Chapter 16
Early Mathematics in Play Situations: Continuity of Learning

Hedwig Gasteiger

Abstract In recent years, many concepts for early mathematics education have been developed. Taking a closer look at these concepts, it can be seen that they differ considerably in pedagogical background and in quality. During the transition from kindergarten to school, it is extremely important to guarantee consistency and continuity in mathematical learning processes. All early mathematics education should be mathematically correct, 'intellectually honest' and ensure that children acquire the essential prerequisites for further mathematical learning. Additionally, mathematical learning should be designed according to children's specific age. Based on scientific findings, this chapter specifies why early mathematics education in natural learning situations, like play activities, meets these requirements of subject- *and* child-orientation. Play situations can foster the development of mathematical learning in kindergarten and in school sustainably. Results of an intervention study about learning mathematics while playing traditional board games ($n = 95$, average age: 4.8 years, control and intervention group) confirm this claim. The intervention shows significant effects. Video analyses of the play situations illustrate the findings and allow investigating in detail the role of the teachers and the mathematical learning processes which occurred during the play activities.

16.1 Early Mathematics Education—But How?

There is a common consensus that mathematics is a necessary component of early education (Australian Association of Mathematics Teachers and Early Childhood Australia 2006; Kortenkamp et al. 2014; National Association for the Education of Young Children (NAEYC) 2002). However, there are different concepts and ideas about how mathematics should be integrated into the daily work of kindergarten. Approaches range from instructional concepts or training to the idea that mathematical learning happens in many activities every day—and we can find advocates for all positions. Higher quality instructional interactions are positively associated

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with basic mathematical skills (Mashburn et al. 2008). Therefore, recommendations like “all early childhood programs should provide high-quality mathematics curricula and instruction” (Cross et al. 2009, p. 3) stand to reason. On the other hand, some require that learning in early childhood should be initiated by the situation, the environment or children’s play. A very extreme position—Lee and Ginsburg (2009) describe it as a misconception about early mathematics education—is the claim that “mathematical learning occurs incidentally, through exploration during free play, with little teacher participation” (Lee and Ginsburg 2009, p. 40). Moreover, the role of instruction in early childhood education differs considerably between countries (Hauser 2013). So it still seems to be an open question how early mathematics education should be organised best in order to succeed in this difficult domain. To help clarify this question, some key issues of early mathematics education will be discussed.

16.2 Some Key Issues of Early Mathematics Education

Scientific findings from different disciplines support the following key issues of mathematics education in the early years.

- First of all, it is necessary to choose carefully the mathematical content taught during the early childhood. It is recommended to “align all efforts of early mathematics education with the ‘big ideas’ of mathematics” (NAEYC 2002, p. 6). These are “overarching clusters and concepts and skills that are mathematically central and coherent, consistent with children’s thinking, and generative of future learning” (Sarama and Clements 2009, p. 16). Numbers, operations, relations, geometry, spatial relations and measurement are some of the main content areas where early mathematics education should focus on (Cross et al. 2009; Sarama and Clements 2009; Wittmann and Müller 2009). Beyond that, learning contexts should provide opportunities for problem-solving, communication and reasoning (Lee and Ginsburg 2009). Hunting (2010) proposes a more detailed provisional list of big ideas with content and mathematical skills like class inclusion, composition and decomposition, representing, imagining, and naming. Although there are different lists of big ideas, it seems to be a broad consensus that early mathematics education should orient itself towards mathematically central, coherent and consistent content.
- To ensure continuity of learning, it is necessary to make clear the “broader fundamental structure of a field of knowledge” (Bruner 1999, p. 31). Fuson refers to “big coherent conceptual chunks” (Fuson 2004, p. 106). If the mathematical structure is clear, children have a chance to understand what they learn (Fuson et al. 2005) and only then early mathematical learning can be related to mathematical learning in school and life contexts. Teaching mathematical content in a simplified manner, which is supposed to be appropriate for children, can be counterproductive if the fundamental ideas, “the underlying principles that give

structure to that subject" (Bruner 1999, p. 31), get lost. In Germany, for example, there is a method where numbers are presented in a personified manner—"the two" is a character with two feathers on its hat and it repeats every word twice (Friedrich and de Galgóczy 2004). The intention is to design mathematical learning especially suitable for children at a young age, but the fundamental idea of numbers—the mathematical structure—disappears in conceptualisations like these. However—as Bruner (1999) stated,—"any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). This should be a standard for early childhood mathematics education.

- Continuity of learning means to take into account the hierarchical structure of mathematical content—but continuity of learning also means to respect children's learning processes. It is known that children differ considerably in their mathematical achievement in the early years due to social background, family factors and the quality of home learning environment (Anders et al. 2012, p. 207; Starkey and Klein 2008). If children show low mathematical achievement in kindergarten, they are more likely than other children to have difficulties while learning mathematics at school (Dornheim 2008). We even know that children who focus on numerosity in their early years have better subitising and counting skills at the age of 5 years (Hannula et al. 2008). Therefore, early mathematics education should assess and attend to children's individual stages of mathematical development (Clements 2004, p. 13).
- Organising learning processes in early childhood means to respect that children construct their knowledge actively (Fthenakis et al. 2009). Van den Heuvel-Panhuizen describes learning in early years as "a process that occurs primarily 'from inside'… driven by the child's own natural curiosity, its urge to find out how things fit together" (2001, p. 25). So explicit teaching is seen as less effective than creating opportunities which offer children possibilities to discover mathematical concepts and solution strategies for different mathematical problems (Baroody and Wilkins 1999, p. 62).
- Particularly for early childhood mathematics education this constructivist view of learning is complemented with a social component. Learning processes need social interaction with adults and peers (Reusser 2006; van Oers 2004). Results of the EPEY (*Effective Pedagogy in the Early Years*) study show that early education settings are effective if they encourage "sustained shared thinking" (Siraj-Blatchford et al. 2002, p. 10), which is defined as "an episode in which two or more individuals 'work together' in an intellectual way to solve a problem, clarify a concept […]" (Siraj-Blatchford et al. 2002, p. 8). The important role of adult-child-interaction is almost always mentioned in the context of early mathematics education (Anderson et al. 2008; Hunting et al. 2012; Montague-Smith 2002).
- Results of research in developmental psychology state that children at early ages are very motivated to learn, but have difficulties with explicit and intentional learning as it is normally practised in school contexts (Hasselhorn 2005). Therefore, early mathematical learning should be appropriate for children's development.

> Otherwise, long-term effectiveness is not guaranteed (Siraj-Blatchford 2002, p. 29). Moreover, there are indications that direct instruction in early childhood causes more anxiety and lower self-esteem (Sylva and Nabuco 1996). While child-initiated learning activities in early childhood education support the development of "requisite skills and dispositions to become responsible adults" (Schweinhart and Weikart 1997, p. 140), direct instruction has no such preventive value.

To synthesise these statements, instructive learning settings with strong-guided interactions or narrow perspectives of mathematical learning, which are not in line with the big ideas of mathematics and which are not appropriate for children's development, are not suitable for early mathematics education.

16.3 The Concept of Early Mathematics Education in Natural Learning Situations

A concept of early mathematics education that tries to integrate all these key issues is "Early Mathematics Education in Natural Learning Situations" (Gasteiger 2010, 2012, 2014, Fig. 16.1). It is a theoretically based concept (Gasteiger 2010) which focuses primarily on play and everyday activities. These situations deal with mathematical content in a straightforward manner and in a broader context which allows children to recognise relations between mathematics and reality. Learning in complex situations—like everyday situations—and not only in carefully arranged step-by-step units, gives children a chance to understand. Moreover, play and everyday

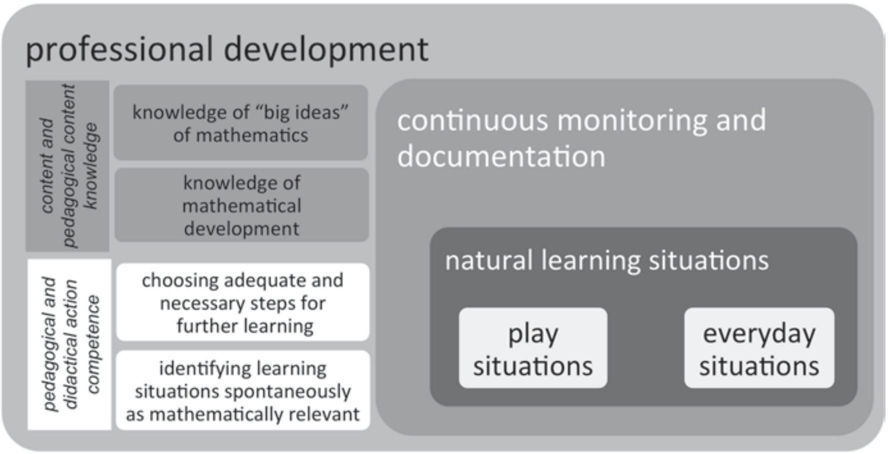

Fig. 16.1 Concept of early mathematics education in natural learning situations. (Gasteiger 2010, 2012, 2014)

situations stimulate children's natural curiosity and open interactive and constructive learning opportunities.

However, mathematical learning does not just occur incidentally (Lee and Ginsburg 2009). Hence, the concept of "Early Mathematics Education in Natural Learning situations" includes two other components.

To guarantee continuity in children's individual learning processes, a continuous monitoring of children and a documentation of their learning processes have to accompany mathematical learning in play and everyday situations. Only then can the next steps of learning be planned, and children be fostered individually in their mathematical development.

Due to the fact that all these efforts of early mathematics education are very challenging for the educators, professional development is an important part of the overall concept. Educators have to know the big ideas of mathematics and milestones in children's mathematical development. They need pedagogical and didactical action competence to identify learning situations as mathematically relevant and to choose adequate and necessary steps for further learning. Without this knowledge and competence they cannot successfully use play or everyday situations for mathematical learning and support children in their individual learning.

There is already some empirical evidence for different components of this concept. The results of a small evaluation study indicate that a professional development program with a focus on natural learning situations, on monitoring and documenting of children's development, and on the underlying content/pedagogical content knowledge and pedagogical/didactical action competence can have effects on children's mathematical achievement (Gasteiger 2010, 2014).

The effectiveness of different play or everyday situations as natural situations for children's mathematical learning in kindergarten still has to be studied in detail. In a first step, the role of number-dice games for mathematical learning was examined in an experimental study. Results of this study will be reported here. Prior to this, detailed argumentation will show why play activities can guarantee continuity of learning in the above-mentioned aspects.

16.4 Play Situations as Natural Learning Situations

16.4.1 Meeting the Key Issues

A definition of play is seen as nearly impossible (Hauser 2013), but there are many attempts to find criteria to describe play. Joyful, child-chosen and child-invented activities which focus on the process, not on the product and which require active involvement, are seen as play activities (Wood and Attfield 2005).

First of all, play as characterised in this way is not instructional learning as often met in school contexts. It seems to be a developmentally appropriate form of learning for children in the early years, because it allows children to discover themselves and their environment actively, they learn to change perspectives—and all this in a kind of protected space (Fröbel 1838; Kunze and Gisbert 2007).

While playing children construct knowledge, they communicate with others and often start a meta-cognitive process (Pramling and Asplund Carlsson 2008). So play situations can be described as co-constructive processes (Jordan 2009). They meet the requirement of sustained shared thinking (Siraj-Blatchford et al. 2002) if children either play together or with adults.

Vygotsky (1978) mentions that "play creates a zone of proximal development of the child" (p. 102), because in play the child behaves beyond its age. Play is "a free activity" (Huizinga 1949, p. 13), which means that children can guide the process of playing on their own. These statements reveal that continuity concerning the individual learning processes of children is guaranteed in play situations, because challenges that play situations offer can be used by children to take a step further, but they need not if it is not suitable for the individual learning process.

Guaranteeing continuity concerning the big ideas of mathematics and the underlying structure of mathematical content in play situations is more difficult. To meet this requirement, it is necessary to focus on the role of the adults. Wood and Attfield (2005) use a "play-non-play continuum" (p. 6) to define play. Play as a meaningful, voluntary, pleasurable, and rule-governed activity can be seen on the far left of the continuum, and playful situations that educators use to provide opportunities to learn can be placed on the far right of the continuum. Talking about using play situations for early mathematics education requires us to agree that adults stimulate learning processes by providing playful opportunities to learn or by encouraging children to think or talk about their actions during play from a mathematical point of view. At this point, we are rather right of the middle in the described continuum. However, even in the context of mathematical learning, these situations should meet the criteria that characterise play. The educators have to ensure the continuity of mathematical learning in the above-mentioned sense, but on the other hand, they should ensure that play situations remain play situations (Perry and Dockett 2010). The important role of the educator is obvious: The "future of mathematical thinking in young children strongly depends on the quality of early years teachers to recognise mathematical actions in children, to see the mathematical potential of play activities and play objects, and to guide children into the future where they can still participate autonomously and creatively in mathematical communications" (van Oers 2013, p. 271).

Play situations can ensure continuity in early mathematics education—both continuity concerning mathematics and continuity concerning children's learning processes. How does this balance with Lee and Ginsburg's (2009) notion that mathematical learning does *not* just occur incidentally in play situations? "As long as [children's] actions are not intentionally and reflectively carried out, we cannot say that children perform mathematical actions" (van Oers 2010, p. 28). Hence, the role of adults, early childhood educators or teachers is very important. They see or create mathematical opportunities to learn in play situations, stimulate children's learning by giving inspiring comments or additional material and by knowing the relations between mathematics in the early years and later on (Gasteiger 2014)

16.4.2 Mathematical Development and Play—Some Scientific Findings

Many scientific results show that using play situations is an effective way to foster children's mathematical development.

McConkey and McEvoy (1986) analysed whether moderately mentally handicapped children (mean age 12.3 years) enhanced their counting abilities by a 6 week intervention of playing dice and card games specially designed for the study. While the control group made almost no progress over the 6-week period, the students in the experimental group showed significant improvement.

Peters (1998) conducted an intervention study with 5 year-old children with low to average number knowledge. These children played mathematical card and board games in small groups with parental support. The games were played in the classroom once a week for 8 months. Children taking part in this intervention ($n=14$) performed better in counting tasks than a control group ($n=37$).

That number games and story books can improve were examined in a study by Young-Loveridge (2004). Over a 7-week period, 23 lower achieving children attended daily intervention sessions (30 min each) in school over 2 months. Two children played (modified) commercial dice and card games with a teacher, heard a number story and talked about the numbers in the accompanying pictures. Each session started and ended with a number rhyme. The teachers were advised to engage the children in mathematical activities and to support them in their individual development. The control group ($n=83$) continued their mathematical lessons. Their teachers used a special program based on Piaget's ideas of matching, sorting, comparing, and classifying. The intervention program had significant effects over time, even 15 months after intervention. Intervention children made greater gains in knowledge of number, number patterns, numeral identification, making small collections of objects, and in addition of two collections.

The effect of a very short intervention was examined by Ramani and Siegler (2008) who randomly assigned 124 preschool children (mean age 4.9 years) from low-income backgrounds to two groups. They played in four 15–20-min sessions one-on-one with an adult within a 2-week period and in a fifth session 9 weeks later. Children in the experimental condition played a linear board game with the numbers of 1–10 in 10 squares and dice with the numbers 1 and 2. Children in the control condition had a linear board game with colours in the 10 squares and dice with colours. The child should say the number/colour on the squares they passed while moving their token. After intervention, children in the experimental condition improved their results in number line estimation, in counting, numerical magnitude identification and in numeral identification considerably, while children in the control condition showed no improvement. A second study of Ramani and Siegler (2008) indicated that children who had more experience in playing board games at home or at other people's home showed greater numerical knowledge, whereas experience in playing card and video games was not related to numerical knowledge.

Rechsteiner et al. (2012) compared the mathematical achievement of children in kindergarten (mean age 6.3 years) who received a play-based approach for mathematical learning ($n=89$) with that of children who were given an instructional training ($n=110$) and with children in a control group with no intervention ($n=125$) (Stebler et al. 2013). Children in the play-based approach played three times a week (30 min. per session) in an 8-week period using commercial and specially designed card and board games. They played on their own in small groups. Children in the instructional training group were given a commercial training program with the same duration and timetable and the control group had no explicit intervention. The mathematical development of children in the play-based approach was significantly better than that in the control group, whereas children in the instructional training group performed not significantly better or worse than children in the two other groups. The play-based approach seemed to be comparable with the instructional training, but considerably better than the regular daily work in kindergarten.

These findings support the idea that play (board and card games, games with or without dice) can enhance the mathematical development of children.

16.5 Fostering Early Mathematical Development with Traditional Board Games—Results of an Intervention Study

The majority of the above-mentioned studies focus on children with mathematical achievement below average or with difficult conditions for further learning, and most of the investigated children were already in school or at least 6 years of age. The games used in these studies were mostly modified or specifically designed for the interventions.

Opportunities to learn at home and with parents seem to influence children's mathematical knowledge even before they enter kindergarten (Anders et al. 2012; Ramani and Siegler 2008). Hence, one should focus on younger children and on traditional games because these games can be found at children's home and as they are sometimes/often played in normal play situations at home without a special learning focus.

The question therefore arises whether children in their early years stand to benefit from play—irrespective of their social background and their previous mathematical knowledge—and if their engagement in "normal" play situations with traditional games guarantees sustainable, continuous mathematical learning processes. The intervention study MaBiiS (elementare **ma**thematische **Bi**ldung **i**n **S**pielsituationen) examined this question.

16.5.1 Method

16.5.1.1 Participants

A total of 95 children (52 girls and 43 boys, 31 with migration background and 64 without) took part in the study. They were recruited from five German kindergartens, their mean age was 4.8 years (4.5–6.2 years) and they all had 1 year and a half until their school enrolment. The children were randomly assigned to an intervention and a control group.

16.5.1.2 Intervention

The children attended the intervention sessions in groups of two or three over a three-and-a-half-week period. Each child had seven intervention sessions of 30 minutes each. During the intervention sessions a trained adult played dice games with the children.

Children in the intervention group played board games with normal number dice. These were the traditional games ludo[1] and coppit[2], and a game called "Collecting Treasures"[3]. Playing this game, children have to move their token forward, some squares show an amount and if the token is on one of these squares, the child has to collect the right number of coloured treasures—the winner is the one who can collect most of the treasures. Children in the control group played dice games as well, but with colour or symbol dice. They played a game very similar to ludo[4] but with symbol dice: the symbol on the dice shows to which square the player should move. A second game used colour dice to choose small parts in different lengths which form together a worm—the player with the longest worm wins the game[5]. Counting was not necessary in either game played with children in the control group.

The adults who played with the children were trained. The most important point in their training was that they should 'play' and not instruct the children in mathematics. However, they were trained to play while remaining alert to everything that happened. They functioned as a role model by counting out loud, when they moved forward, by naming the number or colour/symbol the die showed or by giving verbal stimuli like "Count again. Your token was here" or "I think you can catch someone". They remained attentive so that the children had enough time for their moves, and guaranteed that no one forestalled the players' actions or answers.

1 Mensch ärgere dich nicht—Schmidt-Spiele.

2 Fang den Hut—Ravensburger.

3 Schätze sammeln—ZahlenZauberei, Oldenbourg-Schulbuchverlag.

4 Der Maulwurf und sein Lieblingsspiel—Ravensburger.

5 Da ist der Wurm drin—Zoch.

16.5.1.3 Measures

Children's mathematical competencies were assessed before (pretest), immediately after (posttest) and 1 year after the intervention (follow-up-test) using an individual standardised test for children between kindergarten and third grade (TEDI-Math: Kaufmann et al. 2009). The counting principles, enumeration, numeral identification, number word identification, and calculating subscales were utilised. The standardised test was supplemented by a subscale structure knowledge and structure use. Data from the follow-up-test has not yet been analysed. Children's intelligence was measured with the WPPSI (Petermann and Lipsius 2011), and the quality of the day-care centres was assessed using the KES-R (Tietze et al. 2005).

One play session for each adult, with children of the intervention group, was videotaped to analyse communication and activities between the children and the adult during the play situation.

16.5.2 Results

16.5.2.1 Effectiveness

An ANCOVA was performed on children's posttest score of mathematical achievement using pretest score as covariate. Posttest score is influenced significantly by the pretest score ($F(1,92)=291.88$, $p<0.001$), and by the intervention-condition ($F(1,92)=13.57$, $p<0.001$) with an effect size of 13 % (partial eta squared). The intervention group shows greater gains in the posttest than the control group as the solution rates in Table 16.1 show: in the pretest, both groups performed nearly equally, while in posttest the solution rate of children in the intervention group (72 %) was better than of children in the control group (67 %).

The impact of the play intervention was found to be independent of gender, migration background, intelligence and day-care centre. The results indicate that children who played number-dice games showed significantly higher learning gain from pre- to post-test than children in the control group who played with colour- or symbol-dice.

The subscale in which children of the intervention group performed substantially better than children of the control group was enumeration ($F(1,92)=9.96$, $p<0.01$, $\eta^2=0.10$). An explanation for this is that counting and respecting one-to-one-correspondence is often experienced when children move their tokens forward during their play.

Table 16.1 Comparison of solution rates

	N	M (SD)	
		Pretest	Posttest
Intervention group	48	0.60 (0.16)	0.72 (0.14)
Control group	47	0.61 (0.15)	0.67 (0.16)

16.5.2.2 Mathematical Action and Communication During Play

To analyse the mathematical learning opportunities during play situations, nine intervention sessions were videotaped. These data were analysed to detail the mathematical content in which children were engaged and how much time they used for mathematical activities or dialogues. Another question was whether there were differences between the adults, or the three games (Sedlmeier 2013).

Therefore, the periods of time of all activities and comments were coded in separate categories for adults and for children (see Tables 16.2 and 16.3). Verbal/non-verbal, and mathematical/non-mathematical categories were differentiated. Category 10 (non-verbal play activity) was coded when the game had been prepared, dice were rolled or passed to the next player, and other similar activities. These are non-verbal and non-mathematical activities.

Category 11 (not defined non-verbal activity) is the category most often used for the adults' activities and comments. This is not surprising, considering that each child spent time on play activities and comments while the adults probably listened and observed. Analysing active time of the adults (categories 1–10), we can see that

Table 16.2 Categories for adults' activities and comments (mathematical categories in italics)

Verbal	Non-verbal
1 Mathematical stimuli or questions	10 Non-verbal play activity
2 Mathematical explanation, correction	11 Not defined non-verbal activity
3 Mathematical comment on one's own play activity	
4 Confirmation of mathematical comment of children	
5 Mathematical, verbal accompanied play activity	
6 Disciplinary comment	
7 Comment concerning rules	
8 Other non-mathematical comment	
9 Incomprehensible comment	

Table 16.3 Categories for children's activities and comments (mathematical categories in italics)

Verbal	Non-verbal
1 Enumeration (right/wrong)	*10 Silent enumeration (right/wrong)*
2 Comparing amounts (right/wrong)	*11 Non-verbal subitising (right/wrong)*
3 Subitising (right/wrong)	12 Non-verbal play activity
4 Part-whole (right/wrong)	
5 Calculating (right/wrong)	
6 Comments based on mathematical thinking	
7 Comment concerning rules	
8 Other non-mathematical comment	
9 Incomprehensible comment	

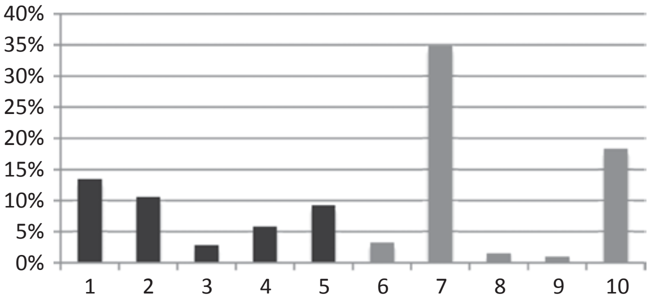

Fig 16.2 Adults activities and comments (mathematical categories: *dark*, others: *light*)

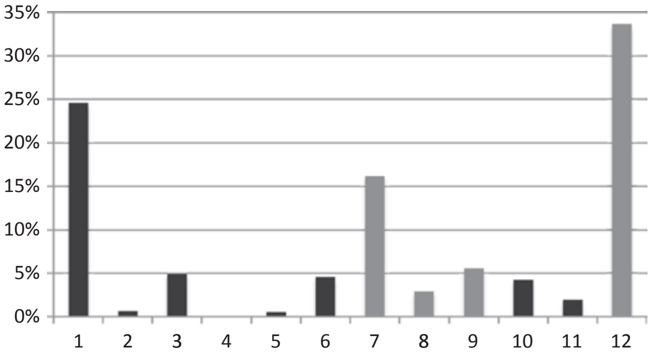

Fig. 16.3 Children's activities and comments (mathematical categories: *dark*, others: *light*)

35 % of this time was spent on comments concerning rules (Fig. 16.2, category 7), and during 42 % of their active time adults gave mathematical stimuli, explanations, comments, or accompanied their play activity verbally with a mathematical intention (categories 1–5).

There were almost no differences between the three games, but some differences between the adults. While one adult had only 29 % verbal time during the whole intervention sessions, the others spent between 41 and 55 % on talking or commenting. As we videotaped only one session with each adult, it is not reasonable to examine correlations between children's mathematical achievement and adults' verbal activity.

The mathematical activities and comments of the children on the videotapes were exactly characterised, and different codes were used if the comment or activity was correct or incorrect. Table 16.3 overviews the categories. Active time of each child was analysed, and for each session those data were summed in the different categories. Two of the children's non-verbal categories were coded as mathematical activity: if a child recognises the dice-pattern and/or moves its token forward correctly without speaking, this can be seen as a mathematical activity.

As can be seen in Fig. 16.3 children spent most of their active time on non-verbal play activities (category 12: 34 %).

They were mathematically active in 42 % of their active time (all categories except 7, 8, 9, 12). Most of that time was used for enumeration (25 % verbal, 4 % non-verbal) and subitising (5 % verbal, 2 % non-verbal). Less than 1 % of children's active time was used for comparing amounts, part whole activities or calculating. Five percent of the active time was used for comments based on mathematical thinking such as "To catch me, you need six" or "You should go with this token" (if it was a good piece of advice for a tactical move when it was necessary to reflect on a number of moves).

The coding of correct or incorrect statements or activities showed problems with the number sequence or the one-to-one-correspondence in individual cases, but all in all, 91 % of the time that children were mathematically active, they acted or verbalised correctly.

The analyses of the video data showed the potential of traditional board games for mathematical learning: adults and children spent in equal measure 42 % of their active time on mathematical comments, discussions, activities, or thinking. The qualitative analysis of the video data showed that children also commented on activities of their peers or the adult player. They were involved in play activities even when it was not their turn.

16.6 Discussion

This chapter tried to give a theoretical foundation for the use of play situations for early mathematics education with a special focus on continuity of learning and reported empirical evidence for the effectiveness of play.

Play situations can be used successfully to foster the mathematical development of under-achieving children—in kindergarten and likewise in school (Peters 1998; Ramani and Siegler 2008; Rechsteiner et al. 2012; Young-Loveridge 2004). The results of our study show the potential of number-dice games for all children—regardless of gender, migration background, intelligence and the day-care centres children attended. We played traditional games in 'normal' play situations in which our adult players were requested to play like—for example—alert parents. This approach offers another big chance for mathematical learning: learning situations like those can be carried out very easily within the family environment as well. With respect to the considerable influence of learning situations at home before entering kindergarten (Anders et al. 2012; Ramani and Siegler 2008), these results are of great importance.

Focusing on continuity of mathematical learning, the appropriateness of play situations can be reflected even more soundly. Therefore, it is reasonable to consider mathematical competencies in the early years, which are predictive for mathematical learning in school. In a longitudinal study Dornheim (2008) determined verbal counting (counting on, counting in steps included), enumeration, subitising, using structures, and simple calculations as predictive for further mathematical learning. Children who have difficulties in these domains at an early age are more likely to have problems with mathematical learning in school compared to children who perform well. Our studies show that verbal counting, enumeration and subitising can be trained in play situations. The study of Young-Loveridge (2004) even shows effects 15 months after the intervention. The follow-up-test data of our study has to be analysed in detail, but there are signs that children in the intervention group can profit from the board game intervention 12 months later. This means that board games can help to foster especially those mathematical competencies which are predictive for further learning. This shows once again how play situations can contribute to continuity in the learning of mathematics.

In conclusion, one major implication for mathematics at transition from kindergarten to school is to respect play as a successful approach for mathematical learning in kindergarten, school and even family. For practice it can be recommended to analyse the mathematical potential of different games, to choose them carefully for application in kindergarten and school and to be alert for children's learning processes in play situations.

Early mathematics learning in play situations also suggests future work. This chapter focused primarily on numbers, but play situations have a great potential for mathematical learning for other content areas as well. Seo and Ginsburg (2004) showed in an observation study that children engage in many different mathematical activities in their free play. Such play is sometimes quite complex and includes different content such as pattern and shape, classification, spatial relation, enumeration, magnitude, or dynamics. There is still a lot of research to do to support children's mathematical learning in play situations in domains other than number. It is necessary to get more insight in children's mathematical development in these domains, and the effectiveness of these play situations for mathematical learning has to be studied.

The video data analysis of our study shows that almost all mathematical comments have been correct. Closer consideration of the qualitative data shows that some children are more active than others. Can it be assumed that children with less mathematical prerequisites took less part in the conversation while playing than the others? Here, the important role of the adults becomes obvious: their role "is crucial, as the adult will introduce and use new language, and encourage discussion to help the child to understand a concept or acquire a skill" (Montague-Smith 2002, p. 140). Stimuli, comments or questions of adults are necessary to let children see the mathematics, think about it and move a step further in their own development. Substantiate findings and good programmes of professional development are necessary to help the educators see the mathematical development in children's play and to act or react in a co-constructive manner. Not till then, will play situations unfold their whole potential for early mathematics education.

References

Anders, Y., Grosse, C., Rossbach, H.-G., Ebert, S., & Weinert, S. (2012). Preschool and primary school influences on the development of children's early numeracy skills between the ages of 3 and 7 years in Germany. *School Effectiveness and School Improvement: An International Journal of Research, Policy and Practice*, *24*(2), 195–211.

Anderson, A., Anderson, J., & Thauberger, C. (2008). Mathematics learning and teaching in the early years. In O. N. Saracho & B. Spodek (Eds.), *Contemporary perspectives on mathematics in early childhood education* (pp. 95–132). Charlotte: Information Age Publishing.

Australian Association of Mathematics Teachers & Early Childhood Australia. (2006). *Position paper on early childhood mathematics*. http://www.aamt.edu.au/content/download/721/19509/file/earlymaths_a3.pdf. Accessed 23 Nov 2013.

Baroody, A. J., & Wilkins, J. L. M. (1999). The development of informal counting, number, and arithmetic skills and concepts. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 48–65). Reston: The National Council of Teachers of Mathematics.

Bruner, J. (1999). *The Process of education*. London: Harvard University Press.

Clements, D. H. (2004). Major themes and recommendations. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 7–72). Mahwah: Lawrence Erlbaum Associates.

Cross, C. T., Woods, T. A., & Schweingruber, H. (2009). *Mathematics learning in early childhood: Paths towards excellence and equity*. Washington, DC: The National Academies Press.

Dornheim, D. (2008). *Prädiktion von Rechenleistung und Rechenschwäche: Der Beitrag von Zahlen-Vorwissen und allgemein-kognitiven Fähigkeiten*. Berlin: Logos.

Friedrich, G., & de Galgóczy, V. (2004). *Komm mit ins Zahlenland. Eine spielerische Entdeckungsreise in die Welt der Mathematik*. Freiburg: Christophorus, Herder.

Fröbel, F. (1838). Ein Ganzes von Spiel- und Beschäftigungskästen für Kindheit und Jugend. Erste Gabe: Der Ball als erstes Spielzeug des Kindes. In E. Blochmann (Ed.), *Fröbels Theorie des Spiels I*. (2nd ed., pp. 16–38). Langensalza: Thüringer Verlagsanstalt.

Fthenakis, W. E., Schmitt, A., Daut, E., Eitel, A., & Wendell, A. (2009). *Natur-Wissen schaffen. Band 2: Frühe mathematische Bildung*. Troisdorf: Bildungsverlag EINS.

Fuson, K. C. (2004). Pre-K to grade 2 goals and standards: Achieving 21st-century mastery for all. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 105–148). Mahwah: Lawrence Erlbaum Associates.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In S. Donovan & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 215–256). Washington, DC: The National Academies Press.

Gasteiger, H. (2010). *Elementare mathematische Bildung im Alltag der Kindertagesstätte. Grundlegung und Evaluation eines kompetenzorientierten Förderansatzes*. Münster: Waxmann.

Gasteiger, H. (2012). Fostering early mathematical competencies in natural learning situations. Foundation and challenges of a competence-oriented concept of mathematics education in kindergarten. *Journal für Mathematik-Didaktik, 33*(2), 181–201.

Gasteiger, H. (2014). Professionalization of early childhood educators with a focus on natural learning situations and individual development of mathematical competencies: Results from an evaluation study. In U. Kortenkamp, B. Brandt, C. Benz, G. Krummheuer, S. Ladel, & R. Vogel (Eds.), *Early mathematics learning. Selected papers of the POEM 2012 conference* (pp. 275–290). Berlin: Springer.

Hannula, M. M., Räsänen, P., & Lehtinen, E. (2008). Development of counting skills: Role of spontaneous focusing on numerosity and subitizing-based enumeration. *Mathematical Thinking and Learning, 9*(1), 51–57.

Hasselhorn, M. (2005). Lernen im Altersbereich zwischen 4 und 8 Jahren: Individuelle Voraussetzungen, Entwicklung, Diagnostik und Förderung. In T. Guldimann & B. Hauser (Eds.), *Bildung 4- bis 8-jähriger Kinder* (pp. 77–88). Münster: Waxmann.

Hauser, B. (2013). *Spielen. Frühes Lernen in Familie, Krippe und Kindergarten*. Stuttgart: Kohlhammer.

Huizinga, J. (1949). *Homo ludens*. London: Routledge.

Hunting, R. P. (2010). Little people, big play, and big mathematical ideas. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education. Proceedings of the 33rd* *annual conference of the Mathematics Education Research Group of Australasia*, (pp. 715–718). Fremantle: MERGA.

Hunting, R., Mousley, J., & Perry, B. (2012). *Young children learning mathematics. A guide for educators and families*. Camberwell: ACER.

Jordan, B. (2009). Scaffolding learning and co-construction understandings. In A. Anning, J. Cullen, & M. Fleer (Eds.), *Early childhood education. Society and culture* (pp. 39–52). London: Sage Publications.

Kaufmann, L., Nuerk, H.-C., Graf, M., Krinzinger, H., Delazer, M., & Willmes, K. (2009). *TEDI-MATH. Test zur Erfassung numerisch-rechnerischer Fertigkeiten vom Kindergarten bis zur 3. Klasse*. Bern: Hans Huber, Hogrefe.

Kortenkamp, U., Brandt, B., Benz, C., Krummheuer, G., Ladel, S., & Vogel, R. (Eds.). (2014). *Early mathematics learning. Selected Papers of the POEM 2012 Conference*. Berlin: Springer.

Kunze, H.-R., & Gisbert, K. (2007). Förderung lernmethodischer Kompetenzen in Kindertageseinrichtungen. In BMBF (Ed.), *Auf den Anfang kommt es an: Perspektiven für eine Neuorientierung frühkindlicher Bildung* (pp. 15–117). Berlin: BMBF.

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood, 34*(4), 37–45.

Mashburn, A. J., Pianta, R. C., Hamre, B. K., Downer, J. T., Barbarin, O. A., Bryant, D., Burchinal, M., Early, D. M., & Howes, C. (2008). Measures of classroom quality in prekindergarten and children's development of academic, language, and social skills. *Child Development*, *79*(3), 732–749.

McConkey, R., & McEvoy, J. (1986). Games for learning to count. *British Journal of Special Education, 13*(2), 59–62.

Montague-Smith, A. (2002). *Mathematics in nursery education* (2nd ed.). Oxon: Routledge.

National Association for the Education of Young Children (NAEYC). (2002). Early childhood mathematics: Promoting good beginnings. http://www.naeyc.org/positionstatements/mathematics. Accessed 23 Nov 2013.

Perry, B., & Dockett, S. (2010). What makes mathematics play? In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics eduction. Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia*, (pp. 715–718). Fremantle: MERGA.

Petermann, F., & Lipsius, M. (2011). *Wechsler preschool and primary scale of intelligence III, German version*. Frankfurt a. M.: Pearson Assessment.

Peters, S. (1998). Playing games and learning mathematics: The results of two intervention studies. *International Journal of Early Years Education*, *6*(1), 49–58.

Pramling, I., & Asplund Carlsson, M. (2008). The playing learning child: Towards a pedagogy of early childhood. *Scandinavian Journal of Educational Research, 52*(6), 623–641.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*(2), 375–394.

Rechsteiner, K., Hauser, B., & Vogt, F. (2012). Förderung der mathematischen Vorläuferfertigkeiten im Kindergarten: Spiel oder Training? In M. Ludwig & M. Kleine (Eds.), *Beiträge zum Mathematikunterricht 2012* (pp. 677–680). Münster: WTM.

Reusser, K. (2006). Konstruktivismus—vom epistemologischen Leitbegriff zur Erneuerung der didaktischen Kultur. In M. Baer, M. Fuchs, P. Füglister, K. Reusser, & H. Wyss (Eds.), *Didaktik auf psychologischer Grundlage. Von Hans Aeblis kognitionspsychologischer Didaktik zur modernen Lehr- und Lernforschung* (pp. 151–167). Bern: h.e.p. verlag.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research. Learning trajectories for young children*. New York: Routledge.

Schweinhart, L. J., & Weikart, D. P. (1997). The high/scope preschool curriculum comparison study through age 23. *Early Childhood Research Quarterly, 12*(2), 117–143.

Sedlmeier, S. (2013). Würfelspiele als Möglichkeit zur Förderung mathematischer Kompetenzen in der Vorschulzeit—eine theoretische und praktische Auseinandersetzung. Unpublished thesis.

Seo, K.-H., & Ginsburg, H. P. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 91–104). Mahwah: Lawrence Erlbaum Associates.

Siraj-Blatchford, I., Sylva, K., Muttock, S., Gilden, R., & Bell, D. (2002). *Researching effective pedagogy in the early years*. Norwich: Queen's Printer.

Starkey, P., & Klein, A. (2008). Sociocultural influences on young children's mathematical knowledge. In O. N. Saracho & B. Spodek (Eds.), *Contemporary perspectives on mathematics in early childhood education* (pp. 253–276). Charlotte: Information Age Publishing.

Stebler, R., Vogt, F., Wolf, I., Hauser, B., & Rechsteiner, K. (2013). Play-based mathematics in kindergarten. A video analysis of children's mathematical behavior while playing a board game in small groups. *Journal für Mathematik-Didaktik, 34*(2), 149–175.

Sylva, K., & Nabuco, M. (1996). Research on quality in the curriculum. *International Journal of Early Childhood, 28*(2), 1–6.

Tietze, W., Schuster, K.-M., Grenner, K., Roßbach, H.-G. (2005). *Kindergarten-Skala (KES-R): Feststellung und Unterstützung pädagogischer Qualität in Kindergärten*. Berlin: Cornelsen Scriptor.

van den Heuvel-Panhuizen, M. (2001). *Children learn mathematics. A learning-teaching-trajectory with intermediate attainment targets for calculation with whole numbers in primary school*. Utrecht: Freudenthal Institute.

van Oers, B. (2004). Mathematisches Denken bei Vorschulkindern. In W. E. Fthenakis & P. Oberhuemer (Eds.), *Frühpädagogik international* (pp. 313–330), Wiesbaden: VS Verlag für Sozialwissenschaften.

van Oers, B. (2010). Emergent mathematical thinking in the context of play. *Educational Studies in Mathematics, 74*(1), 23–37.

van Oers, B. (2013). Challenges in the innovation of mathematics education for young children. *Educational Studies in Mathematics, 84*(2), 267–272

Vygotsky, L. (1978). *Mind in society. Development of higher pedagogical processes*. Cambridge: Harvard University Press.

Wittmann, E. C., & Müller, G. N. (2009). *Das Zahlenbuch. Handbuch zum Frühförderprogramm*. Stuttgart: Klett.

Wood, E., & Attfield, J. (2005). *Play, learning and the early childhood curriculum*. London: Sage Publications.

Young-Loveridge, J. M. (2004). Effects on early numeracy of a program using number books and games. *Early Childhood Research Quarterly, 19*(1), 82–98.

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