and professional development was driven by both prevailing theoretical frameworks in education and technological innovations. She stated that

> … changes in the use of video over this time have been driven to a large extent by changes in leading theoretical frameworks in education. In particular, the shift in perspective from behaviourism to cognitive views of teaching was reflected in changes in the role of video in teacher education… technological innovations have also influenced the ways in which video has been used with teachers. Just as the initial availability of portable cameras stimulated a variety of uses, the possibility of viewing video as digitalized on a computer was followed by new innovation. (pp. 9–10).

However, she argued that there was a neglect of how certain features of video make it particularly useful for teacher learning and professional development. Therefore, it is important to attend to the unique affordances of video in designing the ways in which teacher educators engage pre-service and in-service teachers with video.

Affordances of Video

Despite a growing use of video in mathematics teacher education and professional development, the literature contends that the use of video does not always reflect a precise understanding of what it is about video that might provide support for teacher learning. In other words, it is important to understand how some of the key features of video might support teachers' learning of new instructional practices and reflections on existing practices and student learning. It is believed that the ultimate goal of using video in mathematics teacher education and professional development is to enhance teachers' pedagogical knowledge, for example, how to interpret and reflect on classroom practices. Sherin (2004) has identified three affordances of video: (1) video is a lasting record; (2) video can be collected, edited, and recombined; and (3) video sustains a set of practices that are very different from teaching.

Video Is a Lasting Record With the advent of video technology, teacher practices and classroom interactions can be captured permanently as well as be listened and observed unobtrusively. The video records can be replayed and viewed repeatedly and from different angles. As such, teachers do not have to rely on his or her memory to recall a specific episode of teaching. With the aid of voice recording, video also allows a focus on particular video segments for analysis, discussion, and reflection.

Video Can Be Collected, Edited, and Recombined Video can be collected, edited, and reorganized into video libraries. With digitized technology, pre-service and in-service teachers can jump between different video segments or focus their discussions on particular teaching episodes or classroom interactions based on their learning goals in a course or professional development settings. Video can be combined with other media such as graphics and text. It can also store artefacts of classroom practice such as lesson plans, assessment tasks, students' work, and reflective journals. Such data can be used to generate productive conversations in a course or professional development settings because they serve as the concrete examples of classroom practice and student learning.

Video Affords a Different Set of Practices Video enables teachers to engage in new types of learning experiences and new thinking about instructional practices. Sherin (2004) has provided three examples of using video to engage teachers in a different set of practices. First, when watching a pedagogical dilemma on video, teachers can revisit the video segments later and reflect on it prior to giving their responses or taking any actions. Second, watching video opens up the possibility of teachers' seeing alternate pedagogical practices or strategies in others' classrooms. Both pre-service and in-service teachers can learn new instructional strategies, assessment methods, curricula, classroom management, and classroom cultures. Third, video allows teachers to engage in analysing their instructional practice and student thinking, for example, what is underlying a student's misconception of an algebraic equation. Such valuable information will enable teachers to provide quality feedback (Hattie & Timperley, 2007) to the student. If such a misconception is shared by most of the students in a class, a teacher can focus on adjusting his or her instructional plan and on re-teaching of the concept. Formative assessment is made possible by teachers' fine-grained analysis of student thinking captured in video.

In addition to these, another affordance of video is its feasibility for qualitative data analysis. There is an increasing use of video for qualitative data analysis in educational research. For example, videotaped lessons provide a rich set of data for lesson study and analysis of pedagogical practices.

The Role of Video in Teacher Education and Professional Development

The affordances of video have led to many applications of videos in teacher education and professional development over the past two decades. Brophy has pointed out that videos are able to capture the complexity and subtlety of classroom teaching and learning as they occur in real time (as cited in Sherin & van Es, 2009). Therefore, it is believed that both pre-service and in-service teachers can benefit from watching videos that provide authentic representations of practice and making use of such opportunities to reflect on teaching and learning. There is a body of literature focusing on using video to improve mathematics teachers' instructional practices through practice-based professional development. The literature has shown that the use of video enables teachers to participate in a shared and supportive learning culture, to engage in productive conversations, reflections, and critical examination of their own instructional practices, and to examine and analyse student thinking. However, there are also some constraints in the use of video.

There are three prominent ways of using video in teacher education and professional development: video clubs, video representations, and video annotation tools.

Video Clubs

Sherin and van Es's (2009) study examined how video clubs could be used to support mathematics teacher learning in professional development. In a video club, a group of teachers met to watch and discuss excerpts of videos from each other's classrooms. The selected video excerpts were used "to question, reflect on, and learn about teaching" (Sherin & Han, 2004, p. 165). The study is interesting because it focused on how video clubs support the development of mathematics teachers' professional vision, that is, their ability to notice and interpret significant features of classroom interactions. Two factors are deemed to support teachers' professional visions, namely selective attention and knowledge-based reasoning. For selective attention, teachers must choose to focus their attention on only one of the many things happening in a classroom. Similarly, teachers need to be able to carefully listen to the range of students' ideas. Knowledge-based reasoning refers to teachers' interpretation and reasoning of a particular classroom event based on their knowledge of the curriculum and students.

The study took place in a middle school and an elementary school for a year. Four mathematics teachers in the middle school met monthly for seven video club meetings while seven mathematics teachers in the elementary met once or twice a month for ten video club meetings. Across both video clubs, a researcher videotaped in one teacher's classroom and selected a 5-min clip for the group to watch at the next meeting. The only difference between the two schools was that only one clip was watched per meeting in the middle school. In the elementary school, the teachers watched two video clips per meeting. They also participated in individual pre- and post-noticing interviews as well as classroom observations. In the video club meetings, the researcher facilitated the teachers' discussions on what they had noticed in the videos and helped teachers to attend to student thinking.

Sherin and van Es (2009) found that video clubs had influenced the participating teachers' professional vision not only in the video club meetings, but also in interviews outside of the video club meetings, and in the teachers' instructional practices.

The teachers were able to increase their capacity to notice and attend to student mathematical thinking. They also develop new analytic skills, which might be transferable to their actual classrooms.

In another related study, Sherin and Han (2004) examined the learning that occurred in the middle school's video club meetings. They analysed the transcripts recorded from the seven video club meetings based on the following topics: pedagogy, student conceptions, classroom discourse, mathematics, and other issues. Over time, they found that the discourse in the video clubs shifted from a primary focus on the teacher to increased attention to student thinking. In addition, the teachers' discussions of student thinking changed from simple restatements of students' ideas to fine-grained analyses of student thinking. Teachers' ability to focus on student thinking and make meaningful connections between pedagogy and student thinking has become increasingly important for successful implementation of mathematics education reforms. To improve the quality of teaching and learning in mathematics, teachers need to learn to respect students' thinking and to make sense of students' mathematical understandings (Ball, 1997). For example, once teachers became aware of students' thinking and understanding of multiplication and division, they were able to support students' learning of these concepts.

Video Representations

Borko et al.'s (2011) study examined how the use of video representations of teaching in practice-based professional development programmes could serve as a focal point for mathematics teachers' collaborative exploration of the central activities of instruction and learning. By comparing two professional development programmes in mathematics, namely the Problem-Solving Cycle (PSC) and Learning and Teaching Geometry (LTG), Borko et al. (2011) found that the affordances and constraints of video were different between the two programmes. Although both programmes focused on promoting teachers' pedagogical content knowledge, their use of video fell along a continuum ranging from highly adaptive to highly specified. The PSC was an example of adaptive professional development because the facilitators selected and used video clips and resources from the participating teachers' classrooms, and made design decisions by taking into account the local school context. The PSC was an iterative, long-term professional development approach to supporting teachers' learning. Each iteration consisted of three interconnected workshops in which the mathematics teachers shared a common pedagogical experience and their conversations were organized around a rich mathematical task.

On the contrary, the LTG was an example of highly specified professional development wherein the facilitators used commercially available materials that specified in advance particular learning goals and video clips, provided resources from outside of the participating teachers' classrooms, and made design decisions that did not take into consideration the local school context. Borko et al. (2011) found that the learning goals and topics in the LTG might not fully meet the specific needs of the participating teachers as compared to the PSC approach to professional development. This is because the latter tailored the focus of the professional development programme to the needs, interests, and concerns of the participants.

Using a situative lens, Borko et al. (2006) examined the use of video as a tool for fostering mathematics teachers' productive conversations about teaching and learning. A group of mathematics teachers participated in a summer algebra institute and attended monthly, full-day professional development workshops for two academic years. The PSC model was used in the study. Over the 2 years, there were three iterations of workshops. In each of the iterations, the teachers were involved in solving a single, rich mathematical task in algebraic reasoning and engaged in professional conversations about mathematical content, pedagogy, and student thinking. Prior to viewing video footage from one of their colleagues' classroom and from their own classrooms, the facilitators helped the participating teachers develop norms for viewing and analysing classroom videos. This emphasis on building a supportive community of practice had enabled the participating teachers to generate productive conversations about their own instructional practices and student thinking. Over time, the videos helped the participating teachers learn important mathematical content and delve deeply into issues around teaching and learning a specific mathematical problem.

Video Annotation Tools

Rich and Hannafin (2009) investigated the changing role of video in teacher education by looking at studies wherein teachers used a video annotation tool for reflections and analysis of their own teaching. They noted that the use of video annotation tools offers pre-service and in-service teachers the potential to redirect their effort to specific teaching activities or events. Based on a comprehensive review of the existing studies, they identified the following video annotations tools: VAST/video callout, VITAL, VAT, video traces, video paper, MediaNotes, and StudioCode. One of the affordances of these tools was that teachers could create video clips and select portions of video for making narrative comments or for linking it to associated rubrics. They concluded that such video annotation tools offered the potential to support teachers' reflections and analysis of their own teaching with minimal video editing as well as the ability to associate captured video with related student and teaching evidence. A majority of the pre-service teachers found that simply knowing that video clips would be used as evidence of thinking in their VITAL essays influenced how they watched videos. The use of video traces allows for synchronized feedback at precise points in teachers' instructional practices whereas video paper synchronizes the appearance of images (e.g., student work) within the video timeline. VAST provides a portfolio-like connection to external resources, such as student work samples, lesson plans, and other sources of student and teaching evidence. Both MediaNotes and StudioCode allow advanced data mining and coding across different videos. They are powerful research tools in teacher education and professional development.

In summary, video technology has brought considerable benefits to teacher education and professional development in mathematics. A review of the literature reveals that the affordances of video have enabled teachers to: (1) engage in productive conversations, (2) analyse and reflect on own instructional practices, (3) analyse student mathematical thinking, and (4) question and reflect on teaching and learning.

Rubric for Evaluating the Effectiveness of Video Technology

Based on the western literature on the use of video in mathematics teacher education and professional development, a rubric was co-constructed by the author and her editor to evaluate the effectiveness of using video technology as a teaching tool to support pre-service and in-service mathematics teacher education and professional development in six Asian countries.

The rubric is presented in Table 14.1 below.

Results and Discussion

The effectiveness of the use of video technology in mathematics teacher education and professional development across all the empirical studies as presented in the chapters in Sections I and II was evaluated and discussed according to each of the criteria in Table 14.1.

Identification of Research Problem—Achieve Objective(s) of the Study

Overall, all the studies had identified research problems clearly and the research problems were stated logically based on the relevant body of literature. Only one out of the nine studies had focused on the use of video technology in pre-service teacher education. The Ho, Leong, and Ho study examined the effects of the use of video suites on Singaporean pre-service teachers' buying-in to an innovative teaching of factorization using the Algecards. Four of the nine studies examined the use of video recordings in building primary mathematics teachers' capacity in the following domains: facilitation of model-eliciting activities, teaching of part-part-whole concept of numbers and its representation, teaching of fractions and conversion of units, and noticing students' thinking in mathematical investigations. Ng, Cheng, and Ng, Widjaja, Chan, and Seto had focused on the use of video recordings in the teaching of a particular mathematical content. The study conducted by Lim and Kor with a group of Malaysian primary mathematics teachers only used the videos of Japanese Lesson Study. In Widjaja and Dolk's study, videos, photos, and

Table 14.1 Rubric for evaluating the effectiveness of video technology

	Levels of performance			
Criteria	Novice	Apprentice	Practitioner	Expert
Identification of research problem—achieve objective(s) of the study	No problem was identified; the chapter appeared to deviate from the objectives of the study.	Identified problem at a surface level; objectives of the study were ambiguous in the chapter.	Identified problem clearly; objectives of the study were clearly stated in the chapter.	Identified the problem clearly and at an in-depth level; objectives of the study were clearly and logically explained in the chapter.
Quality of use of video in PD	Video was not well used in PD. Participants were not sure how to maximize the affordances of the video. Professional development/course could have proceeded without the use of the video.	Video was used adequately in PD. However, other means of data collection were necessary to supplement the affordances of the video, e.g., audio technology, photos.	Video was well used in PD. Other means of data collection were also used to support the affordances of the video, e.g., audio technology.	Video was used superbly in PD. Affordances of the video were sufficient. Other technologies or artefacts were not necessary.
The affordances of the video enabled teachers to engage in productive conversations about mathematical content and delve deeply into issues around teaching and learning of a specific mathematical problem	Contents of the video clips were selected haphazardly and teachers were not able to engage in productive conversations.	Contents of the video clips were shallow; teachers' professional conversations were limited to only some aspects of mathematical content.	Contents of the video clips were appropriate and enabled teachers to engage in productive conversations about mathematical content.	Contents of the video clips were rich and selected carefully; teachers were able to engage in productive conversations about mathematical content and delve deeply into issues around teaching and learning of a specific mathematical problem.

(continued)

Table 14.1 (continued)

Criteria	Levels of performance			
	Novice	Apprentice	Practitioner	Expert
The affordances of the video helped teachers analyse and reflect on their own instructional practices	Contents of the video clips were selected haphazardly and teachers were not able to engage in analyses of their own or others' instructional practices.	Contents of the video clips were shallow; teachers' analyses of their own or others' instructional practices were superficial.	Contents of the video clips were appropriate and enabled teachers to engage in meaningful analyses of their own or others' instructional practices.	Contents of the video clips were rich and selected carefully; teachers were able to engage in meaningful analyses of and reflections on their own instructional practices.
The affordances of the video helped teachers analyse their student thinking	Contents of the video clips were selected haphazardly and teachers were not able to engage in analysis of their student thinking.	Contents of the video clips were shallow; teachers' analysis of their student thinking was superficial.	Contents of the video clips were appropriate and enabled teachers to engage in meaningful analysis of their student thinking.	Contents of the video clips were rich and selected carefully; teachers were able to engage in in-depth and insightful analysis of their student thinking.
The affordances of the video helped teachers question their beliefs about teaching and learning	Contents of the video clips were selected haphazardly and teachers were not able to question their beliefs about teaching and learning.	Contents of the video clips were shallow; teachers questioned their beliefs about teaching and learning to a limited extent.	Contents of the video clips were appropriate and enabled teachers to question their beliefs about teaching and learning to a moderate extent.	Contents of the video clips were rich and selected carefully; teachers were able to question their beliefs about teaching and learning to a great extent.

students' work were used to not only enhance primary mathematics teachers' noticing skills of students' mathematical thinking, but also to develop students' socio mathematical norms.

The three studies conducted with mathematics teachers at the secondary school level varied in terms of the mathematical contents and objectives of using video. Yap and Leong examined how the use of video club influenced teachers' instruction of mathematical problem solving in a private secondary school in the Philippines. They were very clear that their focus of study was on teachers' instruction rather than on students' thinking. Ho's study focused on the use of video to increase a mathematics teacher's reflective thinking while she was teaching the graphs of quadratic functions and the application of quadratic functions. The Mok study in Hong Kong was conducted with an experienced mathematics teacher in the teaching of factorization of linear and quadratic expressions. Using a video-stimulated recall technique, the grade 8 mathematics teacher was asked to comment on his lesson and instructional practice as well as his rationalization for his teaching actions based on the teacher metacognition framework (Artzt & Armour-Thomas, 1998).

Quality of Use of Videos in PD

Using Borko et al.'s (2011) continuum of video use, we can conclude that most of the mathematics educators and researchers in Asia had used high adaptive videos in their professional development programmes/workshops. The videos used in seven of the nine studies were highly adaptive because they came from the teachers' own classrooms. In addition, the foci of the professional development programmes were tailored to the needs, interests, and concerns of the participants. Only the studies by Ho et al. and Lim and Kor had used highly specified videos in working with pre-service and in-service mathematics teachers. Given that the teachers in Lim and Kor's study had encountered language barriers, the affordances of the video of the Japanese lesson study were not maximized in the professional development sessions, albeit English subtitles were provided in the videos. The researchers also used power point slides and multi-lingual response sheets to support the mathematics teachers' understanding of the lesson study videos. Although Lim and Kor had mentioned about the use of local examples of lesson study with the mathematic teachers, there was no substantial evidence for the affordances of the use of local lesson study videos. Ho et al. had used pre-packaged, highly specified video suite with a group of pre-service teachers at the National Institute of Education Singapore to influence their buy-in for the innovative teaching approach (i.e., Algecards) to factorization. In addition, milestone tasks were used to identify which aspects of the video suites had brought about pre-service teachers' shifts in their beliefs about the usefulness of the innovative teaching method.

The use of video club was quite successful in the Yap and Leong study. A maximum of two video clips were shown to the secondary mathematics teachers in every professional development session and the length of the video clips were 2–7 min. In three other studies with primary mathematics teachers, other means of data collection were used together with video recordings of the participating teachers' lessons. In both Cheng and Ko studies with mathematics teachers, the researchers served as critical commentators for the teachers while they were engaged in reflections on their own teaching. The teacher in Ko's study found that the experience of watching her own videotaped lessons might not be meaningful unless she was able to engage a critical friend/mathematics educator to work with her. This critical friend was important in the process of her intentional reflective thinking about her own teaching. This is because the critical friend not only listened to her talk about her lesson, her rationalization of her choice of pedagogical actions, and her concerns about her students, but also challenged her with cognitively demanding questions.

In the Cheng study, no videos were used in the first cycle of professional development. Instead, the mathematics teachers reflected on a fellow teacher's lesson using photographs. They found the limitations of not using any videos in their reflections once they were unable to recall specific details of their colleague's lesson. In the second cycle of professional development, other artefacts of practice such as student work samples and photographs of the conversion lesson were used together with videos. However, the teachers were asked to reflect on students' work. Although the teachers had watched the videotaped lesson conducted by the critical commentator, they were reluctant to use it for their reflections in the professional development session. Only one out of the seven mathematics teachers, who was also the Head-of-Department, had benefited from the use of video recording during the second cycle of professional development.

In the context of mathematics reform in Indonesia, Widjaja and Dolk's study focused on building primary mathematics teachers' capacity in noticing key mathematical teaching and learning moments as well as students' mathematical thinking through the use of videos and other artefacts such as photos and students' work. The authors found that the limitations of video were compensated by other observation tools.

The Affordances of the Video Enabled Teachers to Engage in Productive Conversations

As pointed out by Borko et al. (2006), one of the affordances of video is to enable mathematics teachers to engage in productive conversations about mathematical content and delve deeply into issues around teaching and learning of a specific mathematical problem. Among the nine studies, only the Yap and Leong study showed that video clubs provided a platform for the mathematics teachers to discuss collaboratively about issues related to the teaching of mathematical problem solving. Productive conversations among mathematics teachers were not the focus in the rest of the studies. In a few studies, productive conversations were not possible because only a single mathematics teacher was involved in the professional development.

The Affordances of the Video Helped Teachers Analyse and Reflect on Their Own Instructional Practices

In Ho et al.'s study, the use of highly specified video was found to engage pre-service mathematics teachers in high-level reflections on their buy-in for an innovative teaching approach. Although the video was highly specified and based on the instructional practice of established teachers, it helped situate the teaching of algebraic factorization using the Algecards in authentic classroom context. The 'buy-in' group of pre-service teachers were able to engage in high-level reflections such as theorizing about the suitability of the innovative teaching method in their own future classrooms. In their study, Lim and Kor had encouraged primary mathematics teachers to reflect on their colleagues' teaching based on edited video clips.

In a few studies (e.g., Ng, Mok, Yap & Leong, Ng et al.), the use of video recordings had enabled the mathematics teachers to analyse and reflect on their own instructional practices. All the researchers concurred that the use of video made playback references easy in the process of teachers' comments and reflections on their own instructional practice. In the Yap and Leong study, video clubs were effective to encourage the mathematics teachers to reflect on their own instructional practices. In addition, the teachers were able to compare their own classroom actions to that of their colleagues and to view their own instruction more impartially by listening to their colleagues' comments. Some teachers also found that when their video clips were featured in video clubs, they received affirmation from their colleagues about their instructional practices. Furthermore, the teachers also learned and tried out new teaching strategies by watching their colleagues' video clips. The Ng et al. study also focused on the use of carefully selected video recordings to help the mathematics teacher identify and reflect on her competence in incorporating modelling tasks. The use of a selected videotaped lesson for post-lesson interview was conducted by Mok with an experienced mathematics teacher in Hong Kong. The contents of the video had the potential to help other teachers understand his pedagogical content knowledge and teaching actions, his choice of examples, and questions he posed to students. In Ng's study, the primary mathematics teacher used feedback provided by the video of herself at work to improve her teaching of the part-part-whole concepts of numbers and its attending representation known as the model drawing to 12 Grade 2 students who were identified as having difficulties learning mathematics.

The Lim and Kor study seemed to be limited in terms of the use of the video to help mathematics teachers analyse and reflect on their own instructional practices. The limitations were due to two reasons: teachers' language barriers and sociocultural backgrounds. Most of teachers in the study could not comprehend the contents of the Japanese lesson study fully. Moreover, this group of Asian teachers is less vocal and critical due to their sociocultural backgrounds and educational experiences.

The Affordances of the Video Helped Teachers Analyse Their Student Thinking

Many of the studies had focused on the use of video to help mathematics teachers analyse their student thinking. Mathematics teachers' competence in analysing student thinking is important because it allows them to discuss how specific instructional strategies may help students overcome their misconceptions (Rich & Hannafin, 2009). According to Broko et al. (2011),

> One important component of teaching expertise is the ability to observe and interpret classroom events as a lesson unfolds, and to make instructional decisions based on those interpretations. To promote mathematical inquiry and fostering deep understanding of important mathematical ideas, teachers must be able to attend to the mathematics in what students say and do, interpret students' mathematical thinking, and respond in ways that build on their mathematical knowledge and reasoning." (p. 185).

Borko et al.'s emphasis on the importance of teachers' ability to analyse and interpret students' mathematical thinking was echoed by Cohen (2004) who noted that teachers who analysed student thinking via video became more effective at responding to student ideas during instruction.

Five of the studies in Sections I and II had focused on mathematics teachers' ability to analyse students' mathematical thinking and ideas while they watched the selected video clips or recordings. Table 14.2 presents the focus of analysing student thinking across those studies.

Table 14.2 Teachers' focus on analysing student thinking

Authors	Mathematical contents	Instruction	Students' mathematical thinking
Ng et al.	Mathematical modelling	Designing, facilitating, and evaluating students' mathematisation (Model-Eliciting Activities)	Using video recordings of teacher-student interactions during a modelling task, the teacher reflected upon her competencies in navigating her students through the potentials and challenges of the modelling task toward advancing students' mathematisation and fostering students' metacognition.
Ng	Development of part-part-whole concept of numbers	Teaching of the part-part-whole concept of numbers and its attending representation (Model Drawing)	The Mediational Intervention for Sensitising Caregivers approach was used as a tool to enrich the quality of the interaction between the mathematics teacher and her students. The videotaped lesson is believed to provoke the teacher to think and question deeply how children learn mathematical modelling to solve word problems.
Cheng	Fractions and conversion of units	–	One of the primary mathematics teachers used video recording to reflect on and discuss how the students build connections between mathematical concepts. Other teachers also analysed, interpreted, and reflected on students' errors based on the photographs of students' work and the critical commentator's questions. But the use of video recording as an affordance for teachers' analysis and reflections was more superior than photographs.
Mok	Factorization of linear and quadratic expressions	Teaching of factorization	The secondary mathematics teacher's metacognitive framework played a useful role to help him understand his rationale for the lesson events and how these events might be conducive for the students' learning of factorization. His understanding of his own students and their difficulties with the concept of factorization helped him plan and execute his instruction to overcome students' obstacles.
Widjaja & Dolk	Addition strategies and multiplicative thinking	Mathematical investigations	The primary mathematics teachers were able to focus more on noticing students' mathematical thinking than their own actions.

The Affordances of the Video Helped Teachers Question Their Beliefs About Teaching and Learning

The use of video as a tool to help mathematics teachers question their beliefs about teaching and learning is essential, especially in the twenty-first centruy teaching and learning context. Cochran-Smith and Lytle (2009) had called for twenty-first century teachers to engage in inquiry about their teaching and to be reflective practitioners. Part of the success of the Finland education is that the Finnish teachers play an active role as teacher-researcher (Sahlberg, 2011). Hence mathematics teachers' ability to question their beliefs about teaching and learning through the use of videotaped lessons helps improve both their instructional practices and students' learning.

In some of the studies, the researchers discussed how the use of video had enabled pre-service and in-service teachers to question their beliefs about teaching and learning. For example, Ho et al.'s study had examined the effects of video suites in the teaching of factorization of quadratic expressions on pre-service mathematics teachers' beliefs about an innovative teaching approach. The Mok study in Hong Kong showed that by watching the videotaped lessons of an experienced mathematics teacher, other mathematics teachers and educators could expand their understanding of his pedagogical beliefs and knowledge as well as how these determine his teaching actions, his choice of examples, and the questions he posed to the students.

Conclusion

In conclusion, there is a growing demand for using effective video technology to building pre-service and in-service mathematics' capacity in the contexts of teacher education and professional development. The studies conducted in Sections I and II have shed some light into the affordances of video in mathematics teacher education and professional development in the Asian teaching and learning contexts. In future studies, the selection of particular video clips will need to be planned carefully and be negotiated with the participating teachers. This will enable the teachers to have a shared learning experience and common goals embedded within a strong professional learning community. It is also important to select video clips based on the learning goals set in professional development programmes. This will enhance the quality of mathematics teachers' professional conversations and reflections during the professional development sessions. Although most of the studies are well planned and executed within a short timeframe, a situative lens of professional development can be considered in the future effort to improve the quality of mathematics teachers' professional learning experiences. There is a novelty effect in most educational innovations. One of the limitations in most of the studies is a lack of critical reflections on the constraints of video technology.

Reflections on the Chapter

The use of video technology in teacher education and professional development programmes enables both pre-service and in-service mathematics teachers to engage in productive conversations and reflections on mathematical contents, their own instructional practices, and students' mathematical thinking.

In teacher education and professional development programmes, mathematics teacher educators and researchers need to ensure that the contents of a video are relevant to the local mathematics curricula and sociocultural context.

Highly adaptive videos are more effective than highly specified videos because the former allows facilitators to select and use video clips and resources from the participating teachers' classrooms. The latter uses ready-made videos that may represent only the best practices of teachers who have taught in a different sociocultural context. For effective teacher learning and professional development, it is important that facilitators make design decisions by taking into account the local school context. As such, the participating teachers could see the relevance between the contents of professional development and the needs in their mathematics teaching.

References

Artzt, A. F., & Armour-Thomas, E. (1998). Mathematics teaching as problem solving: A framework for studying teacher metacognition underlying instructional practice in mathematics. *Instructional Science, 26*, 5–25.

Ball, D. (1997). From the general to the particular: Knowing our own students as learners of mathematics. *The Mathematics Teacher, 90*(9), 732–737.

Borko, H. (2004). Professional development and teaching learning: Mapping the terrain. *Educational Researcher, 33*(8), 3–15.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2006). Video as a tool for fostering production discussions in mathematics professional development. *Teaching and Teacher Education, 24*, 417–436.

Borko, H., & Koellner, K. (2008). *Situativity: A theoretical lens for designing and studying programs of professional development*. Paper presented at the Symposium on the Occasion of the 100th Anniversary of ICMI, Rome.

Borko, H., Koellner, K., Jacobs, J., & Seago, N. (2011). Using video representations of teaching in practice-based professional development programs. *ZDM Mathematics Education, 43*, 175–187.

Cochran-Smith, M., & Lytle, S. L. (2009). *Inquiry as stance: Practitioner research in next generation*. New York: Teachers College Press.

Cohen, S. (2004). *Teachers' professional development and the elementary mathematics classroom: Bringing understanding to light*. Mahwah, NJ: Erlbaum.

Garet, M. S., Porter, A. C., Desimone, L. M., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38*(4), 915–945.

Hall, I., & Wright, D. (2007). *Literature review of the use of video as a resource for professional development of mathematics teachers*. Newcastle upon Tyne: National Centre for Excellence in the Teaching of Mathematics.

Hargreaves, A., & Fullan, M. (2012). *Professional capital: Transforming teaching in every school*. New York: Teachers College Press.
Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81–112.
Janik, T., Seidel, T., & Najvar, P. (2009). Introduction: On the power of video studies in investigating teaching and learning. In T. Janik & T. Seidel (Eds.), *The power of video studies in investigating teaching and learning in the classroom* (pp. 7–19). Berlin: Waxman.
Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.
Rich, P. J., & Hannafin, M. (2009). Video annotation tools: Technologies to scaffold, structure, and transform teacher reflection. *Journal of Teacher Education, 60*(1), 52–67.
Sahlberg, P. (2011). *Finnish lessons: What can the world learn from educational change in Finland?* New York: Teachers' College Press.
Santagata, R. (2009). Designing video-based professional development for mathematics teachers in low-performing schools. *Journal of Teacher Education, 60*(1), 38–51.
Sherin, M. G. (2004). New perspectives on the role of video in teacher education. In J. Brophy (Ed.), *Using video in teacher education* (Advances in Research on Teaching, 10, pp. 1–27) Oxford: Elsevier Science.
Sherin, M. G., & Han, S. Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education, 20*, 163–183.
Sherin, M. G., & van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education, 60*(1), 20–37.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.
Silver, E. A., Mesa, V. M., Morris, K. A., Star, J. R., & Benken, B. M. (2009). Teaching mathematics for understanding: An analysis of lessons submitted by teachers seeking NBPTS certification. *American Educational Research Journal, 46*(2), 501–531.
The Partnership for 21st Century Skills. (2002). *Learning for the 21st century skills: A report and MLE guide on 21st century skills*. Retrieved from http://www.p21.org/storage/documents/P21_Report.pdf
Wiliam, D., & Thompson, M. (2008). Integrating assessment with learning: What will it take to make it work? In C. A. Dwyer (Ed.), *The future of assessment: Shaping teaching and learning* (pp. 53–82). Mahwah, NJ: Erlbaum.

Chapter 15
Going Forward: Encouraging Teachers to Embrace Video Technology for Self-Development

Overall the studies reported in this book found that teachers, researchers and mathematics educators have benefitted from the affordances of videos. In particular, teachers who overcame their reluctance to be videoed learnt much more about their practice. The reflections of three particular teachers showed how they became more critical about their practice after watching themselves at work. Teacher-T from Singapore who taught model drawing to a group of Primary 2 children challenged herself to design more meaningful instructional materials to address the intellectual needs of the children. More importantly the grounded images of children volunteering information that they had difficulties engaging with different representations challenged her assumptions of how children engage with the different representations used to capture the same set of mathematical relationships. Teacher-K from Korea became more reflective with viewing of each of her five lessons. Her reflections showed that she became increasingly more concerned for those students who were not engaging with the mathematics lessons. Bessie from the Philippines learnt that confidence about her teaching could be misplaced. To ensure that she was able to identify students' misconceptions, she had to review sharing of her colleagues and to value those shared knowledge. Although teachers from Lu Pien Cheng's chapter finally embraced video technology, none agreed to be videoed. The teachers were willing to watch a video of the critical commentator, who was also the author of the chapter, conducting a lesson on conversion between different units of measurement. Grounded images of the critical commentator at work were used as a catalyst to promote richer reflections amongst the participating teachers.

Comparing and contrasting the nature of the feedback proffered by teachers, it would seem that those who chose to take the risk of being videoed, namely, the risk takers, appeared to have gained more from their experiences. The risk takers studied their actions and the reactions and responses of the students to their actions in greater detail than those who chose to remain part of the audience, but not the actors. Thus risk takers may have more opportunities to learn as the grounded images showed how specific actions may elicit certain responses from students. However,

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4_15

those who remain as members of the audience may not have such grounded images of themselves at work. Thus the wish to be more reflective may prompt teachers to find ways to invest more of themselves in the reflection process. Such risk takers may be more open to listening to constructive criticisms of their work which may cause them to reflect further about their practice. Whether they choose to act upon the feedback provided by the critical commentator is another matter. The willingness to be open suggests an important first step towards self-development, and the willingness to be videoed must be made by the teachers.

There are ways to encourage teachers to video their lessons for self-reflection. By serendipity, I found that the pre-service teachers who participated in mathematics courses conducted by me were using their cell phones to videotape their lessons for their own consumption. When approached, a few of them were willing to share their video recordings. In that request I would list down what were particularly good about their presentations and what would be highlighted to the audience. Such requests were often well received. Perhaps teachers' reluctance to be videoed could be overcome in two ways. First, encourage teachers to video their lessons for their own consumption. They could be encouraged to review their lessons privately and to use the following list of questions to reflect on their teaching.

In my lesson

1. What pedagogical practices were effective in my lesson and why?
2. What pedagogical practices were not so effective and why? How can I improve these areas of my teaching?
3. What pedagogical practices were not effective at all and should not be repeated?

Next ask them to edit the video to select examples of their own practices which they are willing to share with others. Perhaps when they are free to edit what they wish to share with others, teachers may be encouraged to make use of the affordances of videos to improve their practice. But it is equally important to provide a list of guiding questions that could encourage teachers to think deeper about their practice.

Videos are one of many forms of digital communications. Social media is a common means of interactions amongst communities of people, in which they create, share and/or exchange information and ideas in virtual communities and networks. Popular social media tools and platforms include Facebook, YouTube and Vimeo, Flickr and Instagram. There are many who do not hesitate to post videos of themselves engaging in various daily activities on such social media. People post videos of themselves cooking their special recipes. Would the current generation of people be open to posting videos of themselves teaching good lessons and sharing good pedagogical practices with others?

Contributors

Chun Ming Eric Chan (Ph.D.) is a Lecturer with the Mathematics and Mathematics Education Academic Group, National Institute of Education, Singapore. He lectures on primary mathematics education in the pre-service and in-service programmes. He is the author of several mathematics resource books (primary levels) and is also the co-author of the Targeting Math series.

Lu Pien Cheng received her Ph.D. in Mathematics Education from the University of Georgia (USA) in 2006. She is an Assistant Professor with the Mathematics and Mathematics Education Academic Group at the National Institute of Education (NIE), Nanyang Technological University, Singapore. She specialises in mathematics education courses for primary school teachers. Her research interests include the professional development of primary school mathematics teachers and tools and processes in mathematics education programmes for pre-service teachers. Children's mathematical thinking in problem solving and problem posing is also another key area of research.

Maarten Dolk is a researcher and developer of mathematics education at the Freudenthal Institute in the Netherlands, where he has been involved in the development of in-service materials for teachers and of multimedia learning environments for teachers and student teachers. He has also directed an in-service project in the Netherlands for teacher educators and staff developers. In Indonesia he has supported the work by the PMRI-team. His work in the United States and in other countries is related to the in-service of primary school teachers.

Foo Him Ho obtained his M.Sc. in Mathematics from University of Sheffield (UK). He is an experienced mathematics teacher and a prolific writer of school mathematics textbooks and guidebooks. His interests span both mathematics and mathematics education, e.g., analysis, dynamic geometry and problem solving. He is currently attached to the Curriculum Planning and Development Division, Ministry of Education, Singapore.

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4

Weng Kin Ho received his Ph.D. in Computer Science from The University of Birmingham (UK) in 2006. His doctoral thesis proposed an operational domain theory for sequential functional programming languages. He specializes in programming language semantics and is dedicated to the study of hybrid semantics and their applications in computing. Notably, he solved the open problem that questions the existence of a purely operationally-based proof for the well-known minimal invariance theorem of (nested) recursive types in fixed point calculus. His research interests also include domain theory, exact real arithmetic, category theory, algebra, real analysis and applications of topology in computation theory. In addition, he is also interested in the use of technology in teaching and learning of mathematics, history of mathematics and the impact of mathematical pedagogical content knowledge on teaching and learning of mathematics.

Ho Kyoung Ko is an Associate Professor at the Graduate School of Education, Ajou University, Korea. Her major is Mathematics Education, and her research interests include teacher education, curriculum and technology.

Kim Koh is an Associate Professor at the Werklund School of Education, University of Calgary, Canada. She holds a Ph.D. in Educational Measurement and Evaluation, and Research Methodology from the University of British Columbia, Canada. She is involved in teaching assessment courses at both undergraduate and graduate level. Her research interests include understanding pre-service and in-service teachers' assessment literacy and conceptions of assessment, and building teachers' capacity in authentic assessment and assessment for learning through teacher education and professional development.

Liew Kee Kor is a Ph.D. graduate from the Universiti Sains Malaysia. She is an Associate Professor with the School of Educational Studies (Mathematics Education) Universiti Teknologi MARA Malaysia. She has presented numerous papers at national and international conferences and has written journal articles pertaining to the use of educational technology in the teaching of mathematics. Her Ph.D. research project on the use of graphics calculator in teaching statistics won an award in the Universiti Sains Malaysia Graduate Research Exposition 2004. Since then, she has been actively researching and publishing articles on technology in mathematics education.

Yew Hoong Leong is an Assistant Professor with the Mathematics and Mathematics Education Academic Group, National Institute of Education, Nanyang Technological University. He engages in design research that places emphasis on mathematics teacher development, problem solving, and teaching for low achievers.

Chap Sam Lim is a Professor in Mathematics Education at the Universiti Sains Malaysia, Penang, Malaysia. She obtained a Ph.D. in Mathematics Education from the University of Exeter, United Kingdom. She is currently teaching mathematics education research course and qualitative research method courses at graduate level. Her research interests focus on public images of mathematics, teaching mathematics in second language, cross-cultural study and particularly Lesson Study as a professional development for mathematics teachers. She is an active collaborator of several international research projects especially on Lesson Study and cross-cultural comparative study of mathematics teaching and learning in schools.

Ida Ah Chee Mok is an Associate Professor with the Faculty of Education, University of Hong Kong. She obtained her B.Sc. and M.Ed. from the University of Hong Kong, and her Ph.D. from King's College, University of London. She has been awarded the Diamond Jubilee International Visiting Fellowship (2013–2016), University of Southampton. Since 1990, she has been a teacher educator active in both local community and research in the area of mathematics education. She has a broad research interest which includes: mathematics education, technology in mathematics education, comparative studies in mathematics education, teaching and learning, teacher education, pedagogical content knowledge, and lesson study. She is co-chair of The ICME 11 Topic Study Group: Research on classroom practice, and co-editor of the book, *Making Connections: Comparing Mathematics Classrooms Around the World.* She is also the author of the book, *Learning of Algebra: Inspiration from Students' Understanding of the Distributive Law*; and co-author of the book, *Polynomials and Equations*.

Kit Ee Dawn Ng is a Lecturer in the Mathematics and Mathematics Education Academic Group at the National Institute of Education. She holds a Ph.D. in Mathematics Education from The University of Melbourne, Australia. She is involved in teacher professional development at both primary and secondary levels. Her research interests include the use of contextualised tasks in the teaching and learning of mathematics and fostering students' mathematical thinking and metacognition.

Swee Fong Ng is an Associate Professor at the National Institute of Education, Nanyang Technological University, Singapore. She holds a master and a Ph.D. in Mathematics Education, both from the University of Birmingham, United Kingdom. Her general interests include how teacher's pedagogical content knowledge influence the nature of questions and the choice of examples used to support the teaching and learning of mathematics across the curriculum. The teaching and learning of algebra is her special interest.

Cynthia Seto is a Master Teacher (Mathematics) with the Academy of Singapore Teachers, Ministry of Education. She has taught mathematics for more than 30 years and her teaching experience ranges from Lower Secondary Mathematics to Primary Mathematics. She has published and presented several papers at international conferences and is an author of the mathematics book, *Teaching Fraction, Ratio and Percentage Effectively*. She has received several innovation awards, such as Microsoft-MOE Professional Development Award, Hewlett-Packard Innovation in Information Technology Award etc., for mathematics teaching with technology. Her research interest is in classroom learning environment, learning with technology and teachers' learning communities.

Wanty Widjaja is a Lecturer in Mathematics Education at Deakin University. Prior to her present appointment, she worked as mathematics educator in Indonesia for 14 years and spent 1 year as a visiting academic in Singapore. She is a recipient of two scholarships for her graduate studies: Kelly Stephen scholarship (Boston University, 2000–2002) and AusAID scholarship (The University of Melbourne, 2004–2008). She has collaborated with international researchers from Indonesia, The Netherlands, Singapore and Australia in various research projects. Her theoretical approach straddles social constructivism and socio- cultural perspectives. Her research interests centre on design-based research, mathematical reasoning, and professional learning of mathematics teachers.

Romina Ann S. Yap holds an M.S. degree in Applied Mathematics from the University of the Philippines and is currently completing her Ph.D. in Mathematics Education at the National Institute of Education (NIE), Nanyang Technological University, Singapore. She has worked as a secondary school mathematics teacher in the Philippines and as a mathematics instructor at the Ateneo de Manila University. Her research interests include mathematical problem solving and mathematics teacher development.

Index

S.F. Ng (ed.), *Cases of Mathematics Professional Development in East Asian Countries*, Mathematics Teacher Education 10,
DOI 10.1007/978-981-287-405-4