Chapter 15 Lerman's Perspectives on Information and Communication Technology

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Introduction

In many of his recent publications, Steve Lerman discusses the use of information and communication technology (ICT) in mathematics teaching and learning, addressing several issues such as numeracy, classroom interactions, scaffolding, teacher education, pedagogy, online education, the use of whiteboards, and so forth (Lerman 2004; Lerman and Zevenbergen 2006, 2007; Zevenbergen and Lerman 2005, 2006, 2007, 2008; Crisan et al. 2006; Rosa and Lerman 2011).

On the one hand, within our work in mathematics education (e.g., Borba and Villarreal 2005; Borba 2012; Scucuglia 2012), we have addressed sociocultural perspectives to conceptualize the role of ICT, or digital technologies as we have been calling it lately, as cultural artefacts in mathematical learning and activity (Borba et al. 2010). We have built on the very notion of humans-with-media to emphasize cognition and mathematical knowledge production as a social, collective, and object-directed undertaking (Borba 2009). On the other hand, we have not properly deeply addressed Steve Lerman's perspectives in our theorization as we should. Thus, in this chapter, we present (a) the way Steve Lerman dealt with ICT in different publications and (b) potential links between his perspectives and part of the work of our research group on computers, other media and mathematics education – GPIMEM (http://www.rc.unesp.br/gpimem/) at Sao Paulo State University, Campus of Rio Claro, in Brazil. We also emphasize potential theoretical insights to our current interest on the use of digital technology and the performance arts for multimodal mathematical communication (Scucuglia 2012).

Sociocultural Perspectives for the Use of ICT as the Resistance Towards Constructivism

We find an interesting similarity on the use of sociocultural perspectives in theorizing pedagogies for the use of ICT in mathematics education when we compare some of Steve Lerman's ideas and the notion of humans-with-media proposed by Borba and Villarreal (2005). This notion has been paramount for most of research developed by GPIMEM. In order to emphasize a social, cultural, and collective nature of the use of ICT in education, authors such as Lerman, and Borba and Villarreal have pointed out some aspects that expose a kind of *resistance* toward the localism of the individual-biological nature of the being or subject in constructivist points of view. According to Lerman (1996):

Rejecting constructivism, the individual is integrally part of the social world, and thinking is a dialect relationship with that world. Individual mental structures are not the fundamental unit of cognition; meanings, which are first on the social plane, perform this function. Inevitable biological development is not seen to lead to human functioning; the development of consciousness, which only takes place in social life, is the essence. (Lerman 1996, p. 148)

Steve Lerman also creates an argument to point out a tension within constructivist perspectives in terms of intersubjectivity, mainly through the movement involving radical and social constructivism.

The extension of radical constructivism toward a social constructivism, in an attempt to incorporate intersubjectivity, leads to an incoherent theory of learning. A comparison of Piaget's positioning of the individual in relation to social life with that of Vygotsky and his followers is offered, in support of the claim that radical constructivism does not offer enough as an explanation of children's learning of mathematics... Constructivists, whether radical, weak, or social, draw their inspiration from Piaget, for whom the individual is the central element in meaning-making.... Vygotsky attempted to develop a fully cultural psychology by which I mean placing communication and social life at the center of meaning-making, which is a challenge to Piaget's ideas. (p. 133)

It is not a surprise to argue that sociocultural perspectives point out that reality, knowledge, and meaning are socially, historically, and culturally produced through language. Socioculturalism connects activity to participation in cultural practices (Cobb 1994). Instead of focusing on the individual processes of learners' meaning-making and knowledge construction (e.g., cognitive conflict and equilibrium in Piagetian constructivism), sociocultural perspectives emphasize the social interaction and enculturation of subjects in (mathematical) learning, development, and activity.

Vygotsky (1978) investigated children's development and learning and how these processes are conditioned by the role of culture and language. According to Vygotsky, higher mental functions are historically developed within particular cultural groups, through social interactions with the significant people in children's lives, particularly parents and teachers. Through these interactions, children learn the habits of the culture, including patterns of speech, verbal and written language,

and other symbolic representations. Thus, Vygotsky emphasized (a) the social interaction with more knowledgeable others in the zone of proximal development and (b) the role of culturally developed sign systems and languages as psychological tools of thinking.

In fact, we do agree with Gadanidis and Geiger (2010) when they state that:

Sociocultural theories of learning are founded on a position that intellectual development originates in, and so is not just facilitated by, social interaction. Learning is a process of enculturation into the practices of a learning community. Enculturation into the community requires the appropriation of modes of reasoning, discourse and knowledge creation that are accepted by the discipline around which the community is based. Learning mathematics in such a community means a learner must participate in debate about new ideas and practices, offer critique of others' ideas and defend their own propositions via explanations and justifications. (p. 96)

Central in our research, sociocultural theories actually lead us to an object-oriented view of cognition. Goos et al. (2000) clarify that "a central claim of sociocultural theory is that human action is mediated by cultural tools and is fundamentally transformed in the process" (p. 306). In our perspectives, technologies can be conceptualized as cultural artefacts of thinking (Papert 1980; Noss and Hoyles 1996). Borba and Villarreal (2005) thus argue that technologies are not neutral in mathematical knowledge production. Media are actors that (re)organize mathematical thinking. Not only humans, but humans-with-media (e.g., students-and-teachers-with-computers) produce mathematical knowledge.

Humans-with-Media

Borba and Villarreal (2005) use the expression *humans-with-media* as a metaphor to theorize the cognitive "inter-shaping" between humans and technologies in mathematical knowledge production. The inter-shaping relationship stresses the mutual shaping that there is between humans and artefacts. Artefacts are produced by collectives of humans-with-other-artefacts with a certain goal. Such a goal is transformed by others who use and shape it to social perspectives of a historically dated collective of humans-with-media. So artefacts and in particular digital artefacts are transformed and transform different collectives of humans-with-media, in the sense that artifacts are always seen as communicating device.

The authors build their perspectives using the notion of *technologies of intelligence* (Levy 1993): a historical-cognitive perspective of technologies. According to Levy (1993), there are three main technologies of intelligence associated with memory and knowledge. They are: orality, writing, and information technology. In oral societies, humans produced knowledge through myths and rituals, cyclically and locally, transmitting information from one generation to another. However, this *circularity* was reorganized into *linear* ways of reasoning in writing societies, mainly through the popularization of books, due in large part to the invention of Gutenberg's printer press.

ICT can be understood in the same way. The linearity of memory conditioned by the temporality of writing has been assuming a "web or net design" through the plasticity of digital technology. Computers and online tools combine multiple modes of communication. They shape the ways that contemporary societies interact and communicate. The "linear reasoning" of writing has been challenged by ways of thinking involving orality, writing, images, simulation, experimentation, and instantaneous communication. Regarding current technological innovations, there are innovating ways to communicate, extend memory, store information, represent, simulate, and produce meanings and knowledge.

Borba and Villarreal (2005) argue that "our individual consciousness and cognitive process are always subject to interaction with the technologies of intelligence" (p. 26). That is, "knowledge is produced with a given medium or technology of intelligence" (p. 23).

Humans-with-media, humans-media or humans-with-technologies are metaphors that can lead to insights regarding how the production of knowledge itself takes place.... This metaphor synthesizes a view of cognition and of the history of technology that makes it possible to analyze the participation of new information technology 'actors' in these thinking collectives. (Borba and Villarreal 2005, p. 23)

Borba and Villarreal (2005) discuss sociocultural perspectives (Tikhomirov 1981) to develop the notion of humans-with-media. According to Tikhomirov (1981), computers do not replace, substitute, or merely complement humans in their intellectual activities. Processes mediated by computers *reorganize* thinking. Tikhomirov, who was Luria's student, argues that computers play a mediating role in thinking as language does in Vygotsky's theory. Regarding the nature of human-computer interaction in terms of feedback, the dimensions involving computational mediation provide new insights in terms of learning, development, and knowledge production. Tikhomirov claims that:

With regard to the problem of regulation we can say that not only is the computer a new means of mediation of human activity but the very reorganization of this activity is different from that found under conditions in which the means described by Vygotsky are used. (p. 273)

Borba and Villarreal (2005) use Tikhomirov's ideas to argue how the notion of mediation by computers is qualitatively different to the mediation involving paper and pencil, for instance. Through digital mediation, information technologies reorganize mathematical thinking. Media shape knowledge production and transform mathematics.

Levy (1993) defines *cognitive ecology* as "the study of technical and collective dimensions of cognition" (p. 137). He sees technology not simply as a tool used by humans, but rather as an integral component of the cognitive ecology. Further Levy (1998) claims "as humans we never think alone or without tools. Institutions, languages, sign systems, technologies of communication, representation, and recording all form our cognitive activities in a profound manner" (p. 121). According to Levy, technologies *do not determine* thinking. Technologies *condition* thinking (Levy 1993, 2000). He (1993) uses the term *thinking collectives* to

discuss the collaboration between human and non-human actors in the cognitive ecology. Levy (1993) argues that thinking collectives of humans-technologies form the cognitive ecology.

Levy (1997) relates cognitive ecology and thinking collectives to *collective intelligence*, defined as "a form of universally distributed intelligence, constantly enhanced, coordinated in real time, and resulting in the effective mobilization of skills" (p. 13). By intelligence, Levy (1998) means "the canonical set of cognitive aptitudes, namely the ability to perceive, remember, learn, imagine, and reason" (p. 123). More recently, Borba (2009, 2012) has proposed that media does not only change the way collectives think but it has changed the very nature of what "being human" means. Media such as mobile phone and computers are not only merging among themselves but are deeply transforming our very perception of who we-are-with-technology.

In fact, it is important to clarify that the resistance toward constructivist views, in the context of supporting a collective-sociocultural perspective for the educational use of ICT, does not exclude the symbiosis involving contextual and *personal* dimensions of classroom activities. In this direction, Crisan et al. (2007) propose a framework to theorize teachers' practices on use of ICT at the secondary level considering the involvement between ICT content and curricular conceptions, pedagogic and mathematical conceptions. The personal ICT pedagogical construct emerges from that involvement through teachers' learning and experiences with ICT (see Fig. 15.1). In fact, the authors argue that:

Learning to teach with ICT is a process. It demands doing and practice and ... teachers developed their own 'expertise' with ICT, which we call here personal ICT pedagogical construct, consisting of conceptions of how the ICT tools and resources at their disposal

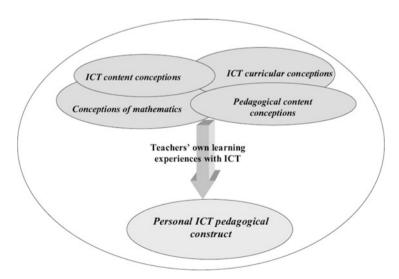


Fig. 15.1 Crisan, Lerman, and Winbourne's (2007) framework about teachers' practices on use of ICT

benefited their teaching of mathematics and their pupils' understanding and learning of mathematics (Crisan et al. 2007, p. 33).

On the Risk Zone for Teaching with ICT

Lerman (2004) presents some aspects of a research project that focused on key policy initiatives in Queensland, with emphasis on numeracy and the use of ICT in the curriculum. In his conclusions, Lerman (2004) points out some problems toward *innovation* in teaching, highlighting that "most teachers are worried about being seen to be less competent in computer use than their students and fear the loss of power and status if students see they know more than their teachers" (p. 622). Similarly, Lerman and Zevenbergen (2006) argue that "there may be some resistance to change pedagogy in mathematics classrooms in response to the potential of ICTs and to the call for improving achievement amongst traditionally failing students" (p. 49).

Focusing specifically on some results of an analysis of the project called New Basics, Lerman and Zevenbergen (2006) state that:

Of course there are many ways of using ICTs and not all of them enhance the learning of mathematics in the same way ...[but some classrooms and pedagogies] may in fact offer the opportunity for successful learning by more students. We conjecture, however, that, without explicit awareness by teachers of the implications of different forms of pedagogy on different social groups the aims of the New Basics in terms of more equitable outcomes are not likely to be met. (p. 55)

In regard to these issues, Borba and Pentado (2001) suggest that the use of computers in education brings teachers to a "risk zone", because the use of ICT challenges the typical lesson structures in which teachers could predict and control every single event of the dynamics of the classroom such as the nature of the questions as well as students' answers for them. The use of ICT challenges the authoritative and unidirectional interaction between teachers and students and reorganizes the nature of mathematical problems. ICT offer ways to both to explore open-ended tasks and highlight the collective intelligence in the classroom. However, teachers tend to stay in a kind of "comfort zone" (Borba and Pentado 2001), resisting or ignoring the presence of ICT in the world or simply conducting a "domesticated" use of ICT, reproducing typical pedagogies of right or wrong, yes for no for the control of the classrooms.

Borba and Zulatto (2010) present examples of how mathematics teachers collaborate in an online course when they explore activities with the software Geometricks. The design of the activity emphasizes collective experimentation-with-technology and proof in dynamic geometry. In this scenario, the authors believe that:

Teaching in online environments situates the teacher within a new model of risk zone with respect to the use of ICT in the teaching of mathematics. New challenges arise: How to follow the progress of my students who are physically distant? How to discuss mathematics

online? How to express my reasoning? What resource is more appropriate for each situation? (Borba and Zulatto 2010, p. 114)

Borba and Zulatto (2010) also propose that there is also a dynamic of the risk zone, in which there may be teachers who actually feel comfortable running risks as they explore technology in the classroom. Teaching with technology "in online environments require teachers who are more comfortable working in the risk zone while learning together with their students/peers! Like engaging in 'radical sports,' with practice, the risk zone can become comfortable" (p. 124).

On the one hand, we do acknowledge the variety and diversity of pedagogies toward the use of ICT in mathematics education, the problem of accessibility of computers in education (mainly in the global south), and issues concerning technical support for teachers in schools as well as teachers' "computational literacy." On the other hand, the reorganization of thinking emergent with the use of ICT in mathematics (education) cannot be ignored in terms of cognition and affectivity, although the research conducted by GPIMEM has not properly addressed discussions on affectivity and the use of technology. Among several aspects, the use of ICT in education disrupts the power relations that see the teacher as the iconic symbol of knowledge in the classroom. As Doll (1993) points out, when the linear and sequential pedagogic dynamics become less ordered and more fuzzy, "the relations between teachers and students... change drastically" (p. 3). That is, "these relations ... exemplify less the knowing teacher informing unknowing students, and more a group of individuals interacting together in the mutual exploration of relevant issues" (p. 4), and it has a direct influence in terms of curriculum.

On the Use of Interactive Whiteboards

Lerman and Zevenbergen (2007) mention how the affordances of interactive whiteboards (IWB) may offer possibilities for "rich communications and interactions in the classroom as teachers are seduced by the IWB's ability to capture pupils' attention" (p. 175). The authors highlight that:

Teachers' advance preparation for using the IWB, often via the ubiquitous PowerPoint package or pre-prepared lessons for the IWB, are leading to a decreased likelihood that teachers will deviate in response to pupils' needs and indeed might notice pupils' needs less frequently through the possibility to increase the pacing of mathematics lessons. (p. 175)

Zevenbergen and Lerman (2007, 2008) explore teachers' use of IWB in class-rooms through the various lenses of activity theory. These lenses help the authors to "understand the tensions and contradictions in teachers' use of the IWB and to identify possible developmental trajectories for realising some of their potential to change pedagogy for the better" (Zevenbergen and Lerman 2008, p. 124). The authors thus address the synergy between pedagogy and the use of ICT, seeing the

classrooms as fruitful social environments, focusing the nature of the interaction of students-teachers-ICT in classrooms.

The potential and rhetoric of IWB supporters, the ways in which it is used in the classroom may inhibit learning... The two dimensions that focus on knowledge production – intellectual quality and relevance – suggest that the scaffolding around the use of IWBs can be enhanced through higher expectations of learning... These aspects of pedagogy may be one way in which higher levels of intellectual quality may be facilitated. Aspects of the social environment – supportive school environment and recognition of difference – may also be challenged. The whole class interaction may stifle participation (and engagement) of students. Reorganising pedagogy so as to foster interaction, collaboration in smaller groups, or to employ other tools alongside the IWB may encourage greater interaction among learners. (Zevenbergen and Lerman 2008, p. 124)

The members of the research group GPIMEM did not conduct a research about the use of IWB in classrooms yet. However, Mazzi et al. (2012) conducted an exploratory study toward potential affordances of IWB for teaching and learning of Calculus and Geometry and produced a guide in Portuguese for a math-oriented use of a specific type of IWB (a guide is available at http://tidia-ae.rc.unesp.br/portal).

Exploring only the applications offered that IWB, Mazzi et al. (2012) identified some limitations such as (a) small dimension of the actual board interaction; (b) restrictions in transferring videos directly from websites; (c) imprecision of measurement tools and (d) higher costs of the IWB in Brazil. However, the authors highlighted the support of the IWB in running dynamic geometry software and CAS, that is, how typical software can be used with IWB. Thus, as mentioned by Zevenbergen and Lerman (2008), all those pedagogic issues regarding the use of ICT in mathematics education (see Tall 1991; Borba 1993; Noss and Hoyles 1996; Laborde 2000; Borba and Villarreal 2005) are also important issues toward the use of IWB.

We also see an interesting aspect of IWB in terms of multimodality (The New London Group 1996). The traditional modes for human-computer interactions happen through the use of screen, keyboards, mouse, speakers, microphone, webcams, and so on. In an IWB, one interacts directly by touching the screen instead of using a mouse. Thus, we do see a change from clicking to touching in terms of multimodality when we use an IWB, and when we use tablets. Since experimentation and visualization are fundamental aspects of mathematical exploration and thinking, we do conjecture that hands-on manipulation of virtual mathematical objects in an IWB has an impact in terms of heuristics and cognition. The transformation from clicking to touching is being properly addressed through the notion of humans-with-media in a current research project conducted by members of GPIMEM (Mazzi et al. 2012).

Humans-with-Internet: Performance and Identities

The use of the Internet in mathematics education (Borba 2004, 2009) and, more recently the use of performance arts and digital technology (Scucuglia 2012) have become an important research focus of our group GPIMEM. At this point in the chapter, we would like to highlight some conceptions we hold toward ICT in mathematics education:

- ICT has reorganized mathematical thinking. New problems and investigative possibilities have emerged with the use of computer algebra systems and dynamic geometry software in pedagogic scenarios. Fallibilistic trends in philosophy of mathematics have been consolidated with focus on heuristics and challenged more strict notions of "formal proof" or "mathematical true" (e.g., the four colors theorem).
- Online distance education has offered new possibilities for in-service and pre-service mathematics teacher education (Gadanidis and Borba 2008)
- The internet has become an actor in mathematics classrooms and reorganized (a) the nature and structure of the mathematical content explored in schools or the design of lesson plans and (b) the nature of the collaboration and power relations between teachers and learners.
- The internet has a potential to make mathematics popular or accessible as a social endeavor. School mathematics usually stays inside the classrooms. Students do not have conversations with their relatives and friends about their favorite math ideas as they do when they talk about their favourite song or TV show. When parents ask to their children "what did you learn in math today?", typical responses are "nothing" or "I don't know" (Gadanidis 2009). As the Internet has a potential for democracy, the Internet has the virtual conduction to become a global stage in which students and teachers. The use of arts and the production of digital texts are fundamental to consolidate such a pedagogic/social practice.
- The genre of the online communication is similar to the genre of the performance (art). It involves multiple modes of communication, improvisation, and interaction with the "audience".

We conceptualize the *cyberspace* is a privileged educational nexus for creativity and collective intelligence. Levy (2001) defines cyberspace as the space of communication opened by the world interconnection of computers and memories of computers. This space is unique, because digital codifications shape the plastic, interactive, hypertextual, multimodal, and virtual nature of information in this context. Levy (2001) also defines *cyberculture* as the set of (materials and intellectuals) technologies, practices, attitudes, modes of thinking, and values developed through the growth of the cyberspace. The cyberculture redefines the notions of economy and knowledge, bringing up new possibilities to several areas such as education and the arts. Levy (2001) claims "the genres of cyberculture are similar to performance art, such as dance or theatre [or] the collective improvisations of jazz,

the commedia dell'arte, or the traditional poetry competitions of Japan" (p. 135). Interestingly, Levy (2001) uses the term *cyberart* to discuss the artistic-aesthetic dimension of cybercultures, suggesting the possibilities for (collective) collaboration and the continuous creation as a fundamental aspect of cyberart. In other words,

The virtual work is 'open' by design. Every actualization reveals a new aspect of the work ... Thus the creation is no longer limited to the moment of the conception or realization; the virtual system provides a machine of generating events. (Levy 2001, p. 116)

In the *Math* + *Science Performance Festival* (www.mathfest.ca), students, teachers and artists have shared videos in which they use the performance arts (e.g. music, drama, and poetry) to communicate their mathematical ideas. These videos are conceptualized as *digital mathematical performances* (Gadanidis and Borba 2008; Scucuglia 2012). Gadanidis and Geiger (2010) have referred to the Festival as "one example that helps bring the mathematical ideas of students into public forums where it can be shared and critiqued and which then provides opportunity for the continued development of knowledge and understanding within a supportive community of learners" (p. 102). Gadanidis and Geiger (2010) also posit the Festival "offers a glimpse into how collaboration in mathematics learning might be extended to include math performance, or perhaps how collaboration in a media-rich digital environment might be reconceptualized as collaborative performance" (p. 101).

In fact, from a narrative point of view (Bruner 1996), when students produce texts (such a video file) of a skit or a song performed in the classroom to produce a digital mathematical performance for the Festival, they are not only presenting mathematical ideas to their classmates and teachers. They are performing, communicating and representing their mathematical activity, learning, and discourses for a wide audience, because, potentially, the digital performances will be publicly available on the Festival's website. Both the classroom and the cyberspace are social/cultural settings in which students, teachers, and other agents interact, collaborate, and produce meaning and knowledge. The playful nature of digital mathematical performances offers ways of expressing ideas collaboratively, with creativity and imagination. The playfulness may also help students to make sense of mathematics through narrative because when they produce a digital mathematical performance they are seeking to communicate a mathematical story through a digital narrative/text to the audience. The process of producing a digital mathematical narrative to be published is a process in which (elementary school) students construct identities as performance mathematicians (Gadanidis et al. 2008; Scucuglia 2012).

Mauricio Rosa, an associate member of GPIMEM conducted part of his doctoral research under the supervision of Steve Lerman, when a visiting PhD student at the London South Bank University. In his doctoral thesis developed in our research group GPIMEM, Rosa (2008) explores the relations between the construction of online identities and the teaching and learning of calculus in an online course, when pre-service teachers perform role play games (RPG). Rosa and Lerman (2011)

re-examine these data focusing on issues about research methodology. According to the authors:

(a) [the] cyberspace is a natural environment in an online RPG context; (b) the playful process in online learning in mathematics education brings important new aspects to our understanding of mathematical knowledge as a social construction; (c) the investigation becomes a game; (d) research subjects are who they want to be while they are in flow, that is, there is intentionality; (e) the challenge of research methodology inside cyberspace must be faced by researchers; and (f) the researcher needs to consider those different identities as integral to the research process. (Rosa and Lerman 2011, p. 69)

Humans-with-Digital-Technology: Multimodality

Communication is a fundamental endeavour within sociocultural perspectives that supports mathematical classroom activity. Lerman (1998, p. 40) states that:

Learners come to the classroom as persons of multiple, overlapping subjectivities.

Different aspects of those subjectivities are called up by different aspects of the practices of the classroom, and are expressed through identities of powerfulness or powerlessness.

At the same time, new subjectivities are constituted in the social relationships and forms of communication which make up the activities of the classroom. Rather than the intension of teaching mathematics as the handing over, or the individual construction, of ultimately decontextualized mathematical concepts by the teacher or by the pupil respectively, teaching can be conceived of as enabling pupils to become mathematical actors in the classroom and beyond. The goals and needs of pupils, and the ways of behaving and speaking as mathematicians, become the focuses of the teacher's intentions. (Lerman 1998, p. 40)

Issues on mathematical communication also involve the socio-political context of mathematics classroom (Lerman and Zevenbergen 2004). We do recognize that students bring very different discursive rules and practices into schools and such a process "influence how they act and how actions are interpreted" (Lerman and Zevenbergen 2004, p. 32). That is,

In considering the different discursive backgrounds of students, teachers' perceptions of their students' learning styles – that frequently correlate with the social background of the students -, and the ways in which classrooms and curricula are organized for students depending on their backgrounds, it is also important to take into account interactions within a classroom. (Lerman and Zevenbergen 2004, p. 33)

We have argued that the nature of communication based on the use of Internet is multimodal. In some scenarios of our research, students have produced multimodal texts in their classrooms to disseminate their mathematical ideas in the cyberspace. Scucuglia (2012), for instance, used the notion of multimodality in literacy to form a lens to interpret how students-with-media produce digital mathematical performances, that is, to analyze the role of digital technology in shaping students'

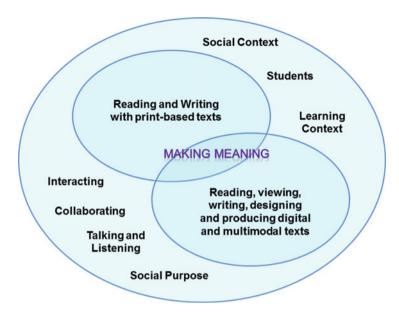


Fig. 15.2 Classroom interaction in a multimodal perspective (Walsh 2011)

mathematical thinking when they produce digital narratives using the performance arts to communicate their mathematical ideas.

In our research (Gadanidis et al. in press), we have used a model proposed by Walsh (2011) (see Fig. 15.2) to emphasize the mathematics classrooms as *social* environments with potential to form rich scenarios for multimodal learning environments when students-and-teachers-with-digital-technology produce digital mathematical performances. Walsh (2011) theorizes how classrooms can become multimodal learning environments when students interact, collaborate, and produce multimodal texts in schools. Walsh actually emphasizes the role of inter-textuality (the combination of print-based and digital/multimodal texts) and dialogue in meaning production within educational and social purposes.

Let us clarify what we mean by multimodality. Pahl and Rowsell (2005) posit that the word *multimodal* "describes the way we communicate using a number of different modes to make meaning" (p. 27). Rowsell and Walsh (2011) state that "multimodality is the field that takes account of how individuals make meaning with different kinds of modes" (pp. 55–56). According to Walsh (2011), *multimodality* is "a study of the communicative process, particularly how meaning is communicated through different semiotic or meaning-making resources and in different social contexts" (p. 105).

Multimodality as in comprehension and competence with language through a variety of modes such as image, sound, touch, multi-dimensions, is the principle upon which digital environments work. This principle of multimodality needs to be understood for educators to apply and assess new modes of learning as a part of everyday classroom practice. (Rowsell and Walsh 2011, p. 54)

According to Gadanidis et al. (2011), "the use of multimodal expression changes the feel of the learning environment" (p. 425). In online environments, students and teachers can use text, drawings, and images, and various tools and representations.

Different modalities – aural, visual, gestural, spatial, and linguistic – come together in one surround in ways that reshape the relationship between printed word and image or printed word and sound. Thinking with and communicating through multiple representations is a common expectation in current mathematics curriculum reform documents. (Gadanidis et al. 2011, p. 425)

Kress (2003) posits that "mode is the name for a culturally and socially fashioned resource for representation and communication" (p. 45). That is, modes are "the various forms used to construct signs" (Kress 1997, p. 7). Pahl and Rowsell (2005) state that "a mode could be visual, linguistic, aural, or tacit" (p. 27). Authors like Jewitt (2006) argue that the modalities are aural, visual, gestural, spatial, and linguistic. The New London Group (1996) discusses language within multiliteracies based on the notion of design, that is, "a language for talking about language, images, texts, and meaning-making interactions ... [including] the key terms 'genres' and 'discourses,' and a number of related concepts such as voices, styles, and probably others" (p. 77).

Based on these notions, in his study, Scucuglia (2012) analyzed digital mathematical performance produced by elementary school students from Canada. The performances are available at www.mathfest.ca. Scucuglia defines a digital mathematical performance as a multimodal text/narrative (e.g., a video or a virtual learning object) in which one uses the performance arts to communicate their mathematical ideas. As part of the findings of his study, Scucuglia states that:

The multimodal nature of [students' digital mathematical performances] is one of its most significant pedagogic attributes. Mathematics is traditionally communicated through print-based texts through the use of writing, charts, diagrams, and graphs. Digital media affordances offer ways to represent mathematical ideas through multiple modes, which adds non-usual layers of signs in communicating mathematics (e.g. audio, gestures, space). However, multimodality does not guarantee the conceptual nature of the idea explored in the [performances]. (Scucuglia 2012, p. 216)

Moreover, when students produce digital narratives, they are immersed in contexts in which they can see mathematics as stories (Gadanidis and Hoogland 2003). The synthesis between these two different modes of thinking – the paradigmatic and the narrative – to use Bruner's terms, offers ways to students to address emotions and sensations to their mathematics discourses. In doing so, students incorporate their social and cultural backgrounds into these discourses and develop communication skills (Scucuglia et al. 2011, Scucuglia 2012). We do not think that all mathematics should be communicated through digital performance, but the production of multimodal mathematical texts with emphasis on the arts is a possibility to bring representational diversity and aesthetics into the pedagogic practice of mathematics. It offers ways to challenge instructional discourses that seek "to control the content of the mathematics lesson" (Lerman and Zevenbergen 2004, p. 35).

Conclusions

We present our final remarks in terms of theory and practice. We believe that the pragmatic dimension of the use of ICT in mathematics education is not as central in Steve Lerman's work as is the theorization of sociocultural perspectives for mathematical activity. Steve uses ICT to show how theories may work in practice. In contrast, the research developed by GPIMEM shows examples and possibilities of actual use of ICT for mathematics teaching and learning and uses its research with students and teacher in order to contribute to social cultural perspectives with constructs such as humans-with-media, inter-shaping relationship and digital mathematical performance. However, in both cases, we do see theory and practice in reciprocal synergy, that is, theoretical lenses being refined based on teaching and learning experiences and pedagogic practices being conducted and reorganized based on the theoretical refinements. We believe that both research – the one developed by Steve Lerman and colleagues and the one developed by GPIMEM – has in common the notion that historically dated technology, such as Internet, may change the way communication works. Since both research approaches believe that communication is fundamental for meaning make, it can inferred that digital technology – using our terminology – is an active actor in meaning make and an actor in the process of making knowledge historically dated.

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