Chapter 5
Tracing the Advances in the Field of Mathematics Education

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Introduction: Setting the Scene

The 1980s was a particularly important period for the coming of age of the field of mathematics education, which had just entered its second decade of developing as an autonomous scientific community. The mainly psychological and theoretical perspectives and methodologies dominating the previous years started being considered critically and the social, cultural and political issues began to be seen as significant for understanding pupils' success or failure in learning mathematics within a society and across societies. As a result, diversified theoretical models coming from a range of disciplines such as philosophy, sociology, anthropology and linguistics crept into the young discipline of mathematics education and were exploited by the research community in its attempt to formulate more effective analytical tools to make sense and intervene in the ways mathematics is learnt. Thus, sociocultural approaches little by little entered the field, fighting to become accepted and respected.

The 1980s was the time Steve Lerman first appeared in the community, studying originally at Chelsea College, then King's College London for his doctoral thesis (Lerman 1986), where one can trace the beginning of his interest in epistemological issues of mathematics education. In order to situate the course of his contribution to the field, I discuss some contextual determinants of its future direction at a national and an international level as well as of Steve's main contributions. In particular, I begin with a discussion of some influential state initiatives for improving teaching and learning mathematics in schools and research projects originated by these initiatives. Then, I present research activities which have had an impact on the field's developments and a commentary on scientific activities that signalled new directions in thinking about research and practice in mathematics teaching and

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learning. Unavoidably, the above are mostly related to Anglo-Saxon countries, as relevant activities outside these countries made their appearance only from the 1990s, when socio-cultural approaches began to fight their way into the community.

Throughout the 1980s reports on low achievement in mathematics and concerns expressed publicly for the standards of the mathematics education offered in schools in the USA and in Europe led to the drawing up of agendas for action and to the setting up of committees to deal with these issues, thus establishing the need for change in the way mathematics is taught. The report "Curriculum and Evaluation Standards for School Mathematics" published by NCTM (1989) in the USA and the setting up of the Cockcroft Inquiry into the teaching of mathematics in UK schools (DES 1982) are two typical and very influential international examples of the actions undertaken to improve mathematics education standards. The former provided new objectives for students' learning, e.g., valuing mathematics, becoming confident in doing mathematics, reasoning and communicating mathematically and cooperating. The Cockcroft Report, on the other hand, recommended an emphasis on the uses of mathematics in everyday contexts, a broad description of a common core curriculum to prepare students for employment, a recognition of the need for curriculum differentiation, and a variety of teaching methods to include problem-solving, investigation, discussion and practical work, as well as exposition and practice. Both reports had considerable influence on mathematics teaching and assessment at all levels, but more importantly, fuelled a stream of research projects broadly following constructivism principles, the then emerging 'grand theory' for mathematics education (diSessa and Cobb 2004). Most of these projects, in fact, fit into one of the three constructivism traditions reported by Confrey and Kazak (2006), namely 'misconceptions, critical barriers and epistemological obstacles' (the other two being 'problem solving' and 'theories of cognitive development').

The relevant studies in this period carried out at Chelsea College (which merged with King's College in 1985) were among the most influential. The Concepts in Secondary Mathematics and Science (CSMS) project in particular investigated secondary students' mathematical and scientific reasoning aspiring to identify aspects of their thinking, rather than just measure achievement. The results made a very significant empirical and theoretical contribution to the documentation of children's understanding and misconceptions in school mathematics (Hart 1981) and subsequent projects sought to understand better the relationship between what was taught and what was learned (Johnson 1989). Subsequent national initiatives directed at improving mathematics teaching and learning drew on the CSMS study (e.g., the National Curriculum and the National Numeracy Strategy and Secondary Strategy). Nevertheless, the results of the above studies were criticized for a number of weaknesses. For example, the view of mathematics projected by CSMS findings as "an ordered hierarchy, and of pupils' mathematical abilities as correspondingly hierarchical" (Noss et al. 1989, p. 111), thus assuming that there exists a hierarchy of mathematics understanding, was seen as poorly justified and provocative. The criticism questioned the appropriateness as well as the limitations of the adopted theoretical and empirical approaches, thus keeping alive the quest for perspectives

that would provide better understandings of the highly complex processes of learning and teaching mathematics.

Despite the efforts reported above, subsequent studies indicated that students' mathematical learning and thinking improved overall very little and in some cases even deteriorated abated. Large numbers of pupils kept failing in and being poorly related to mathematics (e.g., Hodgen et al. 2009). What is more, failure in and negative attitudes to mathematics were shown to be more persistent in minority populations, including girls and socially disadvantaged children (e.g., Walkerdine 1998). Thus, the challenge for theoretical frameworks and research methodologies that would go beyond psychological considerations to examine mathematics learning, incorporating social, cultural and political factors was open to be pursued. This quest began to be mirrored in contributions to scientific meetings organized around the world as well as in publications responding to calls for syntheses of past research and the setting up of research directions advancing the field.

For example, research reports and plenary/invited speeches underlying such a quest appear in PME's conference programmes. To mention just few but indicative occasions: (PME-10, 1986, London, UK), Seymour Papert /Beyond the Cognitive: the Other Face of Mathematics, Christine Keitel/Cultural Perspectives and Presuppositions in Psychology of Mathematics Education; (PME-12, 1988, Veszprem, Hungary), Terezinha Nunes/Street Mathematics and School Mathematics; (PME-14, 1990, Oxtabec, Mexico), Valerie Walkerdine/ Difference, Cognition and Mathematics Education. More importantly, a special day devoted to Mathematics Education and Society was reserved in the programme of the sixth International Congress in Mathematical Education in Budapest in 1988. The set of proceedings published afterwards (Keitel et al. 1989) is the first international collection of research papers on social factors in mathematics education.

Other publications indicating increasing awareness of sociocultural issues in mathematics education appeared in the Anglo-Saxon world. For instance, the first Handbook of Research on Mathematics Teaching (Grows 1992) included chapters on ethnomathematics (Nunes), gender (Leder) and race, social class and language (Secada). It was followed later by the International Handbook of Mathematics Education (Bishop et al. 1996) which tried to cover work not necessarily published in English. Lerman (2000) adds to this list publications referring to Vygotsky's work for the first time in Mathematics Education, e.g., Crawford (1981, 1988) and Cobb (1989).

The above reflect the community's efforts right from the beginning to identify theoretical and empirical approaches to understanding and interpreting the processes of learning and teaching mathematics in their entire complexity. It also shows the gradual, sometimes reluctant, awareness of the need for these approaches to incorporate psychological as well as sociocultural elements. Steve Lerman entered in the field as a member of a very active research community (British), exactly at the time that the movement to new perspectives was taking shape, enhanced by matured social demands (e.g., education for all), technological advances, such as microcomputers, as well as significant developments in related disciplines, like psychology (Vygotsky), sociology (Bernstein, Bourdieu) and

philosophy (Foucault, Derrida, Habermas, Heidegger) to mention just a few. His work responded to the research and practice concerns of the community alike in an imaginative and knowledgeable manner, providing some challenging and occasionally provocative ideas. In the next section, an attempt is made to highlight some of his most influential ideas, particular those that had a significant impact on research in mathematics education around the world.

Some Influential Contributions

Three sets of ideas stand out in Steve Lerman's contribution to the field of mathematics education: epistemologies of mathematics and mathematics education, the turn to a socio-cultural view of mathematics education; and, mathematics teacher education and professional development. Of course, Steve's scientific activity covers a much wider range of issues, the bulk of which, however, I would argue, constitute developments along these ideas. In the following, I discuss some fundamental aspects of these central sets of ideas.

Epistemologies of Mathematics and Mathematics Education

At the time Steve Lerman entered the field, mathematics education research drew mainly upon two disciplines, mathematics and psychology, especially cognitive psychology. Mathematics provided a framework for analysing the knowledge to be pursued at school and psychology a rationale for the way to be effectively accomplished by children and teachers. An interest in epistemologies of mathematics and in the dominant constructivist paradigm as well as their influence on learning and teaching mathematics emerging at the time preoccupied Steve's early work.

Seeking to examine the influence of epistemologies of mathematics on mathematics education, Lerman (1990) identified two contrasting views of mathematical knowledge reflected in research and practice in mathematics education, that is, absolutism at one extreme and fallibilism at the other. In the former, mathematics knowledge is seen as absolute, certain, abstract and value-free, a discovery of timeless truths; the latter accepts the uncertainty of the mathematical knowledge, which is understood as a process of conjectures, proofs and refutations:

> Fallibilism, a view which accepts the potential refutation of all theories, and counter-examples to all concepts, allows one to ask how does one know that this answer is better than that one, what might constitute a notion of 'better', might they not both be possible, as with Euclidean and non-Euclidean geometries, or arithmetics with or without the Continuum Hypothesis. (Lerman 1989, p. 217)

For fallibilism, mathematics is the outcome of social processes, relative to time and place, characterized by its activity, which includes engaging in interesting problems, conjecturing, testing, reflecting, evaluating and communicating.

Recognizing that theories of mathematics have an important influence on theories of mathematics education, Steve focused analytically on relevant issues in a later work (Sierpinska and Lerman 1996). In particular, the authors attempted to elaborate critically the origins and to make explicit the basic assumptions underlying epistemologies in mathematics and in mathematics education. With respect to the latter, the subjective-objective character of mathematical knowledge, the role of social and cultural contexts in cognition and relations between language and knowledge were scrutinized. Also, dominant mathematics learning theories were compared and relationships between epistemology and a theory of instruction were explored. The authors concluded that "epistemologies of mathematics could find their way to mathematics education only via genetic, social, cultural and historico-critical epistemologies. Moreover . . .epistemologies do not translate directly into theories of instruction and do not make recommendations for the practice of teaching" (p. 867).

Based on the above, Lerman (1990) argued for the need for a closer interaction between philosophy and psychology of mathematics education, neglected for long because of the "predominance of interest in psychological aspects of what is taught and how" (p. 53). Trying to justify the strong association between mathematics and psychology characterizing the field of mathematics education, he identified two reasons (Lerman 2000). The first is related to the high status of the two disciplines, which offers a legitimization to the research carried out in the latter. The second reason refers to the fact that mathematics and mathematics education are strongly linked with the construction and preservation of the dominant systems of reason in the Western world. Mathematics is seen "as a marker of general intellectual capacity", allowing "its gendered and Eurocentric character, creating through its discursive practices the reasoning logical norm" (Lerman 2000, p. 21). Valero (2004), much in agreement with Lerman, attributes to psychology a crucial role in the process of reducing the student to a cognitive subject via mathematics education and the relevant research interest in his/her mathematical thinking processes. She concludes that "the discourses of mathematics education have resonated with the discourses of mathematics and psychology . . . in the construction of a particular research discipline, with particular theories and methods, supporting the constitution of practices in the classroom that fulfil essential social functions, which help in sustaining a certain kind of social organisation" (p. 5).

Among the psychological perspectives exploited in mathematics education, constructivism has been especially influential. Concentrating on its two hypotheses, that knowledge is actively constructed by the cognizing subject and coming to know is an adaptive process organizing the experiential world, Lerman (1989) noted that the shift of attention to teaching for understanding that constructivism brought about provided no answer to the question of how to make this happen and know that has happened. He attributed this to the notion of 'understanding' remaining tied to certainty and absolutism:

> . . .the process of coming to understand a concept is one that takes place in the mind of an individual, and the final step of achieving that full understanding of a timeless, universal notion is a very private, almost mystical one. It is certainly beyond the power of any outsider, such as a teacher, to know that the process has taken place in full (p. 221).

He argued that accepting the hypothesis that coming to know is an adaptive process organizing one's experiential world locates objectivity in the social domain: concepts and their meaning are public and so too is understanding: "theories and concepts are rooted in practice, and obtain their meaning from use. They gain their objectivity in their public nature, in that theories written down become public property, subject to dispute, negotiation and adaptation. Their objectivity does not lie in their being the ultimate truths" (p. 223).

In a later work, arguing against the central position of constructivism that the individual is the source of meaning, Lerman (1993) juxtaposed the work in cultural psychology, situated cognition, classroom studies and so on, arguing:

> Knowledge isn't in the individual's mind, nor 'out there' in objects or symbols. Knowledge is as people use it, in its context, as it carries individuals along in It and as it constructs those Individuals. Knowledge is fully cultural and social. And so too is what constitutes human consciousness. Communication drives conceptualisation (p. 23).

Thus, Steve progressively moved to a position of rejecting constructivism in favour of a fully socio-cultural view of the mind, the individual, learning and knowledge offering a much richer view of teaching and learning.

Sociocultural Perspectives and the Social Turn

In the 1990s, the shortcomings of drawing predominately on mathematics and cognitive psychology to understand the complexity of mathematics learning and teaching attracted the interest of a notable number of researchers. Constructivism had fueled the query of what it means to know and teach mathematics, placing the learner as an agent in the world. However, its individualistic approach to meaning-making (the individual was seen as autonomously building his/her own subjectivity) started being challenged on the grounds of the little attention paid to interpersonal and social characteristics of the learning context.

Steve Lerman joined the discussion expressing concerns about the adequacy of constructivism but also about attempts to consider the social and individual meaning making approaches as complementary. He argued that the main difference between Piaget's and Vygostky's theories is not that they down-play either the social or the individual, but that the one identifies the cognizing individual and the other cultural and discursive practices as the source of meaning respectively. That is, they rely on different premises with respect to the meaning making process (Lerman 1996). Shortly afterwards, espousing views on the individual as constituted in and through the social world, he suggested that:

> . . .we might want to talk of the individual as a fragmented self at the intersection of a unique collection of overlapping identities constituted in different practices, as lived out through class, race, ethnic, sexual, gendered, regional and other positions. Thus 'we cannot fully specify the psychological subject/agent as an object whose nature can be defined in isolation from a context'. (Harre and Gillett 1994, p. 26 quoted in Lerman 1998a, p. 41)

Gradually but systematically Lerman's work was adopting a sociocultural perspective, where cultural and discursive practices become the source of meaning. In an overview of studies in mathematics education acknowledging 'social' factors, he referred to the growing body of such research as the 'social turn' in the field (Lerman 2000), describing it as "the emergence into the mathematics education community of theories that see meaning, thinking, and reasoning as products of social activity" (p. 23). These theories, like, for example, cultural psychology, theories of cognition in practice and sociological theories, recontextualized within the field of mathematics education led to the production of new knowledge. This knowledge differed from the one developed in mainstream mathematics education in that coming to know mathematics was not seen as emerging "from and within the mind of decontextualized cognitive subjects", but as "constituted in the encounter between contextualized, historically grounded human beings and their activity in particular settings and spaces that are socially constructed" (Valero 2004, p. 6). Lerman (2000) attributed the receptivity of the community to social theories to political concerns related to the continuing exclusion of some children from learning mathematics and the emerging impact of developments in disciplines like psychology – particularly of Vygotsky's ideas – and sociology, anthropology, political science, cultural studies and others on mathematics education community.

For Lerman (2000), the greatest challenge for research in mathematics education offered by theories within the sociocultural perspective is "to develop accounts that bring together agency, individual trajectories . . . and the cultural, historical and social origins of the ways people think, behave, reason and understand the world" (p. 36). He proposed the metaphor of a zoom lens for research whereby the focus of study is seen as:

> . . .a moment in socio-cultural studies, as a particular focusing of a lens, as a gaze which is as much aware of what is not being looked at . . . Draw back in the zoom and the researcher looks at education in a particular society, at whole schools or whole classrooms; zoom back in and one focuses on some children, or some interactions. . . Research must find a way to take account of the other elements which come into focus throughout the zoom, wherever one chooses to stop. (Lerman 1998b, p. 67)

The move of psychology in the last three decades on learning in social practices and the way in which physical and cultural tools mediate learning turned the focus in mathematics education on discursive practices and on the social factors as constitutive of learning. In this view, learning is about participating in practices and becoming a member of a community where individuals are seen as discursively constituted. Such an approach enables the link between the actions of individuals and groups in the classroom, history and culture placing it within sociocultural tradition. As a person joins in a practice, the regulating effects of the practice begin, positioning the person in that practice. Thus, in searching for evidence of

mathematical understanding, the focus moves on the students' developing identities as "speakers and actors of mathematics in school classrooms" (Lerman 2001a, p. 98). Hence, mathematical activities, texts, experiences, social relationships, positions and voices as well as histories and functions of mathematical artefacts acquire particular importance, being constituent elements of the students' identities.

The 'social turn' (Lerman 2000) in mathematics education brought to the fore new understandings and new concerns that gradually de-emphasise cognitive psychology as the only interpretative framework, favouring instead sociocultural theories. Our understanding of children's learning moved from a focus on the cognizing subject to being seen as a product of social activity, where not only the cognition of the subject but also his/her relations with other individuals and their shared discourses matter. According to Lerman, this shift was also the result of growing political concerns about the ways mathematics education is related to reproduction of inequalities through the structures of school, highlighting the critical role of mathematics education in society.

Mathematics Teachers' Education and Professional Development

A substantial part of Steve's work from the end of 1990s onwards is related to research on mathematics teachers' education, an emerging area of interest at the time and a pivotal concern of the community since then. In a review of the relevant studies Lerman (2001b) raised questions about the mainly implicit, not necessarily unproblematic assumptions of these studies with regard to teachers' learning. To substantiate his criticism, Steve focused on some central issues of the relevant research: teachers' beliefs and practices, reflective practice and teachers' knowledge.

In particular, he argued against the then identified trend to map teachers' beliefs to their practices as separated and stable entities. He suggested that they were both influenced by contexts; questioning the prevailing view that reflection promotes teachers' autonomous learning, critical judgement and freedom of bias. This prior view implicitly assumed a direction of development, ignoring the relationships of power which are present. Given that reflective practice takes place in social contexts Steve doubted the legitimacy of treating teachers' knowledge as the result of cognitive conflict situations, thus, implicitly extending learning through adaptation into adult learning. Summing up, he advocated that viewing teachers' learning as their growing awareness rests on a highly individualistic theory of learning which assumes that what is to be learnt is in one's head.

Based on his critique, Lerman (2001b) suggested an alternative route to mathematics teachers' learning, reflecting the emerging interest in social and cultural aspects of leaning in mathematics education at the time. According to this, learning is understood not as a cognitive organization and reorganization but as emerging identities through participation in various communities. This participation produces

and not simply reflects beliefs, practices, purposes and goals, constituting the social settings which shape teachers' learning. Lerman indicated three theoretical perspectives as suitable for studying mathematics teachers' learning in socio-cultural terms: Lave's work (1988), activity theory and postmodernism.

In Lave's approach, the teacher-learner is seen as apprentice, who learns at the same time as becoming part of the social practice (e.g., teaching), thus developing identities. Wenger's (1998) description of identity as "a way of talking about how learning changes who we are" fits the dynamic character attributed to the notion of identity here as "something that is constantly negotiated through the interplay of one's lived experience in the world and how we and others discursively interpret that experience" (Goos 2013, p. 522). Within an activity theory perspective, human activity is seen as a system: a group of people who share a common object and motive over time, as well as the tools they use to act on that object and realize that motive; activity systems are constrained by division of labour and by rules. Tensions and contradictions within the system and between systems constitute sources of learning (Engeström 2001). Here teachers' learning is seen in relation to the social practices in which power and knowledge are situated. Finally, postmodern theory argues that teachers hold multiple, overlapping subjectivities. Each time, certain aspects of these subjectivities are called upon and are expressed by identities of powerfulness or powerlessness. Teachers develop new subjectivities as a result of their social and professional activities. Learning to teach here is viewed as becoming able to deconstruct practice and re-inscribe it into a language recognizing difference and enabling students' voices.

Lerman (2001b) argued that to study teachers' education we need theories that address the complexity of teaching as a social practice, recognize that research settings are themselves learning sites and to grant agency to teachers. Postmodern pedagogy, he advocates, appears to satisfy these requirements, encouraging the expression of difference, promoting methods of critique and encouraging learning and teaching theorizing. In a recent article, more than 10 years ahead and with a significant body of research on mathematics teachers' education within the sociocultural perspective, he appears more careful, arguing for the need to explicate the focus of our theoretical lenses, warning, however, that expectations for coherence across research approaches and impact on learning and teaching practices should be kept low, because of varied social and cultural traditions as well as interpretations of educational research (Lerman 2013).

Sociocultural Research in Greece

Research activity in mathematics education in Greece was initiated by the end of 1980s. Among the first members of the community were researchers who had completed their postgraduate studies abroad and were already active in the international research scene. This group of people started to gradually build cooperation with teachers and researchers from Greece and abroad, creating the conditions for

the establishment of today's vibrant community, with notable contributions in international meetings.

The timing of the emergence of Greek research in mathematics education in the international scene, the emphasis on developing partnerships with the international community, and the rigidity of the country's mathematics education system urging for reforms explain the choices shaping the relevant national research activity at the time. In particular, until early 2000s, although the dominant paradigm is constructivism, socio-cultural considerations and concerns enter the research agenda, mirroring a resonance to the 'social turn' emerging in the world community. Lerman's but also others' work undoubtedly contributed to this direction, as evidenced by his collaboration with Greek researchers (e.g., Lerman et al. 2002), the contributions to national meetings, the postgraduate Greek students and his frequently referenced publications.

In an effort to synthesize the Greek research activity in mathematics education of nearly 20 years at the 33rd IGPME in Thessaloniki, the broadening of the research perspective during the 1990s allowed viewing mathematics also as a sociocultural process was notified (Kynigos et al. 2009). Most of the studies reported under this research paradigm fall in the three main areas identified as crucial for Lerman's contribution to the field (see Section "Some Influential Contributions"). In particular, some of the studies examined teachers' instructional practices from an epistemological point of view (e.g., Tzekaki et al. 2002), identifying aspects of the way teachers manage the epistemological features of mathematics, which distort the mathematical meaning emerging in the classroom; other studies employed a variety of theoretical lenses, such as Vygotskyian or situated learning theories, shedding light on the decisive role of social and cultural practices and tools in the establishment of shared mathematical meanings (e.g., Kynigos and Theodosopoulou 2001). Finally, some studies focused on teachers' professional development, highlighting aspects of the crucial role of the interaction between teachers and researchers for this development (e.g., Sakonidis et al. 2007). In the following, I discuss three representative examples, one from the first and the other two from the third group of the above studies, aiming to exemplify the research developed along central dimensions of Steve's contribution to the field in the Greek context.

The first study reports on an attempt to examine the epistemological status of the mathematical knowledge interactively constituted in the classroom (Kaldrimidou et al. 2008). The basic position adopted in the study is that school and scientific mathematical knowledge differ (Sierpinska and Lerman 1996) and that the epistemological status of the former cannot be deduced only from the latter. It needs to be studied also in relation to the social contexts of the teaching and learning processes. To this direction, the authors focus on the classroom phenomena that determine the nature of the meaning emerging in the classroom and characterized as "mathematics". Three theoretical constructs were employed to investigate this nature: the idea of sociomathematical norms (Yackel and Cobb 1996), the notion of the epistemological triangle (Steinbring 2006) and the analysis of the management of the epistemological features of mathematics (Kaldrimidou et al. 2000). These constructs were used to analyse two mathematics lessons, provided by two secondary

school teachers, in an attempt to examine the different features of the mathematical knowledge shaped in the classroom that each one of these constructs allows to identify. The results show that each of these three perspectives allows access to specific features of this knowledge, which do not coincide. Moreover, when considered simultaneously, the three perspectives offer a rather informed view of the status of the knowledge at hand. The authors concluded that the comparative and sometimes complementary use of different theoretical tools enables the sharpening of the analysis related to the mathematical status of this knowledge.

The second study is on professional learning developed in the context of longitudinal collaboration between three secondary mathematics teachers and two academic researchers on mathematics teaching practice (Potari et al. 2010). Conceptualising teaching as learning in practice and teacher change as a process of shifting participation in a community of inquiry (Jaworski 2006), the study focused on the reflective activity developed by its members as well as on the tensions and the conflicts that emerged in the shaping of an inquiry identified by them. The analysis of the data, mainly transcribed meetings and electronic communication, shows that during the years of working together, the collaboration between the teachers and the researchers changed in content (shifting from focusing on local practices to tackling broader issues of mathematics teaching) and in form (being transformed from a student–supervisor relationship based on a common interest in systematically inquiring into mathematics teaching to one of co-enquirers of its development). These changes can be attributed to the participants' commitment to explore each other's perspectives in relation to teaching mathematics, with the ultimate goal of arriving at a deeper understanding of it. This brought forward issues of 'belonging' and 'becoming', that is, of identity, and thus of identification with existing meanings and negotiation of new ones. As a consequence, tensions and disputes emerged, particularly at the beginning, usually related to the interplay between local mathematics teaching practice and its global context. However, at the same time, coalitions and alignments were enabled, forced by the requirement for some form of consensus in order for the whole process to be socially effective.

The third study concerns again professional learning but in a different site and through a different process compared to the above (Potari 2013). Specifically, the focus here was on the interactions between the research and the teaching activity of a group of prospective and practicing secondary mathematics teachers developed in the context of a Master's course in mathematics education. Taking the view that teachers' professional development is predominately a continuous process of critically aligning with the norms of the teaching community of practice, the author adopted a- third generation – Activity Theory perspective (Engeström and Sannino 2010) to identify links made by the participating teachers between the above two activity systems. The teachers were assigned a number of research-like and teaching tasks during one of the Masters modules aiming to initiate them to research in mathematics education as a tool for inquiring into teaching. The analysis of a group of five teachers' actions connected to the design and evaluation of a teaching intervention showed contradictions related to the rules and the division of labor in power

in each of the two communities; also hesitant convergences emerging as the course was progressing with respect to a shared object of the two activity systems, that is, understanding students' thinking and exploiting research to inform teaching. To this direction, the role of the other teachers and especially of the teacher educator was crucial in promoting critical alignment with research practices, challenging reflective thinking on the interactions between researching and teaching and facilitating these interactions through the provision of appropriate tools.

The above studies are indicative of the research activity developed by members of the Greek research community within the sociocultural paradigm and along some of the central dimensions of Steve Lerman's contribution to the field. Of course, one should not neglect to mention work related to sociopolitical issues, which sprung amid the 'social turn', like, for example, identity and its political orientations (Chronaki 2013), or to cultural dimensions of mathematics education, e.g., language and culture interconnections (Stathopoulou and Kalabasis 2006). This body of research contributes to furthering the exploration of theoretical constructs and the accumulation of empirical evidence that advance our understanding of the dynamics of sociocultural readings of the "what" and "how" of mathematics education.

Concluding Remarks

Over the last three decades, the field of mathematics education addressed a number of important questions concerning research and practice in learning and teaching mathematics. As Jaworski (2006) points out:

> . . .it has become common to think of mathematics in fallibilistic terms . . . to consider learning as a constructive process . . . to situate knowledge and learning relative to communities of practice . . .and to debate the commensurability of constructivist and sociocultural learning theories . . . Looking back . . . we might refer to 'big theories' such as constructivism and sociocultural theories, that have been highly influential in addressing mathematical knowledge and the learning of mathematics. The mathematics education discipline has become mature in such theoretical considerations. . . (p. 188).

It is evident that Steve Lerman's work has been highly influential for this maturity to come about. He challenged individualistic, psychological theorizations to account for learning and teaching in mathematics classrooms, suggesting that these are social activities, situated within nested levels of socialization (e.g., schools, traditions) and influenced by a multitude of social factors (e.g., culture, race and gender). Within this perspective, he promoted a participatory model to account for coming to know and teach mathematics. This model focuses on the use of discourse and its contents (e.g., norms, values) as crucial mediating tools in order to interpret the mathematical learner in context.

Projecting in the future, Lerman (2006a, b) identifies three emerging key areas within the sociocultural paradigm which will develop further: learning as identity formation, activity theory and ethnography. In particular, learning and a sense of identity are the same phenomenon and the development of a school mathematics

identity adds another layer to students' multiple identities, some of which are more important for them. Sociocultural perspectives appear to offer valuable frameworks to examining how the various modalities of teaching mathematics shape school mathematics identities. Activity theory on the other hand, places emphasis on the notion of human activity, incorporating cultural artefacts, social settings and the acting persons' goals and motives into a whole and on meaning as mediating the world for every individual through tools and signs, as well as through community, rules of activity and division of labour within the activity system. Finally, the interaction between practical and formal mathematical knowledge has attracted much research attention in the last decades. Relevant studies indicate that mathematics classrooms, like workplaces, are learning-in-practice sites characterised by tacit as well as explicit features and forms of participation and identity, thus needing ethnographic engagement to make sense of the emergent learning.

It is evident that the enormous complexity of mathematical learning and teaching processes leaves open the challenge of seeking to adequately understand them. Steve Lerman's work has resulted in a growing interest for exploring the social aspects of these processes, with the use of theoretical and methodological frameworks from a range of disciplines, such as sociology, anthropology, political science and cultural studies, contributing substantially to the direction of unfolding their complexity.

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