Chapter 13
Learning as Participatory Transformation – A Reflection Inspired by Steve Lerman's Papers and Practice

João Filipe Matos

> *Constructions of reality, ideologies, world views, are not merely alternative theories with metaphysical implications only, to be discussed, compared, refuted, or supported in the ivory towers of philosophy. Knowledge and Power are inseparably linked, and knowledge is used as and for power, the domination of one group over another, the oppression of people, the legitimation of that oppression and the rationalization of values.*
>
> (Lerman 1996, 1992, p. 173)

Introduction

In his seminal paper *The Social Turn in Mathematics Education Research* (Lerman 2000a) Steve Lerman provides examples of indicators of the receptivity of the mainstream mathematics education community to social theories of mathematics learning. In that he also stresses the idea that perhaps the reception of that article was due more to political concerns that inequalities in society were reinforced and reproduced by differential success in mathematics, than to social theories of learning. He suggests (Lerman 2000b):

> the greatest challenge for research in mathematics education (and education/social sciences in general) from perspectives that can be described as being within the social turn is to develop accounts that bring together agency, individual trajectories, and the cultural, historical and social origins of the ways people think, behave, reason and understand the world. (Lerman 2000b, p. 368)

In parallel Steve acknowledges that the work of Vygotsky and Vygotskian researchers represented a growing source of theoretical inspiration in mathematics education research. Now 15 years after the appearance of that article, the influence of Steve's theoretical elaboration in a variety of topics and issues in mathematics education research is still apparent.

J.F. Matos (✉)
Institute of Education, University of Lisbon, Lisbon, Portugal
e-mail: jfmatos@ie.ulisboa.pt

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In several publications Steve Lerman discusses the developments of situated learning theories and their relevance to mathematics education research. For example, in 1998 (Lerman 1998) he critically acknowledged and discussed the revolutionary notion of the situatedness of knowing, meaning and acting and the centrality of learning from Lave and Wenger (1991) and then 10 years later made a deep and challenging analysis and discussion of the concept of a community of practice (Kanes and Lerman 2008).

Steve played a major role in my own particular trajectory in mathematics education research both in inspiring and in stimulating my work, but also in encouraging me to take risks. The continuous interaction I had with Steve for 3 years in a Portuguese funded project on Mathematical Thinking, made it possible for me to understand the rigour, the extremely deep scientific honesty and the creativity shown by Steve.

In this chapter I pursue a reflection and discussion started in 2010 and strongly influenced by several papers (some on them crucial, such as Lerman 2006) and many moments of discussion with Steve. Putting the focus on mathematics learning, I discus possibilities of a conceptual articulation between theoretical views with rather different socio-historical developments and the relevance to mathematics learning research. I draw on previous work I have undertaken with Madalena Santos (Santos and Matos 1998, 2000, 2008) and from more recent developments (Matos 2010).

A considerable body of work has been and is still being produced within the international mathematics education community focusing on learning in practice, ranging from professional education and development of teachers of mathematics (e.g. Even and Ball 2009; Matos et al. 2009) to mathematics learners in transition in a variety of contexts of mathematical practices (e.g. Abreu et al. 2002; Matos et al. 2002). In several papers published in a variety of journals and book chapters during the last 15 years, Steve Lerman discusses what constitutes the rationale for the 'social turn' and points to three main intellectual resources (Lerman 2001): anthropology (as situated theories of communities of practice), sociology and cultural/discursive psychology with its roots in Vygotsky's theories. His brief review (Lerman 2001) leads to the need to encompass in a more significant form, a sociological orientation.

My intention in this chapter is to contribute to the analysis and discussion of social theories of mathematics learning, articulating and putting in dialogue a view of learning-in-practice taking in particular the notion of learning as participation (drawing on situated learning theoretical perspectives) and learning as transformation (drawing on expansive learning perspective in activity theory). I will concentrate on discussing and exploring the very idea of "*learning as an integral part of generative social practice in the lived-in world*" (Lave and Wenger 1991, p. 35) positioning subject and community within a system of activity (Engeström 1987), acting upon (conceptual) objects and transforming them into mathematically acceptable and legitimated outcomes. In doing so I hope to contribute to the discussion and analysis challenging the possibility of conceptual dialogue between

Jean Lave and Etienne Wenger's social theories of learning and Yrgo Engeström's activity theory-based ideas on expanded learning (Engeström 2007).

First, I should start by underlying, as we have in Santos and Matos (2008), that there are common roots between the situated perspective of Lave and Wenger (1991) and the activity theory as it is recognized by those authors and explicitly referred for example by Engestrom and Cole (1997) and Miettien (1999). Jean Lave finds in the socio-cultural approach (and in particular in activity theory) key issues that serve the way she addresses activity:

(i) a way of conceptualizing activity that makes possible the analysis of its intrinsic organization through the definition of a categorization of levels of activity but that simultaneously recognizes and considers its holistic nature and dynamics;
(ii) the emphasis given to the relational nature of meaning (localized in the relations between the different levels of activity) and the activity system (which operates between the levels of activity such as in the interface action-operation); and
(iii) the dialectic analytical approach to activity and its meaning in the relations constitutive of the activity system.

Secondly, the concept of social practice is more and more explicit in the work of activity theory. For example, Toulmin (1999) elaborates on the idea of knowledge and makes a comparative analysis of the epistemological ideas of Vygotsky and late Wittgenstein. Here he identifies in both a concern with the concept of practice and that 'practice' should be the key central notion in wherever new theory of knowledge is developed in the future. In the Foreword of the book *Activity Theory and Social Practice*, Hedegaard et al. (1999) give an account of how the concept of social practice is brought into the context of the discussion of activity theory. Furthermore they identify the relations that seem to be possible between 'social practice' and 'activity' indicating implications for further research in the area aiming to clarify those relations. As Santos and Matos (2008) indicate, although Hedegaard et al. (1999) underline the importance of the concept of practice as it *"provides a way to characterize those aspects of social practice that are believed to provide the conditions for psychological development"* (p. 19) they recognize the need to think more profoundly about the possibility of existing wider meanings of social practice that exceed the notion of activity. According to Jensen (1999) activity theorists

> have not applied their insight about the situated nature of practice and the practice-situatedness of concepts reflexively, only rarely have activity theorists accounted for their own concepts and theories as embedded in activities and practices. (p. 84)

Concepts in Action Involved in the Idea of Learning-in-Practice

Taking "*learning as an integral part of generative social practice in the lived-in world*" (Lave and Wenger 1991, p. 35) means that participation in social practices does not merely influence otherwise autonomous cognitive processes. From this point of view, learning means changes in the ways that a person *participates* in social practices. Therefore, understanding how learning occurs and what is involved in learning mathematics, implies addressing the social practices where learners are engaged as it is the "*primary, generative phenomenon and learning is one of its characteristics*" (Lave and Wenger 1991, p. 34). This fundamental idea takes as crucial the "*integral nature of relations between persons acting (including thinking and learning) and the social world, and between the form and content of learning-in-practice*" (Lave 1997, p. 20) identified in the work of activity theorists such as Engeström (1987, 2001, 2007) and Davydov and Radzikhovskii (1985).

Santos and Matos (1998, 2002) reported, from a situated learning point of view, how the concepts of artefact and resource were useful in the analysis of learning. The idea of artefact is quite strongly used in Activity Theory in parallel with the notion of *tool*. The concept of resource is explicitly used by Lave and Wenger (1991) in a situated approach to learning. However, the discussion of the social nature of the human construction of mathematical artefacts deserves additional attention as suggested by Lerman (1994).

Activity in Activity Theory

Within the conceptual framework of dialectic materialism, the notion of activity is an initial abstract (Davydov 1999)., Ilyenkov was one of the authors who contributed to the development of this notion, through reflection on the relations between activity and consciousness – as the reflection of subjects on reality, their own activity and themselves–addressing consciousness as co-knowing. According to Bakhurst (1997), Ilyenkov elaborated on a theory of the ideal in which activity becomes literally part of the mind as the ideal constructs the ability to think. He goes on to argue for the capacity to act in accordance with what is proper in a cultural context and therefore he identifies thinking as a kind of activity. Hence, activity is no more seen as an abstraction but as the basic unit of analysis of consciousness (Santos and Matos 2002).

Leont'ev (1978) presents an approach to the concept of activity drawing on the idea of the structure of activity with several components establishing basic categories of human activity and allowing the possibility of researching the way individual consciousness is organized through particular and specific activities – the analysis of activity abstracted from the forms it takes while running. One basic principle for Leont'ev is the recognition of the social and cooperative nature of human activity.

He assumes human individuality is emergent from social activity thus conducive to the need to reflect upon the relation between individual consciousness and the specific activities. He sees activity as a molar unit, not an additive one in the life of the person, but a system with its own structure, its transitions and its internal transformations, and importantly its own development. He identifies non-additive elements linked to central concepts: activity (linked to motives), action (linked to a goal) and operation (linked to conditions). The motives of the activity are intimately connected to the needs felt by the individual, thrown to the activity as a form of responding to those needs. Activity involves different actions aiming to specific results intimately related to the activity and, in this way, directing the activity. Action can be made concrete in different ways and forms through operations according to the conditions available but always aligned with the goal that is supposed to be achieved.

Two key implications emerge from the approach to activity proposed by Leont'ev which are coherent with the idea of learning-in-practice: (i) activity cannot be reduced to a set of simpler stand-alone additive parts or processes, and (ii) its structural and functional unit can only be examined looking at the phenomenon in its active or live state. Goal and motives are the peculiarities of a given activity that allow us to distinguish one activity from another. But those elements have only a potential character in the activity; they are neither deterministic nor definitive as activity can only be realized through development that involves transformations given its dynamic nature.

Artefacts in Activity Theory

Taking the model of the structure of the system of activity proposed by Engeström (1999) I will concentrate on one of its elements – the artefacts, in relation to other elements addressing in particular the idea of mediation that is one of the key concepts of the socio-historical-cultural approaches. The concept of *artefac*t attracts a variety of researchers in mathematics education and is frequently used, for example, in studies that focus on the use of digital technology in education. It is common to see research considering the notion of artefact in two different forms. On the one hand artefacts are referred as tools and signs that mediate action and on the other hand we find researchers who consider external (or physical) artefacts and internal (conceptual or cognitive) artefacts. The key issue is that in both approaches it is the internal character of the artefact that makes its classification, independently of the kind of activity where the use of the artefact takes place.

Engeström (1987) considers that none of those highly dichotomized forms of conceptualizing artefacts is useful and discusses that problem in the context of the non-definitive nor rigid nature of activity. In activity, functions and uses of artefacts are in constant dynamic transformation. Elements that seem to be internal in a certain moment are externalized (for example through speech) as much as the external processes in certain occasions can be internalized. Freezing and splitting

those processes seems to be a poor basis to understanding artefacts and their role in activity (Engeström 1999). The functions and use of artefacts are in a constant fluidity and transformation that goes along with unfolding activity. In this sense, the artefacts are not something fixed and external to the practices but are constitutive of practices; its usefulness is not revealed in the characteristics identified independently of its use in the practices where they are put in action. Artefacts are artefacts-in-the-practice should be understood in interaction with the forms of use that users develop in those practices.

Engeström proposes a differentiation in regard to the uses of artefacts:

> The first type is **what** artefacts, used to identify and describe objects. The second type is **how** artefacts, used to guide and direct processes and procedures on, within or between objects. The third type is **why** artefacts, used to diagnose and explain the properties and behaviour of objects. Finally the fourth type is **where to** artefacts, used to envision the future state or potential development of objects, including institutions and social systems. (Engeström 1999, p. 382, emphasis in the original)

This original classification highlights that an artefact is not considered by itself in isolation instead it is conceptualized in relation to a specific use and is always inserted in a system of activity. As Engeström points out above, the construction and transformation of (conceptual) objects mediated by artefacts is a collaborative process in its nature and dialectic in its core and where different perspectives and voices meet, collide and mix. This framework gives visibility to some characteristics that draw on the collective essence of activity – and thus potentially conflicting – not isolated nor harmonious.

A Situated Perspective and the Concept of Activity

In analysing shopping at the supermarket, Lave et al. (1984) ask "*what is it about grocery shopping in supermarkets that might create the effective context for what is constructed by shoppers as 'problem solving activity'?"* (p. 68). Grocery shopping is seen as an activity that occurs in a specialized setting designed to support it (the supermarket) constituting the arena of grocery shopping as an institution at the interface between consumers and suppliers of grocery commodities. This way of conceptualizing the relations between two layers of activity: grocery shopping and problem solving – shopping helping to shape problem solving through the setting intentionally. It may be thought of as the locus of articulation between the structured arena and the structured activity. Theorizing about the interrelations between activity and setting, Lave et al. (1984) recognize the value of the conceptualization of the idea of a setting but go against a unidirectional, setting-driven relation between activity and setting. This would reduce activity to a passive response to the setting precluding the analysis of the internal relationships within the activity. It is, though, pertinent that Lave recognizes the way activity theorists conceptualize the idea of activity as system with structures, internal transformations and self-development as it allows and creates a basis to the study of the intrinsic

organization of activity. It is also recognized that the studies of Zinchenko of the holistic nature of activity (developed in the framework of activity theory) help to support the idea that understanding the nature of learning in mathematics requires a contextualized understanding of its role within that activity. This is a strong argument of the need for analysing any segment of activity in relation to the flow of activity of which it is a part.

Another relevant aspect of activity theory that deserves the attention and reflection of situated learning authors is the relational emphasis of the activity theorists underlining the parallelism found in the distinction made by Leont'ev between (personal) sense and (public, societal) meaning and the distinction Lave proposes between the constructs of (personal) setting and (public, non-negotiable) arena. In addition, the dialectic character of the analysis of activity is central to the situated perspective assumed by Lave (Santos 2004).

Lave (1996) uses the term '*ongoing activity'* to refer to activity and this form of talk orientates our attention to the strongly fluid and dynamic character of activity. It induces the interrogation of the continuity and of trajectories within the activity – where does it come from, where does it go? This relates to the holistic but local character of activity with the resources, the constraints and the actors in place present in the situation. The ongoing character of activity introduced by Jean Lave seems to be consistent with the view of Leont'ev about the notion of activity that took him to defend that it should be analysed in its active state.

Thus, the option of Jean Lave for an analytical focus on direct experience in a lived-in-world, in a way induces

> reformulating the role of direct experience raising the question of how activity is made accountable while ongoing. An analytic focus on direct experience in the lived-in world leads to emphasis on a reflexive view of the constitution of goals in activity and the proposition that goals are constructed. (Lave 1988, p. 183)

This does not seem to be compatible with a linear view of action as directed towards established goals – "*action is not 'goal directed' nor are goals a condition for action*" (p. 183). Taking as support the idea from Wittgenstein and Giddens that it is through the recursive character of social life that it is possible to capture the nature of social practices as a continuous process of production and reproduction, Lave concludes that "*the meaning of activity is constructed in action*" (p. 184). Where the intentional character of activity comes from?

> In this perspective, motivation is neither merely internal to the person nor to be found exclusively in the environment. That is, even as goals are not 'needs' (hunger or sexual desire are socially constituted in the world), they are not prefabricated by the person-acting or some other goal-giver as a precondition for action. And activity and its values are generated simultaneously, given that action is constituted in circumstances which both impel and give meaning to it. Motivation for activity thus appears to be a complex phenomenon deriving from constitutive order in relation with experience. (Lave 1988, p. 184)

More than adding a typical approach from activity theory (for whom the external world is determinant) with a phenomenological reading (that gives the 'power' to individuals) there is a possibility to dialectically integrate aspects of the two

theoretical fields that allow one to argue that setting and activity connect with the mind through its constitutive relations with person-acting (Santos and Matos 2000). Thus, instead of talking of goals (as in activity theory) a situated perspective refers to "*expectations, dialectically constituted in gap-closing processes, enable activity while they change in the course of activity backward and forward* ***in*** *time at the* ***same*** *time*" (Lave 1988, p. 185, emphasis in the original). This is closely related to the way Jean Lave conceptualizes intentions of actors in ongoing activity as they are "*engaged in what they are doing. When that activity poses conflicts, difficulties, in short dilemmas, they engage in resolving them*" (Lave 1992, p. 80). The procedures adopted in solving them gain form and meaning in relation to those dilemmas that are finally what motives their practices. It is the specific character of certain conflicts more adequate to concrete action that shapes what are problems to be solved. What makes a certain situation be seen as a dilemma would be what makes it be seen as a problem deserving effort in its resolution – what is seen as problematic in the activity emerges from and within that activity. This echoes the notion of contradiction (Engeström 1991) and its role in the activity that I will discuss later.

Mediating Artefacts in Activity Theory and Structuring Resources in Situated Learning

Resources are ways through which transformative relations are incorporated in the production and reproduction of social practices (Giddens 1996). This means that resources are intimately connected to power, be it seen either as an ability that transforms activity or adopting specific sense of domination or ability to intervene. Resources are always means through which social power is implemented; they are the basis and the vehicles of power. Given that resources are equally structural components of social systems, they become also the means through which the structures of domination are reproduced. It is within this framework that Giddens considers that exerting power is not a type of action; power is instantiated in action as regular and routine phenomenon. In this sense, power is not a resource but it depends on resources (Santos and Matos 2008).

A strong claim of the mediating role of artefacts seems to be clear in the introduction of the book edited by Dorothy Holland and Jean Lave in 2001 (Holland and Lave 2001). The authors assume a theoretical perspective grounded in a theory of practice that emphasizes the processes of social formation and cultural production and look with particular attention to cultural forms (close to the conceptualization of cultural artefact with its materiality) given the power of inscription they have. This echoes previous developments of Lave (1998) and Lave and Wenger (1991) on structuring resources that constitute authentic mediating artefacts.

This discussion has family resemblances to the notion of zone of proximal development (zpd). In an extremely clarifying analysis, Meira and Lerman (2009)

criticized the way zpd is conceptualized as a field, a sort of physical space that children get in and that the adult (the teacher) is supposed to reach to be able to successfully teach the child. They present zpd as a symbolic space emerging from learning as a product of dialogic interaction. Thus zpd is thought of as a future oriented structure. Meira and Lerman (2009) conceptualize zpd as a tool to analyse teaching and learning environments at school and they consider a need to make further steps in conceptualizing zpd "from being thought of as a physical entity, towards the notion of a sign-mediated, intersubjective space for analysing how people become actors and communicators within any given activity or social practice" (p. 1). They argue that the revolutionary role of zpd is related to the idea of symbolic space where learning leads development and dialectic thinking and speech is manifested and where the individual's meanings encounter social meanings and purposes. "This implies that the opportunity and possibility for learning does not exist prior to the event or activity" (p. 1).

Participation in Social Practices

The concept of artefact and its relation to the idea of resource brings along the need to discuss the concept of participation which is central in situated learning perspectives. The concept of participation in social practices (within a situated view) and the idea of transformation in activity systems (from activity theory) must be addressed dialogically and unpacked. Sfard (2006) makes an insightful approach to the notion of participation juxtaposing a participationist discourse in mathematics learning against an acquisitionist metaphor claiming that the way we frame learning has powerful consequences both in research and in teaching practices.

It is important to note how researchers in education implicitly associate learning with the ability to participate, avoiding a positioning that puts learning as acquisition but bringing in the concepts such as apprenticeship, guided participation and scaffolding (Rogoff 1990). A number of researchers in mathematics education use, in a more or less explicit form, those ideas to study a variety of learning contexts and problems. However, it is not apparent that the notion of participation is assumed as embedding human agency in the social world in a constitutive form.

The concept of participation is present in all perspectives that claim a situated nature of learning (Santos 2004). It is in fact, as Santos argues, in that common understanding of the centrality of participation that most situated perspectives connect into learning. Lave and Wenger (1991) claim learning as situated in "*legitimate peripheral participation in communities of practice*" (p. 122); Greeno (1993) considers learning as a process of people becoming more capable of participating in practices; and Rogoff (1990) views participation as a process and a product, clamming that it is through guided participation (in systems of apprenticeship) that cognitive development occurs as participatory appropriation. Within those perspectives, the units of analysis include person, activity and the contexts where activity takes place.

Lave and Wenger (1991) identify two rather useful elements to characterize participation in order to reflect on learning:

(i) the social organization, where power relations shape the categorization and forms of participation of people, and
(ii) the relationships between participation within the activity and the recognition of the relevance that participation has to their life projects.

They widely explored the three dimensional concept of legitimate peripheral participation arguing it can be fruitfully expanded and introduced here for its potential to open space to articulate the idea of learning-in-practice with the notion of acting in an activity system. The three dimensions are as follows.

First, legitimacy of participation is a characteristic of participation that refers to the possibilities and degree of openness for action within the community on the part of the participant. The possibilities of participation are not exclusively dependent on rules and norms (both explicit and implicit), they articulate to the affordances and constraints offered by the community; it thus defines belonging not only as a crucial condition of learning but also as constitutive of learning. There are several (although equally legitimate) forms of belonging to a community, linked to more or less inclusive ways of being, located (by the collective) in the field of participation implicitly defined by the community. Participating in what is peculiar and essential in the practice (and not necessarily oriented towards its learning) confers legitimacy to participation. In fact participating is the legitimate way of accessing the practice and of being recognized as a participant. But the concept of legitimacy of participation opens four ways to dynamically conceptualize participation in a community (Lave and Wenger 1991): full participation (as an insider); full non-participation (as an outsider); 'inbound' participation (heading towards full participation); and 'outbound' participation (heading towards full non-participation). Those four spaces are to be understood as conceptual categories that do not categorize nor classify participants but that instead create horizontal landscapes allowing trajectories of participation. Mediation artefacts – such as language – may play a major role in the process of gaining legitimacy of participation. As Lerman (1994) states "*language is specific to particular social practices and is associated with power as knowledge and knowledge as power; language structures what we can talk about*" (p. 193).

Second, the peripherality of participation refers to the positioning the subject (or collective) takes in a certain practice; that is, it localizes the subject/participant in the activity systems where participation occurs; peripherality is related to the nature of the engagement of the participant and to its several forms; understanding the positioning of the person in the field of possibilities of participation opens ways to dynamically clarifying where the participants are heading and that's why it is associated to the idea of 'trajectory of participation'. Although it brings in a topological metaphor, peripherality of participation does not refer to a metric in relation to a standard form or degree of participation and therefore it is not opposed to the idea of 'central' participation (which has no meaning in a community of practice). The topological metaphorical space of participation has multiple

dimensions turning 'central' a meaningless idea and in fact a misleading metaphor. Peripherality translates into multiple forms of participation and to the possibility of several and different forms of involvement; however, it is both participant and community who dynamically define peripherality whereby apparent changes in the positioning and perspective are seen as natural (e.g. typical patterns of the specific practice) both in terms of the trajectories of participation and in the development of participants' identity.

Third, legitimacy of peripherality is a notion implicated in social structures thus involving relations of power. In an activity system, power and associated mediation is constituted according to the legitimacy of participation and it is inherent to the trajectory of participation (and thus to the learning curriculum) (Wenger 1998). The issue of the legitimacy of peripherality and the mechanisms implicated in its development stand as conditions that allow participation. The concept of legitimacy of peripherality makes explicit an inherent ambiguity in participation: if peripherality is legitimated through the access to an increasing and more intense participation, the subject faces a position that progressively gives power to those who learn; if, on the contrary, participation does not develop (e.g. because there is legitimacy in avoiding a stronger engagement and participation) the subject faces a positioning that closes the access to a more powerful stance; the ambiguity in peripheral participation links to the matter of legitimacy, of social organization of resources and control over them (Lave and Wenger 1991).

Learning as Participatory Transformation

Within activity theory, learning can be addressed as an integral form of development that is materialized in qualitative transformations of the activity system, on a macro-level of analysis (e.g. within the social world where the students' practice unfolds) or/and of the subject, from a micro-analytical perspective (e.g. assuming the perspective of the student). This movement is mainly related to progression towards a wider and expansive field, for both the subject and the context. The need to bring into dialogue the analysis of collective activity systems and the point of view of individual subjects (Engeström and Sannino 2010) can be addressed through the exploration of the idea of learning as participatory transformation (Matos 2010).

Learning as transformation is inherently connected to the idea of learning as an activity, or more likely, as learning activity. Learning activity only gets sense and meaning when understood within a system (the activity system), which is representative of the established relationships between the subject and the social world. It is central to consider and analyze how such activity systems change and get transformed over time.

From the point of view of learning as transformation, knowledge is seen and considered as unstable, volatile, diffuse, emergent and in constant evolution. Assuming knowledge as existing in the relations of person to the artefacts, to the

other (in the practice of the community) within the activity system, we should consider both the vertical and hierarchical processes of learning (which are not denied) as well as processes of "*horizontal and sideways learning and development*" (Engestrom 2001, p. 153), where the boundaries of knowledge are open, not imprisoned but crossed.

Learning is understood here as expansive transformation (Engeström 1991) as the activity systems move up through cycles of qualitative change, through which the motive of the activity is (re)conceptualized and new and radically broader horizons of possibilities are embraced.

In consonance with the perspective of Meira and Lerman (2009), a complete cycle of expansive transformation can be seen as a collective journey through the system's zone of proximal development conceptualized as:

> (. . .) the distance between the present everyday actions of the individuals and the historically new form of the societal activity that can be collectively generated as a solution to the double bind potentially embedded in the everyday actions. (Engeström 1991, p. 174)

Among the components of the activity (learning) system continuing and constant changes are happening. The activity system is incessantly rebuilding itself, and these internal reconstructions are seen as attempts of reorganization or (re)mediation of the system, that take place in order to resolve internal and external contradictions.

This perspective on learning – coined by Engeström as *learning as expansive transformation* – emerges as an historically more advanced view on learning. It reveals other dimensions of learning in connection with forms of participation showing the driving-forces that seem to be responsible for the processes through which humans transcend their given contexts.

Expansive learning puts into question the sense and meaning of the context and established norms that are questioned, leading to the emergence and construction of alternatives. An important implication is that learning itself produces culturally expansive new patterns and forms of activity. The changes in human activity (and by extension, in the organization where they take place) are considered instances of expansive transformation.

Taking learning as participatory transformation is configured in six characterizing issues (LEARN 2010) as follows.

Learning Develops in Collectives

Learning-in-practice is conceptualized as a collective endeavor taking place within activity which has and reveals a social nature. Human development is seen as arising from social interactions. Although this assumption does not deny individual learning, it goes beyond an individualistic perspective and integrates learning as acting for and by collective purposes. Consequently, human development is seen as

resulting from collective transformation, which is historical and culturally contextualized and shared.

Learning Is a Contextual Phenomenon

In the analysis and understanding of learning it is essential to consider the socio-historical context in which it unfolds. The socio-historical dimension of human learning is central emphasis being put on the ecological character of learning. Context is considered not only the space-time frame that is directly incorporated in the activity, but also, in a wider perspective, the historic social political and economic time, where activity takes place. This is crucial to understand the learning-in-practice.

Learning Has a Dynamic Nature

Learning as (expansive) participatory transformation assumes a dynamic character through the creative and expanding movements resulting from the reconstruction of subjects – individual and/or collective. In an activity system, relationships between elements evolve leading to changes in the structure of the system. Central to the movements and processes of expansive transformation (as source of human learning and development) is the notion of contradiction (Engeström 1987). Internal and external contradictions constitute the driving forces of change in human activity as learning is linked to the dynamic resolution of emerging contradictions in activity. Contradictions do not show directly but appear as disturbances, disruptions, innovations and changes in activity systems. Transformation takes place by cyclical movements of resolving contradictions, which are typically associated with the development of activity (Engeström 1987).

Learning Is Necessarily Intentional

This is because it has an intentional basis. Behind any human activity there are always motives that drive, orient and maintain subjects' activity. The motives are connected to existing needs (sometimes expressed as desires) which are seen as being fulfilled acting over particular objects (material or imagined, but possibly explicit). Objects and needs by themselves do not produce activity, therefore motive should be necessarily integrated in the learning process.

Learning Is Intrinsically Linked to Production

This is because learning activity reveals always a productive nature. It's linked to the transformation of an object into a given outcome or result. The essence of learning activity is the production of new structures of social activity, which includes new objects, new tools, and new activities. The objects of human learning activity can be pointed out to be their own social productive practices. The object of learning activity is the societal productive practice (or the social world) in its full diversity and dynamic complexity (Engeström 2001). Thus, considering learning as transformation imposes the avoidance of seeing learning as reproduction, and adopting learning as creation and innovation.

To Conclude

Approaching mathematics learning from a situated point of view and locating learning-in-practice within an activity system, relates to the very idea of learning as participatory transformation. This reinforces three key ideas about the human social role:

(i) the person is a systemic, social and historically registered being;
(ii) the person is a creator and a transformer of collective subjectivity;
(iii) the person cannot be fully understood without its cultural means (artefacts).

Jean Lave (1996) presents three dimensions that every theory of learning should include:

(i) a learning telos, meaning a direction of changing and transformation,
(ii) learning mechanisms, as the ways that learning happens, and
(iii) the relation subject-social world as the general specification of relationships between subjects and social world (not as learners and learned stuff) which represents the key issue in social theories of learning.

I understand learning telos in relation to the issue of the relevance of the point of view of the subject (individual or collective) in transforming processes assuming that subjects (and collectives) are oriented towards recognition and identity and act in order to become participants in forms of distributed knowledge within the community. Learning is seen as transformation while subjects engage in goal oriented activity towards the transformation of objects that reify their needs-based motives and wills. Perhaps one could say as Wenger has (personal communication, 18 May 2010) that the telos of activity theory is more focused on the transformation of the object while that of participation is more focused on the transformation of the person.

(continued)

Learning mechanisms are conceptualized as the different forms of becoming a participant in social practices and includes the mechanisms of legitimate peripheral participation, engagement, alignment, imagination – components of ways of belonging, according to Wenger (1998), and the development of meaning. Learning mechanisms are seen as processes (that drive moves into change) such as internalization and externalization and mediation. The discussion in Lerman (2000b) when Steve asks "*to what extent, though, does Vygotsky's perspective provide the mechanism to which Lave refers?*" (p. 367) is extremely relevant and insightful. While Lave suggests that the need for learning mechanisms disappears into practice and people becoming kinds of persons, Steve stresses that "*becoming kinds of persons still calls for a mechanism*" (p. 368) and he proposes that "*internalization through semiotic mediation in the zpd is a suitable candidate*" (p. 368). At another level, to elaborate on the issue of the learning mechanisms we could look at them as actions that open learning possibilities (e.g. dialogue, reflection, intention, critique) and participation in dialogue, acting and producing meaning (Alrø and Skovsmose 2004). This seems to be a challenging topic that certainly deserves further study.

Addressing learning as transformation, as subject and community act upon an object (which goes from something potential into an outcome charged with new meanings and new forms of talk) it is crucial in this process that contradictions are identified (from conflicts and perturbations) and efforts are made to handle them (Engeström 2001). It is in the process of dealing and overcoming the contradictions that crucial action is taken and learning occurs.

If we take learning as participatory transformation the subject is viewed as agent in the socio-historical construction of the world and as product of that construction and culture. The subject is only understood in relation to cultural means (artefacts) of access to knowledge. This view puts value both on vertical and horizontal relationships. The way the relationships between person and social world are expressed assumes the subject as agent in a social world in conflict and thus inviting strategies for control and success. As to specify relationships between subjects, communities and the social world, person and world are not separable entities. Persons within their practices and the social world are mutually constitutive.

I hope that this contribution to articulate different dimensions of learning within different historically situated theories stimulates the emergence of interrogations about mathematics learning. I started this article with a quotation from Steve Lerman that reflects my positioning towards all issues in mathematics education research and practice. It certainly suggests relevant implications emerging from the discussion I presented in this article.

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