

Chapter 8

Mathematics Teachers Responding to Children's Resources to Create Learning for All

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Abstract The focus of this chapter is on how teachers respond to children's resources and their mathematical thinking as they transfer from preschool to primary school. The theoretical framework builds on sociocultural theories. The area of investigation is individuals changing their ways of understanding, perceiving, noticing, and thinking as they collaborate with others. Thus, the emphasis is on classroom cultures and learning environments that promote mathematical learning where all children have a voice and are supported to develop their understanding. The methodological approach comprises narrative inquiry and analysis. Through focus group interviews, narratives are gathered from teachers who work with children in preschool and the early primary grades. We learned that what characterises these teachers is their belief that all children can learn mathematics if learning spaces are created that respect the children's resources. The teachers analyse children's mathematical resources and respond to what they bring with them to school as they organise classroom cultures and develop supportive mathematical learning environments.

8.1 Introduction

The diversity of pupils in Icelandic schools has increased in the last two decades as pupils with disabilities have entered their neighbourhood schools and immigration has brought in students for whom Icelandic is not their native tongue. These changing social conditions have put increasing pressure on teachers to modify their practices and take into account the diverse group of learners that forms their learning communities. From our earlier research and work as teacher educators, we have learned that many teachers find it challenging to teach mathematics. Their own experience as mathematics learners was typically as passive receivers who practiced rules and procedures, introduced by teachers and textbooks. Teachers lack

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experience in investigating, communicating, reasoning and making connections in mathematics. Additionally they feel incompetent in using these approaches in inclusive schools (Guðjónsdóttir and Kristinsdóttir 2011).

For 10 years we have collaborated with our colleague Edda Óskarsdóttir on developing the graduate course *Mathematics for all*. We conducted a self-study of teacher education practices as we planned, implemented and reflected on our teaching, together with the students. Originally this course was developed for special education teachers and the focus was on children's difficulties with mathematics and how teachers could support individual children based on their analysis of the children's difficulties with mathematics. As we learned about these teachers' lack of confidence in using flexible approaches in mathematics teaching we modified the course. The new focus was on children's mathematical development and understanding and teachers' capacity to evaluate and promote pupils' learning through analysis of their engagement in authentic mathematical problems. Additionally, we have considered the teachers' own explorations with mathematics and discussions about their mathematical thinking.

Findings from our ongoing study suggest that if teachers are given opportunities to integrate their experience in their studies and to relate theory and practice, their ability to make informed decisions about teaching and learning increases (Guðjónsdóttir and Kristinsdóttir 2011).

In search for more knowledge about how teachers respond to diverse learners as they organise the learning environments, we contacted nine teachers. We discussed the theories the teachers build on and the way data were collected and analysed. In this chapter we report on their work as mathematics teachers and conclude by discussing the implication of our work for future research with teachers who teach young children mathematics during the transition from preschool to primary school.

8.2 Children's Mathematical Thinking

In this chapter we will discuss the theories on which the participants in this project base their work. As the teachers searched for understanding young children's developmental processes and how mathematical thinking matures, they have revisited their former studies on children's learning while studying and digesting new theories and research findings. Piaget's theories of children's development and the belief that knowledge is an ongoing constructive process, as well as the fact that it varies when children reach the developmental stages he identified (Piaget 1969), support these teachers' work. They are also guided by Vygotsky's theories on children's learning about solving practical tasks with the help of their speech, as well as their eyes and hands and the dynamic relation between speech and action. His description of the zone of proximal development that defines those functions that have not yet matured but are in the process of maturation (Vygotsky 1978) has also influenced their work with children in preschool and the early primary grades.

The work of Dewey (1966), and his emphasis that learning situations must be flexible to enable children to be active learners, has influenced the teachers' beliefs

and actions. Bruner's descriptions of children as active learners and the influence of culture in education (Bruner 1996) have shaped their ideas of teaching and learning and the social culture of the classroom. The theory of multiple intelligences (Gardner 1993) has also supported them in respecting children's diversity.

In the early 1990's the work of Pratt (1948), founder of the City and Country School in New York, and the 'unit blocks' she devised, were introduced to pre-school teacher education in Iceland. In 1991, Cuffaro (1996) participated in a research project with a teacher educator, Tryggvadóttir, and preschool teachers in two preschools in Iceland. The experience gained from the project had major impact on the work in many preschools and also in the lower grades in primary schools. The blocks are now available in most preschools in Iceland and many primary schools.

The Cognitively Guided Instruction (CGI) research project (Carpenter et al. 1995) was introduced at a course for teachers in early primary grades in Iceland in 1995. Some of the teachers in this project participated in the course and so did the authors of this chapter. We learned about the findings of the research team at the University of Wisconsin-Madison on children's thinking about whole numbers and addition, subtraction, multiplication and division problems. By the end of the course the participants were eager to use what they learned about children's mathematics thinking in their work with children. This enthusiasm resulted in workshops where we met to discuss our work and establish collaboration. The results from the CGI project impacted all our teaching and as the research was continued we have followed the writings of the research team and used their books (Carpenter et al. 1999; Carpenter et al. 2003; Empson and Levi 2011; Hiebert et al. 1997).

Sarama and Clements' (2009) research on learning trajectories has also guided our work. Their book *Early childhood mathematics education research: Learning trajectories for young children* has been one of the main readings at a course about young children's mathematical development that many of the participants in the project have attended. The book *Engaging young children in mathematics: Standards for early childhood mathematics education* (Clements et al. 2004) has also been a part of our course material. In their work with young children the teachers draw on research findings presented in these books.

Research on early childhood education in Australia has also impacted our work. The participants have been impressed by descriptions of children's powerful mathematical ideas as presented in *The Numeracy Matrix* (Perry et al. 2007). They can identify with how important it is for teachers to acknowledge children's mathematical thinking.

In her search for supporting herself and her co-teachers in first grade in learning how to listen to children and assess their numerical knowledge, one of the participants in this project found that *Early Numeracy: Assessment for Teaching and Intervention (the Mathematics Recovery Project, MRP)* (Wright et al. 2006) was helpful. The assessment interview and the information gained through discussion with the child have helped them to listen to children and learn to appraise their numerical knowledge.

The teachers who participated in this research project have all engaged with literature on young children's mathematical learning and revisited their understanding

of children's development at the transition stage between preschool and primary school. The focus is on the resources the children bring with them to school, respect for children's diverse backgrounds, as well as supportive learning environments.

8.3 Teacher Professionalism

Inclusive educational practices are responsive to diversity, concerned with and value all pupils equally. Booth (2010) introduces values that he believes are important for schools as they develop learning communities for all pupils. These values involve issues of equality, rights, participation, learning, community, and respect for diversity, trust and sustainability and also the qualities of compassion, honesty, courage, and joy. A commitment to such values prepares teachers and teacher educators to increase the participation of all, overcome discrimination and create an environment that ensures learning for all children. As we understand more about the focus of learning and how people learn, we also understand the critical combination of intellectual and social development and the need to continue learning in the face of constant change and societal complexity. The ability to think, present ideas and work with others is recognised by education and businesses alike as central to the world's future (Fullan 1999).

Schools that make progress towards inclusive ways of working develop the capacity for teachers to learn from one another so that they share ideas and practices and spend time discussing how teaching can be improved. Teacher educators who embrace diversity and inclusion also need to learn how to observe carefully so that they continue to understand practice as it is carried out in their own classrooms and countries. Such processes become starting points to continue the journey of new learning (Ainscow 2007).

Moore (2005) discusses the importance of transformation from theory to practice and concludes that if teachers are expected to teach for diversity and understanding, they need opportunities to develop and enhance their mathematical pedagogical knowledge. It is important for them to experience their own mathematics learning in an environment that reflects the one they are expected to create in their classroom. Teachers are empowered to practice a culturally responsive and socially relevant pedagogy as they begin to look critically at their classroom environment. The practitioner becomes the action researcher, transforming theory into practice and researching on that practice.

In a research project with teachers, Guðjónsdóttir (2000) identified the following diverse roles that professional educators embrace depending on their circumstances and opportunities.

- Pedagogues and experts in teaching and learning: Activist teachers share their knowledge and understandings in an ongoing professional dialogue.
- Reflective and critical problem solvers: Teachers continuously monitor pupils' progress and learning within the classroom. Outside that environment they

reflect both as individuals and as communities of practice on their practice and pupils progress.

- Researchers and change agents: In seeking a deeper understanding of their practice, or in seeking to plan for change, teachers use a variety of evaluation and action research techniques to collect and interpret findings to inform their thinking and decision making.
- Creators of knowledge and theory builders: In the process of reflective practice and action research, teachers develop new understandings of learning, teaching and educational change.

Similarly, Cochran-Smith and Lytle (2009) continued their long discussion of teacher research by identifying five critical elements for consideration in the future discourse on teacher professionalism:

- Emphasis on the teacher as knower and agent of change.
- Creation of new ways to theorise practice.
- Participation of teachers and colleagues in intellectual discourse about critical issues.
- Linking teaching and curriculum to wider political and social issues.
- The creation of inquiry communities that focus on the positive, rather than negative, aspects of what teachers know.

Learning to learn from one's own practice requires active engagement and reflection in communities with others. This was reflected in the results of the research on the development of the teachers who participated in the CGI program discussed above. In their conclusions Fennema et al. (1996) reported that developing an understanding of children's thinking provides a basis for change, but change occurs as teachers attempt to apply their knowledge to understand their own pupils. The CGI study provides strong evidence that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction. It also appears that this knowledge is dynamic and ever-growing, and can probably only be acquired in the context of teaching mathematics (Fennema et al. 1996).

In a longitudinal study of four teachers' collaboration and reflective discussions, Jónína (Kristinsdóttir 2010) found that teachers' discussions about their own pupils' ways of learning mathematics and reflections on their teaching can influence teaching in diverse classrooms. Through constant discussions on the children's solution strategies and reflection on their teaching, the teachers developed their understanding of their pupils' learning. According to Mason (2002), such systematic reflection on mathematical interactions that focus on student's learning and understanding of processes, as well as on one's own interaction with the pupils, represents an essential professional competence of teachers.

Commonly schools respond to diversity in ways that divide and separate children into hierarchies of value and perceived aptitude. Evidence of these approaches can be seen in labelling and/or sorting of pupils by ability and limited consideration of the potential of all learners (Booth 2010). It is only when teachers open their minds and their classrooms to diverse groups of pupils that they will be enabled to

responsively develop learning communities in which teachers and pupils engage in a spectrum of different learning needs.

Perspectives and attitudes of teachers towards diverse groups of pupils are more positive if they have been involved in inclusive education, compared to teachers who have not had that opportunity (Pugach 2005). The most important role for the teacher is creating a classroom in which all students can reflect on mathematics and communicate their thoughts and actions. Building a community of mathematical practice requires teachers to take the lead in establishing appropriate expectations and norms. In classroom cultures that promote mathematical learning, all students have a voice and are supported in developing their understanding of mathematics through exploring, investigating, discussing, reflecting and drawing conclusions.

8.4 Research with Teachers

Narrative inquiry is a way of understanding and researching experience through collaboration between a researcher and participants. It is based on the premise that as human beings we come to understand and give meaning to our lives through stories. It is a common model for research with practitioners; they receive the opportunity to open up their practice, telling their stories as they use their lived experience as a source of their knowledge and understanding (Clandinin 2013).

The purpose of this research was to develop an understanding of how teachers create mathematical learning environments that respond to young learners during the transition from preschool to primary school. Our intention is to give examples from the teachers' community of practice and their classroom cultures, highlighting collaboration and co-learning with colleagues both within their school and between school levels. The goal was to gain knowledge and understanding of how teachers draw on children's resources and how they use children's mathematical thinking as they plan their teaching for children in preschools and the early grades of primary school.

Rodriguez (2007) defines resources as personal qualities and strengths emerging from and shaping life experiences. She drew on the work of Wertch (1998), in which cultural resources are the mediational tools for people to make meaning and act in the world, and Gonzales et al. (2005) who envision culture as 'funds of knowledge' that can be seen as resources for pupils to draw upon to enhance learning. Thus by resources we refer to the personal qualities and strengths that emerge from and are shaped by children's life experiences.

The research questions that guided this narrative inquiry were the following:

- What kind of learning environment do teachers create that supports all children in learning mathematics?
- How do they draw on the mathematical resources children bring with them as they transfer from preschool to primary school?

In our search to understand more about teachers' professionalism in teaching young children mathematics, we decided to collaborate with nine teachers in preschools and primary schools with whom we have been connected through different projects. We invited them to have a conversation with us and tell us stories that were meaningful to them. Our intention was to reflect with them in order to understand their work and how and why they respond to children in the way they do. Thus, we chose to create focus groups that would come together so that teachers could discuss their evolving experiences (Heikkien et al. 2007). We use their stories to explore the way they respond to children's resources and their mathematical thinking as they develop supportive mathematical learning environments. Gathering narratives that are written, oral or visual and focusing on the meanings that people ascribe to their experiences is both the method and the phenomena of this study (Trahar 2009). Our plan was to examine these issues in depth through exploratory, open-ended conversations, prioritising holistic understanding situated in teachers' lived experiences.

In this research, data consists of documents from the nine teachers and transcripts of discussions in focus groups. We conducted three, two-hour discussions each with two, three or four teachers, inviting them to tell their professional stories, opportunities and challenges in their teaching. During the discussion, teachers shared their work with children not only through telling, but also by sharing pictures, videos, projects and tasks by their pupils. Using teachers' stories to explore their practices, narrative inquiry allowed us to understand their representations of their educational settings and their actions and interactions within them. All discussions were audio recorded and transcribed.

In order to understand how teachers were using students' resources, data were analysed by exploring the transformational dimensions of storytelling from different perspectives. As we looked at the different experiences and backgrounds these teachers brought into the settings, the data were deconstructed in order to reveal discourses that foreground how teachers utilise students' resources in their learning. Vital events and scenarios related to the research questions were extracted. The analytical lens was based on the narrator's voice and the verbal action and choices, as well as the ways the narrative was constrained by social circumstances. From different perspectives within the social, cultural and historical context, the data were brought together again into narratives (Hunter 2010). In so doing the participants had opportunity to respond to the narratives chosen and decide how to tell their stories.

The narratives reported illuminate how teachers build on children's resources in developing supportive mathematical learning environments as they transfer from preschool to the early grades of primary school. The collaborative reflection gave a picture of how they use the personal qualities and strengths that emerge from and are shaped by the children's life experiences.

The teachers participating in this project are: **Ásta**, who is educated both as a preschool and primary school teacher and taught for several years in preschool but is now teaching young children in the first grades in *grunnskóli* (compulsory school in Iceland for 6–16 year-old children). **Birna** is a teacher in *grunnskóli* and has taught mathematics at all levels. Her experience is mainly from teaching in the early

primary grades. **Díana** is educated both as a preschool and special education teacher. She is responsible for developing supportive learning communities for all children in her preschool. **Dóróþea** who is educated both as a general and special education teacher. She teaches in *grunnskóli* and her main focus has been on the early primary grades. **Edda** who is educated both as a general and special education teacher. She teaches in a compulsory school and is responsible for planning meaningful learning opportunities for all children in her school. **Guðrún** is educated both as a preschool and primary school teacher and taught for several years in preschool but is now teaching young children in the first grades in *grunnskóli*. **Kristjana** is a teacher in *grunnskóli* and has taught mathematics in the primary grades. **Margrét** is educated both as a preschool and primary school teacher and taught for several years in preschool but is now teaching young children in the first grades in *grunnskóli*. **Þórunn** is a teacher in *grunnskóli* and has taught mathematics at all levels. Her experience is mainly from teaching in the early primary grades.

All of the teachers in the study have experience with a diverse set of responsibilities. Some, but not all, have had experience in teaching both in preschools as well as compulsory schools and some have only taught in either preschool or primary school.

8.5 Teachers' Reflections

Four recurring themes grew from our analysis. The teachers were concerned with respecting children's resources and their powerful mathematical thinking. They also found it important to be aware that the transition from preschool to primary school can be difficult for children and therefore teachers at both school levels need to collaborate. The teachers believe that they are responsible for creating a supportive learning atmosphere where children feel free to explore, discuss and collaborate. Collaboration with colleagues and the children's parents is equally important and the teachers realise that such collaboration supports their professional development.

8.5.1 Children's Resources

When children start primary school they bring with them experiences from preschool. Diana is aware of young children's explorations with mathematical ideas and the importance of having the opportunity to develop them in preschool.

From Diana's journal:

Rut, Birna and Ásta, 5-year-old girls, are playing together with two-dimensional shapes in different colours. They have grouped all the yellow shapes together and the red shapes, too. Then Rut says: "Let's just mix it all together again". When they have put all the shapes in one pile they fetch a cardboard square with two mirrors placed diagonally on two sides. "Now let's do something smart", Birna says and places a yellow rectangle on the cardboard square. "Let's first put all the boxes", Rut adds, as she places another rectangle on the

cardboard. "Here is another box"; Ásta hands Rut a rhombus. "No, not a broken box" Rut replies. They finish one row with rectangles and then begin another row with rhombuses and the third again with rectangles. "And now we can put what we like, all kinds of blocks and colours and everything", Ásta says. They help each other to cover the whole area and Ruth adds: "We just put all kinds, yellow, blue and green and make a kind of a pattern". When they have finished their work they proudly show me their pattern and how it reflects in the two mirrors.

The girls are playing with shapes in different colours and decide to make a pattern that reflects in the mirrors they use. Although they do not use mathematical nomenclature to name the shapes, they are consistent in using the word 'box' for a rectangle and saying that a rhombus is not a box, it is a broken box. This story tells us that the girls have their own image of the shapes and are consistent in what concepts they apply to them. Their teacher respects their thinking, is present and does not interfere in the process, but is willing to discuss their work.

In her work in the primary grades Guðrún collaborates with the neighbouring preschool and teachers of older children in her own school. Once a month children of different ages spend a day together outdoors and work on different kinds of projects where mathematics plays an important role in their explorations and children support each other in their work.

In a focus group meeting Guðrún explained her vision for teaching:

The preschool teacher is within me and I try to meet the children where they are. ... The outdoor education project is in collaboration with the community and we collect information that can be of use for the development of our community. ... There is a lot of measurement and counting, we look for patterns and regularities and we register our findings in different ways, by collecting things, writing and taking photos. We also make new things from our collections and thus integrate with other subjects.

Guðrún expresses clearly that what she brings with her from preschool to primary school is respect for children. She is also aware of how important it is for children in neighbouring preschools and primary schools to work together and to respect the learning community within each school.

8.5.2 Transition from Preschool to Primary School

For 20 years emphasis has been on building a bridge between preschools and primary schools in Iceland. It started with a developmental project between the Nordic countries where emphasis was placed on smooth transitions from preschool to primary school and teachers learning from working together (Menntamálaráðuneytið 1997). The children pay mutual visits to schools in their neighbourhood; the preschool children then learn to know the school they will attend later and when they have started primary school they have a chance to visit their former preschool. The teachers work closely together to make the transition between the schools smooth and learn from each other's work.

Ásta got acquainted with Caroline Pratt's unit blocks through collaboration with a preschool in her neighbourhood and asked permission to buy them for her school.

She used the unit blocks in her teaching in the primary grades and supported other teachers in using them. She has developed her work with the blocks and published a website where she reports on her work, <<http://astaegils.is/>>. She has written for teachers about the values of working with the unit blocks.

In a focus group meeting Ásta told us:

... and as Caroline Pratt says about the unit blocks, the children internalise the blocks and their forms, and when they start to learn formally about this, then it is there, and then they only have to put the name on it. Thus they, and it is a part of this construction, that in their construction in working with the blocks they give them names, as they perceive them. ...The teacher though needs to use the proper concepts when naming the blocks.

This excerpt shows that Ásta encourages children to develop and expand their own mathematical ideas, while gradually linking more formal concepts, such as standard names for the blocks, to the children's informal understandings.

On her website Ásta writes about the mathematics children learn by playing with the blocks:

The mathematical properties of the unit blocks and their internal relationships make them a practical learning tool in mathematics. While building with the blocks the children get multiple opportunities for mathematical reflections and learning mathematical concepts in a way that is natural to them. In their work with the blocks they encounter many problems they need to solve to succeed. The repetition and the perseverance that is so rich in the development of the building process help the children gradually learn to organise their work and think creatively in solving the problems they meet when building with the blocks and discovering their mathematical properties.

Thus, in addition to linking formal and informal knowledge through the block play, Ásta also stresses the importance of practice and of children constructing and re-constructing their understanding of mathematical concepts. By drawing on her experience as a preschool teacher and collaborating with the neighbouring preschool, Ásta supports the children in the transition from preschool to primary school. Her understanding of children's development and her respect for their need to explore the world is reflected in her work.

8.5.3 Responsive Classrooms

In planning their teaching, the teachers reported that they draw on what they have learned about the teaching and learning of mathematics. The teachers emphasise problem solving in their classrooms. They often plan these lessons with the think-pair-share lesson approach. The work usually begins with a whole class discussion about the problem and then children work in pairs or in small groups and by the end of the lesson they discuss their solutions with the whole class. Kristjana and Margrét have written about their work in a journal for mathematics teachers (Ásgeirsdóttir 2009; Skúladóttir 2009). Margrét writes about small group discussions: "The advantage of small group discussions is that they are democratic and all the children are active participants." Kristjana writes: "The teacher urges the pupils to collaborate and thus supports them in the solution process and urges them not to give up."

The end of class discussion is important, too, and Kristjana emphasised that:

Everyone in the class listens to the discussions and if someone does not understand what is being discussed he/she gets a chance to ask for more information and probe for further explanation. The children feel safe to ask questions to gain further understanding and are not afraid of discussing their mistakes because we all learn from discussing them.

When Þórunn got the opportunity to participate in the planning of the teaching in a new school she introduced her idea of thematic mathematics units with mixed age groups. Her belief that positive experience of mathematics learning is important for all children supported her in planning this project and involving her colleagues. The emphasis is on concrete objectives in mathematics and strong relationships with the pupils' environment. The endeavour is to respond to pupils' diversity by multiform methods and resources. The connection between mathematics and daily lives is endless and the teachers' responsibility is to find ways to make this both simple and interesting but at the same time effective.

The teachers often plan their mathematics teaching around activities that can occur in learning stations, that is, designated spaces where pupils can work on mathematical tasks. Usually teachers divide the class into groups of two or four pupils. Each group goes from one station to another until all of the stations are completed. Math learning stations are designed to benefit diverse learners, and therefore teachers often offer more than one task at each station. There are clear opportunities to work on the tasks in different ways and tasks for independent work. Sometimes the teacher works at one station with pupils and facilitates discussions around pupils' work. The emphasis is often on various real life projects designed to engage pupils in authentic tasks relevant to their daily lives. Other times they are designed to be locally relevant so that children can directly relate to them.

8.5.4 Collaborating with Colleagues

The teachers reported repeatedly on collaboration with colleagues and parents. They say that it is as helpful to establish a good relationship with them as it is to have a good relationship with their pupils. The teachers find it important to establish a community of learning for teachers: a space for discussions, sharing, and supporting each other. Kristjana and her colleagues have managed to create a space for teachers with different backgrounds and beliefs, and report that they all participate in these discussions although they don't always agree. She said: "We discuss everything, how we work with children and their contributions, how different children solve the problems in various ways and what they say and what they think". These discussions made those teachers more aware of their own thinking and influenced their professional language. Margrét, who teaches at the same school, feels that by becoming a part of these discussions she builds on her professional development to change her practice.

Birna is a divisional manager and runs a mathematical facility at her school. She meets with teachers to gain information or requests for a focus and then she structures learning stations for different groups. In other circumstances she team-teaches.

She has compiled a mathematical kit with which she travels from classroom to classroom depending on who invites her to work with their class. Edda, a special educator, organises collaboration between the classroom teachers and the special education teacher so that the teachers can divide the class into smaller groups. The teachers are satisfied because not only do they work with smaller groups of children, but they also plan their teaching together, learn from each other and gain an opportunity to discuss teaching and learning. Birna, Kristjana and Margrét also offer workshops for teachers where they address mathematical learning from different viewpoints and propose examples of tasks that are likely to stimulate children's mathematical thinking.

8.5.5 Collaborating with Parents

As Dóróþea worked with Pétur, a 6-year-old boy, it surprised her how well he understood the tasks she gave him. When she asked him why he is so strong in mathematics he replied: "Because my mom and dad are always making problems for my brother and me". Although Pétur is only 6 years old, he realises how his parents are supporting him in his learning. The work parents do with their children benefits them by making them more confident in working with mathematics. The teachers found it important to collaborate with parents and they provided some examples of their practice. To introduce mathematical learning in first grade, Birna finds it important to inform parents of how children learn mathematics and the way they develop their mathematical thinking. She also gives parents ideas about how they can play with mathematics at home by counting, measuring, and looking at different shapes, and how they can refer to math in their surroundings or in children's books and stories.

Children who struggle with mathematics in school often need support at home to develop their understanding and capability in mathematics. One way that Dóróþea and Birna support parents is to prepare a kit with suggestions of tasks to work on at home. The contents of each kit are different and depend on pupils' interest and strength. It can focus on counting forward or backward, counting by fives or tens, reading and writing numbers, playing cards or learning about money. The teachers find it important that the tasks are actual tasks or from daily life, and not bookwork. The kit is offered to parents, but as it is introduced to them it is emphasised that it is for support not obligation. Although a sheet for marking when the child has mastered all the tasks is a part of the kit, it is only for organisation and not required for use.

8.6 Learning from Teachers

The fundamental basis of this research was learning in partnership with teachers who have all developed their practice in collaboration with children and colleagues while reflecting on their understanding of young children's mathematical thinking. These teachers have developed a community of practice where they can both reflect

on their practice and share their knowledge and understanding in a professional dialogue (Cochran-Smith and Lytle 2009; Guðjónsdóttir 2000). The teachers are empowered as they look critically at their classroom environment, realise student progress and learning in mathematics and communicate their new understanding and learning in a professional community of practice.

The teachers show respect for children's thinking and are capable of relating their knowledge of children's development to their work. They have learned from their own practice through active engagement and reflection in communities with their colleagues and by participating in developmental projects and further education. Our results are consistent with the results from the CGI study that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction (Fennema et al. 1996).

The teacher's understanding of how children learn has made them aware of children's diverse needs. They have developed educational practices that are concerned with and valuing of all pupils equally. The teachers are empowered to practice a culturally responsive and socially relevant pedagogy as they begin to look critically at their classroom environment.

The new teacher professionalism focuses more on learner-centred practice, informed practice, critical reflection, collaboration, and commitment to professional development and knowledge creation (Reeves 2009). Without collaboration, teachers' knowledge is not always recognised and often remains tacit, staying within the teacher.

In this research we learned about responsive teachers who create learning environments that foster mathematical understanding and creativity for all children as they move from preschool to primary school. These teachers report that taking part in this research project has given them an opportunity to participate in professional dialogue and they sense the efficacy of this experience. The teacher education community can, through partnership with teachers, gain understanding and knowledge about what teachers need to develop and grow. The findings indicate the importance for primary school teachers to understand the challenges children meet as they transfer from preschool to primary school but also that they acknowledge the resources they bring with them. It is important that these findings are considered both in practice and in teacher education. The next step for this particular research topic could be continuing with this group as they are more aware of how to make the transition a learning moment.

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