Chapter 83
On the Ochiai Index with Hurwicz Criterion in Ranking Fuzzy Numbers

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Introduction

In a fuzzy environment, the ranking of fuzzy numbers (FNs) plays an important role in practical use and has become a prerequisite procedure for a decision-making problem. In fuzzy decision analysis, FNs are employed to describe the performance of alternatives, and the selection of alternatives will eventually lead to the ranking of corresponding FNs. However, ranking of FNs is not an easy task since FNs are represented by possibility distribution and they can overlap with each other.

Various methods for ranking fuzzy numbers (RFNs) have been developed such as distance index by [1], signed distance by [2, 3], area index by [4], and centroid index by [5]. However, no method can rank FNs satisfactorily in all cases and situations [6]. Some methods are limited to normal and trapezoidal shapes of FNs and only consider neutral decision-makers' view. There are also methods that cannot distinguish the ranking of FNs having the same mode and symmetric spread, and some methods produce non-discriminate and nonintuitive results.

In this paper, a new method for RFNs based on Ochiai index and Hurwicz criterion is proposed. Ochiai is a similarity measure index and Hurwicz is a criterion for decision-making that compromises between the optimistic and pessimistic criteria. Thus, the proposed ranking method considers all types of decision-makers' view such as optimistic, neutral, and pessimistic which is crucial in solving decision-making problems.

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This paper is organized as follows. Section "Preliminaries" contains basic concepts and notations used in the remaining parts of the paper. In section "New Method for Ranking Fuzzy Numbers," the new ranking method based on Ochiai index and Hurwicz criterion is proposed. Two observations on the new ranking method are presented in section "Observations on the Ochiai with Hurwicz Criterion Ranking Index." Section "Numerical Examples" presents some numerical examples to illustrate the advantages of the proposed method. The paper ends with a conclusion in section "Conclusion."

Preliminaries

In this section, some basic concepts and definitions on FNs are reviewed from the literature.

Definition 1

A fuzzy number is a fuzzy set in the universe of discourse X with the membership function defined as [7]

$$\mu_A(x)=\begin{cases}\mu_A^L(x) & , a\leq x\leq b\\ w & , b\leq x\leq c\\ \mu_A^R(x) & , c\leq x\leq d\\ 0 & , \text{otherwise}\end{cases}.$$

where $\mu^L{}_A:[a,b]\rightarrow[0,w]$, $\mu^R{}_A:[c,d]\rightarrow[0,w]$, $w\in(0,1]$, $\mu^L{}_A$ and $\mu^R{}_A$ denote the left and the right membership functions of the fuzzy number A.

The membership function μ_A of a fuzzy number A has the following properties:

1. μ_A is a continuous mapping from the universe of discourse X to $[0,w]$.
2. $\mu_A(x)=0$ for $x<a$ and $x>d$.
3. $\mu_A(x)$ is monotonic increasing in $[a,b]$.
4. $\mu_A(x)=w$ for $[b,c]$.
5. $\mu_A(x)$ is monotonic decreasing in $[c,d]$.

If the membership function $\mu_A(x)$ is a piecewise linear, then A is called as a trapezoidal fuzzy number with membership function defined as

$$\mu_A(x)=\begin{cases}w\left(\dfrac{x-a}{b-a}\right) & , a\leq x\leq b\\ w & , b\leq x\leq c\\ w\left(\dfrac{d-x}{d-c}\right) & , c\leq x\leq d\\ 0 & , \text{otherwise}\end{cases}$$

and denoted as $A=(a,b,c,d;w)$. If $b=c$, then the trapezoidal becomes a triangular fuzzy number denoted as $A=(a,b,d;w)$.

Definition 2

Let A_1 and A_2 be two fuzzy numbers with $A_{1_\alpha}=\left[a_\alpha^-,a_\alpha^+\right]$ and $A_{2\alpha}=[b_\alpha^-,b_\alpha^+]$ be their α-cuts with $\alpha\in[0,1]$ [8]. The fuzzy maximum of A_1 and A_2 by the α-cuts method is defined as

$$\left[MAX\left(A_1,A_2\right)\right]_\alpha=\left[\max\left(a_\alpha^-,b_\alpha^-\right),\max\left(a_\alpha^+,b_\alpha^+\right)\right].$$

The fuzzy minimum of A_1 and A_2 is defined as

$$\left[MIN\left(A_1,A_2\right)\right]_\alpha=\left[\min\left(a_\alpha^-,b_\alpha^-\right),\min\left(a_\alpha^+,b_\alpha^+\right)\right].$$

Definition 3

Let $A_1=(a_1,b_1,c_1,d_1;h_1)$ and $A_2=(a_2,b_2,c_2,d_2;h_2)$ be two trapezoidal fuzzy numbers [9]. The fuzzy maximum of A_1 and A_2 by the second function principle is defined as

$$MAX\left(A_1,A_2\right)=\left(a,\ b,\ c,\ d;\ h\right)$$

where

$h=\min\{h_1,\ h_2\}$, $T=\{\max(a_1,a_2),\max(a_1,d_2),\max(d_1,a_2),\max(d_1,d_2)\}$, $T_1=\{\max(b_1,b_2),\max(b_1,c_2),\max(c_1,b_2),\max(c_1,c_2)\}$, $a=\min\ T$, $b=\min\ T_1$, $c=\max\ T_1$, $d=\max\ T$, $\min\ T\le\min\ T_1$ and $\max\ T_1\le\max\ T$.

The fuzzy minimum of A_1 and A_2 is defined as

$$MIN\left(A_1,A_2\right)=\left(a,\ b,\ c,\ d;\ h\right)$$

where

$h=\min\{h_1,\ h_2\}$, $T=\{\min(a_1,a_2),\min(a_1,d_2),\min(d_1,a_2),\min(d_1,d_2)\}$, $T_1=\{\min(b_1,b_2),\min(b_1,c_2),\min(c_1,b_2),\min(c_1,c_2)\}$, $a=\min\ T$, $b=\min\ T_1$, $c=\max T_1$, $d=\max\ T$, $\min\ T\le\min\ T_1$ and $\max\ T_1\le\max\ T$.

Definition 4

The cardinality of a fuzzy number A in the universe of discourse X is defined as [10]

$$|A|=\int_X\mu_A\left(x\right)dx.$$

New Method for Ranking Fuzzy Numbers

The new ranking method is developed based on [11] with similarity measure index defined as

$$S_O(X,Y) = \frac{f(X \cap Y)}{\sqrt{f(X \cap Y) + f(X - Y)}\sqrt{f(X \cap Y) + f(Y - X)}}$$

and reduced to $S_O(X,Y) = \frac{f(X \cap Y)}{\sqrt{f(X)}\sqrt{f(Y)}}$ or known as Ochiai index.

Typically, the function f is taken to be the cardinality function. The objects X and Y described by the features are replaced with FNs A and B which are described by the membership functions. The fuzzy Ochiai is defined as

$$S_O(A,B) = \frac{|A \cap B|}{\sqrt{|A|}\sqrt{|B|}},$$

where $|A|$ denotes the scalar cardinality of fuzzy number A. $\cap$ and $\cup$ are the t-norm and s-norm, respectively. The fuzzy Ochiai ranking index with Hurwicz criterion is presented as follows:

Step 1: For each pair of the FNs A_i and A_j, find the fuzzy maximum and fuzzy minimum of A_i and A_j. The fuzzy maximum and fuzzy minimum can be obtained by the α-cuts method for normal FNs and the second function principle for non-normal FNs.

Step 2: Calculate the evidences of $E(A_i \succ A_j)$, $E(A_j \prec A_i)$, $E(A_j \succ A_i)$ and $E(A_i \prec A_j)$ which are defined based on fuzzy Ochiai index as

$$E(A_i \succ A_j) = S_O(MAX(A_i,A_j),A_i),$$

$$E(A_j \prec A_i) = S_O(MIN(A_i,A_j),A_j),$$

$$E(A_j \succ A_i) = S_O(MAX(A_i,A_j),A_j),$$

$$E(A_i \prec A_j) = S_O(MIN(A_i,A_j),A_i),$$

where $S_O(A_i,A_j) = \frac{|A_i \cap A_j|}{\sqrt{|A_i|}\sqrt{|A_j|}}$ is the fuzzy Ochiai index and $|A_i|$ denotes the scalar cardinality of fuzzy number A_i.

To simplify, C_{ij} and c_{ji} are used to represent $E(A_i \succ A_j)$ and $E(A_j \prec A_i)$, respectively. Likewise, C_{ji} and c_{ij} are used to denote $E(A_j \succ A_i)$ and $E(A_i \prec A_j)$ respectively.

Step 3: Calculate the total evidences $E_{total}(A_i \succ A_j)$ and $E_{total}(A_j \succ A_i)$ which are defined based on the Hurwicz criterion concept as

$$E_{total}\left(A_i \succ A_j\right) = \beta C_{ij} + \left(1-\beta\right)c_{ji} \tag{83.1}$$

$$E_{total}\left(A_j \succ A_i\right) = \beta C_{ji} + \left(1-\beta\right)c_{ij} \tag{83.2}$$

$\beta \in [0, 0.5)$, $\beta = 0.5$ and $\beta \in (0.5, 1]$ represent pessimistic, neutral, and optimistic criteria, respectively.

To simplify, $E_O(A_i, A_j)$ and $E_O(A_j, A_i)$ are used to represent $E_{total}(A_i \succ A_j)$ and $E_{total}(A_j \succ A_i)$, respectively.

Step 4: For each pair of the FNs, compare the total evidences in Step 3 which will result the ranking of two FNs A_i and A_j as follows:

1. $A_i \succ A_j$ if and only if $E_O\left(A_i, A_j\right) > E_O\left(A_j, A_i\right)$ (83.3)
2. $A_i \prec A_j$ if and only if $E_O\left(A_i, A_j\right) < E_O\left(A_j, A_i\right)$ (83.4)
3. $A_i \approx A_j$ if and only if $E_O\left(A_i, A_j\right) = E_O\left(A_j, A_i\right)$ (83.5)

Observations on the Ochiai with Hurwicz Criterion Ranking Index

The ranking results of the proposed method were observed based on the values of d_{ij}, β_{ij}, n_{ij} and Equations (3.1), (3.2), (3.3), (3.4), and (3.5), with $d_{ij} = C_{ij} - c_{ji} - C_{ji} + c_{ij}$, $n_{ij} = c_{ij} - c_{ji}$ and $\beta_{ij} = \dfrac{c_{ij} - c_{ji}}{d_{ij}}$. The observation can be divided into six cases as follows.

Case 1:

Let $d_{ij} \neq 0$, $\beta = \beta_{ij}$ and $\beta_{ij} \in [0, 1]$.

Since $\beta_{ij} = \dfrac{c_{ij} - c_{ji}}{d_{ij}}$ we have

$$\beta = \frac{c_{ij} - c_{ji}}{d_{ij}} = \frac{c_{ij} - c_{ji}}{C_{ij} - c_{ji} - C_{ji} + c_{ij}},$$

$\beta(C_{ij} - c_{ji} - C_{ji} + c_{ij}) = c_{ij} - c_{ji}$, and rearranging the equation will give

$\beta\ C_{ij} + (1-\beta)\ c_{ji} = \beta C_{ji} + (1-\beta) c_{ij}$ which implies $E_O(A_i \succ A_j) = E_O(A_j \succ A_i)$

and, therefore, $A_i \approx A_j$.

Thus, if $d_{ij} \neq 0$, $\beta = \beta_{ij}$ and $\beta_{ij} \in [0, 1]$, then $A_i \approx A_j$.

Case 2:

Let $d_{ij} \neq 0$, $\beta > \beta_{ij}$ and $\beta_{ij} \in (-\infty, 1)$.

Since $\beta_{ij} = \dfrac{c_{ij} - c_{ji}}{d_{ij}}$ we have $\beta > \dfrac{c_{ij} - c_{ji}}{d_{ij}} = \dfrac{c_{ij} - c_{ji}}{C_{ij} - c_{ji} - C_{ji} + c_{ij}}$.

For $d_{ij} < 0$,
$\beta(C_{ij} - c_{ji} - C_{ji} + c_{ij}) < c_{ij} - c_{ji}$, and rearranging the inequality will give
$\beta\, C_{ij} + (1-\beta)c_{ji} < \beta\, C_{ji} + (1-\beta)c_{ij}$ which implies $E_O(A_i \succ A_j) < E_O(A_j \succ A_i)$
and, therefore, $A_i \prec A_j$.
For $d_{ij} < 0$,
$\beta(C_{ij} - c_{ji} - C_{ji} + c_{ij}) > c_{ij} - c_{ji}$, and rearranging the inequality will give
$\beta\, C_{ij} + (1-\beta)\, c_{ji} > \beta\, C_{ji} + (1-\beta)\, c_{ij}$ which implies $E_O(A_i \succ A_j) > E_O(A_j \succ A_i)$
and, therefore, $A_i \succ A_j$.
Thus,

1. If $d_{ij} > 0$, $\beta > \beta_{ij}$ and $\beta_{ij} \in (-\infty, 1)$, then $A_i \succ A_j$.
2. If $d_{ij} < 0$, $\beta > \beta_{ij}$, and $\beta_{ij} \in (-\infty, 1)$, then $A_i \prec A_j$.

Case 3:

Let $d_{ij} \neq 0$, $\beta < \beta_{ij}$ and $\beta_{ij} \in (0, +\infty)$.

Since $\beta_{ij} = \dfrac{c_{ij} - c_{ji}}{d_{ij}}$ we have $\beta < \dfrac{c_{ij} - c_{ji}}{d_{ij}} = \dfrac{c_{ij} - c_{ji}}{C_{ij} - c_{ji} - C_{ji} + c_{ij}}$.

For $d_{ij} > 0$,
$\beta(C_{ij} - c_{ji} - C_{ji} + c_{ij}) < c_{ij} - c_{ji}$, and rearranging the inequality will give
$\beta\, C_{ij} + (1-\beta)\, c_{ji} < \beta\, C_{ji} + (1-\beta)\, c_{ij}$ which implies $E_O(A_i \succ A_j) < E_O(A_j \succ A_i)$
and, therefore, $A_i \prec A_j$.
For $d_{ij} < 0$,
$\beta(C_{ij} - c_{ji} - C_{ji} + c_{ij}) > c_{ij} - c_{ji}$, and rearranging the inequality will give
$\beta\, C_{ij} + (1-\beta)\, c_{ji} > \beta\, C_{ji} + (1-\beta)\, c_{ij}$ which implies $E_O(A_i \succ A_j) > E_O(A_j \succ A_i)$
and, therefore, $A_i \succ A_j$.
Thus,

1. If $d_{ij} > 0$, $\beta < \beta_{ij}$, and $\beta_{ij} \in (0, +\infty)$, then $A_i \prec A_j$.
2. If $d_{ij} < 0$, $\beta < \beta_{ij}$, and $\beta_{ij} \in (0, +\infty)$, then $A_i \succ A_j$.

Case 4:

Let $d_{ij} = 0$ and $n_{ij} > 0$.
Then, for all $\beta \in [0, 1]$,

$$\beta d_{ij} < n_{ij}.$$

Since $d_{ij} = C_{ij} - c_{ji} - C_{ji} + c_{ij}$ and $n_{ij} = c_{ij} - c_{ji}$, then
$\beta(C_{ij} - c_{ji} - C_{ji} + c_{ij}) < c_{ij} - c_{ji}$, and rearranging the inequality will give

$\beta C_{ij} + (1-\beta)c_{ji} < \beta C_{ji} + (1-\beta)c_{ij}$ which implies $E_O(A_i \succ A_j) < E_O(A_j \succ A_i)$ and, therefore, $A_i \prec A_j$.

Thus, if $d_{ij} = 0$ and $n_{ij} > 0$, then for all $\beta \in [0,1]$, $A_i \prec A_j$.

Case 5:

Let $d_{ij} = 0$ and $n_{ij} < 0$. Then, for all $\beta \in [0,1]$,

$$\beta d_{ij} > n_{ij}.$$

Thus, $\beta(C_{ij} - c_{ji} - C_{ji} + c_{ij}) > c_{ij} - c_{ji}$, and rearranging the inequality will give $\beta C_{ij} + (1-\beta)c_{ji} > \beta C_{ji} + (1-\beta)c_{ij}$ which implies $E_O(A_i \succ A_j) > E_O(A_j \succ A_i)$ and, therefore, $A_i \succ A_j$.

Thus, if $d_{ij} = 0$ and $n_{ij} < 0$, then for all $\beta \in [0,1]$, $A_i \succ A_j$.

Case 6:

Let $d_{ij} = 0$ and $n_{ij} = 0$.

Then, for all $\beta \in [0,1]$,

$$\beta d_{ij} = n_{ij}.$$

Thus, $\beta(C_{ij} - c_{ji} - C_{ji} + c_{ij}) = c_{ij} - c_{ji}$, and rearranging the equation will give $\beta C_{ij} + (1-\beta)c_{ji} = \beta C_{ji} + (1-\beta)c_{ij}$ which implies $E_O(A_i \succ A_j) = E_O(A_j \succ A_i)$ and, therefore, $A_i \approx A_j$.

Thus, if $d_{ij} = 0$ and $n_{ij} = 0$, then for all $\beta \in [0,1]$, $A_i \approx A_j$.

The ranking result of the proposed method can be classified as having two main observations which are Observations 4.1 (covers cases 1–3) and 4.2 (covers cases 4–6). The two main observations are presented as follows.

Observation 4.1

For two FNs A_i and A_j with $d_{ij} \neq 0$, the ranking results for Ochiai index are as follows:

1. If $d_{ij} \neq 0$ and $\beta = \beta_{ij}$, then $A_i \approx A_j$.
2. If $d_{ij} > 0$ and
   (a) $\beta > \beta_{ij}$, then $A_i \succ A_j$.
   (b) $\beta < \beta_{ij}$, then $A_i \prec A_j$.
3. If $d_{ij} < 0$ and
   (a) $\beta > \beta_{ij}$, then $A_i \prec A_j$.
   (b) $\beta < \beta_{ij}$, then $A_i \succ A_j$.

Observation 4.2

For two FNs A_i and A_j with $d_{ij}=0$, the ranking results for Ochiai index are as follows:

1. If $n_{ij}>0$, then for all $\beta\in[0,1]$, $A_i \prec A_j$.
2. If $n_{ij}<0$, then for all $\beta\in[0,1]$, $A_i \succ A_j$.
3. If $n_{ij}=0$, then for all $\beta\in[0,1]$, $A_i \approx A_j$.

Numerical Examples

In this section, four sets of numerical examples are presented to illustrate the validity and advantages of fuzzy Ochiai ranking index. For two FNs A_1 and A_2, $d_{12}=C_{12}-c_{21}-C_{21}+c_{12}$, $\beta_{12}=\dfrac{c_{12}-c_{21}}{d_{12}}$, and $n_{12}=c_{12}-c_{21}$ and C_{12}, c_{21}, C_{21} and c_{12} denoted the evidences $E(A_1 \succ A_2)$, $E(A_2 \prec A_1)$, $E(A_2 \succ A_1)$ and $E(A_1 \prec A_2)$ respectively.

Example 1

Consider the FNs in [12], i.e., $A_1=(0.1, 0.3, 0.5)$ and $A_2=(0.2, 0.3, 0.4)$.

Since A_1 and A_2 have the same mode and symmetric spread, a number of the existing ranking methods cannot discriminate them, such as [1–5, 13–16]. However, [12, 17–21, 22] produce $A_1 \prec A_2$. By the proposed method, we obtain $d_{12}=0.098>0$ and $\beta_{12}=0.5$.

Thus, by Observation 4.1 the ranking order is produced as $\begin{cases} A_1 \prec A_2 & ,\beta\in[0,0.5) \\ A_1 \approx A_2 & ,\beta = 0.5 \\ A_1 \succ A_2 & ,\beta\in(0.5,1] \end{cases}$, where $A_1 \prec A_2$ for pessimistic decision-makers, $A_1 \approx A_2$ for neutral decision-makers, and $A_1 \succ A_2$ for optimistic decision-makers. The ranking result is affected by decision-makers' perspective, and this shows that the proposed method has strong discrimination ability.

Example 2

Consider the FNs in [15], i.e., $A_1=(0.3, 0.5, 0.9)$ and $A_2=(0.155, 0.645, 0.8)$. References [1, 4] rank them as $A_1 \prec A_2$, while [12, 15] produce $A_1 \succ A_2$. By the proposed method, we obtain $d_{12}=-0.006<0$ and $\beta_{12}=0.833$, and by Observation 4.1 the ranking order is produced as $\begin{cases} A_1 \succ A_2 & ,\beta\in[0,0.833) \\ A_1 \approx A_2 & ,\beta = 0.833 \\ A_1 \prec A_2 & ,\beta\in(0.833,1] \end{cases}$. Both neutral and pessimistic decision-makers rank $A_1 \succ A_2$ while optimistic decision-makers rank them in three different results. The result shows that the equal ranking does not necessarily occur for neutral decision-makers.

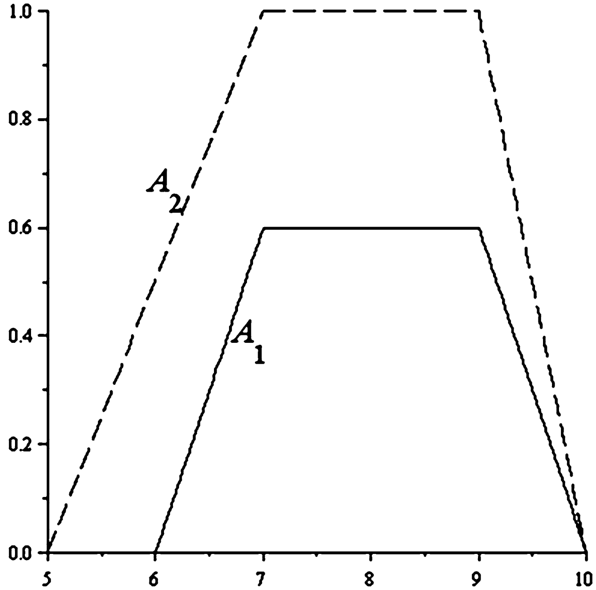

Fig. 83.1 Fuzzy numbers in Example 3

Example 3

Consider the FNs in [5], i.e., $A_1=(6,7,9,10;0.6)$ and $A_2=(5,7,9,10;1)$ as shown in Fig. 83.1.

Some of the existing ranking methods such as [2, 3, 15, 16, 23–25] can only rank normal FNs and, thus, fail to rank the FNs A_1 and A_2. Moreover, [1, 5] rank them as $A_2 \prec A_1$, while [4] ranks them as $A_1 \prec A_2$. By the proposed method, $d_{12}=0.434>0$ and $\beta_{12}=0.350$, thus, obtain the ranking result as $A_1 \prec A_2$ for $\beta \in [0, 0.350)$, $A_1 \approx A_2$ for $\beta=0.350$ and $A_1 \succ A_2$ for $\beta \in (0.350, 1]$. Similarly, the ranking result is affected by decision-makers' perspective.

Example 4

Consider the FNs in [24], i.e., $A_1=(1,2,5)$ and $A_2=(1,2,2,4)$ as shown in Fig. 83.2, with the membership function of A_2 defined as

$$\mu_{A_2}(x)=\begin{cases}\sqrt{1-(x-2)^2} & ,[1,2]\\ \sqrt{1-\frac{1}{4}(x-2)^2} & ,[2,4]\\ 0 & ,\text{else}\end{cases}.$$

Some of the existing ranking methods such as [12, 17, 18] can only rank trapezoidal FNs and, thus, fail to rank the FNs A_1 and A_2. By using the proposed method, we have $d_{12}=-0.015<0$ and $\beta_{12}=3.88$. Therefore, the ranking order is $A_1 \succ A_2$ regardless of the decision-makers' perspective, as shown in Table 83.1. The ranking result of the proposed method is consistent with human intuition and other ranking methods in Table 83.1.

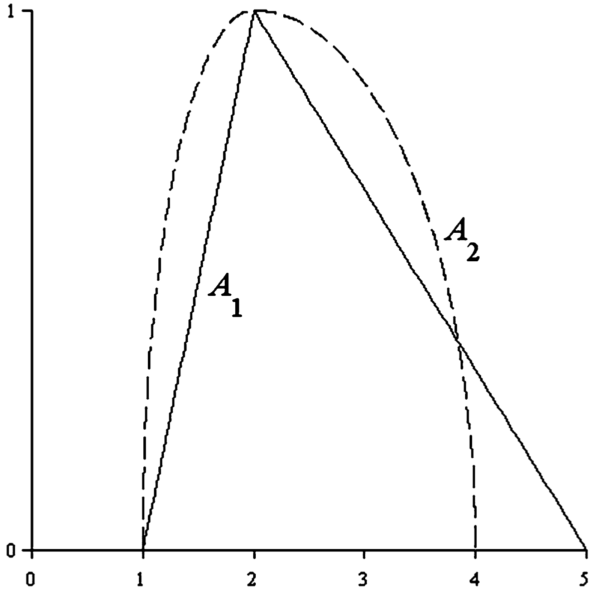

Fig. 83.2 Fuzzy numbers in Example 4

Table 83.1 Ranking results of Example 4

Method	Fuzzy numbers	Index value	Ranking results
Proposed method	(d_{12}, β_{12})	$(-0.015, 3.88)$	$A_1 \succ A_2$, $\beta \in [0, 1]$
[2]	A_1	3.162	*
	A_2	*	
[17]	A_1	*	*
	A_2	*	
[18]	A_1	0.371	*
	A_2	*	
[26]	A_1	0.274	$A_1 \succ A_2$
	A_2	0.190	
[24]	A_1	0.2154	$A_1 \succ A_2$
	A_2	0	
[16]	A_1	2.5	$A_1 \succ A_2$
	A_2	2.360	
[4]	A_1	1.245	$A_1 \succ A_2$
	A_2	1.182	
[1]	A_1	2.717	$A_1 \succ A_2$
	A_2	2.473	
[15]	A_1	0.890	$A_1 \succ A_2$
	A_2	0.806	

*, the ranking method cannot calculate the ranking value.

Conclusion

This paper presents a new method for RFNs using Ochiai index and Hurwicz criterion. Two observations that can simplify the ranking procedure are produced. The observations have rendered the proposed ranking index as an advantageous method since the ranking results can be obtained for all continuous values of $\beta \in [0, 1]$. The proposed method can overcome certain shortcomings that exist in the previous ranking methods such as can rank both non-normal and general shapes of FNs and can discriminate the ranking of FNs having the same mode and symmetric spreads which fail to be ranked by the previous ones. The proposed method can be highly applied in solving decision-making as it has strong discrimination ability which is a crucial criterion in solving decision-making problems.

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