

# Improving the Khosla Method of Estimating Subsurface Flow Properties in Hydraulic Design Using MODFLOW



Rath Prayas, K. K. Khatua, and K. C. Patra

**Abstract** The Khosla theory of flow nets has stood the test of time. It has been in use for designing the hydraulic structure for nearly a century by estimating the seepage properties like the pressure heads, seepage discharge, and exit gradient. In this method, flow nets, comprising equipotential lines and streamlines, are implemented to predict the movement of the water below the hydraulic structure to estimate the various seepage properties. The major limitation of the method lies in the important assumption of homogeneity and isotropy in the foundation of the soil. In this paper, we have aimed to address this limitation by using a commonly used finite difference groundwater flow model MODFLOW. We have created a 2D theoretical barrage model and tested for the various seepage properties in different hydrogeological conditions including anisotropy and different thickness of the saturated zone below the structure. The results showed that by increasing the anisotropy, the exit gradient increased manifold. When the depth increases, the exit gradient and discharge increases up to a certain limit, upon further increasing the depth, there was minimal effect on the exit gradient. The results shall help us to evaluate the uncertainty in estimating the seepage properties and improve the Khosla method in designing the hydraulic structures.

**Keywords** Khosla method · Hydraulic structure · Seepage · Exit gradient · Anisotropy · MODFLOW

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## 1 Introduction

Hydraulic structures like dams and barrages have been in use to control the flow of water for flood control, recharging groundwater, store water for irrigation, and human consumption since time immemorial. Hydropower and industries also depend on these hydraulic structures for their water demand. In the last few decades, additional stress for water resources due to increase in population and climate change has resulted in higher need water storage and flood control structures. Also, in this span of time, a number of these hydraulic structures have failed. Studies and surveys of major failures in the last century have shown that piping is one of the major causes. Piping failure occurs when the seepage water has sufficient force to lift the soil at the downstream end of the structure resulting in progressive removal of the soil from beneath the foundation. This leads to subsidence of the structure. The force of the flow of the seepage depends on exit gradient that depends on properties of the subsurface flow. Thus, the need to study the subsurface flow beneath a hydraulic structure is of paramount importance. Over the last hundred years, engineers and scientists have studied the properties of seepage flow below these structures.

The theory of creep length [1] was introduced for the flow passing under hydraulic structures. The creep length was defined as the route of the first line of seepage which is in contact with the foundation of the hydraulic structure. Along the creep line, the hydraulic gradient is constant, and the energy loss is linearly proportional to the creep length. Hence, the uplift pressure distribution beneath the hydraulic structure is linear. It was discovered that there is difference between vertical and horizontal creep paths [2]. As a result, he introduced the weighted creep path theory. He added factors of 0.33 and 1 to the total horizontal and vertical lengths, respectively. A method based on flow nets [3] was established to estimate the pressure distribution and exit gradient. This method assumes the seepage flow which occurs in streamlines perpendicular to the equipotential heads. This method can be construed to be a special case of Darcian flow system, where heterogeneity and anisotropy are neglected. Khosla used complex power function to solve for the hydraulic heads (pressure) below the structures. Of all the three methods discussed, Khosla method is used extensively by hydraulic engineers in India. But the basic assumption of homogenous and isotropy in Khosla method can limit its applications. In the last few decades, various other computational methods to estimate flow through the porous media have been established. With increase in computational efficiency finite element [4–6], finite difference methods and isogeometric have been implemented to solve the flow system equations. MODFLOW is one such computer code that implements finite difference scheme to solve for groundwater flow problem. MODFLOW solves the following equation:

$$\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) + Q'_s = SS \frac{\partial h}{\partial t} \quad (1)$$

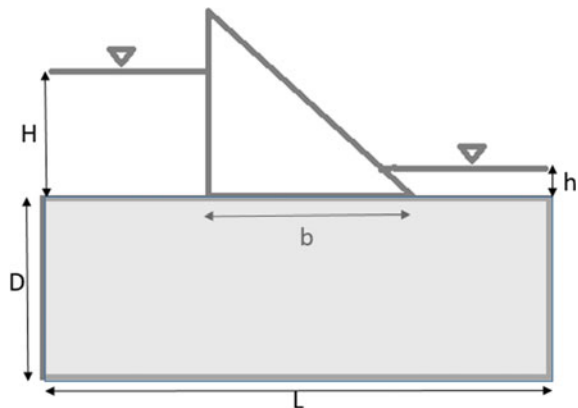
where  $K_{xx}$ ,  $K_{yy}$ , and  $K_{zz}$  are values of hydraulic conductivity along the  $x$ ,  $y$ , and  $z$  coordinate axes, which are assumed to be parallel to the major axes of hydraulic conductivity ( $L/T$ );  $h$  is the potentiometric head ( $L$ ),  $Q'_s$  is a volumetric flux per unit volume representing sources and sinks of water, with  $Q'_s$  being negative for flow out of the groundwater system, ( $T^{-1}$ );  $SS$  is the specific storage of the porous material ( $L^{-1}$ ); and  $t$  is time ( $T$ ).

## 2 Methods

In the present study, the seepage flow system is 2D and steady state with no sinks or sources. Hence, the terms in  $z$  direction,  $Q'_s$ , and the RHS term containing the time function in Eq. (1) are all reduced to zero. The model is implemented using MODFLOW NWT [7] in a Python environment using FloPy [8]. The MODFLOW NWT is the Newton–Raphson formulation of MODFLOW which handles drying and rewetting nonlinearities of the unconfined groundwater flow to allow more stable water table solutions than the previous versions [9]. The models were bounded by constant head boundaries on the vertical edges (i.e., flow divides) and the no flow boundary along the bottom (i.e., impermeable barrier) for all geometries (Fig. 1). The length of the hydraulic structure is  $b$ , the distance to the impermeable layer below is  $D$ , and the hydraulic conductivity of the flow system is in  $x$ - and  $y$ -directions which are denoted by  $K_x$  and  $K_y$ .

The whole domain is discretized into 5000 columns horizontally and 1000 rows vertically resulting in 50,000 cells. This fine scale of discretization helps us to avoid any errors due to grid sizes in estimating the seepage properties. The upstream head  $H$  is at 70 m and downstream head is set at 30 m. The width of the hydraulic structure (b) is set to be 40 m. The length of the domain  $L$  is 200 m. The base case thickness or depth to the impervious layer ( $D$ ) is 25 m. The exit gradient is calculated from the output of heads calculated by MODFLOW at the different layers.

**Fig. 1** Model domain and the schematization of the different parameters used

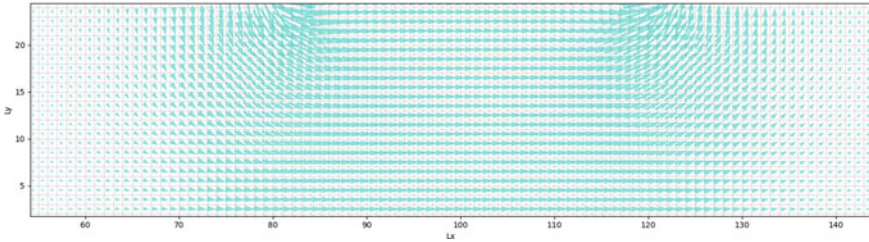


The model is tested with different thickness to length ratios ( $D/L$ ) and anisotropies ( $K_y/K_x$ ) to find the head distribution and upwards seepage discharge and exit gradient. The range of  $D/L$  tested is 0.025 to 2.5. And the range tested for anisotropies is  $10^{-5}$  to  $10^5$ .

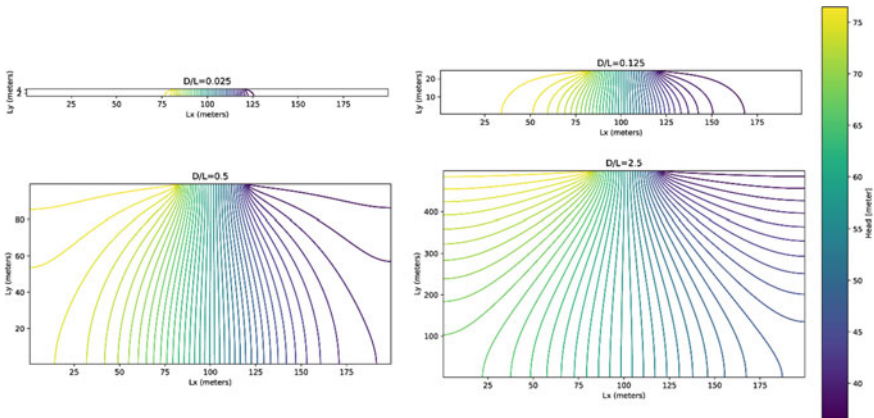
### 3 Results

#### 3.1 Effect of Thickness of the Saturated Layer

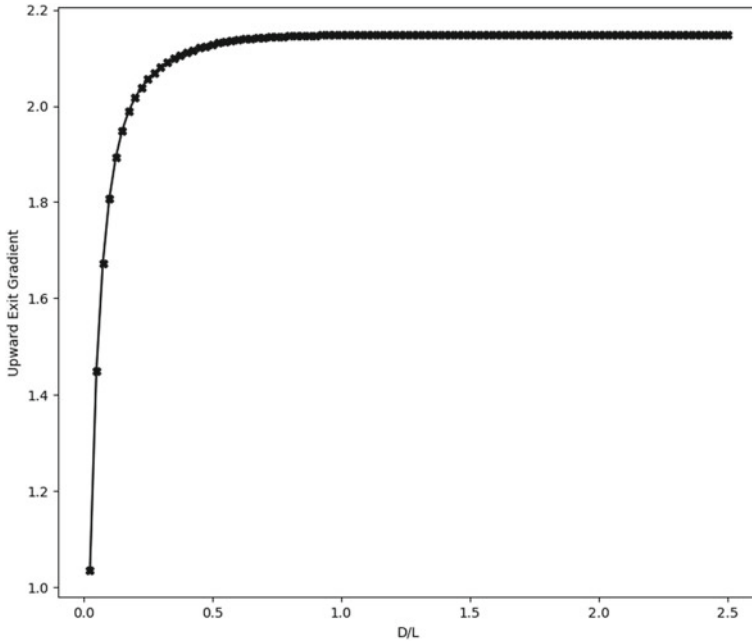
Figure 2 shows a simulation of the calculated heads with the structure. We evaluated the effect of the  $D/L$  by changing the values of  $D$ . We found that increasing the  $D/L$ , the head contours gradually became more distributed in the  $x$ -direction in the whole domain and the gradient along the  $y$ -direction became more noticeable (Fig. 3).



**Fig. 2** Model cross section showing the direction of flow under the hydraulic structure. The cyan colored arrows are the flow vectors



**Fig. 3** Contour plot of head distribution under the hydraulic structure for different depths to length ratios

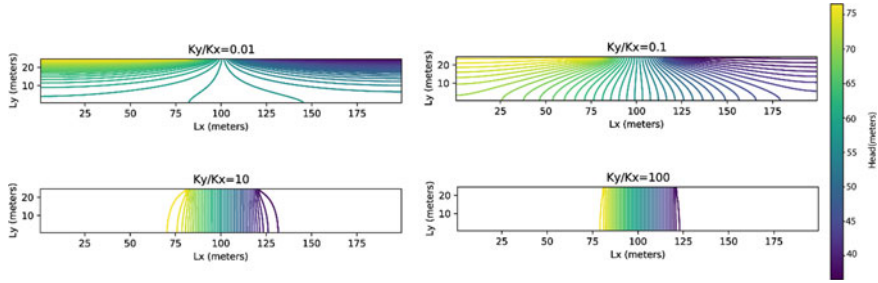


**Fig. 4** Variation of upward exit gradient with change in thickness ( $D$ ) of the saturated layer

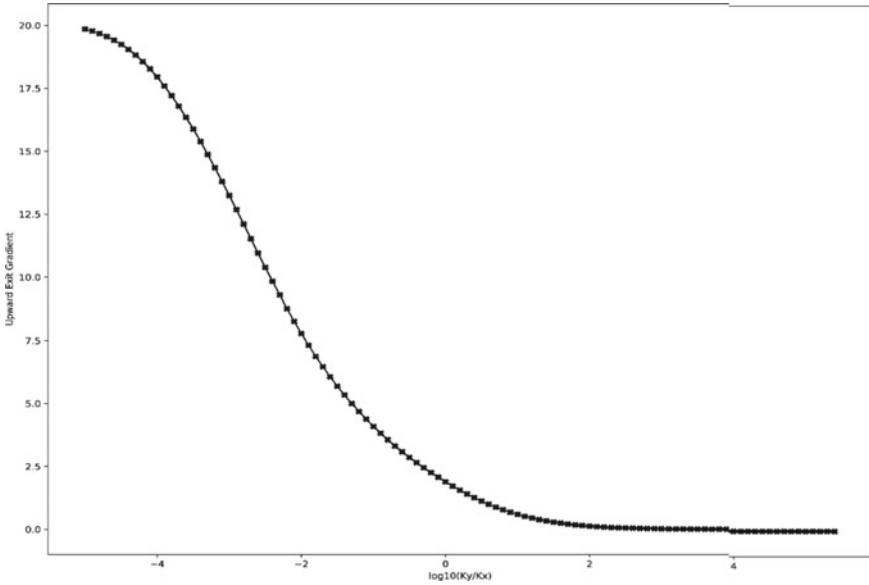
The upward exit gradient thus increases with increase in the depth of the permeable layer. However, this increase is plateaued after a certain depth is reached (Fig. 4). Upon further increasing the depth, the exit gradient remains unaffected. Thus, the assumption of infinite thickness may affect the estimation of safe exit gradient by overestimating it for thinner saturated zones.

### 3.2 Effect of Anisotropy

Another main assumption of Khosla theory is the isotropic medium. We found that increasing the anisotropy, i.e., increasing the ratio of  $K_y/K_x$  increased the direction of flow in the  $y$ -direction (Fig. 5). Hence, the gradient in the upward direction decreases (Fig. 6). But the flow or the rate of specific discharge increased with increase in the anisotropy as the  $K_y$  increased (Fig. 7).



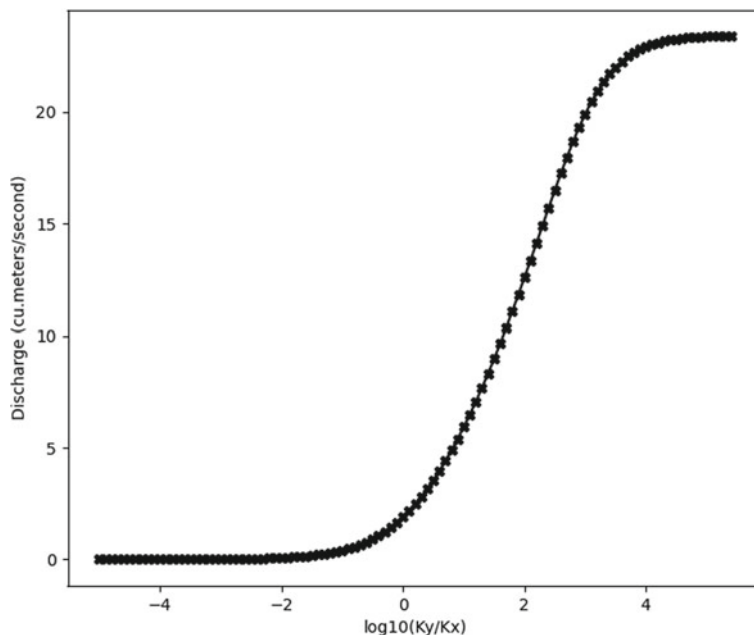
**Fig. 5** Contour plot of head distribution under the hydraulic structure for different depths to length ratios



**Fig. 6** Variation of upward exit gradient with change in anisotropy ( $K_y/K_x$ ) of the saturated layer

### 4 Conclusions

The anisotropy and the thickness of the saturated layer affect the groundwater flow features. Hence, the underlying assumption of Khosla theory of flow nets may render its wide applicability in doubt especially when the anisotropy is high, or the thickness is low. We found at higher anisotropy, although the exit gradient is low, but the specific discharge (product of hydraulic conductivity and hydraulic gradient) increases. Hence, theoretically the consideration of exit gradient as the primary driver of discharge and hence the factor to be considered for safety should be treated with bit of doubt. For practical purposes, such high anisotropies may not be existing on



**Fig. 7** Variation of specific discharge with change in anisotropy ( $K_y/K_x$ ) of the saturated layer

riverbeds, but we should always be aware of these imitations while applying. This paper also shows how MODFLOW can be used to improve the Khosla theory and can be helpful for design. Future works will be done on the inclusion of cut-off walls in the present model and the effect of boundary conditions, i.e.,  $D/L$ .

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