Early Mathematics Learning and Development

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Amy MacDonald
Ann Gervasoni *Editors*

Mathematics and Transition to School

International Perspectives

Springer

Early Mathematics Learning and Development

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Preface

With pride and appreciation I welcome the first book of the Springer series, *Early Mathematics Learning and Development*. The editors, Bob Perry, Amy MacDonald, and Ann Gervasoni, and all the contributing authors have done a sterling job of producing the significant and timely book, *Mathematics and Transition to School: International Perspectives*.

Early childhood development and mathematics learning are active, substantive fields of research. Insufficient attention, however, has been devoted to early childhood mathematics learning with increased calls internationally for reform in this area. The *Early Mathematics Learning and Development* series provides a platform for international educators and researchers to bring together various perspectives on what needs reforming and how such reforms might best be implemented.

As the editors indicate, this first book presents an inaugural, international collection of studies drawing on two important components of young children's lives—mathematics learning and transitioning to primary or elementary school education. Numerous debates exist on how we might define such transitioning and what it entails. In their introductory chapter, the editors provide a theoretical framework that conceptualises transition as "opportunities, aspirations, expectations, and entitlements" for all who are involved including children and their families, communities, educators, and educational bodies. Across the three main sections (The Mathematics Young Children Bring to the First Year of School, Continuity of Mathematics Curriculum and/or Pedagogy as Children Begin School, and Informal and Formal Mathematics and the Transition to School), the authors explore the various roles of mathematics in these transitional components.

Ways in which we can capitalise on the mathematics children know and can apply on starting school are illustrated in chapters addressing opportunities in the home that help lay the foundations for subsequent school learning (e.g., Skwarchuk and LeFevre, Chap. 7). Other examples are presented in chapters that report on mathematics intervention programs for early childhood educators (e.g., MacDonald, Chapter 6; Gervasoni and Perry, Chap. 4), as well as in chapters that examine ways in which educators might identify the nature of young children's mathematical strengths (e.g., Clarke, Chap. 3; Wager, Graue, and Harrigan, Chap. 2; Carruthers, Chap. 19; Cheeseman, Chap. 17).

Exploring the aspirations component of transition are chapters that consider ways in which partnerships between educators and families might be fostered (e.g., Goff and Dockett, Chap. 11), together with chapters examining the expectations of education systems designed to improve achievement through system-wide initiatives (e.g., Lee and Lomas, Chap. 13). Adult aspirations for improved early childhood education include those by Papic and her colleagues (Chap. 14), whose professional learning program for educators of young Australian Indigenous children has made substantial inroads into enhancing their early mathematics learning. Equally important are the inspirational factors of young children themselves, such as what they hope for and expect to learn on entering school. Examples of such aspirations appear throughout the book but especially in Chap. 14 (Papic et al.,) and Chap. 18 (Dunphy).

Various chapters consider the impact of expectations on behaviour and achievement, including those that consider cultural expectations of learning (e.g., Ng and Sun, Chap. 15), as well as chapters that look at the expectations imposed by new curriculum on both children and educators (e.g., Lee and Lomas, Chap. 13). As the editors note in their introductory chapter, a significant observation identified in several chapters is that young children already "know" the mathematics that they are being "taught" on entering school. Sarama and Clements (Chap. 10) highlight this apparent mismatch, supported by Gervasoni and Perry (Chap. 4), who stress that educators must expect and recognise the mathematical strengths young learners bring to school.

Another important aspect of expectations relates to the nature of student assessment administered on beginning school. Several chapters demonstrate the benefits of assessment that extends beyond paper-and-pencil testing to include, for example, one-on-one interactions with young children as they relate their mathematical understandings (e.g., Peter-Koop and Kollhoff, Chap. 5). Families have further expectations of their children's learning as they transition to school, including the recognition and nurturing of their children's strengths and the expectation that their family will play a role in their children's learning. At the same time, families sometimes anticipate being labelled as inadequate in their children's education because of various background factors. Targeting such issues through establishing partnerships between families and educators is an increasingly important endeavour, requiring further research. Geoff and Dockett (Chap. 11) explore some of these issues.

Lastly, but equally important in young learners transition to school are entitlements. Every young child deserves a quality program in early mathematics education, one which encourages them to thrive in a rich, non-threatening environment. Expert educators with future-oriented curricula recognise and build on the mathematical capabilities of young children, strengthen less developed talents, and capitalise on children's natural propensity for exploring problems in their world. Many chapters address the importance of quality programs and teacher expertise as well as other entitlements needed for advancing the mathematics learning of all children. For example, Carruthers (Chap. 19) and Cheeseman (Chap. 17) stress the importance of time to listen to and talk with children, with other authors also emphasising time needed to assess young children's learning meaningfully to identify appropriate starting points for teaching (e.g., Peter-Koop and Kollhoff, Chap. 5; Wager et al., Chap. 2).

Collectively, this rich set of chapters presents a broad range of international perspectives and themes that draw together the two core fields of transition to school and early mathematics learning. Addressing timely and significant issues, *Mathematics and Transition to School: International Perspectives* provides powerful foundations for future research, curriculum development, professional learning, and policy decisions. It is indeed a fine book to initiate the new series.

Lyn English

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Chapter 1
Mathematics and Transition to School: Theoretical Frameworks and Practical Implications

Bob Perry, Amy MacDonald and Ann Gervasoni

Abstract This edited book brings together for the first time an international collection of work built around two important components of any young child's life—learning mathematics and starting (primary or elementary) school. The chapters take a variety of perspectives, and integrate these two components in sometimes explicit and sometimes more subtle ways. This chapter provides a theoretical framework for transition to school and investigates possible places for mathematics in that transition. It stresses the importance of considering the strengths of all involved in the transition to school and how these strengths can be used to assist children learn increasingly sophisticated mathematics. The chapter concludes with an analysis of each of the book chapters in terms of their links into the theoretical framework for transition to school and young children's mathematics learning.

1.1 Introduction

The significance of the early childhood years has received increasing international recognition over the last 20 years (Organisation for Economic Cooperation and Development (OECD) 2001, 2006; Woodhead and Moss 2007). Recognition has grown through the impact of research into brain functioning and the importance of the early years in brain development (National Scientific Council on the Developing Child 2007; Shonkoff and Phillips 2000), perceived links between young children's literacy and mathematics development with later success in schooling (Bowman et al. 2000; Claessens and Engel 2013: Clements 2013; Linder et al. 2013) and

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economic arguments about the positive return in later years on investment in quality early childhood education (Heckman and Masterov 2004). Within this recognition, transition to primary or elementary school has taken a key position. While much of this discussion has centred on the disputed notion of school readiness (Dockett and Perry 2009; Kagan 2007) and the belief that if a child knows more at the beginning of school, then she/he will be more successful at school (Sabol and Pianta 2012), critical transition scholars have begun to offer alternative approaches to considering transition to school (Dockett 2014; Dockett and Perry 2013; Graue and Reineke 2014; Perry 2014; Peters 2010; Petriwskyj 2014; Petriwskyj and Greishaber 2011).

'Transition to school' is a contested term but it often incorporates children's readiness for school and their abilities to adjust to the new context of school. Other definitions focus on "transition as a set of processes as individuals move from one … context to another or change their role in educational communities" (Dockett et al. 2014). Such definitions consider transitions as both individual and social experiences within educational organisations, families and communities.

Other conceptualisations of transition to school involve notions of 'border crossing' (Giroux 2005; Peters 2014) and 'rites of passge' (Garpelin 2014; van Gennep 1960) in which the values of both continuity (across the borders of prior-to-school settings and schools) and change (moving from one stage of life to another, with no likelihood or desire for return) are emphasised.

There are many debates about what constitutes 'transition to school'. Major components include the adjustment of children, families and educators (Griebel and Niesel 2009; Margetts 2009, 2014; Ritchie et al. 2010); readiness of children, families, schools and communities (Centre for Community Child Health 2008; Dockett et al. 2010); continuity and change (Brooker 2008; Fabian and Dunlop 2002); time period over which transition processes are appropriate (Graue 2006; Peters 2010); practices that promote effective transitions (Dockett and Perry 2013; Fabian and Dunlop 2007); and agency of all involved (Brooker 2008; Dunlop 2014; Ecclestone 2010). Dockett et al. (2014, p. 3) provide the following summation:

> While there is no universally accepted definition of transition, there is acceptance that transition is a multifaceted phenomenon (Petriwskyj et al. 2005) involving a range of interactions and processes over time, experienced in different ways by different people in different contexts. In very general terms, the outcome of a positive transition is a sense of belonging in the new setting.

The major aim for this book has been to investigate how mathematics learning and teaching best fits within this understanding of transition to school.

1.2 Transition to School Position Statement

In 2010, a group of 14 of the leading transition to school researchers and six higher degree candidates from eight countries came together to share their theoretical and practical approaches and to develop a common position on what they believed was important about transition to school. The results of that meeting have been explored in an international research text on transition to school (Perry et al. 2014) and the

Transition to School Position Statement (Educational Transitions and Change Research Group (ETC) 2011). The process of development of the *Transition to School Position Statement* is described by Dockett and Perry (2014).

> The position statement reconceptualises transition to school in the context of social justice, human rights (including children's rights), educational reform and ethical agendas, and the established impact of transition to school on children's ongoing wellbeing, learning and development…
>
> Transition to school is taken to be a dynamic process of continuity and change as children move into the first year of school. The process of transition occurs over time, beginning well before children start school and extending to the point where children and families feel a sense of belonging at school and when educators recognise this sense of belonging.
>
> *Transition to school is characterised by:*
>
> - *opportunities*
> - *aspirations*
> - *expectations*
> - *entitlements.* (ETC 2011, p. 2)

The *Transition to School Position Statement* argues that each of the participants in the transition process—children, families, communities, educators, and educational organisations—can supply opportunities, aspirations, expectations and entitlements and can benefit from or be hindered by these. For example, children can benefit most from the opportunities provided by transition activities when they are assumed to be competent and capable. If families are expected not to be interested in their children's learning, it is unlikely that educators will engage the families in that learning. Families hope that their children will 'do well' at school and often aspire to 'higher outcomes' for their children than they were able to achieve. Educators want all of their children to do well, to continue to learn, and thrive. Systems aspire to show that their children are growing in their learning and that everyone is able to reach their potential. Teachers are entitled to respect from all involved in the transition to school, including their employers, communities and the families of the children they teach. Equally, parents are entitled to respect as the children's first teachers and people who know the children well.

Within this conceptualisation of transition as opportunities, aspirations, expectations, and entitlements for all involved, mathematics education plays a major role. This book is an opportunity for the chapter authors to explore the contribution of their research and scholarship in this field.

1.3 Transition to School and Mathematics Learning and Teaching

1.3.1 Opportunities

Young children are powerful mathematicians (Perry and Dockett 2008). They know a great deal of mathematics as they start school (Clarke et al. 2006; Clements and

Sarama 2014; MacDonald and Lowrie 2011). Sometimes, they know much of what it is scheduled for them to 'learn' in the first year of school (Gervasoni and Perry 2013; Gould 2012). There are wonderful opportunities available to the children, their teachers and their families to build on what children know in mathematics, and how they know it, as they start school. This strength-based approach is more likely to benefit everyone involved, especially the child, than an approach which is driven by rigid curriculum or textbook expectations. Many of the chapters in this book consider this matter but those that have been placed in the section of the book entitled *The Mathematics Young Children Bring to the First Year of School* do so explicitly. Skwarchuk and LeFevre (Chap. 7) and Gasteiger (Chap. 16) illustrate the important opportunities provided by home experiences and the way in which these help establish a foundation for later mathematics or numeracy learning. In other examples, MacDonald (Chap. 6) and Gervasoni and Perry (Chap. 4) draw their evidence from a current mathematics intervention program that sees early childhood educators working with families to help develop children's mathematics ideas from play and other familiar activities. Both Ng and Sun (Chap. 15) and Hemmi and Ryve (Chap. 12) consider the mathematical knowledge that children bring with them to school through cultural lenses, and consider the opportunities that different cultural contexts might provide for young children and their learning of mathematics.

In Chap. 8, Kristinsdóttir and Guðjónsdóttir consider the opportunities afforded for children's mathematics learning as they start school, provided the early childhood educators around them notice the children's strengths and build upon these. Linked to this notion are the chapters of Clarke (Chap. 3), Wager, Graue, and Harrigan (Chap. 2), Carruthers (Chap. 19) and Cheeseman (Chap. 17) that consider ways in which educators might idenitfy the nature of children's mathematical strengths. All three of these chapters debate the relative value of different assessment regimes for young children. For reasons linked to children's development and the pedagogies they are likely to have experienced in prior-to-school settings, including home, the authors regard one-to-one interviews, conversations or playful encounters as more appropriate than many of the more formal assessments that the children might encounter when they move into school.

Clearly, there are many opportunities afforded for enhanced mathematical experiences as children start school. Mathematical experiences in the home, in prior-to-school settings and in schools also afford opportunities for educators and family members to come together, explore and discuss children's learning. By building on the strengths of all involved in the child's transition to school, opportunities abound to enhance the experience for all.

1.3.2 Aspirations

The *Transition to School Position Statement* (ETC 2011) uses the idea of the aspirations of all involved in the transition to school endeavour to highlight that everyone hopes for the very best for children as they start school. However, there is more to the overall endeavour than this. For example, "Educators aspire to the development of strong partnerships with families, other educators, professionals and communities as part of strong and supportive educational environments" (ETC 2011, p. 3). The aspirations to build partnerships between educators and families is advocated strongly by Goff and Dockett (Chap. 11) who see this as a social justice challenge for many families as their children start school. Parents want to be part of their children's mathematics education, and Skwarchuk and LeFevre (Chap. 7) provide two extreme examples of such enthusiasm to participate.

Educators also aspire to meaningful partnerships with their colleagues. Often these are built around particular challenges, such as those related to new curricula or external pressures arising from externally imposed assessment regimes. In Chap. 9, Aubrey and Durmaz investigate the aspirations of an education system looking to improve standards through system-wide initiatives. This theme is also taken up by Lee and Lomas (Chap. 13) through their analysis of the impact of a 'child-centred' curriculum in New Zealand implemented with the aspiration of children reaching their potential in mathematics learning. Through a comparison of the two systems, Hemmi and Ryve (Chap. 12), consider the aspirations of both Finnish and Swedish education and the ways in which these aspirations have impacted on policy and practice. In another cultural turn, Ng and Sun (Chap. 15) consider the aspirations for academic success that emanate from a Confucian-based society.

There are many 'interventions' in early childhood education that are motivated by the aspirations of adults—mainly educators and policy makers—for children to 'do better' in their mathematics learning. Papic and her colleagues (Chap. 14) have worked extensively on the development of a patterns and algebra professional learning program for educators working with Australian Indigenous children (Papic 2013; Papic et al. 2011). Both aspirations and expectations of Indigenous children have traditionally been seen to be quite low (Howard et al. 2011; Howard and Perry 2011) and the program has made some inroads into this deficit view. Similarly, the *Let's Count* program in Australia has been implemented in low socioeconomic sites and is built upon aspirations for improving both the mathematical learning of preschool children and the skills and knowledge of preschool educators. MacDonald (Chap. 6) and Gervasoni and Perry (Chap. 4) report on the successes of *Let's Count.*

Sarama and Clements (Chap. 10) provide an interesting counterpoint to early intervention in the mathematics education of young children. This team has been involved in many interventions in both preschools and the early years of school and have reported widely on the success of these (Clements and Sarama 2007; Sarama and Clements (2013)). In Chap. 10, they question the longer-term efficacy of such interventions and "hypothesise that most present educational contexts are unintentionally and perversely aligned against early interventions" (this volume, p. 155). While the representatives of education systems are unlikely to agree with this summation, their aspirations for successful mathematics learners may be, unwittingly, damaged through their own actions.

Adults –both educators and parents—and systems all have high aspirations for the mathematics learning of 'their' children and there are many examples of these in this book. Children also have aspirations or hopes for their own learning as they start school (Dockett and Perry 2007). They hope that they will learn all sorts of things as they start school—especially reading, writing and mathematics. In this book, the aspirations of children are considered in many chapters but, particularly, in Papic et al. (Chap. 14) and Dunphy (Chap. 18). In both of these chapters, it is clear that the children are looking towards enhanced mathematics learning experiences as they get older and move into school.

1.3.3 Expectations

Ever since the work of Rosenthal and Jacobson (1968), it has been demonstrated that expectations can impact on behavior and achievement. The impact of expectations on young children's mathematics learning is an ongoing theme throughout this book.

The impact of curriculum expectations is considered in a number of chapters. Cultural expectations can be imposed on curriculum in some interesting ways as demonstrated by Ng and Sun (Chap. 15) and, comparatively, by Hemmi and Ryve (Chap. 12). The impact of expectations on children and educators that arise from the implementation of new curricula are canvassed by Aubrey and Durmaz (Chap. 9) and Lee and Lomas (Chap. 13).

An important observation made in a number of chapters is that young children starting school may encounter a great deal of mathematics that they already know but which the teacher is determined to 'teach' them. For example, Carruthers (Chap. 19, p. 326) suggests that "There is a mismatch between the planned mathematical curriculum in reception classes in comparison with children's mathematical ability" while Sarama and Clements (Chap. 10, p. 153) argue that

> Primary curricula assume little mathematical competence, so only low-level skills are taught. Most teachers are required to follow such curricula rigidly and remain unaware that some of their students have already mastered the material they are about to "teach."

Data from Gervasoni and Perry (Chap. 4) corroborate these statements as do all the other chapters which argue for educators to expect that children will start school knowing some mathematics and to recognise children's strengths in both knowing and learning.

Another take on the expectations that children encounter as they start school encompasses changes in assessment of their learning. Many chapters in this book stress the importance of educators listening to and working on a one-on-one basis with young children in order to assess or enhance the mathematics learning of these children. For example, Carruthers (Chap. 19), Gasteiger (Chap. 16), Cheeseman (Chap. 17) and Kristinsdóttir and Guðjónsdóttir (Chap. 8) all consider the expectation that educators, including parents, will interact meaningfully with individual children as they learn mathematics. Clarke (Chap. 3) stresses the importance of using a one-on-one assessment interview with young children, but also highlights the difficulties arising from an expectation that all children will interact with the tool successfully. Peter-Koop and Kollhoff (Chap. 5) use such one-on-one interviews to ascertain the mathematical knowledge of over 400 young children in order to help the researchers predict later difficulties in mathematical development. Wager et al. (Chap. 2) argue that the expectation that children's mathematical knowledge can be measured through the formal assessment methods often used in schools, including standardised testing, is inappropriate. "We wonder if broadening our assessment resources might broaden our knowledge as well" (p. 29).

Families also have expectations as their children make the transition to school. They expect that they will be respected and that their knowledge of their children will be listened to and acted upon. They expect that they will play a part in their children's continuing education, even though they may not be sure about what that part might be. Families expect that their children's strengths, including those in mathematics, will be recognised and developed. Sometimes, unfortunately, they expect that they will be labeled as 'poor' parents because of their cultural or socioeconomic backgrounds and circumstances (ETC 2011). Goff and Dockett (Chap. 11) explore the development of partnerships between educators and families so that expectations can be shared. Other aspects of parental or family expectations about children's mathematics learning are considered by Skwarchuk and LeFevre (Chap. 7) and Gasteiger (Chap. 16).

1.3.4 Entitlements

The United Nations Convention on the Rights of the Child (United Nations 1989), to which most countries in the world are signatories, states, in Article 27, that signatories to the Convention should "make primary education compulsory and available free to all" and, in Article 3, that "the best interests of the child shall be a primary consideration". Taken together, these statements proclaim that young children are entitled to a quality mathematics education as they make the transition to school. The stimulus for this book has been a strong belief in this entitlement for young children.

Education systems work in many ways towards the provision of quality mathematics education experiences for young children. One way is through the provision of quality curricula and expertise within the teaching profession. Many chapters in the book consider the importance of mathematics curricula and teachers during the transition to school. As well, they consider other entitlements for teachers that enable them to undertake their jobs effectively. In particular, the chapters of Aubrey and Durmaz (Chap. 9), Lee and Lomas (Chap. 13) and Hemmi and Ryve (Chap. 12) investigate various issues concerning mathematics curricula. Dunphy (Chap. 18) and Papic and her colleagues (Chap. 14) consider particular aspects of mathematics that they argue should be in the mathematical programs of all young children and, therefore, in the relevant curricula. The enhancement of expertise within the teaching profession is also dealt with in many chapters of the book including those of Kristinsdóttir and Guðjónsdóttir (Chap. 8), Sarama and Clements (Chap. 10) and Cheeseman (Chap. 17).

Educators also have entitlements in terms of their teaching of young children as they start school. Not only are they entitled to the resources to do their job effectively but they are also entitled to the respect of children, families and systems as they undertake their professional role. One of the resources educators need is time, particularly time to get to know the children as they start school. They need time to listen to the children (Carruthers Chap. 19), to talk with the children (Cheeseman Chap. 17) and to assess the children's mathematics learning in ways that are meaningful to all concerned, so that they know from where to start their teaching (Sarama and Clements Chap. 10; Peter-Koop and Kollhoff Chap. 5; Clarke Chap. 3; Wager et al. Chap. 2).

Families have entitlements as their children start school. They are entitled to know that their children are safe—physically, culturally, socially and academically—at school.

> Families are entitled to be confident that their children will have access to education that promotes equity and excellence and that attends to the wellbeing of all children. Families have a right to be respected as partners in their children's education. (ETC 2011)

In all of the chapters in this book, issues of equity and excellence in young children's mathematics education are explored. Via many routes, the chapter authors have sought to reach the one goal of providing all children with the resources and support to thrive mathematically and to claim their right to an excellent mathematics education.

1.4 Using This Book

This has been a very enjoyable book to develop. Not only have we, as editors, been able to bring together some of the leading early childhood mathematics educators in the world but we have also been able to ask them to consider the particular period of time when children start school and to consider the implications of their work for enhancing the mathematics learning and teaching of these children. Thus, we have been able to bring together two key driving forces for our past and current work: transition to school and early childhood mathematics education.

From the perspectives of the editors, the key themes in this book are:

- recognition of the mathematical, and other, strengths that all participants in the transition to school bring to this period of a child's life
- recognition of the opportunities provided by transition to school for young children's mathematics learning
- The importance of partnerships among adults, and among adults and children, for effective school transitions and mathematics learning and teaching
- the critical impact of the expectations of all involved as children start school on children's mathematics learning, and the importance of providing meaningful, challenging and relevant mathematical experiences throughout the transition to school

- the clear entitlement of children and educators to have assessment and instructional pedagogies match the strengths of the learners and the teachers
- the importance for the aspirations of children, families, communities, educators and educational organisations to be recognised as legitimate and key determinants of actions, experiences and successes in both transition to school and mathematics learning
- the overriding belief that young children are powerful mathematics learners and that they can demonstrate this power as they start school.

We hope that these themes make the book appealing to a wide audience of researchers, early childhood educators, policy makers, doctoral candidates and preservice teachers. While the book is not necessarily designed for parents, we hope those who read it are inspired about the role that they can play in their children's mathematics learning during transition to school and beyond.

Given the innovative nature of this book in bringing together the two areas of mathematics learning and transition to school, the story has only just begun. Hopefully, many readers will be inspired to conduct further research and undertake further dissemination of their ideas in what we believe is an important arena for further research and scholarship. We eagerly anticipate such ongoing research and hope that this book provides stimulus to endeavours designed to assist young children as they start school.

We suggest to any reader that they read the entire book, though not necessarily in the order in which it is presented. Chapters have been grouped together into three sections but almost all of the chapters could have been placed in at least one other section. This is a book to be dipped into, thought about, and enacted. We hope you enjoy it, find it challenging, and, most of all, use it to build on the considerable strengths young children possess while they continue to learn mathematics as they start school.

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Part I
The Mathematics Young Children Bring to the First Year of School

Chapter 2
Swimming Upstream in a Torrent of Assessment

Anita A. Wager, M. Elizabeth Graue and Kelly Harrigan

Abstract Growing attention to preK mathematics and increased focus on standards in the US may be leading policy makers, administrators, and practitioners down the wrong path when it comes to assessing young children. The temptation to rely on standardised assessment practices may result in misguided understandings about what children actually know about mathematics. As part of a larger study of professional development with teachers focused on culturally and developmentally responsive practices in preK mathematics, we have found that our understanding of children's mathematical knowledge varies greatly depending on the form (what), context (where), assessor (who), and purpose (why) of assessment. Drawing on findings from three cases, we suggest that in the transition to school, shifting to more a formalised 'school-type' assessment is fraught with obstacles that vary greatly by child.

2.1 Introduction

This chapter would have been a story situated in a particular time and place—of the challenges involved in helping teachers learn new things about their preK students' mathematical experiences. However, the story we'll tell here is slightly more complicated. In the process of urging teachers to traverse boundaries—between classroom and home, preschool and elementary school, parent and teacher, formal and informal—we realised that our story of mathematics and transitions is essentially a story about assessment. It is about understanding children and our capacity to take up what they know in ways that are culturally responsive and mathematically rich. It is about using that knowledge to help students as they transition from one

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institutional context to another. It is about finding out more than what they show us in static classroom contexts. And, it is about swimming upstream.

> Determining what counts, and what does not, as evidence of young children's development has become an increasingly complex issue for early childhood educators. A broad range of stakeholders in the education of young children, from parents and teachers to administrators and policy-makers, have their own views about how children develop and how learning should be supported and assessed. (Casbergue 2011, p. 13)

We begin by describing the context for our work, a time of transition for early childhood programming and for the use of assessment. We recognise that this is a U.S. centric story but we are confident that many of the threads salient in the telling are relevant in international contexts as well (Black and Wiliam 2001; Perry et al. 2014).

Programming and curriculum in early childhood contexts have changed dramatically in the last 50 years, with higher proportions of children attending preK programs and multiple pressures on teachers and administrators to escalate learning. The traditional kindergarten curriculum has migrated to preschool and kindergarten has become a colorful version of first grade, focusing on literacy and mathematics (Graue 2009). This escalation has sped up in the last decade with standards-based curriculum and assessment systems benchmarking what had previously been soft developmental expectations due to the sanctions schools would suffer if their students missed proficiency. Play-based pedagogy, a critical attribute of traditional teaching in the U.S., is fading, seen as a waste of time in schools that are measured in terms of the gains their students make on academic tests (Miller and Almon 2009).

The tenor of escalation has taken on a new urgency in the current U.S. policy context. In an effort to affect education reform, the federal government recently offered funding to states to join an unprecedented movement to: a) establish a coherent set of expectations for students across the nation, and b) build data systems that would allow local, state and federal governments to follow student progress across time. These two actions were unusual because the U.S. constitution frames education as a local responsibility, and until recently national standards were antithetical to the local control aspects of education (Bagnato et al. 2011). Data systems were also new to the early childhood community as services for young children were scattered across multiple agencies and assessing children younger than five was seen as fraught with problems. These two trends, national standards and development of data systems, have changed how early educators conceptualise their practice; their work dictated by later achievement goals rather than the needs of the child. The developmental approach that framed early childhood as a process has given way to a more assessment-driven, intervention-mediated, and content-oriented curriculum (Sophian 2004).

At the same time, mathematics' role in early learning has received significant attention in the research community, with recognition that children are capable of learning 'everyday mathematics' that includes abstract and concrete concepts (Ginsburg et al. 2008). This learning potential is often minimised by limited opportunities to learn mathematical content, particularly compared to home and school practices that support literacy. Most early childhood curricula have thin threads of mathematics and teachers often have little support to transform these into rich experience for children (National Research Council 2009). As a result, mathematics is often a secondary and less intentional theme in teaching.

It is within this national (and global) context that we tell our story of how local preK teachers are swimming against this rush of assessment to teach in developmentally and culturally responsive ways.

2.2 Background

In 2011, a medium-sized school district in the Midwestern United States implemented a public preK program for 4-year-olds (4K) following a national shift toward a preK-12 system. The teachers in this new program included early childhood educators with years of experience teaching preschool and veteran elementary teachers interested in play-based pedagogy. Based on this wide range of expertise and research suggesting that mathematics is a greater predictor of future academic success than early literacy skills (Duncan et al. 2007; Romano et al. 2010), we partnered with the district and designed a professional development program (PD) to provide culturally and developmentally responsive teaching and learning in counting and number.

Working to develop professional learning communities of preK teachers, we created courses that integrated best practices in early education, funds of knowledge, and early number. Teachers met weekly to discuss readings from these three domains and engaged in a series of reflective activities. One of these activities was a child study project that required each teacher to learn about a child in multiple contexts, including the home, over the course of a school year. The goal of this exercise was to support teachers to identify and understand the multiple mathematical resources children access from their families and homes. Teachers conducted home visits, interviewed families, developed instructional plans based on home practices, and regularly observed the focal child to identify mathematical activities as they emerged in play.

Public preK programs are relatively new transitional spaces in early education, a bridge between the private realm of home and child-care and the public realm of official school. Our goal was to support teachers in making preK a gentle launching pad for children to enter the world of school mathematics. To smooth this transition, we worked with teachers to link mathematics content with children's home experiences using play as the primary site for learning. This required teachers to create a play-based environment, make connections to home resources, and mathematise children's everyday activities. These elements formed the foundation for culturally and developmentally responsive early mathematics when teachers used them simultaneously to build on children's experience.

2.3 Teachers and Researchers Transitioning to New Reasons for Assessing

One of the transitions we experienced in planning and implementing the PD was a shift in the purpose of assessments. Our conceptualisation of assessment was to communicate with families and plan for instruction, yet new mandates from the local district shifted the purpose to performance reporting. We were asking teachers to take a holistic approach to assessments that reached across the boundaries between home and school by learning about children's funds of knowledge and mathematics engagement in play. In contrast, the district was requiring teachers to use the standardised assessment sold for use with the commercial curriculum product, Creative Curriculum Gold, and to complete quarterly progress reports that were used to report to families and to identify children for supplementary education programs. As a result, the teachers felt pressure from the district to provide detailed assessment data using these unaligned tools, and our focus on culturally and developmentally responsive mathematics practices seemed at odds with the district's multi-layered and multi-purposed assessment requirements. The teachers felt as if they were swimming upstream. They were trying to reach the goal of responsive practice but had so many assessments to do they were often diverted.

To complicate matters, our grant advisory board asked us to add an assessment component to document children's learning of counting and number and provide evidence of the PD's efficacy. In response we did number interviews with six children in each teacher's classroom in the fall, and then conducted the same interviews in the spring. We asked the teachers to use a similar assessment with their focal child.

The PD was designed on the assumption that children bring to school a diverse set of resources. Further, culturally and developmentally responsive practices require teachers to draw on these multiple mathematical resources. We define children's multiple mathematical resources as experiences in homes and communities, play experiences that provide natural engagement with mathematics, and children's mathematical thinking (Wager and Delaney 2014). Understanding these elements requires teachers to engage in a variety of assessment practices that provide a more holistic picture of the child and learning so that they can do assessment-informed instruction and communicate with families. These assessment practices extend from working with families to recognising the resources from home (funds of knowledge) to ongoing open observation narratives (learning stories) to conducting skills-based assessments (such as interviews).

Funds of Knowledge, defined as "historically accumulated bodies of knowledge and skills essential for household functioning and well-being" (Moll et al. 1992, p. 133), is an anthropologically-based process for recognising the rich knowledge in low-income and minority households (González et al. 2005). Teachers access funds of knowledge through interactions with families during home visits and interviews. Although, not historically described as an assessment tool, the knowledge that teachers gain about the ways children are involved in daily activities can help illuminate the ways that mathematics is central to everyday family practice (Moll et al. 1992). We argue that this process is a form of assessment as it provides information to be used to modify teaching and learning activities. When the evidence is used to inform teaching, it is a kind of *formative assessment* (Black and Wiliam 2001)

Learning Stories, a narrative assessment tool, were developed by Margaret Carr and Wendy Lee (Carr 2011), as a way of adapting the oral documentation traditions of Maori people (Dreaver 2004; Reisman 2011). Grounded in trust in children's agency in the learning process, learning stories are narratives that document children's learning within the context of their learning community and the play environment. More traditional assessment approaches that use checklists based on developmental benchmarks decontextualise children's learning and only represent part of the whole learning process (Reisman 2011). In contrast learning stories are designed to document the teaching-learning process and focus on how the child displays and develops learning dispositions.

Clinical interviews are flexible questioning practices that assess children's mathematics knowledge. They were developed by Piaget to understand unanticipated responses and "establish the child's cognitive competence" (Ginsburg 1981, p. 4). Drawing on Piaget, Ginsburg argues that clinical interviews have three possible goals: discovering, identifying, and evaluating. Our initial goal for the clinical interviews was to respond to our advisory board's request for a measure of PD efficacy by measuring growth in children's understanding of early number (evaluating competence). However, they also provided insight into understanding the cognitive activity in which the children were engaged during the interviews. The interviews we designed reflected the mathematics focus of the PD and incorporated selected story problems from the problem-solving interviews developed through the research in Cognitively Guided Instruction (CGI) (Carpenter et al. 1989) and some basic counting skills. In CGI, teachers use interviews to understand how children construct and solve problems as well as typical misconceptions they have. Teachers then use this information to plan instruction.

2.4 Methods

Fifteen teachers participated in the PD, but for this story, we are focusing on Birdie, Wanda, and Marley[1] and their respective focal children Tommy, Mikey, and Bernadette. We selected these three teachers and their focal children as representative of various ways in which children's understanding is evidenced. We have drawn on multiple sources of data to explore these children's assessment experiences, including number interviews, teachers' interview narratives, learning stories, and reflections on home visits.

To measure changes in children's knowledge, we developed a protocol to interview six students in each of the 15 teachers' classes in the fall and spring. The same questions were used in the fall and the spring. The eight-question interviews assessed the following skills: rote counting, one-to-one correspondence, counting out, cardinality, comparing two sets, and problem solving of selected problem types (separate result unknown, multiplication, and partitive division). For purposes of this chapter, we are focused on five questions from the protocol (verbal counting, one-one correspondence, and three CGI problem types). Although these problems may seem advanced for 4-year olds, we wanted to understand what was possible rather than only assess what might be expected. The children did not know the interviewers who were members of the research team.

[1] All names used in this chapter are pseudonyms.

We asked the teachers to conduct similar interviews with their focal child in the fall and provide a narrative reflection of the interview. The form was much like the one developed for the clinical interviews but allowed teachers to modify the problems. Teachers transformed the interviews into learning stories to capture interview observations, what they thought that meant about the child's understanding, and how they would support future learning.

Data also included a learning story that teachers completed in mid-fall to document an observation of their focal child's mathematical interactions during play, and a reflection on a visit to their focal child's home. These reflections documented each teacher's experience during the visit and what she learned about the child's experiences in the home.

We aggregated the data into a table organised by type of mathematical understanding (rote counting, one-one correspondence, etc.) and type of assessment (number interview, teacher reflection on interview, observation of math in play, and reflection on home visit). Two of the authors examined data to identify evidence of children's mathematical understanding across data sources. We then developed narratives articulating where, what, how, and with whom children provided information about their understanding. We then compared the narratives to identify themes and develop case narratives.

2.5 What [Do] Assessments Tell Us About Children's Mathematical Knowledge?

We first present findings from the number interviews of all the students to provide a context for the three cases. We then provide a narrative of each case study child based on our analysis of the data from the interviews and the teachers. The narratives include: (a) background information on the child, school, and family; (b) information about the child's skills with regard to counting and story problems; and (c) a brief summary of the connections between the child, home, school, and assessment.

2.5.1 Number Interviews

The results from the number interviews are presented to provide evidence of the often unexpected change in responses from fall to spring. Our purpose here is to raise questions about the use of these instruments as evidence of children's mathematical understanding rather than attempt to unearth the reasons for unexpected changes in individual children's responses.

2.5.1.1 Counting

The first thing children were asked was to count aloud as high as they could. We assumed that the combination of development and learning would produce a higher

Table 2.1 How high can you count? Aggregate result

Change from fall to spring	Number of children	Percent (%)
Counted higher	34	65
Counted lower	9	17
No change	9	17

Table 2.2 One-one correspondence

	Fall		Spring	
Object placement		Random	Linear	Neither
Random	36 (69 %)	32	4	–
Linear	7 (13 %)	5	–	2
Neither	9 (17 %)	3	3	3
Total-spring	–	40 (77 %)	7 (13 %)	5 (10 %)

number in the spring than the fall, yet in comparing the results we discovered that 17 % counted lower in the spring and 17 % saw no change (see Table 2.1).

To assess one-to-one correspondence we asked children to count the number of checkers randomly arranged on a paper. If their answer was wrong, they were then asked to count the number of checkers arranged in a line. Table 2.2 sets forth the number of students who demonstrated one-to-one correspondence based on object placement (random/linear) in the fall. Results from spring are shown based on students' fall results. For example, of the seven students able to count using a linear arrangement in fall, two were not able to use either arrangement and five were able to count using a random arrangement.

2.5.1.2 Story Problems

We purposefully selected a range of story problems to explore how 4-year olds responded and whether there was a change in their response over the year. We found that students who got the correct answer in the fall sometimes got an incorrect answer in spring, especially for separate result unknown problems. The number of correct and incorrect responses for each problem type in fall and corresponding responses for spring are provided in Table 2.3.

2.5.2 Children's Stories

In this section we provide a narrative of Tommy, Mikey, and Bernadette and compare the information about each child's mathematics learning available from various sources.

Table 2.3 Story problems

		Fall	Spring	
Problem type			Correct	Incorrect
Separate result unknown	Correct	18 (36%)	9	9
	Incorrect	32 (64%)	*13*	*19*
			22 (44%)	28 (56%)
Multiplication	Correct	19 (38%)	17	2
	Incorrect	31 (62%)	*8*	*23*
			25 (50%)	25 (50%)
Partitive division	Correct	7 (15%)	5	2
	Incorrect	40 (85%)	*14*	*26*
	–	–	19 (40%)	28 (60%)

2.5.2.1 Tommy

> Tommy is a 4 year old boy who takes ownership in being 4. He tells me almost daily, 'My number is 4. I am 4.' while holding up 4 fingers. (Birdie)

Tommy is a White boy in Birdie's 4K classroom in a large public elementary school that serves an ethnically and linguistically diverse population of students, 80% of whom are designated as low income. Tommy lives with his mother and father and a younger brother in a single-family home they rent near the school. Tommy's family believes it is important to follow routines and to spend time together playing games, exploring the city, and being outdoors.

During a home visit Birdie learned that Tommy's parents worried that since he often played alone in his room, he wasn't learning to share his toys and that he displayed some tendencies toward obsessive compulsive disorder. Tommy was very focused on lining things up and would notice when toys were missing. He was such a rule follower that he often worried others were angry with him if he didn't get things right. Some of the mathematical practices that Birdie noticed in the home included playing games that required counting, comparing numbers, and cooking. His mother shared that Tommy had always been interested in numbers. Birdie wrote in her home visit reflection,

> He learned to recognise numbers from magnets on the fridge, a toy wooden clock his grandfather made, and flashcards. Since he was a little over a year old he has liked to line up his toys and cars and count them. …His mom said he counts everything—toys, stickers, candy, fingers, toes, spokes on bike wheel, etc. His mom thinks part of his interest in counting comes from a show he watches on Nickelodeon called Team Umizoomi which does a lot with counting and shapes.

Birdie observed that during her home visit, Tommy counted the teeth on his toy dinosaur and pieces from a game he played with his dad. Tommy's enthusiasm for numbers and counting is reflected in Birdie's interview narrative and learning story.

Counting Birdie often noticed Tommy spontaneously counting in class. When she interviewed Tommy she noted,

Table 2.4 Tommy's counting

	Number interview	Teacher interview	Classroom observations	Home visit
Counting	23	20	Counts spontaneously; has counted to 39	Counts regularly
One-one correspondence	Random	N/A	Both	

Table 2.5 Tommy's story problem interviews

	Research Team	Teacher
Separate Result Unknown	No	Yes
Multiplication	No	Yes
Partitive division	No	No

> When asked to rote count he spoke softly and was pointing to something. When I realised he was trying to count the vehicles in the bucket, I hid the bucket and then asked him to count out loud in a loud voice. The second time he was able to rote count to 20 before getting confused. He started saying numbers such as 23, 21, 22, 40, 60, 70, 80, 21, 30–60, 100. However, during other observations I have seen him count as high as 39 correctly.

Although Birdie had witnessed Tommy counting at home and his mother had shared the many things he counts, when she asked him what he counted at home Tommy said, "fire trucks, school buses, and race cars". Birdie was "unsure if he understood the question about what do you count at home because it sounded like he was just naming off vehicles and possibly trying to recall what was in the bucket of vehicle counters."

Birdie observed Tommy's use of cardinality in play and during her interview. When Birdie asked Tommy to count out a set of nine, Tommy was able to count out the correct amount and immediately answer how many were in the set without recounting. Birdie also noted in her learning story that after counting how many friends were at the art table, Tommy counted five and then told her "five" without recounting.

In comparing the results of the data collected in the fall (Table 2.4), we notice when the task is natural and meaningful Tommy is more likely to utilise his number skills.

Story Problems When Birdie posed the problems during her interview, she changed the context "to helicopters and landing pads because that is what he was interested in that day." She had to provide support by modeling how to set up the problems but Birdie found that, "Tommy was able to do the Separate Result Unknown and Multiplication story problems after she showed him how to set up the problems using counters, but he was unable to do the Partitive Division problem." In Table 2.5 we compare the results of the number interviews conducted by the research team and teacher.

Overall, Tommy demonstrated greater understanding in familiar and meaningful contexts, such as in play and at home. This could be due to any number of things. As Tommy is transitioning to school and school-like assessments it may be that familiarity with the interviewer, problem context, or approaches to questioning affect the mathematical understanding he demonstrates. Further, his teacher's awareness of his interests and willingness to scaffold him enabled him to feel successful in responding. One thing is evident: a clinical interview alone would have provided a limited window on Tommy's mathematical understanding.

2.5.2.2 Mikey

> Mikey loves to take things apart and put things back together. His mom told me that he took all the knobs off the door and put them back on. (Wanda)

Mikey is a 4-year-old White boy in Wanda's 4K classroom. Their school is ethnically diverse and 70% of the students are designated as low income. The school is set in a neighborhood with single-family homes and apartments, and is predominately working class. Wanda chose Mikey as her focal child because he frequently offered to help out in the class and Wanda wondered what that reflected about Mikey's experiences at home. Wanda learned that Mikey lived with his mother, father, sister, and cousin. Mikey's mother runs a home daycare so there are often other children in the house during the day for whom Mikey helps to care. Although there was no explicit mention of mathematics, Wanda learned from her home visit that Mikey was a tinkerer and was always exploring things. Mikey's parents described him as a people pleaser who frequently offers to help.

Counting Unlike Tommy, Mikey did not seem to randomly count things at home. In her reflection on her interview with him Wanda stated,

> When I asked Mikey what he could count at home, he responded with, "I count numbers." I felt like I needed to guide him more with an example. When I asked if he ever counted his toys, he said, "No, I don't know how." After I suggested that he could take one of his bins off his shelf and count how many toys were in it, he gave me other ideas of things he could count in his house.

During his interview with Wanda, Mikey counted to 19. Wanda said she noticed in other contexts that he had the general idea of one-to-one correspondence but during the interview, she had to show him strategies such as moving objects as he counted them. Wanda also observed Mikey's approach to number during puzzle play. The goal was to match the numeral on one piece with the correct set of objects on the other. Mikey, the tinkerer, had another plan.

> He sat down across from me and picked up some pieces. "I don't need to count them, I look at the pieces," Mikey says as he grabbed a piece with a number and one with the object pictures. He tried to fit them together without looking at the number or objects on the two pieces. The pieces did not fit together, so he put one down and grabbed another. Mikey did this many times without success. He was obviously not counting the objects pictured on the piece, but was looking to match the two parts together at the edge.

To Mikey the goal of the activity was to match the edges of the puzzle pieces–a skill important for a puzzle-doer–and a component of developing spatial awareness. The assessment task in Wanda's mind was matching numeral to set. To orient him to her task, Wanda suggested that Mikey try counting the pictures, which he did

Table 2.6 Mikey's counting

	Number interview	Teacher interview	Classroom observations	Home visit
Counting	31	19	Used other strategies to get to answer rather than count	No evidence of counting
One-one correspondence	Linear	Neither		

Table 2.7 Mikey's story problems

	Research Team	Teacher
Separate Result Unkown	No	Yes
Multiplication	Yes	Yes
Partitive division	No	–

with trouble at first but then had success after Wanda showed him how to point to each item he counted. A description of the evidence of Mikey's counting in various contexts in the fall is provided in Table 2.6.

Story Problems In responding to the separate unknown problem—"[Child's name] made six cookies, he ate two of them, how many cookies were left?"—Wanda found,

> Mikey did what many children his age would do. He heard me say "[Mikey] ate 2" in the story. He forgot about the part where he starts with 6. He took 2 from the original pile and proceeded to count the remaining from the original pile. When I guided Mikey, he was able to do the problem.

Like most of the teachers in the PD, Wanda felt that story problems were not developmentally appropriate for 4-year-olds, so labeling Mikey's initial response developmentally makes a lot of sense[2]. For the multiplication problem, Wanda posed the problem using baskets and pumpkins. Mikey put four pumpkins in each of three baskets and proceeded to count all the pumpkins. He made an error counting initially but after Wanda encouraged him to pull the objects away after he counted them, Mikey answered correctly. Wanda found similar results to that of the interviewer in fall, although Mikey was able to answer the separate result unknown problem with assistance (see Table 2.7).

Mikey provided us with the clearest example of a lack of agreement in the problem space of assessing. Wanda was asking one question (what he counts at home or how many dots were on the puzzle piece) and Mikey construed the question another

[2] As we suggested earlier, we posed problems that assessed what might be possible for children to solve, not necessarily what we expected them to solve. This was done in an effort to identify the breakdown between early childhood and developmental knowledge.

way (the counting at home question seemed non-sensical and he approached the puzzle the way he wanted to). Mikey showed us the importance of how questions are framed so that the assessor and assessee have agreement.

2.5.2.3 Bernadette

> Although Bernadette is enthusiastic about everything she does, clearly she would rather have been filling up pots with stones and dumping them out, than counting them. (Marley)

Bernadette is one of 17 students in Marley's 4K classroom in the day care centre at a local community centre. The centre participates in the district's 4K program offering 'free' 4K in addition to wrap around care. Bernadette attends 4K in the morning and goes home or to an in-home day care in the afternoon. Like Bernadette, the children who attend 4K here are predominately from White, working class homes. Bernadette is an only child who lives with her two mothers, two large dogs, a hamster, and various other pets. She loves animals and at home likes to play with balls and board games

Bernadette is a happy-go-lucky child who finds fun everywhere. This is how Marley described what Bernadette was doing during a conversation with Bernadette's mother during the home visit.

> She hopped off [the arm of the couch] several times to point different things in the room out to me. They had a nativity set of four bears. Bernadette pointed to the bears and said, "That is the mommy, that is the daddy, that is the drummer, and that is the baby!" Bernadette brought over a small megaphone that Mom said she uses at football games, she shouted into the megaphone and Mom took it away. Bernadette also brought over a glass snowman bell. She continually rang it while we were speaking.

This description of Bernadette at home was similar to how she behaved in her 4K classroom and may provide some insight into the way she engaged with questions about counting and number.

Counting Marley is the only adult in her room so she interviewed Bernadette in the midst of the busy classroom. She got out a bag of 'magic gemstones' and dumped them on the maths table during free choice time for children to explore, sort, and count. Bernadette was one of seven children who chose to work with Marley at this time. While children were exploring their stones, Marley asked Bernadette if she would count out loud for her. She counted to 13 but Marley believed that Bernadette was able to count higher, which she attributed to her observations of Bernadette's participation in a daily routine of counting the 17 children in the classroom. When asked if she ever counted things at home, Bernadette's answer was "nope." Although not explicitly discussed, there was nothing that Marley noticed during the home visit that would contradict this statement. In some ways, it seemed that Bernadette didn't necessarily think about counting things, which was affirmed in the classroom when the following occurred:

> During free choice time, Bernadette was playing in the dramatic play area. She was dressed up in a kimono and fire hat and playing with the "puppies" [stuffed animals]. She brought

Table 2.8 Bernadette's counting

	Number interview	Teacher interview	Classroom observations	Home visit
Counting	13	13	17	No evidence of counting
One-one correspondence	Random	Neither	Neither	

> the puppies over to me and said, "Look at my puppies!" She had her arms full, so I asked her, "How many puppies do you have?" Bernadette looked at me with surprise.

From Marley's perspective, it hadn't occurred to Bernadette that she might count the puppies. Marley suggested to her that they count the puppies, which provided an opportunity to explore Bernadette's understanding of one-to-one correspondence. As Bernadette put the puppies down, she counted each "one, two, three, five!" Marley suggested they try again pointing to each as Bernadette provided the same response. Marley found similar results when she asked Bernadette to count a set of seven gemstones randomly arranged on a piece of paper, "she counted, by pointing at each stone" and said six. When Marley lined up the seven stones on the paper, Bernadette counted each again and said, "eight." This suggests that at least in these contexts Bernadette is not yet able to demonstrate an understanding that counting involves assigning a single number to each item in a sequence. However, when interviewed by the project team in the fall, Bernadette was able to correctly count the chips in a random order. Interestingly, she required that the chips be set up in a linear fashion when she counted them in the spring. Table 2.8 provides a comparison of the results of Bernadette's counting in various contexts in the fall.

Story Problems Marley posed three problem types to Bernadette and provided quite a bit of scaffolding to set up and answer the problems. She asked Bernadette to pretend the stones were cookies. For the first problem,

> I told her that she had six cookies and I counted them out for her. Then I pretended to eat two of the cookies and asked her how many were left. She joyfully said, "Three!"

Marley found similar results with the other story problems—in the multiplication problem with four cookies on each of three plates, Bernadette said there were "five." For the partitive division problem, Bernadette distributed eight cookies on four plates as follows: five on the first, three on the second, and none on the others. She then counted them all and stated there were nine cookies. At this point, other children noticed the plates and 'cookies' and brought materials over from the dramatic play area and the counting activity "became more about filling and dumping."

During her interactions with the gemstones and the puppies, Marley had to do a lot of 'redirection' to keep Bernadette focused on Marley's version of the activity. Bernadette's responses in both situations suggest that she took great pleasure in whatever she was doing but wasn't particularly concerned about getting the 'right answer' or even an answer. Bernadette was an enthusiastic player who was not focused on finding a shared understanding of a task. She did not need to succeed in the assessment activities in the traditional school way. This was confirmed in the interview our project team completed during which Bernadette did not correctly answer any of the story problems.

2.6 Conclusion

What do the stories of Tommy, Mikey, and Bernadette tell us? Not surprisingly, as the literature suggests, the wider the variety of assessments, the more we know about a child. Yet, the assessments typically used with young children don't tell the whole story and are likely misrepresenting all that children are capable of doing. Young children who have yet to be socialised into the rules of schooling can be precocious, curious, and willing to follow their own interests. As a result, they provide very different responses depending on who asks the questions, the context of the questions, and the setting. Although this is not particularly surprising, we raise it here as a warning that standards-based assessments may be more a measure of a child's agreement about a task than of a child's knowledge and skill. As we look across the three cases, the degree to which the children transitioned to school maths might be attributed to what makes the most sense to the child. For Tommy, his interest in number might position him as more knowledgeable about maths. Mikey's interest in tinkering might suggest that he is more focused on finding an approach to answering problems that does not require him to count. And, because Bernadette was so interested in play the questions became part of the play, which might not have had a connection to the assessor's reality.

Even when teachers take the time to observe children in play, it is important to recognise that she can still only get a partial view of the child from what happens in school. The child brings to the classroom a life full of experience, knowledge and skills. Tapping perspectives that illuminate a child's funds of knowledge through home visits and family interviews provides complementary information about what a child knows. Assessment with young children is more a matter of triangulating multiple sources of evidence than thumbs up or down for a right answer. As a result, decisions about whether a child is proficient in some skills need to be based on diverse sources of information in multiple contexts, recognising that we might not be asking the right question to have access to a child's knowledge. Unfortunately, the environment that seems to 'matter' the most is the assessment environment/results, while the environment that we are maybe most likely to see proficiency from is the classroom or home. As we found, the children displayed different levels of competency depending on whom and how they were assessed, and these differences varied by child.

Traditional assessment is based on the idea that if the questions are standardised, variation in response can be attributed to differences in skills and knowledge. Based on our experiences with 4-year-olds, a different way to conceptualise assessment suggests that to understand *what* children know, we must play close attention to *how* they know. If their responses are contingent on where they are, what they are doing, who they are talking with, and why the task is being presented, we wonder if the goal of assessment is to find out the question rather than finding out the answer. While this might seem a throwback to Piaget, we do not use the variation as an indicator of misconceptions or lack of development. Instead, we see this approach as one that is based on the assets that children bring to school and therefore one that places responsibility on adults to do the hard work of finding out what and how children know. This is most important in thinking about the transition to school because it represents a major shift in the resources available to support children's learning. In an environment pushing teachers toward increasingly structured assessments, they may have less opportunity to learn about a child's funds of knowledge or how s/he constructs a problem space. We recognise that there are developmental patterns in mathematics learning and that they have been used fruitfully to build curricula for young children. However, they tell only one story, and we wonder if broadening our assessment resources might broaden our knowledge as well. To facilitate children's transitions to the culture of school, we might think of maximising the tools we have to better understand children—tools that look across social contexts, actors, and purposes.

We hope to continue to develop a deeper understanding of how young children perceive tasks and when, in children's minds, a task transitions from the relationship between child and adult to the problem being asked. Further, we will continue to use this line of work to identify what, how, and where children demonstrate what they understand and work to advance these ideas into public policy.

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Chapter 3
Assessing Young Children's Mathematical Understanding: Opportunities and Expectations at the Transition to School

Barbara Clarke

Abstract One-to-one interviews have been used extensively in Australia by both researchers and teachers to assess young children's mathematical understanding. This chapter discusses the use of a one-to-one task based interview developed as part of the Early Numeracy Research Project. The First Year of School Mathematics Interview component has been used in a range of research contexts, both prior to school and in the early years. A recent study, using the interview with children with Down syndrome where the interview was presented in a more flexible manner, raises important questions regarding its use both in research and practice. The opportunities and expectations during the transition to school and how these may be enhanced by the use of one-to-one assessment interviews is also discussed.

3.1 Introduction

When assessing children as teachers or researchers, we have access to their mathematical understanding through watching, listening or their documentary productions. These can be produced through a range of techniques or provocations. Any insights into the mathematical understanding or thinking of young children which emerge are a product of these provocations and the interpretation of the educator.

Assessment in mathematics has long been associated with pen and paper methods with the traditional mathematics test dominating assessment practices in school including the early years (Clements and Ellerton 1995). However, observational techniques have traditionally been the main focus of assessment of individual children in early childhood settings and these have been general in nature with an emphasis on cognitive, social-emotional, physical and language development (Fleer and Quinones 2013). There is an increasing focus on assessment within curriculum disciplines in early childhood with the implementation of more specific articulation of curriculum requirements.

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In the transition to school, the differences in forms and roles of assessment have potential to impact children, educators and other caregivers. External forms of assessment including those that are used to evaluate students for special programs or interventions are often given during this transition period. Specialist practitioners provide data on individuals that are often used to determine funding for care and support, particularly in school systems where such information provides a measure of need. While such data are valued for their reliability, they may be of limited use to the classroom teacher or educator. In addition, parents are often expected or required to engage in decision-making based on the results of assessment. Difficulties in this process can lead to increased disadvantage for those who are less informed or empowered to advocate.

There is tension between the different roles of assessment and the form of assessment used should reflect the purposes of its use. However in the busy life of an educator it is necessary to make decisions based on manageability as well as meaningfulness.

Sometimes, politicians and other educational policy makers seem to believe that it is the act of assessment that will lead to improved learning, when in fact it is the action that follows, using the information gained from the assessment that is potentially most powerful (Clarke 1989). For the educator, the information needs to be valid and provide potential for action.

In this chapter, experiences of the use of one-to-one task based interviews as a tool for gaining valid insights into the mathematical thinking of young children in the transition to school will be shared.

3.2 Task-Based Interviews as an Assessment Tool for Mathematics

The power of a one-to-one, task-based interview as a tool for both teachers and researchers to notice young children's mathematics has been well documented (Bobis et al. 2005; Clarke et al. 2011; Ginsburg 2009). Different approaches to the conduct of an interview can provide different insights into children's mathematics learning and thinking. They can show what children can do through well designed tasks and questions. Of course, researching and understanding young children's mathematical thinking is challenging, as much of what we want to know are cognitive processes or mental strategies.

Following the work of Piaget, clinical interviews have been used for many years in mathematics education research (Ginsburg et al. 1998). Typically, such research had been conducted with relatively small numbers of children, and the results not always communicated well to the teaching profession. However, the late 1990s, in Australia and New Zealand, saw the development and use of research-based one-to-one, task-based interviews with large numbers of children, as a professional tool for teachers of mathematics (Bobis et al. 2005). The interview that was developed as part of the Early Numeracy Research Project (ENRP) was typical of these.

3.3 The Early Numeracy Research Project

The ENRP was conducted from 1999 to 2001 in 35 project ('trial') schools and 35 control ('reference') schools, and involved 353 teachers and over 11,000 students aged 5–8 years in the first 3 years of school, in Victoria, Australia (Clarke et al. 2002). There were three main components to the project: a framework of research-based growth points as a means for understanding young children's mathematical thinking; a one-to-one assessment interview used by all teachers at the beginning and end of the school year as a tool for assessing knowledge and strategies for particular individuals and groups; and a multi-level professional development program geared towards developing further such thinking.

The interview was structured with specific instructions for administration and recording. It allowed for more conversation and recording of varied strategies than more formal psychological assessment protocols. Such strict protocols are arguably more reliable for comparison but do not provide the same richness of data for either the researcher or the teacher.

Of course a structured interview can provide surprising insights. A favourite anecdote from the interviews for the ENRP came from a teacher and related to the "draw a clock" task, in which the children were instructed to simply "draw a clock." The child's clock was then used to initiate a discussion of their understanding of how time and clocks work.

> I asked the child "What are the numbers on the clock doing?" The child looked strangely at me and said "the numbers are doing nothing; they are waiting for the arrows to come around. Don't you know that? Are you stupid or something?" (ENRP teacher)

3.3.1 A Research-Based Framework of "Growth Points"

To underpin the task-based interview, it was decided to create a framework of key 'growth points' in mathematics learning. Students' movement through growth points could then be tracked over time. The project team studied available research on key 'stages' or 'levels' in young children's mathematics learning (Carpenter and Moser 1984; Fuson 1992; Mulligan and Mitchelmore 1996; Wright 1998), as well as frameworks developed by other authors and groups.

Within each mathematical domain, growth points were stated with brief descriptors in each case. There are typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children 'count-all' to find the total ("1, 2, 3, …, 11, 12, 13"), even though they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at nine and counting on ("10, 11, 12, 13"), they can solve the problem in an easier way. Counting All and Counting On are therefore two important growth points in children's developing understanding of addition.

1. Count-all (two collections)
   Counts all to find the total of two collections.
2. Count-on
   Counts on from one number to find the total of two collections
3. Count-back/count-down-to/count-up-from
   Given a subtraction situation, chooses appropriately from strategies including count-back, count-down-to and count-up-from.
4. Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts)
   Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident.
5. Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies)
   Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident.
6. Extending and applying addition and subtraction using basic, derived and intuitive strategies
   Given a range of tasks (including multi-digit numbers), can solve them mentally, using the appropriate strategies and a clear understanding of key concepts.

Fig. 3.1 ENRP growth points for the domain of addition and subtraction strategies

The six growth points for the domain of addition and subtraction strategies are shown in Fig. 3.1.

These growth points informed the creation of assessment items, and the recording, scoring and subsequent analysis.

We do not claim that all growth points are passed by every student. For example, growth point 3 involves ‘count-back’, ‘count-down-to; and ‘count-up-from’ in subtraction situations, as appropriate. There appears to be a number of children who view a subtraction situation (say, 12 − 9) as “what do I need to add to 9 to give 12?” and do not appear to use one of those three strategies in such contexts. This student is using a ‘fact family’, one of what we call ‘derived strategies’ (see Growth Point 5).

The growth points should not be regarded as necessarily discrete. As with Wright’s (1998) framework, the extent of the overlap is likely to vary widely across young children, and “it is insufficient to think that all children’s early arithmetical knowledge develops along a common developmental path” (p. 702).

3.3.2 Early Numeracy Research Project Interview

A one-to-one interview in Number, Measurement and Geometry was developed to be used with every child in grades K-2 in ENRP schools at the beginning and end

18) Counting on

a) Please get four green teddies for me.
Place 9 green teddies on the table

b) I have nine green teddies here *(show the child the nine teddies, and then screen the nine teddies with the ice-cream lid).*
That's nine teddies hiding here and four teddies here *(point to the groups).*

c) Tell me how many teddies we have altogether... Please explain how you worked it out.

d) *(if unsuccessful, remove the lid).* Please tell me how many there are altogether.

Fig. 3.2 An excerpt from the addition and subtraction interview questions

of the school year. The interview was expected to take around 40 min per child. The disadvantages of pen and paper tests have been well established by Clements and Ellerton (1995) and others, and these disadvantages are particularly evident with young children, where reading issues are of great significance. The face-to-face interview was an appropriate response to these concerns. Many writers have commented on the power of the one-to-one assessment interview as providing powerful insights into student thinking (Schorr 2001).

Although the full text of the ENRP interview involved around 60 tasks (with several sub-tasks in many cases), no child moved through all of these. The path was specified, in accordance with a student's response to each task. Figure 3.2 shows a question, involving little plastic teddy bears, from the section on Addition and Subtraction Strategies. Words in italics are instructions to the interviewer. In normal type are the words the interviewer uses with the child.

Question 18 provided information on whether the child was able to count-on or use a known fact, needs to count-all, or was unable to find the total by any means. The aim in the interview was to gather information on the most powerful strategies that a child accesses in a particular domain. However, depending upon the context and the complexity of the numbers in a given task, a child (or an adult) may use a less powerful strategy than they actually possess, as the simpler strategy may "do the job" adequately in that situation.

Of particular interest when considering young children and the transition is the First Year of School Mathematics Interview (FYSMI), a component of the larger ENRP interview. Details of the FYSMI including data from a large sample of children are reported in Clarke et al. (2006). The teachers in a specialist school for children with specific learning needs within the ENRP found it to be a very valuable tool that was easily used and interpreted in their context (see Clarke and Faragher 2004). The FYSMI has also been used by researchers in preschool settings with considerable levels of engagement (Clarke and Robbins 2004). It also enabled comparison with the larger ENRP data set.

A further feature of the ENRP one-to-one task based interview was that the children in the early years had the opportunity to go beyond the mathematics dictated by the curriculum and there was very limited ceiling effect. It provided both teachers and researchers with unexpected insights as illustrated by the following teacher quote:

> I have to admit I was really surprised when I did the testing on them, at how much two or three of them knew, they knew far more than I realised. A couple of them are being held back because they still can't do the counting, one, two, three, they go wrong. But when we go beyond that it's just amazing how much understanding they've got. I was just blown away by a couple of the results, I really was. (Special school teacher interview, as reported in Clarke and Faragher 2004)

While instruments such as the ENRP interview including the FYSMI provide opportunities for individual children's thinking and strategies to be evidenced, they generally assume a traditional trajectory of mathematics learning and may limit options. They evidence a 'moment in time' rather than a definitive assessment of an individual child's mathematical understanding. This is particularly relevant and possibly limiting when interviewing children with specific learning difficulties. In a recent project that attempted to map the mathematical development of young children with Down syndrome (Faragher et al. 2008; Faragher and Clarke 2014), the ENRP interview was adapted and a slightly different approach taken to its application.

Literature indicated that children with Down syndrome interviewed in unfamiliar contexts by people they did not know reduced performance on literacy tasks (Brown and Semple 1970}. Therefore, we interviewed children with Down syndrome in their home or school, in the presence of their parents (or teacher) who watched from behind the child. The adults were invited to comment on the performance of the child, either by taking notes during the interview, or in a discussion following the interview. The interviews were videotaped and the 'semi-structured' approach that was used is discussed in the next section.

3.4 One-to-One Mathematics Interviews with Young Children with Down Syndrome

With a limited research base, methods to chart the mathematical learning of children with Down syndrome are still developing. The choice of task-based, one-to-one interviews was appropriate. The ENRP interview (Clarke et al. 2002) and Extending Mathematical Understanding (EMU) interview (Gervasoni 2004) were used as the basis of an interview with children with Down syndrome. While these instruments were already demonstrably effective, necessary modification, trial and development was undertaken.

In the Down syndrome project, the interview was implemented in a more flexible form than in the ENRP and associated project to ensure maximum opportunities for individual children to show what they knew and could do rather than as a protocol driven instrument. Tasks were first asked in the same form of wording as the original instrument but follow-up questioning, instructions or guidance were provided at the discretion of the interviewer. This allowed the interviewer to follow up on responses from the child, to double back to earlier tasks, to ask a similar task in a different way and to add tasks, such as counting stickers that had been given as rewards during an interview. In order to do this, the interviewer needed to know the purpose behind the interview questions as well as be able to make preliminary judgments about what was being observed in the interview while it was in progress. The interviews were video-taped to allow more detailed analysis.

Sometimes, children with Down syndrome exhibit behaviours that hinder the assessment of their mathematical understanding. In the case of one child, Gina, giving an answer "one" seemed to be 'avoidance' behaviour, a well-established aspect of behaviour in children with Down syndrome (Wishart 1996). This is a learned (albeit potentially unhelpful) behaviour, and not in any sense misbehaviour. There were seven occasions during the interview when Gina gave an answer "one". On only one occasion was this an appropriate response. It appeared from the analysis of the video that it was her 'default' response. It would seem to be an attempt to disengage with the question, perhaps to effectively avoid thinking about the question, or maybe to provide a response when knowing what to do was unclear. A particular example is quite enlightening:

> Gina was presented with some dot cards and numeral cards and asked to find the number to match the dots. She did not show evidence of matching but pointed to the numeral 3 and said "three." The interviewer used this as a cue to ask if she knew any other numbers. During this sequence, the interviewer picked up the card with the numeral 4 and asked Gina what number it was. Gina responded quickly by saying "one" and then said "four" quietly. It was as if "one" was her standard answer and then she realised that she actually could read the numeral.

Gina was an engaging child but struggled with much of the interview. She was one of the youngest of the children that was interviewed. However, the flexible approach gave greater insights into her thinking than would have been the case following the script per se. A more traditional protocol driven assessment interview where the first answer is used or where restatement or adaption by the interviewer is not permitted would have limited what was found. Of course, differences in methodology are generally due to different purposes, but for this project we wanted to expand the opportunities for the children to show what they knew and could do.

A further example was when one of the questions from the FYSMI that focused on location language was asked. The original task asked children to place a small plastic teddy in a specified position relative to another teddy. Maggie was asked to place a green teddy behind the blue teddy that was in front of her on the table. She did not do this so the interviewer got out of her seat, moved over to the clear space with Maggie and asked her to stand behind her. Maggie did this successfully, showing some understanding of the concept 'behind'. This additional task became a feature of future interviews within the Down syndrome project providing additional information on the mathematical understanding of the children.

3.4.1 Strategies for Dealing with Avoidant Behaviour

A major reason for the use of the semi-structured approach to the task-based interview was in response to the behaviour of the children. As previously mentioned, avoidant behaviour has been extensively documented even in very young children with Down syndrome. Therefore, we were not surprised (though we were certainly entertained!) by the many instances where children were using strategies to avoid attempting the tasks such as changing the tasks, playing with the equipment, using behaviours to distract the interviewer (burping, being 'cute,' changing the subject) and refusing to participate. It is important to note that children used avoidant strategies even when they were able to do the tasks. Our interview protocol and flexible technique allowed us to work around these antics to gather data we could trust. Some studies on mathematics performance by children with Down syndrome give a more pessimistic view than the experiences of parents and teachers would suggest (Abdelhameed and Porter 2006). The discrepancy may be due to the use of research methods which are unable to take account of the avoidant behaviours and therefore limit opportunities.

Modification to the interview became necessary for some participants when it appeared that the presentation of the tasks themselves was distracting. The standard interview protocol makes use of objects such as plastic teddy bears with the deliberate purpose of engaging participants. For some of our children, though, these objects seemed to be a distraction. Some children needed to arrange all the teddies to be facing the same way, but took so much time that they forgot what they needed to do for the task. Others engaged in the story of the teddies going to the beach and lying on beach towels (as a context for division), to the point of missing the mathematics. It could be that the children were glad of an alternative task to pursue or it could be that they were genuinely distracted from the mathematics. In either case, however, it became clear that small blocks could be used instead, making explicit the mathematics required.

Mary was one of the older children interviewed and confidently worked on the first few tasks in the interview. She was then asked to take five blue teddies from a mixed collection which she did successfully. Next the interviewer spread the teddies out and asked how many there were. Mary counted again, successfully. The intent was to see if she would identify the quantity *without* counting and arguably evidence conservation of number (see Clarke et al. 2006, for discussion of the difficulties in interpreting this task). The interviewer again repeated the process and Mary again counted. While we would have expected that she could conserve number and understand that the count indicated the numerosity of the set, this task had not provided the necessary evidence.

As the interviewer packed up the teddies she had the blue teddies in a group under her hand. She then asked Mary how many there were and Mary quickly answered "5." She was clearly demonstrating understanding of the cardinality of the set and conservation though not in response to the question intended to elicit this knowledge, but rather from an incidental question. This interaction again illustrates the challenges of more protocol driven interviews with the behavioural practices of children with Down syndrome and the value of flexibility in the hands of a knowledgeable researcher in providing insights into mathematical understanding and thinking.

The interviewer needs to be flexible and highly skilled in understanding the mathematics underlying the interview questions in order to probe appropriately and provide valid data on individual understanding. This approach provided greater insight into the mathematical thinking and processing of the children with Down syndrome we were studying.

3.5 Interviewing as Enhancing Teachers' Knowledge

One of the key findings of the ENRP was the value of the interview for enhancing teachers' knowledge (Clarke et al. 2011). Along with the growth points, the interview provided teachers with insights into children's mathematical thinking and a way of describing what they were seeing and hearing when children are engaged in mathematics. They evidenced improved questioning techniques including the opportunity to see the benefits of increased wait time. It provided "a clearly evidence-based understanding of student thinking in mathematics and what students know and can do" (p. 907).

Portions of the broader ENRP interview have been used by student teachers in a range of contexts. In a study to investigate the effectiveness of using the task based interview to build pre-service teachers' understanding of what children know and can do in the early years of school, McDonough et al. (2002) found that the use of the interview enhanced the knowledge and skills of pre-service teachers in the following ways:

- Pre-service teachers are more aware of the kinds of strategies that children use, including their variety and level of sophistication.
- Pre-service teachers have seen the power of giving children one-to-one attention and time, without the distraction and influence of their peers.
- The interview provides a model of the kinds of questions and tasks that are powerful in eliciting children's understandings.
- The interview and subsequent discussion stimulate pre-service teachers to reflect on appropriate classroom experiences for young mathematics learners. (p. 223)

Interviews have been conducted by future teachers as part of their teaching experience in the context of preschools as well as the early years of school. These have provided insights into the mathematical thinking of young children as well as a shared language for describing and discussing this thinking. Carpenter and Lehrer (1999) highlighted the importance of this linking:

> Knowledge of mathematics must also be linked to knowledge of students' thinking, so that teachers have conceptions of typical trajectories of student learning and can use this knowledge to recognize landmarks of understanding in individuals. (p. 31)

Bobis and Gould (1999), reporting on the Count Me In Too (CMIT) project in New South Wales, also found that the provision of a research-based learning framework enhanced teachers' knowledge of how children learn mathematics. In a major project in New Zealand, the National Numeracy Project, the learning framework gave teachers "direction for responding effectively to children's learning needs" (Higgins et al. 2003, p. 166). It is not the interview on its own but the interpretation and possibilities for learning that such assessment creates that are particularly powerful.

3.6 Implications for Transition

In addition to being a tool for researchers, this work has highlighted the value of a task-based one-to-one assessment interview for educators in the early years of school. As previously discussed, we can assess children through watching, listening or interpreting documentation. Is the interview just about listening? It is more than just listening, as it is the form of the questions—the provocations that are linked to important mathematical ideas, which provide a direction for subsequent questioning as well as future planning. There is structure based on the research on children's mathematics learning to enable purposeful assessment. The interview can provide 'eyes and ears' for the educator to see, hear and interpret the mathematical thinking of the child. Their experience with the interview means that they know what to look for.

For many early years' teachers, the role of the interviewer was novel, but this brought challenges. They were less the teacher and more the observer and in some cases this was a struggle. In the early stages of the ENRP when the child was not successful a teacher would comment, "but they could do it yesterday." Or they would claim that the child would have been successful if the question was asked in a different way. It was a shift from success being measured by a correct answer to a deeper focus on finding out what the child really knew, the strategies they used and the 'edges' of their learning.

Such an interview provides a range of opportunities for the children that are important as they transition to the generally more formal school setting. It provides a balance between structure and openness. Not structure for its own sake, but to enable a focus on the important mathematics. It needs to provide opportunities for extending and surprising the interviewer, whether in the role of educator or researcher. It can also provide an opportunity to challenge the expectations of the teacher.

3.6.1 Expectations of Children

From very early in the ENRP, teachers observed what for many were unexpected levels of mathematical understanding among their children. The first set of interviews provided teachers with information about their individual children that had not been previously obtainable, and initially many were surprised by what their children knew. Several quotes from ENRP teachers capture the spirit of many teachers' comments, as they reflected on highlights and surprises that emerged from the first set of interviews.

> My greatest surprise was that most children performed significantly better than I anticipated. Their thinking skills and strategies were more sophisticated than I expected.
> Working with a gifted five year-old who actually worked out the answers quicker than I did.
> Reading 24 746 154 on the calculator. Amazing!

It should be noted however, that the raising of expectations was not across the board. There were several areas where teachers were surprised with the difficulty that many children appeared to have on particular tasks:

> Many children had difficulty with the task involving sharing 12 teddies between 4 teddy mats, and with the tasks relating to abstracting multiplication.

> Quite a few children were able to read and write two- and three-digit numbers, but were unable to order one-digit numbers.

> Reading clocks was more difficult for children than many teachers expected, given its emphasis in their programs.

Overall, the expectations were more realistic and linked directly to the children rather than the teaching and curriculum expectations. The following quote from a teacher at the end of the project illustrates this:

> I expect more and I extend horizons more. I'm not as structured in my approach and I realise that there might be more than one way of solving a problem. I am interested in how children 'think'.

The importance of the interview to enable children to show what they know is evident in this quote:

> Four years ago this is what we taught preps [first year of school] and that's what I taught and that's what I tested so at the end of the year I could say 'that child can do that' but I wouldn't be able to tell you what else he could do.

The interview also clearly showed those aspects with which the children were still struggling, but not in a judgmental or comparative approach but intended as joint exploration of their thinking. The role of the interviewer whether an educator or researcher was to elicit mathematical thinking—to find the 'edge' of their current understanding, some might argue their zone of proximal development.

In the Australian state of Victoria, an on-line adapted version of the ENRP interview is used extensively in the early years. It is time consuming but provides an opportunity for young children to show what they know and can do to their teachers in a comfortable and safe environment.

We are extending our work with children with Down syndrome to focus on teachers in inclusive settings in primary schools and plan to produce, trial and refine a version of the interview that enables teachers to develop a record of the mathematical thinking of the child with greater flexibility and an on-going record that builds over time. One of the challenges is the challenge of gathering the fine-grained information that might be useful in the context of children with special needs as well as early indicators of mathematical understanding in content other than number.

3.7 Concluding Comments

Structured task-based one-to-one interviews are an important methodology for researchers to notice the mathematics of young children. Highly structured protocols provide reliable comparisons but limit the opportunities for children to evidence the richness of their mathematical understanding. Structured interview protocols that are designed to elicit different strategies, encourage conversations and highlight children's thinking (such as the ENRP interview) provide greater insights about individual children. A more flexible approach in the form of semi-structured interviews has provided richer and more valid data for children with Down syndrome and has much potential for researching the mathematics of young children in general. A knowledgeable interviewer is required for this method to be effective. It requires sophisticated knowledge of the mathematical development of young children as well as the skills to engage the children, to intervene or stay silent, to persist or know when to move on.

In the context of the early years of schooling, a structured interview given by an informed educator is an important tool in their assessment repertoire. As we move from the largely observational strategies of early childhood to the paper-based assessments of the school system, the semi-structured, one-to-one task based interview provides a transition in form that is accessible to the children and responsive to the individual. For the school teacher, at the beginning of the school year, the interview can provide important insights into the mathematical thinking of the children in their class enabling opportunities for more focused and appropriate teaching.

One of the challenges in the transition for educators is the identification and articulation of the mathematics through having a common understanding and language. In the early years of school, there has been a shift to the valuing of the strategies rather than the specific answer. For example, in the past we may have recorded if a child could successfully add 4 and 5, with value placed on quick recall of this calculation. The shift has been to the strategies, so that a child who can tell you that $4+5$ is 9 because they know that $4+4$ is 8 and this is one more, is evidencing a quality of mathematical thinking that is likely to be built on constructively. The previous practice often developed a rote approach to teaching and an emphasis of the answer rather than the thinking. However change takes time and such limited approaches are still emphasised in many school settings. Is there a similar emphasis in preschools? Are we able to link the expectations of curriculum documents across the transition in ways that will enable the effective development and use of the assessment tools to which our children are entitled?

In assessment, providing opportunities and tools for the child to demonstrate what they know and can do as part of the regular learning process, and not just in formal testing procedures will be important. Assessment using one-to-one task based interviews in a flexible way could be considered by teachers as well as researchers. These do not need to be administered formally but can be incidental during teaching sessions. However, the more formal approach enables the teacher to focus solely on one child at a time, without the distractions of the busy classroom.

Children are rich mathematical thinkers. They are entitled to experience assessments that provide opportunities to show what they know and can do to researchers and educators. In advocating a place for more flexible approaches to interviewing I would argue that it provides greater richness and validity in terms of results for individual children. However, listening is vital.

Downs and Strand (2006) suggested the following guiding principles be used when evaluating assessment methods in both regular early childhood settings and early childhood special education:

- As a general rule, assessment should be restricted to variables that are responsive to intervention on the part of teachers.
- The value of an assessment is the function of its capacity for generating novel or unexpected information; therefore, assessments should be questioned to the extent that they are a source of redundancy, regardless of psychometric considerations.
- Outcomes assessment should be prioritised over fidelity assessments.
- The timing and frequency of assessment should allow for intervention changes in cases in which performance changes are inadequate or less than anticipated.
- In the service of generating and sharing ideas about effective instruction, forums should be established in which teachers present to their peers data reflecting the cumulative education attainments of students under their change. (p. 678)

The one-to one task based mathematics interview would seem to reflect these principles well. Ginsburg (2009) argued that the clinical interview method is an essential component of formative assessment and "indeed, what is the alternate to obtaining detailed understanding of children's knowledge and using it to inform instruction" (p. 126).

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Chapter 4
Children's Mathematical Knowledge Prior to Starting School and Implications for Transition

Ann Gervasoni and Bob Perry

Abstract Research over the past 10 years has established that many children starting school are more mathematically capable than teachers, mathematics curricula and text book writers assume. This issue and implications arising for children's transition to school are explored in this chapter through examining data for 125 children who participated in the Australian *Let's Count Longitudinal Evaluation Study* in 2012, 1438 children who participated in the Australian *Early Numeracy Research Project* (ENRP) in 2001, and the new *Australian Curriculum—Mathematics*. The children's mathematics knowledge was assessed using the *Mathematics Assessment Interview.* The findings suggest that large numbers of children in both the *Let's Count* preschool group and the ENRP Beginning School group met the new *Australian Curriculum—Mathematics Foundation Standard* prior to beginning school. This suggests that many children may be inadequately challenged by the mathematics tasks and instruction they experience in their first year of school.

4.1 Introduction

Children making the transition to school have a diverse range of backgrounds and experiences. As a result, teachers expect that children will differ with respect to their confidence, knowledge, skills, and disposition to learning mathematics. It is commonly assumed also that children living in economically and socially disadvantaged communities are over-represented in the group of children with the least formal mathematics knowledge when they begin school, and that it is important that prior-to-school experiences help to overcome this disadvantage. An ongoing challenge for education authorities is providing suitable guidelines for mathematics instruction and curricula that, on the one hand, respond well to children's differences to

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ensure that all will thrive mathematically, whilst on the other hand, reflect and build upon the mathematics that children typically know when transitioning to school. Research over the past 10 years has established that many children starting school are more mathematically capable than teachers, mathematics curricula and text book writers assume. This finding suggests that many children may be inadequately challenged by the mathematics tasks and instruction they experience in their first year of school, and that this may have a negative impact on their mindsets and opportunity to thrive mathematically. This chapter provides insight about this issue by examining data about children's mathematical knowledge around the time that they transition school. The data are drawn from two Australian studies. The first is the *Let's Count Longitudinal Evaluation Study* that assessed children just prior to their beginning school (Gervasoni and Perry 2013). The second is the *Early Numeracy Research Project* (ENRP) (Clarke et al. 2002) that assessed children's mathematics just after they began school.

The 125 children participating in the *Let's Count* study in 2012 were from economically disadvantaged communities and were assessed in December 2012 in order to provide baseline data about the range of children's mathematical knowledge in these communities prior to an intervention planned for 2013–2014, and prior to their beginning school in 2013. The 1438 children participating in the *Early Numeracy Research Project* in 2001 were assessed in March 2001 just after they started school and were from 34 Victorian schools that were selected to provide a representative sample of the Victorian population. We decided to compare the mathematical knowledge of the *Let's Count* group with that of the more representative ENRP cohort to gain insight about any apparent differences in the mathematical knowledge of the two groups, as measured by the Mathematics Assessment interview. The mathematics knowledge of both groups was also compared to that assumed by the new *Australian Curriculum—Mathematics* (Australian Curriculum, Assessment and Reporting Authority, ACARA 2013) in order to determine how adequately the new mathematics curriculum responds to children's mathematics knowledge when they begin school, and whether any issues for children living in 'disadvantaged' communities were apparent.

In order to appreciate the context and data examined in this chapter, some details about early childhood education in Australia are provided. Australian children most commonly begin primary school in February each year when they are between 4 years and 6 months and 5 years and 9 months of age, although some differences exist between the States and Territories. The first year of primary school is referred to in the *Australian Curriculum* as the Foundation Year. Primary school teachers teach all curriculum areas and have completed a 4-year degree in Primary Education that includes several units focused on mathematics education. In the year prior to beginning school, most Australian children attend preschool for at least 15 h per week. Their preschool teachers mostly have completed a 3- of 4-year degree in early childhood education. Within the past 5 years, new national curricula documents for both preschools and primary schools have been introduced. These documents will be discussed later in this Chapter.

4.2 Expectations of Mathematics Performance and 'Disadvantaged' Communities

In communities nominated by governments as 'disadvantaged', there can be expectations that, on average, children will not perform as well academically as same age children from more 'advantaged' communities (Caro 2009). For children in the early years of school, similar results concerning the relationship between a child's mathematical performance and their family or community's socio-economic status have been reported (Carmichael et al. 2013; Rimm-Kaufman et al. 2003). As a result of their work, Carmichael et al. (2013, p. 16) felt confident to make the statement "the socio-economic status of the community in which the family resides was the strongest home microsystem predictor of numeracy performance, explaining 10.5 % of the variance in the home-community microsystem model".

However, there is also evidence from Australian and international research that many young children begin school as capable mathematicians who already exceed many of the first year mathematical expectations of mandated curricula or textbooks (Bobis 2002; Clarke et al. 2006; Ginsburg and Seo 2000, Gould 2012; Hunting et al. 2012). For example, Gould (2012, p. 109) concludes from his study of the results of the mandated *Best Start* assessment in New South Wales (NSW Department of Education and Communities 2013) that the expectation in the *Australian Curriculum—Mathematics* that students can make connections between the number names, numerals and quantities up to 10 by the end of the first year at school "would be a low expectation for at least half of the students in NSW public schools". Even in 'disadvantaged' communities (Ginsburg and Seo 2000) and rural and regional communities (Hunting et al. 2012), many children demonstrate that they are powerful mathematicians before they start school. The examination of children's knowledge presented in this Chapter will consider whether this is also true for children in the *Let's Count* and ENRP groups.

4.3 The Early Years Learning Framework and the Australian Curriculum—Mathematics

The current introduction of an Australian curriculum for the first time signifies a time of great change in the Australian education scene. Previously, the federal structure of the Australian constitution has ensured that states and territories have held responsibility for school education while they have shared responsibility for prior-to-school education and care with the Australian government. From 2006, through a cooperative agreement between the eight State and Territory governments and the Australian government, more national approaches have been developed. One result has been the implementation of national mathematics curriculum approaches in both prior-to-school and school sectors. The *Early Years Learning Framework* (Department of Education, Employment and Workplace Relations, DEEWR

2009) was developed for the prior-to-school sector, and the *Australian Curriculum—Mathematics* (ACARA 2013) for the school sector. These curricular documents provide an unprecedented opportunity in Australia to consider and explore just what mathematics children starting school know and bring with them and how their mathematics knowledge developed prior-to-school.

The *Early Years Learning Framework* (DEEWR 2009) promotes preschool children's achievement of five broad learning outcomes. The most relevant to mathematics learning (with the most relevant key components) are:

> Outcome 4: Children are confident and involved learners
> (Children develop a range of skills and processes such as problem solving, enquiry, experimentation, hypothesising, researching and investigating)
> Outcome 5: Children are effective communicators
> (Children begin to understand how symbols and pattern systems work).

The *Early Years Learning Framework* does not specify particular content achievement levels in mathematics but makes the following general statement.

> Children bring new mathematical understandings through engaging with problem solving. It is essential that the mathematical ideas with which young children interact are relevant and meaningful in the context of their current lives. Educators require a rich mathematical vocabulary to accurately describe and explain children's mathematical ideas and to support numeracy development. Spatial sense, structure and pattern, number, measurement, data argumentation, connections and exploring the world mathematically are the powerful mathematical ideas children need (DEEWR 2009, p. 38).

In contrast, the *Australian Curriculum—Mathematics* (ACARA 2013) provides a content-based achievement standard for children at the end of their first year at school:

> Students make connections between number names, numerals and quantities up to 10. They compare objects using mass, length and capacity. Students connect events and the days of the week. They explain the order and duration of events. They use appropriate language to describe location.
> Students count to and from 20 and order small collections. They group objects based on common characteristics and sort shapes and objects. Students answer simple questions to collect information.

This achievement is supported by the Foundation Year proficiencies that focus on understanding, fluency, problem solving and reasoning:

> Understanding includes connecting names, numerals and quantities.
> Fluency includes readily counting numbers in sequences, continuing patterns, and comparing the lengths of objects.
> Problem Solving includes using materials to model authentic problems, sorting objects, using familiar counting sequences to solve unfamiliar problems, and discussing the reasonableness of the answer.
> Reasoning includes explaining comparisons of quantities, creating patterns, and explaining processes for indirect comparison of length.

These Foundation standards and proficiencies indicate what Australian children are expected to know and do in mathematics at the end of their first year of school.

4.4 *Let's Count*

Let's Count (Perry and Gervasoni 2012) is an Australian early mathematics program designed by The Smith Family and the authors to assist parents and family members to help their children aged 3–5 years play, investigate and learn powerful mathematical ideas in ways that develop positive dispositions to learning, and learning mathematics. A key focus of the program is to notice, explore and discuss mathematics as part of everyday activities. The Smith Family is a children's charity "helping disadvantaged Australian children to get the most out of their education, so they can create better futures for themselves" (The Smith Family 2013). *Let's Count* is supported by the Origin Foundation and developed in partnership with Blackrock Investment Management. It was piloted in 2011 in five communities designated as experiencing social and economic disadvantage across Australia. In 2012/2013, The Smith Family introduced a refined *Let's Count* program in six additional sites and further sites were included in 2013/2014. All sites are designated as 'disadvantaged' with high proportions of children classified as starting school 'at risk' developmentally, as measured by the Australian Early Development Index (AEDI) (Centre for Community Child Health 2013). For further details about *Let's Count*, see MacDonald (Chap. 6 of this volume).

4.5 The *Let's Count* Longitudinal Evaluation

The authors of this chapter are responsible for the longitudinal evaluation of *Let's Count.* One aim of this evaluation is to determine whether the *Let's Count* approach has an impact on the formal mathematical knowledge children construct prior to beginning school. Over 2012/2014, data will be gathered at multiple points from early childhood educators (surveys and interviews), parents and other adult members of families (interviews) and children in the year before they start school (one-on-one assessment interview). In this chapter, we consider only the assessment interview data for 125 children in 2012 who formed the study's comparison group.

4.5.1 Assessing Children's Knowledge of School Mathematics

The tool selected to assess children's mathematical knowledge for the *Let's Count Longitudinal Evaluation* was the *Mathematics Assessment Interview* (Gervasoni et al. 2010; Gervasoni et al. 2011). This assessment was designed for young children, is task-based and interactive, derived from extensive research, and enables mathematical learning to be measured in nine domains. One section of the assessment focuses on early mathematics concepts for children beginning school. This assessment was originally developed as part of the *Early Numeracy Research Project* (ENRP) (Clarke et al. 2002; Department of Education, Employment and Training

2001) and following refinement during the *Bridging the Numeracy Gap* project was renamed the *Mathematics Assessment Interview* (MAI) (Gervasoni et al. 2010; Gervasoni et al. 2011).

The principles underlying the construction of the tasks and the associated mathematics growth point framework were to:

- describe the development of mathematical knowledge and understanding in the first 3 years of school in a form and language that was useful for teachers;
- reflect the findings of relevant international and local research in mathematics (e.g., Fuson 1992; Gould 2000; Mulligan 1998; Steffe et al. 1983; Wright et al. 2000);
- reflect, where possible, the structure of mathematics;
- allow the mathematical knowledge of individuals and groups to be described; and
- enable a consideration of children who may be mathematically vulnerable (Gervasoni and Lindenskov 2011).

The interview includes four whole number domains (Counting, Place Value, Addition and Subtraction, and Multiplication and Division); three measurement domains (Time, Length and Mass); and two geometry domains (Properties of Shape and Visualisation). The assessment tasks in the interview take between 30–45 min for each child and were administered in this evaluation by independent, trained assessors who followed a detailed script. Each child completed about 30 tasks in total, and given success with one task, the assessor continued with the next tasks in a domain for as long as a child was successful, according to the script. The processes for validating the growth points, the interview items and the comparative achievement of students are described in full in Clarke et al. (2002).

A critical role for the assessor throughout the interviews was to listen and observe the children, noting their responses, strategies and explanations while completing each task. These responses were noted on a detailed record sheet and then independently coded to

- determine whether or not a response was correct;
- identify the strategy used to complete a task, and
- identify the growth point reached by a child overall in each domain.

This information was entered into an SPSS database for analysis. Of particular interest for this study were the children's responses to tasks in the early mathematics concepts section and the initial tasks in the other domains. Links between the tasks and the *Australian Curriculum—Mathematics* (ACARA 2013) will be made.

4.5.2 The 2012 *Let's Count* Comparison Group

The 125 children in the *Let's Count* comparison group were assessed in December, 2012. They did not participate in the *Let's Count* program but provided a measure of the level of mathematics known by children in *Let's Count* communities prior to

Table 4.1 Percentage of *Let's Count* children assessed as developmentally 'at risk' in one or more AEDI domains (2012 data)

State	*Let's Count* Centres	State average	Australian average
1	10.8–21.0	19.9	22.0
2	20.5–36.8	19.5	22.0

Table 4.2 Percentage of *Let's Count* children assessed as developmentally 'at risk' in two or more AEDI domains (2012 data)

State	*Let's Count* Centres	State average	Australian average
1	2.7–11.0	9.2	10.8
2	11.3–24.3	9.5	10.8

the program commencing. All were eligible to begin school in January, 2013 and aged between 4.5 and 5.5 years. They attended preschool programs in ten centres in two large regional Australian cities. By chance, more boys (56 %) were assessed than girls (44 %).

All of the preschool centres were situated in, and drew children from, communities identified as 'disadvantaged' through community measures such as the Australian Early Development Index (AEDI). The AEDI assesses 'disadvantage' through calculating the percentage of children starting school in a particular district who are deemed to be developmentally 'at risk' in one or more, or two or more, of the following domains:

- physical health and wellbeing;
- social competence;
- emotional maturity;
- language and cognitive skills; and
- communication skills and general knowledge (Centre for Community Child Health 2013).

Tables 4.1 and 4.2 provide an overview of AEDI data concerning the 'at risk' levels from the *Let's Count* communities in the two states. They show that State 1 communities are tracking near the State and National averages while State 2 communities are tracking more 'at risk' compared to the State and National averages.

Data from both levels of developmentally 'at risk' measures show that although the two state sites have been deemed to be 'disadvantaged', their results on the AEDI measures are quite different. We shall return to this later in the paper.

4.6 Children's Mathematical Knowledge

The *Mathematics Assessment Interview* results for the 125 children in the *Let's Count* comparison group are presented in tables in the following section of this chapter. The results have been grouped to match the associated components of the

Table 4.3 Percentage success on tasks with small sets (usually small plastic teddies)

Tasks	*Let's Count* ($n=125$)	ENRP ($n=1438$)	Australian Curriculum Foundation standard
Tasks with small sets			
Count a collection of 4 teddies	95	93	*Students make connections between number names, numerals and quantities up to ten*
Identify one of two groups as "more"	90	84	
Make a set of five teddies when asked	77	85	
Conserve five when rearranged by child	79	58	
Combine 5+3 blue teddies and total	75	na	
Make collection of seven (when shown number 7)	63	na	
Knows one less than seven when one teddy removed	61	na	
Knows one less than seven without recounting	25	na	
Part part whole tasks			
Show six fingers (usually five and one)	79	78	
Six fingers second way	27	20	
Six fingers third way	10	8	
One to one correspondence task			
Know five straws needed when asked to put one straw in each of five cups	88	92	

Australian Curriculum—Mathematics Foundation Year standard. This enables an assessment to be made about the appropriateness of the curriculum standard for children at the end of their first year at school. Each table shows the percentage of children who were successful with each task for the *Let's Count* prior-to-school group in December 2012 and the 1438 children in the ENRP beginning school group in February/March 2001 (Clarke et al. 2006). Due to the refinement of some assessment tasks in 2009, results for some tasks were not available for the ENRP group. These have been indicated with 'na' in the tables. It should be noted that the ENRP cohort are representative of first year of school children across State 2. The average age of these children is approximately 3–4 months greater than the average age of the *Let's Count* cohort.

Table 4.3 focuses on children's success with tasks involving small sets of objects.

For this set of tasks, the *Let's Count* and ENRP cohorts have performed similarly, with both showing that about 75% of the children were able to demonstrate the curriculum standard before they begin school (*Let's Count)* or shortly thereafter (ENRP).

Table 4.4 shows the percentage of children able to recognise the number of dots on a card (either in a standard pattern or a random collection) without counting them, and also their ability to match a numeral to the number of dots. High percentages of children from both the *Let's Count* and the ENRP cohorts were able to subitise small numbers of dots in both random and standard configurations. Not

Table 4.4 Percentage success in subitising tasks and matching numerals to dots

Tasks	*Let's Count* (*n* = 125)	ENRP (*n* = 1438)	Australian Curriculum Foundation standard
Subitising tasks			
Recognise zero without counting	81	82	*Students make connections between number names, numerals and quantities up to ten*
Recognise two without counting	94	95	
Recognise three without counting	83	84	
Recognise random three without counting	86	na	
Recognise four without counting	70	71	
Recognise random four without counting	50	na	
Recognise five without counting	44	43	
Recognise nine without counting	16	9	
Matching numerals to dots tasks			
Match numeral to zero dots	73	63	
Match numeral to two dots	90	86	
Match numeral to three dots	73	79	
Match numeral to three random dots	82	na	
Match numeral to four dots	73	77	
Match numeral to four random dots	69	na	
Match numeral to five dots	65	67	
Match numeral to nine dots	38	41	

surprisingly, standard configurations led to higher success rates than random arrangements of the dots as subitising is known to be a pattern recognition activity (Wolters et al. 1987).

Perhaps more surprising is that about one-sixth of the *Let's Count* cohort could subitise nine dots while less than one-tenth of the ENRP cohort could do so. It should be noted that the nine dots were presented as in Fig. 4.1 which is the logo for a popular television network in Australia. Perhaps young children are watching more television in 2012 than they were in 2001?

The majority of students could also match numerals to the number of dots, although nine was much harder to match than the other numbers. The ability to recognise quantities without counting and match numerals to numbers of objects is important for future number work and it would seem that the majority of children from both the *Let's Count* and ENRP cohorts are well on their way to achieving the Foundation standard.

Subitising is an example of the importance of pattern and structure in young children's mathematical learning. The Foundation proficiencies of fluency and reasoning focus on continuing and creating patterns. The data presented in Table 4.5 suggest that about three-quarters of children from both the *Let's Count* preschool cohort and the ENRP school cohort can match patterns at the time of their transition to school, and about one-third of children from both cohorts can continue and explain a pattern.

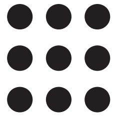

Fig. 4.1 Nine dots

Table 4.5 Percentage success in pattern tasks

Tasks	*Let's Count* (*n* = 125)	ENRP (*n* = 1438)	Australian Curriculum Foundation standard
Pattern tasks			
Name colours in pattern	98	94	*Fluency proficiency includes: continuing patterns. Reasoning proficiency includes: creating patterns.*
Match pattern	72	76	
Continue pattern	34	31	
Explain pattern	34	31	

The results concerning patterning suggest that many children will need more than an 'ABAB' pattern to either match or continue in order for there to be sufficient challenge in this important aspect of mathematics development.

The Foundation standard of the *Australian Curriculum—Mathematics* also focuses on students counting to and from 20 and ordering small collections. Several tasks in the MAI focused on sequence counting, counting a larger collection of at least 20 items and ordering numerals. The percentage of students able to complete these tasks is presented in Table 4.6.

The data suggest that the majority of the *Let's Count* cohort can rote count to 10 and at least one-quarter can complete the rote forward count to 20, indicating that they have already met this component of the Foundation standard. This result is reinforced by Gould (2012) who found that 16 % of students in New South Wales could rote forward count to at least 30, the standard in that state for the end of the

Table 4.6 Percentage success with counting and ordering numerals

Tasks	*Let's Count* (*n* = 125)	ENRP (*n* = 1438)	Australian Curriculum Foundation standard
Counting tasks			
Rote count to ten	87	na	*Students count to and from 20 and order small collections*
Rote count to 20	29	na	
Count a collection of at least 20 and, when one item is removed, knows total without recounting	8	na	
Ordering numbers tasks			
Order numeral cards 1–9	48	46	
Order numeral cards 0–9	32	38	
Orders three one digit numbers	47	na	
Orders three two digit numbers	28	na	

Table 4.7 Percentage success with length and time measurement tasks

Tasks	*Let's Count* (*n* = 125)	ENRP (*n* = 1438)	Australian Curriculum Foundation standard
Length Measurement Tasks			
Ordering three candles smallest to largest	73	61	*Students compare objects using mass, length and capacity*
Ordering four candles smallest to largest	54	50	
Accurately compares two lengths—string and stick	65	na	
Measures length using informal units	8	na	
Time measurement tasks			*Students connect events and the days of the week*
Aware of the purpose of a clock	83	na	
Knows some days/months	17	na	

first year of school under the previous state syllabus (NSW Department of Education and Training, DET 2002). Few of the *Let's Count* children could both count 20 teddies successfully and identify how many teddies remained when one teddy was removed. It appears that a focus on the cardinal value of numbers to 20 would be a profitable area for instruction in the first year at school, though this is only connected vaguely with the Foundation standard "connecting names, numerals and quantities".

The absence of ENRP cohort comparisons for most of these counting and ordering tasks is unfortunate and results from the ENRP data entry being less differentiated for these tasks. On the two ordering questions that are comparable, both cohorts perform similarly.

Several tasks in the interview focused on measuring length and time. Table 4.7 highlights that many children beginning school are able to compare and order lengths, in line with the Foundation standard, and are also aware of the purpose of a clock. Seventeen percent of children knew the names of some days of the week and months. In the cases where comparisons are available with the ENRP cohort, the *Let's Count* cohort is on or above par.

Spatial reasoning is a key aspect of learning mathematics (Clements and Sarama 2004; Perry and Dockett 2008). The data presented in Table 4.8 show the success rates of both the *Let's Count* and ENRP cohorts with tasks involving describing and interpreting locations, recognising the properties of shapes and using mental imagery to manipulate shapes.

The data suggest that the *Let's Count* cohort was proficient in these spatial tasks and almost all children met the Foundation standard prior to beginning school. In the three tasks for which there is comparable ENRP data, the *Let's Count* cohort succeeded to at least at an equivalent level. These data provide impetus for teachers in the first year of school to consider how they can engage children in more probing tasks than the shape recognition and naming experiences that occur frequently in preschools and the first year of school.

Table 4.8 Percentage success on spatial tasks

Tasks	*Let's Count* ($n = 125$)	ENRP ($n = 1438$)	Australian Curriculum Foundation standard
Language of location tasks			
Beside	94	88	*Students use appropriate language to describe location*
Behind	87	87	
In front of	91	83	
Properties of shapes tasks			*Students group objects based on common characterist-ics and sort shapes and objects*
Knows square	85	na	
Knows circle	92	na	
Knows rectangle	74	na	
Knows some triangles	83	na	
Knows all triangles	63	na	
Visualisation Tasks			
Identifies a reoriented rectangle in room	89	na	
Identifies and traces possible shapes when a shape is partially hidden	16	na	

Peeking Over Task

Close your eyes for a moment while I get the next task organised. ... Now open your eyes. [Hold the green piece of paper with the partially hidden yellow shape in front of the child].

I have a yellow shape that is peeking over thispiece of paper. We can only see part of the yellow shape. [Place the paper down on the table]. What do you think the shape might be?

Show me with your finger how that yellow shape "goes" underneath. [If necessary for understanding, ask can you draw aroundthe outside of the shape with your finger?]

Fig. 4.2 Example of the peeking over task from the visualisation section of the interview

The most difficult geometry task involved the recognition of hidden shapes and required children to use spatial imagery. The task is reproduced in Fig. 4.2 to illustrate the level at which the *Let's Count* cohort were successful.

Such performance is well beyond that expected by the Foundation standard and alerts first year of school teachers to the possibility that, for a sizeable portion of their class, more advanced experiences are required than is typically suggested in curriculum guidelines.

The MAI also includes a range of tasks involving calculations, although few of the *Let's Count* cohort progressed far in these domains. The results from four calculation tasks (Table 4.9) show that many of the children were capable of completing the initial addition, multiplication and division tasks, thus providing a school starting level which might be viewed as surprising. All tasks were presented orally and involved the use of materials. There are no ENRP comparison data available for these tasks.

Table 4.9 Percentage success on calculation tasks involving materials

Tasks	*Let's Count* ($n=125$)	Australian Curriculum Foundation Standard
Calculation tasks		
Adds 5+3 when screen over five removed	49	*Problem solving proficiency: using materials to model authentic problems, sorting objects, using familiar counting sequences to solve unfamiliar problems, and discussing the reasonableness of the answer*
Adds 9+4 when screen over nine removed	25	
Calculates total for two teddies in four cars	48	
Divides 12 teddies between four mats	31	

Most children who were successful with the first three tasks worked out the answers by counting all the items one by one. A small number of students used the counting on strategy. Most children solved the division task through grouping rather than sharing by ones. The results and the children's strategies indicate that a large group of children are well on their way to meeting the Foundation problem solving standard before beginning school.

4.6.1 Performance Differences Between Girls and Boys

One question of interest for the study was whether there was any difference in performance between girls and boys. For the most part, data from the MAI is categorical (mainly Yes/No). So, for all such items, χ^2 tests were run to ascertain differences across gender. In only one case was a statistically significant result returned (at the 5% level). This was for the question "What colour is the 3rd teddy (in a line of teddies)?" and a higher percentage of boys answered correctly than girls. However, with $p=0.046$, this single result is only marginally statistically significant and probably not educationally significant, given that boys and girls performed equally on identification of the fifth teddy.

4.6.2 Performance Differences Between the Two States

A similar χ^2 analysis for all suitable MAI items was used to ascertain if there were any statistically significant differences across the geographical origin of the data from State 1 or State 2. Sixty-four children (51.2% of the cohort) attended preschool in State 1 while 61 (48.8%) were from State 2. On 14 of the individual MAI questions, statistically significant differences (at the 5% level) were found across the two states. Table 4.10 provides details of these.

The findings in terms of state differences cluster in interesting ways. On the items that have delivered statistically significant differences between the states, State 2 children in the *Let's Count* cohort have performed more ably on the counting, subitising

Table 4.10 Statistically Significant Differences across Geographical Location

MAI Item	χ^2 value	*p*	Better performing state
Please get five blue teddies	4.770	0.024	State 2
(After changing arrangement of five teddies) tell me how many teddies now	6.309	0.011	State 2
Five teddies and three teddies. How many teddies altogether: 5+3	4.515	0.027	State 2
Put a green teddy behind the blue teddy	5.041	0.023	State 1
Please make the same pattern	5.566	0.015	State 1
I'm going to show you some cards quite quickly. Tell me how many dots you see. (4)	3.928	0.037	State 2
Add in question for subitise 3–2 as I do not seem to be able to find it	4.134	0.035	State 2
Find the number to match the dots. (2)	8.261	0.004	State 2
Find the number to match the dots. (3)	5.056	0.020	State 2
Please show me six fingers	4.272	0.032	State 2
Add in question for enumerates #2	10.664	0.005	State 1
Add in question for enumerates #1	21.711	0.000	State 1
Add in question for enumerates #0	15.233	0.000	State 1
Here are some numbers (*two, five, nine on separate cards)*. Order these from smallest to largest. Please point to the largest. Please point to the smallest	5.927	0.012	State 1

and matching numerals to sets of dots while State 1 children have performed more ably on the location, enumeration and ordering tasks. In particular, the enumeration tasks are enlightening in terms of the methods used by the children. In all three of the enumeration tasks listed in Table 4.10, State 2 children are more likely to obtain the correct answers but State 2 children who do get correct answers are much more likely than State 1 children with correct answers to subitise rather than count.

It is tempting to suggest that there may be curriculum or pedagogical differences between the preschool programs in each state but we do not have any evidence for this as the specific programs undertaken by the children in the *Let's Count* comparison group were not studied.

4.7 Implications for Transition

It is well established that the play, exploration and engagement of young children in everyday activities in the preschool and other venues involves much informal mathematical activity (Ginsburg and Seo 2000; Hunting et al. 2012; Perry and Dockett 2008; Wager 2013). Nevertheless, the data presented in this chapter highlight the broad range of formal mathematics knowledge that many children construct prior to beginning school, perhaps through their own play, perhaps through more intentional teaching instigated by their early childhood educators. This finding supports the findings of earlier research (Bobis 2002; Clarke et al. 2006; Gervasoni and Perry 2013; Ginsburg and Seo 2000, Gould 2012; Hunting et al. 2012).

The *Early Years Learning Framework for Australia* (DEEWR 2009) extols the virtues of play as an important pedagogy in early childhood education. While it is not the only pedagogy used in preschools, it is well accepted and widely adopted.

> Play provides opportunities for children to learn as they discover, create, improvise and imagine. When children play with other children they create social groups, test out ideas, challenge each other's thinking and build new understandings. Play provides a supportive environment where children can ask questions, solve problems and engage in critical thinking. Play can expand children's thinking and enhance their desire to know and to learn. In these ways play can promote positive dispositions towards learning. (DEEWR 2009, p. 15)

While the results presented in this chapter highlight the diversity of children's mathematical knowledge, it is also apparent that children's everyday home and preschool experiences prepare a large proportion of them well for the transition to learning mathematics at school. It also appears that there is little relationship between the extent of young children's mathematical knowledge before they start school and the rating of 'at risk' status given to their communities by first year of school teachers. While both geographical sites were deemed to be 'disadvantaged', they had quite different AEDI profiles. The State 2 sites measured as 'more disadvantaged' on the AEDI than the State 1 sites but the mathematical performance of the children in the *Let's Count* comparison group were mostly consistent across the sites. Even when there were statistically significant differences across the sites, the message was mixed with the children from State 1 performing better on some tasks and less well on others. There is a body of research that suggests a strong link between levels of community disadvantage and the academic performance of the children from communities (Carmichael et al. 2013; Caro 2009) as they move through school. Data from the study reported here suggest that either this might not play out in the same way for preschool children or that the AEDI is not a very reliable predictor of preschool children's mathematical performance. This is an important finding for education authorities and teachers to consider.

Comparison of children's mathematics knowledge with the *Australian Curriculum—Mathematics* Foundation standard and proficiencies suggest that large numbers of children in both the *Let's Count* preschool group and the ENRP Beginning School group met the end of year Foundation Standard in Number, Measurement and Geometry prior to or just after beginning school. An implication of this finding is the critical need for teachers to find out what mathematics children know when they begin school and extend the Foundation mathematics curriculum right from the first day of school to challenge and engage many children in mathematics learning. While teachers are skilled in differentiating instruction for children, these findings highlight the importance of this role.

Overall, it appears that the new *Australian Curriculum—Mathematics* Foundation standard is neither sufficiently challenging for children nor adequate for signaling to teachers the type of experiences and instruction that are important. Whilst acknowledging that the *Australian Curriculum—Mathematics* encourages teachers to adjust curriculum and instruction to match children's knowledge, it must also adequately reflect the mathematical capabilities of children when they begin school. The data presented in this chapter suggest that Australian education authorities need to undertake more fine-tuning to set the Foundation standard at a level that sufficiently engages and challenges children at the time of transition.

4.8 Conclusion

The findings reported in this chapter indicate that, prior to beginning school, many Australian children have constructed powerful mathematical ideas that involve number, measurement and geometry. It is essential that both preschool and primary school teachers notice the extent of the mathematics that children know and use so that they can build upon and extend this knowledge during children's play and explorations. The findings also suggest that it is essential for first year of school teachers to examine curriculum documents critically and in light of their own assessment of children's mathematics knowledge and capabilities, with the intention of refining and extending these frameworks. Only then can our community ensure that all children have the opportunity to thrive mathematically during the transition to school and beyond.

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Chapter 5
Transition to School: Prior to School Mathematical Skills and Knowledge of Low-Achieving Children at the End of Grade 1

Andrea Peter-Koop and Sebastian Kollhoff

Abstract Recent psychological studies as well as research findings in mathematics education highlight the significance of early number skills for the child's achievement in mathematics at the end of primary school. In this context, the ongoing 3-year longitudinal study discussed in this chapter, investigates the development of early numeracy understanding of 408 children from 1 year prior to school until the end of Grade 1. The study seeks to identify children who struggle with respect to their mathematics learning after the first year of school and compare their achievements with their number concept development 1 year prior to school as well as immediately prior to school entry (Grade 1). Initial findings suggest that children's understanding and skills with respect to number and counting are important precursors for later school success. The children who were identified as low-achievers in mathematics at the end of Grade 1, also demonstrated less knowledge and skills than their peers prior to school.

5.1 Introduction

Children start developing mathematical knowledge and abilities a long time before they enter formal education (Anderson et al. 2008 ; Ginsburg et al. 1999). In their play, their everyday life experiences at home and in child care centres they develop a foundation of skills, concepts and understandings about numbers and mathematics (Anderson et al. 2008; Baroody and Wilkins 1999). However, the range of mathematical competencies children develop prior to school obviously varies quite substantially. While most preschoolers manage to develop a wide range of informal knowledge and skills in early numeracy, there is a small number of children who, for various reasons, struggle with the acquisition of number-skills (Clarke et al. 2008; Peter-Koop and Grüßing 2014). Furthermore, clinical psychological studies suggest

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that children potentially at risk in learning mathematics can already be identified 1 year prior to school entry by assessing their number concept development (Aunola et al. 2004; Krajewski 2005). Findings from these studies also indicate that these children benefit from an early intervention prior to school helping them to develop a foundation of knowledge and skills for successful school-based mathematics learning. This seems to be of crucial importance as findings from the SCHOLASTIK project (Weinert and Helmke 1997) suggest that students who are low achieving in mathematics at the beginning of primary school tend to stay in this position. In most cases a recovery does not occur. In addition, Stern (1997) emphasises that subject-specific knowledge prior to school is more important for later success at school than general cognitive factors such as intelligence. Hence, the development of early numeracy skills should be included in early childhood education prior to school entry in kindergarten or preschool programs.

5.2 Theories on Number Concept Development

While pre-number activities based on Piaget's *logical foundations model* are frequently still current practice in first year school mathematics (Anderson et al. 2008), research findings as well as curriculum documents increasingly stress the importance of children's early engagement with sets, numbers and counting activities for their number concept development. Clements (1984) classified alternative models for number concept development that deliberately included early counting skills (Resnick 1983) as *skills integrations models*. Piaget (1952) assumed that the development of number concept builds on logical operations based on pre-number activities such as classification, seriation and number conservation. He emphasised that the understanding of number is dependent on operational competencies. In his view, counting exercises do not have operational value and hence no conducive effect on conceptual competence regarding number. However, since the late 1970s this theory has been questioned due to research evidence suggesting that the development of number skills and concepts results from the integration of number skills, such as counting, subitising and comparing (Fuson et al. 1983, Clements 1984; Sophian 1995).

Krajewski and Schneider (2009) provide a theoretical model that is based on the assumption that the linkage of imprecise nonverbal quantity concepts with the ability to count forms the foundation for understanding several major principles of the number system. The model depicts how early mathematical competencies are acquired via three developmental levels (see Fig. 5.1). In the *first level* (basic numerical skills) number words and number-word sequences are isolated from quantities. In the sense of Resnick's "proto-quantitative comparison schema" (1989, p. 163) children compare quantities without counting by using words like 'less', 'more' or 'the same amount'. At the age of 3–4 years most children start to link number words to quantities, i.e. they develop awareness of numerical quantity (Dehane 1992) and hence enter the *second level* (quantity number concept). The understanding of the linkage between quantities and number words is acquired in two phases:

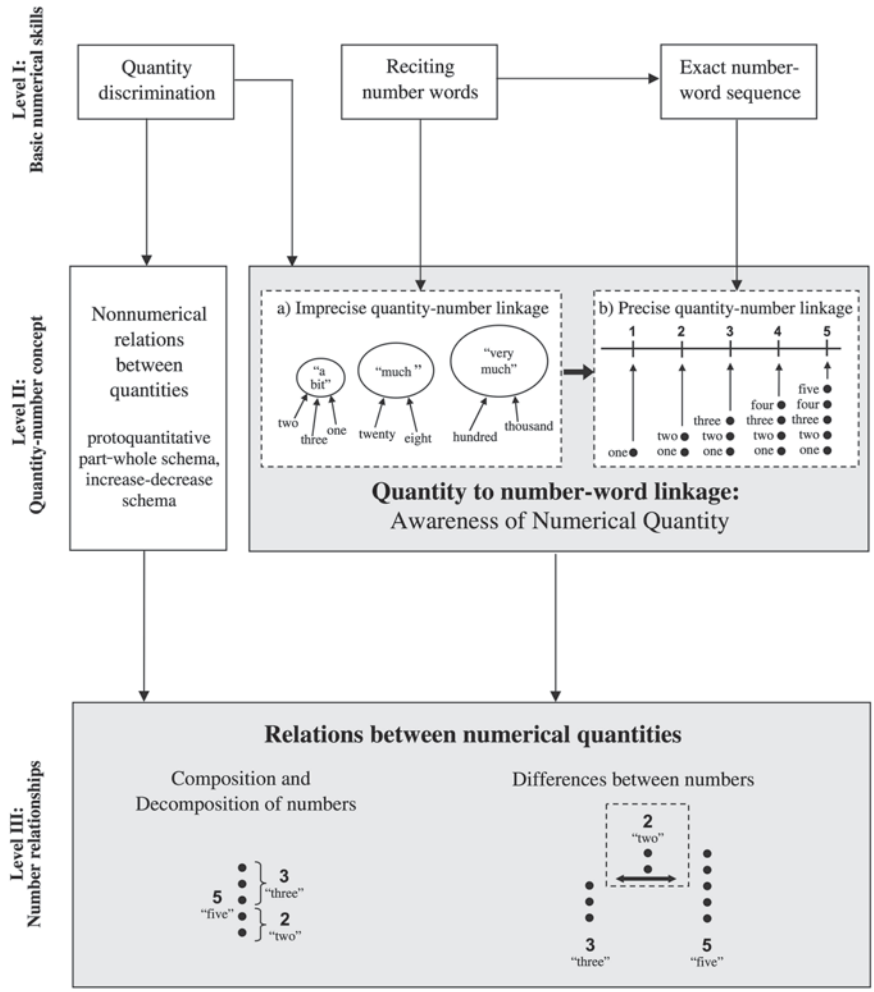

Fig. 5.1 Model of early mathematical development. (Krajewski and Schneider 2009, p. 515)

> First they develop an imprecise, vague conception of the attribution of number words to quantities and assign number words to rough quantity categories (Level IIa: imprecise quantity to number-word linkage)…. The ability to distinguish close number words develops in the second phase of the quantity to number-word linkage, when number words are also linked to exact quantities (Level IIb: precise quantity to number-word linkage, where counting is linked with quantity discrimination). (Krajewski and Schneider 2009, p. 514)

Furthermore, at this level children also gain experiences with non-numerical relations between quantities as they increasingly understand "proto-quantitative part-whole schema", i.e. the understanding that a quantity can be split into pieces which, taken together, make up the whole quantity, as well as "proto-quantitative increase/decrease schema", i.e. the insight that quantities change if something is taken away or added (Resnick 1989, p. 163).

At the *third level* (linking quantity relations with number words) children then understand “that the relationship between quantities also takes on a number-word reference. They realise that numerically indeterminate quantities, e.g., “all” lollies, can be divided into smaller amounts, e.g., “a few” lollies, and “also understand that this can also be represented with precise numbers” (Krajewski and Schneider 2009, p. 516), e.g., five lollies can be split into two lollies and three lollies, which then again make five altogether. Fuson (1988) described this as *composition* and *decomposition* of numbers. Furthermore, children discover that two numerical quantities (e.g., three lollies and five lollies) differ by a third numerical quantity (two lollies).

However, it is important to note that children are not necessarily at the same developmental stage with respect to number words and number symbols. Furthermore, a child might have already reached the third level when dealing with smaller numbers, while s/he still operates with larger numbers on the second level. The use of manipulatives also affects the children’s performances, so that a child might already reach the second or third level when using concrete objects, while still not able to deal with tasks based on iconic or symbolic representations. Hence, with respect to numerical development, it is very difficult to classify a child exactly to one level (Krajewski et al. 2009).

In summary, Krajewski (2008) states that the quantity-number-competencies that children develop up to school entry build the foundations for later understanding of school mathematics. While competencies on the third level (i.e. number relationships) reflect first computation skills and in this respect initial arithmetic understanding, the first two levels (i.e. basic numerical skills/ quantity-number concept) can be accounted as “preparatory mathematical skills” (pp. 208–281).

5.3 Number-Quantity Competencies and their Influence on the Transition to School Mathematics

In a longitudinal study Krajewski and Schneider (2009) investigated the predictive validity of the quantity-number competencies of these developmental levels for mathematical school achievement. The results of the studies indicate that quantity-number competencies related to the *second level* (see Fig. 5.1) measured in kindergarten predict about 25 % of the variance in mathematical school achievement at the end of grade 4. Moreover, a subgroup analysis indicated that low-performing fourth graders had already shown large deficits in their early quantity-number competencies (p. 523). It can be concluded that these early quantity-number competencies constitute an important prerequisite for the understanding of school mathematics. These results conform to other longitudinal studies (Aunola et al. 2004, Kaufmann 2003).

Furthermore, a previous intervention study by the first author in 2005–2008 indicates that (at least in Germany) children with a migration background[1] are overrepresented in the group of preschoolers potentially at risk in learning school mathematics (Grüßing and Peter-Koop 2008; Peter-Koop and Grüßing 2014; Peter-Koop et al. 2008). A total

[1] Migration background in this context means that the children speak at least one language other than German at home.

of 854 children were interviewed/tested 1 year prior to school with three different instruments—an early numeracy interview, a standardised test as well as an intelligence test for preschoolers and the individual results led to the identification of 73 children potentially at risk in learning school mathematics based on the current stage of their number concept development. Following an 8 months long, primarily play-based, intervention (for details see Peter-Koop and Grüßing 2014, pp. 311–313) all participants of the study were interviewed/tested again immediately before entering Grade 1. In order to monitor long-term effects of the intervention, follow-up tests were conducted at the end of Grade 1 and Grade 2. The intervention for the 73 preschoolers identified to be potentially at risk learning school mathematics was conducted in two treatment groups: Children in group 1 were visited weekly by a pre-service teacher who had been prepared for this intervention as part of a university methods course. The intervention for the children in group 2 in contrast was conducted by the kindergarten teachers. While the intervention for group 1 was carried out one-on-one at a set time each week, the kindergarten teachers working with the children in group 2 primarily tried to use every day related mathematical situations, focussing on aspects such as ordering, one-to-one correspondence or counting, as they arose in the children's play or everyday routine. In particular, they challenged the children identified to be at risk in these areas. Children in both groups were not aware of the fact that they took part in an intervention. However, the parents of all children who took part in the intervention had been informed and had given their written permission. It is important to note that for ethical reasons it was not possible to establish a control group, i.e. children identified to be potentially at risk who did not receive special support prior to school, as parents would not have agreed for their children to be part of this group.

Key results of the study can be summarised as follows (Peter-Koop and Grüßing 2014):

- The data clearly show short-term effects of the intervention. The children potentially at risk in particular have increased their competencies in those areas that were addressed during the intervention, i.e. knowledge about numbers and sets as well as counting abilities, and performed significantly better in the post-test, especially in tasks related to ordinal numbers, matching numerals to dots, ordering numbers, knowing numbers before/after and part-part-whole relationships (p. 314).
- With respect to the substantial increase in achievement demonstrated by the children of the two intervention groups, no significant difference between the group of children who experienced a weekly one-on-one intervention and the group of children who received remedial action within their groups was found (p. 316).
- While children with a migration background were over-represented in the group of preschoolers who were identified as at risk with respect to learning school mathematics (see above), this group also demonstrated the highest increase in mathematical achievement in the test interval. While the achievement of both groups, i.e. migrant children and children with a German speaking background increased ($p < 0.001$) within the test interval, the children with migration background demonstrated an increase of 3.6 points between pre- and post-test compared to an increase of 2.9 points in the remaining group of children from German families (p. 315).

- Further analyses of data collected at the end of Grade 1 and Grade 2 suggest that for more than 50% of the children from the two treatment groups the increase in their mathematical achievement prior to school entry proves to be of lasting effect at the end of Grade 1 (Grüßing and Peter-Koop 2008, pp. 77–78). However, this percentage drops significantly after year 2 (Peter-Koop and Grüßing 2014, p. 317). One possible explanation for this finding relates to curriculum. In Grade 2 mathematics in Germany the focus shifts from number work to operations—a concept area that was not included in the intervention.

While overall the results of the intervention study are encouraging, there are a number of questions that cannot be addressed on the basis of the data collected. Since the study lacks a control group (see above), it is not clear how many of the children identified to be potentially at risk learning school mathematics based on their number concept development 1 year prior to school would have shown at least average achievement at the end of Grade 1 without participating in the intervention. In order to optimise early intervention for children at risk, it is necessary to understand which of the skills contributing to children's number concept development and counting, the children who are low achieving in mathematics at the end of Grade 1 particularly struggle with before school entry in comparison to their higher achieving peers. Research suggests that knowledge and skills with respect to number word sequences, subitising and part-whole understanding are key predictors for the identification of children with dyscalculia[2] in Grade 1 or Grade 2 (Dornheim 2008).

Considering the findings from the SCHOLASTIK project (Weinert and Helmke 1997) indicating that low achievers in mathematics at the beginning of primary school in general tend to stay in this position, an early intervention for these children seems to be of crucial importance. Hence, a screening instrument to be applied 1 year prior to school would help to identify those children who should receive special support prior to school entry, i.e. Grade 1. In this context, the OTZ, i.e. the standardised test used in the study, proved to be very difficult for non-German speaking background children due to its demands on German language comprehension. The data from the 2005–2008 study suggests that the EMBI-KiGa (see methodology) is a suitable instrument for the collection of information on preschoolers' individual number learning and respective identification of children that need special support.

These aspects are addressed in a recent longitudinal study (2011–2014) using the same instruments and the same measuring points (1 year prior to school, immediately before school entry, at the end of Grade 1 and at the end of Grade 2) as in the previous study, while the focus of this new study is different. It is recursive in nature, which means that rather than identifying children potentially at risk learning mathematics 1 year prior to school, the lower-achieving learners at the end of Grade 1 are identified[3]. For these children the longitudinal data from two previous measuring points will be analysed to investigate whether these children already showed

[2] However, it is important to note that not all arithmetic learning difficulties can be put on a level with dyscalculia.

[3] A fourth measuring point was included in order to acknowledge the fact that the group of low-achieving children might change towards the end of junior primary school, i.e. that children who

less knowledge with respect to numbers, quantities and counting than did their more successful peers in Grade 1 and if this is the case, to identify the areas that these children—in contrast to their peers—struggled with prior to school.

Furthermore, the new study seeks to validate the EMBI-KiGa as a suitable screening instrument as well as use data from the first measuring point to match profiles of children at risk from the first study and to create a control group of children who did not receive any intervention prior to school and compare their development with children from the intervention group. However, this paper will focus on the identification of low achievers at the end of Grade 1 and address the following research questions:

1. Which children have clearly below average achievement at the end of Grade 1 with respect to their early numeracy skills?
2. Which content areas do these children struggle with most?
3. What number-quantity competencies did these children demonstrate 1 year prior to school and immediately before school entry?
4. Which content areas did they struggle most with before school entry?

5.4 Methodology

The data collection involves four measuring points MP1–MP4 (an overview about the design of the study is provided in Table 5.1). During each measuring point all children participating in the study were given a standardised test on number concept development suitable for their respective age as well as a task-based interview. For the two measuring points prior to school entry (MP1 and MP2) the following two instruments were used, with each individual interview lasting 15–30 min.

- the German version of the *Utrecht Early Numeracy Test* (OTZ; van Luit et al. 2001)—a standardised individual test in interview form aiming to measure children‘s number concept development that involves logical operations based tasks as well as counting related items,
- the *Elementarmathematisches Basisinterview* for use in kindergarten (EMBI-KiGa) based on the *First Year at School Mathematics Interview* (FYSMI)[4] developed in the context of the Australian *Early Numeracy Research Project* (Clarke et al. 2006)—a task-based one-on-one interview for 5-year-olds allowing children to articulate their developing mathematical understanding through the use of specific materials provided for each task, which has been published by Peter-Koop

show slower (mathematical) development than the majority of their peers might perform more weakly at the end of Grade 1 than at the end of Grade 2.

[4] The FYSMI is conducted in the first year of school, which in Australia is the preparatory grade preceding Grade 1. This preparatory year is compulsory and children are aged between 4 years 9 months and 6 years. In Germany in contrast, formal schooling starts with Grade 1 when children are 6 years old. While the vast majority of German five-year-olds attend kindergarten, this is not compulsory and involves fees to be paid by the parents.

Table 5.1 Measuring points, instruments and participants of the study

Measuring points	Instruments	Participants	Ages of the participants (years)
June 2011 MP 1	OTZ	Children participating in the study ($n=538$)	4–5
	EMBI-KiGa	Children participating in the study ($n=538$)	
June 2012 MP 2	OTZ	Children participating in the study ($n=495$)	5–6
	EMBI-Kiga	Children participating in the study ($n=495$)	
June 2013 MP 3	DEMAT 1+	All grade 1 classes with children participating in the study ($n=2250$)	6–7
	EMBI	Children participating in the study ($n=408$)	
June 2014 MP 4	DEMAT 2+	All grade 2 classes with children participating in the study	7–8
(to be conducted)	EMBI	Children participating in the study	

and Grüßing (2011) while the original Australian document is published by the Department of Education, Employment and Training (DEET) (2001).

Data on student achievement in mathematics after the first and second year of primary school is collected with the following instruments:

- *Deutsche Mathematiktests für 1. und 2. Klassen* (DEMAT 1+; Krajewski et al. 2002/DEMAT 2+; Krajewski et al. 2004)—German curriculum based standardised paper and pencil tests conducted at the end of the school year with the whole class.
- *Elementarmathematisches Basisinterview Zahlen und Operationen* (EMBI; Peter-Koop et al. 2007)—a task- and material-based one-on-one interview assessing children's developing mathematical understanding in the four areas counting, place value, addition/subtraction strategies, multiplication/division strategies[5].

The data reported in this chapter only involve the first three measuring points while the third measuring point is the basis for the following analyses. Since the study aims to monitor long-term development, a fourth measuring point at the end of Grade 2 is planned in order to investigate whether the group of low achievers identified at the end of Grade 1 is still low achieving at the end of Grade 2 or whether the number of children low achieving in school mathematics will increase or decrease and with which areas they are (still) struggling. Furthermore, this paper only focuses on the

[5] This instrument is a German adaptation of the Australian Early Years Interview (Department of Education, Employment and Training 2001).

children participating in all three measuring points ($n = 408$). At this point the analysis of the data from the DEMAT 1+, i.e. additional data from all Grade 1 classes with children participating in the study ($n = 1842$), is still in progress. The analysis aims to specify and diminish possible intra- and inter-group effects related to mathematics instruction in Grade 1. Hence, this information cannot be included in this paper.

For a total of 408 children (206 male, 202 female), complete data sets from the first three measuring points are available[6]. Concerning the migration background of the children, the sample includes 193 children (47.3 %) with and 215 children (52.7 %) without a migration background. This set of data provided the basis of the quantitative analysis with the use of SPSS.

In order to identify low achieving children at the end of Grade 1 based on their performances in the EMBI, the growth points[7] that are used to describe student achievement were translated into number scores counting one point for each growth points > 0 in each of the four interview parts—A: Counting, B: Place Value, C: Strategies for Addition and Subtraction, D: Strategies for Multiplication and Division. Based on this scoring the maximum number of points is 23.

5.5 Results

In the following section key results of the first three measuring points MP 1 to MP 3 of the study will be presented with respect to the four research questions guiding the study. However, it is important to note that more detailed and complex analyses will be conducted after the completion of the data collection in 2014.

5.5.1 Identification of Low-Achieving Children in the Sample

In order to identify the children in the sample who are low achieving in mathematics at the end of Grade 1 a cross mapping of the results in the DEMAT 1+ and EMBI

[6] In order to base the statistical analyses on a complete and coherent data set, all student data that was incomplete with the respect to all measuring points or clearly incorrect due to mistakes during the data collection and recording were omitted.

[7] The framework of "growth points" reflects the analysis of "available research on key stages of levels in young children's mathematics learning, as well as frameworks developed by other authors and groups to describe learning" (Clarke et al. 2002, p. 12). The framework was developed to describe mathematical growth of children from 5 to 8 years of age. According to the ENRP researchers "growth points can be considered primary stepping stones along the way to understanding important mathematical ideas" (Clarke et al. 2003, p. 69). To illustrate this concept, the growth point descriptors for counting (interview part A) are given below (Clarke et al. 2002, p. 124).

A. Counting: 0. Not apparent; 1. Rote counting; 2. Counting collections up to 20 objects; 3. Counting by 1 s (forward/backward from variable starting points between 1 and 100; knows numbers before/after); 4. Counting from 0 by 2, 5, and 10 s; 5. Counting from x (where x > 0) by 2, 5, and 10 s; 6. Extending and applying counting skills.

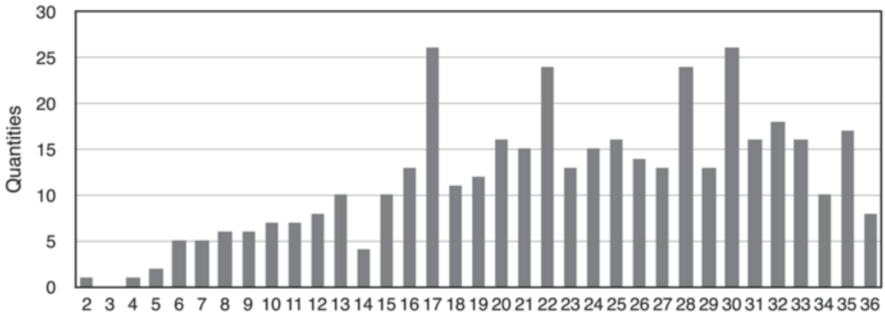

Fig. 5.2 DEMAT 1+ raw values at MP 3

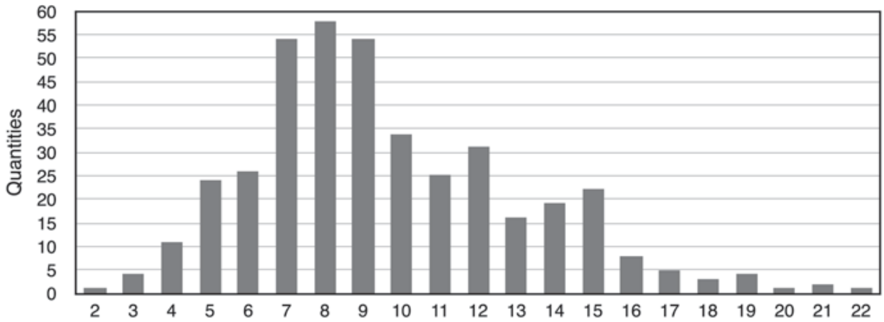

Fig. 5.3 EMBI score sums at MP 3.

was used to eliminate the children with low performance in only one of both tests. In this respect the standardised DEMAT 1+ values provided a pre-selection of the lowest 20 % (DEMAT 1+ raw value <20), which was further validated with the children's performance in the EMBI.

For this validation the overall scores in the DEMAT 1+ (see Fig. 5.2) were analysed and compared with the overall scores in the EMBI (see Fig. 5.3). The analysis of the performances in the EMBI showed that the lowest 16 % did not reach 7 points or more and there was a significant break ($p<0.001$) between the groups scoring 6 and 7 points respectively, which was used as a further criterion to identify the low achievers at the end of Grade 1. As a result 49 children (12 % of the complete sample) performed low in both the standardised test and the interview as well, while the majority of the children tested and interviewed (88 %) demonstrated elaborate abilities and knowledge as described by Anderson et al. (2008, pp. 126–127). This group of 49 children provides the basis for all further analyses.

With respect to the overall sample children with migration background are significantly ($p<0.001$) overrepresented in the group of low achievers (35 of 49 children, 71.4%), while there is no major difference in the gender distribution (21 male, 28 female) to the overall sample.

5.5.2 Performance on the DEMAT 1+ Subtests and EMBI Interview Parts

The group of low achieving children showed significant ($p < 0.001$) differences in their performance on the DEMAT 1+ and on the EMBI in comparison to the remaining group ($n = 359$). With respect to the standardised test the low-achieving first graders ($n = 49$) reached significantly lower scores ($p < 0.001$) in all nine DEMAT 1+ subtests (see Table 5.2). Their results on all four interview parts of the EMBI (see Table 5.3 for) were also lower than for the remaining first graders; the median growth points for the low achievers were one to two growth points less in each of the four domains.

Apart from domain A (*counting*) the low achieving first-graders reach only the first growth point in each domain. The greatest difference between the remaining first graders and the group of low-achievers is shown in domain C (*strategies for addition and subtraction*), where the difference in medians is two growth points (see Table 5.3).

Table 5.2 DEMAT 1+ subscales at MP 3

Subtest	DEMAT 1+ Subscales at MP 3			
	Remaining children in the sample ($n = 359$)		Low achieving first-graders ($n = 49$)	
	Mean	SD	Mean	SD
Sets—numbers	2.715	0.581	2.163	0.799
Number-line activities	3.799	1.105	2.375	1.248
Addition	3.086	1.086	1.469	1.234
Subtraction	2.150	1.498	0.734	1.106
Finding the 2nd addend	2.891	1.288	1.163	1.328
Part-whole	2.217	1.629	0.489	0.844
Addition with more than one addend	2.459	1.375	0.918	0.975
Understanding of "<, >, ="	2.838	1.237	1.857	1.172
Word problems	2.476	1.592	1.163	1.027

Table 5.3 Median growth point EMBI at MP 3

Content domains	MP 3 EMBI growth point scores	
	Remaining first graders in the sample ($n = 359$)	Low achieving first-graders ($n = 49$)
	Median	Median
A. Counting	3	2
B. Place value	2	1
C. Strategies for addition and subtraction	3	1
D. Strategies for multiplication and division	2	1

5.5.3 Achievement Prior to School (MP1 and MP2)

The basis for the analysis of the data collected at MP 1 and MP 2 is the identification of low-achieving first graders at MP 3 ($n = 49$). This group of children is compared to the remaining children in the sample ($n = 359$).

In order to assess the children's performance on the EMBI-KiGa, the interview results were translated into number scores (0 to 1 point for each of the 11 items in order to balance the influence of each item on the total score, acknowledging the fact that the number of sub-items varies).

The analysis of the data from MP1 and MP2 showed that the group of low-achieving first graders already performed lower prior to school entry. Their total scores on the OTZ and their overall scores on the EMBI-KiGa (MP 1: Low achieving first-graders: Mean: 3.159, SD: 1.775—Remaining sample: Mean: 6.632, SD: 2.230; MP 2: Low achieving first-graders: Mean: 6.693, SD: 1.978—Remaining sample: Mean: 8.972, SD: 1.337) show significant ($p < 0.001$) differences. While the overall scores at MP 2 are higher for both groups as expected, the significant difference between the groups remains at an average difference of about 2 points.

5.5.4 Analysis of the Performance with Respect to the Different Content-Specific Items in the OTZ and the EMBI-KiGa

The analysis of the low achieving first-grader's performance on all eight subtests of the OTZ (see Table 5.4) shows significant ($p < 0.001$) differences for both MP 1 and MP 2. While the low achieving first-graders show moderate improvement on most subtests, they achieve major improvement in the subtests *comparing*, *number-line activities* and *one-to-one correspondence*.

In addition, Table 5.5 shows the mean scores on each of the 11 content specific items for each group at MP 1 and MP 2. For MP 1 *numbers before and after* appears to be the most difficult item overall (mean = 253), followed by *ordering numbers 0–9* (mean = 470), *subitising* (mean = 502), *matching numerals to dots*

Table 5.4 OTZ mean scores at MP 1 and MP 2

Subtest	MP 1				MP 2			
	Children not at Risk (n=359)		Children at Risk (n=49)		Children not at Risk (n=359)		Low-achieving first-graders MP 3 (n=49)	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Comparing and counting small sets	4.18	0.962	3.12	1.235	4.70	0.598	4.43	0.890
Number-line activities	3.75	1.067	2.84	1.106	4.47	0.772	4.00	1.000
One-to-one-correspondence	3.03	1.243	1.80	1.307	3.99	0.860	3.29	0.913
Ordering/seriation	2.14	1.562	1.08	1.057	3.51	1.355	1.92	1.397
Using number words	3.12	1.445	2.22	1.031	3.59	1.195	2.33	1.281
Counting all/Counting on	3.10	1.278	2.09	1.033	3.27	1.139	2.22	1.177
Counting (un-) structured sets	2.49	1.255	1.32	1.097	2.61	1.349	1.61	1.133
Word problems	3.47	1.355	2.31	1.979	3.52	1.202	2.08	1.115
Total scores	21.04	6.889	12.96	5.156	29.67	5.473	21.92	5.235

Table 5.5 EMBI-KiGa Mean scores MP 1 and MP 2

Content domains	MP 1				MP 2			
	Children not at Risk (n=359)		Children at Risk (n=49)		Children not at Risk (n=359)	Children at Risk (n=49)		
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Comparing and counting small sets	0.688	0.338	0.377	0.298	0.812	0.267	0.663	0.344
Language of location	0.785	0.411	0.387	0.492	0.949	0.218	0.775	0.421
Pattern	0.540	0.349	0.295	0.304	0.825	0.255	0.602	0.288
Ordinal number	0.612	0.423	0.122	0.298	0.901	0.243	0.653	0.397
Subitising	0.522	0.181	0.357	0.250	0.626	0.217	0.520	0.175
Matching numerals to dots	0.547	0.322	0.193	0.246	0.779	0.254	0.622	0.260
Ordering numbers	0.523	0.500	0.063	0.247	0.857	0.349	0.551	0.502
Part-whole	0.562	0.300	0.255	0.252	0.686	0.245	0.489	0.161
Numbers before/after	0.280	0.328	0.051	0.152	0.635	0.345	0.326	0.298
One-to-one correspondence	0.919	0.272	0.836	0.373	0.958	0.200	0.898	0.305
Ordering by length	0.614	0.453	0.224	0.368	0.938	0.225	0.591	0.475

(mean = 504), *pattern* (mean = 511), *part-whole* (mean = 525), *ordinal number* (mean = 553), *ordering by length* (mean = 567), while *comparing and counting small sets* (mean = 650), *language of location* (mean = 737) and *one-to-one correspondence* (mean = 909) clearly appear to be the least difficult items.

The group of low achieving first-graders 1 year prior to school severely struggles with *numbers before and after* (mean = 051) *ordering numbers 0–9* (mean = 063), and *ordinal number* (mean = 122). Overall this group performs significantly worse ($p < 0.001$) in all content specific items apart from *one-to-one correspondence* ($p > 0.1$) which is also the case for MP 2 (see Table 5.5).

While the group of low achieving first graders overall showed improvements in all categories of the EMBI-KiGa from MP 1 to MP 2, they still score significantly ($p < 0.001$) lower than the remaining sample (apart from *one-to-one correspondence*). There is still a major difference on their performance in the areas *ordinal number* (0.653), *ordering numbers* (0.551), *part-whole* (0.489), *numbers before/after* (0.326) and *ordering by length* (0.591).

5.6 Discussion and Implications

The analyses of the data collected in MP 1 to MP 3 suggests that low-achieving first-graders already demonstrate a significantly lower understanding of sets and numbers and significantly less elaborate counting skills than their peers prior to school at both measuring points—1 year before school and immediately before school entry. These results conform to studies by Aunola et al. (2004). Furthermore, children with a migration background are clearly overrepresented among the low-achieving first-graders (Peter-Koop and Grüßing 2014, p. 315). With respect to their performance on the EMBI and the DEMAT 1+ the low-achieving children demonstrate significantly lower achievement in all four content domains (EMBI) and all subtests (DEMAT 1+). They particularly struggle with respect to the DEMAT 1+ items on *subtraction*, *part-whole relationships*, *addition with more than one addend* and *finding the second addend*. However, the subtests on part-whole relationships, subtraction, addition with more than one addend and word problems proved to be the most difficult items for their higher achieving peers. With respect to subtraction a longitudinal study by Cooper et al. (1996) indicated that second graders were overall more successful on addition tasks of varying difficulty than on respective subtraction tasks.

In contrast to the standardised DEMAT 1+ that focuses on correct results, the EMBI seeks to identify strategies that children apply when given mathematical tasks and problems. With this respect the identified group of low-achieving first-graders demonstrates less elaborate strategies for addition and subtraction. This can be seen in relation to their understanding of number and their number skills prior to school. In order to solve problems such as 8 + 6 with strategies other than counting, an understanding of part-whole schema (Resnick 1989) is necessary to be able to add up to 10 and then on (e.g., 8 + 2 + 4). While they still struggle with part-whole

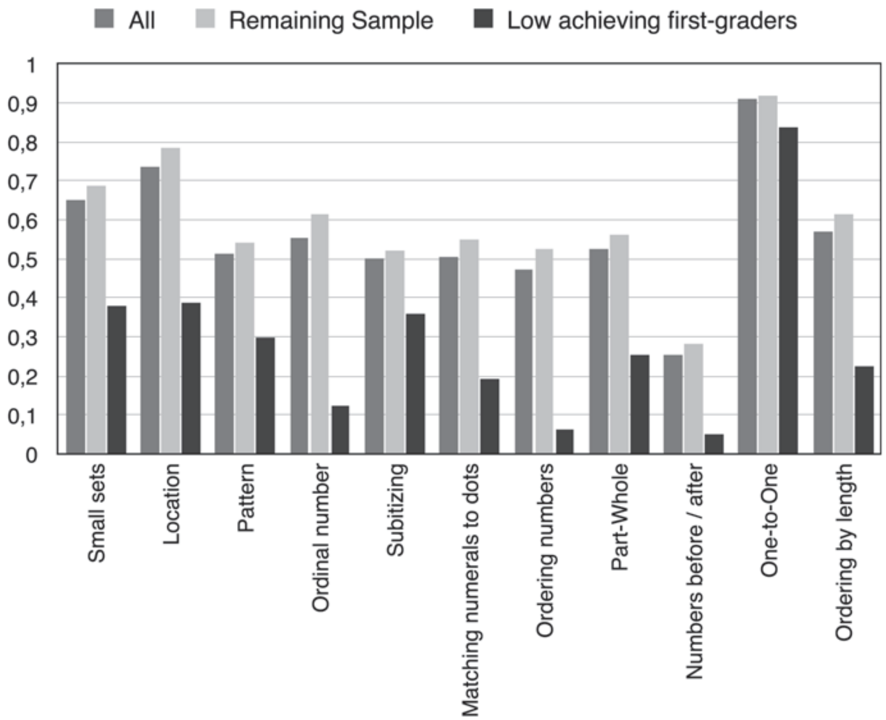

Fig. 5.4 EMBI-KiGa subcategory mean scores MP 1

relationships in Grade 1 (see Table 5.2), they already demonstrated less insight into this concept than their peers prior to school at both MP 1 and MP 2 (see Table 5.5). Furthermore, the low-achieving first graders demonstrate less insight in counting and place value (see Table 5.3). This means that their higher achieving peers have significantly more elaborate knowledge and skills with respect to higher numbers. How far this can be compensated for at the end of Grade 2 so far remains unclear.

When considering the performance of the children who are identified as low-achievers in mathematics at the end of Grade 1, it is also interesting to note that they obviously experience special difficulties with respect to items that require more elaborate language skills, i.e. language of location, numbers before/after and ordinal numbers (see Figs. 5.4 and 5.5). This might explain the overrepresentation of children with a migration background.

However, since the assessment of German language competencies has not been included in the study design, this possible relationship needs to be further investigated.

Moreover, the low-achieving first-graders prior to school also demonstrated significantly less knowledge and understanding of number symbols, which suggests that their command of the German language might only be one factor among others that would explain why they tend to struggle with the development of number skills and counting much more than their peers.

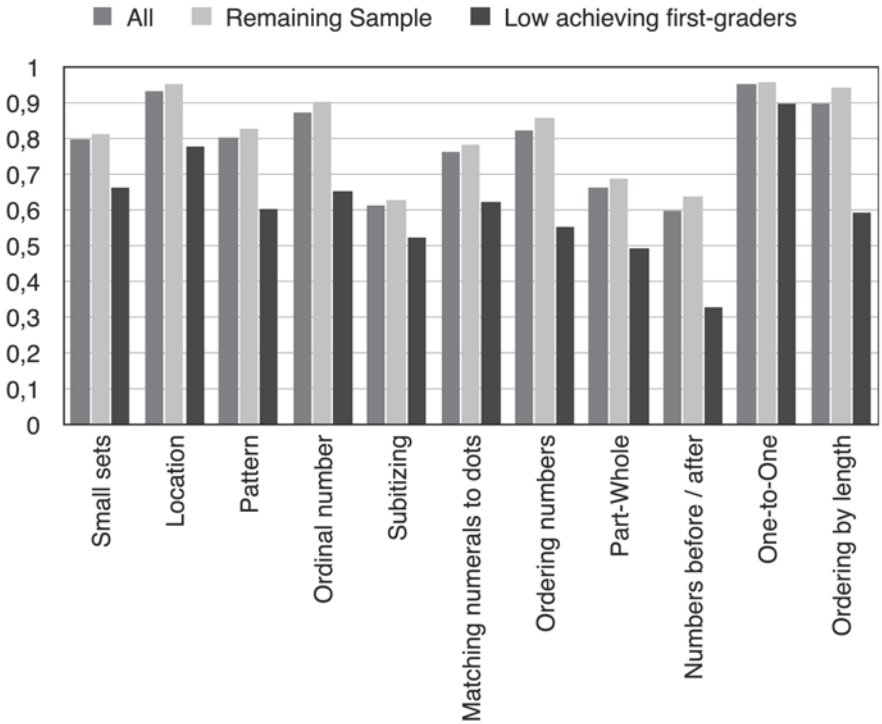

Fig. 5.5 EMBI-KiGa subcategory mean scores MP 2

However, as Table 5.5 as well as the comparison of Figs. 5.4 and 5.5 suggest, this group of children does improve from MP 1 to MP 2. Immediately before school entry they show about the same average scores on the EMBI-Kiga (mean = 6.693) as their peers did 1 year before school entry (mean = 6.632). This complies with findings of a longitudinal study conducted by Aunola et al. (2004). They describe the cumulative effects of children having little number-related knowledge and skills prior to school, i.e. preschoolers who demonstrated low competences in dealing with numbers and sets clearly showed slower development of their mathematical competencies in primary school with an increasing gap with respect to their peers who started school with higher number skills and knowledge.

In summary the study in progress reported in this chapter confirms previous findings that understanding and skills with respect to number and counting are important precursors for later school success. The children who were identified as low-achievers in mathematics at the end of Grade 1 demonstrated significantly lower knowledge and skills than their peers prior to school. However, the results presented and discussed here provide only first insights into the development of number skills and counting ability.

Furthermore, the data suggest that the EMBI-KiGa is a suitable screening instrument for the identification of children potentially at risk learning mathematics, especially because of its focus on strategies and skills as well as the fact that it is conducted as a one-to-one interview that allows for children to use concrete objects/manipulatives to demonstrate and articulate their mathematical understanding in addition or even as a replacement for verbal explanations.

In addition, more detailed analyses of the individual development of the children will help to better understand and describe the factors that explain the differences in achievement in the transition from kindergarten to school. Hence, further in-depth analyses will include qualitative approaches in the form of individual case studies. With respect to the model of early mathematical development by Krajewski and Schneider (see Fig. 5.1) first broad analyses suggest that the children who later struggle in Grade 1 mathematics, prior to school entry only demonstrate competencies that can be assigned to the first level and partly to the second level, while their better achieving peers show competencies that comply with level two and three. Further in-depth analyses of the development, which has been recorded in the study, will provide more detailed insight into the transitions between the levels and their influence on school mathematics learning.

Ultimately, a more extensive competence model of children's developing mathematical skills is required that not only focuses on numerical skills and understanding but includes children's language abilities and comprehension as well as their spatial and structural abilities.

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In his personal research he is working on transfer processes in mathematics learning.

Chapter 6
Let's Count: Early Childhood Educators and Families Working in Partnership to Support Young Children's Transitions in Mathematics Education

Amy MacDonald

Abstract *Let's Count* is an early mathematics program designed by The Smith Family and researchers from Charles Sturt University and the Australian Catholic University as a means of assisting parents and other family members to help their young children (aged 3–5 years) play with, investigate and learn powerful mathematical ideas. *Let's Count* involves early childhood educators in the role of mentors to the parents and family members of the children in their settings, providing assistance in noticing and exploring mathematics in everyday life. In 2011, I was responsible for developing *Let's Count* into the form of a distance education subject for offer to students enrolled in an early childhood teacher education degree at Charles Sturt University, as a means of sustaining the *Let's Count* initiative and achieving a wider impact on the early childhood community. In this chapter, I report on a project which followed up with former participants in the subject, and the families with whom they have worked, to ascertain the success of the *Let's Count* program in bringing together early childhood educators and families to support positive transitions in children's mathematics education. This chapter explores the ongoing effects of educators' and families' engagement with the program, and shares examples of *Let's Count* activities in prior-to-school, school, family and community contexts.

6.1 Introduction

For the better part of a decade, I have been researching with children, families and early childhood educators in the area of educational transitions. My earlier work was around transitions to school in a more general sense (MacDonald 2008, 2009), but for the most part my research endeavours have focused specifically on *mathematics* and transitions to school (MacDonald 2013; MacDonald and Lowrie 2011).

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In particular, I have been very interested for some time now in the mathematical experiences and understandings children encounter in the prior-to-school years, and the majority of my work has been devoted to gaining some understanding of the mathematics children bring with them in their transitions from prior-to-school settings to school settings. A recent project in this area has been the development of *Let's Count*, an early mathematics program designed by The Smith Family and researchers from Charles Sturt University and the Australian Catholic University as a means of assisting parents and other family members to help their young children (aged 3–5 years) play with, investigate and learn powerful mathematical ideas (Perry and Gervasoni 2012). *Let's Count* involves early childhood educators in the role of mentors to the parents and family members of the children in their setting, providing assistance in noticing and exploring mathematics in everyday life. In 2011, I was responsible for developing *Let's Count* into the form of a distance education subject for offer to students enrolled in an early childhood teacher education degree at Charles Sturt University, as a means of sustaining the *Let's Count* initiative and achieving a wider impact on the early childhood community. In this chapter, I report on a project which followed up with former participants in the subject, and the families with whom they have worked, to ascertain the successfulness of the *Let's Count* program in bringing together early childhood educators and families to support positive transitions in children's mathematics education. This chapter explores the ongoing effects of educators' and families' engagement with the program, and shares examples of *Let's Count* activities in prior-to-school, school, family and community contexts, both nationally and internationally.

6.2 Background

The past decade has seen a significant shift in thinking, with young children now celebrated as capable mathematical thinkers and learners (Balfanz et al. 2003; Lee and Ginsburg 2007). In a survey of Australian early childhood educators (Hunting et al. 2008), there was overwhelming agreement that young children were capable of mathematical activity and thought well before they started school. This result has been echoed in many other studies with the general agreement that all children in their early childhood years are capable of accessing powerful mathematical ideas and that they should be given the opportunity to access these ideas through high quality child-centred activities in their homes, communities, and prior-to-school settings (Balfanz et al. 2003; Lee and Ginsburg 2007; Perry and Dockett 2008).

However, there is a significant body of research which suggests that many early childhood professionals are reluctant to engage in intentional teaching of mathematics (Sarama and Clements 2002; Perry and Dockett 2008), and that this reluctance may be explained by concerns about overly didactic programs, privileging other parts of the curriculum (namely, language and literacy), and teachers' anxieties about their own mathematics knowledge (Cohrssen et al. 2013). A further challenge is that those early childhood educators who *do* include mathematics education as part of their curriculum typically hold a very narrow view of what constitutes mathematics, stressing the ability to count and knowledge of numbers (Department for Education and Child Development 2012, cited in Carrington and Feder 2013). Indeed, "many pre-school educators consider practising the number-word sequence and identifying basic shapes to be sufficient in the preschool context" (Cohrssen et al. 2013, p. 96).

Doig et al. (2003) have suggested several reasons for the importance of understanding children's mathematical development in the years prior to school, including the increasing number of children participating in early childhood programs and growing recognition of the importance of mathematics in general. The work of Baroody (2000) and Klibanoff et al. (2006) has indicated that children who enter primary school with high levels of mathematical knowledge maintain these high levels of mathematical skill throughout, at least, their primary school education. Furthermore, the development of mathematical concepts in the prior-to-school years is likely to increase a child's potential for later school achievement. In a study of school readiness and later school achievement, Duncan et al. (2007) found a strong correlation between early mathematics skill and later mathematics achievement, as well as associations between early maths and other competencies such as reading and writing abilities.

Relationships among family members, children and educators can have a substantial influence on learning, including the learning of mathematics. Studies have shown a positive relationship between parental involvement in their children's learning and the achievement of these children (Civil et al. 2005; Young-Loveridge et al. 1997). However, some family members will be reluctant to 'get involved' with mathematics and early childhood educators might have to provoke such involvement. One of the aims of such provocation will be to assist the families realise the mathematical potential of their everyday activities with their children.

The importance of early childhood educators working in partnership with families in order to assist children's learning is recognised within Australia's national early childhood curriculum, the *Early Years Learning Framework for Australia* (EYLF; Department of Education, Employment and Workplace Relations [DEEWR], 2009). As stated in the EYLF:

> Learning outcomes are most likely to be achieved when early childhood educators work in partnership with families. Educators recognise that families are children's first and most influential teachers. They create a welcoming environment where all children and families are respected and actively encouraged to collaborate with educators about curriculum decisions in order to ensure that learning experiences are meaningful (p. 12).

As explained by Perry and Gervasoni (2012, p. 5):

> In order for such partnerships to become a reality, early childhood educators need to engage with families and communities in ways that are relevant, meaningful and culturally appropriate. Relationships are built upon mutual trust and respect and these need to be earned by all parties to any relationship.

As such, the *Let's Count* program is built upon the skills and knowledge of early childhood educators in mathematics and upon their skills in developing trusting and respectful relationships with the families with whom they work.

6.3 Overview of *Let's Count*

As Perry and Gervasoni (2012) explain, *Let's Count* is not a mathematics teaching program; however, it does involve early childhood educators in the role of advisers to the parents and family members of the children in their settings about ways they can notice, discuss and explore mathematics with their children. *Let's Count* includes a professional learning program for educators to assist them in their critical role of advising parents and family members. Additionally, this professional learning enables educators to consider their own pedagogical approaches in mathematics and add to their repertoire of successful practices (Perry and Gervasoni 2012).

In short, *Let's Count* has been developed with the following underlying characteristics:

- Partnerships among early childhood educators and families;
- Play and investigation for all;
- Recognition of all as potentially powerful mathematicians;
- Realisation that mathematics learning can be fun for all when it is undertaken in a relevant and meaningful context;
- Mentoring and advising of families by early childhood educators;
- Meaningful documentation of learning; and
- Strong links to the theoretical and practical bases of the EYLF (Perry and Gervasoni 2012).

6.3.1 The Importance of the Prior-to-School Years

Bredekamp and Copple (1997, p. 97) note that the prior-to-school years are "recognised as a vitally important period of human development in its own right, not as a time to grow before 'real learning' begins in school". Indeed, what children learn about mathematics in the early years is important in their transition to learning at school. The rationale for *Let's Count*'s focus on the prior-to-school years draws on the work of Duncan et al. (2007), who performed a coordinated analysis of six longitudinal data sets relating changes in early skills to later teacher ratings and test scores of school reading and mathematics achievement. They found that school-entry mathematics, reading, and attention skills were associated with later achievement, and noted that the predictive power of early mathematics skills was particularly impressive. However, Duncan et al. (2007) also cautioned that their findings did not support the adoption of 'drill-and-practice' curricula, and argued that play-based curricula designed with the developmental needs of children in mind can easily foster the development of academic and attention skills in ways that are engaging and fun. This stance is echoed in recent national statements on mathematics learning in early childhood (Australian Association of Mathematics Teachers and Early Childhood Australia 2006; National Association for the Education of Young Children and National Council for Teachers of Mathematics 2002).

The Australian Association of Mathematics Teachers and Early Childhood Australia (2006, p. 1) state that:

> All children in their early childhood years are capable of accessing powerful mathematical ideas that are both relevant to their current lives and form a critical foundation to their future mathematical and other learning. Children should be given the opportunity to access these ideas through high quality child-centred activities in their homes, communities, prior-to-school settings and schools.

Key to this is the critical importance of early childhood educators and families holding high expectations of all children's potential as mathematical learners. However, this may be inhibited by adult perceptions of, and past experiences with, mathematics. As Perry and Gervasoni (2012, p. 16) explain:

> Many adults have struggled with learning mathematics at school. Many consider mathematics to be a collection of facts to learn that do not connect to real life. Consequently, their attitudes toward mathematics are often negative, with their own experiences crowding out any possibility of seeing mathematics as beautiful and joyful. On the other hand, many adults do see the beauty, joy and usefulness of mathematics and these people most often harbour very positive attitudes towards mathematics and its learning.

As such, one of the aims of *Let's Count* is to build adults'—both educators and parents'—confidence with, and positive attitudes towards, mathematics so that young children can see that they are not only able to learn some powerful mathematics, but also *want* to do so (Perry and Gervasoni 2012). It is particularly important to foster this in the prior-to-school years because, as Henderson and Mapp (2002, p. 64) note, there is "a positive and convincing relationship between family involvement and benefits for students, including improved academic achievement".

In summary, *Let's Count* specifically targets children in the prior-to-school years for the following key reasons:

- To acknowledge that powerful mathematical ideas are developed prior to school;
- To support positive transitions from mathematics learning in home and early childhood contexts to school contexts; and
- To enhance the confidence of parents in identifying and supporting their children's mathematical development, particularly as they start school.

6.3.2 Let's Count Program Pedagogies

The professional learning within *Let's Count* has been offered in two forms:

1. a face-to-face mode, in which early childhood educators participate in two full-day professional development workshops; and
2. a distance education mode, in which early childhood educators complete six online modules and associated tasks.

The first of these modes, the face-to-face offering, was developed, implemented and evaluated by program authors Bob Perry and Ann Gervasoni (refer to Chap. 4 in this volume for further details). Following the successful pilot of the face-to-face program in 2011, I was invited to take responsibility for the development, implementation and evaluation of a distance education form of *Let's Count*. The development of a distance education mode provided a means of sustaining the *Let's Count* initiative as well as achieving a wider impact on the early childhood community—beyond what might be possible in a face-to-face workshop mode. To date, the online offering has been completed by 184 educators.

As a means of 'enacting' the *Let's Count* principles and practices, the distance education form of the program requires early childhood educators to engage with two key pedagogical approaches—family gatherings and learning stories.

6.3.2.1 Family Gatherings

A key pedagogy of *Let's Count* is encouraging early childhood educators to implement 'family gatherings' with the children, parents, and other caregivers in their setting. Family gatherings are essentially workshops designed to allow early childhood educators to have conversations about mathematics with parents, and to assist parents to help their children learn mathematics. Family gatherings are an opportunity for educators to work with families to assist them in recognising the opportunities for mathematical development in their everyday family life. They are also an opportunity for educators to learn about, and appreciate, the unique capacities and resources of each family. Key to the family gatherings is a focus on the 'everydayness' of mathematics and the use of everyday activities and resources—no specialised games or tools are required. Family gatherings are also used in *Let's Count* as a way of engaging families and developing positive relationships within early childhood settings.These gatherings take any number of forms: for example, *Let's Count* educators have brought together a small group of families for a face-to-face workshop; they have worked individually with a small selection of families; they have gathered both physically and virtually, capitalising upon the potential of online social networks; they have held brief meetings over a period of time, or have come together for one, more extended, block of time. By way of an example, the following account from Jody, one of the *Let's Count* educators, explains the approach she took with her family gathering activities:

> I initially set up a facebook group conversation with seven parents to see if they would like to be involved. Once I had my head around what was required I set up a meeting with all the parents to discuss the project, however, I did ask parents if they could provide me with their children's interests at home so that before our meeting I could plan some mathematical games that would be of interest to the children so they could do them with their parents. It was like giving them some ideas but it wasn't something they had to do, it was just a precursor for the parents to see how they could incorporate mathematics into their children's time with them at home. There was no minimum amount required and they were all asked to let us know in the group conversation how they were going to help spur on each other and perhaps provide ideas to other families on what they were doing and what worked and didn't work for them [Jody, NSW].

Educators have been encouraged to think about what might work best for the families in their service, and to think creatively about how they might 'gather' families around the topic of early numeracy development. The main thing educators were encouraged to remember—and emphasise to families—is that family gatherings are an opportunity for early childhood educators and parents to work together to explore the mathematics in children's lives.

6.3.2.2 Learning Stories

A second pedagogical approach employed in *Let's Count* is the writing of mathematical learning stories. Participants in *Let's Count* are asked to use learning stories to document the mathematical learning of children who have participated in the family gathering. While no set format for the learning stories is given, the educators are encouraged to include three key features in their stories:

- Description of the context and what happened;
- Analysis of the child's mathematical learning; and
- Suggestions for how this learning might be further developed (with a focus on what families can do).

Educators are encouraged to *unpack* the mathematics learning which has taken place, attending closely to the mathematical concepts being developed. For example, rather than just saying "Audrey was counting the blocks", educators would practice explaining the *concepts* involved in counting, for example "As Audrey counted the blocks, she demonstrated developing understandings of one-to-one correspondence, numeral names, and the stable order principle." An example of a mathematical learning story produced by a *Let's Count* educator can be seen in Fig. 6.1.

The intention of this approach was to assist early childhood educators in noticing, naming and explaining mathematical development in the early childhood years. In this way, educators attended more closely to the potential for mathematical development in children's play and investigation, and also honed their skills in communicating children's mathematical learning to others. Furthermore, the learning stories were a communication tool for discussing children's mathematical learning with families, and provided an additional medium for offering support and advice to families as to how they might explore mathematics at home with their child.

6.4 Evaluating the Impact of *Let's Count*

In 2013, after four offerings of the *Let's Count* distance education subject, I began implementing a small-scale evaluation of the program. The aim of the study was to ascertain the impact of *Let's Count* on early childhood educators and families' capacity to support young children's numeracy development prior to starting school.

Child's Name: E **Age:** 4 years old **Educator:** Stephanie	**Date**:23/4/13
EYLF Outcomes	**Learning Story: Block Play**
Outcome 1: Children have a strong sense of identity. a) Children feel safe, secure and supported b) Children develop their emerging autonomy, inter-dependence, resilience and sense of agency c) Children develop knowledgeable and confident self-identities d) Children learn to interact in relation to others with care, empathy and respect **Outcome 2: Children are connected with and contribute to their world.** a) Children develop a sense of belonging to groups and communities and an understanding of the reciprocal rights and responsibilities necessary for active civic participation b) Children respond to diversity with respect c) Children become aware of fairness d) Children become socially responsible and show respect for the environment **Outcome 3: Children have a strong sense of wellbeing.** a) Children become strong in their social, emotional and spiritual wellbeing b) Children take increasing responsibility for their own health and physical wellbeing **Outcome 4: Children are confident and involved learners.** a) Children develop dispositions for learning such as curiosity, cooperation, confidence, creativity, commitment, enthusiasm, persistence, imagination and reflexivity b) Children develop a range of skills and processes such as problem solving, inquiry, experimentation, hypothesising, researching and investigating c) Children transfer and adapt what they have learnt from one context to another d) Children resource their own learning through connecting with people, place, technologies and natural and processed materials **Outcome 5:Children are effective communicators.** a) Children interact verbally and no-verbally with others for a range of purposes b) Children engage with a range of texts and get meaning from these texts c) Children express ideas and make meaning using a range of media d) Children begin to understand how symbols and pattern systems work e) Children use information and communication technologies to access information, investigate ideas and represent their thinking	**<u>What happened:</u>** E was sitting alone when she had set up her blocks onto the table. Whilst there I noticed she was talking to herself whilst building. I tried to stand in closer to see what she was saying when another child joined her. During this experience E appeared to be making a little town and talking herself through the steps- "we need 1 block for the house, 1 block for the roof, we also need more houses so we will need more blocks for this." E appeared to be devising a plan before she started her work. She knew exactly what she wanted to build but when the other child came into the area and saw what she was building they started to build together. E led this play and showed great verbal direction and communication with her peer. E was able to name the blocks and give each a number she didn't go higher than 3 at a time, and her blocks were systematically lined up next to each other along the table. **<u>Mathematical learning that took place:</u>** E developed this play situation from her own ideas and sense of agency and autonomy (Outcome 1b). Once again she shows great understanding of persistence, enthusiasm, commitment, and imagination (Outcome 4a). E's problem solving and hypothesising skills is once again a key in her play skills and the high level of understanding she has with these two very abstract terms (Outcome 4b). She is able to count the blocks and predict which ones will make the houses. She demonstrated the concept of understanding and naming numbers and that by placing each on top of the other she identified the outcome of this process (it makes a house). E can interpret and perceive different levels within her building. E shows sound knowledge in shapes and patterns and she does this by turning a triangular block into a roof or a square block into a base for a house (Outcome 5d). E's spatial and measurement awareness when constructing the building out of the blocks shows she is experimenting with height, width, length and three dimensional equipment. **<u>What next:</u>** Add pictures of multi-story towns, so she can expand her awareness of shapes, patterns, and levels. This will then encourage and extend on her counting abilities, shape naming and recognition skills. E's parents could take her to the city to see some high rise buildings or show her pictures on their computer or in magazines to encourage her imagination.

Fig. 6.1 Example of a mathematical learning story

6.4.1 Evaluation Design

All past *Let's Count* participants were invited to participate in an email interview, or "EView" (Fenton 2012), about their experiences, reflecting upon their initial engagement with *Let's Count* and how this has impacted their current beliefs and practices. Educators who agreed to participate were then asked to extend an invitation to participate in the research to any parents with whom they have worked as part of their *Let's Count* activities, both former and current. Parents who wished to be involved also participated in an EView about their experiences with *Let's Count*. Participants were also invited to share examples related to *Let's Count* activities, if they felt comfortable in doing so. For educators, these examples included documentation of mentoring approaches such as presentations, newsletters, curriculum planning documents, or learning stories; while parents shared examples of numeracy explorations they have undertaken at home, and ways in which they have communicated with educators about their child's numeracy development.

6.4.1.1 EViews

Email interviews, or 'EViews', is an approach developed by Fenton (2012) as a means of undertaking interviews via email communications. The EView approach was chosen as it was in keeping with the online approach taken during the delivery of the *Let's Count* professional learning program; hence, it could be reasonably assumed that participants felt a degree of comfort in operating in this online mode.

The EViews were semi-structured; participants were provided with an initial interview schedule to read and respond to, but were encouraged to respond only to those questions with which they felt comfortable. Additionally, participants were encouraged to add additional comments and foci if they wished, as well as respond in alternate ways (such as the sharing of anecdotes and photographs). Because of the flexible nature of the EViews, communication with participants has been ongoing, with no fixed endpoint to the conversations. The sustained nature of these EViews is testament to the relationships which have been developed through engagement with *Let's Count*.

6.4.1.2 Participants

In this chapter, I report on the EView data from the educator participants in the evaluation study. Eighteen early childhood educators participated in the evaluation study. The average age of the educators was 36 years, and the participants were all female—reflective of the demographics of early childhood educators in Australia. Indeed, only a very small number of male educators have participated in *Let's Count* more broadly. Participants in the evaluation study were drawn from communities across Australia (though, predominately New South Wales and Victoria), as well as internationally—specifically, Brunei Darussalam and the United Arab Emirates—

as a result of the participation of international students in the degree program housing the *Let's Count* subject.

6.4.1.3 Data Analysis

Analysis of the EView data was informed by grounded theory approaches (Strauss and Corbin 1990). Open-reading of the EView transcripts was undertaken before coding of the data in order to identify emergent themes, with verbatim excerpts from the data chosen to exemplify these themes. These themes and excerpts are reflected in the presentation of data which follows.

6.5 Insights from the *Let's Count* Evaluation

6.5.1 Noticing Mathematics in the Prior-to-School Years

As noted in the work of Cohrssen et al. (2013), early childhood educators' understanding of, and confidence with, mathematics is a significant issue in early childhood education. This is a point that was reflected in the educator data, with several participants commenting on the fact that prior to their participation in *Let's Count* they struggled with mathematics, both personally and professionally. However, through their engagement with the program, these educators reported increased confidence with, and understanding of, mathematics, as the following comments illustrate:

> How I feel about mathematics has absolutely changed…. I feel confident as an early childhood educator in being able to teach mathematics to the children in my care [Annette, VIC].

> The priceless knowledge that I have learned from this is the way I 'see' maths now. I couldn't seem to understand maths when I was young, even when I first started to teach, but now I am proud to say I have gained a massive understanding in maths [Apple, Brunei Darussalam].

This changed perception of mathematics has meant that these educators are now more attuned to the mathematics in everyday life and, importantly, are noticing the opportunities for mathematical exploration in the prior-to-school settings in which they work. Educators have reported noticing opportunities for mathematical learning—for both themselves, and for the children with whom they work—as the following quotes demonstrate:

> I am now more interested in seeking the mathematics in life for myself. I see maths as something that can be approached and tackled rather than avoided [Sarah, NSW].

> I'm not confident with maths but after undertaking the course I felt I benefitted as well as the children. It gave me the confidence to implement more 'maths' type activities and to talk confidently about maths [Stephanie, VIC].

> I actually silently mention to myself at certain times, "That was maths you just used. See, you did need it when you grew up" [Carissa, NSW].

In addition, educators have reported that a significant impact of *Let's Count* is that they now are 'seeing' the maths in young children's activity, which had previously gone unnoticed. As the following quotes demonstrate, educators perceive there to be a direct relationship between their increased knowledge of mathematics and their ability to recognise mathematics in children's everyday activity:

> I've learned so much from this subject and it deepened my knowledge in maths. I can understand maths better through children's play and I discovered that I can 'see' mathematics all around me every day [Apple, Brunei Darussalam].

> I never used to acknowledge the children's learning styles and how they used these to engage with mathematics in play. I see more maths learning in the children than I have ever seen before [Carissa, NSW].

> I learned that maths is all around us, and that armed with knowledge we can more clearly see children's maths play [Sarah, NSW].

Importantly, this new ability to notice the mathematics in everyday activity has been reported by educators to have had an impact on their practice in early childhood education settings. As the following quotes illustrate, educators are now attending to the use of mathematical language, and noticing and naming mathematical concepts:

> Mathematical language is being used more by the educators—what was a sensory activity of pouring rice has turned into a mathematical activity identifying mathematical concepts such as full and empty, weight and quantity [Annette, VIC].

> The way I talk to the children has changed, using more appropriate vocabulary and actions to scaffold their abilities and knowledge [Carissa, NSW].

> A change that I made which stemmed from this project was pushing more for children to find their own answers to questions or questioning them from a different perspective to what I normally would have [Jody, NSW].

Furthermore, educators are now recognising 'mathematics' as being more than just number recognition and counting, and have begun to incorporate a wider range of concepts and processes in the mathematical experiences they provide for children:

> As my confidence grew the children's opportunity to take part in maths grew. I would always have counting and problem solving activities, but now I can use more types of experiences to give the children the opportunity to participate with maths. I can see experiences differently now, therefore able to pass these on to the children [Stephanie, VIC].

> Many of the other educators in my service stick to plain counting and number games, whereas I now use nature, the environment and the children themselves to collect, gather, count, classify and hypothesise to extend their mathematical knowledge [Melanie, VIC].

Let's Count has also provided educators with 'reinforcement' of the importance of mathematics education in early childhood, and completion of the program has equipped them with the confidence and strategies to bring about change in their workplaces:

> One of the highlights was the opportunity to show the educators the *Let's Count* project to prove to them I do know what I am teaching about (often childcare whispers and historical teaching beliefs cause doubt and reluctance to change)…. I believe *Let's Count* has supported me to support other educators at the service to incorporate mathematical concepts through everyday practice in informal and formal teaching opportunities [Valerie, NSW].

The changes made to their mathematics education practices as a result of *Let's Count* have, importantly, been reported by educators to have had a positive impact on children's interest in, and engagement with, mathematics in both their early childhood education settings and home environments:

> Before *Let's Count*, the children only participated in maths activities when they were asked to, but now they have built interest in maths and we need to extend their skills in deeper maths concepts such as counting, sorting, classifying, etc…. One of the children said, 'I can do measuring'. He successfully counted the number of cups needed to make play dough. After the family gathering, his parents were so proud of him. He helped his mum bake a cake at home and he could measure the ingredients needed to make the cake. The parents and I still keep in touch [Apple, Brunei Darussalam].

> The children are more interested in learning about maths now so instead of me doing structured maths activities we are doing maths all the time … the children have created their own positive dispositions for learning maths [Carissa, NSW].

> A highlight was the conversations with the child, and the child's insightful comments revealing her apparently innate interest in numbers and 'mathematising' in order to make sense of her world—fascinating! [Sarah, NSW].

6.5.2 Supporting Parents to Explore Mathematics at Home

A key function of *Let's Count* is to develop early childhood educators' skills in supporting the parents of children in their service to explore mathematics with their children at home. There are many contexts for learning about mathematics, and one of the most meaningful learning contexts is children's homes (MacDonald 2012). Indeed, engagement with mathematics outside of school may have a profound impact on the knowledge that children bring to the classroom (Guberman 2004). In reflecting upon their engagement with the *Let's Count* program, educators talked about how the program helped them to develop their ability to act as mentors to parents and support the exploration of mathematics in home environments:

> I was aware that there wasn't much going, not through not trying, however through not knowing how to introduce mathematics to their children. This was why I decided to use all house hold items and daily activities to show families just how easy it is to incorporate mathematics into everyday routines [Annette, VIC].

> I enjoyed doing the learning stories, in particular giving advice to the parents on how they can extend on mathematics learning at home…. I encourage parents to be more hands on in their child's learning and recognise that they are the number one teachers of their child and

> they may be missing out on important opportunities because they think counting 1–10 is what we will teach them and is the only maths they need to learn [Carissa, NSW].

Several educators in the study talked about *Let's Count* as highlighting the importance of the mathematical learning opportunities provided by parents, and how the program has reinforced the role of parents as children's first teachers of mathematics. In many cases, it was reported that *Let's Count* assisted parents actually recognising themselves as teachers of their children; indeed, for some parents, *Let's Count* was a revelation as to the important role they play in their child's mathematics education. Educators who participated in this study reported increases in parents' confidence with this role, as well as a new appreciation for the mathematical activity they were already doing with their children—often without realising it:

> I loved the fact that parents actually realised that they were doing these great things with children already but didn't actually know it or understand the benefits of it. I also loved the fact that parents were into it just as much as the children…. Parents also said they would continue on or, more to the point, be more aware of how mathematical concepts could be introduced in so many ways for children [Jody, NSW].

> One of the highlights was hearing a parent say "Thank you, I'm doing that but it's just nice to realise I'm doing the right thing" [Stephanie, VIC].

6.5.3 Fostering Partnerships between Educators and Families

As described in the EYLF (DEEWR 2009, p. 12), "learning outcomes are most likely to be achieved when early childhood educators work in partnership with families", and it is on this basis that *Let's Count* was formed. Indeed, the process of fostering relationships/partnerships between educators and families features strongly among the reflections of *Let's Count* educators. In particular, educators commented on how the sense of partnership between themselves and the families created different understandings of each others' roles, and fostered mutual knowledge and appreciation of each other:

> The parents that attended the night got to see me as an educator, not a manager, which has created a different level of understanding and respect we both now have for each other [Annette, VIC].

> I got to know the child better through the family gathering. Parents enjoyed the gathering and asked for the next gathering! I've learned a lot from parents and shared strategies and give positive advice on what and how they can introduce maths at home too [Apple, Brunei Darussalam].

> Through working on such projects with children and families as equal partners we are enabled to share and celebrate children's learning. The family I worked with were clearly proud of the child's numeracy understanding and thinking. The child was seen as competent by all and her family expressed an intention to further extend on her numeracy learning in their everyday lives [Sarah, NSW].

Key to the success of the relationships built during *Let's Count* was parents being acknowledged as the children's first, and most important, teachers. Educators reported that this acknowledgement—and the communication of this to families—changed the dynamic of their relationships with parents, with both parties being seen as of importance in children's mathematics education:

> Although we had positive relationships with families, *Let's Count* has promoted the discussion about learning and teaching from our educator perspective and the role of families in working in partnership together and has encouraged confidence in parents in their role as the child's most influential teacher [Valerie, NSW].

> The discussions which have taken place with the parents at my centre involved in this project have built our relationships immensely. It has provided or stimulated many more conversations that probably have come more from their end on how well they are 'teaching their child'. I mean this by the fact that of course parents are the child's first teacher, however, they often send them to preschool to learn and develop and it was like 'look what we have done'. It was my turn to say how wonderful they are working with their children and hear excitement in their voices telling you about the children's 'homework', which is what the parents called it [Jody, NSW].

6.5.4 Promoting Positive Transitions to School Mathematics

Educators who participated in *Let's Count* have expressed their belief that the program has helped to promote positive transitions to school mathematics, for both the children and also for the parents. In particular, educators cited increased parental confidence and familiarity with the mathematics in everyday life as being beneficial as their children start primary school:

> The gathering helps parents to better understand how they can include maths at home as well as in school [Apple, Brunei Darussalam].

> I think our conversations may have increased the parents' confidence in their child's mathematics abilities, in the months before starting school [Sarah, NSW].

> I think some of the parents will now be a bit more comfortable when they have to teach their child school mathematics [Vicki, NSW].

Educators also felt that it was beneficial for the children to be starting primary school with a positive disposition towards mathematics and a sense of how they use it in their everyday life. Educators expressed their enjoyment in helping to promote this interest and understanding in the children with whom they have worked, and felt that this would be of assistance in their transition to school:

> I enjoyed seeing children develop, and be able to move forward through their school life with an understanding of maths [Apple, Brunei Darussalam].

> Mathematical concepts have really been a major part of our school readiness program this year and more so than in previous years due to the fact that I have completed this program…. I do believe personally that these children have had such a positive start to mathematics that [school mathematics] will certainly be a positive experience for them all [Jody, NSW].

Finally, educators again emphasised the importance of relationships between families and educators, particularly as child transition to primary school, expressing a belief that these relationships will assist children's learning and development as they encounter school mathematics:

> I think *Let's Count* helped to promote parents' understanding of how important and valuable their role is in the education of their child. It also promotes how effective relationships with their child's educator can assist their child's learning and development [Valerie, NSW].

6.5.5 Sustaining **Let's Count** *Initiatives*

Testament to the success of the *Let's Count* program was the educators talking at length about their intentions for sustaining their *Let's Count* initiatives. As discussed earlier in the chapter, *Let's Count* has had a significant impact on the mathematics education practices of these early childhood educators, and the data from the EViews suggests that educators see benefits in sustaining—and further developing—these new practices:

> I intend to regularly revisit the *Let's Count* learning resources to help me remember the many facets of mathematics that are observable and extendable in children's work and play. I have made myself a little revision document summarising some of the mathematical concepts, processes and ideas. I also intend to create some posters with examples of children playing with maths to inspire myself and others to continue seeking and providing for mathematics learning [Sarah, NSW].

> We continue to take every opportunity to promote relationships with families that encourage family input about children's learning at home that can be extended in the service then flow back and forth. We continue to make learning visible to families and children to promote further learning and celebrate children's achievements [Valerie, NSW].

> I am going to implement something similar with my next group of school readiness parents. I will on this occasion involve all the parents within this group of children and use the information I am given back from the parents to put into the children's portfolios…. I may need to tweak it each year but do what works best for the parents at the centre. The first thing needed is to be able to communicate with them about the importance of mathematics for these children [Jody, NSW].

Parents, too, have shown interest in sustaining the activities they have undertaken as part of *Let's Count*, and some of the educators involved in the study have reported that parents have maintained their communication about their children's mathematics:

> I know some of the families are still continuing with the *Let's Count* activities in their home environments. I often have emails and phone calls from parents asking for suggestions of different activities and receive photos and videos of them participating [Melanie, VIC].

It would seem that a strength of *Let's Count* has been its flexibility and adaptability, and its ability to influence practices beyond the scope of the initial program offering. Indeed, the data from the evaluation would suggest that *Let's Count* has been merely a 'springboard', and that educators and families have taken ownership of the ideas inherent in the program and adapted these to suit their own contexts and purposes. This is key to sustaining the effects reported in this study.

6.6 Implications of *Let's Count* for Mathematics and Transitions to School

Interviews with *Let's Count* educators have shown that the program has provided many opportunities for children, families and early childhood educators to explore mathematics in the years before school. The approach of the program has promoted partnerships between families and educators to support young children's mathematical development. The program has highlighted the importance of noticing, naming and celebrating the mathematical understandings which young children develop prior to starting school, and data from the interviews would suggest that the program has resulted in mathematics being given greater attention and higher priority than it may otherwise have received. Making mathematics a priority in early childhood education is critical because, as Cohrssen et al. (2013, p. 95) argue, "Children who have not mastered foundational mathematical ideas prior to commencing formal education may be disadvantaged as they start school (Chien et al. 2010; Mazzocco and Thompson 2005; van de Heuvel-Panhuizen and van den Boogaard 2009), and the gap may never be closed (Entwisle and Alexander 1990)".

The evaluation of *Let's Count* has highlighted several key imperatives for mathematics education in the years prior to starting primary school. These imperatives include:

- Making mathematics visible in the years before school;
- Celebrating the mathematical experiences of children before school;
- Children, parents and educators noticing, naming and talking about mathematics before school;
- Educators developing strategies for communicating about children's mathematics—both with families, and also for the purpose of inter-setting communication (i.e. between service and school) at the time of transition to school;
- Empowering parents to engage with their child's mathematics education; and
- Raising parents' and early childhood educators' awareness of their important role in children's mathematics education, and the impact of this early engagement with mathematics on later outcomes.

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Chapter 7
The Role of the Home Environment in Children's Early Numeracy Development: A Canadian Perspective

Sheri-Lynn Skwarchuk and Jo-Anne LeFevre

Abstract Experiences that children have at home can establish a foundation for numeracy learning, and serve as an important transition toward school entry. However, Canadian parents and other caregivers do not often have a good understanding of numeracy learning, they may not be prepared to provide appropriate activities, and some may avoid numeracy activities because of their own negative views of mathematics. Accordingly, when parents and caregivers do focus on academic preparation, they typically emphasise literacy over numeracy activities. In this paper, we describe how children's home experiences support numeracy learning, in preparation for school. Our research has shown that young children who are involved frequently in numeracy activities in both formal and informal contexts are better prepared for numeracy learning in school than their peers who have fewer numeracy experiences. These results support the view that parents and other caregivers should be encouraged to take an active interest in children's early learning, and to help children to make appropriate connections between intuitive understandings of numeracy concepts and the formal knowledge that is emphasised in school. Our work has also shown that parents can use their children's personal interests to foster numeracy knowledge. In this chapter, we summarise our findings, present two extreme cases, and provide recommendations to show how caregivers can be involved in providing stimulating numeracy opportunities for children.

7.1 Introduction

In most areas of the world, mathematics abilities are strongly linked to the economic success of individuals and of nations. Butterworth and colleagues described the results of one large UK cohort study showing that compared to literacy abilities,

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people with poor numeracy abilities “earn less, spend less, are more likely to be sick, are more likely to be in trouble with the law, and need more help in school” (Butterworth et al. 2011, p. 1049). Researchers from around the world have been studying the core competencies associated with mathematical, numerical, and number sense proficiencies, and the developmental progression of these competencies into adulthood (Lyons et al. 2014; Purpura et al. 2013). We describe our contribution to understanding children’s development of mathematically related competencies before the start of school, and how parents are involved in the learning process. We discuss the Canadian cultural practices and values that frame our research, provide a summary of our research program, and explore some of our specific findings as they pertain to transitions to schooling. Finally, two case studies are described to show (in an extreme way) how numeracy opportunities may be fostered in the home.

Although the terms *mathematics* and *numeracy* are often interchanged, differences exist. The Oxford online dictionary (2014), describes *mathematics* as “the abstract science of number, and space;” whereas *numeracy* is a subset of the mathematics discipline defined as “the ability to understand and work with numbers.” Given that our current work involves the developmental study of young children, our focus is on understanding how children acquire numeracy skills first, which presumably lead into more general mathematical competencies.

7.2 Our Lived Canadian Perspective

Our research was framed under a Canadian cultural backdrop with our values and lived experiences. As this article is part of an international collection, it is important to share some of our assumptions regarding cultural norms and expectations in Canadian society. These personally lived experiences may or may not be the same as those experienced in other Western countries, and may account for differences in student achievement that have emerged cross-nationally (Programme of International Student Assessment 2013).

Canadian children are part of a pluralistic society represented by over 100 ethnicities (Statistics Canada 2009). Before school onset, families often enjoy paid government funded maternity and/or parental leaves, usually for the child’s first year of life (Government of Canada 2014). After this point, many Canadian children attend a range of childcare programs and/or are in home-based cared with relatives or nannies while one or both of their parents work. Preschool curricula are designed to be play-based with an emphasis on developing socialisation skills (Cromwell 2000). Although there may be some content that is directed toward school-based academics, there is little direction on early numeracy pedagogy (Manitoba Child Care Program 2005), and literacy activities receive preference over numeracy ones (Skwarchuk 2009). Children are required to start school by age 6 (Government of Manitoba n.d.), but most school boards offer half or full day programming starting at age 4 or 5. Education is typically a provincial/territorial responsibility in Canada, and thus each province develops and maintains its own curricula in mathematics and other areas, with some interprovincial collaboration. Variations are common.

Canadian parents have eclectic views on childrearing, and diversity is celebrated, resulting in a varied range of exposure to number-related concepts before their children start school. A few specific examples highlight the variability that exists in Canadian children's numeracy experiences. On the same residential street in one community, one neighbour teaches her child to count in Korean (the parent's native language) and uses workbooks to teach basic number facts. Another neighbour explains addition strategies to her interested Kindergarten child, using her fingers to illustrate the procedures. Another child's home numeracy exposure includes singing counting songs and watching educational TV. Another child in the neighbourhood learns how to say the names of big number words with his grandparents who care for him while his parents are at work. A fifth child practices the counting sequence as a group activity when he attends an early childcare program with his friends. In all cases, family and childcare providers are using their personal beliefs and background knowledge to enhance numeracy opportunities for children in their care but the experiences are very different for the individual children.

There has been recent concern that Canadian children are not being adequately prepared for a numerate world. Although Canada ranks within the top 15 countries on cross-national comparisons of mathematics abilities (Programme for International Student Assessment 2009, 2013), mathematics anxiety is often reported among adult Canadians. There is current debate on the ways in which mathematics should be taught in schools (Reynolds 2012; Stokke 2013); and there has been a slow increase in the dissemination of resources by school divisions, public libraries, and government-funded health and welfare agencies on developmentally appropriate numeracy practices to encourage families and child care facilities to become involved in both formal and informal numeracy educational opportunities (Healthy Child Manitoba n.d.; Canadian Language and Literacy Research Network n.d.).

Changes in technological advances are also affecting the ways in which Canadians are exposed to number concepts. Readily available technological products, such as phones with calculators and applications that allow easy conversions or calculations (e.g., "Tip Me" and "Retail Markdown" phone applications) allow Canadians to avoid performing numerical computations that were once common actions. Furthermore, as parents rely on banking debit and credit cards rather than cash, and on-line purchasing, children have few opportunities to observe monetary transactions occurring in their immediate surroundings compared to the previous generation. Finally, traditional board and card games are now available in digital form eliminating many natural learning opportunities that once existed for learning and practicing numerical operations in collaboration with parents or other children (e.g., rolling dice, counting spaces, adding money and scores, number exposure on basic playing cards). All of these societal changes have been operating in the background over the past decade and have influenced the kinds of questions we have asked parents in our research. As experiences with numeracy concepts change, children's exposure will also evolve, and hopefully, numeracy opportunities with new technologies and in new naturalistic settings will emerge.

7.3 Understanding Children's Numeracy Development

In 2003, there was a strong focus on numeracy development in Canada and around the world. Questions such as "What skills are pre-requisite to learning mathematics in school?" and "How can parents and early educators provide positive numeracy experiences for children?" were unanswerable given the current state of knowledge. Furthermore, focus groups with teachers revealed that they knew what to do with children who could not read, but they were not confident about how to help children with numeracy and/or mathematical difficulties at school. In this context, we initiated a large longitudinal project. The goal of the *Count Me In* project was to study children's numeracy development from preschool until age 10. The project included many cognitive and academic measures. Over 500 children were tested individually in their schools each year across two Canadian provinces, for a maximum of 4 years (some children did not participate in all testing years). The children completed a range of numeracy assessments relevant for their age level (so not all assessments were completed at all grades). Based on this research, the *Numeracy Pathways Model* was developed (LeFevre et al. 2010), which explains the importance of three general competencies, namely verbal abilities, quantitative awareness and spatial working memory in predicting numeracy development. Emergent projects from this dataset focused on specific topics such as the development of children's counting skills (LeFevre et al. 2006) and counting principles (Kamawar et al. 2010), children's use of inversion in solving arithmetic problems (Watchorn et al. 2014), number line development (LeFevre, Jiminez et al. 2013), and the role of executive attention in children's numeracy development (LeFevre, Berrigan et al. 2013). As a part of the *Count Me In* initiative, we also developed a questionnaire for parents to determine how home experiences contribute to numeracy knowledge (LeFevre, Schwarchuk et al. 2009) and an opportunity emerged to compare these results with a Greek sample (LeFevre, Polyzoi et al. 2010). With these studies, we came to appreciate the variety of different skills required for proficiency in numeracy, and the importance of the early home numeracy environment in fostering the development of this knowledge. Similar findings about numeracy trajectories have been supported by the work of other numeracy researchers from around the world (Cirino 2011; Krajewski and Schneider 2009; Vukovic et al. 2014).

Our second line of research focused on the characteristics of the early learning environment that are important for numeracy development. Our *Parents Count Too* project included over 100 parents (mostly mothers, despite our efforts to recruit fathers), representing geographically and economically diverse participants from a new cohort of children about to begin formalised schooling. The parents completed an online or paper survey about: their child's current interests and play preferences, involvement in preschool or childcare programs and other extracurricular activities, as well as parents' involvement styles, and learning expectations for their child. The children were tested individually one year later to determine whether previous activities reported by parents predicted children's numeracy skills. The results of this project led to the development of the *Home Numeracy Model* (Skwarchuk et al. 2014), which shows how advantages in numeracy skills development accrue to children who have a stimulating early learning environment. Furthermore, we explored the importance of parental involvement and fostering children's interests (Lukie et al. 2013) in numeracy skills development. An additional related study described the numeracy opportunities provided by early childhood educators (LeFevre, Sowinski et al. 2009). The findings from these two lines of research (i.e., *Count Me In* and *Parents Count Too* project initiatives) will be described in detail in the next section with particular consideration of how they pertain to transition to school.

7.4 Transitions to School

In understanding the developmental precursors affecting numeracy learning in school, three main findings have been substantiated by research:

1. Children begin learning numeracy concepts much earlier (in preschool and infancy) than was predicted by Piaget's theory of cognitive development (McCrink and Wynn 2009; McEvoy and O'Moore 1991);
2. Early numeracy experiences are critical because children who start school lacking in basic numeracy skills rarely catch up to their peers (Duncan et al. 2007; Geary et al. 2013; Siegler et al. 2012); and
3. Home or preschool environments, with parents (and early childhood educators) as children's first teachers may provide excellent opportunities for numeracy exposure and growth (Blevins-Knabe 2008; LeFevre, Fast et al. 2010; LeFevre, Schwarchuk et al. 2009; Skwarchuk et al. 2014).

When parents ask about how they can prepare their child for entry into school, the answer from our research seems to suggest establishing a solid numeracy base filled with rich early experiences, at home, in child care centres and/or in other learning settings.

What knowledge is relevant for early numeracy learning? According to the *Numeracy Pathways Model*, three kinds of knowledge/skills correlate with children's numeracy skills acquisition in school. Understanding the development of these skills is important for school transition, as many of the skills start to develop before school entry, and parents and educators can be exposing children to the requisite skills for numeracy acquisition.

The first developmental trajectory that has been shown to relate to children's numeracy development from our model is *verbal or language abilities*. One of the ways in which number is represented is as words (e.g., 'four'; Skwarchuk and Anglin 2002). Number words are used to describe the cardinal number sequence, to represent quantities, and to describe quantitative problems. Language abilities were assessed in our longitudinal work by having children identify numbers, provide the next number in a series, and complete place value tasks by naming the digit and positional values in numbers. Language-related numeracy skills such as these were strongly correlated with children's performance on subsequent mathematical tasks. Thus, language extension activities for young children should support their learning. Such activities could include: using number words while counting in sequence by ones, twos, tens, etc.; using relational words (more, less, largest, smallest); and having children practice the rules for counting.

The second factor that predicts numeracy abilities from our model is *quantitative awareness*. Quantitative awareness was assessed in the project by having children: compare two quantities to determine which has more (i.e., magnitude comparisons); determine the quantities of sets through *subitising* (i.e., knowing how many objects are in a small set without counting them); and perform basic addition and subtraction operations without the presence of formal symbols (+, −, =). These skills may be used in activities where children compare quantities, share objects, match items, and note approximate amounts or quantities.

The final factor affecting numeracy acquisition, according to the model, is *spatial and executive working memory abilities*. These skills help children to attend to relevant aspects of the task, inhibit distracting information, and interpret visual information correctly, in a timely fashion. One way in which spatial attention was assessed in the research occurred when children identified the sequence in which a frog jumped from lily pad to lily pad on a computer screen. The children pointed to the order in which the frog moved from one lily pad to the next, increasing in sequence length with each successive trial. In numeracy tasks, visual spatial information may be relevant in order to keep track of digit positions in calculation problems, interpret geometric relations, and maintain information in memory while computing numeracy problems. Although the application of this model is still speculative, some tasks that may implicate the use of *spatial working memory* include: playing games where visual and musical sequences need to be repeated such as Simon or Bop It; or completing games with visual or procedural sequences (e.g., Memory, Chess, Rubics cube, Traffic Jam). One way of ensuring a successful school transition is to ensure that children have had exposure to these three knowledge areas before the start of school.

7.5 Formal Versus Informal Numeracy Activities

How should children be exposed to requisite numeracy skill and knowledge domains? Our second longitudinal project, *Parents Count Too,* attempted to determine the kinds of activities that are predictive of numeracy development, and the ways in which numeracy content is exposed to children in their homes. In this work, parents of preschool children were asked to complete a questionnaire about their child's learning environment. Questions were designed to provide information on the kinds of activities typically enjoyed by preschoolers (e.g., blocks, pretend play, puzzle books such as dot-to-dot) as well as parental demographic information, and parental beliefs and expectations about numeracy learning. Thus, just as learning the alphabet letter names and sounds is important for reading, and linking musical notes with keys on a piano is important for learning to play music, we wanted to determine which activity (or set of activities) contributes to numeracy learning. In line with other research (Huntsinger et al. 2000), we studied children's home experiences to identify the early learning variables associated with numeracy development.

Based on our previous questionnaire data, we selected a range of formal and informal activities in which Canadian children enjoy, and regressed their involvement in these activities on children's numeracy outcomes once they had reached Kindergarten. We defined formal activities as those activities in which an adult has a direct intention to teach children about numeracy concepts or skills, such as when parents report helping children learn simple sums, recognise digits, do math in their head, or talk about time with clocks and calendars. Parents were asked to indicate how frequently they provided these activities for their child. Conversely, informal activities were defined as those in which numeracy learning was not the goal of the activity, but the context and content provided opportunities for children to practice or at least be exposed to numeracy-relevant experiences. Although it seems that informal opportunities are harder to conceptualise than formal numeracy activities, some examples could include: cooking or carpentry activities, playing card/board games or buying something with money. In our study, informal numeracy exposure was assessed by having parents identify the names of real board games from a list of real and distracter names. If parents are playing these board games with their children, they should be able to identify more of the games than parents who infrequently chose such activities. This practice eliminates participants from responding according to how they feel they should respond, and has been used successfully in literacy research where parents' knowledge of children's books consistently correlates with children's vocabulary knowledge (Sénéchal and LeFevre 2002).

Our results revealed that both formal and informal experiences are related to children's numeracy acquisition, but through different mechanisms. Formal numeracy knowledge was related to children's basic number knowledge in kindergarten (as assessed by the KeyMath standardised test measuring counting skills, number recognition abilities, and the ability to define numeracy related vocabulary words). This finding paralleled the work with early literacy, where parents' reports of teaching their child to read or write words predicted children's learning to read words in grade 1.

Conversely, informal numeracy exposure (measured using parents' knowledge of board games) was associated with the ability to mentally represent and manipulate small quantities, as measured by children's abilities to perform non-symbolic arithmetic in numeracy research. This non-symbolic arithmetic measure requires children to keep track of the number of plastic animals added or subtracted from a toy barn, by reporting the total number of animals contained in the barn. Children show their informal knowledge of addition or subtraction principles without the use of any digit or operational symbols.

In sum, our project supported a model in which the contributions of both formal and informal ways of delivering numeracy content to young children were predictive of children's numeracy performance. The *Home Numeracy Model* illustrated how parents' intent to teach numeracy concepts correlates with children's basic numeracy content knowledge in Kindergarten; whereas informal exposure via board games and other activities having some numeracy content, contribute to children's understanding of informal principles and quantitative awareness. Thus, home experiences can be used to support the early exposure to concepts shown to be important in the *Numeracy Pathways Model*.

7.6 Cultivating Children's Interests and Encouraging Parent Involvement

According to early childhood philosophies on learning, parents and and early educators may feel it is important to balance directed early learning opportunities (formal exposure) with the philosophy of child-directed learning (which could align with informal exposure), where the child plays a leadership role in determining what they will learn based on their interests (Cromwell 2000). But what about the child who prefers to play with miniature dolls or cars instead of participating in board games, shared book reading, or number learning? Do children's preferences exert a strong influence on the activities that parents provide or encourage in their homes? In our *Parents Count Too* longitudinal study, parents were also asked to identify their child's preferred activities and these activities were linked with parents' reports of the opportunities they provided for their children's numeracy learning (Lukie et al. 2013). We asked parents about two specific home activities that children and parents often share, that is, cooking and making Valentine or birthday cards. We were interested in how parental involvement styles in these specific situations (situations that invoke collaborative versus adult-directed approaches) might relate to parental reports about the frequency of other numeracy activities. In addition to the questionnaire data collected in the *Parents Count Too* project, interviews were also requested with eight of the families who completed the initial questionnaire to provide further insights into the varieties of home experiences occurring in children's homes, to validate and extend the survey data.

Factor analyses grouped child interest items into several categories: cognitive exploratory play, active play, crafts and screen time (television, computer and video game activities); and then these composite measures of children's interests were correlated with the frequency of children's numeracy-related activities (as reported by parents). Parents who reported that their children preferred exploratory, active or craft activities had children who frequently engaged in numeracy activities. Furthermore, parents who used more collaborative approaches (as opposed to adult directed) in their interactions with their children in the cooking activity also reported frequent participation by their children in numeracy activities. Exposure to various screen time activities was not related to numeracy learning.

From the interview data, we learned more about the ways in which parents incorporated numeracy learning in activities while respecting their children's interests. For example, one parent taught her son how to use a spreadsheet and graphing program to keep track of NASCAR racing statistics. Another parent described playing board games such as Monopoly with her son as the banker. Finally, a third mother traded stickers with her daughter, which were categorised and organised as a collection on a hallway wall. These examples are included because all of these children in our sample obtained high scores on our numeracy assessments. But all families reported some numeracy involvement with their children. For example, parents reported collaborative involvement with their children counting dinner plates, writing numbers, playing with dice, skip counting while walking down basement stairs, spending money and counting change while buying gummy candies, and discussing practical numeracy problems as they occur in everyday contexts. In the next section, two case studies are used to illustrate how two Canadian families fostered the numeracy learning of their children when mathematics was a favorite passion of the child, and how parents provided numeracy opportunities because they were motivated to do so by their child's interests in mathematics. These cases may provide numeracy ideas for all families, whether children have mathematics as a passion or not.

7.7 Two Cases of Children with a Passion for Numbers

What happens in the extreme case where a child shows an early interest in and fascination with numbers and mathematical concepts? Developing a passion for a specific topic has been found to contribute to the development of cognitive skills and provide positive learning outcomes (Hidi et al. 2004). Furthermore, parents may play a role in developing and maintaining a child's interest. Although it is difficult to make causal links between parents' behaviors and children's mathematical learning, the following examples are illustrative of collaborative numeracy activities that may contribute to positive learning experiences in mathematics. Reflections from two sets of Canadian parents of boys (now attending school) with early interests in mathematics are included as a starting point to show examples (at the extreme end) of how numeracy content could be included in early childhood. Although there are ordinary examples documenting how children 'do mathematics' in natural learning contexts with adult facilitators (Anderson et al. 2004, Vandermaas-Peeler et al. 2012), the extreme examples described herein show ways in which mathematics can be extended to children when interests and parental involvement intersect. Other parents and educators may be empowered by these examples to encourage the mathematics learner in their lives.

Darren (pseudonym) is the youngest of three children living in a middle-class neighbourhood with his two sisters and his parents. Darren's father, who is a math teacher, spent lots of time pointing out patterns and shapes that seemed mutually interesting. On one occasion, Darren noticed a rainbow, which could have evoked discussions on colour naming and leprechaun folklore, but instead, his father discussed wavelengths of light. As a preschooler, Darren enjoyed jigsaw puzzles, games involving cards or dice, snowflake cutting, number printing, magic tricks and science "experiments." Once he started school, Darren sought out activities involving arithmetic computations, and due to his persistence, his father would show him new ways to compute numbers. Darren particularly loved the process of doubling numbers.

During informal conversations his father taught him about addition and subtraction first, and curiosity and mutual interest led them into topics about multiplication, division, and factorials. Darren memorised some math facts that are not usually taught to young children (e.g., $6! = 720$). He invented his own mathematical symbol called "ab-string" (which was a shortcut symbol for adding two numbers and dividing the result in half) and he assigned it a special symbol. He also enjoyed printing and saying large numbers and wrote and illustrated stories about "the mathematiser," a boy who is good at math. Darren continues to enjoy working with numbers and has above-average numeracy scores. His teacher recognised his strengths and interests in mathematics and invited him to participate in a lunch hour mathematics club intended for older children.

In the second case study, Dylan (pseudonym) is an only child living in a middle-class neighbourhood with his two parents. Both of Dylan's parents have doctoral degrees in STEM (Science, Technology, Engineering, Mathematics) related disciplines. Dylan is a precocious learner; he has always been interested in alphabet letters and numbers and he learned to read at age two years. Dylan played with anything number or letter related (e.g., foam numbers, magnetic letters, Scrabble letter game tiles, books, puzzles, computer games and applications with a number theme). By the age of just over two-and-a-half, he spent one afternoon filling the driveway (and the neighbour's driveway) with chalk-written numbers up to about 160. He counted each piece of food before eating, or his parents would arrange food on a plate in the shape or denomination of a certain specified quantity. His parents looked for materials and had discussions to stimulate his interests. His mother stated:

> When he was in preschool and kindergarten, we showed him things like how to use a number line, counting, counting by multiples and simple times on a clock. Then he moved on to using tools like adding machines, calculators, slide rules (brief discussion of logarithms and powers), times tables, and flash cards. When he was 4 (Father) explained to him how addition was commutative, while subtraction was not, and a couple of days later (Dylan) pointed out that rhyming was also commutative. He's very good at that sort of thing—taking a new concept he has learned and applying it to another topic or situation. We don't have to do much to foster that—he just seems to take new ideas, process them and stores them away for later use.

Psychological testing has confirmed that Dylan has very high achievement and intellectual scores, supporting his parent's reports about his interests and abilities in both mathematics and reading. Now in school, his favourite subject is mathematics, and he prefers to discuss anything physics-related.

Although both of these cases represent extreme cases in which children's interests and parents' involvement can enhance numeracy learning, they nevertheless are good examples of ways that children acquire mathematical concepts that are consistent with our research. To reiterate, we cannot conclude that the behaviours of parents in these vignettes caused positive mathematical trajectories. However, for both children, parents' retrospective reports suggest that (perhaps because of their own personal background experiences) they knew how to promote numeracy learning, and provided both formal and informal numeracy exposure to their children. The vignettes also showed that these parents were highly involved in their children's learning and provided collaborative experiences. Thus, these case studies support the more general conclusions that we drew from the survey data and also suggest that interactions between children's interests and parents' involvement can contribute to a rich early numeracy experience.

7.8 Recommendations for Parents and Early Childhood Educators

Although some Canadian parents have expressed frustration over their children's mathematics learning (Reynolds 2012), and believe that the way mathematics is taught in schools needs to change (Stokke 2013), little attention is paid to opportunities available at home that may set children on the right track for numeracy learning. Just as researchers have shown the importance of reading storybooks and learning alphabet letters for children's early literacy acquisition (Sénéchal and LeFevre 2001), the results of our work show that children may also benefit when they are exposed to similar activities for numeracy. Exposure to formal activities such as mental arithmetic, writing numbers, or constructing large numbers may be important for developing number system knowledge. In addition, informal exposure to number concepts through activities such as game playing, cooking or completing jigsaw puzzles may help children to develop an awareness of numbers and quantities. Both types of exposure may be relevant to children's number knowledge abilities, and affect vocabulary and quantitative awareness abilities once children start school. Thinking about numeracy in relation to children's interests and involving them collaboratively in the learning process may enable numeracy and other related cognitive skills to enhance development in their natural environment.

Parents and educators with young children also need to pay attention to their own biases and expectations concerning numeracy learning. In one of our offshoot studies involving early childhood educators, those caregivers who had high expectations for children's learning tended to provide advanced numeracy activities and opportunities for children in their care (LeFevre, Sowinski et al. 2009). Because parents often report that they dislike mathematics activities, and that they do not know how to introduce numeracy concepts to their children (Blevins-Knabe 2008; Skwarchuk 2009), further resources may be needed to increase public awareness of the importance of early numeracy exposure, to avoid the transmission of negative adult messages that may affect children's numeracy learning. In our opinion, parents and educators do not require doctoral degrees in STEM-related disciplines to create numeracy opportunities for their children. Parents who have strong mathematical backgrounds may be in advantageous situations to know about and take interest in numeracy opportunities, as those situations occur. Using the extreme cases as examples in this chapter, it is clear that these parents were not doing anything special that required extra training or materials. Reflective practice and parental involvement in home contexts to pull out important numeracy concepts is all that is required.

Despite the considerable developments in research on the role of home experiences in children's early numeracy learning, further research is needed to improve assessment of children's early mathematical skills and to facilitate the development of remediation programs for children struggling in mathematics (Grégoire and Desoete 2009). For example, how do we help the child who cannot count? How can pre-numeracy skills (beyond counting ability) be assessed before school onset? What if a child is lacking in one or more of the developmental trajectories? It is not possible to provide a comprehensive slate of appropriate remedial activities that are evidence-based given the current state of numeracy research. Furthermore, most findings in the early numeracy field are correlational (our work included), and so caution is warranted when attributing cause. Although some intervention work is promising in that it has a solid empirical base (Ramani et al. 2012), and there are now evidence-based recommendations available for parents of young children (Kotsopoulos and Lee 2014), more empirical work is needed to resolve the uncertainties associated with providing children a solid numeracy base before school onset.

In summary, our research program over the past decade has resulted in two models that account for children's early numeracy learning from preschool throughout the school years. Our first model from the *Count Me In* longitudinal project has been called the *Numeracy Pathways Model* (LeFevre, Fast et al. 2010). It shows how children's numeracy abilities are related to three pathways of development: language abilities, quantitative awareness and spatial working memory skills. The second model from the *Parents Count Too* project has been labelled the *Home Numeracy Model* (Skwarchuk et al. 2014). It parallels trajectories in literacy development, showing how the early (home) learning context provides an opportunity for numeracy exposure in both formal and informal contexts. When numeracy content is related to children's interests, and caregivers are involved in fostering numeracy knowledge, such as in the two case studies and the many other examples described herein, children may have considerable opportunities to develop and connect numeracy ideas in natural contexts, affecting the early skills acquisition described in the first model. Both of these models are relevant to providing appropriate numeracy transitions as they highlight the knowledge that is required (Pathways Model) and the ways to deliver that knowledge (Home Numeracy Model) ensuring a successful transition into formal schooling.

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Chapter 8
Mathematics Teachers Responding to Children's Resources to Create Learning for All

Jónína Vala Kristinsdóttir and Hafdís Guðjónsdóttir

Abstract The focus of this chapter is on how teachers respond to children's resources and their mathematical thinking as they transfer from preschool to primary school. The theoretical framework builds on sociocultural theories. The area of investigation is individuals changing their ways of understanding, perceiving, noticing, and thinking as they collaborate with others. Thus, the emphasis is on classroom cultures and learning environments that promote mathematical learning where all children have a voice and are supported to develop their understanding. The methodological approach comprises narrative inquiry and analysis. Through focus group interviews, narratives are gathered from teachers who work with children in preschool and the early primary grades. We learned that what characterises these teachers is their belief that all children can learn mathematics if learning spaces are created that respect the children's resources. The teachers analyse children's mathematical resources and respond to what they bring with them to school as they organise classroom cultures and develop supportive mathematical learning environments.

8.1 Introduction

The diversity of pupils in Icelandic schools has increased in the last two decades as pupils with disabilities have entered their neighbourhood schools and immigration has brought in students for whom Icelandic is not their native tongue. These changing social conditions have put increasing pressure on teachers to modify their practices and take into account the diverse group of learners that forms their learning communities. From our earlier research and work as teacher educators, we have learned that many teachers find it challenging to teach mathematics. Their own experience as mathematics learners was typically as passive receivers who practiced rules and procedures, introduced by teachers and textbooks. Teachers lack

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experience in investigating, communicating, reasoning and making connections in mathematics. Additionally they feel incompetent in using these approaches in inclusive schools (Guðjónsdóttir and Kristinsdóttir 2011).

For 10 years we have collaborated with our colleague Edda Óskarsdóttir on developing the graduate course *Mathematics for all.* We conducted a self-study of teacher education practices as we planned, implemented and reflected on our teaching, together with the students. Originally this course was developed for special education teachers and the focus was on children's difficulties with mathematics and how teachers could support individual children based on their analysis of the children's difficulties with mathematics. As we learned about these teachers' lack of confidence in using flexible approaches in mathematics teaching we modified the course. The new focus was on children's mathematical development and understanding and teachers' capacity to evaluate and promote pupils' learning through analysis of their engagement in authentic mathematical problems. Additionally, we have considered the teachers' own explorations with mathematics and discussions about their mathematical thinking.

Findings from our ongoing study suggest that if teachers are given opportunities to integrate their experience in their studies and to relate theory and practice, their ability to make informed decisions about teaching and learning increases (Guðjónsdóttir and Kristinsdóttir 2011).

In search for more knowledge about how teachers respond to diverse learners as they organise the learning environments, we contacted nine teachers. We discussed the theories the teachers build on and the way data were collected and analysed. In this chapter we report on their work as mathematics teachers and conclude by discussing the implication of our work for future research with teachers who teach young children mathematics during the transition from preschool to primary school.

8.2 Children's Mathematical Thinking

In this chapter we will discuss the theories on which the participants in this project base their work. As the teachers searched for understanding young children's developmental processes and how mathematical thinking matures, they have revisited their former studies on children's learning while studying and digesting new theories and research findings. Piaget's theories of children's development and the belief that knowledge is an ongoing constructive process, as well as the fact that it varies when children reach the developmental stages he identified (Piaget 1969), support these teachers' work. They are also guided by Vygotsky's theories on children's learning about solving practical tasks with the help of their speech, as well as their eyes and hands and the dynamic relation between speech and action. His description of the zone of proximal development that defines those functions that have not yet matured but are in the process of maturation (Vygotsky 1978) has also influenced their work with children in preschool and the early primary grades.

The work of Dewey (1966), and his emphasis that learning situations must be flexible to enable children to be active learners, has influenced the teachers' beliefs and actions. Bruner's descriptions of children as active learners and the influence of culture in education (Bruner 1996) have shaped their ideas of teaching and learning and the social culture of the classroom. The theory of multiple intelligences (Gardner 1993) has also supported them in respecting children's diversity.

In the early 1990's the work of Pratt (1948), founder of the City and Country School in New York, and the 'unit blocks' she devised, were introduced to preschool teacher education in Iceland. In 1991, Cuffaro (1996) participated in a research project with a teacher educator, Tryggvadóttir, and preschool teachers in two preschools in Iceland. The experience gained from the project had major impact on the work in many preschools and also in the lower grades in primary schools. The blocks are now available in most preschools in Iceland and many primary schools.

The Cognitively Guided Instruction (CGI) research project (Carpenter et al. 1995) was introduced at a course for teachers in early primary grades in Iceland in 1995. Some of the teachers in this project participated in the course and so did the authors of this chapter. We learned about the findings of the research team at the University of Wisconsin-Madison on children's thinking about whole numbers and addition, subtraction, multiplication and division problems. By the end of the course the participants were eager to use what they learned about children's mathematics thinking in their work with children. This enthusiasm resulted in workshops where we met to discuss our work and establish collaboration. The results from the CGI project impacted all our teaching and as the research was continued we have followed the writings of the research team and used their books (Carpenter et al. 1999; Carpenter et al. 2003; Empson and Levi 2011; Hiebert et al. 1997).

Sarama and Clements' (2009) research on learning trajectories has also guided our work. Their book *Early childhood mathematics education research: Learning trajectories for young children* has been one of the main readings at a course about young children's mathematical development that many of the participants in the project have attended. The book *Engaging young children in mathematics: Standards for early childhood mathematics education* (Clements et al. 2004) has also been a part of our course material. In their work with young children the teachers draw on research findings presented in these books.

Research on early childhood education in Australia has also impacted our work. The participants have been impressed by descriptions of children's powerful mathematical ideas as presented in *The Numeracy Matrix* (Perry et al. 2007). They can identify with how important it is for teachers to acknowledge children's mathematical thinking.

In her search for supporting herself and her co-teachers in first grade in learning how to listen to children and assess their numerical knowledge, one of the participants in this project found that *Early Numeracy: Assessment for Teaching and Intervention (the Mathematics Recovery Project, MRP)* (Wright et al. 2006) was helpful. The assessment interview and the information gained through discussion with the child have helped them to listen to children and learn to appraise their numerical knowledge.

The teachers who participated in this research project have all engaged with literature on young children's mathematical learning and revisited their understanding of children's development at the transition stage between preschool and primary school. The focus is on the resources the children bring with them to school, respect for children's diverse backgrounds, as well as supportive learning environments.

8.3 Teacher Professionalism

Inclusive educational practices are responsive to diversity, concerned with and value all pupils equally. Booth (2010) introduces values that he believes are important for schools as they develop learning communities for all pupils. These values involve issues of equality, rights, participation, learning, community, and respect for diversity, trust and sustainability and also the qualities of compassion, honesty, courage, and joy. A commitment to such values prepares teachers and teacher educators to increase the participation of all, overcome discrimination and create an environment that ensures learning for all children. As we understand more about the focus of learning and how people learn, we also understand the critical combination of intellectual and social development and the need to continue learning in the face of constant change and societal complexity. The ability to think, present ideas and work with others is recognised by education and businesses alike as central to the world's future (Fullan 1999).

Schools that make progress towards inclusive ways of working develop the capacity for teachers to learn from one another so that they share ideas and practices and spend time discussing how teaching can be improved. Teacher educators who embrace diversity and inclusion also need to learn how to observe carefully so that they continue to understand practice as it is carried out in their own classrooms and countries. Such processes become starting points to continue the journey of new learning (Ainscow 2007).

Moore (2005) discusses the importance of transformation from theory to practice and concludes that if teachers are expected to teach for diversity and understanding, they need opportunities to develop and enhance their mathematical pedagogical knowledge. It is important for them to experience their own mathematics learning in an environment that reflects the one they are expected to create in their classroom. Teachers are empowered to practice a culturally responsive and socially relevant pedagogy as they begin to look critically at their classroom environment. The practitioner becomes the action researcher, transforming theory into practice and researching on that practice.

In a research project with teachers, Guðjónsdóttir (2000) identified the following diverse roles that professional educators embrace depending on their circumstances and opportunities.

- Pedagogues and experts in teaching and learning: Activist teachers share their knowledge and understandings in an ongoing professional dialogue.
- Reflective and critical problem solvers: Teachers continuously monitor pupils' progress and learning within the classroom. Outside that environment they

reflect both as individuals and as communities of practice on their practice and pupils progress.

- Researchers and change agents: In seeking a deeper understanding of their practice, or in seeking to plan for change, teachers use a variety of evaluation and action research techniques to collect and interpret findings to inform their thinking and decision making.
- Creators of knowledge and theory builders: In the process of reflective practice and action research, teachers develop new understandings of learning, teaching and educational change.

Similarly, Cochran-Smith and Lytle (2009) continued their long discussion of teacher research by identifying five critical elements for consideration in the future discourse on teacher professionalism:

- Emphasis on the teacher as knower and agent of change.
- Creation of new ways to theorise practice.
- Participation of teachers and colleagues in intellectual discourse about critical issues.
- Linking teaching and curriculum to wider political and social issues.
- The creation of inquiry communities that focus on the positive, rather than negative, aspects of what teachers know.

Learning to learn from one's own practice requires active engagement and reflection in communities with others. This was reflected in the results of the research on the development of the teachers who participated in the CGI program discussed above. In their conclusions Fennema et al. (1996) reported that developing an understanding of children's thinking provides a basis for change, but change occurs as teachers attempt to apply their knowledge to understand their own pupils. The CGI study provides strong evidence that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction. It also appears that this knowledge is dynamic and ever-growing, and can probably only be acquired in the context of teaching mathematics (Fennema et al. 1996).

In a longitudinal study of four teachers' collaboration and reflective discussions, Jónína (Kristinsdóttir 2010) found that teachers' discussions about their own pupils' ways of learning mathematics and reflections on their teaching can influence teaching in diverse classrooms. Through constant discussions on the children's solution strategies and reflection on their teaching, the teachers developed their understanding of their pupils' learning. According to Mason (2002), such systematic reflection on mathematical interactions that focus on student's learning and understanding of processes, as well as on one's own interaction with the pupils, represents an essential professional competence of teachers.

Commonly schools respond to diversity in ways that divide and separate children into hierarchies of value and perceived aptitude. Evidence of these approaches can be seen in labelling and/or sorting of pupils by ability and limited consideration of the potential of all learners (Booth 2010). It is only when teachers open their minds and their classrooms to diverse groups of pupils that they will be enabled to responsively develop learning communities in which teachers and pupils engage in a spectrum of different learning needs.

Perspectives and attitudes of teachers towards diverse groups of pupils are more positive if they have been involved in inclusive education, compared to teachers who have not had that opportunity (Pugach 2005). The most important role for the teacher is creating a classroom in which all students can reflect on mathematics and communicate their thoughts and actions. Building a community of mathematical practice requires teachers to take the lead in establishing appropriate expectations and norms. In classroom cultures that promote mathematical learning, all students have a voice and are supported in developing their understanding of mathematics through exploring, investigating, discussing, reflecting and drawing conclusions.

8.4 Research with Teachers

Narrative inquiry is a way of understanding and researching experience through collaboration between a researcher and participants. It is based on the premise that as human beings we come to understand and give meaning to our lives through stories. It is a common model for research with practitioners; they receive the opportunity to open up their practice, telling their stories as they use their lived experience as a source of their knowledge and understanding (Clandinin 2013).

The purpose of this research was to develop an understanding of how teachers create mathematical learning environments that respond to young learners during the transition from preschool to primary school. Our intention is to give examples from the teachers' community of practice and their classroom cultures, highlighting collaboration and co-learning with colleagues both within their school and between school levels. The goal was to gain knowledge and understanding of how teachers draw on children' resources and how they use children's mathematical thinking as they plan their teaching for children in preschools and the early grades of primary school.

Rodriguez (2007) defines resources as personal qualities and strengths emerging from and shaping life experiences. She drew on the work of Wertch (1998), in which cultural resources are the mediational tools for people to make meaning and act in the world, and Gonzales et al. (2005) who envision culture as 'funds of knowledge' that can be seen as resources for pupils to draw upon to enhance learning. Thus by resources we refer to the personal qualities and strengths that emerge from and are shaped by children's life experiences.

The research questions that guided this narrative inquiry were the following:

- What kind of learning environment do teachers create that supports all children in learning mathematics?
- How do they draw on the mathematical resources children bring with them as they transfer from preschool to primary school?

In our search to understand more about teachers' professionalism in teaching young children mathematics, we decided to collaborate with nine teachers in preschools and primary schools with whom we have been connected through different projects. We invited them to have a conversation with us and tell us stories that were meaningful to them. Our intention was to reflect with them in order to understand their work and how and why they respond to children in the way they do. Thus, we chose to create focus groups that would come together so that teachers could discuss their evolving experiences (Heikkien et al. 2007). We use their stories to explore the way they respond to children's resources and their mathematical thinking as they develop supportive mathematical learning environments. Gathering narratives that are written, oral or visual and focusing on the meanings that people ascribe to their experiences is both the method and the phenomena of this study (Trahar 2009). Our plan was to examine these issues in depth through exploratory, open-ended conversations, prioritising holistic understanding situated in teachers' lived experiences.

In this research, data consists of documents from the nine teachers and transcripts of discussions in focus groups. We conducted three, two-hour discussions each with two, three or four teachers, inviting them to tell their professional stories, opportunities and challenges in their teaching. During the discussion, teachers shared their work with children not only through telling, but also by sharing pictures, videos, projects and tasks by their pupils. Using teachers' stories to explore their practices, narrative inquiry allowed us to understand their representations of their educational settings and their actions and interactions within them. All discussions were audio recorded and transcribed.

In order to understand how teachers were using students' resources, data were analysed by exploring the transformational dimensions of storytelling from different perspectives. As we looked at the different experiences and backgrounds these teachers brought into the settings, the data were deconstructed in order to reveal discourses that foreground how teachers utilise students' resources in their learning. Vital events and scenarios related to the research questions were extracted. The analytical lens was based on the narrator's voice and the verbal action and choices, as well as the ways the narrative was constrained by social circumstances. From different perspectives within the social, cultural and historical context, the data were brought together again into narratives (Hunter 2010). In so doing the participants had opportunity to respond to the narratives chosen and decide how to tell their stories.

The narratives reported illuminate how teachers build on children's resources in developing supportive mathematical learning environments as they transfer from preschool to the early grades of primary school. The collaborative reflection gave a picture of how they use the personal qualities and strengths that emerge from and are shaped by the children's life experiences.

The teachers participating in this project are: **Ásta**, who is educated both as a preschool and primary school teacher and taught for several years in preschool but is now teaching young children in the first grades in *grunnskóli* (compulsory school in Iceland for 6–16 year-old children). **Birna** is a teacher in *grunnskóli* and has taught mathematics at all levels. Her experience is mainly from teaching in the early primary grades. **Díana** is educated both as a preschool and special education teacher. She is responsible for developing supportive learning communities for all children in her preschool. **Dóróþea** who is educated both as a general and special education teacher. She teaches in *grunnskóli* and her main focus has been on the early primary grades. **Edda** who is educated both as a general and special education teacher. She teaches in a compulsory school and is responsible for planning meaningful learning opportunities for all children in her school. **Guðrún** is educated both as a preschool and primary school teacher and taught for several years in preschool but is now teaching young children in the first grades in *grunnskóli*. **Kristjana** is a teacher in *grunnskóli* and has taught mathematics in the primary grades. **Margrét** is educated both as a preschool and primary school teacher and taught for several years in preschool but is now teaching young children in the first grades in *grunnskóli*. **Þórunn** is a teacher in *grunnskóli* and has taught mathematics at all levels. Her experience is mainly from teaching in the early primary grades.

All of the teachers in the study have experience with a diverse set of responsibilities. Some, but not all, have had experience in teaching both in preschools as well as compulsory schools and some have only taught in either preschool or primary school.

8.5 Teachers' Reflections

Four recurring themes grew from our analysis. The teachers were concerned with respecting children's resources and their powerful mathematical thinking. They also found it important to be aware that the transition from preschool to primary school can be difficult for children and therefore teachers at both school levels need to collaborate. The teachers believe that they are responsible for creating a supportive learning atmosphere where children feel free to explore, discuss and collaborate. Collaboration with colleagues and the children's parents is equally important and the teachers realise that such collaboration supports their professional development.

8.5.1 Children's Resources

When children start primary school they bring with them experiences from preschool. Díana is aware of young children's explorations with mathematical ideas and the importance of having the opportunity to develop them in preschool.

From Díana's journal:

> Rut, Birna and Ásta, 5-year-old girls, are playing together with two-dimensional shapes in different colours. They have grouped all the yellow shapes together and the red shapes, too. Then Rut says: "Let's just mix it all together again". When they have put all the shapes in one pile they fetch a cardboard square with two mirrors placed diagonally on two sides. "Now let's do something smart", Birna says and places a yellow rectangle on the cardboard square. "Let's first put all the boxes", Rut adds, as she places another rectangle on the

> cardboard. "Here is another box"; Ásta hands Rut a rhombus. "No, not a broken box" Rut replies. They finish one row with rectangles and then begin another row with rhombuses and the third again with rectangles. "And now we can put what we like, all kinds of blocks and colours and everything", Ásta says. They help each other to cover the whole area and Ruth adds: "We just put all kinds, yellow, blue and green and make a kind of a pattern". When they have finished their work they proudly show me their pattern and how it reflects in the two mirrors.

The girls are playing with shapes in different colours and decide to make a pattern that reflects in the mirrors they use. Although they do not use mathematical nomenclature to name the shapes, they are consistent in using the word 'box' for a rectangle and saying that a rhombus is not a box, it is a broken box. This story tells us that the girls have their own image of the shapes and are consistent in what concepts they apply to them. Their teacher respects their thinking, is present and does not interfere in the process, but is willing to discuss their work.

In her work in the primary grades Guðrún collaborates with the neighbouring preschool and teachers of older children in her own school. Once a month children of different ages spend a day together outdoors and work on different kinds of projects where mathematics plays an important role in their explorations and children support each other in their work.

In a focus group meeting Guðrún explained her vision for teaching:

> The preschool teacher is within me and I try to meet the children where they are. …. The outdoor education project is in collaboration with the community and we collect information that can be of use for the development of our community. …. There is a lot of measurement and counting, we look for patterns and regularities and we register our findings in different ways, by collecting things, writing and taking photos. We also make new things from our collections and thus integrate with other subjects.

Guðrún expresses clearly that what she brings with her from preschool to primary school is respect for children. She is also aware of how important it is for children in neighbouring preschools and primary schools to work together and to respect the learning community within each school.

8.5.2 Transition from Preschool to Primary School

For 20 years emphasis has been on building a bridge between preschools and primary schools in Iceland. It started with a developmental project between the Nordic countries where emphasis was placed on smooth transitions from preschool to primary school and teachers learning from working together (Menntamálaráðuneytið 1997). The children pay mutual visits to schools in their neighbourhood; the preschool children then learn to know the school they will attend later and when they have started primary school they have a chance to visit their former preschool. The teachers work closely together to make the transition between the schools smooth and learn from each other's work.

Ásta got acquainted with Caroline Pratt's unit blocks through collaboration with a preschool in her neighbourhood and asked permission to buy them for her school.

She used the unit blocks in her teaching in the primary grades and supported other teachers in using them. She has developed her work with the blocks and published a website where she reports on her work, <http://astaegils.is/>. She has written for teachers about the values of working with the unit blocks.

In a focus group meeting Ásta told us:

> … and as Caroline Pratt says about the unit blocks, the children internalise the blocks and their forms, and when they start to learn formally about this, then it is there, and then they only have to put the name on it. Thus they, and it is a part of this construction, that in their construction in working with the blocks they give them names, as they perceive them. ….The teacher though needs to use the proper concepts when naming the blocks.

This excerpt shows that Ásta encourages children to develop and expand their own mathematical ideas, while gradually linking more formal concepts, such as standard names for the blocks, to the children's informal understandings.

On her website Ásta writes about the mathematics children learn by playing with the blocks:

> The mathematical properties of the unit blocks and their internal relationships make them a practical learning tool in mathematics. While building with the blocks the children get multiple opportunities for mathematical reflections and learning mathematical concepts in a way that is natural to them. In their work with the blocks they encounter many problems they need to solve to succeed. The repetition and the perseverance that is so rich in the development of the building process help the children gradually learn to organise their work and think creatively in solving the problems they meet when building with the blocks and discovering their mathematical properties.

Thus, in addition to linking formal and informal knowledge through the block play, Ásta also stresses the importance of practice and of children constructing and re-constructing their understanding of mathematical concepts. By drawing on her experience as a preschool teacher and collaborating with the neighbouring preschool, Ásta supports the children in the transition from preschool to primary school. Her understanding of children's development and her respect for their need to explore the world is reflected in her work.

8.5.3 Responsive Classrooms

In planning their teaching, the teachers reported that they draw on what they have learned about the teaching and learning of mathematics. The teachers emphasise problem solving in their classrooms. They often plan these lessons with the think-pair-share lesson approach. The work usually begins with a whole class discussion about the problem and then children work in pairs or in small groups and by the end of the lesson they discuss their solutions with the whole class. Kristjana and Margrét have written about their work in a journal for mathematics teachers (Ásgeirsdóttir 2009; Skúladóttir 2009). Margrét writes about small group discussions: "The advantage of small group discussions is that they are democratic and all the children are active participants." Kristjana writes: "The teacher urges the pupils to collaborate and thus supports them in the solution process and urges them not to give up."

The end of class discussion is important, too, and Kristjana emphasised that:

> Everyone in the class listens to the discussions and if someone does not understand what is being discussed he/she gets a chance to ask for more information and probe for further explanation. The children feel safe to ask questions to gain further understanding and are not afraid of discussing their mistakes because we all learn from discussing them.

When Þórunn got the opportunity to participate in the planning of the teaching in a new school she introduced her idea of thematic mathematics units with mixed age groups. Her belief that positive experience of mathematics learning is important for all children supported her in planning this project and involving her colleagues. The emphasis is on concrete objectives in mathematics and strong relationships with the pupils' environment. The endeavour is to respond to pupils' diversity by multiform methods and resources. The connection between mathematics and daily lives is endless and the teachers' responsibility is to find ways to make this both simple and interesting but at the same time effective.

The teachers often plan their mathematics teaching around activities that can occur in learning stations, that is, designated spaces where pupils can work on mathematical tasks. Usually teachers divide the class into groups of two or four pupils. Each group goes from one station to another until all of the stations are completed. Math learning stations are designed to benefit diverse learners, and therefore teachers often offer more than one task at each station. There are clear opportunities to work on the tasks in different ways and tasks for independent work. Sometimes the teacher works at one station with pupils and facilitates discussions around pupils' work. The emphasis is often on various real life projects designed to engage pupils in authentic tasks relevant to their daily lives. Other times they are designed to be locally relevant so that children can directly relate to them.

8.5.4 Collaborating with Colleagues

The teachers reported repeatedly on collaboration with colleagues and parents. They say that it is as helpful to establish a good relationship with them as it is to have a good relationship with their pupils. The teachers find it important to establish a community of learning for teachers: a space for discussions, sharing, and supporting each other. Kristjana and her colleagues have managed to create a space for teachers with different backgrounds and beliefs, and report that they all participate in these discussions although they don't always agree. She said: "We discuss everything, how we work with children and their contributions, how different children solve the problems in various ways and what they say and what they think". These discussions made those teachers more aware of their own thinking and influenced their professional language. Margrét, who teaches at the same school, feels that by becoming a part of these discussions she builds on her professional development to change her practice.

Birna is a divisional manager and runs a mathematical facility at her school. She meets with teachers to gain information or requests for a focus and then she structures learning stations for different groups. In other circumstances she team-teaches.

She has compiled a mathematical kit with which she travels from classroom to classroom depending on who invites her to work with their class. Edda, a special educator, organises collaboration between the classroom teachers and the special education teacher so that the teachers can divide the class into smaller groups. The teachers are satisfied because not only do they work with smaller groups of children, but they also plan their teaching together, learn from each other and gain an opportunity to discuss teaching and learning. Birna, Kristjana and Margrét also offer workshops for teachers where they address mathematical learning from different viewpoints and propose examples of tasks that are likely to stimulate children's mathematical thinking.

8.5.5 Collaborating with Parents

As Dóróþea worked with Pétur, a 6-year-old boy, it surprised her how well he understood the tasks she gave him. When she asked him why he is so strong in mathematics he replied: "Because my mom and dad are always making problems for my brother and me". Although Pétur is only 6 years old, he realises how his parents are supporting him in his learning. The work parents do with their children benefits them by making them more confident in working with mathematics. The teachers found it important to collaborate with parents and they provided some examples of their practice. To introduce mathematical learning in first grade, Birna finds it important to inform parents of how children learn mathematics and the way they develop their mathematical thinking. She also gives parents ideas about how they can play with mathematics at home by counting, measuring, and looking at different shapes, and how they can refer to math in their surroundings or in children's books and stories.

Children who struggle with mathematics in school often need support at home to develop their understanding and capability in mathematics. One way that Dóróþea and Birna support parents is to prepare a kit with suggestions of tasks to work on at home. The contents of each kit are different and depend on pupils' interest and strength. It can focus on counting forward or backward, counting by fives or tens, reading and writing numbers, playing cards or learning about money. The teachers find it important that the tasks are actual tasks or from daily life, and not bookwork. The kit is offered to parents, but as it is introduced to them it is emphasised that it is for support not obligation. Although a sheet for marking when the child has mastered all the tasks is a part of the kit, it is only for organisation and not required for use.

8.6 Learning from Teachers

The fundamental basis of this research was learning in partnership with teachers who have all developed their practice in collaboration with children and colleagues while reflecting on their understanding of young children's mathematical thinking. These teachers have developed a community of practice where they can both reflect on their practice and share their knowledge and understanding in a professional dialogue (Cochran-Smith and Lytle 2009; Guðjónsdóttir 2000). The teachers are empowered as they look critically at their classroom environment, realise student progress and learning in mathematics and communicate their new understanding and learning in a professional community of practice.

The teachers show respect for children's thinking and are capable of relating their knowledge of children's development to their work. They have learned from their own practice through active engagement and reflection in communities with their colleagues and by participating in developmental projects and further education. Our results are consistent with the results from the CGI study that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction (Fennema et al. 1996).

The teacher's understanding of how children learn has made them aware of children's diverse needs. They have developed educational practices that are concerned with and valuing of all pupils equally. The teachers are empowered to practice a culturally responsive and socially relevant pedagogy as they begin to look critically at their classroom environment.

The new teacher professionalism focuses more on learner-centred practice, informed practice, critical reflection, collaboration, and commitment to professional development and knowledge creation (Reeves 2009). Without collaboration, teachers' knowledge is not always recognised and often remains tacit, staying within the teacher.

In this research we learned about responsive teachers who create learning environments that foster mathematical understanding and creativity for all children as they move from preschool to primary school. These teachers report that taking part in this research project has given them an opportunity to participate in professional dialogue and they sense the efficacy of this experience. The teacher education community can, through partnership with teachers, gain understanding and knowledge about what teachers need to develop and grow. The findings indicate the importance for primary school teachers to understand the challenges children meet as they transfer from preschool to primary school but also that they acknowledge the resources they bring with them. It is important that these findings are considered both in practice and in teacher education. The next step for this particular research topic could be continuing with this group as they are more aware of how to make the transition a learning moment.

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Part II
Continuity of Mathematics Curriculum and/or Pedagogy as Children Begin School

Chapter 9
The Relationship Between Policy and Practice in the Early Mathematics Curriculum for Reception-Class Children in England

Carol Aubrey and Döndü Durmaz

Abstract This chapter considers the relationship between policy and practice in the Early Years Foundation Stage (EYFS) mathematics curriculum in England, with a particular focus on reception-class (RC) children aged 4–5 years. It explores what the policy requires teachers to do in terms of curriculum implementation; what teachers' views and understanding of the EYFS mathematics curriculum are; and how RC teachers implement EYFS mathematics policy. A case-study design included policy text analysis, interviews with EYFS teachers and observations of EYFS mathematics practice. International comparison studies appeared to have had an important influence on early childhood mathematics policies by creating a top-down pressure for higher standards. Document analysis revealed that despite claims of a reduction and a simplification of early learning goals in the EYFS, in fact the mathematical content had substantially increased. Moreover, the teaching guidance provided to support RC teachers through this change in requirements was wholly inadequate. Tensions in policy text were reflected in mixed and ambivalent views and practices. Whilst RC teachers applauded the principle of a play-based pedagogy in the EYFS, the mathematical content required was regarded as complex and confusing and, in some cases, planned by colleagues teaching the national curriculum to 6–7 year-olds. Observation revealed predominantly child-initiated small-group work with little mathematics in nursery classes for 2–3 year-olds. By 3–4 years, a growing emphasis on large-group work for literacy and numeracy was apparent and by 4–5 years, children were receiving a structured daily mathematics lesson reminiscent of the old National Numeracy Strategy. Hence, teachers brought their own values, experience and understandings to practice. The study revealed the interplay of global influences of educational comparison and national fear of falling standards, with RC teachers and young children caught in a nexus of forces.

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9.1 Introduction

This chapter was stimulated by a series of investigations of English children's early numeracy in the broader context of concern over low school mathematics achievement. Our European study of numeracy development of 5–6-year-olds revealed that performance differences between countries that took part were negligible. This was surprising since English children had been in formal schooling throughout the three testing cycles, Belgian, German, Greek and Dutch children from the midpoint, and Slovene children not at all (van de Rijt et al. 2003). The English sample of 300 boys and girls was then tracked through primary school. Nothing much disturbed levels of performance over children's primary years. Those making almost no progress by 7 years, however, were distinguished less by low initial scores than by swift decline during the earliest years of schooling. Analysis of the original sub-test scores at ages 7 and 11 years revealed that general number knowledge or problem-solving had the most stable predictive value (Aubrey and Godfrey 2003; Aubrey et al. 2006). In other words, children's early problem-solving skills predicted best their later mathematics achievement at the end of primary school.

A study of similarities and differences in young children's early numeracy at age 5 years in England, Finland and the People's Republic of China (Aunio et al. 2008) had revealed that young Chinese children out-performed those from England and Finland and that, in turn, Finnish children out-performed English children. This is much in line with the latest test results for the Programme for International Student Assessment (PISA) for 2012 that the Organisation for Economic Co-operation and Development (OECD) 2013 announced in the week in December that this chapter was being prepared.

Given the continuing overall poor performance of English reception-age (RC) children at 5 years there was an incentive to explore the policy context to changes in curriculum goals and pedagogical practices over a period of 10 years that have been associated with English policy-makers' drive to raise attainment judged in national and international terms. This led to a further study that explored what policy required RC teachers to do in terms of curriculum implementation, their views and understanding of the early mathematics curriculum, how RC teachers implemented early years mathematics policy, and how RC children responded. A case-study design included interviews with élite participants who had influenced the early mathematics curriculum and assessment policy-making process at the time, a survey of RC teachers; and a detailed investigation of RC classrooms on three school sites over a year. As élite interviews emphasised, international comparison studies have had an important influence on early childhood mathematics policies in England by creating top-down pressure for improvement of standards. Élites and practitioners drew attention to policy tension between a play-based pedagogy that professionals were 'invited' to value and a standards agenda. Tensions in policy text were reflected in mixed and ambivalent policy messages and reported practices by élites and practitioners. RC teachers did not merely receive and implement policy requirements but brought their own values and understandings to their practice. This study revealed an interplay between local and global influences in the context of changing views of early childhood, early mathematics learning and early years pedagogy (Aubrey and Durmaz 2012).

Education has been centrally positioned in social welfare policy in England as is the case for many other countries and has been seen as the major means for achieving economic productivity and competitiveness. Accordingly, early childhood education has become a prominent topic on the policy agenda with significant changes in preschool provision and practices over the last 15 years.

The latest PISA results (OECD 2013) showing no improvement in English standards in numeracy over 3 years do suggest 'no real sustained improvement over time' (Garner 2013) despite increased resources and continual curriculum change. As Ball (2013) has suggested, English education has played a particular role in the development and dissemination of a notion of education as a social laboratory of experimentation, with the OECD having its impact on public sector educational reform. In terms of early mathematics for 3–5 years, this entailed a curriculum originally intended to articulate with the primary National Numeracy Strategy (NNS) (Department for Education and Employment (DfEE) 1999), that was introduced in 2000, revised in 2008 to include children birth to 5 years, and then revised again following an independent report on its working in 2012.

Accordingly, the following research questions were posed for the present study.

1. What is the relationship between policy and practice in the early childhood mathematics curriculum in England?
2. What are teachers' views and understandings of the early childhood curriculum and how do they interpret policy for early mathematics?
3. How do they implement early childhood policy in the context of classroom practice?

9.2 Background

The method and approach was one of policy sociology (Ball 1999, 2013). Concepts are used as tools for making sense of policy with a particular focus on language, that is the way policy discourses operate to privilege certain ideas, interests and 'speakers' and to exclude others. The specific tool in this case was the policy trajectory model (Ball 1993; Bowe et al. 1992; Mainardes and Marcondes 2009), used to interrogate the way policies are represented, enacted and disseminated. Policies are enacted in specific, yet interrelated contexts of policy influence, policy text and professional practice.

The *context of influence* is where public policy is initiated, ideas are circulated and policy discourses are constructed through a process of struggle between agencies, public and private, interest groups and individuals, in an attempt to influence definitions and meanings, and eventual compromise if consensus is not reached.

The *context of policy text production* concerns the documents and speeches that embody policies and policy ideas translated through public sector reform into roles, responsibilities and practices. Policy texts are authoritative, presented as reasoned, reasonable, a self-evident 'truth' or 'good' intended to bring about change or improvement. This can only be 'read' or understood in the light of production in particular social, political or economic circumstances in which it was created.

The *context of practice* is where national policy is re-interpreted and enacted within localities and institutions. Hence, policies are contested, reinterpreted and remade in different sites. Not only is official policy reproduced and reworked but it is resisted, rejected or misunderstood at the local level. Policy is thus a process that is unstable and changing over time.

9.3 Methodology

A case-study design was adopted as a flexible research strategy to focus on the policy-to-practice context of early childhood mathematics, using different sources of evidence (Yin 2003). It sought to understand how the curriculum was understood nationally in the context of influence; in terms of the context of policy text that reflected struggle and compromise; and in the context of practice, where early childhood practitioners were charged with reinterpreting and recreating policy, locally in relation to available resources. Hence, the study was predominantly interpretive. Whilst a study of the particular, it allowed consideration of policy over time, through change of policy text and as interpreted in different physical settings by informants through whom practice could be known (Stake 2000). Triangulation was achieved through bringing together multiple perspectives on policy; interviews with key professional informants and constant comparison of their different perspectives; accurate observation and careful representation of classroom and preschool setting.

9.3.1 Sampling

For policy-text analysis, successive versions of the Early Learning Goals (ELGs) for mathematics in the English Early Years Foundation Stage (EYFS) were scrutinised by the authors, with a focus on number.

Four settings were selected for observation of children of a variety of ages (between 2 and 5 years):

- two state primary schools with reception-aged children (4–5 years) and two nursery classes (for 3–4-year-olds);
- two nursery classes, one state and one private (for 2–3- year-olds); and
- one children's centre nursery class (for 3–4-year-olds).

A minimum of 2 days was spent at each setting. Two target children were selected for each age group in each setting, one girl and one boy.

9.3.2 Materials

For interviews with teachers, three open-ended questions were asked:

- what did they think about the new EYFS (Department for Education (DfE) 2012);

- what was their view of mathematics learning and the expectations of the EYFS for this area; and
- what did they see as the main differences between new EYFS framework and the previous one (Department for Children, Schools and Families (DCSF) 2008).

For observations, a timeline was constructed and field notes kept of activities, resources available and, where possible, interactions. A well-structured target-child observation schedule (originally developed by Sylva et al. (1980) and used in a variety of studies since (Adams et al. 2004; Sylva et al. 1999)), explored the way particular children responded to planned curriculum activities over a set period, 40 min for each target child. This comprised:

- an *activity* code, to record what the child was doing;
- a *language* code, to record interactions that involved the child;
- a *task* code, to identify planned curriculum activity; and
- a *social* code, to specify the social context of the activity.

A 40 min tracking period provided 20 samples (at 2 min intervals) through the observation period. This technique was regarded as a reliable instrument for investigation of activities in early childhood settings (Aubrey and Durmaz 2012).

9.3.3 Analysis

Content analysis of the Statutory Frameworks for the EYFS (DfE 2012b; DCSF 2008) focused on the early learning goals for Number in the mathematics area, with reference made to the original, *Curriculum Guidance for the Foundation Stage* (DfEE 2000). Analysis of interviews relied on *a priori* coding at the first level (based on the three questions asked), that allowed flexibility for emergent codes at the second level from any issues or surprises that arose.

Observation field notes provided contextual information related to classroom layout, resources/materials, grouping procedures. For target-child observation, frequencies were generated from coding of the activity, language, task and social elements of the schedule.

9.4 Context of Influence

Preschool providers are charged with showing that the majority of children reach so-called early learning goals (ELGs) by the end of RC in their first year of compulsory education. The level of progress children should have reached by the end of the EYFS is defined by the ELGs. These are embedded in seven areas of learning and development that comprise the educational programmes of early years setting: three *prime* areas of communication and language; physical development; and personal, social and emotional development; and four *specific* areas of literacy; mathematics; understanding the world; and expressive arts and design. The framework is mandatory for all providers who are regularly inspected by the Office for Standards in Education (Ofsted). As noted by Her Majesty's Chief Inspector in the first Ofsted report dedicated to early years (Wilshaw 2014):

> I have been pressing policy-makers, practitioners and the public to do more to tackle the long tail of underperformance that blights too many of our poorest children and our country… the importance of the early years in setting the pattern of a lifetime was highlighted … By age five, children should show a 'good level of development' and be ready for formal schooling. In 2007… the gap was around 20 percentage points, six years later in 2013, the gap had not closed …

Inevitably preschool inspections and an increasing emphasis on preparation for the ELGs has created a preschool curriculum and pedagogy geared to improvement. Early intervention across health, welfare and early childhood education has become a policy meme.

9.5 Context of Policy Text

The content of the ELGs for mathematical development remained unchanged for more than 10 years. They were originally published as key objectives for RCs in the NNS (DfEE 1999). The key objectives for RC were in line with the ELG and so by the end RC year 'most children' would be able to:

- say and use number names in order in familiar contexts;
- count reliably up to ten everyday objects;
- recognise numerals 1–9;
- use language such as 'more' or 'less', 'greater' or 'smaller', 'heavier' or 'lighter', to compare two numbers or quantities;
- in practical activities and discussion begin to use the vocabulary involved in adding and subtracting;
- find one more or one less than a number from 1 to 10;
- begin to relate addition to combining two groups of objects, and subtraction to 'taking away';
- talk about, recognise and recreate simple patterns;
- use language such as 'circle' or 'bigger' to describe the shape and size of solids and flat shapes;
- use everyday words to describe position; and
- use developing mathematical ideas and methods to solve practice problems.

A whole section in the NNS was devoted to a supplement of examples of suitable activities for teaching counting and recognising numbers, produced by Anita Straker, Director of the NNS, together with a large group of early childhood experts and entirely consistent with levels of addition strategies used by young children to solve simple word problems:

- counting all;
- counting on from the first number;

- counting on from the larger number;
- using known number facts and using derived number facts. (Carpenter and Moser 1984)

Similarly, when the Curriculum Guidance for the Foundation Stage (DfE 2000) appeared, a 'Stepping Stones' section provided practitioners with strategies for moving children on from the most basic 'counting all' strategies for combining two groups to 'counting on':

> Jordan was finding out which fruit (apple or banana) the children wanted… and made a mark on the clipboard. He counted the marks … six apples and nine bananas … when asked how many children he had asked, he looked at his marks and said – 'That's nine, then, 10, 11, 12, 13, 14, 15. There are fifteen people!'

This example illustrates Carpenter and Moser's third strategy. Whilst the first, 'counting all' strategy to determine the cardinal value of a particular collection of objects may emerge quite spontaneously, children may need more help in combining two collections and, as Thompson (2008) p. 99 noted, to recognise that "the number sequence is a breakable chain, where oral counting can begin at any point within this chain". Here it can be seen that the child needs not only to hold in mind the number of children wanting bananas and apples aided by the use of tally marks but also the number words themselves, in this case, the number words that *follow* the number of bananas (nine) and to keep track of the number words that match the tally marks for apples, the last one being the solution.

The Statutory Framework for the Early Years Foundation Stage (DCSF 2008, p. 12) set standards for learning, development and care for children from birth to 5 years with eight ELGs for number to "establish expectations for most children to reach by the end of the EYFS", typically at the end of RC. Again, guidance was provided for practitioners working with young children from birth to 60 months in developing "their understanding of problem-solving, reasoning and numeracy in a broad range of contexts in which they can explore, enjoy, learn, practise and extend their developing understanding", distinction being drawn between "numbers as labels and for counting" and "calculating". Children of 30–50 months, for instance, are expected to "compare two groups of objects" and by 40–60 months "find the total number of items in two groups by counting all of them, first one to five objects and later one to nine". Thompson (2009), however, draws attention to the *lack* of reference to 'count on' strategies in the guidance, referring to Fuson's (1988) distinction between an 'early acquisition' stage in young children's learning of number words and a later 'elaboration' stage, where early operation of a continuous 'string level' of number names as an 'unbreakable chain' gives way to 'breakable chain' level, where they appreciate that it is unnecessary to start counting on from numbers that differ from 'one', with the proviso that the 'counting on' continues as an unbreakable chain. The last planning and resourcing note provided in the guidance states: "use rhymes, songs and stories involving counting on and counting back".

The revised English Early Years Foundation Stage (EYFS) Framework for children aged from birth to 5 years came into effect in September 2012, with a view to taking forward reforms to the previous 2008 framework as recommended in a review (Tickell 2011). The review stressed that there was overwhelming international evidence that weak foundations laid down in the first years of life could have impact on longer-term development. It was stated that there was clear unambiguous evidence that outcomes for children were improving, though less than half of children (44 %) were considered to have reached a good level of development by the end of children's reception year. In terms of mathematics, two aspects were proposed: numbers (that conflated the two former aspects of 'numbers as labels and for counting' and 'calculating'; and shape, space and measures. It was noted that whilst children recognised numbers and numerals, this knowledge was not applied to solving problems). Accordingly, it was proposed that by the end of the EYFS children should "use numbers up to ten to do simple addition and subtraction to solve practical problems … find a total by counting on, and calculate how many are left from a larger number by counting back". An extension to this was "to estimate a number of objects and check quantities by counting up to ten… solve practical problems that involve combining groups of 2, 5 or 10, or sharing into equal groups" (Tickell 2011, p. 75). The reforms aimed to:

- reduce paperwork and bureaucracy;
- strengthen partnerships between parents and professionals;
- focus on the three prime areas of learning most essential for children's readiness for future learning and healthy development;
- detail seven areas of learning and development that must shape education provision in early years settings;
- introduce a progress check at aged two to provide for early intervention as necessary; and
- simplify assessment at age 5, usually at the end of a child's first year at school (reception year).

In sum, the new EYFS set "standards that all providers meet to ensure that children learn and develop well' and promoted 'teaching and learning to ensure children's school readiness" (Department for Education 2012, p. 3).

The seven areas of learning comprised three *prime* areas –

- communication and language;
- physical development; and
- personal, social and emotional development.

It also comprised four *specific* areas, through which the prime areas were to be strengthened and applied –

- literacy;
- mathematics,
- understanding the world; and
- expressive arts and design.

The ELG for number, however, was significantly extended:

> Children count reliably with numbers 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find an answer. They solve problems, including doubling, halving and sharing (DfE 2012b, p. 9).

Suffice it to say, the appropriateness of such a change has already been extensively questioned (Thompson 2013a; 2013b; 2013c). Instead of reducing and simplifying the ELGs as Tickell's review (2011) had recommended, the number of ELGs increased in both scope and complexity.

The EYFS mathematics curriculum content had remained remarkably stable since the introduction of the NNS (DfEE 1999). The new EYFS number ELG intended for 5-year-olds bore a greater resemblance to the NNS key objectives for Year 1 (for 6-year-olds) (DfEE 1999). At the time the NNS key objectives were created, there was recognition of *progression* in addition strategies that young children needed to learn (for example, counting aloud or silently, using mental calculation strategies and known number facts within ten to solve simple problems). Moreover, there was an extensive supplement of exemplar material available, designed to aid planning and teaching for children with emergent skills, those who were working towards or working at the expected level or indeed those exceeding expectations. By contrast, new non-statutory guidance *Development Matters in the Early Years Foundation Stage* (DfE 2012a) provides little help to RC teachers seeking to progress children's calculation strategies. For instance, it makes little mention of children's 'counting back', learning basic number facts such as halving or doubling, or even of 'sharing out' in equal groups.

The one constant feature of the EYFS over time has been the principle of well-planned play, indoors and outdoors, as a "key way in which young children learn with enjoyment and challenge" (DfEE 2000). Over time, the adult role has become clarified so that now "each area of learning and development must be implemented through planned, purposeful play and through a mix of adult-led and child-initiated activity". There is an expectation of an "ongoing judgement to be made" about the balance between activities led by children and activities led or guided by adults with a gradual "shift towards more activities led by adults, to help children prepare for more formal learning" (DfE 2012b).

9.6 Context of Practice

Since it is in the context of practice that teachers are charged with interpreting and enacting the mathematics curriculum it was important to consider the views and actions on the new EYFS (DfE 2012b) of practitioners who took part in this study.

9.6.1 Unstructured Observation from Field Notes

Field notes revealed the mix of adult-led and child-initiated activity recommended by the EYFS, with a greater emphasis on activities led by adults in the RC classes. In RCs, 4–5-year-olds, for example, took part in whole-class adult-led activity that started mostly with numeracy or literacy. After whole-class teaching, children moved into small groups to carry out planned activities with an adult or by themselves. Finally, children returned to sit on the carpet with the teacher for a plenary session. Interestingly, the observed organisation of whole-class introduction, small-group reinforcement and practice, with final review of main teaching points, still followed the prior expectations of the NNS (DfE 1999). Nursery-aged children of 3–4 years followed a similar régime, starting with a shorter literacy or numeracy activity, followed by small-group activities, freely-chosen or from time-to-time adult-led. In one of the nursery classes, children returned to the carpet for an adult-led plenary. Nursery-aged children of 2–3 years experienced no whole-class activity under the direction of a practitioner. All observed activities took place in small groups. Given generous staff-child ratios, table activities were always supervised by adults, or adults were at least on hand to intervene, where necessary. No number or mathematical activities were observed for this age-group.

9.6.2 Structured Target-Child Observation

Forty-minute timed observation focusing on the *social setting* confirmed that across settings 37.9 % of time was spent in small-group activity, whilst 18.9 % of time was spent in large-group activity, predominantly by older children. Interestingly, 23.9 % of time was spent in paired activity though 19.3 % of time was spent alone or in parallel with others.

Forty-minute timed *task* observation of two target children in each setting revealed that adult-directed art and manipulation, and reading, writing and mathematics activities each occupied nearly 20 % of the time. Informal games and movement (purposeful and large motor activity) each occupied a further 12.5 % of time. Smaller amounts of time were spent on a range of other activities such as domestic play and small-scale toys.

Timed *language code* observation of two target children in each setting revealed that overall adults initiated 48.6 % of interactions with children, whilst children initiated 15.2 % interactions with adults. Nearly one-third of interactions were initiated by children to other children with 5.8 % of language comprising 'self-talk' as children talked aloud to themselves.

9.6.3 Interviews with Practitioners

Practitioners working with the children of 2–3 years were positive about the EYFS. They felt it addressed the needs of young children in terms of care and education and in its emphasis on play. The private nursery practitioner however also stressed that she planned literacy and numeracy activities according to the expectations of the EYFS as many parents wanted their children to know numbers and counting, shape and colours. She also tried to teach children to write their own names, adding that parents of private nursery schools had expectations that their children would be taught basic skills before they started RC at 4 years.

Practitioners working with 3–4-year-olds were also very positive about the EYFS but their responses were more varied. It was a 'very good curriculum for young children's learning', 'child-friendly and emphasising play that was really important for children's learning'. One expressed the view that separating learning areas was 'wrong' and that 'maths is everywhere'. Another ventured the view that the previous EYFS framework was 'a bit complicated'. The third added that the EYFS was 'good for teachers to plan their teaching accordingly'. In her school, the teachers worked to a planned curriculum so activities were almost identical in four RCs and a simplified version of their plans was also used in four nursery classes. The planning however was carried out by Key Stage 1 (KS1) (for 6–7-year-olds) from which RC and nursery teachers made their own long- and short-term plans.

One of the two RC teachers interviewed thought that the EYFS was 'mostly for nursery classes and younger children' and felt that the RC 'curriculum needed to be more structured' (as indeed observation in RCs confirmed.) Only one of the RC teachers interviewed had been teaching long enough to comment on the Curriculum Guidance for the Foundation Stage (DfEE 2000) and to relate this to subsequent EYFS frameworks. She observed that there were 'huge differences between the Curriculum Guidance and the latest EYFS, particularly in the organisation of the learning areas. She noted that children in the EYFS were now expected to count up to 20 and to solve problems, including doubling, halving and sharing, but then observed:

> We [early childhood practitioners] do not know to what extent we need to do extend children to do problem-solving. Which numbers do they [children] need to use for doubling? Is it up to 10 or what? How about halving?

Her questions go straight to the heart of this new and apparently 'simplified' EYFS. As children are now required to count, order and say one more or less than a given number from 1 to 20, to add and subtract two single-digit numbers, counting on or back to find the answer, are they expected to solve problems, including halving doubling and sharing, within 10 or 20? Exemplification material published in *Early Years Outcomes* (DfE 2013) would suggest problem-solving within ten but this is by no means clear.

If individual preschool children do bring to formal education a different range of intuitive arithmetic knowledge and strategies, such as remembering, counting, derived addition facts within ten that include doubling and non-doubling strategies, then practitioners will need to have an equally sophisticated array of observation, questioning and prompting strategies in order to uncover them. Beyond counting reliably forwards and backwards from any small number, children's understanding of the base-10 system and the importance of this for arithmetical development however will take much longer to establish.

9.7 Numeracy in Practice

RC1 had a total of 29 children on register, aged 4–5 years. The class was going to make rice crispy cakes. After the mixture had been prepared, discussion moved to the number of cakes required.

Teacher: How do we find out how many cakes we need to make?
Child: We need 5 cakes.
Teacher: 1, 2, 3, 4, 5 (counting five children by pointing to them). Do you think having 5 is enough for everyone (again pointing to five children and then the whole class).
Child (shaking his head): No.
Another child: We need plenty.
Teacher: How many is plenty?
Another child: We need to count.
Teacher: That is really clever. We need to count to find out how many we need to make.
Teacher (touching children's heads by going around the semi-circle in which children are sitting, all counting): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27! How many children?
Teacher and children: 27!
Teacher: We should not forget counting the adults. We should make one cake for each of them, too. So, one for me … oh, we had 27 children. Let's count on the number of adults to find out how many cakes we need to make in total. One for me. That makes 28, one for Miss Lee …
Teacher and children: 29!
Teacher: One for Mrs Spree … 30 and one for the researcher … 31! 31 crispy cakes we should make. This way, everyone, including adults will have one crispy cake…

RC2 had 30 children. The teacher and children marched to a corner where they were going to set up the calendar.

Look at our 100 square. We had our 22nd maths lesson yesterday and we are having another one, 22 and one more. Yes, everyone tell me together.
Children: 23rd.
Teacher: OK, everyone we need to move back to our maths corner. (She opens the interactive white board and shows, 1p, 2p coins and gives every child a 1p coin.) Yes, everyone has 1p. I am wondering who is going to tell me how many pennies you have altogether. Anyone, who can tell me? (Various guesses are proffered.)
Teacher: Let's count and see. I will collect your1p and we all count whenever I get a 1p from you.
Children (they all count) …. 1p, 2p, 3p, 4p, 5p, 6p, 7p, 8p, 9p, 10p, 11p, 12p, 13p, 14p, 15p, 16p, 17p, 18p, 19p, 20p, 21p, 22p, 23p, 24p, 25p, 26p, 27p, 28p, 29p, 30p. Fantastic. Tell your friends – 30 children have 30p in total.
Teacher (opens a teddy bear picture on the interactive white board) I want to buy this teddy. It is 30p and I have been saving. Count these pennies with me… 1p, 2p, 3p, 4p, 5p, 6p, 7p, 8p, 9p, 10p, 11p, 12p!
Teacher: Oh, so far I have saved? 12p! How much more do I need to save to buy this teddy which is 30p? Let's count on, on top of 12. We need to count till we reach 30 and decide how much more I need to save.
Teacher and children 'counting pennies on top of 12' while the teacher puts additional coins into an empty jar: 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30. So we need to count again to see how my more I need to save. Let's count. (She empties the jar and puts them back again, counting, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18p!
Teacher: I saved 12p and how much more do I need to buy this teddy?

Children are all very quiet.
Teacher: Let's count again … How much?
One child: 18p!

The two examples illustrate the complexity of counting to 20 and above. The first example for RC1 showed a relatively straight-forward count on from 27 to 30. The second illustrated the teacher's attempts to count on from 12p to 30p first by saying aloud the 'pennies that come on top of 12', then continuing the number recitation from 13 and stopping at 30, using a tally system of coins in a jar and, finally, counting these. Whether or not counting on can be taught in this way and whether or not such an activity contributes to children's understanding of number remains to be seen. Suffice to say that the curriculum planning for this RC class of 4–5-year-olds had been carried out by Year 1 and 2 teachers (for 6–7-year-olds), as the teacher interview revealed. Most 5–6-year-olds use counting to solve simple problems, with and without fingers. Using fingers indeed may work well with a difference of 10 or less but not for larger problems. In the meantime, with experience and maturation, our 4–5-year-olds RC children who already use counting to solve simple problems will adapt these strategies to solve addition and subtraction problems with small numbers.

9.8 Discussion

Returning now to the questions posed. First—what is the relationship between policy and practice for the English mathematics EYFS? The ELGs have changed three times in the last 12 years. Whilst the principle of play has continued to structure thinking about adult-led and child-initiated learning and stimulated critical thinking about a play pedagogy, the new number ELG has radically changed and extended expectations of what young English children can know and understand. Since the introduction of the NNS, Year 1 teachers have expected to build on a secure set of counting skills acquired in RCs to construct a range of mental calculation strategies. Without a secure base of counting on and back, using doubles and halving, adding and subtracting within ten, is this likely to occur?

Secondly—what are teachers' views and understandings of EYFS mathematics policy and how do they reinterpret ELGs in their own practice? They support a play-based pedagogy that is realised through small-group, adult-led and child-initiated activity. Those teachers working in RCs, however, are uncertain about interpreting the number ELG, are without sound curriculum guidance and may well find their EYFS planning subsumed in planning by colleagues teaching Year 1 and 2 for KS1 of the national mathematics curriculum.

Thirdly—how do teachers implement EYFS policy in the context of classroom practice? Although the NNS and subsequent Primary Strategy are no longer required, RC teachers are still organising their mathematics teaching as a three-part lesson. The content they are required to teach meanwhile has as much in common with a Year 1 curriculum as a RC curriculum.

9.9 Conclusions

In a *context of influence,* continuing concern about mathematics standards and early intervention becoming a current and widely-circulating policy meme have led to the top-down policy imposition of an EYFS mathematics curriculum. In the *context of policy text,* the *number* ELG whilst presented as 'reduced and simplified' and 'much less burdensome' (Tickell 2011) is complex, contradictory and leads to confusion. As one of our RC teachers described it—'EYFS is confusing and too simple. It does not help practitioners'. In the *context of practice,* many early years teachers are not confident in their pedagogical subject knowledge for early arithmetic and problem-solving and policy-makers simply call for more content at an ever earlier age, without reference to the vast literature and expertise that exists in this field.

Ball's trajectory model assumes policy to be a continuous process with feedback from practice, where policy can be reworked. When the ELGs were first introduced, policy-makers of the time consulted and incorporated expert knowledge into their thinking and decision-making. What is new and disturbing about the latest iteration of the numbers ELG of the EYFS (DfE 2012) is its disregard of research evidence concerning young children learning mathematics and the absence of advice from experienced mathematics educators who have been expressing a deepening concern at the structure of the EYFS mathematics curriculum and its transition into KS1.

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Chapter 10
Scaling Up Early Mathematics Interventions: Transitioning with Trajectories and Technologies

Julie Sarama and Douglas H. Clements

Abstract Transitions in the early years have substantial effects on children's success in school. Moreover, lack of consideration of continuity and alignment may mislead both researchers and politicians to assume preschool effects 'fade', when it may be that poor transitions to primary school are to blame. We hypothesise that most present educational contexts are unintentionally and perversely aligned against early interventions. For example, primary curricula assume little mathematical competence, so only low-level skills are taught. Most teachers are required to follow such curricula rigidly and remain unaware that some of their students have already mastered the material they are about to 'teach'. Teachers may be held accountable for getting the largest number of students to pass minimal competency assessments, engendering the belief that higher performing students are 'doing fine'. In this way, we believe the present U.S. educational system unintentionally but insidiously re-opens the gap between students from low- and higher-resource communities. We conducted a large cluster randomised trial of an intervention that evaluated the persistence of effects of a research-based model for scaling up educational interventions, with one control and two intervention conditions. Only the intervention condition that included a follow-through treatment to support the transition to the primary grades maintained substantial gains of the pre-K mathematics curriculum.

10.1 Introduction

Transitions in the early years have substantial effects on children's success in school. This may be especially true in the domain of early mathematics, because many schools fail to encourage, and may even discourage, communication between pre-K, Kindergarten, and primary grade teachers, and because many teachers lack knowledge of and confidence in mathematics. In this chapter, we discuss the

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importance of the transition to primary school and continuity in mathematics education for young children (Perry et al. 2012a). We begin by documenting weaknesses in many countries' early instruction, especially the U.S., and describe our rigorous test of the notion that early mathematics interventions are important and require follow through into the primary grades (at least).

10.2 The Need: Weaknesses in Mathematics Education, a U.S. Example

Countries differ significantly in mathematics achievement (Mullis et al. 2012). Low-performing countries may need to revise their mathematics education systems. For example, the mathematics achievement of U.S. students compares unfavourably with the achievement of students from many other nations, with some cross-national differences in informal mathematics knowledge appearing as early as 3–5 years of age (Sarama and Clements 2009).

Further, children from some groups come to school less prepared in mathematics than others. For too many, these differences increase as they move through the grades (National Mathematics Advisory Panel 2008). In the U.S. this gap is most pronounced in the performance of children living in economically deprived urban communities. The achievement gaps have origins in the earliest years. For example, the percentage of 4-year olds demonstrating proficiency in numbers and shapes was 87% in higher-socioeconomic status (SES) families but only 40% among lower-SES families (Chernoff et al. 2007).

Thus, there is an early developmental basis for later achievement differences in mathematics: Children from different sociocultural backgrounds are provided different foundational experiences. Programs need to recognise sociocultural and individual differences in what children know and in what they bring to the educational situation. These differences should inform planning for programs and instruction, including extra support for those from low-resource communities. We must meet the needs of all children, especially groups disproportionately under-represented in mathematics, such as children of colour and children whose home language is different than that of school. All these children also bring diverse experiences on which to build meaningful mathematical learning. There is no evidence that such children cannot learn the mathematics that other children learn. Too often, children are not provided with resources and support equivalent to middle-class or upper-class majority children. They may have different and inequitable access to foundational experiences, mathematically-structured materials such as unit blocks, technology, and so forth.

This brings us to another equity concern: Transitions to school, recovering from initial gaps in learning, and maintaining more positive trajectories of learning mathematics may be more problematic for African-American children than white children (cf. MacDonald et al. 2012, and efforts to work with indigenous children). In another study (Alexander and Entwisle 1988), African-American children gained less than white children, with the gap widening over a 2-year period. Similarly, African-American children can make real gains in mathematics knowledge in preschool, but over the first 2 years of school, they lose substantial ground relative to other races (Fryer and Levitt 2004). Quality is lower in classrooms with more than 60 % of the children from homes below the poverty line, when teachers lacked formal training (or a degree) in early childhood education, and held less child-centered beliefs (Pianta et al. 2005).

10.3 The Issue of Fade Out and the Need to Plan for Transitions and Follow Through

Some studies indicate that early interventions can have lasting effects. For example, several have shown positive and long-lasting effects of preschool experience (Clements and Sarama 2014; Wylie at al. 2009). However, there is considerable empirical research and resultant (practical) assertions that preschool gains 'fade' in the primary grades. For example, in one study of six cohorts, gains in preschool weakened as children progressed through the primary grades, disappearing by fourth grade (Fish 2003). Other studies show a similar fade (Administration for Children and Families (ACF) 2010; Natriello et al. 1990; Preschool Curriculum Evaluation Research Consortium 2008).

Although an ostensible reason for such fade is that early effects are themselves evanescent, we believe that a contradictory explanation is more theoretically cogent. We hypothesise that present educational contexts are unintentionally and perversely aligned against the persistence of early interventions. Transitions to the primary grades are not planned or implemented well. Consider the educational trajectories of children who benefited from a successful pre-K experience as they move into kindergarten. The kindergarten curriculum they experience likely assumes little or no mathematical competence, so only low-level skills are taught. Their teachers are often required to follow such curricula rigidly and remain unaware that some of their students have already mastered the material they are about to 'teach' (Bennett et al. 1984; Clements and Sarama 2014; National Research Council 2009; Sarama and Clements 2009). Further, biases may negatively affect the subsequent school experiences of children at-risk during pre-K. For example, kindergarten teachers rated Head Start children's mathematics ability as lower than that of other children, even though direct assessments showed no such differences (ACF 2010). Thus, teachers may view children from different SES or ethnic groups as lacking knowledge or the ability to learn and thus overlook their competencies and potential for growth. Even if the children are assigned to a kindergarten teacher who recognises their competencies, pressure to increase the number of children passing minimal competency assessments may lead this teacher to work mainly with (and/or mainly at the level of) the lowest performing children. Within this context and without continual, progressive support (especially given that children from low-resource communities attend low-resource schools), early gains may fade. In this way, we believe the present

U.S. educational system inadvertently but insidiously re-opens the gap between students from low- and higher-resource communities.

For these reasons, we designed and evaluated the effectiveness of TRIAD's follow-through intervention, testing our hypothesis that such follow through is the 'missing piece' in many early interventions whose longitudinal evaluations have found less positive effects (cf. the effects of Te Mahere Tau, The Number Framework, MacDonald et al. 2012; Trinick and Stevenson 2009). Although this might appear to be an issue of simple 'educational engineering', the issue has implications for both theory and policy. Interpretations of this fade often call for *decreased* funding and attention to pre-K (Fish 2003). Although this may appear reasonable (with logic such as, if effects fade out, why fund that intervention?), we believe this mistakenly treats initial effects of interventions as independent of the future school contexts. Instead, we believe children's trajectories must be studied as they experience different educational courses. If such effects fade in traditional settings but do not in the context of follow-through interventions, then attention to and funding for follow-through efforts for both pre-K and the primary grades should arguably increase.

10.4 Intervention: The *Building Blocks* Curriculum and TRIAD Scale-Up Model

To begin to address these needs, we designed the *Building Blocks* preschool (mainly for 4-year-olds) mathematics curriculum (Clements and Sarama 2013) as a set of tools that would enable all young children to build a solid foundation for mathematics, and especially that would increase the mathematical knowledge of children from low-resource communities. *Building Blocks* is a National Science Foundation-funded mathematics curriculum designed using a comprehensive Curriculum Research Framework (CRF) (Clements 2007) to address numeric/quantitative and geometric/spatial ideas and skills. Woven throughout are mathematical subthemes, such as sorting and sequencing, as well as mathematical processes. General processes include communicating, reasoning, representing, and problem solving and the overarching mathematising. Specific mathematical processes include number and shape composition and patterning. We considered these to be critical mathematical building blocks based on our previous work (Clements at al. 2004).

At the core of the CRF are empirically-grounded learning trajectories. We define learning trajectories as "descriptions of children's thinking and learning in a specific mathematical domain, and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesised to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain" (Clements and Sarama 2004, p. 83). Our learning trajectories' are not simply 'educated guesses' but are based on empirically-supported developmental progressions (more so for more heavily researched topics, of course). These share many similarities with the "growth points" of the Early Numeracy Research Project (ENRP) (Clarke et al. 2002; Perry et al. 2008) and other projects in Australia, New Zealand, and other countries (Bobis et al. 2005; Perry 2010). As an example, children's developmental progression for shape composition advances through levels of trial and error, partial use of geometric attributes, and mental strategies to synthesise shapes into composite shapes. The sequence of instructional tasks requires children to solve shape puzzles off and on the computer, the structures of which correspond to the levels of this developmental progression (Clements and Sarama 2007; Sarama et al. 1996).

Building Blocks' basic instructional approach is finding the mathematics in, and developing mathematics from, children's activity. Children are guided to extend and mathematise their everyday activities, from block building to art to songs to puzzles, through sequenced, explicit activities (whole group, small group, centers, including a computer center, and 'throughout the day'). Thus, off-computer and on-computer activities are designed based on children's experiences and interests, with an emphasis on supporting the development of mathematical activity at the next level of thinking within the learning trajectory. Although the complete *Building Blocks* is a preschool curriculum, the computer activities extend into kindergarten and the primary grades.

Results from our early summative evaluations (Clements and Sarama 2007) were satisfying, but also revealed that similar successes would be unlikely at a large scale without a complete scale-up program. Our scale-up model is called TRIAD, for Technology-enhanced, Research-based, Instruction, Assessment, and professional Development. The model's acronym suggests that successful scale-up must address the triad of essential components of any educational intervention and that the model is based on research and enhanced by the use of technology. However, TRIAD is a general model for scaling up varied educational interventions, based on successful efforts to take such interventions to scale. The following are the 10 research-based guidelines in the TRIAD model.

1. Involve, and promote communication among key groups around a shared vision of the innovation (Bobis et al. 2005; Hall and Hord 2001; see Sarama et al. 2008, for a complete review for all guidelines). Emphasise connections between the project's goals, educational standards (Perry et al. 2012b), and greater societal need. Promote clarity of these goals and of all participants' responsibilities. School and project staff must share goals and a vision of the intervention (Bryk et al. 2010). This is especially important for teachers from pre-K through the primary grades, as implicit and explicit (policy) barriers often separate age- and grade-level groups (Sarama and Clements 2013; Thomson et al. 2005). These efforts institutionalise the intervention, across grade levels and in the case of ongoing socialisation and training of new teachers (Elmore 1996; Huberman 1992; Kaser et al. 1999; Sarama et al. 1998).
2. Promote equity through equitable recruitment and selection of participants, allocation of resources, and use of curriculum and instructional strategies that have demonstrated success with underrepresented populations (Kaser et al. 1999).

Again, this should be equitable across age levels as well as supportive of the special needs of groups and individuals.

3. Plan for the long term. Recognising that scale up is not just an increase in number, but also of complexity, provide continuous, adaptive support over an extended period of time. Plan an incremental implementation and use dynamic, multilevel, feedback, and self-correction strategies (Bryk et al. 2010). Communicate clearly that change is not an event, but a process (Hall and Hord 2001), and involve teachers at grade $n+1$ (if not more) in understanding the challenges, work, and successes of teachers and students at grade n.
4. Focus on instructional change that promotes depth of children's thinking, placing learning trajectories at the core of the teacher/child/curriculum triad to ensure that curriculum, materials, instructional strategies, and assessments are aligned with (a) national and state standards and a vision of high-quality education, (b) each other, and (c) 'best practice' as determined by research, including formative assessment (Bodilly 1998; Bryk et al. 2010; Kaser et al. 1999; National Mathematics Advisory Panel 2008; Raudenbush 2008). This guideline is important for implementation with fidelity at any scale, although alignment is increasing important at larger scales and across grade levels. That is, learning trajectories can provide the connective tissue that helps teachers from pre-K to primary grades connect and communicate about mathematical goals, children's developmental levels, and instructional activities and strategies (Clements et al. 2013).
5. Provide professional development that is ongoing, intentional, reflective, goal-oriented, focused on content knowledge and children's thinking, grounded in particular curriculum materials, situated in the classroom and the school (Clarke 1994; Perry 2010; Sarama et al. 2008). A focus on content includes accurate and adequate subject-matter knowledge both for teachers and for children. A focus on children's thinking emphasises the learning trajectories' developmental progressions and their pedagogical application in formative assessment. Grounding in particular curriculum materials should include all three aspects of learning trajectories, especially their connections. This also provides a common language for teachers in working with each other and other groups (Bryk et al. 2010). Situated in the classroom does not imply that all training occurs within classrooms. However, off-site intensive training remains focused on and connected to classroom practice and is completed by classroom-based enactment with coaching. In addition, this professional development should encourage sharing, risk taking, and learning from and with peers. It should be based on a specific curriculum and develop teachers' knowledge and beliefs that the curriculum is appropriate and its goals are valued and attainable. Work should be situated in the classroom, with coaches who formatively evaluating teachers' fidelity of implementation and provide feedback and support in real time (Bodilly 1998; Bryk et al. 2010; Kaser et al. 1999). As with guideline #4, guideline #5 is important for implementation with fidelity at any scale. However, the planning, structures, common language, formative evaluation, and school-level context are increasingly important as the implementation moves to larger scales and especially across grade levels, where curricula frequently differ, and thus the connective tissue of learning trajectories is especially important.

6. Build expectations and camaraderie to support a consensus around adaptation. Establish and maintain cohort groups and build cross-age working groups. Facilitate teachers visiting successful implementation sites and each other's classrooms at their own grade level and those before and after. Build local leadership by involving principals and encouraging teachers to become teacher leaders.
7. Ensure school leaders are a central force supporting the innovation and provide teachers continuous feedback that children are learning what they are taught and that these learnings are valued. Leaders, especially principals, must show that the innovation is a high priority, through statements, resources, and continued commitment to permanency of the effort. An innovation champion leads the effort within each organisation (Bodilly 1998; Bryk et al. 2010; Hall and Hord 2001; Sarama et al. 1998).
8. Give latitude for adaptation to teachers and schools, but maintain integrity. Emphasise the similarities of the curriculum with sound practice and what teachers already are doing. Help teachers distinguish productive adaptations from lethal mutation (Brown and Campione 1996). Also, do not allow dilution due to uncoordinated innovations (Huberman 1992; Sarama et al. 1998).
9. Provide incentives for all participants, including intrinsic and extrinsic motivators linked to project work, such as external expectations—from standards to validation from administrators. Show how the innovation is advantageous to and compatible with teachers' experiences and needs (Berends et al. 2001; Borman et al. 2003; Elmore 1996; Rogers 2003).
10. Maintain frequent, repeated communication, assessment ('checking up'), and follow-through efforts at all levels within each school district, emphasising the purpose, expectations, and visions of the project, and involve key groups in continual improvement through cycles of data collection and problem solving (Hall and Hord 2001; Huberman 1992; Kaser et al. 1999). Throughout, connections between teachers following children through the grades and also with parents and community groups is especially important, to meet immediate and long-range (sustainability) goals (for more details, see Sarama and Clements 2013).

10.5 How the TRIAD Guidelines Were Implemented in Pre-Kindergarten

For the pre-K teachers, the first year was a 'gentle introduction' to TRIAD and *Building Blocks*, because our previous experience and others' research suggested that teachers often need at least a year of experience before completely and effectively implementing a curriculum (Berends at al. 2001; Clements and Sarama 2014). They participated in seven full days of professional development, including time to address the 'developmental appropriateness' of the intervention's mathematics education and its importance to the teachers and children, especially in promoting equity. This work focused on the learning trajectories for each mathematical topic, usually as woven into the *Building Blocks* curriculum. Training addressed each of the three components of the learning trajectories. To understand the goals, teachers learned core mathematics concepts and procedures for each topic. For example, they re-learned the geometry of early and primary education. To understand the developmental progressions of levels of thinking, teachers studied multiple video segments illustrating each level and discussed the mental 'actions on objects' that constitute the defining cognitive components of each level (Perry 2010). To understand the instructional tasks, teachers studied the tasks, and they viewed, analysed and discussed video of the enactments of these tasks in classrooms. A central tool to study and connect all three components was the Internet-based software application, Building Blocks Learning Trajectories (BBLT). BBLT provided scalable access to the learning trajectories via descriptions, videos, and commentaries. Two sequential aspects of the learning trajectories—the developmental progressions of children's thinking, and connected instruction—are linked to the others. The coaches joined the teachers in the participated in professional development, as well as several days of training on coaching, most of which focused on the unique aspects of coaching early mathematics education. Coaches worked with teachers during the year to provide continual feedback and support, avoiding dilution of the intervention, while promoting productive adaptations.

In Year 2, teachers and coaches participated in an additional four full days of professional development. They continued to study the learning trajectories, including discussions of how they conducted various curricular activities the previous year. As part of this work, teachers brought case studies of particular situations that occurred in their classrooms to the group to facilitate these discussions; thus, this work included elements of lesson study.

10.6 Results of Implementing the TRIAD Model: Pre-K

These general guidelines were implemented fully for the pre-K intervention (Sarama and Clements 2013; Clements et al. 2011). Findings from that year were positive. Briefly, 42 schools serving low-resource communities were randomly selected and randomly assigned to three treatment groups involving 1,375 preschoolers in 106 classrooms. Two of these groups implemented the TRIAD model in pre-K, the third group was a 'business-as-usual' control. TRIAD teachers taught the *Building Blocks* curriculum with adequate fidelity (Clements et al. 2011). Pre- to post-test scores revealed that the children in the *Building Blocks* group learned more mathematics than the children in the control group (effect size, $g=0.72$). African-American students in the treatment groups scoring significantly better than that of African-American students in the control group (although they scored lower than non-African Americans in all groups). We also checked if there were any deleterious effects on language and literacy scores with the commitment of more instructional time to mathematics. Results showed no evidence that children who were taught mathematics with *Building Blocks* performed differently than control children who received the typical district mathematics instruction on measures of letter recognition, and on two of the oral language (story retell) subtests, and sentence length. However, children in the *Building Blocks* group outperformed children in the control group on four oral language subtests: ability to recall key words, use of complex utterances, willingness to reproduce narratives independently, and inferential reasoning (Sarama et al. 2012b).

10.7 How the TRIAD Guidelines Were Implemented in Kindergarten and First Grade

The results for Kindergarten and 1st grade are directly relevant to the theme of this chapter. In these grades, the two groups randomly assigned to TRIAD in pre-K differed: Only one of them, TRIAD-Follow Through (TRIAD-FT) continued to implement the TRIAD intervention, the other, TRIAD-Non Follow Through (TRIAD-NFT), did not. Transitions were addressed in that the teachers in the schools assigned to TRIAD-FT were introduced to what their students had learned in pre-K and ways to build upon it. That is, they were shown the mathematics many of their entering students had learned from video recordings and through presentations of the pre-K teachers, who shared stories, pictures, and some videos of mathematics that the preschoolers had learned in the previous year. They were also taught about the learning trajectories to their grade level and beyond, including the developmental progressions and how to modify their extant curricula to more closely match the levels of thinking of their students. They also received access to the *Building Blocks software* (Clements and Sarama 2007/2012), which follows the learning trajectories through the primary grades and is the same suite that the students had used previously.

10.8 Results of Implementing the TRIAD Model: Kindergarten and First Grade

At the end of the students' kindergarten year (Sarama et al. 2012a), both TRIAD groups outperformed the control condition ($g = .46$ for the follow- through, $g = .30$ for the non-follow through). One moderator was statistically significant, with African-American students within the TRIAD-FT group scoring significantly better on kindergarten outcomes than African-American students in the TRIAD-NFT group.

At the end of first grade, students in the TRIAD-FT group scored significantly higher than control group, with a higher effect size ($g = .51$) than that of the TRIAD-NFT compared to control ($g = .28$). Furthermore, the TRIAD-FT scored group significantly higher than he TRIAD-NFT group ($g = .24$). Although African-American students continued to lag behind non-African-American students in all conditions, the TRIAD-FT intervention helped them narrow that achievement gap.

10.9 Implications

Before we return to the issue of fade out, we wish to be clear about our position on our follow-through intervention: We believe it was underpowered. That is, although it had a significant and important impact, the effect of the treatment appeared to decrease, and to close the gap between children in low-resource communities—such as those in our study—and those from higher-resource communities, we need interventions that increase the effect each successive year. The TRIAD-FT intervention differed from the TRIAD pre-K intervention in several ways that suggest weaknesses that could be ameliorated in future work. (a) TRIAD pre-K introduced a new, research-based curriculum; TRIAD-FT used the school's existing curriculum. (b) TRIAD pre-K teachers learned and practiced the intervention for a year before data collection; TRIAD-FT teachers did not. (c) TRIAD pre-K were allowed to implement all aspects of the intervention; some TRIAD-FT teachers reported that the 'fidelity police' of their schools insisted they follow their existing curriculum schedule, thus preventing them from condensing or compacting curricula (one of the intervention's strategies). Thus, we believe the evidence strongly supports the need for follow through, as we discuss in the remainder of the chapter, but also believe that the TRIAD-FT implementation was adequate, but not ideal, and that more efficacious TRIAD follow through interventions can and should be implemented and studied.

Nevertheless, even the less-than-ideal TRIAD-FT treatment was important in maintaining children's early gains. This finding has broad implications. We believe interpretations of studies reporting that preschool gains fade (ACF 2010; Fish 2003; Natriello et al. 1990; Preschool Curriculum Evaluation Research Consortium 2008; Turner and Ritter 2004) often mistakenly treat initial effects of interventions as independent of the students' future school contexts. That is, these interpretations reify the treatment effect as an entity that should persist unless it is 'weak' and thus susceptible to fading. Taking this perspective views the gain analogically as a static object carried by the student that, if not evanescent, would continue to lift the student's achievement about the norm, as if it were a platform on which to stand. Our theoretical position and our empirical results support an alternative view. Successful interventions do provide students with new concepts, skills, and dispositions that change the trajectory of the students' educational course. However, these are, by definition, exceptions to the normal course for children in their context (in our case, low-resource communities). Because the new trajectories are exceptions, multiple processes may erode their positive effects. Curricula designed for the typical student from that district or school assume low levels of mathematical knowledge and often focus on lower-level skills. Studies have substantiated that some kindergarten and first grade instruction cover material children already know even without extensive pre-K experience with mathematics (Engel et al. in press; van den Heuvel-Panhuizen 1996). A culture of low expectations for certain groups may support the use of such curricula. Teachers are often required to follow such curricula strictly and may have few means to recognise that students have already mastered or surpassed the content they are about to 'teach' them (Bennett et al. 1984; Clements and Sarama 2014; National Research Council 2009; Sarama and Clements 2009). Even if they do so recognise students' competencies, pressure to increase the number of students passing minimal competency assessments may lead teachers to work mainly with (and/or mainly at the level of) the lowest performing students. Within this context and without continual, progressive support, children's nascent learning trajectories revert to their original, limited course. These arguments and the empirical support proffered by this study suggest that we must further investigate the implied concern, that multiple characteristics of the present U.S. educational system are aligned to unintentionally but perniciously dismantle the benefits of successful early childhood interventions.

An implication is that, students' trajectories must be studied as the students experience different educational courses. Treatment effects are relative, both in contrasting experimental and control groups and, longitudinally, to the nature of educational experiences the students in these groups subsequently receive. There is a cumulative positive effect of students experiencing consecutive years of high-quality teaching, and a cumulative negative effect of low-quality teaching (Sanders and Horn 1998; Wright et al. 1997).

Interpretations of fade out may call for decreased funding and attention to pre-K (Fish 2003), but our position is that a lack of support for transitions to primary school and specific follow-through interventions is responsible. Our position is consistent with that of (a) the authors of the meta-analyses on fadeout, who conclude that because it takes a long time (about 10 years) for impacts to disappear, there is more than enough time for possible follow-through interventions that capitalise on the gains from these programs (Leak et al. 2012) and intervention researchers' notion of environmental maintenance of development (Ramey and Ramey 1998).

In the evaluation of the same students in this study as well as previous studies, the TRIAD implementation was particularly successful for students who identified themselves as African-American. Although African American students continued to lag behind non-African American students in all conditions, the TRIAD-FT intervention helped them narrow that achievement gap. A high quality, consistent mathematics education can make a demonstrative and consistent positive impact on the educational attainment of African American students in the pre-K, kindergarten, and 1st grade years compared to traditional instruction. We interpret these findings as supporting our theoretical interpretation of students' educational courses. We did not hypothesise this interaction, so we proffer explanations that are by necessity post hoc. (a) Centering instruction around learning trajectories may focus teachers' attention on students' thinking and learning of mathematics, and what children can learn to do, avoiding biases, such as views of African-American students' learning from a deficit perspective, that impair teaching and learning (ACF 2010). That is, especially given the significant mediation of the classroom culture, including enthusiastic interaction with children around mathematics they believe children can learn, it may be that the TRIAD interventions changed teachers' views of African-American students' mathematical capabilities (Jackson 2011). The curriculum's learning trajectories are based on the notion that learning is developmental and amenable to instruction, and the curriculum's approach, including specific, sequenced activities and formative assessment strategies, may have offered a way to act on these nascent views. In such action, the productive views are further strengthened. (b) The TRIAD intervention may promote a conceptual and problem-solving approach infrequently emphasised in schools serving low-income children, explicitly supporting African-American students' participation in increasingly sophisticated forms of mathematical communication and argumentation. (c) The TRIAD follow-through intervention may raise several aspects of the quality of mathematics education, lack of which has been suggested as a reason preschool benefits dissipate for African-American children; for example, the language-rich nature of the curriculum and its expectation that all children invent solution strategies and explain them. These and other possible reasons should be evaluated, compared, and combined, especially in interventions targeted to the primary grades.

The TRIAD follow-through intervention's effect was partially due to the increase in the positive classroom cultures teachers develop. Interventions such as TRIAD may help engender a greater focus on mathematics, which in turn can help increase students' mathematics achievement. As other work has shown (Clements et al. 2011; Jacobs et al. 2001; National Research Council 2009), helping primary teachers' gain additional knowledge of mathematics, students' thinking and learning about mathematics, and how instructional tasks can be designed and modified—that is, the three components of learning trajectories—has a measurable, positive effect on their students' achievement. This is particularly important in the early years because teachers often do not recognise when tasks are too difficult, but even when they do, they provide 'more of the same' (Bennett et al 1984). Further, they overlook tasks that provide no challenge to children—that do not demand enough (Bennett et al. 1984; van den Heuvel-Panhuizen 1996). Thus, most children, especially those who have some number knowledge, may learn little or no math in kindergarten (Wright 1991).

Implementing interventions such as TRIAD is therefore important, given that early mastery of concepts and skills in mathematics and literacy is the best predictor of students' successful academic careers (Aunola et al. 2004; Duncan et al. 2004; Duncan and Magnuson 2011). Further, students from low-income communities benefit more relative to students from higher resource communities from the same 'dose' of school instruction (Raudenbush 2009). Thus, comprehensive implementations of research-based models, such as the TRIAD follow-through model, may be especially effective in such lower-resource schools. This speaks to a caveat concerning the effectiveness of the TRIAD follow-through intervention. The intervention maintained, but did not add to, the gains of the more comprehensive TRIAD pre-K intervention. Differences in scores remain statistically significant, but effects were not cumulative. Future design studies might investigate ways to (a) avoid or ameliorate the limiting influence of pacing guides and other school district policies that may have limited the effect of the TRIAD follow-through component, (b) increase the intensity or duration of that component, or (c) implement different and more extensive interventions, such as curriculum replacement (as the TRIAD intervention did in pre-K). That is, future research should evaluate the efficacy and scalability of a fully implemented TRIAD model in the primary grades to see if the pre-K slope can be maintained throughout elementary school. Such an intervention may go beyond "resisting fade out" to show that a positive rate of learning can and should be sustained. In other words, we argue that what should persist is not just a pre-K gain, but also a dramatic trajectory of successful learning.

10.10 Final Words

The best predictor of a successful academic career is early mastery of literacy and mathematical concepts and skills. Students from low-resource communities benefit more relative to students from higher resource communities from the same 'dose' of school instruction (Raudenbush 2009). Thus, comprehensive implementations of research-based models, such as the TRIAD follow-through model, may be especially effective in low-resource schools such as those in this study. Future research should develop and evaluate more effective follow-through interventions that use learning trajectories to support continuity of learning into the primary grades but instantiate the learning trajectories' instructional tasks more explicitly.

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Chapter 11
Partnerships that Support Children's Mathematics during the Transition to School: Perceptions, Barriers and Opportunities

Wendy Goff and Sue Dockett

Abstract In this chapter we share the perceptions of a small number of principals, teachers and parents about children's prior-to-school mathematics. Rather than focusing on the somewhat limited notions of young children's mathematical experiences reflected in some of the comments of these adults, we position the transition to school as a relational context, recognising it as a time when many and varied beliefs, expectations and understandings come together as a cultural interface. We advocate that working collaboratively at this time has the potential to enhance the experiences of young children and the adults with whom they interact, and to provoke both professional and personal reflection and change, particularly in relation to mathematics education.

11.1 Introduction

Prior to starting school, children notice, explore and experiment with the mathematics of their world: mathematics is a tool for discovery, a means of investigation and a way to learn more about the physical and social spaces in which they live and learn. As is now well-documented nationally and internationally, children engage in a wide range of mathematical experiences and develop many sophisticated and powerful mathematical ideas in the years before school (Clarke et al. 2002; Geist 2009; Perry and Dockett 2002; Sophian 2009).

How and to what extent children's mathematical ideas develop depends on a range of social, cultural and geographic factors, including family resources and experiences (Biddulph et al. 2003), and access to early childhood education (National Association for the Education of Young Children and National Council for Teachers of Mathematics 2002). A common element across these influences is the involvement of adults.

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11.2 The Influence of Adults on Young Children's Mathematics Learning

Adults are important in the lives of young children. Their actions afford and/or restrict opportunities, enact choices for and about young children's lives, and contribute to the environments in which young children live and learn. Everyday experiences and interaction between young children and adults have the potential to build and extend rich, embedded understandings of a wide range of mathematical concepts (Civil and Bernier 2006; Civil et al. 2005; de Abreu et al. 1997; Sophian 2009).

The influence of parents on the learning of young children has been well documented (Levine et al. 2009; Plowman et al. 2011; Silinskas et al. 2012). As one example, correlations are reported between home numeracy experiences, such as number talk, board games, shopping and cooking, and the mathematical skills of children in the early years of school (LeFevre et al. 2009).

The influence of educators has also been researched extensively (Bowes et al. 2009; Emilson 2007; Fox et al. 2010; Sammons et al. 2008). Young children's play incorporates a wide range of mathematical concepts, ideas and explorations (Seo and Ginsburg 2004). When supported and extended by educators, young children's play and mathematical understandings have been reported as extensive and complex (Ginsburg 2006). Despite this, some early childhood educators have been reluctant to promote mathematical experiences, based on their own lack of confidence with mathematics (DeVries et al. 2010), concern for encroaching on children's play (Grieshaber 2008; Ryan and Goffin 2008) and their own negative attitudes towards mathematics (Sweeting 2011).

School educators also exert major influences on the development of young children's mathematical skills and understandings. First-year-of-school teachers have opportunities to build upon children's existing knowledge and ways of knowing, as they engage children in the mathematics curriculum of school. Effective teachers draw on their own mathematical content knowledge, understandings of curriculum and pedagogy, as well as skilled observation and their experiences as teachers, to generate the conditions necessary to support and extend young children's mathematical ideas (Fox et al. 2010). However, some primary educators are reported to hold limited views of mathematics which impact on the experiences and opportunities provided in the classroom (Nisbett and Warren 2000).

While there is widespread agreement among researchers and educators that adults play major roles in facilitating young children's mathematical learning, there is also recognition that different parents and educators engage with children and mathematics in different ways, influencing different outcomes (Levine et al. 2009). Key factors influencing the ways that adults—parents and educators—engage with children and their mathematics are attitudes and content knowledge. Where adults are uncomfortable and unfamiliar with mathematics, they tend to promote it less than other areas in either the home or educational setting. For example, the depth and extent of educators' mathematical knowledge impacts not only on the experiences they promote, but also the ways in which children's engagement in the experiences is interpreted and extended (Fox et al. 2010).

In a similar vein, adults' emotional connection to mathematics also influences they ways in which it is promoted (Grootenboer and Hemmings 2007; Stipek and Byler 2001). Whilst literature on the affective domain is plentiful in areas of teacher education, secondary school, and the primary school setting, there is quite a 'noticeable (now glaring gap) in any research in the early childhood sector' (Grootenboer et al 2008, p. 10), and an even bigger gap in the literature of transition to school. This also includes an exploration of the ways in which parent and teacher beliefs and attitudes influence the provision of mathematical experiences for young children and how these beliefs and attitudes might sit in tension or come together to support mathematical learning over the transition to school.

11.3 The Transition to School

In recent years, there has been growing national and international focus on the processes of starting school (Dockett and Perry 2013). Such focus has resulted in wide recognition of its importance, particularly in relation to the impact that experiences of transition might have on the later learning and developmental outcomes of children (Margetts and Kienig 2013). Contemporary research, policy and practice around the transition to school recognises it as a major life transition (Sayers et al. 2012).

Transitions are times when people change their roles in communities (Rogoff 2003). Starting school is a time when children, families and educators change their roles in the educational communities in and around schools (Dockett and Perry 2007). Changing roles affords both opportunities and challenges for all involved (Educational Transitions and Change (ETC) Research Group 2011). When considering young children's mathematics, starting school generates opportunities for them to demonstrate their knowledge and understandings and for educators to notice, recognise and build upon this (Dockett and Goff 2013). At the same time, starting school can be a time when children, parents and educators experience challenges as they navigate new context, experiences and expectations (Perry and Dockett 2008).

A range of research has highlighted the importance of relationships at this time, both between and among adults, as well as children (Birch and Ladd 1997; Hamre and Pianta 2001). Positive home-school relationships (Pianta et al. 2001), and positive connections between educators located in primary schools and prior-to-school settings (Einarsdóttir 2006) can facilitate the sharing of information, which in turn, can promote recognition of children's existing knowledge and interests. Relationships between educators and parents have the potential to support positive transitions to school by building bridges between home and school or prior-to-school settings and school. In particular, positive relationships between educators in different settings provide impetus for the development of partnerships and, through these, spaces for reflection on the 'cultural encounter between school and preschool, as well as the pedagogical possibilities and risks involved in an integration of the two school forms' (Moss 2013, p. 20).

Conceptualising the transition to school in relational terms situates the processes and the experiences of starting school within the complexities of human interactions. It provides opportunities to recognise social and cultural elements of the transition, while at the same time acknowledging the unique experiences of each individual. In such a frame, difference is regarded as positive and to be expected.

Relationships are forged through the coming together of people. However, they do not create the space: relationships are the result of what happens in the space of coming together. Nakata (2002, p. 285), defines the space that is created when people come together as the cultural interface,

> The place where we live and learn, the place that conditions our lives, the place that shapes our futures and more to the point the place where we are active agents in our own lives—where we make decisions—our life world.

It is the space in which relationships can be forged or resisted, and in which differing and similar views, beliefs and culture can be both rejected and harnessed. It is the starting point from which relationships commence and a crucial component of the relational context of starting school.

In the following sections of this chapter, we explore data generated through the first author's doctoral project. Key components of this project were the conceptualisation of the transition to school as a relational context, generating a cultural interface which promotes relationship building; and the importance of adults in supporting children's mathematical learning across and between differing contexts. Findings around other themes that have emerged throughout the project have been reported elsewhere (Goff et al. 2013). However, this is the first time in which the data reported in this chapter have been shared.

The project explored the notion of prior-to-school teachers, families and primary school teachers working together to support the mathematics learning of children as they made the transition to school. It involved the implementation of a small-scale intervention, based on the establishment of two research teams located at two different sites in rural Australia. Each research team consisted of a prior-to-school teacher, a first-year-of-school teacher and parents of children who were attending the prior-to-school setting. The aim of each research team was to investigate the mathematics learning of children, and to then use this information to devise a plan that would support this learning as children made the transition to school. The project drew on a design-based research methodology (Herrington et al. 2007) and utilised the conceptual framework of the cultural interface (Nakata 2002, 2007) to explore the processes employed by each of the research teams as they worked together through this experience.

The focus of the following discussion is data generated during the recruitment stage of the project. It reports the perspectives of a small number of principals, teachers and parents as they reflected upon young children's mathematical learning and capabilities at the time of transition to school.

11.4 Principal, Teacher and Parent Perspectives

Establishing each research team required ongoing liaison with the directors/managers of early childhood services and school principals. While these people were not necessarily directly involved in the study, they did act as gatekeepers, controlling access to educators in different settings and, through this, access to potential parent participants as well. Hence, interactions with school principals and early childhood centre directors were a necessary feature of the recruitment process.

11.4.1 Principals

To promote participation in the project, the researcher met with principals to share the research proposal, garner feedback about the planned research, and seek permission to present the project to first-year-of-school teachers. Ten principals in regional Australian schools were approached: six agreed to a talk about the project. The data reported are drawn from these meetings.

Principal 1 agreed to a telephone conversation about the project. When the researcher explained the project focus on researching and supporting the mathematics learning of young children as they made the transition from preschool to primary school, Principal 1 suggested that "behaviour and emotional development are much better areas to focus on during the transition to school" and added that "all that learning stuff comes when they can sit on the floor and listen, and when they're all settled into their classes". Principal 2, who met with the researcher in person, offered a similar perspective, suggesting that primary school is "all about routines during term 1" and noted that "we get into focusing on mathematics and literacy after they have adjusted to the school environment". Principal 3 opted for a telephone conversation and commented that, "we like to give them some time to get to know the school and then we focus more on learning".

Principal 4 met with the researcher in person and described the schools focus as "simply… making the children feel safe, cared for and comfortable". Principal 5 also met with the researcher in person. She explained that, at her school "numeracy has been a bit neglected", noting that "it's so busy at the start of the year, it's hard to find out what they know. You feel like you're chasing your tail a bit." Principal 6, who also met with the researcher in person, explained that throughout his career he had found that "some teachers are so focused on getting the kids to read in that first year that maths takes a back seat". He elaborated his view that "they are both equally important" and that "you don't want to stop or slow that momentum that's been building in kinder".

During these conversations there were also varying insights into the principals' perceptions of prior-to-school mathematics learning and the capabilities of young children. For example, Principal 1 commented "kids are so used to playing that it's hard for some of them to just sit and concentrate, especially on maths, you know it's hard when they can't sit still." Principal 2 outlined the provision of "little things like puzzles that can help with their mathematics, but nothing too heavy when they first come". When discussing prior-to-school mathematics learning, Principal 4 proposed, "some form of mathematical learning must take place" before children come to school, but indicated that "it is nothing robust". Principal 3 described the approach in his school as setting "up similar little things to preschool but start focusing on learning when they're a bit more comfortable." Principal 6 explained, "we [the school] don't have much to do with the kinder so I can't really tell you what mathematics they do, probably the usual type of things, sand, water, puzzles… not sure". He concluded, "most of the kids that come in are pretty switched on. We can have kids working way above where they should be so something is working." In relation to prior-to-school mathematics, Principal 5 suggested, "the kinder do a great job, the kids amaze us every year, most of them can count and they know their shapes and things…". Principal 5 also suggested that, "they [the kinder] have a tough job given the area that we're in, they do an amazing job really. I'm not sure that much is done with them [the children] before they get to kinder. So yeah they do an incredible job."

Teachers of the first-year-of-school were present during the meetings with two of the principals. In one instance, the teacher and principal opted into the research project immediately after it was presented. In another instance, the principal suggested that the researcher meet with the first-year-of-school-teacher to discuss the project further and to gauge her interest to participate. The remaining principals indicated that they did not wish to pursue involvement in the project. While it can only be supposition, the comments reported earlier suggest that mathematics was not a priority area of focus for these principals.

11.4.2 Teachers

The recruitment phase of the project also involved meetings with several Transition to School Networks—informal gatherings of prior-to-school and first-year-of-school teachers working in the same geographical location. During these meetings, four teachers provided some insights into their perceptions of prior-to-school mathematics learning and the capabilities of young children.

Teacher 1, a prior-to-school teacher suggested that, "my curriculum is play-based but I can adapt it to focus on maths [for the project]". Teacher 2, also a prior-to-school teacher, explained "the problem is in my training I was taught about numeracy not maths, and that kinder is preparation for life skills not school…I don't want to go over that boundary and focus on school skills rather than life skills." This teacher went on to explain that "we support their numeracy by talking to them, we don't have lessons, but we support their learning with the environment and stuff, we still teach them, but just different." Teacher 3, a first-year-of-school-teacher, described prior-to-school mathematics learning as "incidental mathematics, you know nothing too much just those incidental things that happen that are maths during the course of the day". This same teacher elaborated "those incidental things help with the more difficult maths" and "it's good if they have been to preschool 'cause they know nothing if they haven't". Teacher 3 added further that, "you can really tell the kids that have been to preschool cause they're way ahead in maths than those who haven't. Those that haven't are lucky to count to ten." Teacher 4, a first-year-of-school teacher, suggested to a prior-to-school teacher that "it'd be good to have some time to show you [the prior-to-school teacher] what maths they [the children] will need to do, you know, to get them ready…it would help you to see how it sets them up for what's to come."

11.4.3 Parents

The project was also presented to parents and prior-to-school teachers at four prior-to-school settings. During these presentations, six parents referred to their children's mathematical experiences, learning and capabilities.

Parent 1 explained, "we don't really do much [mathematics] at home, we count but that's about it…we don't do anything else like sums or that". Parent 2 asked the prior-to-school teacher, "You don't do much at preschool do you?" and then explained, "there's plenty of time for it all next year when they go to school, no need to rush into maths, they need time to play and be kids first." Parent 3 explained that

> Maths is everywhere but I'm not sure that it really makes sense to them [children] until they start school, like they count but it doesn't really make sense 'til they start doing it all properly, so we can't really support it until then, can we? …They're more interested in playing at this age group, when they get to school they know they've got to learn, so it's different.

Parent 4 suggested that "the preschool do a lot, they're always counting and they read stories with numbers and that, we do the same, well…similar things at home but yeah, it's just fun stuff, you know like at preschool". Parent 5 indicated that

> The kinder give us lots of ideas for home. We've started making puzzles, yeah that's the latest. He draws a picture, we cut it up and he puts it into one of those um snap-lock bags, you know those sandwich bag things, then he pulls them out and puts them together.

Parent 5 also described how her son was, "always counting and watching that show on TV, what's it called, you know the one with Piggly Winks, lots of maths in that show, he learns heaps from that." Parent 6 explained, "I hate maths but his father loves it. He's always telling him to do something, counting, adding things up, minuses you know. They spend ages doing it." In relation to the prior-to-school setting, Parent 6 suggested that "they do so much here, all those songs, drawing around their bodies and lining them up, the cooking yeah just so much, he's doing so much and he loves it, its good."

11.5 Initial Impressions

These data provide a window into the perceptions of young children and mathematics held by this small number of principals, prior-to-school teachers, first-year-of-school teachers and parents. Two main issues were identified from these data:

1. The most nuanced views of young children's capabilities and their mathematics learning came from those who interacted with them most frequently—their parents and prior-to-school teachers.
2. The inference that 'real' mathematics was encountered at school.

The six parents, more so than teachers or principals, recognised their children's engagement with mathematics beyond counting and knowing shapes, mentioning puzzles, singing, drawing and cooking. One of the two prior-to-school teachers emphasised differences between mathematics and numeracy, preferring to focus on 'life skills, rather than school skills'. This contrasted with the view of the two first-year-of-school teachers, one of whom offered to share "what maths they [the children] will need to do" when the children commenced primary school. Most—but not all—principals reported limited views of young children's mathematics before they started school.

Despite the range of mathematical experiences noted by parents, most of those discussing the project regarded the play-based, holistic curricula associated with prior-to-school settings as not facilitating opportunities for intense engagement with complex mathematical ideas.

One of the implications of the expectation that young children do not engage with sophisticated mathematical ideas before they start school is that existing knowledge is neither recognised nor valued. This can mean that the maintenance and enhancement of that knowledge is then compromised. The view that children only encounter 'real' mathematics within school contexts not only devalues the learning that may have occurred prior-to-school, but also contributes to expectations that do not match children's existing understandings and interests. In order to recognise the mathematical learning young children bring with them to school, it would seem important to engage with those who know them best—their parents and prior-to-school teachers.

11.6 The Cultural Interface

At first glance these data could be interpreted as identifying some major barriers to overcome in relation to supporting the mathematics learning of young children as they make the transition to school. However, this is not our intention. In the remainder of this chapter, we consider an alternative approach, focusing on the ways in which these perceptions might be reframed to facilitate the mathematical learning of young children, particularly in the context of relationships but also in the context of starting school. To achieve this, we draw on the conceptual framework of the cultural interface (Nakata 2002, 2007).

The cultural interface provides a framework to consider how and why parents and educators come together as children start school, creating an interface where perspectives, culture, beliefs, values, and knowledges meet. In relation to the data presented in this chapter, it provides a way to re-examine this through exploration of the interface of home, prior-to-school and school contexts, and to explicate potential opportunities and facilitators for growth, learning and change as these different contexts, and the people within them, come together during children's transition to school. This directs our attention to the ways in which these different people and contexts interact, the links they build and the ways in which they confer and enact value and respect. Using the framework of the cultural interface recognises the important contribution of all participants to the co-construction of transition experiences (Griebel and Niesel 2013), as well as the processes of continuity and change that underpin transition to school for all involved (ETC 2011)

Utilising the framework of the cultural interface repositions transition to school as a time when the adults in young children's lives, as well as the children themselves, inhabit spaces of possibility—spaces where there are opportunities for all to work together to support the learning of young children. Essential to this repositioning is the valuing of the contributions that each participant brings, the relationships that are forged, and a commitment to identifying and working with the tensions that will often arise. In Nakata's words, the interface promotes focus on the processes engaged in, as well as the processes experienced or omitted (Nakata 2007).

Identifying the transition to school as a cultural interface provides a means to identify the possibilities and opportunities that arise to scaffold adult interactions and relationships, and to promote recognition of young children's mathematical understandings. It provides a context for learning from one another, and with one another, to support the learning of young children. The following section of this chapter provides a re-examination of the data presented previously, using the lens of the cultural interface.

11.7 Principal, Teacher and Parent Perspective—A Second Look at the Data

Examining the data using the framework of the cultural interface helps to consider alternative interpretations of the data. These readings are based on the expectations that people will bring different views and understandings to bear in new contexts. This framework situates the transition to school as a time for sharing perspectives and forging relationships that help to explore the challenging and the complex, and that generate new possibilities and new learnings (Nakata 2007).

As one example, we consider the views of four of the principals who agreed to discuss the project, but on reflection decided that mathematics was not an area of priority focus as children made the transition to their schools. Rather than addressing any curriculum area, these principals referred to the importance of socio-emotional factors, such as helping children to settle in to school and feel comfortable. While it is possible to interpret the lack of attention to mathematics as a barrier to promoting continuity of learning for the children, it can also be interpreted as a realistic approach to children's wellbeing during this time. However, this focus contrasts with the views of the first-year-of-school teachers, who referred to readiness and the importance of preschool in preparing children for school, to the extent of offering to show the prior-to-school teachers what was required in terms of mathematics. While cautious in interpreting the data from such small groups, it is likely that ongoing interactions between teachers and principals have the potential to generate some common ground about expectations and approaches.

A further example is drawn from the views of parents and prior-to-school educators. While these groups seemed to have a more nuanced views of prior-to-school mathematics that school educators, there remained a strong sense that 'real' mathematics was encountered at school. This suggests both common ground and different perspectives that could inform ongoing interactions.

11.8 Relational Spaces

Framing the transition to school as a cultural interface presumes that adults identify this as a time when they have much to gain, and much to share in interactions with others. Relationships that are forged at this time have the potential to go well beyond the rhetoric of 'readying for school' (Moss 2013, p. 9), establishing a potential meeting place for the sharing of expertise and recognition of different knowledges and beliefs. These, in turn, can generate new or stronger ways of knowing and doing.

In some contexts, adults have already generated spaces where different views may be shared, explored, challenged, tested and contested. In other contexts, some prompt or provocation will be required to create such a space; in others even with such prompts, generating such spaces will present challenges. Despite this, the transition to school is a time recognised by many adults as a time of change and as a time to reach out to others, as well as to become engaged with others.

Much of the research literature about transition to school centres round children and families, the skills they bring and the experiences they encounter (Perry et al. 2014). The data presented in this chapter suggest that the transition to school also affords opportunities for adults—parents and teachers—to forge relationships and work together in ways that provoke examination of pedagogies and practices, as well as expectations and entitlements. Such examination can occur independently, with one another, and with children. It can also occur in varying ways. For example, through ongoing dialogue, frequent interactions, and the sharing of information. Professional and personal growth and change is an important notion: it is important for the adults in the lives of young children to be open to new ways of knowing and doing, and also to the possibilities of and for change. If different views, beliefs and attitudes around children's learning remain isolated, such change might not be realised. Reconceptualising starting school as a relational context affords these opportunities.

11.9 Conclusion

In this chapter we have shared the perceptions of a small number of principals, teachers and parents about children's prior-to-school mathematics, and positioned these as potential barriers and/or facilitators. Rather than focusing on the apparently limited notions of young children's mathematical experiences reflected in some of the comments of these adults, we position the transition to school as a relational context, recognising it as a time when many and varied beliefs, expectations and understandings come together as a cultural interface. Working together at this time has the potential to enhance the experiences of young children and the adults with whom they interact, and to provoke both professional and personal reflection and change.

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Chapter 12
The Culture of the Mathematics Classroom During the First School Years in Finland and Sweden

Kirsti Hemmi and Andreas Ryve

Abstract This chapter elaborates findings from a longitudinal ongoing cross-cultural study comparing the teacher education and classroom practices in Finland and Sweden. The focus is on the cultural scripts of mathematics instruction during the first school years (ages 6–8). Firstly, we present a description of the contexts of each country concerning primary teacher education and the transition from preschool to school. We then characterise the dominating conceptualisations of the mathematics classroom practices for the early years in both countries, building on several analyses of different data sources. We focus especially on the intricate balance between flexibly building mathematics on pupils' ideas of familiar everyday phenomena within a thematic teaching style on the one hand, and on the other, the organisation of learning environments strictly based on a predetermined hypothetical learning trajectory. Finally, we discuss our findings in light of the international literature on early mathematics education and transition from preschool to school.

12.1 Introduction

The aim of this chapter is to describe and compare the culture of teacher education and mathematics classrooms for the first school grades in Finland and Sweden. The chapter is based on the results of an ongoing cross-cultural project in which we have gathered and analysed various kinds of data since 2009 in order to understand the educational cultures in Sweden and in Finland. Both insiders and outsiders have been engaged in the data analysis, in order to capture features within the cultural scripts in each country that might otherwise be lost (Clarke 2013). In this chapter we draw on the various results obtained from the project and take a special look at the mathematics instruction during the first school grades in both countries. The studies employed in the chapter range from curriculum studies (Berg et al. 2013; Hemmi and Berg

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2012; Hemmi et al. 2013b) to studies focusing on teacher educators' views on (good) mathematics teaching (Hemmi and Ryve 2013; Knutsson et al. 2013), textbook studies (Hemmi et al. 2013a) and studies on the character of the teaching practice offered to the prospective teachers in the two countries (Knutsson et al. 2013; Ryve et al. 2013, 2012). We have also recently started analysing classroom data, and use the results of this ongoing analysis as a complementary source for the description of cultural scripts in each country (Andrews et al. 2014; Corey et al. 2010). Comparing the teacher education and classroom cultures in Finland and Sweden is particularly interesting for the research field, because despite many similarities in the school systems (for example, an inclusive compulsory school with no tracking), the results of recent international and national evaluations differ substantially. Finnish students have shown excellent results, especially in PISA assessment, while there has been a downward trend in the results of Swedish students in both international and national evaluations. Moreover, the status of the profession of primary school teacher is exceptionally high in Finland, where it is one of the most popular study programs at universities (Krzywacki et al. 2012). In Sweden, the situation has been the opposite. This chapter adds to research on early classroom teaching in different cultures by characterising and comparing the cultural scripts in Finnish and Swedish first grades mathematics classrooms (e.g., Kilpatrick et al. 2001; Verschaffel et al. 2007).

An important focus in this chapter is on the classroom culture prospective teachers meet in their school-based practice. Several studies suggest that the nature of school practice within teacher education is of crucial importance for the education of prospective teachers (Ebby 2000; Wilson et al. 2001). In Finland, teacher education has been at a Master's degree level for about three decades. This kind of scientific primary teacher education distinguishes Finland from most other countries (Laine and Kaasila 2007). Those studying to be teachers in *preschool* and *primary school* study education as their major subject, and primary school teachers choose one or two specialising subjects. It is possible to specialise in the didactics of the first school grades (Grades 1–2). In Sweden teacher education has been reformed several times in recent decades, and the latest reform established a separate teacher education program for those studying to teach preschool class to Grade 3 and another for preschool teachers. This reform strives to enhance the academic level of the primary school teacher education and preschool teacher education, but the teachers still do not reach the Master's degree level within the program. In Finland, teacher training within teacher education is arranged through university practice schools, where teacher educators and mentors supervise the students. In Sweden, the teaching practice is organised within ordinary schools with practicing teachers as mentors.

School start in Finland and Sweden has traditionally occurred in the autumn, the same year the child turns seven. Both countries offer early childhood education for children after the parental leave, and play, exploration, art, music and physical training are stressed in both countries' steering documents (Skolverket 2014; The Finnish National Board of Education 2010). In the Finnish document it states that "mathematical orientation is based on making comparisons, conclusions and calculations in a closed conceptual system. In ECEC, this takes place in a playful manner in daily situations by using concrete materials, objects and equipment that children know and they find interesting." In the Swedish syllabus for ECEC relatively detailed mathematical content is stated where children should develop their "ability to use mathematics to investigate, reflect over and test different solutions to problems raised by themselves and others". Distinguishing, expressing, examining and using mathematical concepts and their interrelationships as well as putting forward and following reasoning are also stated as goals.

For decades, both countries have organised a so-called 'preschool class' which is optional for all children one year before school start. Most children in both countries take part in preschool class activities. In Sweden, there is no special document for preschool class, only recommendations to combine preschool pedagogy with school-pedagogy. In Finland there are 40 pages of curriculum guidelines for the preschool class. One page is devoted to mathematics (The Finnish National Board of Education 2003). There are similarities in the school curricular development in the two countries, and the current national school curricula in mathematics can only be regarded as frameworks, offering teachers a great deal of freedom to plan and conduct their teaching (Hemmi et al. 2013b). Both countries had a period of decentralisation in the 90s, but in Finland a new, more detailed curriculum was introduced in 2004. In the Finnish compulsory school curriculum, special goals and contents are stated for the first grades (1–2) and there is a description of good mathematics performance for the end of Grade 2. In Sweden, there was a long period since the 1994 curriculum was introduced when only very general goals for Grades 5 and 9 in compulsory school were stated. The fulfillment of these goals was monitored for the first time during Grade 5, through national examinations testing the minimal level of pupils' achievement. During this period, schools and primary teachers had a great deal of space to choose both the mathematics content to be dealt with and the rate of instruction in the classroom, especially in the first grades (Hemmi and Berg 2012). The new Swedish curriculum states core content for Grades 1–3, and offers a definition of minimum "knowledge demands" for the end of the Grade 3. Although the statements of goals and content and the introduction of the national examination during Grade 3 imply more steering than the previous system, the curriculum can still be seen as a relatively general framework as it does not recommend teaching methods, textbooks, lessons plans or tests (Hemmi and Berg 2012). There are differences in transition from preschool class to the first school class. In Finland, preschool class can be organised either in primary schools or in the day-care centers while in Sweden they are organised within schools and the goals stated for Grade 3 should guide the work in preschool class.

Next, we describe the cultural scripts of the mathematics classrooms in each country as identified in our studies and exemplify them with extracts from interviews with the teacher educators. The extracts are based on the analysis of four focus group interviews with teacher educators in two different departments in each country, interviews with eight mentors in Sweden and five in Finland, and documentation of four feed-back discussions with prospective teachers and mentors/teacher educators in each country. In Finland the mentors work in university practice schools but in Sweden in ordinary schools. All of them are acting within primary school teacher education and all the mentors are fully trained teachers. We then discuss our findings in light of the international research on early mathematics education, and describe the likely trajectories for our work over the next few years. We conclude with some examples of how our research results have been applied in practice so far.

12.2 Cultural Scripts in the First Years' Mathematics Classrooms

Based on our studies and ongoing analyses, we claim that there is a striking difference between the mathematics classroom cultures in the two countries. We describe the cultural scripts of each country starting with Finland.

12.2.1 The Case of Finland

The Finnish national curriculum states that the "core tasks of mathematics instruction in the first and second grades are the development of mathematical thinking; practice concentrating, listening and communicating; and acquisition of experience as a basis for the formulation of mathematical concepts and structures" (The Finnish National Board of Education 2004). From the very beginning Finnish children seem to be socialised in the classroom, with certain routines of practice (Franke et al. 2007) that are repeated almost every lesson. The main elements in a mathematics lesson are, according to the mentors, analysis of classroom data and teacher guides, review of the previous day's homework, introduction of new concepts and procedures, mental calculation and homework assignment. Suggestions for how all these activities can be conducted are offered in the mathematics teachers' guides (Hemmi et al. 2013a). These elements are reminiscent of those advocated by the Missouri Mathematics Effectiveness Project conducted by Good and associates in the late 1970s (Reynolds and Muijs 1999), and we note that Finnish teacher education became research-based and started to absorb influences from the international field of mathematics education about the same time as the results from the project were published (Hemmi and Ryve 2013).

Creating a proper balance between routines and variation is one of the foci in several of the interviews with the teacher educators. They maintain that variation is desirable, but within the repeating elements of the lessons; for example, concerning how homework is checked, how new contents are taught or the order in which the different parts are dealt with.

> As in all teaching, using varied activities, for example when checking homework or teaching new stuff, although in mathematics maintaining the routines, one has to learn the routines, it's really important, but it's a kind of balancing act, somewhere one needs to find the variation and in some way it's just working hard. (Finnish mentor 2011)

The role of certain repeated routines in quality mathematics teaching is stressed by several mentors.

> I think that, in mathematics, routines are important, certain recurrent activities in the lessons. (Finnish mentor 2011)

This is also stressed in the feedback discussions.

> It was fun, the store play, children often like it when the same activity is repeated, the rule of the play, no one was left out, all of them eagerly took part. (Finnish feedback discussion 2011)

Specific 'rehearsal' lessons with various group activities like working at different "stations" with games, problem-solving, practicing skills, and so on are also advocated by the mentors.

A specific character of the Finnish national guidelines is that children should learn to concentrate, listen and communicate. This is stressed already in the curriculum for the preschool class: "In Pre-school education, it is important to develop children's concentration, listening, communication and thinking skills" (The Finnish National Board of Education 2010). Various classroom events identified in our studies reflect this goal. Firstly, the lessons seem to consist of quite short lesson events in which concentration, listening and communication are practiced in various manners. For example, with the small sessions of mental arithmetic in almost every lesson, children practice listening and concentration in small proportions. Another aim of the small mental arithmetic tasks for the first grades is to connect mathematics to everyday and fantasy problems as many of the children do not yet read well, though the mental arithmetic tasks can (according to the teacher guides and classroom analysis) also be purely mathematical problems.

Further, communication is also practiced when working with concrete models, play, games and physical activities that are carefully planned, organised and monitored by the teachers in order to enhance the learning of certain concepts and procedures, used regularly in the Finnish classrooms in Grades 1–2. Working with concrete models reflects the goal of "acquisition of experience as a basis for the formulation of mathematical concepts and structures", and this goal is also visible in the classroom events identified in our studies. According to the teacher guides and the classroom observations, certain concrete models are repeatedly used when introducing both new concepts and procedures to the children. There are also different kinds of hands-on material for all children, connected to the textbook models that every child receives with the textbooks.

> Concerning activating material, it's a big issue in itself that I would like to raise, actually from the point of view of the teacher, mastering it as a pedagogical tool, we go from the concrete to the abstract. It's a natural path in mathematics that we've developed different kinds of concrete material for the teaching of various concepts and algorithms. (Finnish teacher educator 2009)

Using the manipulatives and various models as well as visualisation (also with help of ICT) is pointed out as important for enhancing children's understanding by all the Finnish teacher educators we interviewed. It is also raised as an important topic in the feedback discussions with the student teachers practicing in the first school grades.

The children's own responsibility for and reflection of learning are already addressed in the preschool curriculum. "The teacher shall support learning and guide children to become conscious of their own learning and to perceive that they can themselves influence their own success in learning." This is to be developed with the help of small routines, according to the teacher educators. For example, children learn to check the correctness of their tasks by themselves or in pairs from the very beginning of Grade 1, and these checks happen frequently. According to the teacher educators, this helps children reflect on their learning and they also receive feedback quickly, while at the same time the teacher can engage in helping the children who need special support.

Children also receive a small homework assignment after every mathematics lesson (except over the weekends) from the beginning of Grade 1. The benefits of the regular homework are stressed by the school mentors and teacher educators, as it (according to them) extends the learning process beyond the school context and offers a possibility for the children to reflect on their own learning, as well as for the teacher to receive quick feedback on the children's learning process. Further, according to the Finnish teacher educators and mentors, homework offers several opportunities for learning the new topic as children come into contact with it in different contexts within a short time span.

> In principle, there should always be a triple confirmation, so that the new stuff is taught and there's some exercise, then the homework and the checking of it. Through this there are several possibilities to catch it, at the latest when we check the homework. (Finnish mentor 2011)

The importance of regular homework is raised especially for the 'weaker' learners, as according to the mentors it encourages a deeper learning of new contents and skills.

During the feedback discussions, homework is also raised as a possibility for the teacher to follow students' learning continuously and react if there are problems (formative assessment). From the equity point of view, it seems to be important that the obligatory homework is of a kind that all children manage to complete themselves. Yet, some teacher educators advocate differentiation by homework, i.e. giving more challenging tasks to those who need them.

Clarity of presentation and of the concrete models, as well the clearness of the work on the whiteboard, are introduced as important characteristics of effective teaching, especially for small children (Hiebert and Grouws 2007). This is also touched on several times in the feedback discussions with teacher educators, mentors and prospective teachers. Logical presentation is also portrayed as important by both teacher educators and mentors, in order not to hinder children's learning.

> …because if the teacher is somehow illogical or makes, for example, the board work in an illogical manner or leaves it flimsy, certainly all kinds of illogicalities backfire and lead to problems in children's learning. (Finnish mentor 2011)

Not only is clear board work claimed to be important for children when following the presentation, but they should also be able to return to and reflect on the presentation during the exercise session.

Another issue stressed in the feedback discussions is the importance of always writing a relevant title on the board in order to make the goal of the lesson visible to the children, even if it is not expected that they can all read yet. Prospective teachers also have to define the goal of the lesson in their written lesson plans that form the base of their planning and the feedback discussions with their mentors. In the lesson plans students have to define the contents of different lesson events and the central mathematical concepts of the lesson, and describe how they will differentiate and evaluate the learning as well as what they are going to assign as homework.

Concerning problem-solving, all the mathematics teacher guides for the first grades contain small problems for every lesson, usually small mathematical puzzles. Some of the mental arithmetic tasks can also be regarded as problems. The mentors also talk about activities in which children can apply and discover something themselves. Listening to children's explanations is raised as important. The mentors especially stress that even small children can already figure out specific procedures themselves, and even explain them if one focuses on this from the beginning.

> They have unbelievably excellent thoughts, such that I never…, for example, how they accomplish ten transition, […]how it's possible that they can tell such things, that's amazing sometimes, so one just has to give them the space and of course they can't do that if we don't focus on it. (Finnish mentor 2011)

Concerning assessment, the Finnish teacher educators and mentors state that a good teacher "lives with their finger on the pulse" and evaluates pupils' learning all the time in different ways in order to support those who need it so that they can keep up with the rest of the group.

> I think it's important to really be observant all the time, although one doesn't always test but it's also easy to notice by other means in the classroom situation if someone is way off track, one has to make sure that all the pupils keep up. (Finnish mentor 2011)

In all feedback discussions, formative assessment is addressed in various ways, such as discussing the techniques for making children's learning visible during the ordinary mathematics lessons, and in connection with the homework review, observing if someone 'is way off track'. The importance of frequent checking is especially stressed for the teaching of young children, in order to intervene in time if there seem to be problems and thus prevent difficulties later. The mentors advocate written tests as worthwhile from the very beginning of Grade 1. They are not called tests, but still, all tasks are given marks and parents are continuously apprised of the results of these 'repetitions'. An important aspect concerning these diagnoses is that pupils are given as much time as they need to accomplish the tasks.

Although all the mentors we have followed and/or interviewed keep groups of pupils together in the same area from lesson to lesson, both the Finnish mentors and teacher educators stress that different kinds of pupils have to be considered differently in quality mathematics teaching, supporting the weaker learners and challenging the 'gifted' ones.

To sum up, the Finnish cultural script for the first mathematics classroom involves whole class instruction, often connected to concrete models, with some seatwork (whereby children check the correctness of the tasks themselves); a small homework assignment; and lessons following certain patterns within which some variation is desirable. Clarity of the lesson goals and of the board work is raised as important from the first school grades. An important goal for children is to learn to concentrate and take responsibility for learning in small portions. Formative assessment and considering different kinds of learners are considered as important part of primary teachers' work.

12.2.2 The Case of Sweden

A student teacher and a mentor are discussing the mathematics lesson just finished. The mentor states

> This was a very typical lesson, like many others look like. One (the teacher) walks around, one doesn't have to say that this isn't good, because it is good. One walks around and that's actually the teacher's work. One walks around and listens to the pupils, they raise their hands, they ask, you try to listen, what's the problem, how are we going to approach this problem? (Swedish feedback discussion 2011)

This statement captures key characteristics of the cultural script of classroom teaching and the role of the teacher as they are construed in Sweden. The teacher's competence in figuring out children's ways of thinking as a way to build the teaching is one aspect stressed frequently and clearly within the Swedish context. Therefore, a recurring term within the discussions is responsiveness to children's needs. Another mentor from our data suggests

> that it's important to be extremely responsive and see what children need, where they are, what they can do, and what they understand (Swedish mentor 2010)

followed by a third mentor, who stresses

> I agree that responsiveness is extremely important, that one sees the child and sees that the child can think (Swedish mentor 2010).

This conceptualisation is closely connected to ideas in, for example, the Teaching Principles of the National Council of Teachers of Mathematics (NCTM) (2000) in which effective teaching is understood in terms of the teacher basing instruction on the students' thinking (Hiebert and Grouws 2007). However, results from the Swedish national evaluations suggest that this conceptualisation of the relationship between teachers, students and content is typically operationalised into classroom practice by students working individually and the teacher circulating in the classroom. Such an operationalisation does not harmonise with dominant research initiatives aimed at realising the NCTM Teaching Principles (2000) in US classrooms (Cobb and Jackson 2012; Smith and Stein 2011).

Further, building teaching on children's ideas by being responsive refers not only to relating to the individual child's ways of thinking mathematically, but also to connecting mathematics to everyday phenomena. According to the Swedish teacher educators, a good teacher should use everyday situations and connect them to important mathematical ideas. Hence, it is not a question of working with contextualised problems within the textbook but instead of relating mathematics to children's everyday experiences:

> that they use mathematics in some way; before we went to the farm I brought a liter of milk to the class [and said] 'this is milk, is there any mathematics in it?'. The box was just standing there, well it says one and a half here, somebody knew percent and we were reasoning about it, what is it, it's about fat. (Swedish mentor 2010)

This way of using everyday experiences as a starting point for various kinds of mathematics is practically non-existent in the Finnish script, but in Sweden it is strongly emphasised not only in classroom teaching, curriculum material, during school based teacher education (SBTE) and among teacher educators in mathematics, but also in Swedish steering documents. Hemmi et al. (2013a) highlight that a special feature of the Swedish mathematics goals is that the word 'everyday' is repeated 23 times in the ten-page document. Dominating teacher guides used in the Swedish context also promote this kind of working manner. Interestingly, viewing real-life situations as mathematical has been an important feature within the recent Western reform movement in mathematics education (Drake and Sherin 2006; Wilson et al. 2005). Thus, once again, the Swedish scripts of mathematics classrooms seem to incorporate features from the recent Western reform while this pattern is less visible in the Finnish context.

Interestingly, the focus on everyday mathematics has begun to bother some teacher educators as they note that there is a risk of getting stuck in everyday experiences and never making explicit the mathematical ideas for the children. The solution to this problem within the Swedish script is the same solution that is asserted in all kind of situations: the knowledge of teachers. This way of stressing the teachers' knowledge as the main explanation of and solution to students' decreasing mathematical knowledge in TIMSS and PISA is visible not only in our data but also in the political debates. Again, we can see similarities to approaches advocated in recent Western reforms as, for example, in the NCTM, a condition for effective teaching is documenting that a teacher deeply knows and understands the mathematics he or she is teaching and is able to draw with flexibility on this knowledge in teaching tasks (Wilson et al. 2005).

The pattern of constructing early mathematics classrooms, in terms of responsiveness to students' thinking and a teacher with appropriate knowledge, portrays a classroom in which students work individually and the teacher circulates the classroom, as described above. In this respect, the teacher is passive in terms of presenting mathematical ideas, engaging in formative assessment, orchestrating whole-class discussions or directing group work. This picture is further supported in the data from the mathematics teacher education (Ryve et al. 2013). According to the Swedish teacher educators, prospective teachers entering teacher education programs typically request concrete methods for teaching but the teacher educators strongly reject this way of conceptualising classroom teaching:

> the expectations they [the prospective teachers] have when they come to the course, they usually write it down, and it's that one should tell them how to explain mathematics… ha ha… instead of being able to listen and follow others' ways of thinking and see where the child is, they want it to be like 'Say this'. (Swedish teacher educator 2009).

Therefore, and in accordance with the data above, a good teacher derives the necessary procedures from his or her analysis of the students rather than engaging in certain routines of practice (Franke et al. 2007). There are no methods for how to teach different kinds of children mathematics; a good teacher makes many important decisions in a given situation, and such decisions are directed towards individual students.

> Yes, because it's not a question of … there aren't any ready-made formulas but instead it's … you have to be able to analyse what pupils' thoughts mean, I think that's absolutely the most important thing. (Swedish teacher educator 2009)

The picture of a teacher as being responsive rather that proactive, is further strengthened throughout our analyses of data. For instance, systematic or planned assessment as a means to collect information on students' results is not introduced at all as part of the cultural script. That is, none of the Swedish teacher educators or mentors expresses any artifacts, procedures or routines for assessing pupils' knowledge (like diagnostic or formative and summative tests) other than listening to the children. The ideas about assessing students' knowledge in order to adjust the teaching is in close harmony with the ways the teacher educators and mentors talk about being responsive, meaning that one flexibly adapts the teaching to each individual student. Therefore, teachers should take the opportunity, when it arises, to build flexibly on students' interests, everyday experiences and ideas to discuss mathematics. They should thus not be proactive or plan teaching, since teaching differentiation appears spontaneously through working with, for instance, open problems and being responsive to students' everyday experiences.

To sum up, the cultural script of teaching in Sweden emphasises students' thinking, ideas and interests. Within this script the teacher should act with situational flexibility, and a teacher's work is not considered to be understood in terms of setting goals and planning teaching sequences. Instead, the cultural script as expressed within our data analyses constructs the interpretations of individual students' thinking in specific situations as forming the dominating bases for actions in classrooms. A further aspect of flexibility that is introduced is the teacher's processes of connecting everyday situations to mathematics. Good teaching refers to the use of spontaneous everyday situations that the skillful teacher can use for mathematical discussions. The dominating requirement for accomplishing this kind of flexible, responsive and everyday anchored teaching is teachers' mathematical knowledge for teaching (Kilpatrick et al. 2007).

12.3 Discussion and Implications for Future Research and Teaching Practice

Comparing the cultural scripts of the mathematics classrooms in the first school grades in Sweden and Finland reveals interesting differences. Several researchers have highlighted the balance between flexibly building the teaching of mathematics on children's ideas and everyday experiences on the one hand, and organising learning environments strictly based on the hypothetical learning trajectories of an ‘average’ child on the other (Clements and Sarama 2007). This distinction is often drawn between the preschool education and the first school grades (White and Sharp 2007). Perry et al. (2007) state that there is a clear difference in Australian settings between the cultures within the prior-to-school and school settings, with the first years of school characterised by “teacher-centered, syllabus-driven lessons and written group-based assessment while the preschools tend to adhere to their child-centered, play-based approaches” (p. 1). Considering our cases, it seems that the classroom culture in Swedish first school grades is closer to the culture often associated with preschool environments (Samuelsson and Carlsson 2008), as the ideal seems to be that mathematics teaching is quite spontaneously based on children’s ideas and their everyday experiences. In contrast, the analyses of the Finnish cultural script illuminate a practice in which teachers plan and orchestrate teaching focused on developing pupils’ conceptual and procedural understanding, hence closer to the “teacher-centered, syllabus-driven lessons and written group-based assessments” (Perry et al. 2007, p. 1). It is possible that transition from the preschool class to the first school grade is smooth in Sweden as the preschool classes are organised in schools and mathematics in the first school grades seem to be connected to children’s interest and everyday situations. It is also possible that the transition to school in Finland is to be facilitated by incorporating goals of the first school grades into the curriculum guidelines for the preschool. However, this needs more investigation in the form of in-depth analysis of the ECEC- steering documents as well as observations of prior-to-school activities combined with interviews with preschool teachers in both countries. Our future studies will specifically deepen in the character of cultural scripts of prior-to-school activities in order to investigate further the character of transition from ECEC to school in both countries.

Our studies show that the prospective teachers in Finnish teacher education are practicing within a relatively uniform cultural script with respect of the activities of a typical mathematics lesson (Corey et al. 2010). The cultural script is made explicit through, among other things, artifacts such as teacher guides and lesson plans. For the Finnish prospective teachers, this implies that the guidance and feedback from mentors are typically structured and clearly connected to the patterns within the scripts through, for instance, lesson plans (Corey et al. 2011). Finnish teacher education stresses the use of tools in initiating prospective teachers in the profession, a feature recently stressed as central to the effective education of teachers (Cobb and Jackson 2012). In studying the guidance and feedback from mentors to prospective teachers in Sweden, we find a pattern mirroring the relationship between pupils and teachers in the early mathematics classrooms. That is, the mentors are responsive and flexible to the prospective teachers’ thinking, and do not seem to follow any particular routine of practice or use any tools (guidelines, lesson plans, batteries of questions) to confirm that some pre-given aspects are covered (Knutsson et al. 2013).

Supporting tools and cultural scripts will be central in our future research and practical developments. Our ongoing studies show that early mathematics teachers in Sweden receive limited support from artifacts for planning, conducting and reflecting upon classroom teaching. That is, the educative potential of curriculum materials (Davis and Krajcik 2005) in Sweden is almost non-existent, and a future interest for the research group is to work closely with early-years mathematics teachers in developing supporting artifacts for teachers to act in the classroom and collaborate with colleagues (Bartolini Bussi 2011). For instance, we are using and adjusting the Five Practices model, developed by Mary K. Stein, Margret S. Smith and colleagues (Smith and Stein 2011; Stein et al. 2008), to support Swedish early mathematics teachers in establishing classroom practices that enable children to engage in problem-solving activities. In developing this future line of research, we will deepen our conceptualisations of the cultural script of early mathematics classroom practices in Sweden and iteratively adjust, implement and evaluate artifacts that support teachers in acting productively in classrooms. We are currently working closely with more than 200 early mathematics teachers over a 4-year period, and hope this work will add to our knowledge on how to develop educative curriculum material in mathematics education that supports teachers in being active in children's early mathematics transitions. From a practical point of view, the PISA results from 2012 are regarded as a national trauma. As researchers in this politically very complicated debate that implicitly and explicitly blames teachers, we are careful in stressing that the support for teachers in establishing productive classroom practices in early age mathematics classrooms is almost non-existent. We therefore hope this line of research will contribute to research and the development of classroom practice, as well as the position and status of primary teachers in Swedish society.

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Chapter 13
A New Zealand Perspective: Mathematical Progressions from Early Childhood to School Through a Child Centred Curriculum

Shiree Lee and Gregor Lomas

Abstract This chapter will discuss the New Zealand [NZ] approach to mathematics in early childhood settings with a particular emphasis on the foundational mathematics that infants and toddlers gain through play. It will describe ways that children's mathematical knowledge is developed through and with others as they move toward formal schooling. Early years curriculum in NZ encompasses both prior-to school (early childhood) and school education. Early childhood education serves children in education and care services from birth to school entry at (approximately) 5 years of age and is underpinned by *Te Whariki*, the NZ framework for early childhood. The formal (school) sector is, similarly underpinned by *The NZ Curriculum Framework*. These two documents are both grounded in constructivist, socio-cultural theory and provide the basis for a seamless transition from EC to school. Mathematics education in NZ has a particular emphasis on the development of numeracy (number knowledge and understandings) through progressions laid out in national frameworks. This emphasis has established the need for early childhood programmes to ensure that very young children are given opportunities to explore foundational mathematics in a variety of ways. However, the challenge for NZ early childhood teachers is twofold: to provide a play-based mathematics curriculum that builds on children's interests and provides for seamless, child-centred transitions, and to align the informal learning of EC with the more formal requirements of established mathematical transitions.

13.1 Introduction

This chapter will discuss New Zealand approaches to mathematics in early childhood settings and some interactions between early childhood and school settings. It will outline: the shift in early childhood delivery emphasis from a focus of care to

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the current one of education (and care); the development of a national curriculum, and related pre-service early childhood teacher education. The nature of the curriculum and its connections to school curricula and frameworks will be explored briefly. Then vignettes from a recent case study of the foundational or naïve (Wellman and Gelman 1992) mathematics that infants and toddlers (0–3 years) may gain through their play, are presented.

Internationally, many early years curricula encompass both prior-to-school (early childhood) and school education. In New Zealand (NZ) early childhood education serves children in education and care services from birth to school entry at (approximately) 5 years of age and is underpinned by *Te Whãriki* (Ministry of Education [MoE] 1996), the NZ curriculum framework for early childhood. The formal (school) sector is similarly underpinned by *The New Zealand Curriculum* [*NZC*] (MoE 2007).

Mathematics education in NZ for younger school students has a particular emphasis on the development of numeracy (number knowledge and understandings) as evidenced by the higher time allocation for work on number within the time allocated for mathematics (MoE 2007). The development of numeracy (mathematical progressions) is laid out in national framework—*NZC* and *The Numeracy Development Project* [*NDP*] (MoE 2008). The current emphasis on mathematical progressions has identified a potential for early childhood programmes to ensure that very young children are given opportunities to explore foundational or naive mathematics in a variety of ways. However, this is to ensure that there is an alignment of mathematical progressions rather than a preparation for transitioning to school mathematics and the way mathematics might be taught. Thus, the challenge for early childhood teachers is twofold: to provide a child-centred mathematics curriculum that builds on children's interests and provides for seamless, child-centred mathematical progressions, and to align the potential informal learning of mathematics in early childhood with the requirements of more formalised school curriculum mathematics, but not necessarily the way mathematical progressions are taught within schools.

In this chapter we use progression to refer to the development of mathematical ideas while transition is used to refer to shifts in the landscape and between delivery settings, sites and sectors where it is the adult teacher input that varies.

13.2 The New Zealand Early Childhood Landscape

Until the 1980s early childhood education in NZ was focused on children over 3 years of age, and was seen as appropriately occurring in state funded kindergartens or other 'preschool' settings. Alongside this approach childcare, from birth to 5 years of age, was seen as a separate service not necessarily involving education. The separation of care and education was challenged by major educational reforms of the 1980s (Te One 2003). This led to a more unified approach with the responsibility for the then existing early childhood providers of education and care services—

Kindergarten (teacher-led), Playcentre (volunteer parent-led) and home based care, changing from state welfare to state education (May 2001).

While historically children began attending kindergartens from age three (May 2001), from the 1970s increasing numbers of children were accessing early childhood services from infancy. Indeed, the attendance of children under 3 years of age increased by over 35 % from 2001 to 2006 (Rockel and Nyland 2007). The increase in demand for these services, and for greater access to services for older children, led to the development of new types of providers such as private for profit centres, community based centres and home-based (for small numbers of children) catering for children from birth to age five, and all with a primarily educative focus.

As more infants, toddlers and very young children attended all-day full-week education and care services in New Zealand (and internationally), many child advocates, parents and teachers became increasingly concerned that these services should offer high quality programmes and environments rather than just being primarily care facilities (Gonzalez-Mena and Widmeyer-Eyer 2004). The case was made that stimulating and appropriately educative environments, effective pedagogy, curriculum and care were vital in the education and nurturing of children (Hutchins and Sims 1999). Addressing these ongoing concerns underpinned the development of the first national early childhood curriculum framework document, *Te Whãriki* (MoE 1996).

13.3 The Development of the New Zealand Early Childhood Curriculum—*Te Whãriki* (MoE 1996)

The development of the *New Zealand Curriculum Framework [NZCF]*, (MoE 1993) sparked the notion of a national curriculum for early childhood education. This was initially as much a reaction to a possible downward curriculum push from the compulsory primary sector into early childhood as it was any particular desire for a guiding curriculum. The early childhood sector was strongly resistant to a large-scale importation of the primary school curriculum, its subject segregation and a presumed focus on whole class pedagogy. Indeed, early childhood academics argued at the time, that "*Te Whãriki* was developed as much to protect the interests of children before school as it was to promote and define a curriculum for early childhood education" (Carr and May 1997, p. 226). These authors argued, however, that the early childhood sector could gain status by ensuring clear links to aspects of the essential knowledge and skills of the *NZCF* and thus the strength to resist the predicted downward curriculum push. There were implications for both the primary and early childhood sectors in this development as the implementation of a curriculum framework for early childhood could have significant effects on the formal gathering of assessment information about children at age five (school entry) by schools.

Once a draft curriculum document had been developed a nation-wide consultation process was undertaken. Many early childhood teachers at the time were sceptical of the idea of a document outlining the complete curriculum for the sector (May 2001) and much of the discussion during the consultation phase was centred on ensuring that the diversity (the range of different services available and the right of parents to choose which service to use) of the early childhood sector would remain 'un-spoilt'. However, once those that were involved had engaged with the draft document, they were able to see the possibilities for child-centred holistic education and the value to children in having such a curriculum framework. The notion that this new curriculum had the potential to provide a framework against which to measure the quality of education and care being offered in early childhood settings was also a positive move to many in the sector.

Along with the traditional areas of play evident within most early childhood settings (sand, water, visual art, music, books, blocks, etc.), the notion of 'school curriculum subjects' (the seven essential learning areas: language; mathematics; science; social studies; technology; arts; and health and physical education) from the *NZCF* as the basis of learning experiences for children, developed. Teachers, teacher educators and parents involved in early childhood settings began to consider using traditional school curriculum subjects as a partial basis for child-centred curriculum (May 2001). However, while the idea of school curriculum subjects as important foci for development in early childhood settings has been debated widely, and the debate continues, the philosophy behind the *Te Whãriki* (MoE 1996) principle of holistic development within a child centred curriculum, in which school curriculum subjects and related content materials for learning experiences are integrated, remains the driving force in NZ early childhood education.

13.4 The Nature/Structure of the New Zealand Early Childhood Curriculum

Te Whãriki defines curriculum as including "the sum total of activities, and events, whether direct or indirect, which occur within an environment designed to foster children's learning and development." (MoE 1996, p. 10). Within this definition *Te Whãriki* exists as a framework underpinned by the socio-cultural theoretical ideas of Vygotsky (1978) and ecological theoretical ideas of Bronfenbrenner (1979) situating the child at the centre of the curriculum. In addition, teaching approaches and practices are closely aligned with the theory of social constructivism.

The foundations of *Te Whariki* consist of four philosophical principles: family and community; holistic development; empowerment; and relationships. At the next level, are five strands: wellbeing; belonging; communication; contribution, and exploration that state the focus and development expectations for all children prior-to-school in NZ each with a series of 'goals'. It has an overall approach to providing holistic experiences for children to develop from, rather than being taught formally in a more traditional didactic school model. The strands, however, do align with core school skills and learning areas (see Fig. 13.1).

Early Childhood: Te Whãriki **Strands**	**School: NZ Curriculum Framework** **Essential Numeracy Skills**	**School: NZ Curriculum Framework** **Essential Learning Area: Mathematics**
Well-being The health & well-being of the child are protected and nurtured.	Children develop competence in mathematical concepts & using them in daily life	Exploring mathematical concepts encourages creativity, perseverance, and self confidence
Belonging Children & their families feel a sense of belonging.	Children learn to use numbers in relation to family members, children in a group, and ordering the environment in patterns & relationships	Mathematical concepts are used in practical family & social contexts, such as remembering telephone numbers, street numbers and birth dates.
Contribution Opportunities for learning are equitable, & each child's contribution is valued.	Children learn to use number to monitor fair division of resources & equitable sharing of efforts toward a common goal.	Children develop mathematical problem-solving strategies in, for instance, sharing & dividing resources, turn taking, & estimating times.
Communication The languages and symbols of their own& other cultures are promoted and protected.	Children have fun with numbers & begin to understand & respond to information presented in mathematical ways.	Development of mathematical vocabulary to help children communicate complex ideas such as weight, shape, and volume.
Exploration The child learns through active exploration of the environment.	In exploring their world children find reasons to calculate & estimate with increasing accuracy & to use measuring instruments & mathematical concepts.	Children develop & use mathematical concepts when they collect, organise, compare and interpret different objects & materials.

Fig. 13.1 Links between *Te Whãriki* strands and the essential skill and learning areas of the NZ curriculum framework. (MoE 1996, pp. 16–17; MoE 2007, pp. 94–98)

While the connections to numeracy and mathematics generally are stated for each strand (see Fig. 13.1) it is mainly in the communication strand of *Te Whãriki*, for "Goal 3: Children experience an environment where they experience the stories and symbols of their own and other cultures" (MoE 1996, p. 78) where specific numeracy and mathematics learning objectives and activities are focussed. For example, "familiarity with numbers and their uses by exploring and observing the use of numbers in activities that have meaning and purpose for children" (p. 78). The inference here being that teachers in early childhood settings should be providing opportunities for children to engage in meaningful experiences with numbers through child centred experiences and activities.

Effective pedagogical approaches for children in early childhood education are not solely viewed as the achievement of tangible outcomes but more importantly as the formation of learning through engagement in experiences that develop knowledge and skills. That is, engagement is often seen as more valuable than the specific outcomes or products of children's experiences (MoE 1996). Thus, suitable dispositions for engagement and therefore learning are important elements of the curriculum (Carr 2006). Dispositions for learning that are emphasised in *Te Whãriki* are: taking an interest; involvement; persisting with challenge or difficulty; expressing ideas or feelings, and taking responsibility. With this focus on dispositions as tools for learning, children's subject content knowledge is threaded throughout child-centred experience particularly at the infant and toddler levels.

13.5 Cross Curricula Links

The school curricula redevelopment in the1990s mainly occurred prior to the publication of *Te Whãriki* in 1996, for example, *Mathematics in the New Zealand Curriculum* (MoE 1992) and *The New Zealand Curriculum Framework* (MoE 1993). There was a vision of a seamless education system, and thus an implied continuity (MoE 1994). There were, however, no explicit links in the specific school curricula documents of the seven 'essential learning areas' or their 'essential skills', to *Te Whãriki*. However, *Te Whãriki* had explicit links to the NZ curricula from its beginning (Haynes 1999; Lee 2010; Peters 2000), outlining the connections between its five strands, the essential learning areas and essential skills of the school curriculum (see Fig. 13.1). Additionally, *Te Whãriki* argues for a seamless developmental continuum across the early childhood/school divide as "Children moving from early childhood settings to the early years of school are likely to have extensive prior learning and experience which provide starting points for further learning" (MoE 1996, p. 83) and the possibility of the use of *Te Whãriki* with *NZCF* as "There is no developmental cut-off at school entry age. During the early school years, the principles and strands of the early childhood curriculum continue to apply and can be interwoven with those of the NZ curriculum statements for schools." (p. 21). There were, however, no parallel statements in *NZCF* or the individual school curriculum documents suggesting such potential ways of using the early childhood curriculum in unison with school curricula.

It was not until 2007 that the revised NZ curriculum (MoE 2007) was explicitly linked to *Te Whãriki*. There were brief comments on the structure of *Te Whãriki*, its strands, relationship to the key competencies of the revised school curriculum, and that early childhood experience may have some links with school learning: "This new [school] stage in children's learning builds upon and makes connections with early childhood learning and experience." (MoE 1996, p. 41). The extent to which these inclusions have had any impact on primary school teachers overall or on junior school teachers in particular is open to question given the extent of changes elsewhere in the compulsory school curriculum. In mathematics generally, however, there was a much more explicit linking in the *Best Evidence Synthesis* work (an academic review of the international and NZ literature on current and best practice in mathematics teaching) of Anthony and Walshaw (2007) where there is a chapter on mathematics education in the early years. But again the impact on school teachers of this non-compulsory evidential document is open to question.

Infant (0 to 18 months)	**Toddler (12 months to 3 years)**	**Young child (2.5 years to school entry)**
Numbers are used in conversation and interactive times, such as in finger games. Everyday number patterns are talked about, for example, two shoes, four wheels, five fingers.	Adults' conversations with toddlers are rich in number ideas, so that adults extend toddlers' talk about numbers. Adults model the process of counting to solve everyday problems, for example, by asking "How many children want to go on a walk?"	Adults comment on numerical symbols which are used every day, such as calendars, clocks, and page numbers in books. Children have opportunities to develop early mathematical concepts, such as volume, quantity, measurement, classifying, matching, and perceiving patterns.

Fig. 13.2 Early childhood age groups in *Te Whãriki* and examples of possible mathematical focii. (MoE 1996, p. 79)

Anthony and Walshaw (2007) acknowledge the existence and importance of mathematics in early childhood. They accept Pound's (1999) argument for mathematical competencies beginning at birth in which she claims that "in the early months of life, [babies] are busy learning about mathematics" (p. 3) to enable them to fit into the community environment that surrounds them. This is further supported by Ginsburg et al.'s (1998) position that a child's development in their first 5 years is of intrinsic importance as both a time for growing and learning prior to, and independent from, school learning, and Perry and Dockett's (2004) view that children's development, including mathematics, during these years is holistic and occurs within three domains—physical, socio-cultural and cognitive. These positions are consistent with those of *Te Whãriki*, as is the model of shared learning with children's control as the starting point for learning alongside a teacher (adult) role in supporting such learning (Gifford 2005). These authors, alongside the theoretical ideas and the underpinning values of *Te Whãriki*, strengthen the implication that there are mathematical needs of infants, toddlers and young children that require addressing by early childhood teachers in centres

The numeracy examples provided in *Te Whãriki* (MoE 1996) are linked to goals and learning outcomes and discuss ways in which adults can provide experiences for children to: hear and see numbers; count; explore numerical patterns; use mathematical tools for the intended purpose; use the language of measurement, and engage in many other ideas for mathematical play. These examples provide early childhood teachers with ideas on which to base their teaching with children who exhibit an interest in mathematical exploration. For each 'age group' infant (0–18 months), toddler (12 months–3 years) and young child (2–5 years), *Te Whãriki* provides specific examples of possible learning experiences for implementing with children (see Fig. 13.2) in an age group and depending on their stage of development and interest. As always, however, all these examples are applicable throughout the years of early childhood and should be linked to and build upon individual children's previous learning experiences. For example, the continuity of mathematical conversations with significant adult input is a fundamental aspect across all groups (see Fig. 13.2).

13.6 Some Implications of *Te Whãriki's* Introduction

Since the introduction of *Te Whãriki* early childhood teachers have supported the move from a more traditional developmental early childhood practice which tended to dominate western curriculum models (May 2001), with a significant focus on human development and care, to the more educative ethos of *Te Whãriki*.

In particular this move emphasised the need for enhancing teacher curriculum subject content knowledge, including mathematics, as a way of improving teachers capacity (in both content and pedagogy) to assist children's mathematics learning (MoE 2000). In order to ensure this, teachers need to have learning experiences that develop their own curriculum subject content knowledge and make explicit possible pedagogical approaches. This can then be drawn upon in their interactions with children so as to maximise the learning potential of play experiences.

The MoE parental education campaign, *Feed the mind* (MoE 1999) also served to promote this focus on curriculum subject learning, via a child centered approach (mostly through 'play', including games), throughout the wider community via television, radio and printed materials (MoE 1999).

13.7 Foundational Mathematical Knowledge

Early childhood teachers, who understand and articulate the importance of children's play and create an environment that has potential for hands-on discovery, provide opportunities for children to lay the foundations of their mathematical understanding. This notion of children constructing their own knowledge is espoused clearly within the reflective questions for teachers in *Te Whãriki.* For example, "In what ways and for what purposes, do children see mathematics being used, and how does this influence their interest and ability in mathematics?" (MoE 1996, p. 78).

In a holistic and integrated curriculum, the potential mathematical learning of infants and toddlers is not situated discretely; rather, it occurs while they engage in activities and experiences that have meaning for them, which is most often in their play. Considering curriculum in this manner positions all subject domain knowledge in a similar way. While the child's focus in play may be on their artwork or building their sandcastle, they are potentially exploring (with the input of knowledgeable teachers) foundational conceptual understanding in a wide range of subject domains. For example, scientific concepts are explored while the child tips and pours sand; she is exploring the inherent properties of this medium. Alongside this she is exploring concepts of space and weight, and may be considering ways to move the sand from one place to another. This foundational, conceptual understanding by infants and toddlers can be described as the precursor to formal academic learning. It is the simple, 'naïve' understanding that is built through experience (Wellman and Gelman 1992) and is what 'toddlerhood' is all about. This period of rapid social, emotional and cognitive development (MoE 1996) is the time where toddlers are best engaged in play experiences as the primary form of learning (Langston and Abbott 2005).

13.8 The Case Study

A recent NZ case study explored the ways that children aged between 12 months and 3 years of age (toddlers) engaged in outdoor play that encompassed mathematical understanding and skill. The 32 participants were toddlers attending a single early education and care setting in a high socio-economic area of urban Auckland.

This study was underpinned by socio-cultural theory within a qualitative, interpretivist methodology and the data collection methods used included video recording and field notes. Infants' and toddlers' outdoor play experiences were analysed and interpreted in terms of the mathematical aspects and seven mathematical categories were identified aligned with the *NZC* (MoE 2007) and *Te Whāriki* (MoE 1996).

This study of unstructured outdoor play experiences resulted in three major categories of mathematical foundational knowledge and skill being observed; Space; number, and measurement. Four further categories were observed to a lesser, but still significant, extent; pattern; shape; classification and problem solving (Lee 2010).

Play experiences involving concepts of space, particularly those that illustrated the use of the body and movement of the body within space, were highlighted in this study. Other observations of children exploring space showed how they were able to fit objects, such as blocks and duplo, together, take them apart and then reassemble them.

Toddlers' experiences incorporating number concepts were also present in this study, perhaps as a result of conversations, experiences, songs, stories and rhymes that occur daily in most children's lives. One of the interesting factors to note was the competent use of verbal numerical language in most of the observations of those children who had attained verbal language skills.

Another frequent category that arose in the children's play involved concepts of measurement. In contrast to the examples of number concepts, not all of these included the use of verbal language and therefore were often embedded in the infants' and toddlers' play behaviour rather than within their verbal interactions.

Three further categories of mathematical conceptual knowledge were less evident in this study: pattern; shape and classification. Pattern-making skills were seen in only two ways: repeated actions of behaviour and repetitive singing of a popular song. Understanding of shape and concepts surrounding shape, and ideas and understanding surrounding classification occurred both verbally and non-verbally, indicative of the range of language and physical development. In addition to specific mathematical knowledge, examples of problem solving were evident in many of the observations. These findings provide clear evidence of 'naïve' (Wellman and Gelman 1992) mathematical learning further demonstrating the claim that mathematical competency begins at birth (Anthony and Walshaw 2007) and further develops through play experiences.

In addition to the development of skills and language in mathematical domains, albeit 'naïve', the natural progression of mathematical understanding can be situated in the play experiences children engage in. Within *NDP*, the school based nationwide initiative for increased knowledge and skill in number concepts, and *Te Whariki*, there is further supporting evidence that the progression from naive knowledge to tacit understanding occurs at the youngest levels and is built upon through targeted and individualised experiences with young children.

13.9 Numeracy in Early Childhood Settings and in the Numeracy Development Projects

The development of numeracy as a component of early childhood education is undoubtedly seen as a valid and important aspect of all young children's learning. Indeed, the ministry's policy document *Literacy and Numeracy Strategy* (MoE 2000), which provided a focus for government funding in schools, stresses the important nature of early learning experiences as a catalyst for high levels of success in future numeracy. In addition, the *NDP* resource books have acknowledged throughout that "Although the groundwork is laid in mathematics … in addition, … early childhood settings, … assist in the development of numeracy" (MoE 2008, p. 0). However, this is only a brief note on the inside cover of the resource books rather than being positioned as a significant element for consideration by teachers in schools.

Te Whãriki's general descriptions of mathematics and the more formal 'stage' descriptions of NDP are compatible. For example, in *Te Whãriki* there is an emphasis on classifying and matching, which can be seen in the NDP stage one description of matching things in one to one correspondence.

Despite the lack of formal mathematics learning connection between sector documents children learn mathematics in early childhood. Indeed, there is clear evidence that many children tested at school entry have knowledge and skills above the first stages of the *NDP* numeracy knowledge and strategy frameworks (see Fig. 13.3).

The children's knowledge and skills, and the variation in them, reflect the range and varying intensities of children's individual prior-to-school experiences (and the natural variation in children's intelligence).

Because of variations in early childhood teachers' numeracy knowledge (Babbington and Lomas 2008) and the variation in qualifications and the mathematics content addressed within them, alongside the diverse range of providers, New Zealand's young children are not being uniformly exposed to numeracy experiences in early childhood settings. It is argued that if teachers do not have an up-to-date knowledge of *NDP* (MoE 2008) style numeracy they are unlikely to be in a position to achieve the alignment necessary to support children's interest positively and productively. The nature of the early childhood qualification gained (and any subsequent professional development) will have had an impact on the extent of a teachers mathematical and numeracy knowledge (Lee 2010).

Number knowledge framework	Stage and descriptor	Early NP 2002 (n=5491)	Early NP 2003 (n=10101)	Early NP 2004 (n=7793)
Forward Number Word Sequence Stages	0: Emergent	12	11	11
	1: 1 up to 10	29	30	28
	2: 1 to 10	31	35	33
	3: 1 to 20	19	18	20
	4: 1 to 100	9	6	8
	5: 1 to 1000	0	0	1
Backward Number Word Sequence Stages	0: Emergent	37	41	38
	1: Initial to 1	33	23	21
	2: 10 to 1	26	28	30
	3: 20 to 1	9	5	7
	4: 100 to 1	4	3	4
	5: 1000 to 1	0	0	1
Numeral Identification Stages	0: Emergent	24	30	2
	1: 1 to 10	37	37	28
	2: 1 to 20	18	16	36
	3: 1 to 100	17	14	17
	4: 1 to 1000	3	2	15
Strategy framework stages	0: Emergent	26	16	16
'Highest' strategy stage evident	1: One to one counting	31	28	30
	2: Counting from 1 on materials	33	47	44
	3: Counting from 1 by imaging	6	6	9
	4: Advanced counting	4	2	2

Fig. 13.3 Initial evaluations of 5 year olds on entry to school using *NDP* assessment tools showing percentages (rounded to the nearest whole number) achieving stages/levels. (MoE 2002, pp. 17, 20–21; MoE 2003, pp. 10, 18–19; MoE 2004, pp. 10, 18, 20–21)

As only some early childhood degree programmes have school subject focussed courses the exposure to up-to-date mathematics and numeracy among early childhood teachers is quite limited on their entry to the workforce. In addition, professional development opportunities in centres for teachers vary widely depending on the provider institution. Some providers have provided opportunities around numeracy for many of their staff and this was usually well received and aspects incorporated into teachers' and centre practice. So, while other teachers may be able to draw upon their own idiosyncratic personal knowledge this may not be adequate, accurate, or aligned with the new *NDP* frameworks.

The *NDP* were developed as part of the *Literacy and Numeracy Strategy* (MoE 2000) to meet the identified needs of NZ children in an apparent lack of numeracy skills and general number sense, by increasing the professional capability of teachers. The basis of *NDP* is a framework of number strategy skills and one of subject knowledge (accompanied by a strategy teaching model) that children are facilitated through to increase their numeracy competency. It was implemented within changed school guidelines (MoE 2000) which required a priority focus on literacy and numeracy particularly in the first 4 years of schooling (Education Review Office 2000). This was reinforced in the Minister of Education's covering letter accompanying the release of a revised school curriculum, and the greater time allocation assigned to numeracy within the mathematics guidelines (MoE 2007). There was, however, no similar official change (or numeracy development project) for the early childhood sector, (possibly because it was not a compulsory sector, and there was no compulsory early childhood curriculum at this stage). However, some teacher education programmes for early childhood teachers do include *NDP* as part of curriculum courses (e.g., The University of Auckland, Bachelor of Education (Teaching)).

In the 2000s, school based *NDP* explicitly promoted small group work and working with individual children as a major and integral part of classroom teaching (the strategy teaching model) alongside ongoing formative assessment on where individual children were at and 'teaching' in response to the child's identified needs. This represented a significant shift from whole class teaching, which was seen as the antithesis of child-centred (child-initiated) pedagogy by the majority of early childhood teachers. The extent to which early childhood teachers would have been aware of this potential change is probably minimal due to lack of contact, both formal (curriculum and support documents) and informal (individual or institutional connections) between the sectors (Lee et al. 2008).

Such changes in school teaching approaches may have narrowed the gap between early childhood and school approaches with a greater focus on where the child is at in schools as a starting point for planning learning pathways for each child. What it did not address was the change from a child initiated learning interest (in early childhood) to a teacher directed and activated learning focus (at school). The teacher in setting the learning focus needs to generate and enhance the child's interest and motivate engagement as a replacement for a child's intrinsic interest and motivation in child-centred early childhood settings.

13.10 Summarising Transitions in the Early Childhood Sector

The early childhood sector in NZ has gone through a number of transitions: from a mainly care ethos to a more educative one; from a small number of nationwide providers to a range of diverse providers (national and local); from a situation of limited early childhood teacher education (mainly linked to the larger national providers) to a range of provision of certificate, diplomas and degree level qualifications; from primarily implicit provider-based curricula to having a single nationally mandated curriculum with a holistic child-centred philosophy underpinned by socio-cultural theory; from approaches and content that were quite separate from school curricula to more explicit, formal links with school curricula and the subject matter of the seven essential learning areas, and the increased possibility of using some school subject curriculum content material (e.g., the early *NDP* stages focus and activities) as an integral part of early childhood activities.

13.11 Mathematics in Early Childhood Experiences

In early childhood education in NZ, mathematics is not taught as a separate curriculum subject. Rather, the mathematics (and other) subject domain knowledge and development is integrated throughout child centered (play and care) experiences at the infant and toddler levels. This section will present a series of vignettes, from a case study of infant and toddler's mathematical explorations (Lee 2010), to illustrate the mathematical knowledge and understandings within the infants and toddlers age groupings of *Te Whãriki*.

The first set of vignettes, give examples of the range of children's (developing) knowledge and skill in the number domain.

> Steven [1 year 8 months] went into a playroom and brought out two small trucks, which he added to the two trucks he already had been playing with. "Look, more", he said, showing the teacher two new ones. He took these to the small ramp that had been set up on the obstacle course and let them go at the top. He watched carefully as each of the four trucks went down the ramp. He repeated this with all four vehicles, smiling broadly.

> One of the teachers had placed a large water cooler bottle filled with ping-pong balls outside. Steven had picked it up and was shaking it vigorously, then banging it on the ground upside down in order to encourage the balls to come out. Suddenly one popped out. Laughing, Steven continued to shake the bottle shouting "More, more".

Although Steven was not counting the number of cars or balls aloud, he stated that there were more in each of the sets he formed. His enjoyment in succeeding in his task to get the balls out of the bottle was evident. He was confident in his ability to attempt different strategies to remove the balls from the bottle and showed his understanding of the concept of 'more' in each group—cars and balls.

> Conor [1 year 9 months] is sitting in the sandpit and has filled a muffin tray with sand. He counts as he pushes his finger into each muffin hole: "three, six, seben".

In this example, Conor showed some knowledge of the forward word number sequence, and although not accurate, was clearly indicative of this child's knowledge that objects can be counted and that six comes after three and seven comes after six. Other toddlers showed a more complete command of the forward word number sequence, with no gaps or pauses, and in the correct order as they use counting as a timing measure. For example,

> Fraser [2 years 10 months] and Gene [2 years 5 months] stand on a large wooden box watching a balloon from the house next-door shoot across the playground. "One, two, three, four, five, six" shouts Fraser.

Counting also seemed to give some of the older toddlers elements of control over aspects of their play environment. This often occurred when children counted to a certain number before taking action. For example, Aiden repeatedly counted to three before continuing his desired play action.

> Aiden [2 years 6 months] was on the deck area holding a large plastic hula hoop. Aiden drops his hoop on the ground in front of him, pauses, then counts "one, two, three," then steps into the centre of the hoop and spins right holding onto his shark soft toy.

A further example of an older toddler's skill in rote counting was seen on a rope swing. The forward number word sequence from one to ten was used before the child let herself swing. Then a backward number word sequence, from four to one, was used before the third swing, clearly showing the use of counting as a measure of control;

> Anne [2 years 7 months] is on the rope swing and as she goes to swing she counts forwards "1, 2, 3, 4, 5, 6, (pauses) 6, 7, 8, 9, 10" swings then repeats counting 1–10 before swinging again. Before the third swing Anne says "the 4, the 3, the 2, the 1".

Quantification (or cardinal knowledge) was also evident in toddlers' play episodes. The following example shows how Trent correctly identified the number of objects in a group, classified them as the same object, and verbalised the correct word for the number of objects.

> Trent [2 years] in the sandpit picks up a small sand scoop, uses it to dig in the sand once then picks up a second scoop. He holds the two scoops, one in each hand, digs them both into the sand and then tips the sand into a carton, he holds the two scoops up and states "two".

The following vignettes (Lee 2010) give some examples of children's developing knowledge and skill in the measurement domain.

> The teachers had placed two balloons onto long elastic bands hanging from the awning so that one hung down lower than the other. Trent [1 year] noticed the higher one and reached up to it, but it was too high. He stood and watched for a moment, then went and brought over a nearby chair. Trent climbed onto the chair, reached the balloon, and stepped down.

The knowledge of measurement and manipulation of space Trent showed here, exemplifies his understanding that obtaining and standing upon the chair would enable him to reach the higher balloon.

> Ryder [1 year 3 months] had a small sand scoop and was holding it in both hands with the scoop pointing down. He lifted the handle and placed it in his mouth, then he pushed it into the sand, shaking his hands as if to shake off the sticky sand. He then picked up handfuls of sand and sprinkled an equitable amount of it onto his socks, hands, and feet. He picked up the sand trowel and with the point in the sand rotated the handle using both hands. Ryder shared the sand almost equally between his feet.

This exploration of weight and volume shows Ryder's ability to think logically and offer possible solutions to his intention (or problem). He seemed to understand that each handful would contain approximately the same amount of sand.

> Kyle [2 years 6 months] has on a blue lycra dress-up dress. "Off", he says a teacher helped him to remove it and handed it to him. He placed it over the teachers head, laughing and said, "Too big!".

Again this is an example of Kyle's developing measurement knowledge—he articulated to the adult that he knew the adult was to big to fit the dress.

As can be seen by the examples provided in both the Number and Measurement domains our youngest citizens are capable of exploring a wide variety of mathematical ideas. However, the 'formalisation', and further development of these, requires input from knowledgeable adults (van Oers 2010, 2012).

13.12 Conclusion

No one can deny that mathematics is important but it is often only noticed and responded to with regards to older, school aged, children. The progressions in mathematical knowledge and understanding that occur from infancy through toddlerhood and onto the young child stage (and beyond) can be observed from a very young age. This chapter has shown that this time of rapid physical and cognitive growth provides opportunities for child-centred (play-based), age appropriate, numeracy and general mathematical development and progression. Further research into what is occurring in early childhood centres both in NZ and internationally will add to the limited but growing body of literature around mathematics and other curriculum subject areas in early childhood.

NZ approaches to mathematics in early childhood education centres (and in home environments) vary widely, but the numeracy data collected at school entry indicate that early childhood teachers are providing effective support for mathematics learning within the holistic curriculum of *Te Whãriki* for many children. The challenge for us all is to increase the opportunities for acquiring mathematical knowledge and skills for all children irrespective of centre and in a manner that supports both the individuality and uniqueness of each child, and that the philosophy and principles of *Te Whãriki* are upheld. This will require professional development for early childhood teachers in mathematics curriculum subject knowledge to enhance their ability to provide experiences and guide children's explorations.

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Chapter 14
The Impact of a Patterns and Early Algebra Program on Children in Transition to School in Australian Indigenous Communities

Marina M. Papic, Joanne T. Mulligan, Kate Highfield, Judith McKay-Tempest and Deborah Garrett

Abstract This chapter describes a 3-year early numeracy project conducted with 15 Australian Aboriginal Community Children's Services across the state of New South Wales and the Australian Capital Territory. The project involved 66 early childhood educators and 255 children aged 4–5 years in the year prior to formal school. The children were engaged in a preschool Patterns and Early Algebra program previously developed and trialed with young children. Following an interview-based assessment, the Early Mathematical Patterning Assessment (EMPA), educators implemented a 12 week intensive program based on early patterning frameworks that developed young children's early algebraic and mathematical reasoning, communication and problem-solving skills. Data from children's progression on the frameworks and interview data from primary school teachers in the year following implementation, provided evidence of the impact of the program not only on children's mathematics learning but on their transition to school.

14.1 Introduction

A review of research on early mathematics learning found that children from low income families, on average, demonstrate lower levels of competence with mathematics prior to school entry and the gaps persist or even widen over the course of schooling. Providing young children with extensive, high-quality early mathematics instruction can serve as a sound foundation for later learning in mathematics and contribute to addressing long-term systematic inequities in educational outcomes

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(National Academy of Sciences 2009). Further research highlights the importance of the transition to school period and the long-term impact on children's school success that effective transition to school can have (Duncan et al. 2007). Supporting children and families in the years before formal schooling greatly increases their chances of a successful transition to school as well as better learning outcomes while at school. However, there is much evidence to suggest that there is inequity of educational outcomes for Aboriginal children in Australia (Frigo et al. 2004). Further, there is a paucity of focused and longitudinal research into the educational outcomes for Indigenous students. Perry and Dockett's (2004) review calls for studies of successful approaches to the mathematics education of young Indigenous students and Mellor and Corrigan (2004) argue for more qualitative and case study research along with more rigorous evaluation of indicators by quantified research.

The Australian Government is committed to Closing the Gap in Indigenous disadvantage "to improve the lives of Indigenous Australians, and in particular provide a better future for Indigenous children" (Australian Government Department of Social Services 2013). The *Patterns and Early Algebra Preschool (PEAP) Professional Development (PD) Program* focuses on 'closing the gap' in numeracy achievement as children begin to transition to school. Although the program's goal is to work towards closing the gap in numeracy achievement by advancing children's early algebraic and mathematical reasoning skills, the program also aims to develop many of the skills required for effective transition to school including strong, responsive relationships with adults, and social skills such as taking turns, persistence, problem solving and effective communication skills.

14.2 Background

Australian Indigenous children aged 0–14 years make up approximately 40 % of the Australian Indigenous population. Literacy and numeracy data for Indigenous students are consistently below the national average, especially in remote areas. Australian national statistics highlight the unacceptable levels of disadvantage faced by Indigenous Australians in living standards, life-expectancy, education, health and employment (Australian Government 2009). Australian Indigenous children are less likely to participate in preschool programs than non-Indigenous children and they have higher rates of absenteeism in elementary school (Frigo et al. 2004).

In 2011 the Council of Australian Governments endorsed the Aboriginal and Torres Strait Islander Education Action Plan 2011–2014. This document commits both state and federal governments to 'closing the gap' between Indigenous and non-Indigenous Australian children. Outlining six priority areas, this commitment highlights the importance of quality education in prior to school settings and seeks to "establish culturally appropriate and quality early learning" (Ministerial Council for Education, Early Childhood Development and Youth Affairs (MCEECDYA) 2011, p. 9). In part these initiatives aim to promote effective literacy and numeracy programs in Indigenous prior to school settings. Further, these goals aim to increase access to quality early childhood education and increase long-term educational outcomes for Indigenous Australians. The development and evaluation of appropriate programs to promote literacy and numeracy for Indigenous children must be aligned with initiatives to support professionals within their communities (Perso 2009). Raising professional and community expectations of young Indigenous learners and access to effective professional development programs and resources are critical to achieving this goal. The PEAP PD project builds upon current research (Jorgensen et al. 2009; Warren and deVries 2010) focused on improving mathematics learning opportunities for Indigenous children as well as supporting Indigenous early childhood professionals.

14.2.1 Teaching Mathematics in the Context of Indigenous Learners

There have been a significant number of studies internationally of Culturally Responsive Mathematics Pedagogy (CRMP) (Gay 2000), but few studies have examined transition to school settings in Australian Indigenous contexts. Studies conducted in remote contexts have often lacked a connection to mathematics learning in the school system and are fraught with problems associated with adapting western approaches to teaching and learning mathematics (Morris and Matthews 2011). Further there are difficulties associated with mapping Aboriginal languages to western ways of learning mathematics (Warren and Miller 2013). Teacher knowledge, confidence and approach to promoting numeracy for Indigenous students must take into account children's culture, and their mathematical understandings (Perso 2009). However there has been insufficient sustained investigation and little consensus about what effective teaching of mathematics might be in the widely varying contexts of Indigenous learners and in transition to school.

Morris and Matthews (2011) draw on the notion of CRMP and the implications for educators working with Indigenous students in learning western mathematics. Through the *Make it Count: Numeracy, Mathematics and Indigenous Learners* program (Australian Association of Mathematics Teachers (AAMT) 2009) across eight school clusters, Morris and Matthews (2011) describe the "significant and transformative change in teachers and the curriculum" (p. 29). An important feature of *Make it Count* was the collaboration between research teams and professional learning communities at the ground level to construct and review strategies, resources and approaches. Development of an evidence base was central to this approach. Cultural competence of teachers in schools was considered in relation to mathematics and numeracy. The outcomes of the study showed that teachers who engaged in CRMP possessed deep content knowledge, strong pedagogical content knowledge and culturally relevant pedagogy, as well as a commitment to students (Morris and Matthews 2011, p. 33).

The importance of CRMP was highlighted in a study of early childhood professionals in the *Mathematical Thinking of Preschool Children in Rural and Regional Australia* project (Papic et al. 2010). Nineteen Indigenous early childhood professionals described their understandings and beliefs about young children's learning, including mathematics, which they saw as inextricably linked to Indigenous peoples' place in their community. Two other issues emerged from interviews with the early childhood professionals: the need for community-based mentors and local support from community-based Indigenous personnel. The study highlighted that an understanding of traditional ways and raising expectations of young learners in the community could not be appreciated without living and learning within their communities.

14.2.2 Research on Early Mathematics Learning and Professional Development Initiatives in Indigenous Communities

Several current research projects have focused on improving mathematics learning opportunities for young Indigenous children and supporting professional learning (Papic 2013; Warren and deVries 2010). These projects have focused on research-based pedagogy that supports Indigenous students' learning and engages the community. Projects such as *The Count Me In Too Indigenous program* (NSW Department of Education & Training 2003), *Make It Count: Numeracy, Mathematics and Indigenous learners* (Howard et al. 2011) and *Maths in the Kimberley* (Jorgensen et al. 2009*)* focused on the school years, and engaged with Indigenous communities to develop appropriate strategies for improving participation and success.

A broad longitudinal project for Indigenous children*, Representations, oral language and engagement in Mathematics* (*RoleM*), focuses on closing the gap in numeracy for Indigenous students in the first 4 years of schooling (Warren and Miller 2013). The project supports collaboration between the local community, students, parents and Indigenous education workers from diverse communities in urban, rural and remote contexts in the state of Queensland, Australia. The students' understanding of mathematics and how it is mapped onto Aboriginal English informs the development of learning tasks that blend Western mathematical knowledge with culturally-driven ideas. The study focuses on children's mathematical representations and mathematical language through tasks reflecting core concepts in the *Australian Curriculum—Mathematics* and an emphasis on patterns and structure (Papic et al. 2011; Mulligan and Mitchelmore 2012). Although an understanding of mathematical language was found to be a stronger predictor of success than the ability to pattern, it was found that children who could identify a *unit of repeat* scored higher than other students on the general mathematics component. This was consistent with Papic's (2007) earlier work. The study supports the pattern and structure approach to mathematical learning in combination with emphasis on oral language. Importantly it engages teachers in mathematics learning activities "to scaffold and encourage students as they explored their own thinking and made schematic connections" (Warren and Miller 2013, p. 167).

14.3 The Current Study

At an international level, a growing number of studies have investigated children's early learning about mathematical structure and pre-algebraic reasoning (Blanton and Kaput 2005; Carraher et al. 2006; Papic et al. 2011; Swoboda and Tatsis 2009). In particular, a focus on patterning and spatial structure has shifted attention beyond early numeracy to structural development (van Nes and de Lange 2007). There is also increasing evidence that early algebraic thinking develops from the ability to see and represent patterns and relationships (Papic et al. 2011) such as equivalence and functional thinking in early childhood (Warren and Cooper 2008).

Based on the emerging research on young children's early algebraic thinking, an early mathematical patterning assessment (EMPA) and aligned professional development program was developed for the prior to school years, the Patterns and Early Algebra Preschool (PEAP) Professional Development (PD) program (Papic et al. 2011). This was later trialed with both Indigenous and non-Indigenous communities across the state of New South Wales (Papic 2013). In its early development the program focused on an instructional framework based on patterning tasks (Papic and Mulligan 2007). The program that followed was based on this framework but incorporated culturally appropriate resources and materials, pedagogies that involved the use of both concrete and abstract imagery in both play-based and intentional learning environments. Central to this approach was teachers' high expectations of children.

The PEAP PD program focuses on developing young children's awareness of pattern and structure in order to promote structural development, relational understanding and generalisation, albeit emergent, from an early age with a view to laying the foundation for mathematical thinking (Papic et al. 2011). Papic's early studies of preschoolers' patterning investigations highlighted the critical importance of understanding 'unit of repeat' in developing mathematical concepts (Papic and Mulligan 2007). A suite of studies with 5–6 year olds, (Mulligan et al. 2013) showed that the scaffolding of structured tasks over time can significantly advance the development of such mathematical processes as patterning and unitising, spatial structuring, multiplicative reasoning, and pre-algebraic reasoning.

The program employed a community-based approach to learning by utilising existing positive relationships developed between researchers and educators within the selected early childhood centres. The early childhood educators participated in the program enabling new pedagogical and content knowledge to be developed. They were able to extend mathematical thinking through problem solving and questioning in ways that they had not considered previously:

> It is imperative that teachers 'listen to' their children not 'listen for' an anticipated response. In having a genuine interest in what children do, teachers can extend children's learning through appropriate and contextual dialogue rather than getting them to say and do 'what is expected'. (Papic 2013, p. 262)

This chapter provides an overview of the PEAP PD program and its success in developing young Aboriginal children's mathematical thinking and reasoning skills and usefulness in developing skills for effective transition to school.

14.4 Method

14.4.1 Participants

Fourteen Aboriginal-controlled childcare services and two privately operated services with a high percentage of enrolments from Aboriginal families were invited to participate in the study. All 16 services, supported by Gowrie, Indigenous Professional Support Unit (IPSU), had engaged in culturally-appropriate numeracy workshops led by the first author. Gowrie IPSU is an Australian Government funded organisation established to support staff to ensure that all Indigenous children attending eligible Indigenous childcare services have access to high quality care. IPSU provides professional development through mentoring, advice, support, referral and training. A collaborative partnership with Gowrie IPSU, over a period of 6 years, enabled a relationship to be established with these services. Fifteen of the 16 services agreed to participate in the PEAP PD project: seven in 2011–2012 and eight in 2012–2013. All 15 services were retained for the duration of the project.

The Aboriginal-controlled early childhood services provide more than just early childhood education. They provide access to health assessments and specialist services such as speech pathology, as well as family support. Eighty-five percent of the staff and children from these services identified as Indigenous. The privately operated services had a high percentage of enrolments from Aboriginal families (50–60 %). All educators working with 4–5 year old children in each of the services collaborated as a team with the support of the researchers and two Gowrie IPSU staff, one of whom was an Aboriginal person. The 66 educators comprised eight untrained staff, 16 University qualified early childhood teachers and 42 educators with entry level early childhood qualifications such as Certificates or Diplomas.

Two-hundred and fifty-five children aged between 4 and 5 years were selected and comprised two cohorts: 125 in 2011 and 130 in 2012. These cohorts were followed up in the first term of their first year of formal schooling. Of the initial cohort 56 were retained in the follow up year. This was a smaller sample than desired because of difficulties in tracking children and permission not being granted by all school authorities or families. The intended enrolment for Primary School (beginning when children are 5 years) was identified from the outset by the parents/caregiver and consent was obtained at the commencement of the program to allow follow up data to be collected. To ensure informed consent, an Indigenous early childhood educator or director liaised with the parent/caregiver to explain the research project. Primary school teachers agreed to be interviewed by phone and school leaders agreed to provide Best Start Numeracy (NSW Department of Education & Communities 2009) results where available.

14.4.2 Context

The 15 early childhood centres are spread across 12 rural, regional and inner city communities across NSW and one remote area of the ACT. The services are predominantly in low socio-economic communities within public housing estates and/or on old Aboriginal reserves, usually located on the outskirts of towns. Some of the children accessing the services are from single parent families, living with other family members or are in foster care (not necessarily with an Aboriginal family) or with families that have been affected by intergenerational trauma.

14.4.3 Early Mathematical Patterning Assessment (EMPA)

An interview based assessment tool, EMPA, was refined on the basis of earlier studies (Papic 2013; Papic et al. 2011). The EMPA was administered to all 4–5 year old children (enrolling in formal schooling the year after program implementation) in the 15 participating centres ($n=255$) in the week prior to the commencement of the PEAP PD program.

The interview comprised 13 tasks which assessed children's facility with simple repetition (8 tasks) and spatial patterns (5 tasks). Tasks included copying, drawing and continuing patterns and identifying the number of dots or objects in various spatial arrangements (Papic 2013, pp. 268–269). The educators were trained on the purpose, implementation and analysis of the EMPA as part of the initial three-day PEAP PD program. At least one research team member acted as participant observer for every interview and supported the educators in the implementation and analysis of data. Procedures were consistent with those of Papic et al. (2011). At least 90 % of the interviews were co-analysed by a member of the research team to ensure reliability of data collection and analysis. Children's responses were initially coded for accuracy then classified by one of four increasing levels of sophistication, focusing on the structure of the representation and the use of a unit of repeat (Papic 2013, p. 270).

14.4.4 Professional Learning

The PEAP PD program is a professional learning initiative aimed at developing early childhood educators' mathematical content and pedagogical knowledge. Staff at each early childhood centre working with 4–5 year olds engaged in a three full day training session (in their centres) at the start of the study along with one-day support sessions for nine of the 12 implementation weeks.

The 3-day training session focused on developing educators' understanding of mathematical and pedagogical content knowledge, including "an understanding of different types of patterns, early algebraic thinking and approaches to developing children's mathematical thinking e.g., problem solving tasks, seeing similarity and difference, questioning, communicating, justifying, reasoning and generalizing"

(Papic 2013, p. 266). Educators were also introduced to NING™, an online platform that allows participants of the study to create their own social network to communicate both with researchers and other early childhood educators implementing the program. The nine one-day support visits focused on a different area of development each visit.

Educators were also provided with two additional days of support the year after implementation, focusing on an area of development identified by the early childhood educators in each centre. This support was different for each service.

14.4.5 Implementing the PEAP PD Program

Of the 255 children interviewed 202 participated in the 12-week program that followed. Fifty-three children did not participate in the whole program due to various reasons (e.g., the child left the centre after the interview, attendance was sporadic, the child was no longer enrolling in formal schooling). All children in the 4–5 year old room participated in the numeracy experiences integrated into the everyday curriculum. However, those without parental consent or those not attending school the following year did not participate formally in the patterning tasks within the framework. They were however exposed to some of these patterning tasks in their everyday experiences and interactions with educators.

The program comprised three components: (i) repeating and spatial pattern tasks; (ii) patternising and numeracy across the curriculum; and (iii) patterning and numeracy in play.

14.4.6 Repeating and Spatial Pattern Tasks

Repeating and spatial tasks were categorised at five and four increasing levels of sophistication respectively (Papic 2013). This chapter focuses on the Repeating Pattern Framework (refer to Table 14.1), a research-based learning trajectory focused on repeating patterns. Learning trajectories according to Clements and Sarama (2004) are complex constructions that include “the simultaneous consideration of mathematics goals, models of children’s thinking, teachers’ and researchers’ models of children’s thinking, sequences of instructional tasks, and the interaction of these at a detailed level of analysis of processes” (p. 87). The mathematical goal of the Framework was to develop children’s early algebraic and mathematical reasoning skills and critical to this was developing educators’ content and pedagogical knowledge in this domain. Children were placed on a level based on their EMPA responses and educators worked with individual (or small) groups of children on the same level on the Framework for 10–20 min each week, up to 10 sessions over the 12 week period. Children engaged in a number of tasks each session and there was flexibility in the order of the tasks to ensure that the learning for each child was scaffolded. Tasks at each level were designed to develop children’s understanding

Table 14.1 Repeating pattern framework

Level	Description of tasks
Level 1 Pre-structural	Copying, designing and drawing 2 block towers (visible and screened-from memory)
Level 2 Emergent	Copying, designing, drawing and extending 4 block (ABAB) towers (visible and screened-from memory)
Level 3 Structural	Copying, designing, drawing and extending 6 block (ABABAB) towers (visible and screened-from memory)
	Identifying the missing item or error in the pattern
Level 4 Advanced structural 1	Copying, designing, drawing and extending complex single variable patterns (visible and screened-from memory) e.g., ABC; ABB; ABBC
	Identifying the missing item or error in the pattern
Level 5 Advanced structural 2	Copying, designing, drawing and extending complex multi-variable patterns (visible and screened-from memory) e.g., cyclic and hop-scotch patterns. Viewing patterns from different orientations

of patterns and their mathematical thinking with each level more sophisticated than the last, starting with basic 2-block towers (Level 1) and progressing to complex, multi-variable patterns (Level 5) (refer to Table 14.1). Children progressed to the next level if they showed competency at the current level. This was a professional decision made by the educators when they believed the child had a good grasp of all tasks on the relevant level. Educators engaged with the children as they copied and drew patterns, identified the unit of repeat and the number of repetitions, described similarities and differences between patterns and explained their strategies and thinking (refer to Papic 2013, p. 271 for a full list of tasks).

14.4.7 Patternising and Numeracy Across the Curriculum

Educators were supported to "patternise" the regular preschool program and consciously create more opportunities for numeracy development. Educators were assisted to: create learning environments to stimulate mathematical exploration; provide children with materials and resources that have the potential to engage children in problem solving and enhance mathematical development; be more conscious of the everyday opportunities available to integrate patterning; and the possibilities offered to develop numeracy through literature, science, cooking and the arts.

14.4.8 Patterning and Numeracy in Play

Play affords many opportunities for mathematical exploration and numeracy development. Educators were supported to observe and document children's engagement with mathematics in their play, the mathematical language they used and the solution strategies evident when solving problems. Educators were supported to use this valuable information to provide additional learning opportunities that would extend children's current mathematical thinking and exploration.

14.4.9 Data Collection

Data collection comprised eight main data sets:

1. Children's performance on the EMPA.
2. Children's progression on patterning frameworks—photos, observations, children's drawing and teachers' documentation supported the progression data.
3. Teachers' planning documentation—daybooks and planning documentation throughout the duration of implementation.
4. Early childhood educator focus groups—(i) midway through the program and (ii) at the conclusion of the program, to identify perceptions of the program and professional growth.
5. Early childhood educator reflections—(i) immediately following training (before commencement), (ii) midway through the project and (iii) at the conclusion.
6. Educators' level of engagement with NING™.
7. Primary school teacher interviews—teachers' perception of the participant child's mathematical engagement, confidence and transition to school.
8. Best Start Numeracy (NSW Department of Education and Communities 2009) results.

The results reported in this chapter focus on two main data sets (i) children's progression on the Repeating Pattern Framework and (ii) primary school teacher interviews. These data highlight children's mathematical development as evidenced throughout the patterning experiences and are supported by the Primary school teacher interviews. These data also highlight the mathematical concepts and skills and other behaviors children demonstrated that potentially assisted in an effective transition to school for these children.

14.5 Results

14.5.1 Children's Progression on the Repeating Pattern Framework

Of the 202 children who participated in the 12 week program, 123 completed between six and the suggested ten sessions of repeating pattern tasks. All 123 children showed development in their patterning skills. Children identified at Level 1–3 on the initial assessment moved up at least one whole level on the Repeating Pattern Framework (Table 14.2).

Table 14.2 Children's achievement level on the repeating pattern framework pre and post program for children who completed six or more sessions

Level at initial assessment	Level on Framework at post-program (no. of children)				
	Level 1	Level 2	Level 3	Level 4	Level 5
	Tasks 1–4	Tasks 5–9	Tasks 10–15	Tasks 16–25	Tasks 26–32
1 ($n=36$)	0	13	13	8	2
2 ($n=57$)	0	0	11	44	2
3 ($n=28$)	0	0	0	22	6
4 ($n=2$)	0	0	0	2	0

At the initial assessment 36 children (of the 123) were identified at Level 1 on the Repeating Pattern Framework as they gave no response or, solution strategies for drawn tower representations were either scribbles or markings where no units were evident (Papic 2013). Fifty-seven were identified at Level 2 as their solution strategies were not represented in a row or column or, if they were presented in a row or column incorrect number and color of blocks were evident. Solution strategies were frequently inconsistent and do not show repetition of pattern elements (Papic 2013). Children whose representations had structure and contained at least one property (color, number or unit of repeat) in the majority of their representation were identified at Level 3 ($n=28$). Only two children were 100 % accurate on the repeating pattern tasks at the initial assessment and were therefore placed on Level 4.

Children at all levels were supported to view the pattern, describe it and abstract the unit of repeat. The following excerpt exemplifies this:

Educator: How many times have you made your pattern?
Hilda: Four.
Educator: Can you show me?
Hilda: One times, two times, three times, four times (breaks the pattern into the individual units).

At Level 3 and 4 children were confidently abstracting and generalising the pattern. They could justify why the patterns were the same as well as the differences between patterns. In explaining the difference between a green, red pattern repeated three times and a black, white pattern repeated three times Sky explained:

Sky: They are different cos they don't got the same pattern. They have different colors but same number of colors, they are same big [teacher reinforces the term height], and it's same number of times.

Sky could then go on to make three additional towers with the same structure (AB × 3): black, red three times; blue, yellow three times; yellow, green three times.

Children in the study were continuously encouraged to create repeating patterns, explain them and represent them through drawings. In drawing the various patterns children were encouraged to look for structural similarities between the given concrete pattern and their drawn representation as exemplified in the following excerpt:

Belinda: Orange blue one time, orange blue two times, orange blue three times (pointing at the blocks in the block tower two at a time moving up the tower). Orange blue one time, orange blue two times, orange blue three times (pointing to the drawn representation and the two blocks in the pattern each time). There it is all the same!

At Levels 3–5 children engaged in tasks requiring them to solve problems and more importantly justify their solution, thereby developing their reasoning skills. When children began the program they found it more difficult to explain their strategies for solving problems or completing tasks as exemplified in the excerpt below where the educator removed the fifth block in an ABAB*A*B pattern and asked Jane to explain how she knew the yellow block was missing:

Jane: Because I know what was there. Yellow one was there.

Through ongoing engagement in the activities, communication between educators and children and the modelling of language and strategies by the educators, children developed their skills in communicating and reasoning. For example, two weeks after Jane's response (indicated above), she was presented with another pattern with the fifth block removed. After identifying correctly that it was the orange block that was missing she explained how she solved the problem with greater reasoning and confidence:

Jane: The orange one's not there. [Jane breaks her tower up into the three units of repeat]. That's how [I did it]. The orange is missing from under the red. It's an orange and red pattern.

The ten children at Level 5 were working with complex patterns using a variety of materials. Patterns were presented in different orientations such as cyclic patterns where repeating patterns were presented as square borders or circles and hopscotch patterns where patterns had vertical and horizontal components (e.g., three squares placed vertically, two squares placed horizontally, repeated three times). Both cyclic and hopscotch patterns incorporated multiple variables such as color, size, shape, etc. Children successfully copied the patterns, created their own complex patterns and drew the patterns from different orientations (90 and 180°).

As children engaged in the various patterning tasks, educators' documentation identified more than just the development in children's mathematical skills (e.g., counting, multiplicative thinking). They also highlighted the growth in children's confidence, communication and problem solving skills. In a final reflection educator LP1 writes:

> Carly was a child who struggled with significant behavior … had very low confidence in herself as a learner and difficulty in verbalizing her needs and wants, could not count past three and had very poor fine motor skills … gained many benefits from the 12 week program including a significant increase in her confidence. Initially, Carly used language such as 'I can't', 'I don't know', 'I can't do the rest', 'I can't say' and 'I didn't even know'. This began to change … Carly developed her ability to verbalize a pattern: 'Cos red is up the top and white's down the bottom, three times' and gained specific strategies for solving problems and remembering information: 'You need yellow down the bottom and green up

> the top. It doesn't have no yellow down the bottom'. Carly loved the one-to-one time and attention with the educator and did not experience a single behavioral issue whilst engaging in the program … She was able to represent patterns through drawings and confidently use mathematical language such as more and times: Educator: 'If you made the pattern one time, how many more times do you need to make it to have it three times?' Carly: 'Two more times'. (Papic and Carmichael 2013, pp. 21–22)

As children engaged in the various activities and experiences throughout the 12 weeks they developed their skills in: (i) solving problems; abstracting and generalising, and (iii) communicating, justifying and reasoning. In analysing the primary school teacher interviews these skills, among others, were identified by the teachers as being evident in the children in the early stages of primary school.

14.5.2 Primary School Teacher Interviews

Following completion of the PEAP PD program 56 primary school teachers participated in follow up semi-structured phone interviews (duration 15–20 min). Interviews explored the teacher's perception of the participant child's mathematical engagement, confidence and transition to school. Questions were designed to prompt conversational discussion and include items such as:

- Can you tell me how __________ has settled into the first couple of months of formal schooling?
- What do you see as __________ strengths in the area of mathematics and numeracy?
- Does __________ display confidence when engaged in mathematical experiences?
- Are there any specific concerns you have in terms of __________ mathematical knowledge and understanding?

All interviews were transcribed and themes identified, drawing on processes outlined by Rubin and Rubin (2012). While these interviews covered wide ranging responses, a series of main themes were evident: (i) confidence; (ii) happy and settled children; (iii) engagement and participation; (iv) leadership in the classroom; and (v) children demonstrating a range of mathematical skills.

The first identified theme was *confidence*: confidence in class generally and confidence in engaging in mathematical experiences. For example many teachers commented that the child settled in well. Statements such as "*She is very confident which is so good*" (Delilah12) and "*She's always been very confident and very social…She's always just been very enthusiastic*" (Louise12) exemplify this. Teacher comments relating to student confidence were consistent in all interviews with several teachers providing examples of this "*She's very confident. She regularly participates. Even if she's unsure, she's happy to have a go. If she makes an error—obviously, I never tell the kids they're wrong, but she'll listen to what I'm saying and to think a different way, and she'll give it a go … She's confident throughout*" (Tilley12).

The second identified theme was that children were *happy and settled*. Here teachers made comments such as "*He's in the routine and he's really quite happy to be at school*" (Rowan12) and "*So straight away she adjusted really well to all the routines and she was really settled*" (Louise12).

Many of the responses from the primary school teachers also indicated that the children were *engaged* in tasks and actively *participated* in experiences. Comments included: "*She's always actively engaged, whether it's group, individual, or with the teacher or not with the teacher. She'll answer and will attempt, if encouraged. She's quite happy to ask for help if she needs it*" (Tilley12) and "*He often puts his hands up and has a go*" (Zane12). This theme is further exemplified by comments such as "*Definitely engaged. Really happy to participate in any discussions we have. Any task in any subject area, really keen … quite enthusiastic overall. Will happily engage in any task, always interested, asks questions*" (Rylan12).

Although many teachers indicate that children were participating well they also identified *increased participation in mathematics*. While only a limited number of teachers identified the children as particularly gifted in mathematics, with comments such as "*His mathematical thinking, his arithmetic strategies are very advanced for his age … he is my little bright spark at the moment*" (Josh12), many teachers commented on the child's participation in mathematics, providing evidence that the child was confident generally in the key learning area of mathematics.

Some of the children were not only engaged and confident but also displayed *leadership* skills: "*He is a natural leader. He helps others. He will lean over and help those that are struggling at his table*" and "*She enjoyed helping the other person in her group, and—she'd say no that's not quite right, here's what you do, so she—because she loves playing teacher…she enjoys helping others*" (Sky12).

Another strong theme identified in teacher interviews was *demonstrable skills in a wide range of mathematics domains* including, but not limited to: pattern, number, counting, addition and subtraction, subitising, counting on, problem-solving and measurement: "*I was amazed. I was really impressed because I knew that he knew his numbers quite well. I didn't realise that he could count that high, so that was a really nice surprise … he is able to transfer knowledge from one area to another*" (Brodie12).

In a closer examination of the interview data, primary school teachers were also identifying children's strengths in solving problems, communicating, questioning and reasoning. For example

> When we're working in small groups that I notice that he's a bit more willing to pose questions and ask why and I suppose using all those working mathematical skills that we talk about a lot. I see him kind of investigating and even if he's working in a small group with other children I'll see him using some language that we've been talking about. He's definitely shown some good problem solving skills. I'm kind of just thinking of one activity in particular that we did last week. We were doing fractions and they had to roll the dice and share that number to find out what half of that number is and they were using counters and a domino template. Interesting to see how he worked with his partner when it was an odd number and then trying to work out why it couldn't be [certain things] like that. Using the language that had been modelled for him and working with his partner to solve problems together which was good (Rodeny12).

As would be expected, children were often described as having well developed skills in ‘patterning’. For example a teacher described a child as good at *“anything visual, whether it’s pictorial or ordering and that, he will do quite well. Patterns, making patterns, creating patterns, seeing patterns and measurement, that idea of comparing*” (Callum12). Teachers also identified children’s confidence in patterning during play and when working with numbers and the hundreds chart.

Having examined the positive teacher responses in the follow up interviews it must be noted that not all interviews were positive, with a range of issues described by teachers. Although these are not examined here due to the scope of the chapter, it must be noted that many of these responses were about children who had completed less than five individual or small group patterning sessions.

Teachers’ comments outlined above exemplify some of the necessary skills for effective transition to school. From these comments it could be inferred that the structure of the program and the focus on mathematical processes (e.g. communicating, reasoning) gave children the necessary skills to settle in well into primary school and be ready to learn. The tasks in the framework were developmental and built children’s confidence to solve problems and communicate their thinking. They were encouraged to justify their solution strategies and explain similarities and differences. Further, teachers from two centres specifically reported that local elementary school principals indicated that children who completed the PEAP PD program were more prepared, confident and made a more seamless transition to school than was evident in previous cohorts from these centres (that had not completed the PEAP program).

Given the significant issues that many young Indigenous learners face in starting school these teacher reports of increased confidence and engagement and participation are significant. As this study did not use an experimental design it cannot unequivocally be stated that the PEAP PD program resulted in increased confidence and engagement, however, it must be considered at the least a contributing factor, which in turn enabled a more successful transition to school.

14.6 Discussion and Concluding Points

In recent years, there has been increasing interest internationally in enhancing early mathematics curricula and assessment. Although there has been a surge in numeracy programs to assist teachers in identifying mathematical competency, there is still a lack of research on numeracy assessment tools to assess the learning outcomes of young Indigenous students prior to formal schooling. The numeracy assessment tool utilised in this study, EMPA (Papic 2013), highlighted the variations in young Aboriginal children’s ability to view and represent patterns. At the initial assessment children were diverse in their response levels and the strategies used to identify, copy and represent patterns. In analysing children’s responses on the assessment tool teachers were able to scaffold children’s learning and develop children’s mathematical reasoning and problem solving skills using the Repeating Patterning

Framework. “Scaffolding learning is critical if students are to progress. To be effective in scaffolding, and thus to extend students’ zone of proximal development, teachers must have knowledge of students’ current understanding” (Jorgensen et al. 2010, p. 136).

A key component of the PEAP program was the engagement of the children in the sequential patterning tasks within the patterning frameworks. All children made progress on the patterning frameworks. Findings indicate that, given opportunities to engage in mathematical experiences that promote emergent generalisation, such as those in the PEAP program, children are capable of abstracting complex patterns, solving problems and reasoning mathematically before they commence formal schooling. Children could explain patterns and pattern structures, view patterns from different orientations and use various materials to create complex patterns. Early childhood educators recorded the development in children’s mathematical language and thinking. These skills are critical for long term mathematical learning, growth and development (Papic et al. 2011). Early mathematical knowledge requires a varied skill set and young children might need specifically focused support to develop mathematical foundations necessary for success in mathematics in formal schooling (Aunio and Niemivirta 2010).

The learning trajectory focused on patterns used in this study, facilitated early mathematical learning and development for the children in the study. Specifically, it advanced children’s skills in justifying and reasoning and these skills led to greater confidence, engagement and persistence. These skills may have assisted children in transition to school as they were skills specifically identified in children by some of the primary school teachers in the first term of formal schooling. “Supporting children’s early mathematical thinking has implications for school readiness which, in turn, impacts later achievement. A recent analysis of the links between school readiness indicators and school achievement in six large-scale studies revealed a strong correlation between mathematics skills at school entry and later mathematics and reading achievement” (Brenneman et al. 2009, p. 4).

The study presented in this chapter used a pre- and post-intervention design to evaluate the effectiveness of the PEAP PD Program on the development of children’s early algebraic and mathematical reasoning skills. While there was no control group in this study, previous research that informed the design of this current study included a control group (Papic and Mulligan 2007). In the earlier study the Intervention group outperformed the control group across a wide range of patterning tasks and on a standardised numeracy assessment at the end of the first year of formal schooling (Papic 2013). The current study has shown that a learning trajectory focused on patterning, has the potential to advance young children’s mathematical problem solving and reasoning skills. This is turn has the potential to close the gap in numeracy achievement for children from low socio-economic and disadvantaged backgrounds.

While not a focus of this paper, developing educators’ mathematical content and pedagogical knowledge was a critical aspect of successfully enhancing children’s mathematical thinking skills. “These skills are critical for long term mathematical growth and development (Papic et al. 2011) however, they can only be effectively achieved if teachers are given appropriate support to plan and implement rich mathematical tasks and environments" (Papic 2013, p. 278).

We must acknowledge that the positive impact of this program occurred predominantly within the context of Aboriginal-controlled children's services that were supported by Aboriginal early childhood educators and IPSU staff. It is difficult to ascertain whether the same positive outcomes would have been replicated in centres that were not Aboriginal-controlled. For this reason the study does not permit generalisation across Indigenous populations but provides insight into the importance of implementing a program of this kind with multiple layers of support and empowering educators to take ownership of the program. It is, however, important to note that two of the settings were not Aboriginal-controlled but did have a high percentage of Aboriginal children. However, the data presented within this chapter was aggregated across all centres. Further research implementing this type of program in other Indigenous communities is required. This research will assist in identifying whether an intensive, on-site professional learning numeracy program, focused on patterning, that assesses and scaffolds children's learning, can support Indigenous children in their transition to school.

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Chapter 15
Preschool Mathematics Learning and School Transition in Hong Kong

Sharon Sui Ngan Ng and Jin Sun

Abstract The Chinese number system is believed to aid Chinese children in learning mathematics, which may help to explain the differences in mathematics performance between English-speaking and Chinese-speaking children. This chapter discusses Hong Kong Chinese preschool children's mathematics learning and pre-primary and primary transition from the perspective of language and culture. Hong Kong preschool children have been found to perform better in mathematics compared with their English-speaking peers. Although the Chinese number naming system helps to explain the children's better performance in learning individual mathematics concepts, this is insufficient in itself to account for the children's performance. Other factors, such as classroom instruction and cultural beliefs about mathematics learning, may also have an important influence on children's mathematics performance. Further studies involving teachers have shown that there is top-down pressure for Hong Kong preschools to adopt an academically focused curriculum. The Chinese cultural aspiration for academic success and the quest for a smooth pre-primary and primary transition have led to the use of the traditional drill-and-practice approach for teaching particular advanced mathematics concepts in Hong Kong preschools. The implications of this for early childhood mathematics teaching are discussed.

15.1 Background

Cross-national studies have shown that Chinese students perform better in mathematics compared to their Western peers (Aunio et al. 2008; Cheng and Chan 2005; Huntsinger et al. 1997; Leung 2006; Mullis et al. 2004). These findings have drawn attention to the factors that influence children's learning in the Chinese context.

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Studies have indicated that a range of factors contribute to this difference in mathematics achievement, including the school curricula, the different teaching and learning pedagogies, the Chinese number naming system and cultural differences. Specifically, in relation to early childhood mathematics learning, there is some evidence that the Chinese number naming system and culture play some role in accounting for the differences in mathematics achievement between English- and Chinese-speaking children (Ng and Rao 2010). In this chapter, we discuss studies involving Hong Kong Chinese preschool children with the aim of understanding why Hong Kong Chinese children excel in mathematics learning and the issues relating to the transition from pre-primary to primary school.

15.2 The Influence of Number Naming System and Culture on Children's Mathematics Learning

To explain why Chinese children perform better in learning mathematics compared with English-speaking children, some researchers have argued that the difference can largely be attributed to the influence of the different number naming systems and cultures (Ng and Rao 2010). This section provides an overview of studies that have found evidence of the role of the Chinese number system and culture in affording Chinese children advantages in learning mathematics concepts.

15.2.1 The Nature of the Chinese Number System and Children's Mathematics Learning

The abstract nature of mathematics and the use of Arabic numerals are universal but the system of numbers and the expression of mathematical ideas vary across languages. In understanding the potential influence of mathematical language on the learning of mathematics concepts, Ng and Rao (2010) summarised how the nature of the Chinese number system aids Chinese children's learning of number and operation concepts.

The literature examining the nature of different number systems has outlined some of the differences between the Chinese and English systems. The Chinese number system is generally described as clear, conceptually well-designed, and with a good presentation of the base-ten system. In contrast, the English language number naming system is described as non-systematic and relatively opaque, and is perceived to be harder to learn than the Chinese system (Fuson 1992; Fuson and Kwon 1991; Miller and Paredes 1996; Miller and Zhu 1991). This may partly account for why children from countries that use Chinese-based systems of number words are more likely to perform better in mathematics than children who learn mathematics in English or other European languages at similar age levels.

Ng and Rao (2010) proposed that the Chinese-based systems of number words benefit children's learning of basic number concepts in a number of ways. First, Chinese children learn the numbers 11 to 19 relatively more quickly because of the clarity of the Chinese number words and their unproblematic depiction of the teen numbers (Miller and Stigler 1987; Miller and Zhu 1991). Second, Chinese and other East Asian children learn to generalise patterns and rules to larger numbers comparatively faster because of the spoken base-ten number words used in the East Asian number systems (Song and Ginsburg 1988). Third, teaching methods derived from the base-ten concept, such as the make-a-ten method used in Chinese textbooks, can help children visualise spoken number words and understand number concepts more clearly (Fuson and Li 2009). Fourth, the consistent use of groupings of ten in the number words supports children in learning decomposition methods structured around ten more efficiently (Fuson and Kwon 1992a, b). Finally, Chinese children may benefit from the simplicity of the Chinese method of forming ordinal numbers (Miller et al. 2000). Overall, it appears that the nature of the Chinese number system aids Chinese children in learning number and operation concepts. However, other factors are clearly involved. Nonetheless, disentangling the influences of culture and other factors on children's mathematical achievement is a challenging task.

15.2.2 Culture and Children's Mathematics Learning

The importance of cultural factors in explaining cross-national differences in academic achievement has been highlighted by both Western (Geary 1996; Geary et al. 1992; Song and Ginsburg 1988; Stevenson et al. 1986) and Asian scholars (Leung 1995, 2001, 2006; Park 2001). For example, researchers have proposed that Asian children's better academic performance may be attributable to parental practice (Huntsinger et al. 1997), the nature of the instructional cues provided (Saxton and Towse 1998), the time devoted to mathematics learning and the teaching approach (Miura et al. 1994). In addition, teachers' beliefs and conceptions about mathematics can significantly influence their selection of mathematics content and teaching strategies (Fang 1996; Pajares 1992). Against this background, in this section, we discuss the Chinese cultural beliefs and practices that may influence children's mathematics learning.

There is a dearth of empirical evidence on the relationship between culture and the mathematics learning of preschool children. Therefore, we draw on the existing studies involving primary and secondary school children as the basis for further discussion. The literature shows that Chinese children tend to receive more family support in learning mathematics (Cai et al. 2004) than American children (Stevenson et al. 1986). Ethnic Chinese parents have higher expectations of their children's mathematics achievement than their Euro-American counterparts (Chen and Stevenson 1995). These high expectations may in turn lead to Chinese children spending more time and exerting more effort in mathematics learning (Stevenson and Stigler 1992). Parental involvement was also found to be a significant predictor of elementary school children's mathematics achievement, with Chinese parents being more involved than American parents (Cai 2003).

Research shows that school students are well supported in learning mathematics in a number of ways. More time was found to be devoted to explicit mathematics instruction in Taiwanese than American primary school classrooms (Stigler et al. 1982). Moreover, cultural differences have been observed in teachers' beliefs and practices (Cai 2004; Correa et al. 2008; Stevenson et al. 1985; Stevenson and Stigler 1992; Stigler et al. 1982) and textbook presentation of mathematical concepts (Fuson and Kwon 1991). Asian students were also found to be well-supported in terms of well-designed curricula (Park and Leung 2006) and more coherent lesson organisation (Leung 2005; Leung and Park 2002). Nevertheless, it is important to note that no studies have focused on cross-cultural differences in preschool mathematics curricula and pedagogy, which makes it difficult to determine whether the type of number system or the level of cultural support for learning is more important in explaining the differences in preschool children's mathematics achievement.

15.3 The Hong Kong Context

As a former British colony, Hong Kong is a distinct society, despite its proximity to China. Western political, economic and educational ideas and systems were introduced under British rule. This section provides an overview of issues concerning preschool and school transition and children's mathematics learning.

15.3.1 Preschool and Primary School Transition

In Hong Kong, children are required by law to commence primary school at age six. The nine-year free compulsory education scheme gives the government strong control over the curricula and the teaching strategies adopted in primary and secondary schools. The normal class size in primary schools in Hong Kong is between 35 and 40, with one teacher per class. In contrast, pre-primary education in Hong Kong is still excluded from the main education stream, although preschool attendance for children over three years is almost universal (Hong Kong SAR Government 2012). All pre-primary schools are privately run and there is more diversity in the pre-primary curricula, which means the children's learning experiences tend to differ in nature. The normal class size in Hong Kong preschools is around 15 children and one teacher.

With regard to curriculum continuity, there is no official mathematics curriculum for preschools in Hong Kong. The Guide to the Pre-primary Curriculum (Education Department 2006), and the Education Bureau's list of "Dos and Don'ts" (Education Bureau 2010) provide basic principles guiding the orientation of curriculum planning. Although they do not constitute a syllabus, these official documents do provide basic references on the mathematics concepts to be introduced at pre-primary level. Teachers are required to choose relevant mathematics concepts and to follow the basic principles in designing learning activities for their children. Unlike preschools, primary schools in Hong Kong have a clear and systematic mathematics curriculum. The contents of the curriculum are periodically reviewed and updated, with the most recent being published in 2000 (Curriculum Development Council 2000). In the latest mathematics curriculum, it is assumed that children are able to enter primary school without attending preschool because the mathematics concepts introduced during primary entrance start from the basics. For example, in learning number and operation concepts, preschoolers are expected to be able to count to 20 and do simple addition and subtraction calculations. However, the skills of counting to 20 and basic addition and subtraction are also supposed to be introduced in the first term of Primary 1. In fact, the Primary 1 syllabus has been used in preschools to teach classes of five-year-old children, leading to an overlap in the mathematics curricula between the pre-primary and Primary 1 levels (Curriculum Development Council 1999; Ng 2006).

With respect to pedagogy, there is assumed to be continuity between pre- and primary school as both advocate a child-centred play based approach to learning. According to the Guide to the Preschool Curriculum (Education Department 2006), preschools should implement a curriculum that fosters the balanced development of children through play and experience. As a result, the thematic approach is widely adopted in preschools to allow flexible curriculum integration (Ng 2006). Preschool teachers are trained to promote children's initiative, independence and creativity through play and there is a high level of flexibility in scheduling daily learning activities. Ng (2006) observed the mathematics teaching in Hong Kong pre-primary schools and found that preschool teachers made use of concrete and semi-concrete materials in teaching simple addition. The study also found that mathematics concepts were taught through large group, small group and free play learning activities, in which the children used their daily experience to develop their understanding of mathematics. However, Ng (2006) observed that the pre-primary teachers felt uncertain about the appropriateness of the concepts they taught because mathematics was absent from the syllabus.

Ideas acknowledging child-centred and meaningful learning can also be found in the official documents on teaching and learning in the primary sector in Hong Kong. For example, the 1981 "White Paper on Primary Education and Pre-primary Services" recommended the development of an 'activity approach' to primary education, especially in Primary 1 to Primary 3 (Ng 2006). However, pedagogical continuity is also missing in practice. As Wong (2003) pointed out, pedagogical discontinuity is a major issue in school transition in Hong Kong, which is also true for the transition in mathematics learning. Despite the continuity of the teaching philosophy found in the official documents, only around one-third of primary schools reported having adopted the activity approach, while most of the Primary 1 classes adopted a more teacher-centred approach to learning (Education Commission 1990). In terms of mathematics teaching, Ng (2006) pointed out that the mathematics lessons in primary school are subject-based and mainly directed by the teachers.

In summary, literature revealed that preschool children in Hong Kong normally engage in more child-centred activities, while the learning in primary schools is more teacher and knowledge oriented. Although the official curriculum guides highlight the natural continuity between pre-primary and primary studies in Hong Kong, there are concerns about the "downward pressure" on preschools to adopt a formal academic curriculum to prepare young children for primary school. There are concerns on the lack of training in preschool teachers, the lack of a systematic mathematics curriculum for the pre-primary level, and parental expectations on children's academic achievements.

15.3.2 Chinese Cultural Aspirations towards Education

With over 95 % of its residents being Chinese, Hong Kong also has a strong Confucian cultural heritage, which may help to explain how Chinese children learn (Higgins and Zheng 2002). Traditionally, education and academic success are highly valued in the Chinese culture. Such success not only relates to the individual but also to their family and to society as a whole. Together, these factors motivate children to learn (Salili 1996, as cited in Kennedy 2002). First, there is a pragmatic appreciation of the importance of achieving good results in Chinese culture as academic excellence is considered the proper pathway to success and the route to a good job (Kennedy 2002). Second, in Chinese culture, success is more likely to be attributed to effort and willpower, than to inherited ability or intelligence (Cheng 1996; Lee 1996; Li 2000, 2002; Rao et al. 2003). Third, the memorisation of facts is commonly emphasised in learning (Chan 1996), although studies have also found that memorisation and understanding are both regarded as important in achieving high quality learning outcomes (Biggs 1996; Watkins and Biggs 2001). Fourth, Confucianism emphasises order, stability, hierarchy, self-discipline and obedience. Teachers in Chinese societies are usually considered a symbol of authority (Ho 1994), are expected to be more knowledgeable than their students and are responsible for imparting knowledge to their students (Cheng 1996; Ho 1994). Students are required to be obedient, to conform to group norms and to persevere with assigned tasks, even if they appear pointless (Biggs 1996; Ho 1994). Accordingly, Chinese students have been found to be passive in classroom discussions and in response to questions (Kember 2000, as cited in Kennedy 2002; Wilkinson and Olliver-Gray 2006), and have been characterised as diligent and obedient (Cheng 1996; Salili 1996, as cited in Kennedy 2002).

15.3.3 The Performance of Hong Kong Chinese Preschool Children

Past studies involving Hong Kong children's mathematics learning showed that they performed well compared with their English speaking peers (for example, Aunio et al. 2008; Cheng and Chan 2005). Against the background in discussing the potential influence of the Chinese number system on the children's learning, a study on

Hong Kong preschool children's (aged from 3 to 5, $n = 299$) was conducted with reference to the number and operation performance of English-speaking children (Ng 2012). The results showed that the learning sequence of arithmetic concepts of Hong Kong Chinese children was generally aligned with that of English-speaking children. Furthermore, Hong Kong children performed well in general and performed relatively better than English-speaking children on items such as forward counting, skip counting, and recognising, reading and using numerals. They performed exceptionally well in learning ordinal numbers. These results align well with previous cross-national studies, confirming that Chinese-speaking children master mathematical concepts more quickly than English-speaking children. However, in-depth examination of the data revealed that the results were not sufficient to establish a positive relationship between children's mathematics performance and the Chinese number naming system (Ng 2012). For example, the children performed well in counting backwards from 10 but did not perform equally well in counting backwards from 20. Thus, other factors, such as classroom instruction and the cultural beliefs of parents and teachers in relation to mathematics learning, may also be important in explaining Chinese children's better performance (Ng 2012; Ng and Rao 2008, 2010).

15.3.4 Hong Kong Teachers' Beliefs, Parental Expectations and School Transition

An earlier study on teachers' beliefs about mathematics teaching found that while Hong Kong Chinese preschool teachers were deeply influenced by their traditional Chinese cultural values, they were also challenged by the Western ideologies introduced in their continuing professional development and by the ideas promulgated by the educational reforms (Ng and Rao 2008). Accordingly, while a child-centred, play-based approach to learning is evident in the sample preschools, teachers also emphasise discipline, diligence and academic success. The study also revealed that teaching practices in Chinese classrooms reflect both constructive and instructivist pedagogies and that teachers' classroom practices may be influenced through pressure from schools and parents (Ng and Rao 2008). While preschool teachers claimed that they liked to adopt a child-centred approach to learning, they also felt the pressure from parents to achieve a high level of mathematical competence. Accordingly, the teachers felt that they had to use a teacher-directed approach to learning as it was considered more efficient. Inconsistencies between teachers' claimed beliefs and their classroom practices were also identified. Apparently, the study showed that mathematics achievement is considered important in teachers' beliefs, and parents appeared to play a role in influencing teachers' teaching. As such, the study concluded that the education reforms had changed Hong Kong teachers' views about children and learning, and that the teachers' views about early childhood learning were a fusion of traditional Chinese and Euro-American ideas (Ng and Rao 2008).

Ng (2014) also examined Hong Kong preschool teachers' knowledge of number and operation concepts, their mathematics curriculum planning and their views on mathematics teaching and learning. Preschool teachers' mathematics content knowledge was found to be weak, which echoes past studies on the teaching of number and addition in preschools (Ng 2005). If teachers have limited knowledge of a subject, they are more likely to teach according to the textbook (Fennema and Franke 1992). Ng (2014) found that in the case of Hong Kong preschools, commercial learning packages serve as an important teaching aid for teachers. All teachers were found to rely on at least one of these commercial learning packages in planning their mathematics curricula. The teachers felt confident in planning the mathematics curriculum as they claimed that the mathematics concepts and activities suggested in the learning packages were ample and useful. However, despite the adoption of the learning packages, teachers still had limited mathematics content knowledge, especially on the developmental sequences of number and operation, and the factors influencing the acquisition of these concepts. Moreover, teachers were found to have a narrow understanding of mathematics teaching. Mathematical proficiency requires conceptual understanding, procedure fluency, strategic competence, adaptive reasoning and a productive disposition (Sarama and Clements 2009). Nevertheless, teachers' concerns were found to be focused on helping children to compute, to get the correct answers and to learn advanced mathematical concepts.

In addition, teachers' choice of mathematical concepts was found to depend heavily on the perceived expectations of parents (Ng 2014). According to the teachers' reports, academic success was what they and the children's parents were looking for. Accordingly, the children were frequently exposed to mathematics in both structured and naturalistic settings. Various activities were used for mathematics instruction and assessment, including the adoption of structured and unstructured activities, direct instruction, the traditional drill-and-practice approach, and pencil and paper tests. It should be noted that the adoption of pencil and paper tests is not recommended in the official guide to the pre-primary curriculum (Education Department 2006). This echoes previous findings on the blending of traditional drill-and-practice approaches and child-centred approaches to learning in early childhood education (Ng and Rao 2008).

While the teachers claimed they taught according to the children's abilities, Ng (2014) also found that the teachers were preparing the children in advance to help them to adapt more readily to the primary curriculum. From the teachers' perspective, attaining a smooth pre-primary and primary transition translated into the pre-teaching of arithmetic operations, or the copying of horizontal and vertical operational formats that are usually practised during the early primary school years. The case of pre-primary and primary transition in mathematics teaching partly reflects the overall story in preschool teaching and learning. Recently, newspaper reports have shown that several primary schools have taken advantage of loopholes in the government's guideline not to adopt paper-and-pencil tests for Primary One admission. In this case, the children were 'tested' simply by orally replying to multiple-answer questions or pointing out answers on an iPad (Siu 2013). These reports may explain to some extent why teachers and parents are serious about preschool children's academic success and why pencil-and-paper tests are adopted in some preschools.

In regard to the potential influence of the Chinese number system on preschool children's mathematics learning, Ng's (2014) study supports the proposition that the simple rules governing the formation of ordinal numbers in the Chinese number system help children to master the ordinal number concept. However, it is less clear whether the Chinese number system helps children to perform better in learning other mathematics concepts such as counting, writing and applying numerals. In contrast, teachers stated that children frequently practice verbal counting during their classroom routines. The active role of parents in children's learning was also reflected in teachers' reports on how parents taught their children to write numerals at home (Ng 2014). In short, the Chinese number system may only help children to learn particular mathematics concepts. Although it is clear that the number system aided children's learning of ordinal numbers, as teachers claimed, it is also true that Chinese children are well-supported in school and at home in learning mathematics. Moreover, preschool teachers and parents tend to be influenced by traditional Chinese cultural beliefs, that is, they are both serious about and actively involved in pursing academic success for their children.

Although Ng's (2014) study reported teachers' perceptions of the parents' views, the findings are aligned with previous studies showing that Chinese parents have high expectations of academic achievement and tend to play an active role in helping their children learn mathematics (Cai et al. 2004; Rao et al. 2009; Zhang and Zhou 2003), and that Chinese children receive both family and school support in learning mathematics at primary school (Cai et al. 2004). Hong Kong Chinese parents were found active in bringing their two and a half year old children to attend intelligence quotient (IQ) tests and some parents enrolled their children in learning programmes that claimed to boost children's IQ scores (Lee 2013; Ming Pao 2013). Although clinical psychologists and other scholars have voiced concerns about 'too much, too soon' for children, some preschool parents are trying to seek early admission for their K2 kids to Primary 1, one year ahead of the normal schooling pathway (Singtao Daily 2013).

To summarise, Chinese cultural aspirations towards academic success play an important role in accounting for why Hong Kong preschool children excel in learning number and operation concepts. It may probably influence teachers' choice in pedagogy and curriculum content. Moreover, teachers and parents are concerned about the academic performance of their children as it appears to be linked to primary school enrolment. These findings further substantiate the top-down pressure for preschools to adopt an academically focused curriculum.

15.4 Implications for Early Childhood Mathematics Teaching and Learning

Empirical evidence indicates that the Chinese number system contributes to Chinese children's good performance in mathematics compared with English-speaking children. At the same time, Chinese cultural aspirations towards learning appear to play a role in influencing teachers' beliefs and classroom practices, and to contribute to children's mathematics learning. However, the findings also reveal that a number of factors require further examination from the relevant parties. For example, teachers' mathematics content knowledge was found to be rather weak, and their understanding of mathematics teaching and learning was comparatively narrow. Moreover, teachers' curriculum planning was found to be influenced by parents' expectations and the adopted commercial learning packages. Finally, there are concerns about preschool and school transition and whether mathematics teaching in Hong Kong preschools is aligned with children's abilities.

The concern about teachers' inconsistencies between beliefs and classroom practices as well as the lack of mathematics content knowledge have implications for the design of teacher-training and teacher professional development programmes. Limited mathematics content knowledge may result in teaching rigidly according to the textbook, or the learning packages. At the same time, teachers were found to succumb to parental expectations and outside pressure. The teaching of mathematics content knowledge should be strengthened in teacher education programmes. It is also important to help teachers to understand the changing priorities of school mathematics, and to acquire a vision of mathematics teaching and learning. Helping children to become mathematically proficient requires a lot more than simply helping them to compute the correct answers. The proposals for education reform in Hong Kong have recommended a shift from an instructivist to a constructivist approach to teaching and learning. Evidence shows that Hong Kong appears to be in a transition phase and that schools use a blend of traditional Chinese pedagogies and child-centred learning approaches. As Hong Kong is influenced by both Western and Eastern cultures, it is argued that a mixed approach comprising both instructive and constructive pedagogies may be well received by both teachers and parents during the transitional period. Further studies involving direct classroom observation may help to understand the inter-relationship among teachers' beliefs, classroom practices, and children's mathematics performance, and make specific recommendations in developing a culturally defined pedagogy for Hong Kong Chinese preschool children (Ng 2014).

It is apparent that some teachers plan their curricula with reference to commercially published learning packages. However, as no studies have examined the early childhood mathematics curricula and teaching materials, it remains unclear which mathematics concepts are involved and how difficult they are compared with the primary school mathematics curriculum. The mathematics concepts and teaching pedagogies adopted in textbooks reflect the prevailing cultural beliefs about mathematics learning and teaching and the implemented curriculum (Murata 2008; Park and Leung 2006). Thus, it is not clear whether these learning packages benefit preschool children's mathematics learning, as is the case in primary and secondary schools (Fuson and Kwon 1991; Murata 2008; Park and Leung 2006). Therefore, an examination of the suggested content and pedagogies adopted in the learning packages may help to reveal the consequences of using learning packages in pre-primary level and whether they support a smooth pre-primary and primary curriculum transition. It is also possible for us to identify appropriate teaching strategies for Chinese children using the Chinese number naming system when comparing the packages tailored made for English-speaking children in Hong Kong preschools.

The issue of preschool and school transition has gained prominence in recent years. The Hong Kong government is putting in efforts to facilitate the realisation of quality education outlined by its policies and official documents. According to the official documents, the pre-primary curriculum is assumed to correspond with primary, secondary and tertiary education, even though there is no formal mathematics curriculum document for Hong Kong preschools. However, the Guide to Pre-primary curriculum is not mandatory and does not appear to be enough in guiding teaching and learning in preschool level. Therefore, it is highly necessary for policy makers to examine the issues related to pre-primary and primary transition from different aspects.

The market-driven ecology in Hong Kong preschools has to be highlighted. Parents as service buyers have influenced the provision of preschooling directly and indirectly. Accordingly, children's rights to quality education may be threatened by how parents define 'quality'. Empirical evidences are noting that Hong Kong parents' views on quality education are probably not the same as those proposed by the Pre-primary guide. More than that, preschool teachers lack necessary capabilities and confidence in school-based curriculum design, which has made them difficult to resist the pressure from parents. The tension between realisation of policy and market force exists, if the early childhood education sector is excluded from the main education system. Therefore, reviews on the existing funding mode for preschools may help. More than that, new ideas and measures on primary school place allocation and curriculum continuity are needed.

Parents' concerns on academic success and their active role played in child caring and education are the reflection of Chinese cultural aspiration towards learning. The issue was complicated by the fact that children appeared to be performing well in international mathematics studies. Parents may expect a continuation of the current practices and are not willing to change. Therefore, it is important to help parents understand the current focus of mathematics education in response to the information age of the twenty-first century. It is argued that the instructive or the drill-and-practice approach may help children to master some foundation of knowledge and skills but is not enough to develop other capabilities like logical thinking, creativity, and problem-solving skills. To bridge the gap, studies investigating Chinese parents' perceptions of mathematics education are important. They may contribute substantially to parents' education regarding the ways to help and support children's learning under the light of current trends in early childhood learning.

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Part III
Informal and Formal Mathematics and the Transition to School

Chapter 16
Early Mathematics in Play Situations: Continuity of Learning

Hedwig Gasteiger

Abstract In recent years, many concepts for early mathematics education have been developed. Taking a closer look at these concepts, it can be seen that they differ considerably in pedagogical background and in quality. During the transition from kindergarten to school, it is extremely important to guarantee consistency and continuity in mathematical learning processes. All early mathematics education should be mathematically correct, 'intellectually honest' and ensure that children acquire the essential prerequisites for further mathematical learning. Additionally, mathematical learning should be designed according to children's specific age. Based on scientific findings, this chapter specifies why early mathematics education in natural learning situations, like play activities, meets these requirements of subject-*and* child-orientation. Play situations can foster the development of mathematical learning in kindergarten and in school sustainably. Results of an intervention study about learning mathematics while playing traditional board games ($n = 95$, average age: 4.8 years, control and intervention group) confirm this claim. The intervention shows significant effects. Video analyses of the play situations illustrate the findings and allow investigating in detail the role of the teachers and the mathematical learning processes which occurred during the play activities.

16.1 Early Mathematics Education—But How?

There is a common consensus that mathematics is a necessary component of early education (Australian Association of Mathematics Teachers and Early Childhood Australia 2006; Kortenkamp et al. 2014; National Association for the Education of Young Children (NAEYC) 2002). However, there are different concepts and ideas about how mathematics should be integrated into the daily work of kindergarten. Approaches range from instructional concepts or training to the idea that mathematical learning happens in many activities every day—and we can find advocates for all positions. Higher quality instructional interactions are positively associated

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with basic mathematical skills (Mashburn et al. 2008). Therefore, recommendations like "all early childhood programs should provide high-quality mathematics curricula and instruction" (Cross et al. 2009, p. 3) stand to reason. On the other hand, some require that learning in early childhood should be initiated by the situation, the environment or children's play. A very extreme position—Lee and Ginsburg (2009) describe it as a misconception about early mathematics education—is the claim that "mathematical learning occurs incidentally, through exploration during free play, with little teacher participation" (Lee and Ginsburg 2009, p. 40). Moreover, the role of instruction in early childhood education differs considerably between countries (Hauser 2013). So it still seems to be an open question how early mathematics education should be organised best in order to succeed in this difficult domain. To help clarify this question, some key issues of early mathematics education will be discussed.

16.2 Some Key Issues of Early Mathematics Education

Scientific findings from different disciplines support the following key issues of mathematics education in the early years.

- First of all, it is necessary to choose carefully the mathematical content taught during the early childhood. It is recommended to "align all efforts of early mathematics education with the 'big ideas' of mathematics" (NAEYC 2002, p. 6). These are "overarching clusters and concepts and skills that are mathematically central and coherent, consistent with children's thinking, and generative of future learning" (Sarama and Clements 2009, p. 16). Numbers, operations, relations, geometry, spatial relations and measurement are some of the main content areas where early mathematics education should focus on (Cross et al. 2009; Sarama and Clements 2009; Wittmann and Müller 2009). Beyond that, learning contexts should provide opportunities for problem-solving, communication and reasoning (Lee and Ginsburg 2009). Hunting (2010) proposes a more detailed provisional list of big ideas with content and mathematical skills like class inclusion, composition and decomposition, representing, imagining, and naming. Although there are different lists of big ideas, it seems to be a broad consensus that early mathematics education should orient itself towards mathematically central, coherent and consistent content.
- To ensure continuity of learning, it is necessary to make clear the "broader fundamental structure of a field of knowledge" (Bruner 1999, p. 31). Fuson refers to "big coherent conceptual chunks" (Fuson 2004, p. 106). If the mathematical structure is clear, children have a chance to understand what they learn (Fuson et al. 2005) and only then early mathematical learning can be related to mathematical learning in school and life contexts. Teaching mathematical content in a simplified manner, which is supposed to be appropriate for children, can be counterproductive if the fundamental ideas, "the underlying principles that give

structure to that subject" (Bruner 1999, p. 31), get lost. In Germany, for example, there is a method where numbers are presented in a personified manner—"the two" is a character with two feathers on its hat and it repeats every word twice (Friedrich and de Galgóczy 2004). The intention is to design mathematical learning especially suitable for children at a young age, but the fundamental idea of numbers—the mathematical structure—disappears in conceptualisations like these. However—as Bruner (1999) stated,—"any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). This should be a standard for early childhood mathematics education.

- Continuity of learning means to take into account the hierarchical structure of mathematical content—but continuity of learning also means to respect children's learning processes. It is known that children differ considerably in their mathematical achievement in the early years due to social background, family factors and the quality of home learning environment (Anders et al. 2012, p. 207; Starkey and Klein 2008). If children show low mathematical achievement in kindergarten, they are more likely than other children to have difficulties while learning mathematics at school (Dornheim 2008). We even know that children who focus on numerosity in their early years have better subitising and counting skills at the age of 5 years (Hannula et al. 2008). Therefore, early mathematics education should assess and attend to children's individual stages of mathematical development (Clements 2004, p. 13).
- Organising learning processes in early childhood means to respect that children construct their knowledge actively (Fthenakis et al. 2009). Van den Heuvel-Panhuizen describes learning in early years as "a process that occurs primarily 'from inside'… driven by the child's own natural curiosity, its urge to find out how things fit together" (2001, p. 25). So explicit teaching is seen as less effective than creating opportunities which offer children possibilities to discover mathematical concepts and solution strategies for different mathematical problems (Baroody and Wilkins 1999, p. 62).
- Particularly for early childhood mathematics education this constructivist view of learning is complemented with a social component. Learning processes need social interaction with adults and peers (Reusser 2006; van Oers 2004). Results of the EPEY (*Effective Pedagogy in the Early Years*) study show that early education settings are effective if they encourage "sustained shared thinking" (Siraj-Blatchford et al. 2002, p. 10), which is defined as "an episode in which two or more individuals 'work together' in an intellectual way to solve a problem, clarify a concept […]" (Siraj-Blatchford et al. 2002, p. 8). The important role of adult-child-interaction is almost always mentioned in the context of early mathematics education (Anderson et al. 2008; Hunting et al. 2012; Montague-Smith 2002).
- Results of research in developmental psychology state that children at early ages are very motivated to learn, but have difficulties with explicit and intentional learning as it is normally practised in school contexts (Hasselhorn 2005). Therefore, early mathematical learning should be appropriate for children's development.

Otherwise, long-term effectiveness is not guaranteed (Siraj-Blatchford 2002, p. 29). Moreover, there are indications that direct instruction in early childhood causes more anxiety and lower self-esteem (Sylva and Nabuco 1996). While child-initiated learning activities in early childhood education support the development of "requisite skills and dispositions to become responsible adults" (Schweinhart and Weikart 1997, p. 140), direct instruction has no such preventive value.

To synthesise these statements, instructive learning settings with strong-guided interactions or narrow perspectives of mathematical learning, which are not in line with the big ideas of mathematics and which are not appropriate for children's development, are not suitable for early mathematics education.

16.3 The Concept of Early Mathematics Education in Natural Learning Situations

A concept of early mathematics education that tries to integrate all these key issues is "Early Mathematics Education in Natural Learning Situations" (Gasteiger 2010, 2012, 2014, Fig. 16.1). It is a theoretically based concept (Gasteiger 2010) which focuses primarily on play and everyday activities. These situations deal with mathematical content in a straightforward manner and in a broader context which allows children to recognise relations between mathematics and reality. Learning in complex situations—like everyday situations—and not only in carefully arranged step-by-step units, gives children a chance to understand. Moreover, play and everyday

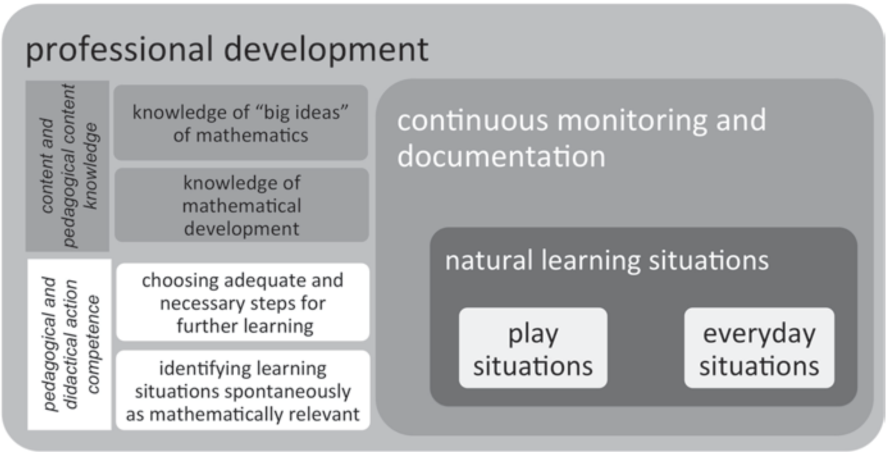

Fig. 16.1 Concept of early mathematics education in natural learning situations. (Gasteiger 2010, 2012, 2014)

situations stimulate children's natural curiosity and open interactive and constructive learning opportunities.

However, mathematical learning does not just occur incidentally (Lee and Ginsburg 2009). Hence, the concept of "Early Mathematics Education in Natural Learning situations" includes two other components.

To guarantee continuity in children's individual learning processes, a continuous monitoring of children and a documentation of their learning processes have to accompany mathematical learning in play and everyday situations. Only then can the next steps of learning be planned, and children be fostered individually in their mathematical development.

Due to the fact that all these efforts of early mathematics education are very challenging for the educators, professional development is an important part of the overall concept. Educators have to know the big ideas of mathematics and milestones in children's mathematical development. They need pedagogical and didactical action competence to identify learning situations as mathematically relevant and to choose adequate and necessary steps for further learning. Without this knowledge and competence they cannot successfully use play or everyday situations for mathematical learning and support children in their individual learning.

There is already some empirical evidence for different components of this concept. The results of a small evaluation study indicate that a professional development program with a focus on natural learning situations, on monitoring and documenting of children's development, and on the underlying content/pedagogical content knowledge and pedagogical/didactical action competence can have effects on children's mathematical achievement (Gasteiger 2010, 2014).

The effectiveness of different play or everyday situations as natural situations for children's mathematical learning in kindergarten still has to be studied in detail. In a first step, the role of number-dice games for mathematical learning was examined in an experimental study. Results of this study will be reported here. Prior to this, detailed argumentation will show why play activities can guarantee continuity of learning in the above-mentioned aspects.

16.4 Play Situations as Natural Learning Situations

16.4.1 Meeting the Key Issues

A definition of play is seen as nearly impossible (Hauser 2013), but there are many attempts to find criteria to describe play. Joyful, child-chosen and child-invented activities which focus on the process, not on the product and which require active involvement, are seen as play activities (Wood and Attfield 2005).

First of all, play as characterised in this way is not instructional learning as often met in school contexts. It seems to be a developmentally appropriate form of learning for children in the early years, because it allows children to discover themselves and their environment actively, they learn to change perspectives—and all this in a kind of protected space (Fröbel 1838; Kunze and Gisbert 2007).

While playing children construct knowledge, they communicate with others and often start a meta-cognitive process (Pramling and Asplund Carlsson 2008). So play situations can be described as co-constructive processes (Jordan 2009). They meet the requirement of sustained shared thinking (Siraj-Blatchford et al. 2002) if children either play together or with adults.

Vygotsky (1978) mentions that "play creates a zone of proximal development of the child" (p. 102), because in play the child behaves beyond its age. Play is "a free activity" (Huizinga 1949, p. 13), which means that children can guide the process of playing on their own. These statements reveal that continuity concerning the individual learning processes of children is guaranteed in play situations, because challenges that play situations offer can be used by children to take a step further, but they need not if it is not suitable for the individual learning process.

Guaranteeing continuity concerning the big ideas of mathematics and the underlying structure of mathematical content in play situations is more difficult. To meet this requirement, it is necessary to focus on the role of the adults. Wood and Attfield (2005) use a "play-non-play continuum" (p. 6) to define play. Play as a meaningful, voluntary, pleasurable, and rule-governed activity can be seen on the far left of the continuum, and playful situations that educators use to provide opportunities to learn can be placed on the far right of the continuum. Talking about using play situations for early mathematics education requires us to agree that adults stimulate learning processes by providing playful opportunities to learn or by encouraging children to think or talk about their actions during play from a mathematical point of view. At this point, we are rather right of the middle in the described continuum. However, even in the context of mathematical learning, these situations should meet the criteria that characterise play. The educators have to ensure the continuity of mathematical learning in the above-mentioned sense, but on the other hand, they should ensure that play situations remain play situations (Perry and Dockett 2010). The important role of the educator is obvious: The "future of mathematical thinking in young children strongly depends on the quality of early years teachers to recognise mathematical actions in children, to see the mathematical potential of play activities and play objects, and to guide children into the future where they can still participate autonomously and creatively in mathematical communications" (van Oers 2013, p. 271).

Play situations can ensure continuity in early mathematics education—both continuity concerning mathematics and continuity concerning children's learning processes. How does this balance with Lee and Ginsburg's (2009) notion that mathematical learning does *not* just occur incidentally in play situations? "As long as [children's] actions are not intentionally and reflectively carried out, we cannot say that children perform mathematical actions" (van Oers 2010, p. 28). Hence, the role of adults, early childhood educators or teachers is very important. They see or create mathematical opportunities to learn in play situations, stimulate children's learning by giving inspiring comments or additional material and by knowing the relations between mathematics in the early years and later on (Gasteiger 2014)

16.4.2 Mathematical Development and Play—Some Scientific Findings

Many scientific results show that using play situations is an effective way to foster children's mathematical development.

McConkey and McEvoy (1986) analysed whether moderately mentally handicapped children (mean age 12.3 years) enhanced their counting abilities by a 6 week intervention of playing dice and card games specially designed for the study. While the control group made almost no progress over the 6-week period, the students in the experimental group showed significant improvement.

Peters (1998) conducted an intervention study with 5 year-old children with low to average number knowledge. These children played mathematical card and board games in small groups with parental support. The games were played in the classroom once a week for 8 months. Children taking part in this intervention ($n=14$) performed better in counting tasks than a control group ($n=37$).

That number games and story books can improve were examined in a study by Young-Loveridge (2004). Over a 7-week period, 23 lower achieving children attended daily intervention sessions (30 min each) in school over 2 months. Two children played (modified) commercial dice and card games with a teacher, heard a number story and talked about the numbers in the accompanying pictures. Each session started and ended with a number rhyme. The teachers were advised to engage the children in mathematical activities and to support them in their individual development. The control group ($n=83$) continued their mathematical lessons. Their teachers used a special program based on Piaget's ideas of matching, sorting, comparing, and classifying. The intervention program had significant effects over time, even 15 months after intervention. Intervention children made greater gains in knowledge of number, number patterns, numeral identification, making small collections of objects, and in addition of two collections.

The effect of a very short intervention was examined by Ramani and Siegler (2008) who randomly assigned 124 preschool children (mean age 4.9 years) from low-income backgrounds to two groups. They played in four 15–20-min sessions one-on-one with an adult within a 2-week period and in a fifth session 9 weeks later. Children in the experimental condition played a linear board game with the numbers of 1–10 in 10 squares and dice with the numbers 1 and 2. Children in the control condition had a linear board game with colours in the 10 squares and dice with colours. The child should say the number/colour on the squares they passed while moving their token. After intervention, children in the experimental condition improved their results in number line estimation, in counting, numerical magnitude identification and in numeral identification considerably, while children in the control condition showed no improvement. A second study of Ramani and Siegler (2008) indicated that children who had more experience in playing board games at home or at other people's home showed greater numerical knowledge, whereas experience in playing card and video games was not related to numerical knowledge.

Rechsteiner et al. (2012) compared the mathematical achievement of children in kindergarten (mean age 6.3 years) who received a play-based approach for mathematical learning ($n=89$) with that of children who were given an instructional training ($n=110$) and with children in a control group with no intervention ($n=125$) (Stebler et al. 2013). Children in the play-based approach played three times a week (30 min. per session) in an 8-week period using commercial and specially designed card and board games. They played on their own in small groups. Children in the instructional training group were given a commercial training program with the same duration and timetable and the control group had no explicit intervention. The mathematical development of children in the play-based approach was significantly better than that in the control group, whereas children in the instructional training group performed not significantly better or worse than children in the two other groups. The play-based approach seemed to be comparable with the instructional training, but considerably better than the regular daily work in kindergarten.

These findings support the idea that play (board and card games, games with or without dice) can enhance the mathematical development of children.

16.5 Fostering Early Mathematical Development with Traditional Board Games—Results of an Intervention Study

The majority of the above-mentioned studies focus on children with mathematical achievement below average or with difficult conditions for further learning, and most of the investigated children were already in school or at least 6 years of age. The games used in these studies were mostly modified or specifically designed for the interventions.

Opportunities to learn at home and with parents seem to influence children's mathematical knowledge even before they enter kindergarten (Anders et al. 2012; Ramani and Siegler 2008). Hence, one should focus on younger children and on traditional games because these games can be found at children's home and as they are sometimes/often played in normal play situations at home without a special learning focus.

The question therefore arises whether children in their early years stand to benefit from play—irrespective of their social background and their previous mathematical knowledge—and if their engagement in "normal" play situations with traditional games guarantees sustainable, continuous mathematical learning processes. The intervention study MaBiiS (elementare **ma**thematische **Bi**ldung **i**n **S**pielsituationen) examined this question.

16.5.1 Method

16.5.1.1 Participants

A total of 95 children (52 girls and 43 boys, 31 with migration background and 64 without) took part in the study. They were recruited from five German kindergartens, their mean age was 4.8 years (4.5–6.2 years) and they all had 1 year and a half until their school enrolment. The children were randomly assigned to an intervention and a control group.

16.5.1.2 Intervention

The children attended the intervention sessions in groups of two or three over a three-and-a-half-week period. Each child had seven intervention sessions of 30 minutes each. During the intervention sessions a trained adult played dice games with the children.

Children in the intervention group played board games with normal number dice. These were the traditional games ludo[1] and coppit[2], and a game called "Collecting Treasures"[3]. Playing this game, children have to move their token forward, some squares show an amount and if the token is on one of these squares, the child has to collect the right number of coloured treasures—the winner is the one who can collect most of the treasures. Children in the control group played dice games as well, but with colour or symbol dice. They played a game very similar to ludo[4] but with symbol dice: the symbol on the dice shows to which square the player should move. A second game used colour dice to choose small parts in different lengths which form together a worm—the player with the longest worm wins the game[5]. Counting was not necessary in either game played with children in the control group.

The adults who played with the children were trained. The most important point in their training was that they should 'play' and not instruct the children in mathematics. However, they were trained to play while remaining alert to everything that happened. They functioned as a role model by counting out loud, when they moved forward, by naming the number or colour/symbol the die showed or by giving verbal stimuli like "Count again. Your token was here" or "I think you can catch someone". They remained attentive so that the children had enough time for their moves, and guaranteed that no one forestalled the players' actions or answers.

[1] Mensch ärgere dich nicht—Schmidt-Spiele.

[2] Fang den Hut—Ravensburger.

[3] Schätze sammeln—ZahlenZauberei, Oldenbourg-Schulbuchverlag.

[4] Der Maulwurf und sein Lieblingsspiel—Ravensburger.

[5] Da ist der Wurm drin—Zoch.

16.5.1.3 Measures

Children's mathematical competencies were assessed before (pretest), immediately after (posttest) and 1 year after the intervention (follow-up-test) using an individual standardised test for children between kindergarten and third grade (TEDI-Math: Kaufmann et al. 2009). The counting principles, enumeration, numeral identification, number word identification, and calculating subscales were utilised. The standardised test was supplemented by a subscale structure knowledge and structure use. Data from the follow-up-test has not yet been analysed. Children's intelligence was measured with the WPPSI (Petermann and Lipsius 2011), and the quality of the day-care centres was assessed using the KES-R (Tietze et al. 2005).

One play session for each adult, with children of the intervention group, was videotaped to analyse communication and activities between the children and the adult during the play situation.

16.5.2 Results

16.5.2.1 Effectiveness

An ANCOVA was performed on children's posttest score of mathematical achievement using pretest score as covariate. Posttest score is influenced significantly by the pretest score ($F(1,92)=291.88$, $p<0.001$), and by the intervention-condition ($F(1,92)=13.57$, $p<0.001$) with an effect size of 13 % (partial eta squared). The intervention group shows greater gains in the posttest than the control group as the solution rates in Table 16.1 show: in the pretest, both groups performed nearly equally, while in posttest the solution rate of children in the intervention group (72 %) was better than of children in the control group (67 %).

The impact of the play intervention was found to be independent of gender, migration background, intelligence and day-care centre. The results indicate that children who played number-dice games showed significantly higher learning gain from pre- to post-test than children in the control group who played with colour- or symbol-dice.

The subscale in which children of the intervention group performed substantially better than children of the control group was enumeration ($F(1,92)=9.96$, $p<0.01$, $\eta^2=0.10$). An explanation for this is that counting and respecting one-to-one-correspondence is often experienced when children move their tokens forward during their play.

Table 16.1 Comparison of solution rates

	N	M (SD)	
		Pretest	Posttest
Intervention group	48	0.60 (0.16)	0.72 (0.14)
Control group	47	0.61 (0.15)	0.67 (0.16)

16.5.2.2 Mathematical Action and Communication During Play

To analyse the mathematical learning opportunities during play situations, nine intervention sessions were videotaped. These data were analysed to detail the mathematical content in which children were engaged and how much time they used for mathematical activities or dialogues. Another question was whether there were differences between the adults, or the three games (Sedlmeier 2013).

Therefore, the periods of time of all activities and comments were coded in separate categories for adults and for children (see Tables 16.2 and 16.3). Verbal/non-verbal, and mathematical/non-mathematical categories were differentiated. Category 10 (non-verbal play activity) was coded when the game had been prepared, dice were rolled or passed to the next player, and other similar activities. These are non-verbal and non-mathematical activities.

Category 11 (not defined non-verbal activity) is the category most often used for the adults' activities and comments. This is not surprising, considering that each child spent time on play activities and comments while the adults probably listened and observed. Analysing active time of the adults (categories 1–10), we can see that

Table 16.2 Categories for adults' activities and comments (mathematical categories in italics)

Verbal	Non-verbal
1 Mathematical stimuli or questions	10 Non-verbal play activity
2 Mathematical explanation, correction	11 Not defined non-verbal activity
3 Mathematical comment on one's own play activity	
4 Confirmation of mathematical comment of children	
5 Mathematical, verbal accompanied play activity	
6 Disciplinary comment	
7 Comment concerning rules	
8 Other non-mathematical comment	
9 Incomprehensible comment	

Table 16.3 Categories for children's activities and comments (mathematical categories in italics)

Verbal	Non-verbal
1 Enumeration (right/wrong)	*10 Silent enumeration (right/wrong)*
2 Comparing amounts (right/wrong)	*11 Non-verbal subitising (right/wrong)*
3 Subitising (right/wrong)	12 Non-verbal play activity
4 Part-whole (right/wrong)	
5 Calculating (right/wrong)	
6 Comments based on mathematical thinking	
7 Comment concerning rules	
8 Other non-mathematical comment	
9 Incomprehensible comment	

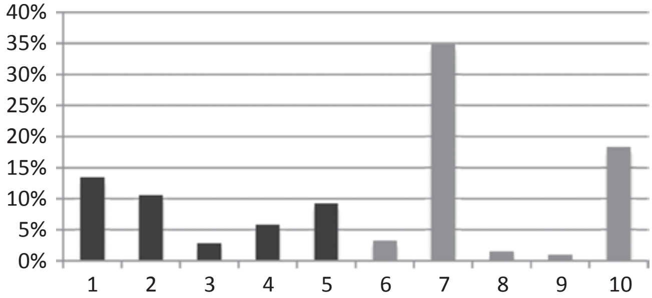

Fig 16.2 Adults activities and comments (mathematical categories: *dark*, others: *light*)

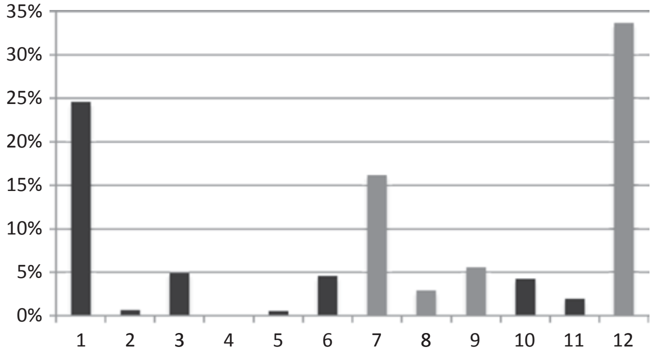

Fig. 16.3 Children's activities and comments (mathematical categories: *dark*, others: *light*)

35 % of this time was spent on comments concerning rules (Fig. 16.2, category 7), and during 42 % of their active time adults gave mathematical stimuli, explanations, comments, or accompanied their play activity verbally with a mathematical intention (categories 1–5).

There were almost no differences between the three games, but some differences between the adults. While one adult had only 29 % verbal time during the whole intervention sessions, the others spent between 41 and 55 % on talking or commenting. As we videotaped only one session with each adult, it is not reasonable to examine correlations between children's mathematical achievement and adults' verbal activity.

The mathematical activities and comments of the children on the videotapes were exactly characterised, and different codes were used if the comment or activity was correct or incorrect. Table 16.3 overviews the categories. Active time of each child was analysed, and for each session those data were summed in the different categories. Two of the children's non-verbal categories were coded as mathematical activity: if a child recognises the dice-pattern and/or moves its token forward correctly without speaking, this can be seen as a mathematical activity.

As can be seen in Fig. 16.3 children spent most of their active time on non-verbal play activities (category 12: 34 %).

They were mathematically active in 42 % of their active time (all categories except 7, 8, 9, 12). Most of that time was used for enumeration (25 % verbal, 4 % non-verbal) and subitising (5 % verbal, 2 % non-verbal). Less than 1 % of children's active time was used for comparing amounts, part whole activities or calculating. Five percent of the active time was used for comments based on mathematical thinking such as "To catch me, you need six" or "You should go with this token" (if it was a good piece of advice for a tactical move when it was necessary to reflect on a number of moves).

The coding of correct or incorrect statements or activities showed problems with the number sequence or the one-to-one-correspondence in individual cases, but all in all, 91 % of the time that children were mathematically active, they acted or verbalised correctly.

The analyses of the video data showed the potential of traditional board games for mathematical learning: adults and children spent in equal measure 42 % of their active time on mathematical comments, discussions, activities, or thinking. The qualitative analysis of the video data showed that children also commented on activities of their peers or the adult player. They were involved in play activities even when it was not their turn.

16.6 Discussion

This chapter tried to give a theoretical foundation for the use of play situations for early mathematics education with a special focus on continuity of learning and reported empirical evidence for the effectiveness of play.

Play situations can be used successfully to foster the mathematical development of under-achieving children—in kindergarten and likewise in school (Peters 1998; Ramani and Siegler 2008; Rechsteiner et al. 2012; Young-Loveridge 2004). The results of our study show the potential of number-dice games for all children—regardless of gender, migration background, intelligence and the day-care centres children attended. We played traditional games in 'normal' play situations in which our adult players were requested to play like—for example—alert parents. This approach offers another big chance for mathematical learning: learning situations like those can be carried out very easily within the family environment as well. With respect to the considerable influence of learning situations at home before entering kindergarten (Anders et al. 2012; Ramani and Siegler 2008), these results are of great importance.

Focusing on continuity of mathematical learning, the appropriateness of play situations can be reflected even more soundly. Therefore, it is reasonable to consider mathematical competencies in the early years, which are predictive for mathematical learning in school. In a longitudinal study Dornheim (2008) determined verbal counting (counting on, counting in steps included), enumeration, subitising, using structures, and simple calculations as predictive for further mathematical learning. Children who have difficulties in these domains at an early age are more likely to have problems with mathematical learning in school compared to children who perform well. Our studies show that verbal counting, enumeration and subitising can be trained in play situations. The study of Young-Loveridge (2004) even shows effects 15 months after the intervention. The follow-up-test data of our study has to be analysed in detail, but there are signs that children in the intervention group can profit from the board game intervention 12 months later. This means that board games can help to foster especially those mathematical competencies which are predictive for further learning. This shows once again how play situations can contribute to continuity in the learning of mathematics.

In conclusion, one major implication for mathematics at transition from kindergarten to school is to respect play as a successful approach for mathematical learning in kindergarten, school and even family. For practice it can be recommended to analyse the mathematical potential of different games, to choose them carefully for application in kindergarten and school and to be alert for children's learning processes in play situations.

Early mathematics learning in play situations also suggests future work. This chapter focused primarily on numbers, but play situations have a great potential for mathematical learning for other content areas as well. Seo and Ginsburg (2004) showed in an observation study that children engage in many different mathematical activities in their free play. Such play is sometimes quite complex and includes different content such as pattern and shape, classification, spatial relation, enumeration, magnitude, or dynamics. There is still a lot of research to do to support children's mathematical learning in play situations in domains other than number. It is necessary to get more insight in children's mathematical development in these domains, and the effectiveness of these play situations for mathematical learning has to be studied.

The video data analysis of our study shows that almost all mathematical comments have been correct. Closer consideration of the qualitative data shows that some children are more active than others. Can it be assumed that children with less mathematical prerequisites took less part in the conversation while playing than the others? Here, the important role of the adults becomes obvious: their role "is crucial, as the adult will introduce and use new language, and encourage discussion to help the child to understand a concept or acquire a skill" (Montague-Smith 2002, p. 140). Stimuli, comments or questions of adults are necessary to let children see the mathematics, think about it and move a step further in their own development. Substantiate findings and good programmes of professional development are necessary to help the educators see the mathematical development in children's play and to act or react in a co-constructive manner. Not till then, will play situations unfold their whole potential for early mathematics education.

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Chapter 17
Mathematical Conversations that Challenge Children's Thinking

Jill Cheeseman

Abstract The interactions young children have with adults are of great importance in developing children's mathematical reasoning. The one-to-one mathematical conversations and interactions young children have with their teachers are memorable to children. In their 1st year at school, children can recall their conversations with the teacher, reconstruct their thinking, and reflect on their learning. Children construct mathematical ideas in the course of their interactions with their teacher and classmates. Interactions in whole class settings have been studied. However, not as much has been written about the interactions between teacher and child in one-to-one conversations during the mathematics lessons of young children. This chapter examines the nature of teacher-child mathematical conversations and how they evolve as children move to the generally more formal setting of school. Interactions that challenge children to think mathematically in their transition year to school illustrate the central characteristics of questioning, listening, and thinking. The mathematical pedagogical behaviours that support and facilitate these interactions are noted.

17.1 Introduction

The nature and quality of mathematical conversations young children have with adults is of vital importance for their development (Cheeseman 2010; Sfard et al. 1998;). Ball (1994, p. 55) says, "play and conversation are the main ways by which young children learn about themselves, other people and the world around them." Children remember mathematical conversations and can recall learning from them (Cheeseman 2008). Of course, the earliest mathematical conversations are likely to be those with their family members in the course of everyday life. In prior-to-school settings mathematical conversations tend to be informal and often in the context of play-based learning. The transition to school can bring changes in the nature of the

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mathematical talk expected of young children. Often the discourse is more formal and less individual. However, the school experience need not be this way. This chapter describes the behaviours of teachers who work in the early years of school yet manage to have daily individual and natural mathematical conversations with children. These mathematical interactions young children have with their teachers challenge the children's thinking.

17.2 Background

My interest in the one-to-one mathematical conversations young children have in the first years of school was stimulated by my involvement in the Early Numeracy Research Project (Clarke et al. 2002). As part of the larger project, the research team investigated the common practices of 'highly effective' teachers of mathematics with young children and wrote case studies of their practice (McDonough and Clarke 2003). These were teachers who could help young children make exceptional learning gains over two consecutive years in the research project as measured by clinical interview (Clarke et al. 2001, 2002). One of the common characteristics of these case study teachers was the vibrant learning community they established with children in the first years of school and the nature of the classroom interactions they promoted (Clarke and Clarke 2004). The case study teachers used a range of question types to probe and challenge children's thinking and reasoning. They would hold back from telling children everything to give children opportunities to think mathematics through for themselves. They would encourage children to explain their mathematical ideas and they encouraged children to listen and evaluate others' mathematical thinking. These teachers listened attentively to individual children and built on children's mathematical thinking and strategies.

17.3 Researching Mathematical Conversations Between Teachers and Young Children

To look in greater depth at this cluster of teacher characteristics I undertook a further investigation of the same teachers to examine their behaviours that challenged children to think mathematically (Cheeseman 2010). The study focused on the one-to-one interactions teachers had with young children in mathematics classrooms in the early years of school. Data were collected using: video of classrooms; field notes; and interviews with teachers and the children with whom they interacted. The perspectives of the participants were collected in what Clarke (2001) called a *complementary accounts methodology*. In this way the close mathematical conversations that the teachers had with individual children were recorded and could be studied in detail.

17.3.1 Conversations

I consider the interactions highly effective teachers have with children in everyday mathematics classrooms as *conversations* about mathematics. Much has been written to recommend mathematical communication in classrooms and many terms have been coined to describe various types of mathematical communication. It is important to distinguish between two of these terms: *conversation* and *classroom talk* to clarify the nature of the interactions that will be discussed in this chapter. Thornbury and Slade (2006) described the characteristics of conversation as: spoken, spontaneous, dialogic, synchronous, interpersonal, and symmetrical in relationship. Classroom talk, they claimed, was neither interpersonal nor symmetrical in relationship. While it is true to say that the mathematical conversations teachers have with young children in their everyday classrooms are not symmetrical in relationship, as the teacher is the adult and in a position of authority, the highly interpersonal nature of the exchanges makes them far more conversational in tone than would be found in classroom talk.

Sfard and her colleagues proposed that the idea of learning through conversation was a natural by-product of the conception of learning as an initiation into a "community of practice" (Sfard et al. 1998, p. 43). Nescher, in her section of the same article, went on to distinguish *conversations between teacher and child* and commented that these conversations "serve many purposes. From the constructivist point of view, it is the major means by which the teacher has the opportunity to learn about the student's thinking and have a real dialogue" (Sfard et al. 1998, p. 43). It was clear from my research that the conversations that teachers had with children during their everyday mathematics lessons were rich opportunities for the teachers to understand children's thinking but also the exchanges created opportunities for teachers to challenge children's thinking to make meaning in the moment and to stimulate new learning right "on the edge" of the child's thinking.

17.3.2 The Interactions

The interactions that were the focus of this study involved exchanges between a teacher and child where:

- The child's mathematical thinking was being challenged by the teacher;
- The teacher asked for an explanation, restatement or justification of thinking;
- The child had to mentally shape a response that required reflection and review;
- The exchange had a number of interactions that built towards a whole, often as a conversation or construction of an idea or examination of a concept; and
- Some learning resulted for the child and was demonstrated by an action or in some work.

17.3.3 The Teachers

Four case studies of teachers were undertaken. These teachers were identified in the original ENRP research as ‘highly effective’ teachers of mathematics with young children (Clarke et al. 2002). They taught children aged from 5 to 8 years old in the first 3 years of primary school. Each of the teachers had more than 20 years of classroom experience and was a specialist in teaching young children. They were also leaders of their early years teams in their schools. Their classrooms were very well organised, interesting and stimulating places which emphasised learning in collaborative communities (Brophy 2010). They planned complex and challenging mathematics programs for children. In addition they all had notable personal qualities such as a quiet determination and a businesslike demeanour together with a gentleness, affection for, and empathy with children. While they had the common features listed above, each was also different in many ways. The teachers will be introduced briefly here with pseudonyms and their specific practices will be elaborated later in the chapter.

- Georgia taught a composite Year 1/2 in a school with a low socio-economic profile and a high number of English as Second Language learners. She had completed her Master of Education in Early Years Mathematics Education and was interested in theoretical aspects of her professional life as well as practical ones.
- Jenny taught a class of children entering school for the first time in a large outer suburban school. Her personal style was quiet and modest and her leadership philosophy was very much one of leading from beside her colleagues.
- Sarah was especially interested and expert in teaching children in the 1st year of school. Her school had a high socio-economic profile. Sarah had maintained her interest in classroom-based research over more than 20 years.
- Maggie was leading her team and teaching a Year 2 in a school with a high socio-economic profile at the time of the research. She had also developed a professional profile beyond her school.

17.3.4 An Overview of the Findings

A cross-case analysis of the practices of the four teachers was undertaken to examine the factors at work (Cheeseman 2009). The main findings of the study showed:

- Mathematically challenging conversations happened on a daily basis in the classrooms of the teachers in this study (Cheeseman 2010).
- These mathematical conversations often formed ‘strings’ of interactions that occurred over lessons, days, and even several days to develop lines of mathematical thinking.
- Teachers and children remembered these intense exchanges.
- Children learned mathematics when challenged to think during these interactions.
- Teacher behaviours were crucial to the quality of the mathematical conversations.

These findings will be discussed in more detail with specific illustrative examples in the sections that follow.

17.3.5 Teacher Behaviours

A common characteristic of the teachers was that they valued listening to children's mathematical thinking and conversing with them about their understandings. This is a style of interaction which may be more commonly associated with preschool settings where children and adults converse about day-to-day events. However, the teachers in my study devoted time to listening intently to children. They chose potentially rich, often open, tasks which provided challenge and would provoke discussion. The skills displayed by the teachers were complex and subtle. They had a clear learning path in mind for the children in general, and for individuals in particular, and it was at the 'cutting edge' of each child's understanding that the teachers were attempting to work. They questioned and evaluated each child's conceptual development, put that knowledge into the context of a broad knowledge of a mathematical framework, then made 'on the run' judgments about how best to press for further understanding. It was this 'press' which created a challenge for the children and which was different in its nature from the more opportunistic conversational exchanges I have witnessed in preschool settings.

Sometimes the teachers offered linking ideas to connect the child to past experiences in mathematics, sometimes they offered a slight variation on the problem, and sometimes they asked a question that required the child to think generally about what had happened. They used a range of techniques to suit the event. Perhaps the most striking quality was the flexibility and the speed with which they considered and responded to situations as they unfolded. One of the interesting findings was the strings of mathematical interactions teachers had with individual children. They often followed a pattern: converse, challenge, withdraw to let the child accept and attempt the challenge, then to revisit to recap and to offer another challenge.

Five main categories of teacher behaviours emerged from the data. Central to the interactions between teacher and child were questioning, listening, and focusing on student thinking. Supporting and facilitating these behaviours were the mathematical pedagogical behaviours of each teacher. In addition there were behaviours that underpinned the functioning of the classroom and served to build a social and emotional climate that fostered successful, optimistic, and resilient children. Only the central behaviours of listening, questioning and focus on thinking will be discussed in detail here.

17.3.5.1 Intense Listening

When the teachers listened they gave the child their undivided attention. They watched what the child did in addition to listening to what was said. In the case of some teachers, their interactions were fairly brief, typically less than 1 min in duration; nevertheless they were concentrated and quite intense events. The intensity of focus of the teacher's attention created a challenge for the child to be coherent and reasoned in the explanation of their mathematical thinking. There were notable exceptions to the pattern of brief interactions when some teachers spent extended periods of several minutes in conversation with an individual child (10–12 % of the total lesson time in some cases). These were occasions when the teachers attended to a child's mathematical thinking privately. Other one-to-one conversations were held with children as a matter of habit, for example, one teacher planned to have a conversation about mathematics with each child every day. This might be during the lesson or at another time during the day.

There were other occasions when the teachers conducted intensive exchanges in public where other children were listener-participants; however, the teacher really was having an individual conversation with one child.

Children in this study expected mathematics discussion and explanation to be regular components of their mathematics lessons. However, the discussion was not a teacher-constructed, vaguely interactive style of reportage but a lively and creative orchestration of the children's ideas. While the mathematical discussions in prior-to-school settings may happen daily, in my experience they are not often expected by preschoolers as a regular part of their day.

Intense interactions where the teacher listened keenly to the child were common to the teachers in this study; however, they were enacted in very different ways by the teachers. The differences are encapsulated by four descriptions:

- Georgia's listening and challenging was often done in public in a whole class setting. The conversation was intense and Georgia turned her undivided attention to each child one at a time; however, other children were also expected to listen and to follow the thinking.
- Jenny made time to listen carefully to every child's individual explanations of their mathematical thinking every day. She worked intensively with individuals during the lesson and then the children she had not talked with were given time to tell her what they had done and what they had been thinking after the lesson. Each child expected a mathematics discussion and explanation session.
- Sarah's interactions with individual children were over an extended time. For instance, she held a pressing conversation with Michael for 6 min. During these long probing interactions Sarah sometimes drew other children into the conversation. She would ask whether the child speaking to her could provide an explanation to another child nearby. Sarah would also press for thinking and leave the child to work on something, returning later to listen to what had been done and issue a further challenge to think. So Sarah's intensive interactions were strung together in chains of interactions with several children each lesson.
- Maggie's interactions were short. However, there were notable exceptions to this general pattern of behaviour. In Maggie's mathematics classroom, incidental exchanges, sometimes as Maggie was nearby, often had the effect of creating a challenge for the child. Children knew they would be listened to, questioned, and tutored by Maggie.

It is worth thinking for a moment about the powerful messages both spoken and unspoken about setting the expectation in the minds of children that their mathematical thinking will be valued every day. For children in transition to formal school settings this is an important precedent to set.

To summarise, the teachers in this study listened intently, in a negotiated and participatory way, to their children's explanations of their mathematical thinking. Often the opportunities to listen were created by the teachers' questioning behaviours.

17.3.5.2 Challenging Questioning

Teachers listened to children talk, explain, and expand on their ideas. Then they would take what the child had said and shape the next question or challenge in light of the child's idea. As they listened to children they evaluated the potential of their thinking and decided whether to intervene by asking a question that required rethinking or reflection. Often the teacher would reframe the child's statement as a question. The teachers differentiated the level of challenge according to their knowledge of each child and the responses they gave to probing questions. Children knew they would be listened to and questioned about their mathematical thinking every day. In some ways this is an extraordinary situation for young children beginning school. Perhaps preschool teachers might be likely to listen and talk to children in depth every day.

Each of the case study teachers used a range of question types and had a different style of questioning. Examples from each teacher participating in the study highlight these differences:

- Georgia's questioning required some recall, for example, "Do you remember when …?" and "What is …?" Some procedural questioning was also often part of her practice, "How did you work it out?" But the main focus of her questions was "Why did you do that?"
- Jenny's questions often required children to think independently. For example, "How did you work that out?" "How do you know?" "Why did you choose …?" "Why do you think …?" She also used leading questions to shape children's thinking and lead them to the response she had in mind. In fact she had a wide repertoire of questioning styles that she used throughout her mathematics lessons as the situation demanded.
- Sarah's questioning was different from the other teachers' questioning styles in that she stayed with individual children for longer in sustained exchanges. She required not only a description of the children's mathematical thinking but a justification of it as well. For example, she asked Jordan: "So how do you know it is half?" Sarah asked repeatedly until Jordan had explained that two sets of counters he had formed were the same in terms of their numerosity. Sarah's questions demanded convincing arguments. Sarah also used questions to form the basis of on-going challenges. She had a thread of an idea that she developed with each child, she asked a challenging question, and then she left the child to get on with the task. A little later she came back to the child and followed up with another question or challenge. In this way her questions interlinked challenging exchanges.

- Maggie used questions in a similar way to Sarah, in that she interlinked them in chains of interactions over the course of a lesson, indeed between lessons as well. For example, Louisa had a conversation with Maggie at the end of one lesson, and Maggie asked her to show the class her idea at the beginning of the lesson on the following day saying, "What was that clever idea you talked to me about yesterday?" Maggie's challenging questions were often part of shorter interactions that lasted a couple of minutes, and she would often leave a child with a question, for example, "Do you think you could work that out another way?"

All of the teachers asked what de Bono described as Socratic, "leading questions" for which "step by step the listener gave the 'expected answer' to a question and so had to reach the conclusion that Socrates wished the listener to reach" (de Bono 2004, p. 78). To distinguish between leading questions and those of a different character, de Bono coined the terms *fishing questions* and *shooting questions*. "Shooting questions have a target and a range of possible outcomes, whereas fishing questions are open-ended and the questioner does not know what answer will be given" (p. 81). The teachers in this study used both types of questions, and a good proportion of them were 'fishing' questions.

Teachers had a wide repertoire of questioning techniques from which to choose in the seamless exchanges that daily took place in the classrooms. They were aware that questioning could be a powerful stimulus to think. Their pedagogical questions were enhanced by *true questions* where the teachers did not know the answer to the question they were posing. These true questions, or higher cognitive questions, were a regular part of the mathematical experience of the children (Burns 1985). For children in the transition to school the use of a range of questioning techniques provides the supportive structuring of mathematical thinking as well as the means of eliciting reasoning and mathematical justification. The listening and questioning by teachers in the study were intended to bring into focus the mathematical thinking of the children and this is the third of the central behavioural characteristics of the highly effective teachers that challenged children's mathematical thinking.

17.3.5.3 Focus on Children's Thinking

To challenge children's thinking it is necessary to understand the nature of children's mathematical thoughts. As Seo and Ginsburg wrote, "to take the child's perspective, understand the child's current intellectual activities and build on them to foster the child's learning" (2004, p. 25). Teachers in this study exhibited a range of behaviours with this goal in mind: eliciting behaviours; challenging thinking behaviours; requiring reflection on thinking; shaping explanation of thinking; and appraising thinking. Each of these behaviours will now be discussed.

Eliciting Behaviours Teachers' eliciting behaviours were intended to bring student thinking to the forefront, that is, to open the mathematics for discussion and consideration. Most of the time they did not 'tell' rather they used eliciting behaviours to lead children's thinking (Lobato et al. 2005). All of the teachers valued and validated student thinking. They challenged children to make decisions, used questions to probe children's thinking, and contested children's ideas. They asked children again and again to think (Cheeseman 2010).

Challenging Thinking Behaviours Children were challenged in different ways during each lesson, and these challenges were set at a high, but realistic, level for each child. Teachers scaffolded children to explore further mathematical thinking. In addition, they often capitalised on children's thinking by discussing children's ideas with the whole group. Strategic thinking was a focus of mathematics lessons, and children were expected to describe their various approaches to a problem. An accurate description of thinking was expected of the children, together with explanations and justifications of their reasoning.

Requiring Reflection on Thinking The teachers required reasoning. Further, children were expected to reflect, consider, and evaluate their thinking. Teachers also challenged the children to consider each other's thinking, to solve each other's problems, and to evaluate each other's thinking. They led children to the recognition of any errors in thinking, they did not 'tell' but guided children to reconstruct and consider their thinking.

Shaping Explanation of Thinking The teachers used a range of techniques to support and shape explanations of thinking. They shaped the story of a child's thinking to engage others, used prompts to support and to extend thinking, remembered specific thinking strategies children had used, required children to correct their thinking, raised questions and left the children to consider the mathematics.

Appraising Thinking Ideas were appraised. Teachers acknowledged successful mathematical thinking, alerted children to a good idea, and praised good thinking. They acknowledged and gave credit for half-formed thinking, and accepted errors as an opportunity to learn.

The way in which the teachers responded to children's thinking encouraged further description of thinking. There was often the straightforward acknowledgement of "Good", followed by the next question or exchange. The idea, not the child, was being evaluated for its mathematical potential. In Sarah's classroom Jordan, aged 5 years, reported his thinking to the class at the end of the lesson. As an aside, Sarah, very quietly and privately, said to him, "I didn't really know you could read these large numbers." Jordan's eyes met hers and a fleeting smile crossed his lips. This was appreciative and genuine praise (Brophy 2010).

Praise was given for hard work. Georgia's classroom, where effort was the most frequent subject of praise, was a prime example. Mueller and Dweck (1998) found that praise for effort had beneficial effects on motivation and led to students making the most of potentially valuable learning opportunities; "being challenged" and "learning a lot" were valued. Children who were praised for effort and hard work displayed more task persistence, task enjoyment, and better task performance.

"The idea of teachers listening to and understanding students' thinking has been widely promoted and supported in the education community" (Crespo 2000, p. 155). The notion of teaching as telling, speaking, and explaining is replaced by listening, hearing, and interpreting. The complexity of adopting a listening approach to teaching was noted by Ball (1993) who said that listening to children was hard work, especially when children's ideas look so different from standard mathematics. "The ability to *hear* [italics original] what children are saying transcends disposition, aural acuity, and knowledge, although it depends on all of these" (p. 388). Listening in order to hear the child's thinking is not the end of it "even when you think you have heard" (p. 388). Deciding what to do is an uncertain matter, and interpreting and responding adds another layer of complexity for the teacher (Ball 1993).

17.3.6 Creating "Learnable" Rather than Teachable Moments

One way to think about the listening approach to teaching is in terms of recognising or creating teachable moments. Hyun and Marshall (2003) claimed that the notion of *teachable moments* typically appears in the definition of the early childhood teacher's role. They described teachable moments as when:

> The learner indicates a readiness or interest through his or her own play, action or expression. The teacher, in turn, captures the moment, observes, recognises and interprets it by filtering it through his or her own personal and professional knowledge and beliefs, then considers, creates and presents some spontaneous, purposeful learning experience. The teacher then observes the child's response and interacts with, or intervenes in the child's learning moment (Hyun and Marshall 2003, p. 121).

Teachable moments in mathematics lessons involve noticing the moment when the child expresses interest or is offering to share some thinking, the teacher spontaneously responds. However, using a teachable moment implies that the teacher waits for the opportunity to arise, then acts. The incidents that were investigated in my study were often initiated by the teachers who were pro-active in the situation; they pressed, they questioned, and they required children's expressions of their thinking. So rather than waiting for teachable moments they created *learnable moments*.

The behaviours of teachers that are intended to challenge young children's mathematical thinking have been described but it is important to know about their effect on children. The next section details children's accounts of the events.

17.3.7 Children's Accounts of the Interactions

The one-to-one mathematical conversations young children have with their teachers are memorable to children. In their first years at school, children can recall their conversations with the teacher, reconstruct their thinking, and reflect on their learning (Cheeseman 2008). Fifty-three children who were involved in conversation with their teachers during mathematics lessons were interviewed about their mathematical thinking and asked to reflect on their learning. Video-stimulated recall was used with a conversational interview to prompt children's recollections and reflections on the same day as the interaction took place. Findings indicated that young children in the first 3 years of schooling were able to recall events in their mathematics lessons, to reconstruct their thinking, and reflect on their mathematical learning. Most of the children interviewed (45 of 53 children) could explicitly describe their mathematical thinking.

An analysis of the children's descriptions of events revealed an interesting three-way split of responses. Some children described only what they did (23 %). For example James (aged 5 years) could be seen on the video interlocking blocks but saying nothing:

Jill: So what was happening here?
James: My brain was counting and I wasn't.

Other children offered a description from their point of view. For example, Ali explained his counting of 5 groups of 5 teddies saying, "It goes 10, 20, 30, 40, 50 You have to count the ears". It is hardly surprising that more than a third of the children who could remember the event described it from their point of view. In fact a large proportion (28 %) described the event with some reconstruction of their reasoning at the time. For example Jessica (aged 6 years) was explaining how to weigh a dog, Joey, who would not stand on bathroom scales:

Jill: Can you tell me about your good idea for maths today please?
Jessica: I thought of holding Joey on the scales. I would know how much Joey weighed. So I hopped on the scales with him and I holded (sic) him. And then we took away 19 [from 28] because I was 19 and he was 9 and so that was 9 kilograms and that's what he weighed.

Forty-five of 53 children (85 %) could explicitly describe their mathematical thinking. For example Tom (aged 7 years) offered a thinking strategy for his classmates who could not count by four. His idea was to use a count by two.

Jill: Now [teacher's name] says that's a really complicated way to work it out I can't really hear what you were saying. She was looking at a page that had 8 legs and 4 things on each leg. How were you trying to work that one out?
Tom: Oh a different way. You know, when there's 8 legs and I was thinking if people didn't know how to count by 4, I was splitting 4 in half to make two on each side. Then I did two times eight equals 16 then I have to count by 2s up to 32 what it equals. I have to count by twos 16 times.

About one-third of the children who remembered facts talked in terms of numbers. For example, Annie (aged 5 years) who had been talking about measuring with a piece of string when asked what she learned said, "I learned that 9 + 11 = 20". While it is not possible to be certain from these data, it raises a question as to what young children think constitutes mathematics learning. Is learning mathematics equated to remembering numbers? Lindenskov (1993) found that students' learning can be influenced by their everyday knowledge of what mathematics is. She was also struck by "the students' perceptions of details, even small ones, both in the teaching and in her/his own learning" (1993, p. 153). Certainly the children interviewed for this research described their learning in detail. For example, Tom talked about his learning saying,

Tom: I think I might have learned some new times tables.
Jill: Oh so you sort of had to figure some out?
Tom: Yes.
Jill: In which times table?
Tom: I think some were like nine times six. I didn't know that but then I knew it because I just counted by 6 nine times.

Some children learned a skill, for example Jordan, who "learned how to count by nines". Importantly about one fifth of the children (11 of 53) reflected on their learning at a conceptual level. For example, Tahani reflected on a lesson where the teacher intended to introduce multiplicative thinking, saying she learned "about groups, to make groups and to count them altogether and I learned to count by sixes."

Young children gave accounts of events from their perspective and could recall at least part of their conversations with the teacher during the day's lesson. These interactions appear to have some lasting effects.

If mathematical conversations that challenge children to think about their mathematical understandings are a critical factor in their learning, then knowing that many young children remember these conversations and can reconstruct their thinking is an important finding.

17.4 An Example of a Classroom Conversation about Mathematics

To illustrate the exchanges that happened between the teacher and child over the course of an hour in the mathematics classroom, the story of Isabella will be recounted here. It was a day towards the end of the child's 1st year of formal schooling and at the time Isabella had just turned 6 years old.

17.4.1 The Story of Isabella

Isabella was very actively involved in the lesson from the outset. She gave lots of eye contact and was eager to participate. The interactions she had with Jenny through this lesson were of several different types. The first was an interaction initiated by Isabella.

17.4.1.1 Interaction 1

In the introduction to the task, Jenny was talking about a catalogue showing things that Dad might like for Father's Day. Isabella volunteered that her daddy wanted a T-shirt. She was clearly connecting the task of the day to her everyday life. This conversation followed:

Isabella: My daddy wanted that T-shirt.
Jenny: Which T-shirt?
Isabella: The one on the right.
Jenny: The T-shirt on the right. The white one? This one?
Isabella: The one in the middle.
Jenny: Oh the one in the middle. Daddy wants that one.
Isabella: He would like the motorbike.
Jenny: Yes I bet he would like the motorbike, [to everyone] but do you think they're selling the motorbike in the shop?
Children: No.
Jenny: No they're just selling the clothes.

Jenny accepted the comment volunteered by Isabella; she sought clarification and joined in the spirit of the joke about the motorbike. She seamlessly connected the exchange to the purpose of the task she was setting up by identifying what catalogues were selling and identifying how much items cost.

17.4.1.2 Interaction 2

During a group work session, Jenny had an exchange with Isabella that began with her asking Isabella to read and interpret the numbers she had selected and quickly moved to an exchange that challenged Isabella's thinking. This was the conversation:

Jenny: What have you cut out for Daddy? And how much are those?
Isabella: Twenty-three dollars, ninety-nine.
Jenny: So how much are you saving? How much are you saving?
Isabella: Six dollars.
Jenny: So they're on special. If they weren't on special do you know much they would be?
Isabella: Twenty-three.
Jenny: No, you have to pay six dollars more. Do you know what six dollars more would be on 23?
Isabella: Twenty nine—$ 29.99.
Jenny: Yes, so is that a good bargain, to buy them now while they are on special?
Isabella: Mm.

Having established that Isabella knew the price and could recognise the advertised saving, Jenny then asked the question, "If they weren't on special do you know how much they would be?" This was a difficult conditional question for a 5–6 year-old. When Isabella couldn't follow the idea Jenny restated and clarified the problem without any conditional reasoning involved in the question. The direct question required Isabella to calculate 23 plus 6. Even stated in a direct form, this was quite a difficult mental calculation and certainly beyond the scope of the intended curriculum (Australian Curriculum Assessment and Reporting Authority 2010).

17.4.1.3 Interaction 3

Another exchange challenged Isabella in a group work setting. Jenny had been having a conversation with Lachlan and knew that Isabella had been listening.

Jenny: [To Lachlan.] Have a look this is $ 99 and this is $ 999. So how much more is this? [Jenny waited for Lachlan. When he looked perplexed she turned to look at Isabella.]
Jenny: Do you know?
Isabella: Nine hundred dollars.
Jenny: Nine hundred dollars more. [Looking at Lachlan] Oh your Dad is getting some expensive presents.

Drawing Isabella into the challenge that was originally meant for Lachlan and crediting her with the knowledge to solve the problem, Jenny acknowledged Isabella's mathematical thinking skills.

17.4.1.4 Interaction 4

In the summary section of the lesson Jenny called on Isabella to make a teaching point about the notation of money. Michael was showing his work (Fig. 17.1) to the class and Jenny wanted to reiterate the purpose of the decimal point in reading and writing money. Michael had made an error and Jenny used this as an opportunity to reteach a skill.

Jenny: Let me see, Aaron can you read that number?
Aaron: Ninety-nine.
Jenny: Not ninety-nine.
Jenny: Isabella?
Isabella: Nine ninety-five.
Jenny: It is nine ninety-five and you know that but by looking at the number that he's written would that tell you it's nine dollars and ninety-five cents?
Isabella: No.

Fig. 17.1 Michael's finished work

Jenny: What would it tell you?
Isabella: Nine hundred and ninety-five dollars.
Jenny: It looks like $ 995. We need the dot again to show us that.

Jenny used Isabella as an authority in the reading and writing of money amounts.

17.4.1.5 Interaction 5

The final interaction between Jenny and Isabella happened while the rest of the children were eating lunch together and Jenny sat with Isabella at a classroom table. Jenny was challenging Isabella to find the total cost of the items that she had selected from the catalogue. It was interesting to note the four different types of notation of prices of the catalogue items that Isabella had selected. From left to right in the photograph (Fig. 17.2) they are; the use of the dollar sign and a decimal point, large numerals signifying dollars with cents written in smaller digits, an exact amount of dollars showing no cents, and the use of a dollar sign and the cents written in small digits. The keying in of money on a calculator to find a total is a challenge for young children under the best of circumstances but this mixed notation created an extra difficulty. Some young children can work with decimal numbers at an early age when they have access to calculators at school (Groves and Cheeseman 1995).

Jenny and Isabella were in conversation over a calculator. Isabella was entering her prices and Jenny was checking the display to see which buttons she had pressed. So the process of entering the figures and adding each in turn progressed. Jenny told Isabella to leave the $99 till last (see Fig. 17.3).

Then Jenny asked Isabella what she was going to press for the $ 99. Isabella said, "ninety-nine". Jenny then asked how many cents were in the price and led Isabella to recognise that they were none and that they had to be entered as "zero, zero". The total was then read and Jenny recorded $ 181.77 on the paper (Fig. 17.4).

Fig. 17.2 Isabella's work sample

Fig. 17.3 Jenny working with Isabella to sum money

17.4.1.6 Isabella's View

In an interview later Isabella talked about what she had learned in mathematics as "adding up the numbers" and described what she did:

Isabella: I was trying to get all of these onto the calculator.
Jill: What were you doing that for?
Isabella: So we could work out how much you pay because I didn't know, because I couldn't work it out in my head.

When Isabella first described how to key the prices into the calculator she read the figures from her paper without any reference to their value, "I just type in 2399 plus 883 plus 88399 plus 495." However, later in the interview when she was asked, "What's that dot about again?" Isabella said, "So we know that they are cents and small we already know." She had learned some of the different notation conventions for writing money and had also remembered how to write money amounts onto a calculator. She showed me how to do that when we went back to the classroom and could get a calculator. This was a wonderful opportunity to see what she had understood from a challenge that had clearly been prompted by Jenny. She handed me the calculator then told me the keystrokes. We began entering the prices on the page from left to right and when I came to the $99 Isabella said that I should "leave that till last". I followed her instructions and having entered all of the other prices we went back to the $ 99. She then explained to me, almost exactly as Jenny had explained to her, that we need to enter 2 zeros for the cents because there were none in $ 99. She then checked the total on the calculator to see that we had the sum correct. Isabella had convinced me that she had certainly learned something in mathematics that day.

During the exchanges with Isabella, Jenny:

- Connected the mathematical task to her life experience;
- Challenged her to read and mentally calculate amounts of money;
- Expected her to explain to others her knowledge of the notation of money;

Fig. 17.4 Jenny recording the total

- Required Isabella to transfer her knowledge of the notation of money to the calculator to find a total; and
- Asked Isabella to describe what she had learned and to teach the same skills to others.

The story of Isabella exemplifies an ongoing conversation where the teacher picked up and left off the mathematical discussion with a child. In this case the conversation stimulated Isabella to engage with the task, challenged her interpretation of and calculation with numbers, invited her to solve another child's problem, asked her to contribute knowledge, and offered an intensive tutorial to apply new skills to solve a difficult problem. Jenny's challenges to Isabella were; to see the connection of the school mathematics task to her life, to mentally calculate, to share her skills and to extend her skills to a new and more difficult context.

17.5 Implications

Based on the research described, I offer three main recommendations for teachers of young children and teacher educators.

1. **Plan time for individual mathematical conversations with children**
   The daily individual mathematical conversations that the teachers had with children took planning and forethought. Not planning in the sense that the teachers needed to know exactly what they were going to say and do, but in the sense that in considering the lesson content and structure, they chose tasks that created *teachable moments* and allowed them to spend time with individuals conversing about their mathematical reasoning and extending their thinking with challenging questions. The planning involved making time within each mathematics lesson, to listen to children. Teachers needed time to reflect during the conversation in order to think about the children's insights and to consider how to frame the next challenge. Therefore planning for mathematical conversations to occur is essential if children are to be challenged to think mathematically.

2. **Expect thinking of children including conjecturing, reasoning, justifying**
   Teachers had high expectations of the children's abilities to conjecture, to reason and to justify their mathematical thinking. These expectations were often expressed as questions. In the *Position paper on early childhood mathematics* the Australian Association of Mathematics Teachers (AAMT) and Early Childhood Australia (ECA) advised teachers to:

   > Encourage young children to justify their mathematical ideas through the communication of these ideas in ways devised by the children that display the appropriate levels of mathematical rigour (2006, p. 2).

The children were expected to use the skills of young mathematicians. These high expectations of children transitioning from prior-to-school mathematical reasoning to more formal school mathematics create a sense of demand from their teachers and children rise to the challenge. Expecting higher order mathematical thinking of young children presents them with an achievable challenge. Therefore expecting rigorous thinking of young children, including conjecturing, reasoning and justifying, is a way to enhance their mathematical development.

3. **Consider tasks and their potential to engage and extend children's thinking**
   The teachers in the study chose tasks for their potential to engage and extend children's thinking then used these tasks skilfully. They took time to let the children explore, investigate and to some extent struggle with the mathematical content of the tasks. If there was no challenge for a child, each of the teachers increased the level of complexity or extended the task, so as to take the child's thinking into new mathematical territory. The teachers were in no hurry for children to reach solutions and were prepared to wait for children to think through the mathematics. Encouraging children to think things through is considered essential:

   > … while materials may be important in young children's development of mathematical ideas, these ideas are actually developed through thinking about action—children need to be encouraged to engage in mental manipulation of mathematical ideas (AAMT and ECA 2006, p. 2)

Teachers who were part of the study showed the need to be flexible in their interactions with children. They adjusted the level of mathematical challenge according to their knowledge of the children's mathematical development. They also allowed some choice by children in their completion of tasks, thereby adjusting the mathematical challenge. The teachers evaluated the mathematical thinking of a child through conversation then formulated a question that would take the child's thinking a little further. This skill required not only a depth of knowledge but also a flexibility of thinking on the part of the teacher. Therefore, the potential of tasks to engage and extend children's mathematical thinking is important. Teachers also need to consider flexible creative ways that they can then interact with children to reach the full potential of the mathematical opportunities offered to the child.

17.6 Future Directions

One of the reasons for embarking on the research in the first place was to understand more fully what highly effective teachers of mathematics with young children do in their everyday practice in school. Having documented some of their behaviours that impact on children's learning, in particular their daily mathematical conversations, one of the natural questions to explore into the future is, "Can this style of one-to-one interaction be learned by others?"

It would also be fascinating to learn how teachers of children who are transitioning to school establish the classroom culture that makes individual mathematical conversations possible and how teachers induct young children into a mathematical community where formal mathematical discourse is expected. Greater knowledge of the informal mathematical language that children bring to school is needed to understand how teachers capitalise on this language and how the language becomes more formalised through experience.

Another future direction would be to see whether extended mathematical conversations happen before children start school. From my experience many of the conversations in prior-to-school settings are opportunistic and to some extent rely on teachable moments. It would be interesting to investigate what teacher-child mathematical conversations occur and to consider the implications of the findings for children in the transition to school.

17.7 Concluding Remarks

It is possible to challenge children's mathematical thinking daily during one-to-one conversations about mathematics whether this is in the home, at preschool, or at school. Children have shown that these conversations are memorable for them and lead to mathematical understanding. Teacher-child dialogue is the essence of teaching and learning mathematics. From my experience, there can be no doubt that the intense interactions that highly effective teachers have with children challenge young children to examine their mathematical thinking. It is at such times that leaps of thinking take place as teachers elicit children's mathematical knowledge and support their construction of new thinking. The behaviours exhibited by teachers reveal a genuine respect for children's thinking, a deep knowledge of the way children typically learn mathematics, and the use of highly skilled mathematical pedagogies. These teacher behaviours enable children to transition from incidental and rather informal conversation to purposeful conversations which begin to shape and to formalise mathematical ideas for young children in the early years of school.

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Chapter 18
Transition to School: Supporting Children's Engagement in Mathematical Thinking Processes

Liz Dunphy

Abstract Internationally, mathematising is now a key focus in mathematics education for children aged 3–8 years (Perry and Dockett, Handbook of International Research in Mathematics Education, 2008; National Research Council (NRC), Mathematics Learning in Early Childhood: Paths Towards Excellence and Equity, 2009). For young children mathematising involves going back and forth between abstract mathematics and real situations in the world around them (NRC, Mathematics Learning in Early Childhood: Paths Towards Excellence and Equity, 2009). It helps children to make sense of mathematics by connecting it to their everyday lives. While mathematical thinking processes associated with mathematising begin in early childhood (Paley, Molly is Three: Growing Up in School, 1986; Tizard and Hughes, Young Children Learning, 2002), educators may not always recognise and promote children's engagement with these processes (Dunphy, International Journal of Early Years Education, 17(1), 3–16, 2009). In this chapter I present examples of children involved in mathematising. The discussion addresses issues that arise in supporting mathematisation during the transition to school, as well as directions for future research.

18.1 Introduction

This chapter explores how 4-year-old children mathematise in a one-to-one interview context. A decade ago, for my doctoral study of four-year-old children's number sense I interviewed a purposive sample of eight boys and six girls (age range 4 years 1 month to 5 years 1 month). The interviews focused on children's perceptions of number (e.g., Dunphy 2005, 2006). The children were in their first month of primary school in Ireland. In this chapter I revisit the data, focusing now on children's engagement in high-level discussion about their mathematical thinking, rather than on their facility with number. The purpose of the chapter is to exemplify the range of ways that the children responded as they engaged with me in the tasks presented.

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In my discussion, I focus on some of the factors at play in early mathematics discourse between children and adults in the context of individual interviews/conversations. I argue that in assessing children's efforts at mathematising, due consideration must be given to their perceptions of the tasks presented, and how this affects the ways that they mathematise in a given situation. I present practical implications for promoting children's mathematising during transition to school. Finally future directions for work in this area are signposted.

18.2 Mathematisation

For Freudenthal (1973) learning of mathematics meant involvement in *mathematisation*. Treffers (1987) distinguished between two different forms of mathematising: horizontal and vertical mathematising. In horizontal mathematisation, the learner develops mathematical tools or symbols and uses them to solve real-life problems. In vertical mathematisation, the learner makes connections between mathematical concepts and strategies and moves within the world of symbols.

Mathematising can involve a number of processes such as reasoning, representing, problem-solving, connecting and communicating (NRC 2009). These are described in considerable detail in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics (NCTM) 2000). Each process is described with reference to one or more of the others, thus demonstrating the close interrelationships between them. For instance, in relation to reasoning and proof, the processes of explaining, justifying and argumentation, are highlighted. Representing is seen to involve processes such as selecting, creating and comparing representations of mathematical situations and problems. In relation to problem-solving, the processes of explaining and generalising are mentioned. Connecting is seen to involve looking for and making connections, and applying previous mathematical learning in new situations. Communicating is linked to the processes of presenting, explaining, arguing and justifying. Perry and Dockett (2008) identify argumentation as being of particular significance in a mathematics curriculum for early childhood, given its importance as the basis of mathematical proof in later years.

The difficulty of distinguishing between the processes is obvious, especially in the case of young children, most of whom are as yet only learning about the sophisticated and complex use of different levels of language (Tizard and Hughes 2002). Efforts to define individual processes, in particular the process of reasoning (NCTM 1999) demonstrate the difficulty of universal definitions. It seems that rather than defining, the more fruitful course is to characterise them, using examples of young children engaging in particular processes.

In the United States, the NRC Report (2009) offers the following description of young children engaged in mathematising:

> Mathematizing happens when children can create a model of the situation by using mathematical objects (such as numbers or shapes), mathematical actions (such as counting or transforming shapes), and their structural relationships to solve problems about the

> situation. For example, children can use blocks to build a model of a castle tower, positioning the blocks to fit with a description or relationships among features of the tower, such as a front door on the first floor, a large room on the second floor, and a lookout tower on top of the roof (p. 44).

However this description does not sufficiently illustrate the breath of responses that children may engage in when invited and supported to mathematise. Also, the role that language and discourse play in children's early efforts to mathematise has not as yet received much attention. Clear explication and exemplification of the *range* of ways in which young children may demonstrate their engagement in mathematising is important in developing and adjusting overarching frameworks, and also in supporting teachers' optimal scaffolding of children's efforts in this regard.

18.3 Opportunities for Young Children to Mathematise

While processes such as numerical reasoning, justifying and representing may be modeled for children by adults in the home when sharing everyday activities such as baking (Tizard and Hughes 2002), as children make the transition to school the situation changes. They may have fewer opportunities for one-to-one access to an interested adult who models and supports important language structures in the context of mathematics (Perry and Dockett 2008). More generally research recognises the need for extended and sustained conversations in small-group and one-to-one situations, to support young children's engagement with thinking processes (Siraj-Blatchford et al. 2002).

During the transition to school rich environments and interactions with adult and peers are important, but the critical intervention of the teacher who not only recognises opportunities to encourage and support mathematisation but who proactively seeks to engage children with mathematisation processes is increasingly seen as a pedagogical imperative. Ginsburg (2009, p. 415) points out that the educator's role is to support children in their efforts "to interpret their experiences in explicitly mathematical form and understand the relations between the two". This support is often offered in the course of everyday activities in the early education settings, perhaps during play time (Paley 1986) or while children are engaged with a picture/story book (van den Heuvel-Panhuizen and Elia 2012). However teachers also need to design activities which seek to proactively offer children opportunities to mathematise.

18.4 Early Mathematical Discourse

The language structures used in mathematising begin to develop in early childhood but need to be supported and developed in the context of mathematics (Ginsburg 2009; Perry and Dockett 2008). As with most of the disciplines that children are introduced to at school (Wells 1992) ways of learning, doing and communicating mathematics are heavily dependent on conversation. In recognition of this, the Australian Association of Mathematics Teachers (AAMT) and Early Childhood Australia (ECA) recommend that early childhood educators should adopt pedagogical practices that focus on the use of language to describe and explain mathematical ideas. They advise that educators should recognise the role that language plays in learning, and encourage young children to justify their mathematical ideas and communicate these in ways that are devised by the children themselves and which show appropriate levels of mathematical rigour (AAMT and ECA 2006). Similarly, in the United States the need to use curriculum and teaching practices that strengthen children's problem-solving and reasoning processes, as well as their ability in representing, communicating and connecting mathematical ideas is emphasised (National Association for the Education of Young Children (NAEYC) and National Council for the Teaching of Mathematics (NCTM) 2002/2010).

Wells (1992) argues that the nature and quality of the conversation between child and teacher needs to be such that it can be described as discourse. He defines discourse as "interactive and constructive meaning—making that occurs in purposeful interaction with others" (pp. 286–287). He sees it as both a means and a goal of teaching. Sfard (2007) distinguishes between language and discourse. She identifies language as a tool and discourse as an activity in which the tool (one of several) is used. For Sfard, knowing mathematics is synonymous with the ability to participate in mathematical discourse. Since it involves symbolic artefacts, this ability has to be learned. From a mathematics point of view what is important here is that the discourse is of a particular form, one that involves reasoning and justifying and other processes associated with mathematisation (Lampert 1998).

For many children mathematising with the teacher may be challenging at first. Some children starting school may have had limited experience with a discourse which utilises complex language structures such as those involved in mathematising. We know that this language style is less likely to be familiar to children from low SES groups (Ginsburg 2009). We know also that young children generally do not describe their mathematical thinking very adequately or very often, and that children from lower SES backgrounds are more likely to have difficulty with understanding problems presented verbally and with expressing thinking. Modeling talk about mathematical thinking is important for promoting mathematising, as is supporting children's efforts to explain their mathematical thinking. For instance, research indicates that children can become more comfortable and more able as explainers when they are supported by a particular social climate. With four-year-old children an environment such as that afforded by the (clinical) interview provides one such context (Pappas et al. 2003).

Gutiérrez et al. (2010) argue that communication in human learning is more than language. They see it rather as the embodied expression of human experience. From this perspective one cannot view language separately from other semiotic means such as artifacts. Rather, mathematical understanding involves multiple modalities and artifacts such as gesture, the body, symbols, representations and so on. While language plays a role in the overall development of mathematical discourse, children may express mathematical reasoning, represent mathematical knowledge, explain mathematical thinking and understanding, and communicate by other means such as movement or gesture. A key point expressed by Gutiérrez et al. (2010) is that there are multiple forms of mathematical discourse, and these are closely connected to everyday discourse and indeed developed from that base. An essential factor in recognising children's engagement in mathematising processes in the transition to school period is the recognition that mathematical discourse for young children is not separate from everyday discourse, but rather a blending of the two. Similarly, Ginsburg (2009) sees young children's engagement in mathematising processes as a bridge between everyday and scientific knowledge. Consequently, we should not expect an easy transition to conventional mathematics discourse when children enter school.

The recent focus on children communicating and mathematising (NRC 2009) prompted me to revisit my data of a decade ago. Foremost in my mind was Ginsburg's (1997) assertion that the form of intensive discourse that characterises one-to-one interviews with children may result in the child being involved in introspection, in understanding the language of cognition and in expressing thought, processes that are at the heart of mathematising. Also Doverberg and Pramling's (1993) suggestion that the interview can serve to stimulate children to think about things that they had not done previously was to the forefront as I reanalysed the data.

18.5 The Data

Here, I report on my analyses of the responses of three children to Tasks 5, 6, and 8 (See Appendix). I selected this particular data since they clearly illustrate the opportunities for mathematisation offered within the interview context. They show the range of responses offered by children in communicating, justifying, explaining, reasoning and argumentation.

The interviews were characterised by sensitivity to individuals, strong efforts to establish intersubjective understandings with children, a desire to capture their thinking in all its nuances, and an ethic of respect for children's ideas, opinions and explanations (Doverberg and Pramling 1993; Ginsburg 1997). The aim of the interviews was to ascertain children's number sense in relation to specific tasks. I built almost all the tasks around a birthday theme (that of the fourth birthday of a then-popular toy character, Coco (a monkey), who was associated with a well-known breakfast cereal). Several tasks revolved around 'games' with Coco. However, even a decade ago, I was aware that this approach was not without its effects in that the discourse associated with such games might not do justice to children's understandings and skills (Walkerdine 1988). A number of the tasks focused on the artefacts associated with Coco's birthday party (e.g., candles, presents). Based on my review of the literature, and from my teaching experience with young children, I felt that overall the theme and the strategies employed offered considerable potential for exploring young children's number sense at the point of transition to school (Dunphy 2005, 2006).

18.6 Analytic Frameworks

In the original study, Rogoff's (1995) three planes of analysis provided me with the lenses within which to view the activity, with analysis focusing on the individual plane (Dunphy 2006). In the intervening decade, the principles of cultural-historical activity theories have been further articulated in both the early childhood literature (Anning et al. 2009), and also in the mathematics education literature (Roth and Lee 2007; van Oers 2010). Activity theory is a development of aspects of Vygotsky's work (Engeström et al. 1999) that has been used particularly in relation to language, language learning and literacy. Its implications for mathematics learning are only now being articulated. Cultural–historical activity theory is characterised as a framework which focuses on culture, diversity, multiple voices, communities and identity (Roth and Lee 2007). It focuses on the joint activity in the learning situation, rather than on individual learners. Activity theorists claim that making activity the focus results in a holistic view of learning (Roth and Lee 2007).

Gutiérrez et al. (2010) suggest new lines of inquiry arising from cultural-historical activity theory. These include thinking about how young children demonstrate and develop competence in the language and discourse of mathematics; how they communicate that for them mathematics is an embodied practice/activity; how language, discourse and other tools work together in a multi-modal and multi-semiotic way to mediate mathematics learning. My analytic approach in preparing this chapter was to examine the data with these issues in mind, and to then consider how the findings might help in recognising and understandings the range of ways in which young children mathematise in an interview situation. I also sought to acknowledge the importance of a diversity of frameworks in interpreting the data.

In the analysis presented below the task is the engagement in the processes of justifying, explaining, reasoning and so on. From this perspective, the children used tools such as language, a particular action or resource to mediate knowledge in interactions with me. My role was to support children's mathematising.

18.7 Findings

Here I show how three children responded to the interview situations in which they are asked to mathematise i.e. to engage in processes such as explaining, justifying, connecting and communicating in relation to specific numerical tasks. The same pseudonyms that were used in the original study are used for the children here.

18.7.1 Shay's Engagement in Mathematising

In Table 18.1 three excerpts from Shay's transcript are presented, one in relation to each of the Tasks 5, 6 and 8 respectively.

Table 18.1 Shay responds to Tasks 5, 6 and 8

Excerpt T5
L: How many presents has he got?
S: Four [Subitises]
L: Are you sure?
S: Yes [Counts]. I was just making proof for you. It's easy for me to do it
L: Now how many has he left now? [I remove one]
S: Three
L: How did you know that?
S: Because you … I counted four to make proof for you
L: That's right
L: Seven presents and we take away that one. How many have we now?
S: Five [subitises]
L: Are you sure?
S: Yes [he counts six presents]. Oh because…eh…you know like …it's confusing
L: Why is it confusing
S: Because…eh…you know when you take presents away from people…and they have so many
L: Right…
S: But you don't need …eh…presents…eh…you don't need presents the same so…the same number as he's going to be
L: No you don't…he doesn't always have to have four presents because he's four…you're absolutely right
S: He doesn't have to have four
L: Right…
S: He can have any size
L: Any number of presents…because when you were four did you have more presents than four or less than four?
S: More than four!
L: Okay…now pretend that he has three presents
S: Because he is three
Excerpt T6
1 and 4
L: How many did I hide?
S: Four
L: How did you know?
S: Because there's one left
L: Right…so what does this mean?
S: Em…That I'm so clever Alec
L: How did that tell you that there were four under the basket?
S: Because after four it's five

Table 18.1 (continued)

3 and 2
L: How many did I hide?
S: Two
Excerpt T8
L: How do you know?
S: I though you put them all in…but when I turned back it nearly gave me a heart attack
L: Did it…how many did I hide?
S: Two
L: How did you know?
S: Because when I turned back I saw that there was five

In his response to Task 5 Shay's awareness of the discourse of mathematics is clear in his use of the word *proof*. His competence in the use of a mathematics discourse to convey mathematical proof is also evident in his response to Task 6. In his response to Task 8 his ability to explain his thinking is demonstrated as is his satisfaction and enjoyment of the activity of mathematising. Also evident is the confusion he experiences as he seeks to reconcile the situation I present about presents being *taken away* from Coco (the monkey character around whom the narrative for the Tasks is constructed), and his everyday understanding about the conventions of present-giving. While not stated overtly, Shay seems to struggle with the idea of presents as something that one can *take away*, perhaps because it contradicts his expectations or experiences. In seeking to make meaning he justifies *taking away* the presents by referencing age. First he argues that since Coco has so many presents and he is only four, then this mismatch between the quantities is the justification to take some away. He clearly communicates his reasoning, though his uncertainty as to the relevance of age is also evident. He continues to puzzle on the interrelationships between age, number of presents and the *taking away* of presents as is evidenced by his final remark *Because he is three*. His response to Task 6 is interesting in that it shows the very different way in which Shay and I understand the word *mean* here. While I use it in a way that I think invites him to reason mathematically, his interpretation is more in tune with everyday usage.

18.7.2 Tom's Engagement in Mathematising

In Table 18.2 below three excerpts from Tom's s transcript are presented, one in relation to each of the Tasks 5, 6 and 8 respectively.

Tom's responses clearly demonstrate the extent to which for him the tasks and the context are implicated. The fact that the tasks are situated within a birthday narrative defines how he responds to them. From his perspective, all efforts to explain, reason, justify and problem-solve must be made with reference to the context of the birthday narrative. This is clear from the ways in which he reflects, connects, argues

Table 18.2 Tom responds to Tasks 5, 6 and 8

Excerpt T5
T: I know what they are. Christmas presents!
L: No, they're Coco's birthday presents. Let's pretend they're his birthday presents. Okay?
T: What's in it? Oh … I think I see…no it's paper Are you going to open them now?
L: No. let's leave them there. Now, how many birthday presents has he now? [I show 4 …then remove one].
[No difficulty with this task … subitises most times but counts anything above 5 items].
T: Are you going to open them now?
L: No. let's play a party game with them.
Excerpt T6
[I explain the game we'll play].
1 and 4
T: Will we play pass the parcel?
L: How many can you see?
T: One
L: So how many are under the basket?
T: Three
L: Why do you say three?
T: Five
L: Which is it now…think really hard about that
T: Five
L: You think there are five under the basket? Why?
T: I saw them…through the holes. Let's…I want to play this game …pass the parcel
3 and 2
L: How many can you see?
T: Three
L: So how many are under the basket
T: Two
L: How did you figure that out?
T: I just knew there was three there
L: How did you know there were three?
T: That's why you said to me to open my eyes
Could we do the tape recorder again? [We listen to the recording]. Let's do that 'Cover my eyes' again
Excerpt T8
L: This is going to be a game that you're going to do in your head. Let's pretend that Coco had three presents and he got one more …how many had he then?
T: Hm?
L: He had three presents and he got one more how many has he now?
T: None
L: We'll just say it again…he had three presents…and then his Mammy gave him one more

Table 18.2 (continued)

T: Yes
L: How many has he now?
T: One
L: Are you sure? Altogether? [He nods].
L: Now he has seven presents…and his Mammy gave him one more…how many has he now?
T: Em…two
L: Are you sure?
T: Yes
L: Okay…now are you listening to me really hard…he has six presents …that's a lot of presents
T: Six
L: And then Daddy came home and gave him another one…now how many has he?
T: Eight
L: Now the next day when he counted his presents he had nine altogether and Granny came and gave him another one…how many has he now?
T: Ten
L: Yes. And then another day he looked at his presents and he had only three.
T: Where are the other ones gone?
L: He must have put them away somewhere…but he has only three presents on the table and then Grandad gave him one more …how many has he now?
T: Four. Do you know all Coco's friends?
L: I know some of them…and then he got another one
T: From who?
[No problem with initial items in this task when repeated at this point]
L: He had six on the table…and one fell off
T: Were they under the table?
L: They were on the table and one fell off…six of them
T: Did one…was there a little hole in the table?
L: No it just fell off the side
T: Yes…and he counted all them there
L: Yes
T: And one fell down and then it went under there
L: Yes…so how many were left on the table?
T: Six

and communicates across all three response excerpts. Tom seeks to apply a situated logic to the problems posed. Such is the distraction, or even attraction of the *presents* scenarios, that Tom's meaning making is focused on these. While he hears and responds to my efforts to engage him in mathematising, it is only after pressing with me a number of aspects of the narrative that he is ready to engage with my focus. In my field notes, I noted his desire and anxiety to clarify aspects of the *Coco* narrative and the way in which he paid such close attention to unexpected aspects of the situation such as what had happened to the presents (T6). It is interesting to note that the introduction of the phrase *another one* in relation to Task 8 seemed to temporarily engage Tom with my focus. Tom's everyday reasoning seemed to dominate his responses to the tasks, as is indicated in his question *Where are the other ones gone*? It is clear that Tom's interest is more on the narrative than on the numerical aspects of the situation. Only occasionally (e.g., T8) did he appear to focus on exclusively numerical aspects of the situation.

18.7.3 Sile's Engagement in Mathematising

In Table 18.3 below three excerpts from Sile's transcript are presented, one in relation to each of the Tasks 5, 6 and 8 respectively.

The case of Sile provides a good illustration of where engaging a child in mathematisation processes makes visible an important aspect of mathematics which needs to be better understood (i.e. counting) in order that her justification and reasoning make sense in a mathematical context. Sile's response to Task 5 clearly indicates the extent to which she uses her everyday experience (of her birthday party) to engage with me about the Task. Sile's ability to use the counting procedure to answer the question *How many?* might be seen as unreliable given her response to the five-item array. However, I suggest that her attention at this point is focused on what for her is central, the issue of justifying the fact that Coco has lots of presents with the argument that he has lots of friends. Her counting is compromised here, or maybe just not as important or interesting to her as the justification of this puzzling situation.

Her follow-up justification to an apparently correct response to the *3 and 2* item (T6), and the justification she offers into the *1 and 4* item together offer interesting insights into her perspective on the use of counting as a response to number-related problems. One interpretation might suggest that during this transition to school period one of the key issues for Sile is that of trying to understand when counting is a useful tool, and when it is not. Her explanation of her response of *none* to Task 8, where no objects were present to count, might suggest confusion for her as to how to respond when the option of counting in response to number-related questions is not available. Could it be that the narrative she presents as a justification for her response is in fact how she reconciles the fact that there are no objects (presents) visible, with a *How many*? question? Alternatively as I reread the transcript once again, it occurred to me that the explanation could equally be a linguistic confusion on her part between *lost one* and *last one.* This illustrates clearly the usefulness of considering the data using different frameworks.

In the examples above, the challenges for children seeking to combine previous discourse experiences with the discourse introduced in the new situation is apparent. This new discourse is characterised by what O'Connor (1998, p. 29) refers to as "school objects and purposes". She argues that "when the learner enters new, more complex forms of school-based discourse, it may be that previous experiences are

Table 18.3 Sile responds to Tasks 5, 6 and 8

Excerpt T5
L: I want you to have a look here at the little presents for Coco. Let's pretend these are what he is getting for his birthday.
S: Presents. But we can't open them.
L: No we're not going to open them. Okay, how many presents has he here?
S: One, two, three, four. Four.
L: Right. Supposing we take away that one. How many has he now?
S: One, two, three. Three. Well, he can have … When I was four I got loads … a hundred presents.
L: A hundred presents? Well let's see how many he has now? [I place 5 presents on the table in front of her].
S: One, two, three, four, five, six. Six. [Here she appeared to have a problem with counting and 1-1 correspondence when she is preoccupied with telling me that she invited a lot of friends so had a lot of presents].
L: Are you sure about that?
S: Yes because when I was four I got loads of presents because I invited nearly all my friends.
L: Right, so they all brought you presents*. [I place 7 presents on the table in front of her]. Can you tell me how many presents now?
S: So he must have lots of friends. One, two, three, four, five, six, seven. Seven.
L: Are you sure?
S: Yes
L: Then he only had that many friends. [I place 3 presents on the table in front of her]. How many presents did he get?
S: One, two, three. three
L: Okay, actually another friend came. How many presents now?
S: One , two, three, four. Four.
[*No problems are evident after I rephrased the question from one about presents to one about friends at the party].
Excerpt T6
1 and 4
When she opens her eyes she counts the four presents visible]
L: How many have I under the basket?
S: Em … 2
L: Are you sure
S: Yes
L: Why do you say that?
S: Because there's …em…these much [She points at the visible ones].
L: How many are under?
S: Two
L: Are you absolutely certain?
S: Yes
L: How did you figure that out?

Table 18.3 (continued)

S: Because I counted I, 2, and 3 and 4 …that's four and we need two more to be five
L: Okay …have a peep
S: There's actually one. [She laughs].
3 and 2
L: How many are under the basket… …how many have we got here …now take your time and think about it?
S: One, two…so there's three!
L: How did you know that?
S: Because there's two
L: And how do you know that there is three under the basket?
S: Because …if there's two and then it goes three
L: Two and then it goes three?
S: Yes
5 and 0
L: How many can you see there?
S: Zero
L: So how many are under the basket?
S: So…em…one…five are under the basket
L: Are you absolutely sure?
S: Yes
L: How do you know that?
S: Because …it's zero out here …because they're all even all gone
4 and 1
L: This is tricky now …okay how many presents can you see?
S: One
L: How many presents are under the basket?
S: One, two, three, four, five
L: So how many underneath
S: Five
0 and 5
S: I see 1,2,3,4,5.
L: How did you know that?
S: Because…em…they're all there and you just put the basket down
Excerpt T8
S: So he'd be…a load of friends
L: Yes…but listen to the story. Coco had 7 presents and he lost 1. How many now?
S: Eight [She appeared to find the idea of lost presents more difficult to deal with. She held up her fingers to indicate to me (or herself) how many now].
L: He had eight and he lost one
S: None

Table 18.3 (continued)

L: Are you sure?
S: See… He lost the number one…and then he might….they be got broken …em…he might… if it was under his couch…pretend this is his couch [She indicates the table].
L: Right
S: And this is under his couch…
L: Okay
S: And if you had one of the presents … with the wrapping paper off
L: Right
S: And pretend that this is like his last one…and pretend it's broken
So he'd have none left
L: No

shaping expectations and guiding interpretations of what goes on" (p. 27). Through consideration of children's responses we can discern how they call on their experiences of everyday discourse in order to engage with the questions put to them. We also see how they begin to participate in the discourse practices associated with school mathematics.

18.7.4 Discussion

As a result of my analyses I argue that the context of the one-to-one interviewing or conversation is an excellent context for supporting children, at transition to school, to the discourse and processes of mathematisation. As suggested in the literature the interview context provides a safe and secure environment in which young children can talk about their thinking and demonstrate and develop the processes associated with mathematisation. O'Connor (1998) argues that discourse patterns learned in the home do not transfer readily into the mathematics activity at school. A difficulty may arise for many children in recognising when the purpose of discourse at school is different, when it is mathematical. While this may be understood by some children, others will need support in moving between the two discourse practices. What the examples from both Shay and Tom show is that during transition, children may need to be inducted into mathematising in situations where they need to suspend their everyday understandings. This is in keeping with O'Connor's (1998, p. 35) assertion that "…increased ability to traverse the mathematical universe would involve both more robust mathematical knowledge and scaffolded practice carrying out everyday discourse routines while operating within the mathematical realm".

The analyses presented here confirm the findings which highlight how the nature of the task presented to young children influences their individual mathematical reasoning. Mathematical tasks that do not make *human sense* (Donaldson 1978) may be particularly confounding for children at the point of transition to school. Asking children to mathematise in such situations is presenting them with a double difficulty; that of making sense of the task, and also of engaging in mathematisation processes in relation to the task. They have not as yet realised that what is required now is a shift in interpretation of the task in the context not of everyday culture and understandings, but in the context of a school mathematics task. While the individual interpretation of mathematical problems is a key issue in analysing how young children are involved in processes associated with mathematisation (Ginsburg 1999), sharing of purpose (i.e. the establishment of intersubjectivity) is key when seeking to involve children in mathematisation. Shay's comment *I counted four to make proof for you* suggests that he understood that I was interested in numerical aspect of the narrative. However, it is clear from some of their comments, that neither Tom nor Sile fully understood my purpose. In both cases, much of the children's efforts were directed at seeking to clarify aspects of the narrative, though both gave responses which at times indicated their efforts to attend to my interests.

During the transition to school it is clearly essential to take into account the fact that children are experienced in everyday discourse practices, but that mathematical discourse practices are new to them. The findings here confirm Schleppegrell's (2010, p. 86) observation that in order to support development "what is required is that teachers need to recognise what a student is trying to say and improve the students ability to articulate it". It is clear also that teachers will need to foreground everyday language during early mathematisation opportunities, with the more technical (i.e. scientific) language gradually introduced as children become more skilled at mathematisation. This is a matter of teacher judgement, but it also relates to the stage of development of the child's knowledge and the task that is being undertaken at a particular time. Teachers must persist in mathematical discussions with children, wherein they challenge them to mathematise using whatever means they have available to them at a given point. It is equally important that they recognise children's efforts as building blocks on which they will learn to mathematise in more conventionally recognised ways (Dunphy 2009). The advice of Barwell et al. (2005, p. 144) that "fuzziness, ambiguity, multiplicity of meanings and exploratory discussion in everyday language should be recognised, not as failure to achieve a truly mathematical degree of precision but as essential to making mathematical meanings and to learning mathematical concepts" seems particularly pertinent here.

18.8 Practical Implications

In scaffolding children's mathematising at the time of transition to school I argue that it is essential for teachers to recognise:

- the role of culture for children's sense making and mathematisation;
- that children's meaning-making is grounded in their out-of-school and prior-to-school experiences;
- children's efforts to combine everyday discourse and mathematical discourse;

- the need to co-construct meaning with young children in mathematising situations; and
- the need to model for children the processes involved in mathematisation.

18.9 Future Directions

These should focus on:

- inviting children to mathematise in structured and planned activities in other areas of mathematics, besides number;
- designing tasks that allow children to concentrate less on the narrative, but more on the dialogue associated with mathematising;
- utilising digital tools such as the iPad to engage and motivate children in articulating mathematical thinking processes;
- utilising digital recordings in order to gain more comprehensive understandings of the child's holistic responses and thinking.

Appendix 1: Details of Tasks

Task 5: How Many Now? (Objects Present)

Look at these nice presents. Let's pretend that they are Coco's presents. How many has he?

Children were asked to count out a linear array of birthday presents (e.g. 4) and say how many. They then were asked to remove one and say how many now. They were also asked to add one to another array (e.g. 5) that they had counted and state how many now.

4 Count/Remove one (Repeat with 7)
3 Count/Add one (Repeat with 5)

Task 6: Understanding of Part–Part–Whole Relationships (Objects Present)

Let's play a game with Coco's presents. First we had better count them to see how many. (Confirm 5)

Children were shown Coco's presents (5) and invited to play a hiding game with these. They first counted the items to confirm that there were five. I then suggested a game of *Hide some/ See some.* While the child's eyes were closed I hid some of the objects and left some visible. We played a number of rounds until the various combinations were exhausted i.e., *1 and 4*, *3 and 2*, *5 and 0*, *4 and 1*, *2 and 3, 0 and 5*.

(Show first amount, hide second).

Task 8: One More Than/One Less Than a Given Numeral (Objects Not Present)

Listen carefully while I tell you what happened about his presents.

E.g. Coco had 4 presents and he got 1 more. How many now?

This was explored with the numbers 3, 7, 6 and 9.

Also *less* explored through the suggestion that we pretend he had x amount of presents but that one got lost.

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Chapter 19
Listening to Children's Mathematics in School

Elizabeth Carruthers

Abstract England's Foundation Stage (birth to five) encourages children's interests to be central in developing the curriculum. As children enter school, in the last year of this stage, political and organisational pressures take over forcing teachers to have uneasy pedagogies. From a postructural stance the 'schoolification' of the child begins and there is a great divide between the mathematics of the child and that of the school. The literature on transitions exposes curricula dissonance not only in England but across Europe, Australasia and North America. Dialogues from teachers highlight their confusion about how and when to teach calculation and especially mathematical notation. Mathematics becomes more teacher centred with strict objectives. The data from England's National Assessments continue to show poor achievement in mathematical problem solving in the Foundation Stage. There needs to be a conceptual shift in the teaching of mathematics to young children in English schools to encompass children's enquiries. This is from a Vygotskian perspective where there is priority given to social and cultural practices stressing the importance of co-participation. This could identify and enhance children's mathematical problem solving. There is, however, much challenge for teachers not only in understanding children's own mathematics that involves children's agency but at the same time they need to confront the organisational walls of opposition.

19.1 Introduction

This research is developed from the author's previous work on *children's mathematical graphics* (Carruthers and Worthington 2005, 2006 and 2011). More recently, an in-depth case study of one teacher and her classroom (Carruthers 2012) highlighted some of the difficulties that this teacher found in uncovering and understanding *children's mathematical graphics*. The research outlined in this chapter builds on the above case study to further investigate teachers' views and reflections on more open ways of working in order to afford children opportunities to develop their own mathematical enquiries.

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This study focuses on the premise that young children can connect their knowledge of mathematics to many life experiences, and that they use this enriched knowledge to create further mathematical thinking in co-participation with others. Taking a socio-cultural stance, I argue that the political landscape in England is a barrier to democratic practices that allow teachers the freedom to listen to children's mathematics. I discuss important aspects of potential pedagogies that enable children's own mathematics to thrive, and describe a recent study centred on school teachers' reflections on their mathematics teaching as they grapple with introducing mathematical opportunities that focus on children's mathematical thinking.

19.2 Why Listen to Children's Mathematics?

The right for children's voices to be heard and for children's opinions to be taken into account was highlighted in the United Nations' *Convention on the Rights of the Child*: "The child has the right to freedom of expression. This right shall include freedom to seek, receive and impart ideas of all kinds, regardless of frontiers, either orally, or in writing, or in print or in the form of art, or through any other media of the child's choice" (UNICEF 2009, Article 13).

My previous research with teachers (Carruthers 2012, 2013) has shown that the ethos and pedagogy of the teachers must be more open to children's mathematical viewpoints so that children can be free to use their own mathematical graphics to support their mathematical thinking. Teachers need to tune into and listen to children's mathematics to understand their rich and diverse viewpoints.

I also put forward that we need to listen to young children because they have much to bring to the learning process; it is well documented that they are curious self-motivated individuals who are able to analyse and hypothesise (Piaget 1952; Vygotsky 1978; Wood 2003). For example, Wells' (1986) study illuminates young children as powerful meaning makers, trying to make sense of their world. Wells reflects that the child has an expert role in their learning, and this learning thrives when those around listen to their ideas and lines of thought. Fleer (2010) also states that children have vibrant minds and she claims that it is the way in which we perceive children that determines how we teach them.

19.2.1 Mathematics and Young Children

Children also have the capability to be highly motivated and inquisitive individuals in their developing understanding of mathematics. A relevant and seminal study of nursery school children in Scotland (Tizard and Hughes 1984) also confirms the persistent and logical manner of the child's thinking before school. Although their investigation focused on how young nursery children learn, an unexpected outcome of the study highlights the children's acute knowledge of mathematics at four years of age. Hughes was surprised by children's mathematical thinking at such an early age, and this provoked him to delve further into their mathematical knowledge (1986). Other studies have also borne evidence of the preschool child's grasp of knowledge, skills and understanding of mathematics (Aubrey 1997, 2004; Ginsburg and Seo 2000; Perry and Dockett 2002). Carruthers and Butcher (2013), through case studies, emphasised how children use mathematics naturally in their spontaneous play; discussing, projecting and imagining. Van Oers (2005) and Worthington (2010) have made connections with children's pretend or imaginary play to abstract thought linked to mathematical imagination. If we understand that young children have mathematical competencies derived from their own self-learning based on their rich cultural experiences at home and in the community, then we could propose that teaching should encompass children's mathematical questions, ideas and lines of thought.

19.3 What is Children's Mathematics?

In this chapter I define children's mathematics as children's own mathematical thinking in play, child-initiated experiences and situations where children are given the freedom to work out mathematical problems in their own ways. Children's mathematics can flourish in classroom cultures and pedagogies that see the child from a positive stance therefore, I am framing children's mathematics through research evidence that may give us an insight into children's own meanings and cultures that promote their own mathematical enquiries. Four areas are considered that could promote children's mathematics within this context:

- Children's mathematical graphics;
- Pedagogy—highlighting the relationship between teacher and child;
- Imaginary play;
- Child-initiated learning.

19.3.1 Children's Mathematical Graphics

In their co-research Carruthers and Worthington (2005, 2006) analysed over 700 examples of children's own forms of written mathematics. These samples were from a few classrooms, including our own that afford children opportunities to use graphic materials: for example, paper and chalk to explore their own mathematical thinking. This research uncovered that children can and do use their own forms of mathematics, if given the chance. *Children's mathematical graphics* are children's own forms of notation; for example symbols, drawings, tallies and scribble-like marks. They use these graphics to express their mathematics or solve mathematical problems and they can be combined with standard methods of computation and notation. Similar to Hughes' (1986) seminal study, Carruthers and Worthington (2005,

Fig. 19.1 Amede writes numerals on his pretend remote control, which he uses in his television play

Fig. 19.2 Billy designs scoring for a game

2006) have shown that children's marks and notations are varied, and further that there is a continuum from young children's earliest marks, to which they attach mathematical meanings, to written calculations. *Children's mathematical graphics* supports children's own lines of mathematical enquiry and aids them in working out mathematical problems. Examples of *children's mathematical graphics* can be seen in Figs. 19.1 and 19.2.

19.3.2 Pedagogy

Many early education mathematics programmes and curricula models, especially in England and the United States, were influenced by the work of Piaget (1952). Piaget, for the most part, did not focus on the importance of social interactions and language (Whitebread and Bingham 2011). In recent years there has been a shift in view, certainly in research, from the child as an isolated thinker to the child's thoughts being influenced by their own communities and social worlds. This has now heavily shaped theoretical standpoints and influenced scholars' understanding of young children and how they learn; particularly from a Vygotskian (1978) perspective, where priority is given to social and cultural practices stressing the importance of co-participation. Rogoff (1990, p. 31) explains,

> For Vygotsky children's cognitive development must be understood not only as taking place with social support in interaction with others, but also as involving the development of skill with socio-historically developed tools that mediate intellectual activity. Thus individual development of higher mental processes cannot be understood without considering the social roots of both the tools for thinking, that children are learning to use, and the social interactions that guide children in their use. This social interaction is between all the humans the child comes into contact with; other children, family, friends, children at school and teachers.

As Lancaster (2003, p. 12) emphasises "In an environment that encourages children's own thinking socially inclusive relationships are part of the ethos". These relationships mean that adults understand how to share power so that children can take part fully in their own play and school learning, and are able to access opportunities and real choices. Children's ideas and thoughts are central to this environment. From a socio-cultural perspective, teachers co-construct learning with the child, and there is an equal share of power (Lancaster 2003).

19.4 Imaginary Play

Vygotsky (1978) placed great emphasis on imagination and symbolic play, seeing them as the highest forms of young children's development. He proposed that children go in and out of real and imaginary events and in doing so they weave in their own cultural knowledge from different aspects of their lives. It is important to acknowledge this, as children will build on this rich cultural knowledge base, linking their previous learning to new learning. Moll et al. (1992) write that children draw on their 'funds of knowledge' as they explore their own meanings from their social and cultural contexts. For example, in the play episode below, children are drawing on their rich experiences from home and nursery. The following is one nursery teacher's observation of spontaneous play.

> **The Context:**
> The children (age three and four in an English nursery school) were interested in calendars, and the teacher had provided a variety of advent calendars. She also had written '11 more sleeps until Christmas' on the whiteboard and this started a conversation which led into a spontaneous play episode.
>
> **The Nursery Teacher's Observation:**
> Alfie's mum read the message on the whiteboard to him when they arrived at nursery.

Alfie: No, it's three more sleeps (knowing this is fewer than 11 and three is a good number because he is three.)

Barley: We already opened number eight and ate the chocolate…I'll be the chocolate maker.

Alfie: I'll be one of the elves making the chocolate.

Barley: We just need to make it all hot so it melts, will you do a job, get a spoon. (Alfie continued to mix the 'chocolate'.)

Alfie: All the people want chocolate. (Alfie used the whiteboard to denote the orders of people wanting chocolate.)

Alfie: Look, all these people want chocolate calendars. (He wiped off my writing on the whiteboard first and then made lines all over the board. Each line represented a person who wanted a chocolate calendar. Other children had also joined this play.)

Barley: This needed to go in the oven.

Alfie: This is to melt the chocolate for the calendars until they are all full to down there (pointing to the calendar from top to bottom). My next job is to sprinkle the stars on to the fairy dust makers, no one else is allowed in the chocolate factory—it's full up. (Alfie pretended to talk over a tannoy, similar to a supermarket.)

Alfie: Excuse me, we are closing the chocolate factory in three minutes so every one of my workers can go home … open the chocolate factory. (Christvie joined the play using the whiteboard next to Alfie making 8s all over the board.)

Christvie: Only eight left over—it's all the chocolate gone. The calendars are all finished. (Alfie watched Christvie writing.)

Alfie: The chocolate takes five months to make—we're making lots and lots of chocolate in three days for Christmas, we're making it all nice for the children.

(Carruthers and Butcher 2013, p. 32)

In this short space of time children were drawing on their experiences from home, school and community. Children's spontaneous imaginary play is within a socio-cultural context, through which the children can generate their own understanding of mathematics. As van Oers (1996, p. 12) emphasises, "mathematical thinking is an emergent property of human communication in socio-cultural contexts". The children in the above scenario were also using mathematics in meaningful contexts, according to their understanding. For example Alfie's knowledge included ideas from watching the film *Charlie and the Chocolate Factory* at home. The teacher provided open experiences to enhance children's own enquiries and the play was spontaneous. Importantly, the teacher listened to the children's mathematics, and this uncovered the children's understanding and knowledge of mathematical concepts: for example, aspects of time, number and data handling. The listening and democratic atmosphere gave children the opportunity to use their mathematical graphics on the child-height whiteboard. In future the children, because the teacher listened to their play and followed their enquiries, will have the confidence to explore and use their own ideas time and time again.

From their study of young children, in play contexts within one English nursery school, Worthington and Van Oers (2015) note that mathematics was found within all the children's spontaneous play. Their work extends beyond number and quantity to span the breadth of the mathematics curriculum. They conclude that teachers should support play environments that give children opportunities to enable them to exploit and explore their cultural understandings, and in Vygotsky's view, to develop spontaneous concepts that will gradually link to mathematics.

19.4.1 Child-Initiated Learning

The term 'child-initiated learning' in this chapter is defined as children choosing their own pathways to achieve their mathematical enquiries. They choose the tools and materials that will help them with their thinking, and select the place where they want to carry out their enquiries. It is about children's agency (Edmiston 2008), which is the facility for children to make personal choices, to direct and shape their own learning. When children are initiating their own learning they are more likely to develop decision-making and problem-solving skills which are so vital for all curriculum subjects, including mathematics, because in this situation they are actively engaged in their own learning.

In summary, children's mathematics thrives in educational environments where:

1. The mathematical problems belong to the children or are open enough to be tackled in a variety of ways;
2. Children's play is valued and teachers understand the mathematical possibilities to develop;
3. Children are given choices and freedoms;
4. Children's home and cultural backgrounds are seen as a rich resource for their learning;
5. Children's ways of thinking are valued and developed;
6. Children's forms of written mathematics are cultivated;
7. Children and teachers co-construct mathematical understandings.

19.5 Factors Influencing Reception and Nursery School Practice in England

It is important to understand the influence of the present political background within which the study described in this chapter is set, and how this background influences the reception teachers' classroom practice.

The Tickell Review (2011) is an assessment of existing early years (birth to five) curriculum policy in England. It has had a huge influence on practice, especially in reception class where children enter school at the age of four. The term 'ready for school' dominates the discourse in this policy, and has resulted in a back to basics approach emphasising phonic drills, writing practice and standard arithmetic approaches. Reception (first year of school) practice is now seen as the vehicle for preparing children for a more formal curriculum and not as a continuation of nursery school (Whitebread and Bingham 2011). Tensions therefore exist between the government, who want children to be prepared to "sit and listen" (Tickell 2011, p. 12), and early years groups, who argue that instead of getting children ready for a more formal curriculum we should adapt the curriculum to the developmental needs of the child (Whitebread and Bingham 2011).

The English school curriculum has moved dramatically from teacher-centred decision making in the last twenty years to more government controlled, and it has also become far removed from the child-centred framework and proposed ethos of England's *Early Years Foundation Stage Framework* (Department for Education 2012). There is therefore a fundamental difference in ethos and principles between primary and early years education. The Organisation for Economic Co-operation and Development (OECD) (2006, p. 61) explains that within many member countries, including England, in an attempt to make smooth transitions between early and primary education the teaching styles and focus of the primary school have been more privileged over those of nursery schools.

> The policy approach has tended to favour teacher centred and academic approaches, resulting in the use of programmes and teaching styles that are poorly suited to the psychology and natural learning strategies of young children. (OECD 2006, p. 62)

Tensions also exist between mathematical advisory groups and English government education policies. In England, the *Williams Mathematics Review* (DCSF 2008) reported on mathematical teaching in early years and primary schools, and in its recommendations emphasised the importance of play in mathematics and *children's mathematical graphics.* The Advisory Committee on Mathematics (England) constantly promotes policies that give teachers greater professional freedom (ACME 2013). However, although more open mathematical approaches are recommended to the government, even by its own advisory and review groups, these recommendations are not implemented.

Pedagogical inconsistencies also exist in other countries across Europe, the United States and Australasia (Brooker 2008; Entwistle and Alexander 1998). The pedagogy of standard curricula, in many cases, marginalises children's views and freedoms to shape their own understandings, and this includes mathematics. For example, Einsarsdottir's (2010) study on children's views of their first experiences of school in Iceland compared with their nursery school experiences highlights that children have little influence on the school curricula, including mathematics.

Teachers cannot listen to children's mathematics if there is no opportunity for children to discuss their own mathematical thinking and solve their own mathematical problems. However, it could be true to say that democracy has to be part of the whole system; for example, if teachers themselves are not listened to, in turn, they may find it difficult to acknowledge children's voices. Sahlberg, the Finnish Director General of the CIMO (Centre for International Mobility and Cooperation) states that:

> The voices of practitioners are rarely heard in the education policy and reform business. Educational change literature is primarily technical discourse created by academics or change consultants. (Sahlberg 2010 p. 104)

In many European countries, government ministers determine the final policies that are eventually handed down to the teachers to implement and there is no real teacher debate or discussion (OECD 2006). Can children's own views be valued if teachers' views are not valued? The study described below is an attempt to listen to school teachers' reflections and discussions as they take on the challenge of listening to children's mathematics.

19.6 Teachers' Views on Listening to Children's Mathematics

19.6.1 Methods and Data

The participants of the study are fifteen teachers in the English school system: seven reception (teaching four and five year olds in the first class of primary school), and eight nursery school teachers (teaching three and four year olds). The participants were part of a Masters in Education degree programme.

This is a qualitative research study drawing upon ethnography (Hoey and Fricker 2007). It is participatory (Manzo and Brightbill 2007) and examines teacher participation and reflection through the teachers' own writings of their classroom practice linked to their own reading, research and writing. Participatory research requires ongoing reflexivity and sensitivity to emergent ethical issues. The study focuses on understanding democratic practices in mathematics teaching that lead to children's lines of mathematical enquiry being acknowledged and responded to (Bomer and Bomer 2001). The teachers' discussions shape the discourse. Teachers read relevant research texts that became a mirror on their practice, and this is pivotal in challenging their thinking about how they teach mathematics in their classrooms, promoting changes in classroom culture. This is seen from a Vygotskian perspective where priority is given to social and cultural practices, stressing the importance of co-participation.

The data are drawn from recorded interviews and the teachers' own writings of their practice as they reflect on the literature and their dialogues with the other participants where they created "collegial confrontation" (Lazzari et al. 2013, p. 12).

Firstly, I will give an account of the school teachers' perspectives as they began to reflect on their practice, and secondly I will focus on the nursery teachers' perspectives in comparison with the first analysis.

19.6.2 Analysis and Findings

19.6.2.1 The Mathematical Pedagogy

The mathematics curriculum was formerly very tightly structured for all these reception class teachers; they worked through the required curriculum without much deviation—for example teaching 0–5 and 5–10, then halving and doubling, introducing adding, then subtracting, with small numbers, usually using 'practical' manufactured mathematical resources. The implication is that the starting point was from a very controlled pedagogy. All of the teachers commented that they were confused about how to teach calculation and when to teach standard written mathematics (Gifford 2005).

19.6.2.2 Planning

Adults' agenda planning was used to maintain tight organisation and control. Play was planned with specific outcomes and curriculum areas to be covered. Perhaps the undercurrent was that a good teacher must be an excellent planner. "Quality maths needed to be well planned" (one teacher in the study). Six of the reception teachers planned straight from the required curriculum and not from children's cultural experiences or self-chosen enquiries.

19.6.2.3 Children's Mathematical Graphics

Most teachers in the study had heard very little of children's own ways of representing their mathematics, but they confused this with children's recording where the child copies the mathematical problem they have already solved. The concept of *children's mathematical graphics* centres on children using their graphics to work out a problem they do not already know the answer to (Carruthers and Worthington 2005, 2006, 2011). Some of the reception teachers did not ask the children the meaning or the context of their mathematical graphics, nor did they understand at first where to take this learning. They found it extremely difficult to uncover the meanings or see the significance, for example in their children's mathematical drawings. At first, one teacher on trialling having blank paper available around her classroom said: "They do not do anything with it—I have tried". However, when teachers had moved on in their understanding through the year (and perhaps because the classroom cultures started to change), then they started to see children use their own graphics more frequently. For some it was a huge conceptual shift in attitude from not noticing, to seeing and then beginning to understand the significance of children's own graphicacy. Anning (2003) states that the reception classes in her study were dominated by getting children to write as quickly as possible, and the power of children's graphicacy was not realised.

19.6.2.4 Child-Initiated Mathematics

Opening up the teaching of mathematics so that children have opportunities to choose problems and find their own strategies to solve them was central to much reflective consideration of pedagogies. An invited reception teacher (see Carruthers

2012 for the case study) who had success in giving children opportunities to do their mathematics explained her thinking to the teachers. They could really identify with her and the journey she had been on. This encouraged the reception teachers to analyse further their adult-initiated group activities and other opportunities for children's mathematics within their classrooms. One teacher commented; "Although it was very difficult to accept, there was very little child-initiated mathematics occurring within my classroom". Two of the reception teachers said that they found mathematics a hard subject in which to be more creative, and that they were much more open in their teaching of literacy.

The teachers trialled various ways of giving ownership of the mathematical enquiries to the children. However, when they opened up the maths, and the children confronted the teachers with their own thinking, this was an uneasy place to be: "I was unsure where to take this… what to do with it" (reception teacher). As Goouch (2010, p. 45) reminds us, "It is a risky undertaking for a teacher to just see what happens and work in ways that they are not sure of the outcome".

The following is an example of a reception teacher's story of trying to listen and support children's mathematics. One five-year-old child asked the question: 'How many children are in the school?' The teacher said she would never have let the children tackle this before, and if she did she would not have asked them for their strategies. So she bravely took this on and said, 'I wonder how we could find this out?' The child said, 'I know—put all the children in the church and count them' (possibly 900). Instead of saying no, as she could see the possible pitfalls of this in terms of organisation, the teacher said, 'You will have to ask the head teacher if all the children can go into the church.' The head teacher then told the child that this was too much organisation but asked whether the child could think of an easier way. This provoked discussion and strategies for counting such a big group. The important point here is that the child's mathematical question was valued and their strategies were not ignored but seriously considered. As Carruthers and Worthington (2011) claim, this is 'creating a culture of mathematical enquiry'. Similarly, Shuard (1986) asserts that the important part of the pedagogy is not doing and investigation but allowing children the freedom to work in an investigative way. In creating an enquiry culture, children practise mathematical skills such as organising, analysing, hypothesising and inference.

The reception teachers in the study understood the benefits of children's mathematics, and when they trialled even more open mathematics they saw the children in a different light; children began to exceed expectations in mathematics (Carruthers 2012). Teachers were surprised that the children knew so much or that they could be creative thinkers in mathematics (Coles 2013). Some of the head teachers were impressed by the achievement of these children and were willing to accept children's mathematics because, as one teacher reflected, "It does not put a ceiling on their learning, they can go as far as they want".

However, the pedagogy can appear complicated at first and not as straight forward as being given a curriculum to follow and targets to achieve. The teachers reflected that they need confidence to support children's own enquiries.

19.6.2.5 Play and Mathematics

The pedagogy of play was a struggle for these reception teachers and whilst all clearly stated they operated a play approach to learning, when they reflected on their practice they were alarmed at the incongruity between their espoused theory and their real, everyday, practice. When the reception class teachers read current research literature on play they faced an uncomfortable reality; for the most part they were "pedagogising play" (Rogers 2010, p. 62). They were planning play with expected curriculum outcomes. In some cases, for example, the role play was themed and children were expected to play within the theme of the role play. The play did not belong to the children—instead it was a dominant teacher agenda. Although the idea of a more structured role play can yield some interesting experiences for the children (van Oers 1996) it may be less likely to uncover children's own mathematics.

The reception teachers all said they really valued play, and they were strong in their confirmation, yet it was not central to their curriculum. As one reception class teacher commented, "We were removing children from valuable play experiences to complete their work". This mismatch between teachers' theories and practice is also a central finding in the Bennet et al. (1998) study of reception class teachers in England who concluded that there is a serious need for adults to understand children's play from a child's perspective.

As the reception teachers in this study discussed and reflected on play in their classroom and school, they all, without exception, said that the area they needed to learn the most about was play. They realised that one of the main drawbacks of having play as a priority in their classrooms was the demands of government initiatives such as phonics. There is an urgency to prioritise phonics because of the national phonics test at six years of age. They reflected that work pressure, which included meeting targets for each child, the expectations from year one (the next year group), and the senior leaders' agenda steered them from seriously understanding the deep intellectual processes involved in play. One teacher lamented "The demands placed on reception class teachers perhaps cannot yield a more open maths curriculum". I believe that is a serious statement to be considered. How do teachers question and stand firm with their own views against a political agenda?

19.6.2.6 Nursery Teachers

In England there is a great tradition of nursery school education based on the works of pioneers like Margaret Mc Millan (1904) and Susan Isaacs (1930). Nursery schools only provide a small proportion of preschool provision for young children in England; there are also private business nurseries, voluntary provision and independent fee-paying schools.

Nursery schools started in the 1920s in an effort to combat poverty and give additional educational opportunities in areas of deprivation, but increasingly the funding for these nursery schools has diminished. Now there are just over four hundred nursery schools left in England.

Nursery schools take children at three to four years of age just before they enter school-age reception class. The nursery school teachers are fully qualified teachers and, traditionally, there have always been qualified teachers in nursery schools. There are also qualified nursery nurses or early years practitioners who specialise in the younger age group. The organisational structure is similar to primary schools with a head teacher and a governing body. However, although the structure of the nursery school is similar to the primary school the two have very different origins. Brooker (2008, p. 28) emphasises: "Elementary education was conceived from its inception as preparation and training for adult work and adult citizenship; nursery education was conceived as an opportunity for the child's innate potential to grow and blossom". Nursery schools are funded by government and local authorities, and they have consistently been rated in the top categories of preschool and school provision by the Office for Standards in Education (Ofsted 2010).

This study revealed the openness of the nursery teachers' approach to children's learning, in which they delved deeper, seeking to understand and providing more open opportunities for children. They also followed children's interests in an authentic way with a view to really acknowledging children's lines of enquiry and persistent interests. The data revealed that nursery teachers had an abundance of examples of children's mathematics and *children's mathematical graphics*—many even before the study began. They work in a way that allows children freedom to express themselves. The nursery teachers' autonomy to explore meant that they grasped really deep issues and could change practice as a result. They constantly questioned other possible ways of analysing and re-structuring. The nursery teachers sought to uncover children's mathematical thinking: the reception teachers, for the most part, covered the curriculum.

19.6.2.7 Teacher's Attitudes Towards Children and Parents

The most striking difference highlighted in this study is the nursery teachers' language when describing the children and the parents. They are more positive about the children, and in their discussions with parents they try to engage parents more in equal dialogue about mathematics rather than to 'help' parents (as was the case for the reception teacher). The nursery teachers seek to genuinely learn about the children and the families' interests and cultures. Hedegaard and Fleer (2013, p. 56) state: "Teachers need to be conceptually and contextually connected with the children so that they can frame the learning activities". Hedegaard and Fleer go on to explain that this deeper knowledge of the children creates a reason and context for their play and learning. It could also be argued that there is certainly a much more challenging and broader maths curriculum in these nursery schools in comparison with school. Reception teachers (perhaps because they are so driven to focus on strict curriculum goals, timetables, over planning and ways of being in their school) cannot easily reflect in different ways, take chances, or risk being too controversial.

There appears to be a ‘school-speak’ that cannot move away from the boundaries of the accepted wisdom.

Brooker (2002, p. 24) talks about the child’s transition to school as “developmentally dramatic”. Is the child’s experience with mathematics also strikingly different? There are studies that point to a dearth of mathematical experiences in nursery schools such as Munn and Schaffer’s (1993). Some of the nursery teachers in this study did write that they were concerned with the lack of mathematical experiences in their settings before the study started. However, it seemed to be much easier for them to take on board the concept of uncovering children’s own mathematics because of the open enquiring culture they provided, and this offered fertile ground for children’s own mathematical thinking and problem-solving.

The importance of attunement was highlighted in the teacher–child pedagogical relationship; to know the child and create a responsive collaborative learning culture was much more embedded within the nursery schools. All the nursery schools, without exception, had adopted the key person approach (Elfer et al. 2003) and this may have be one of the reasons that they were more connected to and positive about parents and children. This is also recommended in the English ‘Early Years Foundation Stage Framework’ (DfE 2012).

The principle of the key person is an adult partnered with a child who is the person the child and the family know that they can engage with over any matters concerning their child. The key person gets to know the child and the family much better than a traditional teacher; they visit the child’s home and regularly share conversations about the child with their family. As a result of this close connection they also tune into the child’s home mathematics. This means that the teacher (key person) can understand the child’s meanings as they talk about their mathematical experiences because they know the child’s home background. Attunement is often connected with the literature on attachment (Berk 2003); however, it may also have aspects that we can draw on to describe important mathematical pedagogical relationships between teacher and child. It is linked to inter-subjectivity (Trevarthen and Aitken 2001), which involves awareness of others’ perspectives, as well as collaboration, communication and empathy.

19.7 Conclusion

There is constant dissonance between school teachers’ beliefs about children’s learning and the political agendas imposed on teachers. The reception teachers in the study seem so restricted by certain ways of teaching and organising their classrooms that their views appear to get strangled by an accepted curriculum which does not yield mathematical freedom for children. This can mean that children are constrained into trying to understand school maths and their own methods of gaining mathematical knowledge are ignored. There is a mismatch between the planned mathematical curriculum in reception classes in comparison with children’s mathematical ability. There is much challenge, especially for school teachers, not only in understanding children's own mathematics that involves children's agency but also in the need to confront organisational walls of opposition. Teachers should have the freedom to make curriculum decisions based on their own reflections, intuitive theories and experimentations. Brooker (2002) expresses the power of teacher research which highlights teacher professionalism and certainly within this study of reception teachers, it was the reading, trialling and reflecting that made the significant difference to their teaching of mathematics.

19.7.1 Implications for Transition

There is a huge gulf between nursery teaching and school/reception teaching in England that has created a discontinuity in children's mathematical experiences as they move from nursery school to school. If we are *really* to listen to children's own mathematics, children need freedom to learn; yet when children enter school in England they seem to be very much more restricted than their previous experience in nursery. This change must be very confusing for children as mathematics takes on a very different and narrower agenda. There appears to be a need for a professional space where nursery and reception teachers can discuss their practice and ethos and perhaps use this to influence the continuity of mathematical experiences as children enter reception classes. For example, the positive connection with parents and children depicted by the nursery teachers in their writing reflections could be a vehicle of discussion for meetings.

Finally, listening to the reception teachers in this study, it appears there needs to be a conceptual shift in the teaching of mathematics to young children in English schools to encompass children's enquiries, children's cultures and children's mathematical thinking. This cannot be accomplished unless government bodies and teacher representatives listen to teachers and develop policy that responds respectfully and knowingly.

19.7.2 Further Research

The study in this chapter highlighted the initial research findings of an ongoing longitudinal study with nursery and reception teachers in England. Possible trajectories of this work could include further analysis of the nursery school pedagogy of mathematical play in comparison to that in reception classes and the implications of this for the continuity of children's learning experience as they transfer to school.

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