Chapter 9
Researching the Role of the Teacher in Creating Socially Productive Classrooms that Facilitate Mathematics Learning

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Introduction

The explication by Steve Lerman of a social perspective on teaching and learning informed a classroom based research program including five funded projects. This chapter elaborates Lerman's perspective, illustrates how it informed each of the projects, and presents a specific lesson to exemplify the key elements of this social perspective. The chapter is a description of ways that aspects of Steve's 1998 plenary address at the conference of the *International Group for the Psychology of Mathematics Education*, titled "A moment in the zoom of a lens: Towards a discursive psychology of mathematics teaching and learning" (Lerman 1998) influenced a program of classroom based research. There are three themes in particular from that address that informed the research program described in the chapter: the search for a descriptive professional language; defining a role for the teacher; and a focus on the social context of the classroom.

In the first theme in that address, Lerman (1998) argued that "the concern of workers in the field is to find a language with which to describe the process of the acquisition of mathematics, and through which to draw inferences for what teachers might do to bring about that acquisition by as many students as possible" (p. 66). Within this quote there are three specific calls to action. The first call is associated with the search for a descriptive professional "language". In many other fields informed by empirical study, there is substantial attention to clarity and meaning of language and technical terms. In the field of mathematics education, there seems to be a startling reluctance by many researchers to build on definitions and terminology of others. The important attempts to introduce clear-planning and teaching terminology such as by Cobb and McClain (1999) on the social norms of the classroom, by Simon (1995) on the hypothetical learning trajectory and by

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Brousseau (1997) on the didactical contract are important exceptions. While researchers can readily contrast comparable terms and consider subtlety in differences in usage, such nuances are often opaque to practitioners, contributing to what seems like a limited uptake of research ideas in classrooms. Indeed, the call by Steve Lerman for the development of descriptive language is probably as relevant now as it was then.

The second call is related to what "teachers might do" indicating that a goal of research in our field is to inform actions by teachers. While it is important that there are researchers exploring theoretical ideas, it is also critical that the focus of much of the research in mathematics education is to inform the practices of curriculum developers, teacher educators and teachers. Indeed such implications should be made explicit in reports of research.

The third call is around "acquisition by as many students as possible" which represents an explicit commitment to inclusiveness. The implication is that learning mathematics can create opportunities, and that the opportunities should be made available to the greatest number of students. There are clearly differences between the achievements of particular groups of students based on familiarity with the language of instruction, culture, socio-economic status, geography and gender. Even within each of these groups there is substantial diversity in achievement indicating that the search for pedagogies, that include as many students as possible, should be the focus of attention by education researchers.

Returning to the address, delivered at the height of the struggle in the field to discern the practical implications from the theory of constructivism, a second theme suggested by Steve Lerman was represented by the following statement: "the metaphor of students as passive recipients of a body of knowledge is terribly limited: so too is the metaphor of students as all-powerful constructors of their own knowledge, and indeed of their own identities" (p. 70). The former of these comments was addressing the all-too-common approach, especially in senior secondary and tertiary mathematics teaching, in which the fundamental planning process seems to be searching for clear explanations and ways of demonstrating mathematics concepts, following by repetitive practice. While this may be moderately successful with students who have elected mathematics study, it is hardly applicable for learners in earlier school years. It also results in a narrow set of experiences for successful learners. The latter metaphor addressed by Lerman was referring to the way that the radical version of constructivism (see, for example, Ernest 1994) was sometimes interpreted by classroom teachers as students working on potentially rich experiences without the benefit of expert teacher direction. Steve elaborated the Vygotskian opposition to pedagogies that seem to require students to "rediscover the development of mankind for themselves" (p. 69). Rather, Lerman argued that mathematics learning is centrally concerned with "the mediation of cultural tools and of metacognitive tools" (p. 69). For both of these, some explicit teacher guidance is needed.

In a third theme in the address, Steve emphasised "the centrality of the social relationships constituted and negotiated during classroom learning" (p. 70). Rather than attention to individual sense-making, he described an emerging process (at that time) in which the psychology of mathematics education is "focused on the way in which consciousness is constituted through discourse" (p. 67). In clarifying this social perspective, Steve elaborated the frequently cited the Zone of Proximal Development (ZPD) metaphor (Vygotsky 1978). Lerman argued that "the ZPD is created in the learning activity, which is a product of the task, the texts, the previous networks of experiences of the participants, the power relationships in the classroom, etc" (p. 71). Even though ZPD is sometimes used to describe teacher choice of an activity to allow students to step onto the next rung on a ladder of many miniscule steps of mathematics learning, Steve argued rather that ZPD is connected to creating classroom environments with conditions that are likely to facilitate student engagement in tasks that they have potential to complete. As he went on to argue "creating ZPD is more about mutual orientation of goals and desires than about the intended content of the interaction" (p. 72).

These three insights from that PME address – development of descriptive professional language, the role of the teacher in supporting student learning, and consideration of the social aspects of the classroom - were important in influencing the development of a program of research focusing on the effective use of tasks in classrooms. Basically it was assumed that the teacher has an active role in choice, adaptation and presentation of tasks, and the management of activity, and that the tasks should stimulate peer interactions as prompts to teaching and learning. The ways that these insights informed the individual projects within a program of research are elaborated in the following sections.

The Overcoming Barriers to Mathematics Learning Project

Supported by two successive grants from the Australian Research Council (ARC), together with Judy Mousley and Robyn Zevenbergen (now Jorgensen) I worked with teachers in their classrooms to describe approaches to teaching that were effective in supporting all students in their learning. One report of this research (Sullivan et al. 2003) described five elements of planning and teaching that they argued can be included in everyday routines of teachers. In our research we found that each of the five elements is manageable in classrooms, that teachers learn them readily, and that the elements can become part of the planning routines of teachers. The five elements are described in the following. The connections to Lerman's themes are considered subsequently.

Building a Communal Classroom Experience

In Sullivan et al. (2003) we argued that all students should have at least some core experiences that can form the basis of later discussions. The expectation is that teachers work with students to develop in them a sense of membership of the class as a whole. This notion is based on Wood (2002) who emphasised the way that "social interactions with others substantially contribute to children's opportunities for learning" (p. 61) and the interplay between children's developing cognition and the "unfolding structure that underlies mathematics" (p. 61). It was assumed that mathematical communications in classrooms that are intended to include all students can best occur if there is some communal experience. If some students in a class are excluded from common experiences and are unable to participate in discussion, this voids the possibility of them feeling affiliated with the class as a whole. Further, such experiences need not only create opportunities for social interaction but also promote thinking about mathematics.

Very much related to building this sense of communal experience is the need for the teacher to address diversity in mathematics awareness and attitudes. Such diversity can be a product of students' prior mathematical experiences, their familiarity with classroom processes (e.g., Delpit 1988), social, cultural and linguistic backgrounds (e.g., Zevenbergen 2000), the nature of their motivation (e.g. Middleton 1995), persistence and efficacy (e.g., Dweck 2000), and a range of other factors.

Some teaching approaches described elsewhere seem either to ignore the diversity of backgrounds and needs of students, or to address diversity in ways that exacerbate differences by, for example, having different goals for particular groups of students. In the Sullivan et al. approach, an expectation is that all students engage sufficiently in the goal or focus task to allow them to participate in a class review of the products of their work on that task so they participate in the social community that is the classroom (see also Askew et al. 1997).

Planning a Trajectory of Mathematical Tasks

Sullivan et al. (2003) argued that there are two considerations for the trajectory of tasks. The first is that there are benefits to inclusivity if at least some of the tasks are open-ended. It has been argued that open-ended tasks engage students in thinking about mathematics exploration, enhance motivation through increasing sense of control, and encourage students to investigate, make decisions, generalise, seek patterns and connections, communicate, discuss, and identify alternatives (Christiansen and Walther 1986; Middleton 1995; Sullivan 1999).

The second consideration relates to sequencing of tasks planned to offer the experiences necessary for students to complete the goal task. Simon (1995) described a hypothetical learning trajectory as one that:

> provides the teacher with a rationale for choosing a particular instructional design; thus, I (as a teacher) make my design decisions based on my best guess of how learning might proceed. This can be seen in the thinking and planning that preceded my instructional interventions . . . as well as the spontaneous decisions that I make in response to students' thinking. (pp. 135–136)

Simon noted that such a trajectory is made up of three components: the learning goal that determines the desired direction of teaching and learning, the activities to be undertaken by the teacher and students, and a hypothetical cognitive process, "a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (p. 136). These predictions are not dependent on students listening to a sequence of explanations but to engaging with a succession of problem-like tasks, based on recognition that learning and knowing is a product of activity that is "individual and personal, and ... based on previously constructed knowledge" (Ernest 1994, p. 2).

The intention is that earlier tasks in the sequence provide experiences that scaffold the student in the solution of later tasks, allowing them to engage in more sophisticated mathematics than would otherwise have been the case.

There are different ways to create sequences of tasks. One of these types of sequence is where the problem formulation remains constant but the numbers used increase the complexity of the task, say moving from small numbers to larger numbers. Another type of sequence is where the problem is progressively made more complex by the addition of supplementary steps or variables, such as in a network task where additional nodes are added. A third type of sequence may be where the concept itself becomes more complex, such as in a sequence of finding areas or progressively more complex shapes from rectangles, to composite shapes, to irregular shapes. The creation of such sequences is a key component of the planning model.

Enabling Prompts that Engage Students Experiencing Difficulty

A third element described by Sullivan et al. relates to supports offered to students who experience difficulty along the way, termed *enabling prompts*. It is common, indeed in places recommended, that teachers gather such students together and teach them (see, for example, Department of Education, Employment and Training 2001). Even worse is the practice of grouping students by teacher perception of their ability. It seems that the consensus is that this practice has the effect of reducing opportunities especially for students placed in the lower groups (Zevenbergen 2003). This can be partly due to self-fulfilling prophesy effects (e.g., Brophy 1983), and partly due to the effect of teacher self-efficacy which is the extent to which teachers believe they have the capacity to influence student performance (e.g. Tschannen-Moran et al. 1998). Brophy argued that, rather than grouping students by their achievement, teachers should: not worry too much about individual differences; keep expectations for individuals current by monitoring progress carefully; let progress rates rather than limits adopted in advance determine how far the class can go; prepare to give additional assistance where it is necessary; and challenge and stimulate students rather than protecting them from failure or embarrassment.

Students are more likely to feel fully part of the class if teachers offer prompts to allow those experiencing difficulty to engage in active experiences related to the initial goal task, rather than, for example, requiring such students to listen to additional explanations or assuming that they will pursue goals substantially different from those of the rest of the class. There are some generic types of prompts. For example, it nearly always helps to draw a diagram or model, to remove one of the constraints, to offer more choice, or to change the form of representation.

Tasks to Extend Thinking

A fourth element relates to anticipating that some students may complete the planned tasks quickly, and can be posed supplementary tasks that extend their thinking on that task. One of the characteristics of open-ended tasks is that they create opportunities for extension of mathematical thinking, since students can explore a range of options as well as considering forms of generalised response. In practice it is arguable that this is the most important and challenging of these planning steps. The premise is that the class progresses together through the lesson contributing to the sense of communal experience. Unless creative opportunities are provided for the students who have completed the tasks along the way then not only might they be bored, and so create difficulties for the teacher, but also they will not be using their time effectively. Note that this offers substantial advantages over the strategy of moving students who finish the work onto the next chapter of the text. Some strategies that teachers used to extend students' thinking included asking them to find all possible answers, to describe the possible answers generally, to create similar problems for other students, and to increase the complexity of the numbers involved or the number of problem steps.

Making Otherwise Hidden Pedagogies Explicit

The fifth element of the framework was related to being clearer about the pedagogies of mathematics teaching. This was informed by the work of Bernstein (1996) who described pedagogies that are hidden from some students. Bernstein argued that, through different methods of teaching, students receive different messages about the overt and the hidden curriculum of schools. He suggested that some students are able to make sense of this "invisible" pedagogy more effectively than others, due to their familiarity with the embedded socio-cultural norms, and hence those students have more chance of success.

As suggested by Delpit (1988), Zevenbergen (1998), and Dweck (2000), it may be possible to moderate the effect of the hidden curriculum by explicit attention to aspects of pedagogies associated with such teaching. Sullivan et al. (2002) listed a range of particular strategies that teachers could use to make implicit pedagogies more explicit and so address aspects of possible disadvantage of particular groups. It seems that teachers are able to make explicit at least some of the key pedagogies associated with such teaching, and that students respond to this explicitness in the direction intended (Sullivan et al. 2003). Zevenbergen et al. (2004) describe three strategies used by one teacher in a school with a high proportion of Indigenous students to make particular pedagogies explicit, either by stating the issue directly or through modelling of a socio-cultural process.

In summary, the three themes from Steve Lerman were foundational in the identification and description of the five elements of this pedagogical approach. There are clearly articulated roles for teachers and students that emphasise teacher/student interactions and student decision-making, there is a explicit intent to include all students in the mathematics learning, key concepts such as classroom social norms and learning trajectory were utilised, and the idea of enabling and extending prompts that drew on Steve's interpretation of ZPD were proposed.

This approach also emphasises the creation of a classroom community and emphasises the social aspect of learning. Various reports of this project (e.g., Sullivan et al. 2009a) not only provide evidence of the feasibility and impact of the approach, but also provide classroom examples that exemplify enacting the planning model.

Students' Responses to Different Types of Mathematics Tasks

Subsequent to the Overcoming Barriers project, I worked with Doug Clarke and Barbara Clarke on the *Task Types in Mathematics Learning (TTML)* project which was a 3 year Australian Research Council funded research partnership between the Victorian Department of Education and Early Childhood Development, the Catholic Education Office (Melbourne), Monash University and Australian Catholic University. The TTML project was strongly influenced by the findings of the earlier project, and also sought to build not only on Lerman's consideration of the role of the teacher, but also on what ZPD might mean in the broader context of converting potentially rich tasks to learning experiences (for a full report on this project, see Sullivan et al. 2013).

The project worked with clusters of middle-years' teachers of mathematics (Grades 5–8) with each cluster typically involving a secondary school and three or four primary schools. These three clusters represented a spread of socio-economic student backgrounds and included schools in both government and Catholic systems. The clusters examined the processes of identifying potentially rich tasks, creating lessons and sequences of lessons from those tasks, and exploring the pedagogies that are associated with the active engagement and support of all students. The notions of enabling prompts and extending prompts, which built directly on Steve's explication of ZPD, were prominent.

Of particular interest was the process of developing and communicating classroom social norms. To explore this further, we sought responses from students to some surveys to gain insights into the types of tasks they value, the nature of the lessons they prefer, and their aspirations in relation to classrooms. The data from the surveys is reported in full in Sullivan et al. (2013), but the following is an attempt to summarise some relevant findings.

The first finding was that while there was not much difference overall in the students' reported confidence and satisfaction over the years 5, 6, 7 and 8 (ages 10–13), even though they move from primary to secondary schools after year 6. At each of these middle-years' levels there is a range of student satisfaction and confidence. There were also substantial and significant differences between classes suggesting that teachers have a major impact of the satisfaction and confidence of their students. In terms of creating a classroom that has socially inclusive norms, teachers should be aware of the views of each of their students. It seems it would be productive for teacher educators to support teachers with strategies for finding out students' levels of satisfaction and confidence, and also for making suggestions on moving students in the direction of being more satisfied and more confident.

A second finding related to different types of tasks ranging from those that focused on mathematical principles to those that were based on realistic contexts to those which were investigative or open-ended. The survey suggested that each of the task types was liked most by some students, and likewise each of the types of task was rated as the one from which they could most learn. The implication is that, in creating an inclusive approach, it is essential teachers use a range of types of tasks in their teaching. This may be particularly relevant to teachers at the secondary schools who seem to use texts with mainly similar types of tasks. This finding also suggests that students need support to gain benefits from tasks that they do not like or do not feel that they can learn from. Teachers may well benefit by making students aware of the purpose of tasks and what it is that teachers are hoping students will learn from them. In terms of the third of Lerman's themes, creating a socially inclusive classroom means that the purpose of pedagogies, the social norms and expectations for effort need to be explicit.

A third finding was drawn from free format essays written by students on the type of lessons they prefer. The essay responses were categorised and quantified. One set of categories, that represented around half of the comments overall, related to lessons that have engaging pedagogies. There was also around half of all the comments that mentioned specific aspects of the mathematical content. Teachers need to find ways to focus on content using engaging and with the purpose and connection of each of these articulated explicitly to the students.

A fourth finding, also from the free format essays, was that the method of grouping is important for students, and nearly all students mentioned grouping in their responses. Most students prefer to work in groups or pairs in their mathematics classes, but around 20 % of the close to 1,000 students indicated unprompted that they prefer to work alone. For similar reasons as with the previous finding, teachers need to explain the method of grouping they are using, and the purpose of those groupings, to the students. It is also helpful to teach specific social skills such as listening to others.

A fifth finding is that the ways that teachers interact with students is important for them. The free format essays indicated that there is a variety in student preferences, so it would be useful for teachers to find out the types of interactions that individual students find helpful.

A sixth finding is that there was tension between students' liking of an approach and the extent to which they felt it helped them learn. This also suggests that teachers need to be explicit about their intentions related to students' satisfaction and confidence, posing a variety of types of tasks, the focus of lessons whether on content or engagement, the method of grouping, and modes of support and interaction.

Connected to the third of Steve's themes, these results elaborate the ways that social considerations inform and influence the classroom community and the learning opportunities of the students. The emphasis in the unprompted responses from the students is on relationships and connections. It seems that descriptive language may well assist students in aligning their efforts with those of the teacher, that teacher guidance is important, and that facilitation of the social dimension of learning is a key challenge for teachers.

The Connection Between Challenge and ZPD

A further ARC-funded project that builds not only on the three key themes from Steve Lerman described earlier but also the above two projects is *Encouraging Persistence Maintaining Challenge* (EPMC) in which I am working with Jill Cheeseman, Doug Clarke, Angela Mornane, Jim Middleton, and Anne Roche. The rationale of the project is described as:

> ...founded on a belief that while it is possible for everyone to learn mathematics, it takes concentration and effort over an extended period of time to build the connections between mathematical ideas, and to be able to transfer learning to practical contexts and new topics. The type of effortful actions that are associated with learning mathematics include connecting, representing, identifying, describing, interpreting, sorting, applying, designing, planning, checking, imagining, explaining, justifying, comparing, contrasting, inferring, deducing and proving. ...All of these actions or activities require effort to initiate and modulate behaviour, and persistence to maintain and direct behaviour towards successful ends.

The project team describes *persistence* as student actions that include concentrating, applying themselves, believing that they can succeed, and maintaining effort to learn. We call tasks that are likely to foster such actions *challenging*.

The project is examining the rationale and processes for encouraging students to persist when working on challenging mathematical tasks, the nature of tasks that provide the appropriate type of challenge, and the actions that teachers can take to support students engaged in challenging tasks.

Primarily the project is exploring student persistence, the opportunities and constraints it creates, and ways of fostering it. We are assuming that this involves teachers posing challenging tasks, encouraging students to take risks in their learning and to work with other students, to allow them make their own decisions on ways of solving problems and to create opportunities for students to explain their reasoning and justify their thinking. We are examining teacher actions that might encourage students to persist.

An aspect of this project that focuses on the student contribution to the social norms of the classroom related to the potential constraining effect that students can sometimes have on other students and the need to address this. One useful model informing our approach to addressing this was proposed by Dweck (2000) who explained that students who have a performance orientation, meaning they seek social affirmation as the goal of their effort rather than understanding of the content, avoid risk taking and challenging tasks due to fear of failure. Sullivan et al. (2009b) described a powerful classroom culture in which students fear censure from peers if they appear to persist, and so avoid the appearance of trying hard.

The first phase of the project involves design projects that included two iterations of observations of a group of teachers who posed challenging mathematical tasks and took actions to encourage students to persist on those tasks. In reporting on the analysis of those classroom observations we noted:

> The teachers were all willing to try the tasks suggested which may have involved departures from their usual practice. While the lessons may not have been perfect, they each created important opportunities for student learning. . . . The task suggestions presented to teachers were welcomed by the teachers and allowed them to adapt the task for themselves. The teachers generally maintained the challenge of the task, and encouraged the students to persist. The teachers adapted the suggested tasks to suit their preferred lesson structure, and it was noted that managing student engagement seems to be important. The lesson review phase is clearly a critical feature of the use of challenging tasks. (p. 44)

Again the three themes from Lerman are evident. The development of a descriptive language around persistence and challenge is important, as is communicating the intent of those terms to both teachers and students. It is clear that the posing of challenging tasks involves neither explicit instruction nor expectations that students create the relevant knowledge for themselves. It is evident that the social context in which challenging tasks are posed both offers opportunities and creates constraints.

The Maths in the Kimberley Project

I was invited to join the *Maths in the Kimberley* project, led by Robyn Jorgensen (see Jorgensen 2009), as a result of this ongoing work on tasks. The project explored three key pedagogical approaches to mathematics teaching and learning:

- ensuring that the focus of learning is both mathematically rigorous and culturally appropriate;
- not only building a sense of community learning and but also accommodating variations to suit particular learners' needs and experiences;
- recognising that teachers' knowledge, beliefs, and expectations have an impact on learners and the key element in any attempt to improve learning is the teacher.

The project examined these issues, both individually and together, in schools in the Kimberley region of Western Australia in which, arguably, the challenge of teaching mathematics is profound due to the multiplicity of variables that impact on the quality and outcomes of learning opportunities. The goal of the research was to investigate the challenges and opportunities afforded by our pedagogical approach.

The project found that being clear about the goals of teaching is helpful both to the teacher and to the students. It appears that "building on what the students know" is an effective strategy. Students are both willing and able to engage with rich tasks that required decision making by them and which allowed the construction of mathematical ideas.

In Conclusion: What Might a Lesson Based on These Ideas Looks Like?
Rather than presenting a summary of the above arguments, the following is an attempt to exemplify what the three themes from Steve Lerman's address might look like in classrooms. Indeed, in the 1998 address, Steve offered some insights into what teaching based on the three themes might look like. He wrote:

> When a teacher offers an activity in a classroom, say to share 2 oranges between 3 children, the different answers offered by the children arise from their previous experiences, what has been called the zone of actual development, and potentially pull the others including perhaps the teacher, into their zones of proximal development. (p. 77)

In other words, the teacher poses a task about which there is some ambiguity, supports student while they engage with the task, and then manages a discussion to which all students have the opportunity to contribute. To elaborate this, and to synthesise the ideas from the projects described above, the following is a description of a lesson that I observed when in Japan.

To establish a context or rationale for the learning, the teacher led a discussion about *tatami* mats (which are traditional rice-straw mats 90 cm

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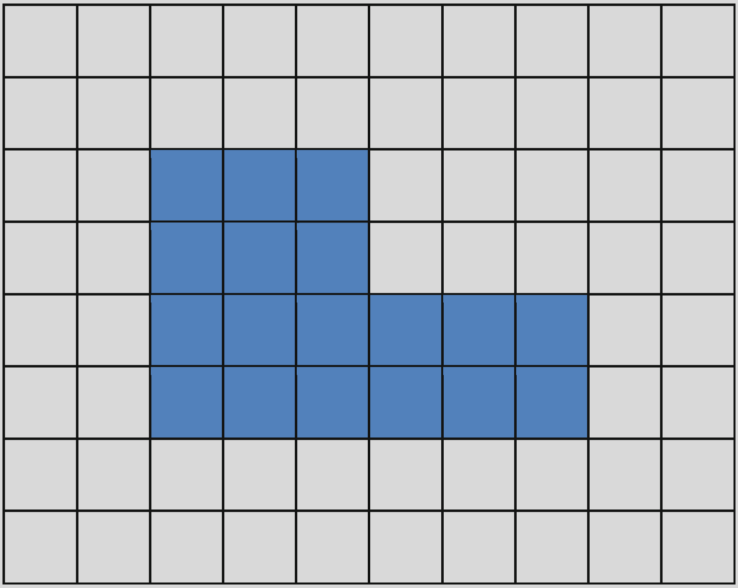
by 180 cm used commonly and sometimes used to describe floor size (area) of rooms or large buildings).

Next the teacher posed the central problem for the lesson, which was to work out how many squares in the following diagram are shaded. The intent of the teacher was to introduce the notion that the number of squares in an array can be calculated by multiplying the number of rows by the number of columns. The teacher had presented students with a worksheet on which there were TWO examples of this diagram,



This had the effect of emphasising to the students that it was not the answer that is important but the method of solution that they would devise. Rather than reproducing the teacher's method, the students could explore their own solution, and so have something to contribute to a classroom discussion later.

While the students were working, the teacher moved around the class identifying students who would contribute to a whole class discussion of their strategies.

The selected students were then given an A3 size version of the diagram, on which they were invited to record their solution. The students then displayed their A3 size versions of their solutions, and wrote the calculation they used. The following photograph records two of the solutions, one of which involved breaking the shape into two rectangles, and in the other the student moved one part to create a new rectangle.

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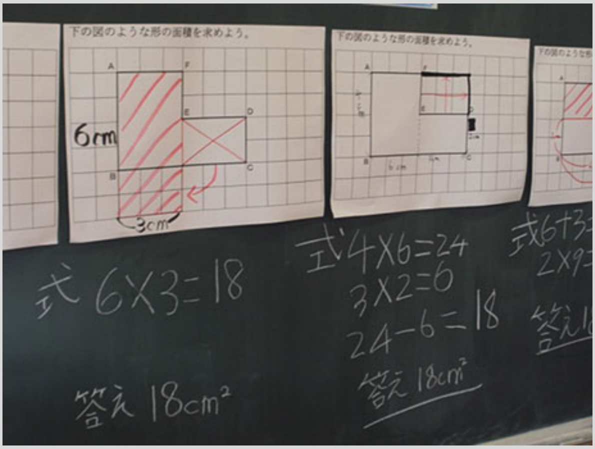

Eventually the board looked like the following with seven different solutions displayed and explained. The teacher had assisted each of the students to explain their strategy, and then spent some time drawing the key themes together.

There are some interesting characteristics of this lesson:

- there was an obvious focus on student generated strategies;
- in the introduction the teacher connected the content to a realistic context, the clarified the task, but did not predetermine the methods that the students would use;
- students were thoughtfully chosen to ensure that a range of strategies were presented and discussed;

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- the task is not complex pedagogically or organisationally, and only requires a willingness to let the students' thinking emerge;
- the task was accessible to all students, and the pedagogies supported the engagement of all students;
- the task was thoughtfully chosen to allow a diversity of strategies and representations, and also to allow students to experience important mathematical ideas such as area conservation (that is useful in the process of calculating the area of parallelograms), breaking a composite shape into parts (that can inform the calculation of the area of trapezia), and subtracting areas (that is used in calculating the area of paths around shapes).

Returning to the first of Steve's three themes, this lesson illustrates the call for the development of descriptive language. Interestingly, there are Japanese words that describe various aspects of this lesson. As described by Inoue (2010), *Hatsumon* refers to the initial problem; *Kizuki* describes what the teacher wants the students to learn; *Kikanjyuski* is the individual or group work on the problem; *Kikanshido* describes the teacher thoughtfully walking around the desks; *Neriage* is the carefully managed whole class discussion seeking the students' insights; and *Matome* is the teacher summary of the key ideas. No doubt this descriptive language is at the basis of the Japanese approach to lesson planning.

In terms of the perspective on how students come to learn mathematics, the content and methods were neither directed by the teacher nor did they involve undirected student activity. In fact, the task on which the students worked was sensitively chosen to facilitate student opportunity to create strategies, and the lesson was structured to facilitate the student contribution to the class learning.

It was clear that the class was operating as a community, and the social interactions at the time of lesson review, especially in the explanation of strategies and the opportunity for questioning of the presented strategy. The selected students clearly had expected this opportunity to explain their thinking, and the other students listened respectfully to the presenters, assuming presumably that this was an opportunity for them to learn.

This lesson illustrates that not only are the approaches recommended by Steve Lerman possible, they can result in meaningful student learning.

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