Chapter 10
Scaling Up Early Mathematics Interventions: Transitioning with Trajectories and Technologies

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Abstract Transitions in the early years have substantial effects on children's success in school. Moreover, lack of consideration of continuity and alignment may mislead both researchers and politicians to assume preschool effects 'fade', when it may be that poor transitions to primary school are to blame. We hypothesise that most present educational contexts are unintentionally and perversely aligned against early interventions. For example, primary curricula assume little mathematical competence, so only low-level skills are taught. Most teachers are required to follow such curricula rigidly and remain unaware that some of their students have already mastered the material they are about to 'teach'. Teachers may be held accountable for getting the largest number of students to pass minimal competency assessments, engendering the belief that higher performing students are 'doing fine'. In this way, we believe the present U.S. educational system unintentionally but insidiously re-opens the gap between students from low- and higher-resource communities. We conducted a large cluster randomised trial of an intervention that evaluated the persistence of effects of a research-based model for scaling up educational interventions, with one control and two intervention conditions. Only the intervention condition that included a follow-through treatment to support the transition to the primary grades maintained substantial gains of the pre-K mathematics curriculum.

10.1 Introduction

Transitions in the early years have substantial effects on children's success in school. This may be especially true in the domain of early mathematics, because many schools fail to encourage, and may even discourage, communication between pre-K, Kindergarten, and primary grade teachers, and because many teachers lack knowledge of and confidence in mathematics. In this chapter, we discuss the

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importance of the transition to primary school and continuity in mathematics education for young children (Perry et al. 2012a). We begin by documenting weaknesses in many countries' early instruction, especially the U.S., and describe our rigorous test of the notion that early mathematics interventions are important and require follow through into the primary grades (at least).

10.2 The Need: Weaknesses in Mathematics Education, a U.S. Example

Countries differ significantly in mathematics achievement (Mullis et al. 2012). Low-performing countries may need to revise their mathematics education systems. For example, the mathematics achievement of U.S. students compares unfavourably with the achievement of students from many other nations, with some cross-national differences in informal mathematics knowledge appearing as early as 3–5 years of age (Sarama and Clements 2009).

Further, children from some groups come to school less prepared in mathematics than others. For too many, these differences increase as they move through the grades (National Mathematics Advisory Panel 2008). In the U.S. this gap is most pronounced in the performance of children living in economically deprived urban communities. The achievement gaps have origins in the earliest years. For example, the percentage of 4-year olds demonstrating proficiency in numbers and shapes was 87% in higher-socioeconomic status (SES) families but only 40% among lower-SES families (Chernoff et al. 2007).

Thus, there is an early developmental basis for later achievement differences in mathematics: Children from different sociocultural backgrounds are provided different foundational experiences. Programs need to recognise sociocultural and individual differences in what children know and in what they bring to the educational situation. These differences should inform planning for programs and instruction, including extra support for those from low-resource communities. We must meet the needs of all children, especially groups disproportionately under-represented in mathematics, such as children of colour and children whose home language is different than that of school. All these children also bring diverse experiences on which to build meaningful mathematical learning. There is no evidence that such children cannot learn the mathematics that other children learn. Too often, children are not provided with resources and support equivalent to middle-class or upper-class majority children. They may have different and inequitable access to foundational experiences, mathematically-structured materials such as unit blocks, technology, and so forth.

This brings us to another equity concern: Transitions to school, recovering from initial gaps in learning, and maintaining more positive trajectories of learning mathematics may be more problematic for African-American children than white children (cf. MacDonald et al. 2012, and efforts to work with indigenous children). In another study (Alexander and Entwisle 1988), African-American children gained less than white children, with the gap widening over a 2-year period. Similarly, African-American children can make real gains in mathematics knowledge in preschool, but over the first 2 years of school, they lose substantial ground relative to other races (Fryer and Levitt 2004). Quality is lower in classrooms with more than 60 % of the children from homes below the poverty line, when teachers lacked formal training (or a degree) in early childhood education, and held less child-centered beliefs (Pianta et al. 2005).

10.3 The Issue of Fade Out and the Need to Plan for Transitions and Follow Through

Some studies indicate that early interventions can have lasting effects. For example, several have shown positive and long-lasting effects of preschool experience (Clements and Sarama 2014; Wylie at al. 2009). However, there is considerable empirical research and resultant (practical) assertions that preschool gains 'fade' in the primary grades. For example, in one study of six cohorts, gains in preschool weakened as children progressed through the primary grades, disappearing by fourth grade (Fish 2003). Other studies show a similar fade (Administration for Children and Families (ACF) 2010; Natriello et al. 1990; Preschool Curriculum Evaluation Research Consortium 2008).

Although an ostensible reason for such fade is that early effects are themselves evanescent, we believe that a contradictory explanation is more theoretically cogent. We hypothesise that present educational contexts are unintentionally and perversely aligned against the persistence of early interventions. Transitions to the primary grades are not planned or implemented well. Consider the educational trajectories of children who benefited from a successful pre-K experience as they move into kindergarten. The kindergarten curriculum they experience likely assumes little or no mathematical competence, so only low-level skills are taught. Their teachers are often required to follow such curricula rigidly and remain unaware that some of their students have already mastered the material they are about to 'teach' (Bennett et al. 1984; Clements and Sarama 2014; National Research Council 2009; Sarama and Clements 2009). Further, biases may negatively affect the subsequent school experiences of children at-risk during pre-K. For example, kindergarten teachers rated Head Start children's mathematics ability as lower than that of other children, even though direct assessments showed no such differences (ACF 2010). Thus, teachers may view children from different SES or ethnic groups as lacking knowledge or the ability to learn and thus overlook their competencies and potential for growth. Even if the children are assigned to a kindergarten teacher who recognises their competencies, pressure to increase the number of children passing minimal competency assessments may lead this teacher to work mainly with (and/or mainly at the level of) the lowest performing children. Within this context and without continual, progressive support (especially given that children from low-resource communities attend low-resource schools), early gains may fade. In this way, we believe the present

U.S. educational system inadvertently but insidiously re-opens the gap between students from low- and higher-resource communities.

For these reasons, we designed and evaluated the effectiveness of TRIAD's follow-through intervention, testing our hypothesis that such follow through is the 'missing piece' in many early interventions whose longitudinal evaluations have found less positive effects (cf. the effects of Te Mahere Tau, The Number Framework, MacDonald et al. 2012; Trinick and Stevenson 2009). Although this might appear to be an issue of simple 'educational engineering', the issue has implications for both theory and policy. Interpretations of this fade often call for *decreased* funding and attention to pre-K (Fish 2003). Although this may appear reasonable (with logic such as, if effects fade out, why fund that intervention?), we believe this mistakenly treats initial effects of interventions as independent of the future school contexts. Instead, we believe children's trajectories must be studied as they experience different educational courses. If such effects fade in traditional settings but do not in the context of follow-through interventions, then attention to and funding for follow-through efforts for both pre-K and the primary grades should arguably increase.

10.4 Intervention: The *Building Blocks* Curriculum and TRIAD Scale-Up Model

To begin to address these needs, we designed the *Building Blocks* preschool (mainly for 4-year-olds) mathematics curriculum (Clements and Sarama 2013) as a set of tools that would enable all young children to build a solid foundation for mathematics, and especially that would increase the mathematical knowledge of children from low-resource communities. *Building Blocks* is a National Science Foundation-funded mathematics curriculum designed using a comprehensive Curriculum Research Framework (CRF) (Clements 2007) to address numeric/quantitative and geometric/spatial ideas and skills. Woven throughout are mathematical subthemes, such as sorting and sequencing, as well as mathematical processes. General processes include communicating, reasoning, representing, and problem solving and the overarching mathematising. Specific mathematical processes include number and shape composition and patterning. We considered these to be critical mathematical building blocks based on our previous work (Clements at al. 2004).

At the core of the CRF are empirically-grounded learning trajectories. We define learning trajectories as "descriptions of children's thinking and learning in a specific mathematical domain, and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesised to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain" (Clements and Sarama 2004, p. 83). Our learning trajectories' are not simply 'educated guesses' but are based on empirically-supported developmental progressions (more so for more heavily researched topics, of course). These share many similarities with the "growth points" of the Early Numeracy Research Project (ENRP) (Clarke et al. 2002; Perry et al. 2008) and other projects in Australia, New Zealand, and other countries (Bobis et al. 2005; Perry 2010). As an example, children's developmental progression for shape composition advances through levels of trial and error, partial use of geometric attributes, and mental strategies to synthesise shapes into composite shapes. The sequence of instructional tasks requires children to solve shape puzzles off and on the computer, the structures of which correspond to the levels of this developmental progression (Clements and Sarama 2007; Sarama et al. 1996).

Building Blocks' basic instructional approach is finding the mathematics in, and developing mathematics from, children's activity. Children are guided to extend and mathematise their everyday activities, from block building to art to songs to puzzles, through sequenced, explicit activities (whole group, small group, centers, including a computer center, and 'throughout the day'). Thus, off-computer and on-computer activities are designed based on children's experiences and interests, with an emphasis on supporting the development of mathematical activity at the next level of thinking within the learning trajectory. Although the complete *Building Blocks* is a preschool curriculum, the computer activities extend into kindergarten and the primary grades.

Results from our early summative evaluations (Clements and Sarama 2007) were satisfying, but also revealed that similar successes would be unlikely at a large scale without a complete scale-up program. Our scale-up model is called TRIAD, for Technology-enhanced, Research-based, Instruction, Assessment, and professional Development. The model's acronym suggests that successful scale-up must address the triad of essential components of any educational intervention and that the model is based on research and enhanced by the use of technology. However, TRIAD is a general model for scaling up varied educational interventions, based on successful efforts to take such interventions to scale. The following are the 10 research-based guidelines in the TRIAD model.

1. Involve, and promote communication among key groups around a shared vision of the innovation (Bobis et al. 2005; Hall and Hord 2001; see Sarama et al. 2008, for a complete review for all guidelines). Emphasise connections between the project's goals, educational standards (Perry et al. 2012b), and greater societal need. Promote clarity of these goals and of all participants' responsibilities. School and project staff must share goals and a vision of the intervention (Bryk et al. 2010). This is especially important for teachers from pre-K through the primary grades, as implicit and explicit (policy) barriers often separate age- and grade-level groups (Sarama and Clements 2013; Thomson et al. 2005). These efforts institutionalise the intervention, across grade levels and in the case of ongoing socialisation and training of new teachers (Elmore 1996; Huberman 1992; Kaser et al. 1999; Sarama et al. 1998).
2. Promote equity through equitable recruitment and selection of participants, allocation of resources, and use of curriculum and instructional strategies that have demonstrated success with underrepresented populations (Kaser et al. 1999).

Again, this should be equitable across age levels as well as supportive of the special needs of groups and individuals.

3. Plan for the long term. Recognising that scale up is not just an increase in number, but also of complexity, provide continuous, adaptive support over an extended period of time. Plan an incremental implementation and use dynamic, multilevel, feedback, and self-correction strategies (Bryk et al. 2010). Communicate clearly that change is not an event, but a process (Hall and Hord 2001), and involve teachers at grade $n+1$ (if not more) in understanding the challenges, work, and successes of teachers and students at grade n.
4. Focus on instructional change that promotes depth of children's thinking, placing learning trajectories at the core of the teacher/child/curriculum triad to ensure that curriculum, materials, instructional strategies, and assessments are aligned with (a) national and state standards and a vision of high-quality education, (b) each other, and (c) 'best practice' as determined by research, including formative assessment (Bodilly 1998; Bryk et al. 2010; Kaser et al. 1999; National Mathematics Advisory Panel 2008; Raudenbush 2008). This guideline is important for implementation with fidelity at any scale, although alignment is increasing important at larger scales and across grade levels. That is, learning trajectories can provide the connective tissue that helps teachers from pre-K to primary grades connect and communicate about mathematical goals, children's developmental levels, and instructional activities and strategies (Clements et al. 2013).
5. Provide professional development that is ongoing, intentional, reflective, goal-oriented, focused on content knowledge and children's thinking, grounded in particular curriculum materials, situated in the classroom and the school (Clarke 1994; Perry 2010; Sarama et al. 2008). A focus on content includes accurate and adequate subject-matter knowledge both for teachers and for children. A focus on children's thinking emphasises the learning trajectories' developmental progressions and their pedagogical application in formative assessment. Grounding in particular curriculum materials should include all three aspects of learning trajectories, especially their connections. This also provides a common language for teachers in working with each other and other groups (Bryk et al. 2010). Situated in the classroom does not imply that all training occurs within classrooms. However, off-site intensive training remains focused on and connected to classroom practice and is completed by classroom-based enactment with coaching. In addition, this professional development should encourage sharing, risk taking, and learning from and with peers. It should be based on a specific curriculum and develop teachers' knowledge and beliefs that the curriculum is appropriate and its goals are valued and attainable. Work should be situated in the classroom, with coaches who formatively evaluating teachers' fidelity of implementation and provide feedback and support in real time (Bodilly 1998; Bryk et al. 2010; Kaser et al. 1999). As with guideline #4, guideline #5 is important for implementation with fidelity at any scale. However, the planning, structures, common language, formative evaluation, and school-level context are increasingly important as the implementation moves to larger scales and especially across grade levels, where curricula frequently differ, and thus the connective tissue of learning trajectories is especially important.

6. Build expectations and camaraderie to support a consensus around adaptation. Establish and maintain cohort groups and build cross-age working groups. Facilitate teachers visiting successful implementation sites and each other's classrooms at their own grade level and those before and after. Build local leadership by involving principals and encouraging teachers to become teacher leaders.
7. Ensure school leaders are a central force supporting the innovation and provide teachers continuous feedback that children are learning what they are taught and that these learnings are valued. Leaders, especially principals, must show that the innovation is a high priority, through statements, resources, and continued commitment to permanency of the effort. An innovation champion leads the effort within each organisation (Bodilly 1998; Bryk et al. 2010; Hall and Hord 2001; Sarama et al. 1998).
8. Give latitude for adaptation to teachers and schools, but maintain integrity. Emphasise the similarities of the curriculum with sound practice and what teachers already are doing. Help teachers distinguish productive adaptations from lethal mutation (Brown and Campione 1996). Also, do not allow dilution due to uncoordinated innovations (Huberman 1992; Sarama et al. 1998).
9. Provide incentives for all participants, including intrinsic and extrinsic motivators linked to project work, such as external expectations—from standards to validation from administrators. Show how the innovation is advantageous to and compatible with teachers' experiences and needs (Berends et al. 2001; Borman et al. 2003; Elmore 1996; Rogers 2003).
10. Maintain frequent, repeated communication, assessment ('checking up'), and follow-through efforts at all levels within each school district, emphasising the purpose, expectations, and visions of the project, and involve key groups in continual improvement through cycles of data collection and problem solving (Hall and Hord 2001; Huberman 1992; Kaser et al. 1999). Throughout, connections between teachers following children through the grades and also with parents and community groups is especially important, to meet immediate and long-range (sustainability) goals (for more details, see Sarama and Clements 2013).

10.5 How the TRIAD Guidelines Were Implemented in Pre-Kindergarten

For the pre-K teachers, the first year was a 'gentle introduction' to TRIAD and *Building Blocks*, because our previous experience and others' research suggested that teachers often need at least a year of experience before completely and effectively implementing a curriculum (Berends at al. 2001; Clements and Sarama 2014). They participated in seven full days of professional development, including time to address the 'developmental appropriateness' of the intervention's mathematics education and its importance to the teachers and children, especially in promoting equity. This work focused on the learning trajectories for each mathematical topic, usually as woven into the *Building Blocks* curriculum. Training addressed each of the three components of the learning trajectories. To understand the goals, teachers learned core mathematics concepts and procedures for each topic. For example, they re-learned the geometry of early and primary education. To understand the developmental progressions of levels of thinking, teachers studied multiple video segments illustrating each level and discussed the mental 'actions on objects' that constitute the defining cognitive components of each level (Perry 2010). To understand the instructional tasks, teachers studied the tasks, and they viewed, analysed and discussed video of the enactments of these tasks in classrooms. A central tool to study and connect all three components was the Internet-based software application, Building Blocks Learning Trajectories (BBLT). BBLT provided scalable access to the learning trajectories via descriptions, videos, and commentaries. Two sequential aspects of the learning trajectories—the developmental progressions of children's thinking, and connected instruction—are linked to the others. The coaches joined the teachers in the participated in professional development, as well as several days of training on coaching, most of which focused on the unique aspects of coaching early mathematics education. Coaches worked with teachers during the year to provide continual feedback and support, avoiding dilution of the intervention, while promoting productive adaptations.

In Year 2, teachers and coaches participated in an additional four full days of professional development. They continued to study the learning trajectories, including discussions of how they conducted various curricular activities the previous year. As part of this work, teachers brought case studies of particular situations that occurred in their classrooms to the group to facilitate these discussions; thus, this work included elements of lesson study.

10.6 Results of Implementing the TRIAD Model: Pre-K

These general guidelines were implemented fully for the pre-K intervention (Sarama and Clements 2013; Clements et al. 2011). Findings from that year were positive. Briefly, 42 schools serving low-resource communities were randomly selected and randomly assigned to three treatment groups involving 1,375 preschoolers in 106 classrooms. Two of these groups implemented the TRIAD model in pre-K, the third group was a 'business-as-usual' control. TRIAD teachers taught the *Building Blocks* curriculum with adequate fidelity (Clements et al. 2011). Pre- to posttest scores revealed that the children in the *Building Blocks* group learned more mathematics than the children in the control group (effect size, $g=0.72$). African-American students in the treatment groups scoring significantly better than that of African-American students in the control group (although they scored lower than non-African Americans in all groups). We also checked if there were any deleterious effects on language and literacy scores with the commitment of more instructional time to mathematics. Results showed no evidence that children who were taught mathematics with *Building Blocks* performed differently than control children who received the typical district mathematics instruction on measures of letter recognition, and on two of the oral language (story retell) subtests, and sentence length. However, children in the *Building Blocks* group outperformed children in the control group on four oral language subtests: ability to recall key words, use of complex utterances, willingness to reproduce narratives independently, and inferential reasoning (Sarama et al. 2012b).

10.7 How the TRIAD Guidelines Were Implemented in Kindergarten and First Grade

The results for Kindergarten and 1st grade are directly relevant to the theme of this chapter. In these grades, the two groups randomly assigned to TRIAD in pre-K differed: Only one of them, TRIAD-Follow Through (TRIAD-FT) continued to implement the TRIAD intervention, the other, TRIAD-Non Follow Through (TRIAD-NFT), did not. Transitions were addressed in that the teachers in the schools assigned to TRIAD-FT were introduced to what their students had learned in pre-K and ways to build upon it. That is, they were shown the mathematics many of their entering students had learned from video recordings and through presentations of the pre-K teachers, who shared stories, pictures, and some videos of mathematics that the preschoolers had learned in the previous year. They were also taught about the learning trajectories to their grade level and beyond, including the developmental progressions and how to modify their extant curricula to more closely match the levels of thinking of their students. They also received access to the *Building Blocks software* (Clements and Sarama 2007/2012), which follows the learning trajectories through the primary grades and is the same suite that the students had used previously.

10.8 Results of Implementing the TRIAD Model: Kindergarten and First Grade

At the end of the students' kindergarten year (Sarama et al. 2012a), both TRIAD groups outperformed the control condition ($g = .46$ for the follow- through, $g = .30$ for the non-follow through). One moderator was statistically significant, with African-American students within the TRIAD-FT group scoring significantly better on kindergarten outcomes than African-American students in the TRIAD-NFT group.

At the end of first grade, students in the TRIAD-FT group scored significantly higher than control group, with a higher effect size ($g = .51$) than that of the TRIAD-NFT compared to control ($g = .28$). Furthermore, the TRIAD-FT scored group significantly higher than he TRIAD-NFT group ($g = .24$). Although African-American students continued to lag behind non-African-American students in all conditions, the TRIAD-FT intervention helped them narrow that achievement gap.

10.9 Implications

Before we return to the issue of fade out, we wish to be clear about our position on our follow-through intervention: We believe it was underpowered. That is, although it had a significant and important impact, the effect of the treatment appeared to decrease, and to close the gap between children in low-resource communities—such as those in our study—and those from higher-resource communities, we need interventions that increase the effect each successive year. The TRIAD-FT intervention differed from the TRIAD pre-K intervention in several ways that suggest weaknesses that could be ameliorated in future work. (a) TRIAD pre-K introduced a new, research-based curriculum; TRIAD-FT used the school's existing curriculum. (b) TRIAD pre-K teachers learned and practiced the intervention for a year before data collection; TRIAD-FT teachers did not. (c) TRIAD pre-K were allowed to implement all aspects of the intervention; some TRIAD-FT teachers reported that the 'fidelity police' of their schools insisted they follow their existing curriculum schedule, thus preventing them from condensing or compacting curricula (one of the intervention's strategies). Thus, we believe the evidence strongly supports the need for follow through, as we discuss in the remainder of the chapter, but also believe that the TRIAD-FT implementation was adequate, but not ideal, and that more efficacious TRIAD follow through interventions can and should be implemented and studied.

Nevertheless, even the less-than-ideal TRIAD-FT treatment was important in maintaining children's early gains. This finding has broad implications. We believe interpretations of studies reporting that preschool gains fade (ACF 2010; Fish 2003; Natriello et al. 1990; Preschool Curriculum Evaluation Research Consortium 2008; Turner and Ritter 2004) often mistakenly treat initial effects of interventions as independent of the students' future school contexts. That is, these interpretations reify the treatment effect as an entity that should persist unless it is 'weak' and thus susceptible to fading. Taking this perspective views the gain analogically as a static object carried by the student that, if not evanescent, would continue to lift the student's achievement about the norm, as if it were a platform on which to stand. Our theoretical position and our empirical results support an alternative view. Successful interventions do provide students with new concepts, skills, and dispositions that change the trajectory of the students' educational course. However, these are, by definition, exceptions to the normal course for children in their context (in our case, low-resource communities). Because the new trajectories are exceptions, multiple processes may erode their positive effects. Curricula designed for the typical student from that district or school assume low levels of mathematical knowledge and often focus on lower-level skills. Studies have substantiated that some kindergarten and first grade instruction cover material children already know even without extensive pre-K experience with mathematics (Engel et al. in press; van den Heuvel-Panhuizen 1996). A culture of low expectations for certain groups may support the use of such curricula. Teachers are often required to follow such curricula strictly and may have few means to recognise that students have already mastered or surpassed the content they are about to 'teach' them (Bennett et al. 1984; Clements and Sarama 2014; National Research Council 2009; Sarama and Clements 2009). Even if they do so recognise students' competencies, pressure to increase the number of students passing minimal competency assessments may lead teachers to work mainly with (and/or mainly at the level of) the lowest performing students. Within this context and without continual, progressive support, children's nascent learning trajectories revert to their original, limited course. These arguments and the empirical support proffered by this study suggest that we must further investigate the implied concern, that multiple characteristics of the present U.S. educational system are aligned to unintentionally but perniciously dismantle the benefits of successful early childhood interventions.

An implication is that, students' trajectories must be studied as the students experience different educational courses. Treatment effects are relative, both in contrasting experimental and control groups and, longitudinally, to the nature of educational experiences the students in these groups subsequently receive. There is a cumulative positive effect of students experiencing consecutive years of high-quality teaching, and a cumulative negative effect of low-quality teaching (Sanders and Horn 1998; Wright et al. 1997).

Interpretations of fade out may call for decreased funding and attention to pre-K (Fish 2003), but our position is that a lack of support for transitions to primary school and specific follow-through interventions is responsible. Our position is consistent with that of (a) the authors of the meta-analyses on fadeout, who conclude that because it takes a long time (about 10 years) for impacts to disappear, there is more than enough time for possible follow-through interventions that capitalise on the gains from these programs (Leak et al. 2012) and intervention researchers' notion of environmental maintenance of development (Ramey and Ramey 1998).

In the evaluation of the same students in this study as well as previous studies, the TRIAD implementation was particularly successful for students who identified themselves as African-American. Although African American students continued to lag behind non-African American students in all conditions, the TRIAD-FT intervention helped them narrow that achievement gap. A high quality, consistent mathematics education can make a demonstrative and consistent positive impact on the educational attainment of African American students in the pre-K, kindergarten, and 1st grade years compared to traditional instruction. We interpret these findings as supporting our theoretical interpretation of students' educational courses. We did not hypothesise this interaction, so we proffer explanations that are by necessity post hoc. (a) Centering instruction around learning trajectories may focus teachers' attention on students' thinking and learning of mathematics, and what children can learn to do, avoiding biases, such as views of African-American students' learning from a deficit perspective, that impair teaching and learning (ACF 2010). That is, especially given the significant mediation of the classroom culture, including enthusiastic interaction with children around mathematics they believe children can learn, it may be that the TRIAD interventions changed teachers' views of African-American students' mathematical capabilities (Jackson 2011). The curriculum's learning trajectories are based on the notion that learning is developmental and amenable to instruction, and the curriculum's approach, including specific, sequenced activities and formative assessment strategies, may have offered a way to act on these nascent views. In such action, the productive views are further strengthened. (b) The TRIAD intervention may promote a conceptual and problem-solving approach infrequently emphasised in schools serving low-income children, explicitly supporting African-American students' participation in increasingly sophisticated forms of mathematical communication and argumentation. (c) The TRIAD follow-through intervention may raise several aspects of the quality of mathematics education, lack of which has been suggested as a reason preschool benefits dissipate for African-American children; for example, the language-rich nature of the curriculum and its expectation that all children invent solution strategies and explain them. These and other possible reasons should be evaluated, compared, and combined, especially in interventions targeted to the primary grades.

The TRIAD follow-through intervention's effect was partially due to the increase in the positive classroom cultures teachers develop. Interventions such as TRIAD may help engender a greater focus on mathematics, which in turn can help increase students' mathematics achievement. As other work has shown (Clements et al. 2011; Jacobs et al. 2001; National Research Council 2009), helping primary teachers' gain additional knowledge of mathematics, students' thinking and learning about mathematics, and how instructional tasks can be designed and modified—that is, the three components of learning trajectories—has a measurable, positive effect on their students' achievement. This is particularly important in the early years because teachers often do not recognise when tasks are too difficult, but even when they do, they provide 'more of the same' (Bennett et al 1984). Further, they overlook tasks that provide no challenge to children—that do not demand enough (Bennett et al. 1984; van den Heuvel-Panhuizen 1996). Thus, most children, especially those who have some number knowledge, may learn little or no math in kindergarten (Wright 1991).

Implementing interventions such as TRIAD is therefore important, given that early mastery of concepts and skills in mathematics and literacy is the best predictor of students' successful academic careers (Aunola et al. 2004; Duncan et al. 2004; Duncan and Magnuson 2011). Further, students from low-income communities benefit more relative to students from higher resource communities from the same 'dose' of school instruction (Raudenbush 2009). Thus, comprehensive implementations of research-based models, such as the TRIAD follow-through model, may be especially effective in such lower-resource schools. This speaks to a caveat concerning the effectiveness of the TRIAD follow-through intervention. The intervention maintained, but did not add to, the gains of the more comprehensive TRIAD pre-K intervention. Differences in scores remain statistically significant, but effects were not cumulative. Future design studies might investigate ways to (a) avoid or ameliorate the limiting influence of pacing guides and other school district policies that may have limited the effect of the TRIAD follow-through component, (b) increase the intensity or duration of that component, or (c) implement different and more extensive interventions, such as curriculum replacement (as the TRIAD intervention did in pre-K). That is, future research should evaluate the efficacy and scalability of a fully implemented TRIAD model in the primary grades to see if the pre-K slope can be maintained throughout elementary school. Such an intervention may go beyond "resisting fade out" to show that a positive rate of learning can and should be sustained. In other words, we argue that what should persist is not just a pre-K gain, but also a dramatic trajectory of successful learning.

10.10 Final Words

The best predictor of a successful academic career is early mastery of literacy and mathematical concepts and skills. Students from low-resource communities benefit more relative to students from higher resource communities from the same 'dose' of school instruction (Raudenbush 2009). Thus, comprehensive implementations of research-based models, such as the TRIAD follow-through model, may be especially effective in low-resource schools such as those in this study. Future research should develop and evaluate more effective follow-through interventions that use learning trajectories to support continuity of learning into the primary grades but instantiate the learning trajectories' instructional tasks more explicitly.

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