Chapter 10
Recent Developments in Fuzzy Dynamic Data Envelopment Analysis and Its Applications

Rajinder Kaur and Jolly Puri

10.1 Introduction

Data envelopment analysis (DEA) is a performance evaluation tool. It is a nonparametric technique based on a linear programming approach to estimate the relative efficiencies of similar decision-making units (DMUs) in terms of multiple inputs–outputs. The DMUs can be educational institutions, banks, bank branches, hospitals, etc. Charnes et al. [1] initially proposed DEA in terms of constant returns to scale and later on extended by Banker et al. [2] to introduce variable returns to scale in DEA. The wide literature on DEA models can be seen in Cooper et al. [3], Tone [4], Tone and Tsutsui [5], Li et al. [6], Kao [7], Emrouznejad and Yang [8], and Contreras [9]. Despite all these extensions and the immense literature on DEA models, it has two key limitations: (i) It measures the performance statically in a particular period and ignores interrelationship present between consecutive periods, and (ii) it entails crisply defined input and output data. However, observed data values are imprecise or vague in real-life applications, e.g., data for customer satisfaction cannot be defined crisply. Fuzzy dynamic DEA (FDDEA) is found to be an emerging area that enables to evaluate a DMU's efficiency by considering interrelationship in the form of carryovers between consecutive periods. Gholizadeh et al. [10] incorporated fuzzy data in dynamic DEA for the first time to measure the efficiency of the investment corporations in the stock exchange. The present study presents a review of fuzzy dynamic DEA (FDDEA) during the last decade by classifying the studies into four categories

R. Kaur · J. Puri (✉)
School of Mathematics, Thapar Institute of Engineering and Technology,
Patiala, Punjab 147001, India
e-mail: jolly.puri@thapar.edu

R. Kaur
e-mail: rkaur_phd19@thapar.edu

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(i) theoretical development of FDDEA models with different fuzzy sets, (ii) FDDEA with network structure, (iii) applications of FDDEA approach, and (iv) integration of FDDEA with other operations research and/or artificial intelligence techniques that facilitates practical situations. As per the available literature, the present work seems to be the first review on FDDEA.

Section 10.2 presents an overview of dynamic DEA models followed by a review of FDDEA in Sect. 10.3. Section 10.4 classifies the FDDEA studies into four categories for a systematic review of the FDDEA. Section 10.5 concludes the present study.

10.2 Overview of Dynamic DEA

Sengupta [11–14] introduced the term dynamic efficiency to overcome the limitation of DEA for not incorporating time effect into the analysis and evaluated the performance of DMUs over different time periods connected through intermediates or links. Nemoto and Goto [15, 16] presented a dynamic approach in which inputs are categorized as variable inputs and quasi-fixed inputs, and later, this approach has been extended by many authors in literature. Many of the studies in literature allocated different weights to the intermediates according to their role of input or output in production system. Based on the idea of Kao [17] of assigning the same weights to the same factor, Kao [18] introduced a relational model to evaluate efficiency in dynamic environment when all the periods are linked through intermediates which are assigned the same weights no matter which period they belong to and are acting as an input or output in that period. Figure 10.1 presents a simple dynamic structure for kth DMU over q periods connected through intermediates or links. Consider n number of DMUs for evaluation over q periods, and each DMU consumes l number of inputs to produce s number of outputs. Let $X_{ij} = \sum_{t=1}^{q} X_{ij}^{(t)}$ and $Y_{gj} = \sum_{t=1}^{q} Y_{gj}^{(t)}$ be the ith $(i = 1, \ldots, l)$ system input and gth $(g = 1, 2, \ldots, s)$ system output for DMU_j $(j = 1, \ldots, n)$, respectively, where $X_{ij}^{(t)}$ and $Y_{gj}^{(t)}$, respectively, denote the lth input and gth output of DMU_j in period t, and $Z_{dj}^{(t)}$ $(d = 1, 2, \ldots, h)$ acts as an intermediate between the two successive periods t and $t + 1$ $(t = 1, \ldots, q - 1)$. Let the initial and final links for DMU_j be denoted by $Z_{dj}^{(0)}$ and $Z_{dj}^{(q)}$, respectively. Model-1 [18] presents an output-oriented model to evaluate dynamic efficiency of DMU_k for the structure depicted in Fig. 10.1.

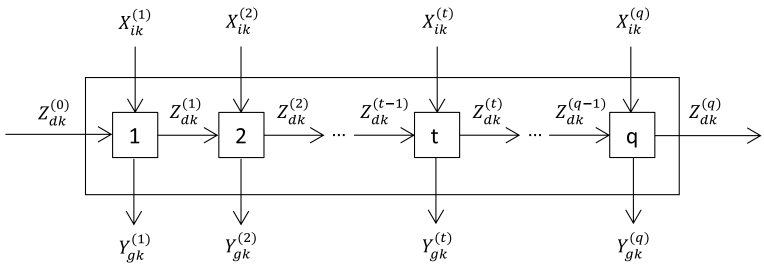

Fig. 10.1 Dynamic structure of DMU_k over q periods

Model-1

$$
\begin{aligned}
&1/E_k^S = \min \sum_{i=1}^{l} v_i X_{ik} + \sum_{d=1}^{h} w_d Z_{dk}^{(0)} \\
&\text{s.t.} \quad \sum_{g=1}^{s} u_g Y_{gk} + \sum_{d=1}^{h} w_d Z_{dk}^{(q)} = 1 \\
&\left(\sum_{i=1}^{l} v_i X_{ij} + \sum_{d=1}^{h} w_d Z_{dj}^{(0)} \right) - \left(\sum_{g=1}^{s} u_g Y_{gj} + \sum_{d=1}^{h} w_d Z_{dj}^{(q)} \right) \geq 0, \quad j = 1, \ldots, n, \\
&\left(\sum_{i=1}^{l} v_i X_{ij}^{(t)} + \sum_{d=1}^{h} w_d Z_{dj}^{(t-1)} \right) - \left(\sum_{g=1}^{s} u_g Y_{gj}^{t} + \sum_{d=1}^{h} w_d Z_{dj}^{(t)} \right) \geq 0, \\
&j = 1, \ldots, n; \quad t = 1, \ldots, q, \\
&v_i \geq \epsilon; \quad u_g \geq \epsilon; \quad w_d \geq \epsilon,
\end{aligned}
$$

where $\epsilon > 0$ is a non-Archimedean infinitesimal.

By using the optimal weights ($v_i^* \; \forall i$, $u_g^* \; \forall g$, $w_d^* \; \forall d$) derived from the Model-1, system efficiency (E_k^S) and period efficiencies ($E_k^{(t)}$) for DMU_k are defined as

$$
E_k^S = \frac{\sum_{g=1}^{s} u_g^* Y_{gk} + \sum_{d=1}^{h} w_d^* Z_{dk}^{(q)}}{\sum_{i=1}^{l} v_i^* X_{ik} + \sum_{d=1}^{h} w_d^* Z_{dk}^{(0)}} \tag{10.1}
$$

$$
E_k^{(t)} = \frac{\sum_{g=1}^{s} u_g^* Y_{gj}^{t} + \sum_{d=1}^{h} w_d^* Z_{dj}^{(t)}}{\sum_{i=1}^{l} v_i^* X_{ij}^{(t)} + \sum_{d=1}^{h} w_d^* Z_{dj}^{(t-1)}} \quad \forall t = 1, \ldots, q. \tag{10.2}
$$

Since the second set of constraints in Model-1 is redundant as it can be obtained by taking summation over the constraints corresponding to all periods, so by using this relation and Eqs. (10.1) and (10.2), a relationship has been established between system efficiencies and the period efficiencies which is defined as follows:

$$1 - E_k^S = \sum_{t=1}^{q} \left(1 - E_k^{(t)}\right) \alpha^{(t)}, \tag{10.3}$$

where $\alpha^{(t)} = \left(\sum_{i=1}^{l} v_i^* X_{ik}^{(t)} + \sum_{d=1}^{h} w_d^* Z_{dk}^{(t-1)}\right) / \left(\sum_{i=1}^{l} v_i^* X_{ik} + \sum_{d=1}^{h} w_d^* Z_{dk}^{(0)}\right)$, i.e., the complement of the system efficiency $(1 - E_k^S)$ can be written as linear combination of the period efficiencies $(1 - E_k^{(t)})$.

10.3 Fuzzy Dynamic DEA

While dealing with real-life problems, it is not always possible to collect precise or crisp data, as in the case of customer satisfaction. The uncertainty or imprecision may exist in the form of interval numbers, linguistic data, ordinal data, or fuzzy numbers. This section is devoted to an overview of fuzzy set theory and its use in dynamic DEA.

10.3.1 Fuzzy Set Theory

Definition 1 [19] A fuzzy set $\tilde{A}$ in a universe of discourse X is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}} : X \to [0, 1]$ is the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ represents the degree of belongingness of x in $\tilde{A}$.

Definition 2 [20] The support of a fuzzy set $\tilde{A}$, denoted by $S(\tilde{A})$, is a crisp set defined by

$$S(\tilde{A}) = \{x | \mu_{\tilde{A}}(x) \geq 0\}.$$

Definition 3 [20] A fuzzy set $\tilde{A}$ in universe of discourse X is said to be convex if and only if

$$\mu_{\tilde{A}}(\lambda x + (1 - \lambda)y) \geq \min(\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)), \text{ for all } x, y \in X \text{ and } 0 \leq \lambda \leq 1.$$

Definition 4 [20] Let $\tilde{A}$ be a fuzzy set in universe of discourse X. Then, it is said to be normal if $\mu_{\tilde{A}}(x) = 1$ for some $x \in X$.

Definition 5 [20] Let $\tilde{A}$ be a fuzzy set in universe of discourse X. Then, it is said to be a fuzzy number if it is both convex and normal.

Definition 6 [21] Let $\tilde{A}$ be a fuzzy set in universe of discourse X. Then, an α-cut of $\tilde{A}$, denoted by $\tilde{A}_\alpha$, is defined as $\tilde{A}_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}$. It is a crisp set of all those elements of X having membership degree greater than or equal to α in $\tilde{A}$.

Definition 7 [22] An l-r fuzzy number, denoted by $\tilde{A} = (\underline{n}, \overline{n}, \rho, \phi)_{lr}$, is a fuzzy number with membership function $\mu_{\tilde{A}}$ given by

$$\mu_{\tilde{A}}(x) = \begin{cases} l\left(\dfrac{\underline{n} - x}{\rho}\right), & \underline{n} - \rho \le x \le \underline{n}, \\ 1, & \underline{n} \le x \le \overline{n}, \\ r\left(\dfrac{x - \overline{n}}{\phi}\right), & \overline{n} \le x \le \overline{n} + \phi, \\ 0, & \text{otherwise,} \end{cases}$$

where $l : [0, 1] \to [0, 1]$ and $r : [0, 1] \to [0, 1]$ are non-increasing continuous shape functions with $l(0) = r(0) = 1$ and $l(1) = r(1) = 0$, $[\underline{n}, \overline{n}]$ is the peak of $\tilde{A}$, and ρ, ϕ are positive scalars.

Definition 8 [19] A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is a fuzzy number with membership function $\mu_{\tilde{A}}$ defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 < x \le a_2, \\ \dfrac{x - a_3}{a_2 - a_3}, & a_2 \le x < a_3, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 9 [23] A fuzzy set $\tilde{\tilde{A}}$ is said to be a type-2 fuzzy set if membership function of its elements is of type-1 fuzzy set.

Definition 10 [24] Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in a universe of discourse X with membership function denoted by $\mu_{\tilde{\tilde{A}}}$, then $\tilde{\tilde{A}}$ is said to be an interval type-2 fuzzy set if $\mu_{\tilde{\tilde{A}}}(x, v) = 1$, for all $x \in X$, $v \in l_x \subseteq [0, 1]$.

Definition 11 [24] A trapezoidal interval type-2 fuzzy set $\tilde{\tilde{A}}$ is defined as

$$\begin{aligned} \tilde{\tilde{A}} &= (\tilde{A}^U, \tilde{A}^L) \\ &= ((d_1^U, d_2^U, d_3^U, d_4^U, h_1(\tilde{A}^U), h_2(\tilde{A}^U)), (d_1^L, d_2^L, d_3^L, d_4^L, h_1(\tilde{A}^L), h_2(\tilde{A}^L))), \end{aligned}$$

where $\tilde{A}^U$ and $\tilde{A}^L$ denote the trapezoidal upper and lower membership functions, respectively, and $h_l(\tilde{A}^U) \in [0, 1]$, $h_l(\tilde{A}^L) \in [0, 1]$ are the membership values of d_{l+1}^U and d_{l+1}^L $(l = 1, 2)$, respectively.

10.3.2 Fuzzy Set Theory and Dynamic DEA

Dynamic DEA is used to evaluate efficiency while keeping in mind the interdependence of periods represented by carryovers from one period to the subsequent period. However, the data for inputs and outputs as well as carryovers are not always in precise or crisp form like customer satisfaction, and environmental pollution [25], airport reputation, and social responsibility [26]. Zadeh [27] introduced fuzzy numbers to represent various imprecise data forms. Since then, several authors have incorporated the concept of fuzzy in DEA and other performance measuring techniques [19, 28]. Let the data for all inputs, outputs, and links be fuzzy numbers and $\tilde{X}_{ik}$, $\tilde{Y}_{gk}$, and $\tilde{Z}_{dk}$ denote the ith fuzzy input, gth fuzzy output, and dth fuzzy link for DMU$_k$, respectively, then dynamic fuzzy efficiency is evaluated by using the following model.

Model-2

$$
\begin{aligned}
&1/\tilde{E}_k^S = \min \sum_{i=1}^{l} v_i \tilde{X}_{ik} + \sum_{d=1}^{h} w_d \tilde{Z}_{dk}^{(0)} \\
&\text{s.t.} \quad \sum_{g=1}^{s} u_g \tilde{Y}_{gk} + \sum_{d=1}^{h} w_d \tilde{Z}_{dk}^{(q)} = 1, \\
&\left(\sum_{i=1}^{l} v_i \tilde{X}_{ij} + \sum_{d=1}^{h} w_d \tilde{Z}_{dj}^{(0)} \right) - \left(\sum_{g=1}^{s} u_g \tilde{Y}_{gj} + \sum_{d=1}^{h} w_d \tilde{Z}_{dj}^{(q)} \right) \geq 0, \quad j = 1, \ldots, n, \\
&\left(\sum_{i=1}^{l} v_i \tilde{X}_{ij}^{(t)} + \sum_{d=1}^{h} w_d \tilde{Z}_{dj}^{(t-1)} \right) - \left(\sum_{g=1}^{s} u_g \tilde{Y}_{gj}^{t} + \sum_{d=1}^{h} w_d \tilde{Z}_{dj}^{(t)} \right) \geq 0, \\
&j = 1, \ldots, n; \quad t = 1, \ldots, q, \\
&v_i \geq \epsilon; \quad u_g \geq \epsilon; \quad w_d \geq \epsilon,
\end{aligned}
$$

where ϵ is a non-Archimedean infinitesimal and v_i, u_g, and w_d are the respective multipliers for ith fuzzy input, gth fuzzy output, and dth fuzzy link.

10.4 Classification of FDDEA Studies

The literature on Fuzzy dynamic DEA can be classified into four categories: (i) Theoretical development of FDDEA models with different fuzzy sets, (ii) FDDEA with network structure, (iii) applications of FDDEA approach, and (iv) integration of FDDEA with other techniques are discussed in detail in subsequent sections.

10.4.1 Theoretical Development of FDDEA Models with Different Fuzzy Sets

Nemoto and Goto [15, 16] proposed an approach in which inputs are classified into two categories, (i) variable inputs and (ii) quasi-fixed inputs, which cannot be immediately adjusted without acquiring an adjustment cost [15, 29]. Based on this idea of Nemoto and Goto [15, 16], Chiang and Tzeng [30] developed a multi-objective DEA model in a dynamic framework which is further extended by Jafarian-Moghaddam and Ghoseiri [21] in fuzzy environment. To solve the given model, they reduced it to a single-objective model by using the membership function suggested in Zimmermann [20].

Kordrostami et al. [31] and Keikha-Javan et al. [32] presented dynamic network DEA models to evaluate interval overall and interval period efficiencies for the whole system and each subunit. The subunits are connected in a parallel structure to each other where the inputs and outputs are not known precisely and are known in the form of interval numbers. Kordrostami et al. [31] also derived a relationship between the interval system efficiency and interval subunit efficiencies in a dynamic environment. Keikha-Javan [32] classified carryovers as desirable carryovers and undesirable carryovers to reflect the interdependence of periods more realistically. Soleimani-damaneh [22] provided a theoretical discussion on fuzzy dynamic DEA approaches for incorporating imprecise data.

Ghobadi et al. [33] extended the models presented by Emrouznejad and Yang [8] and Jahanshahloo et al. [34] to deal with fuzzy inputs–outputs in a dynamic environment and presented an inverse dynamic DEA model to evaluate efficiency when data are in the form of LR fuzzy numbers.

As while evaluating efficiency using the DEA model, it is possible for more than one DMUs to be regarded as efficient, so to further rank these efficient DMUs, Andersen and Petersen [35] presented a concept of super-efficiency in DEA. Li et al. [36] further extended it to incorporate dynamic factors and interval data. Yaghoubi et al. [37] presented a dynamic random fuzzy data envelopment analysis (DRF-DEA) model using a common set of weights methodology with mean chance constraints to evaluate efficiency when the inputs–outputs data are in the form of random triangular fuzzy numbers with normal distribution and to deal with the same type of data, Yaghoubi and Amiri [38] presented a multi-objective stochastic fuzzy DEA model with a common set of weights under mean chance constraints to evaluate efficiency in a dynamic environment. Further, the DDEA model of Emrouznejad and Yang [8] has been extended by Yen and Chiou [39] to handle fuzzy data and is solved by embedding the fuzzy DEA approach of Lan et al. [40].

Zhou et al. [25] developed a goal sequence with the help of a benchmarking model based on dynamic DEA in an uncertain environment (triangular fuzzy number) and used α-cut approach to measure efficiency and presented a layering scheme for the suppliers. Ebrahimi et al. [41] developed a slacks-based approach in dynamic network DEA with free disposal hull in which four types of carryovers (good, bad, discretionary, and non-discretionary carryovers) are considered for the interdependence of two consecutive periods when the data for all the variables are interval numbers. The main feature of their study is that all the inefficient DMUs are projected to the existing DMUs on the frontier. Bansal and Mehra [42] introduced a directional distance function-based model, namely the interval dynamic network DEA model, to estimate efficiency when the data for inputs and outputs are available in the form of integers, intervals, or negative data. Both optimistic and pessimistic approaches were followed to evaluate interval efficiencies when the periods are connected by the desirable and undesirable carryovers.

10.4.2 FDDEA with Network Structure

Although dynamic DEA incorporates the time factor, there is still a limitation that it ignores the internal structure of a DMU. To deal with the issue, many researchers studied dynamic DEA with different types of network structures, which can be seen in Hashimoto et al. [43], Avkiran and McCrystal [44], Tone and Tsutsui [45], Khalili-Damghani et al. [46], and Omrani and Soltanzadeh [47].

Kordrostami et al. [31] and Keikha-Javan et al. [32] presented DNDEA models to study the internal structure of DMUs in a dynamic environment and evaluated interval overall and interval period efficiencies for the whole system and each subunit where the subunits are connected in parallel to each other, and the data are in an imprecise form, particularly interval form. Kordrostami et al. [31] provided a relationship between the interval system efficiency and the interval efficiency of subunits in a manner that the interval dynamic efficiency of all the systems can be derived by taking the sum or average of the interval dynamic efficiency of its subunits.

Considering into account the complexity of structures present in real-life problems, Zadeh [23] introduced type-2 fuzzy sets. Olfat et al. [26] extended dynamic network slacks-based measure (DNSBM) to deal with trapezoidal interval type-2 fuzzy data with undesirable inputs–outputs. Let there be n DMUs with three nodes connected through links, and periods are connected through carryovers as depicted in Fig. 10.2. The trapezoidal interval type-2 fuzzy data are transformed into the interval data by deriving its lower and upper bounds from Eqs. (10.4) and (10.5), respectively. Let $\tilde{\tilde{A}}$ be an interval type-2 fuzzy number [26] written as

$$\begin{aligned}\tilde{\tilde{A}} &= (\tilde{A}^U, \tilde{A}^L)\\ &= ((d_1^U, d_2^U, d_3^U, d_4^U, h_1(\tilde{A}^U), h_2(\tilde{A}^U)), (d_1^L, d_2^L, d_3^L, d_4^L, h_1(\tilde{A}^L), h_2(\tilde{A}^L))),\end{aligned}$$

then lower (M^L) and upper (M^U) bounds of transformed interval number are defined as

$$M^L = \frac{1}{6}(d_1^U + 2d_2^U)h_1^U + \frac{1}{6}(d_1^L + 2d_2^L)h_1^L \tag{10.4}$$

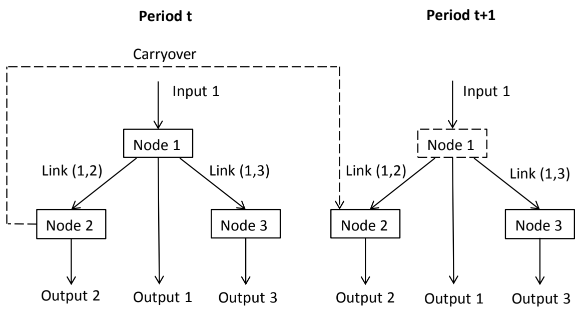

Fig. 10.2 Dynamic network structure with three nodes over two periods

$$M^U = \frac{1}{6}(d_4^U + 2d_3^U)h_2^U + \frac{1}{6}(d_4^L + 2d_3^L)h_2^L \tag{10.5}$$

After transforming all the data into interval numbers, Olfat et al. [26] presented an approach to evaluate the upper bound and lower bound of system efficiency for DMU$_o$ in which they considered constraints related to every input (x), link (z_{link}), carryovers (z_{carry}), and output (y) for all nodes. While writing the constraints, all the undesirable inputs are considered as desirable outputs, whereas all the undesirable outputs are treated as desirable inputs, respectively. Also, a link from node a to node b is both an input (desirable) to node b and output (desirable) of node a. So two sets of constraints related to desirable inputs–outputs for each node (k) and time period (t) are presented in Eqs. (10.6) and (10.7), respectively.

$$\sum_{j=1}^{n} x_j^{Lt} \lambda_{kj}^t + s_{ki}^{t-\text{in}} = x_o^{Lt}, \quad \forall t; \ k = 1, 2, 3; \ i = 1, \ldots, n_{\text{in}}, \tag{10.6}$$

$$\sum_{j=1}^{n} y_j^{Ut} \lambda_{kj}^t + s_{kr}^{t-\text{out}} = y_o^{Ut}, \quad \forall t; \ k = 1, 2, 3; \ r = 1, \ldots, n_{\text{out}}. \tag{10.7}$$

Equations (10.8) and (10.9) depict the continuity of links between nodes and continuity of carryovers between two consecutive periods, respectively. Equation (10.10) represents the variable returns to scale, and Eq. (10.11) represents the non-negativity of weights and slacks.

$$\sum_{j=1}^{n} z_{\text{link }j}^{U(\text{out})} \lambda_{bj}^t = \sum_{j=1}^{n} z_{\text{link }j}^{L(\text{in})} \lambda_{aj}^t, \quad \forall t, \ \forall j \tag{10.8}$$

$$\sum_{j=1}^{n} z_{\text{carry }j}^{U(\text{out})} \lambda_{bj}^t = \sum_{j=1}^{n} z_{\text{carry }j}^{L(\text{in})} \lambda_{aj}^{t+1}, \quad \forall t, \ \forall j \tag{10.9}$$

$$\sum_{j=1}^{n} \lambda_{kj}^t = 1, \quad \forall k; \ \forall t, \tag{10.10}$$

$$\forall s \geq 0;\ \forall \lambda \geq 0. \tag{10.11}$$

The upper bound of system efficiency can be obtained from the objective function defined in Eq. (10.12) subject to the constraints given by Eqs. (10.6)–(10.11).

$$E_o^U = \min\left[\frac{\sum_{t=1}^{T} W^t\left[\sum_{k=1}^{3} w^k\left[1-\frac{1}{n_{\text{in}}+l_{\text{in}}+c_{\text{in}}}\left(\sum_{i=1}^{n_{\text{in}}}\frac{s_{ki}^{t-\text{in}}}{x_o^{Lt}}+\sum_{l=1}^{l_{\text{in}}}\frac{s_{kl}^{t-\text{in}}}{z_{\text{link }o}^{Lt}}+\sum_{c=1}^{c_{\text{in}}}\frac{s_{kc}^{(t,t+1)-\text{in}}}{z_{\text{carry }o}^{L(t,t+1)}}\right)\right]\right]}{\sum_{t=1}^{T} W^t\left[\sum_{k=1}^{3} w^k\left[1+\frac{1}{n_{\text{out}}+l_{\text{out}}+c_{\text{out}}}\left(\sum_{r=1}^{n_{\text{out}}}\frac{s_{kr}^{t-\text{out}}}{y_o^{Ut}}+\sum_{l=1}^{l_{\text{out}}}\frac{s_{kl}^{t-\text{out}}}{z_{\text{link }o}^{Ut}}+\sum_{c=1}^{c_{\text{out}}}\frac{s_{kc}^{(t,t+1)-\text{out}}}{z_{\text{carry }o}^{U(t,t+1)}}\right)\right]\right]}\right] \tag{10.12}$$

Upper bound efficiencies of node k for DMU$_o$ in period t ($E_{ko}^{U(t)}$) and for whole time interval (E_{ko}^U) are evaluated using Eqs. (10.13) and (10.14).

$$E_{ko}^{U(t)} = \min\frac{1-\frac{1}{n_{\text{in}}+l_{\text{in}}+c_{\text{in}}}\left(\sum_{i=1}^{n_{\text{in}}}\frac{s_{ki}^{t-\text{in}}}{x_o^{Lt}}+\sum_{l=1}^{l_{\text{in}}}\frac{s_{kl}^{t-\text{in}}}{z_{\text{link }o}^{Lt}}+\sum_{c=1}^{c_{\text{in}}}\frac{s_{kc}^{(t,t+1)-\text{in}}}{z_{\text{carry }o}^{L(t,t+1)}}\right)}{1+\frac{1}{n_{\text{out}}+l_{\text{out}}+c_{\text{out}}}\left(\sum_{r=1}^{n_{\text{out}}}\frac{s_{kr}^{t-\text{out}}}{y_o^{Ut}}+\sum_{l=1}^{l_{\text{out}}}\frac{s_{kl}^{t-\text{out}}}{z_{\text{link }o}^{Ut}}+\sum_{c=1}^{c_{\text{out}}}\frac{s_{kc}^{(t,t+1)-\text{out}}}{z_{\text{carry }o}^{U(t,t+1)}}\right)} \tag{10.13}$$

$$E_{ko}^{U} = \min\frac{\sum_{t=1}^{T} W^t\left[1-\frac{1}{n_{\text{in}}+l_{\text{in}}+c_{\text{in}}}\left(\sum_{i=1}^{n_{\text{in}}}\frac{s_{ki}^{t-\text{in}}}{x_o^{Lt}}+\sum_{l=1}^{l_{\text{in}}}\frac{s_{kl}^{t-\text{in}}}{z_{\text{link }o}^{Lt}}+\sum_{c=1}^{c_{\text{in}}}\frac{s_{kc}^{(t,t+1)-\text{in}}}{z_{\text{carry }o}^{L(t,t+1)}}\right)\right]}{\sum_{t=1}^{T} W^t\left[1+\frac{1}{n_{\text{out}}+l_{\text{out}}+c_{\text{out}}}\left(\sum_{r=1}^{n_{\text{out}}}\frac{s_{kr}^{t-\text{out}}}{y_o^{Ut}}+\sum_{l=1}^{l_{\text{out}}}\frac{s_{kl}^{t-\text{out}}}{z_{\text{link }o}^{Ut}}+\sum_{c=1}^{c_{\text{out}}}\frac{s_{kc}^{(t,t+1)-\text{out}}}{z_{\text{carry }o}^{U(t,t+1)}}\right)\right]} \tag{10.14}$$

In a similar way, lower bound of system efficiency can be evaluated from objective function defined in Eq. (10.15) subject to the constraints given by Eqs. (10.16) and (10.17) with Eqs. (10.8)–(10.11) and lower bounds of interval efficiencies of node k in period t ($E_{ko}^{L(t)}$) and for whole time interval (E_{ko}^L) are evaluated using Eqs. (10.18) and (10.19).

$$E_o^L = \min\left[\frac{\sum_{t=1}^{T} W^t\left[\sum_{k=1}^{3} w^k\left[1-\frac{1}{n_{\text{in}}+l_{\text{in}}+c_{\text{in}}}\left(\sum_{i=1}^{n_{\text{in}}}\frac{s_{ki}^{t-\text{in}}}{x_o^{Ut}}+\sum_{l=1}^{l_{\text{in}}}\frac{s_{kl}^{t-\text{in}}}{z_{\text{link }o}^{Ut}}+\sum_{c=1}^{c_{\text{in}}}\frac{s_{kc}^{(t,t+1)-\text{in}}}{z_{\text{carry }o}^{U(t,t+1)}}\right)\right]\right]}{\sum_{t=1}^{T} W^t\left[\sum_{k=1}^{3} w^k\left[1+\frac{1}{n_{\text{out}}+l_{\text{out}}+c_{\text{out}}}\left(\sum_{r=1}^{n_{\text{out}}}\frac{s_{kr}^{t-\text{out}}}{y_o^{Lt}}+\sum_{l=1}^{l_{\text{out}}}\frac{s_{kl}^{t-\text{out}}}{z_{\text{link }o}^{Lt}}+\sum_{c=1}^{c_{\text{out}}}\frac{s_{kc}^{(t,t+1)-\text{out}}}{z_{\text{carry }o}^{L(t,t+1)}}\right)\right]\right]}\right] \tag{10.15}$$

$$\sum_{j=1}^{n} x_j^{Lt}\lambda_{kj}^t + s_{ki}^{t-\text{in}} = x_o^{Ut},\quad \forall t;\ k=1,2,3;\ i=1,\ldots,n_{\text{in}}, \tag{10.16}$$

$$\sum_{j=1}^{n} y_j^{Ut}\lambda_{kj}^t + s_{kr}^{t-\text{out}} = y_o^{Lt},\quad \forall t;\ k=1,2,3;\ r=1,\ldots,n_{\text{out}}. \tag{10.17}$$

$$E_{ko}^{L(t)} = \min \frac{1 - \frac{1}{n_{\text{in}} + l_{\text{in}} + c_{\text{in}}} \left(\sum_{i=1}^{n_{\text{in}}} \frac{s_{ki}^{t-\text{in}}}{x_o^{Ut}} + \sum_{l=1}^{l_{\text{in}}} \frac{s_{kl}^{t-\text{in}}}{z_{\text{link } o}^{Ut}} + \sum_{c=1}^{c_{\text{in}}} \frac{s_{kc}^{(t,t+1)-\text{in}}}{z_{\text{carry } o}^{U(t,t+1)}} \right)}{1 + \frac{1}{n_{\text{out}} + l_{\text{out}} + c_{\text{out}}} \left(\sum_{r=1}^{n_{\text{out}}} \frac{s_{kr}^{t-\text{out}}}{y_o^{Lt}} + \sum_{l=1}^{l_{\text{out}}} \frac{s_{kl}^{t-\text{out}}}{z_{\text{link } o}^{Lt}} + \sum_{c=1}^{c_{\text{out}}} \frac{s_{kc}^{(t,t+1)-\text{out}}}{z_{\text{carry } o}^{L(t,t+1)}} \right)} \tag{10.18}$$

$$E_{ko}^{L} = \min \frac{\sum_{t=1}^{T} W^t \left[1 - \frac{1}{n_{\text{in}} + l_{\text{in}} + c_{\text{in}}} \left(\sum_{i=1}^{n_{\text{in}}} \frac{s_{ki}^{t-\text{in}}}{x_o^{Ut}} + \sum_{l=1}^{l_{\text{in}}} \frac{s_{kl}^{t-\text{in}}}{z_{\text{link } o}^{Ut}} + \sum_{c=1}^{c_{\text{in}}} \frac{s_{kc}^{(t,t+1)-\text{in}}}{z_{\text{carry } o}^{U(t,t+1)}} \right) \right]}{\sum_{t=1}^{T} W^t \left[1 + \frac{1}{n_{\text{out}} + l_{\text{out}} + c_{\text{out}}} \left(\sum_{r=1}^{n_{\text{out}}} \frac{s_{kr}^{t-\text{out}}}{y_o^{Lt}} + \sum_{l=1}^{l_{\text{out}}} \frac{s_{kl}^{t-\text{out}}}{z_{\text{link } o}^{Lt}} + \sum_{c=1}^{c_{\text{out}}} \frac{s_{kc}^{(t,t+1)-\text{out}}}{z_{\text{carry } o}^{L(t,t+1)}} \right) \right]} \tag{10.19}$$

Olfat and Pishdar [48] presented an extended version of DNSBM to evaluate both optimistic and pessimistic efficiencies with interval type-2 fuzzy data in the presence of undesirable inputs–outputs.

Although dynamic DEA measures the efficiency of a DMU by taking into account the interdependence of periods, ignoring the internal structure of DMUs may produce misleading results. Tone and Tsutsui [45] proposed a slacks-based dynamic network DEA model to compute system and period efficiencies when there exist four types of links (as input link, as output link, free link, and fixed link) between subdivisions of a DMU, and similarly, the periods are connected through four types of carryovers, namely desirable, undesirable, free, and fixed carryovers. A dynamic network structure with subdivisions linked to each other through intermediate links and periods connected through carryovers is shown in Fig. 10.3. Soltanzadeh and Omrani [49] introduced a dynamic network DEA (DNDEA) model to evaluate efficiency using the α-cut approach when the data for inputs, outputs, and links are of type-1 fuzzy data. They extended the dynamic DEA model proposed by Omrani and Soltanzadeh [47] in the presence of fuzzy data.

Nomenclature

n :	Number of DMUs ($j = 1, \ldots, n$)
K:	Number of divisions in a DMU ($k = 1, \ldots, K$)
p:	Number of time periods ($t = 1, \ldots, p$)
m_k :	Number of inputs of kth division and $i^k \in \{1, 2, \ldots, m_k\}$
s_k :	Number of outputs of kth division and $r^k \in \{1, 2, \ldots, s_k\}$
l_k :	Number of links from kth division to the next division and $l^k \in \{1, 2, \ldots, l_k\}$
d_k :	Number of carryovers at kth division from period t to $t+1$ and $d^k \in \{1, 2, \ldots, d_k\}$

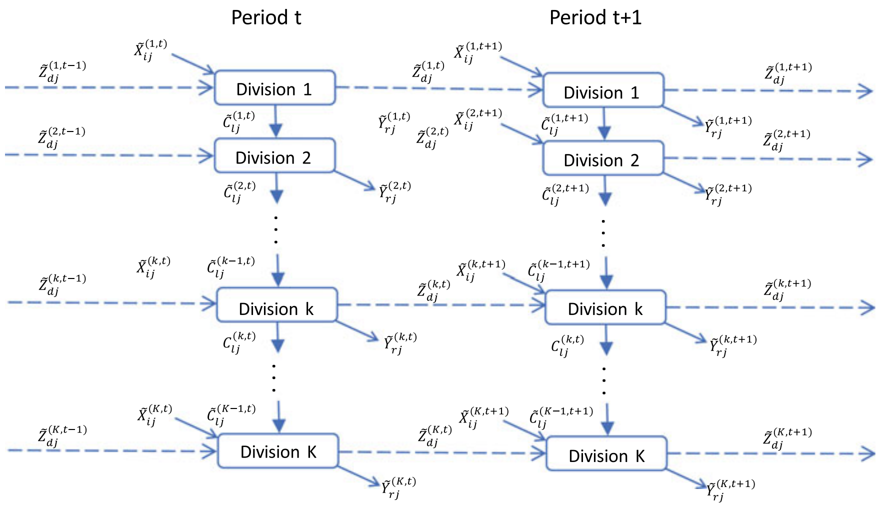

Fig. 10.3 Dynamic structure with K divisions over two periods t and $t+1$

$\tilde{X}_{ij}^{(k,t)}$:	ith fuzzy input of DMU$_j$ for kth division in period t where $i = 1, 2, \ldots, m_k, \ldots, m$
$\tilde{Y}_{gj}^{(k,t)}$:	gth fuzzy output of DMU$_j$ for kth division in period t where $r = 1, 2, \ldots, s_k, \ldots, s$
$\tilde{C}_{lj}^{(k,t)}$:	ith fuzzy intermediate link of DMU$_j$ for kth to $(k+1)$th division in period t where $l = 1, 2, \ldots, l_k, \ldots, L$
$\tilde{Z}_{dj}^{(k,t)}$:	dth fuzzy carry-over of DMU$_j$ for kth division from period t to $t+1$ where $d = 1, 2, \ldots, d_k, \ldots, D$

Let $\tilde{X}_{io} = \sum_{t=1}^{p} \sum_{k=1}^{K} \tilde{X}_{io}^{(k,t)}$ and $\tilde{Y}_{ro} = \sum_{t=1}^{p} \sum_{k=1}^{K} \tilde{Y}_{ro}^{(k,t)}$ be the ith fuzzy system input and rth fuzzy system output for DMU$_o$ and the initial fuzzy carryovers as inputs to division k from period t_0 and the final fuzzy carryovers as outputs from division k at period p be denoted by $\tilde{Z}_{do}^{(k,t_0)}$ and $\tilde{Z}_{do}^{(k,p)}$, respectively. Then efficiency of DMU$_o$ over p periods is evaluated using following model:

Model-3

$$E_o^S = \max \sum_{r=1}^{s} u_r \tilde{Y}_{ro} + \sum_{k=1}^{K} \sum_{d=1}^{D} f_d \tilde{Z}_{do}^{(k,p)}$$

$$\text{s.t.} \ \sum_{i=1}^{m} v_i \tilde{X}_{io} + \sum_{k=1}^{K} \sum_{d=1}^{D} f_d \tilde{Z}_{do}^{(k,t_0)} = 1$$

$$\sum_{r=1}^{s} u_r \tilde{Y}_{rj} + \sum_{k=1}^{K} \sum_{d=1}^{D} f_d \tilde{Z}_{dj}^{(k,p)} - \sum_{i=1}^{m} v_i \tilde{X}_{ij} - \sum_{k=1}^{K} \sum_{d=1}^{D} f_d \tilde{Z}_{dj}^{(k,t_0)} \leq 0 \ \ \forall j,$$

$$\sum_{r\in r^1} u_r \tilde{Y}_{rj}^{(1,t)} + \sum_{l\in l^1} w_l \tilde{C}_{lj}^{(1,t)} + \sum_{d\in d^1} f_d \tilde{Z}_{dj}^{(1,t)} - \sum_{i\in i^1} v_i \tilde{X}_{ij}^{(1,t)} - \sum_{d\in d^{(1)}} f_d \tilde{Z}_{dj}^{(1,t-1)} \leq 0 \forall j;\ \forall t;\ k = 1,$$

$$\sum_{r\in r^k} u_r \tilde{Y}_{rj}^{(k,t)} + \sum_{l\in l^k} w_l \tilde{C}_{lj}^{(k,t)} + \sum_{d\in d^k} f_d \tilde{Z}_{dj}^{(k,t)} - \sum_{i\in i^k} v_i \tilde{X}_{ij}^{(k,t)} - \sum_{l\in l^k} w_l \tilde{C}_{lj}^{(k-1,t)} - \sum_{d\in d^{(k)}} f_d \tilde{Z}_{dj}^{(k,t-1)} \leq 0\ \forall j;\ \forall t;\ k = 2, \ldots, K-1,$$

$$\sum_{r\in r^K} u_r \tilde{Y}_{rj}^{(K,t)} + \sum_{d\in d^K} f_d \tilde{Z}_{dj}^{(K,t)} - \sum_{i\in i^K} v_i \tilde{X}_{ij}^{(K,t)} - \sum_{l\in l^k} w_l \tilde{C}_{lj}^{(K-1,t)} - \sum_{d\in d^{(K)}} f_d \tilde{Z}_{dj}^{K(t-1)} \leq 0\ \forall j;\ \forall t;\ k = K,$$

$$v_i \geq \epsilon\ \forall i;\ u_r \geq \epsilon\ \forall r;\ f_d \geq \epsilon\ \forall d;\ w_l \geq \epsilon\ \forall l.$$

It is obvious that the efficiency obtained from fuzzy numbers will also be a fuzzy number. Let $\mu_{\tilde{X}_{ij}}$, $\mu_{\tilde{Y}_{rj}}$, $\mu_{\tilde{Z}_{dj}}$, and $\mu_{\tilde{C}_{lj}}$ be the membership functions of $\tilde{X}_{ij}$, $\tilde{Y}_{rj}$, $\tilde{Z}_{dj}$, and $\tilde{C}_{lj}$, respectively. Then, membership function $\mu_{\tilde{E}_k}$ for system efficiency of division k of DMU$_o$ denoted by $\tilde{E}_k$ is given by

$$\mu_{\tilde{E}_o}(e) = \sup_{x,y,z,c} \min_{i.r.d.l} \{\mu_{\tilde{X}_{ij}}^{(k,t)}(x_{ij}^{(k,t)}), \mu_{\tilde{Y}_{rj}}^{(k,t)}(y_{rj}^{(k,t)}), \mu_{\tilde{Z}_{dj}}^{(k,t)}(z_{dj}^{(k,t)}), \mu_{\tilde{C}_{lj}}^{(k,t)}(c_{lj}^{(k,t)}) | e = E_k(x_{ij}^{(k,t)}, y_{rj}^{(k,t)}, z_{dj}^{(k,t)}, c_{lj}^{(k,t)})\}$$

Soltanzadeh and Omrani [49] used α-cut approach to solve Model-3. The α-cuts for $\tilde{X}_{ij}$, $\tilde{Y}_{rj}$, $\tilde{Z}_{dj}$, and $\tilde{C}_{lj}$ are defined as follows:

$$(X_{ij}^{(k,t)})_\alpha = \left[\min_{X_{ij}^{(k,t)}} \left\{ X_{ij}^{(k,t)} \in S(\tilde{X}_{ij}^{(k,t)}) | \mu_{\tilde{X}_{ij}}^{(k,t)}(X_{ij}^{(k,t)}) \geq \alpha \right\}, \max_{X_{ij}^{(k,t)}} \left\{ X_{ij}^{(k,t)} \in S(\tilde{X}_{ij}^{(k,t)}) | \mu_{\tilde{X}_{ij}}^{(k,t)}(X_{ij}^{(k,t)}) \geq \alpha \right\} \right] = \left[(X_{ij}^{(k,t)})_\alpha^L, (X_{ij}^{(k,t)})_\alpha^U \right]$$

$$(Y_{rj}^{(k,t)})_\alpha = \left[\min_{Y_{rj}^{(k,t)}} \left\{ Y_{rj}^{(k,t)} \in S(\tilde{Y}_{rj}^{(k,t)}) | \mu_{\tilde{Y}_{rj}}^{(k,t)}(Y_{rj}^{(k,t)}) \geq \alpha \right\}, \max_{Y_{rj}^{(k,t)}} \left\{ Y_{rj}^{(k,t)} \in S(\tilde{Y}_{rj}^{(k,t)}) | \mu_{\tilde{Y}_{rj}}^{(k,t)}(Y_{rj}^{(k,t)}) \geq \alpha \right\} \right] = \left[(Y_{rj}^{(k,t)})_\alpha^L, (Y_{rj}^{(k,t)})_\alpha^U \right]$$

$$(Z_{dj}^{(k,t)})_\alpha = \left[\min_{Z_{dj}^{(k,t)}} \left\{ Z_{dj}^{(k,t)} \in S(\tilde{Z}_{dj}^{(k,t)}) | \mu_{\tilde{Z}_{dj}}^{(k,t)}(Z_{dj}^{(k,t)}) \geq \alpha \right\}, \max_{Z_{dj}^{(k,t)}} \left\{ Z_{dj}^{(k,t)} \in S(\tilde{Z}_{dj}^{(k,t)}) | \mu_{\tilde{Z}_{dj}}^{(k,t)}(Z_{dj}^{(k,t)}) \geq \alpha \right\} \right]$$
$$= \left[(Z_{dj}^{(k,t)})_\alpha^L, (Z_{dj}^{(k,t)})_\alpha^U \right]$$

$$(C_{lj}^{(k,t)})_\alpha = \left[\min_{C_{lj}^{(k,t)}} \left\{ C_{lj}^{(k,t)} \in S(\tilde{C}_{lj}^{(k,t)}) | \mu_{\tilde{C}_{lj}}^{(k,t)}(C_{lj}^{(k,t)}) \geq \alpha \right\}, \max_{C_{lj}^{(k,t)}} \left\{ C_{lj}^{(k,t)} \in S(\tilde{C}_{lj}^{(k,t)}) | \mu_{\tilde{C}_{lj}}^{(k,t)}(C_{lj}^{(k,t)}) \geq \alpha \right\} \right]$$
$$= \left[(C_{lj}^{(k,t)})_\alpha^L, (C_{lj}^{(k,t)})_\alpha^U \right]$$

After using an α-cut approach and some transformations, Model-4(a) and Model-4(b) were presented to measure the lower and upper bounds of system efficiency $\tilde{E}_o$ for each α.

Model-4(a)

$$(E_o^S)_\alpha^U = \max \; E_o^S = \frac{\sum_{r=1}^{s} u_r (Y_{ro})_\alpha^U + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,p)}}{\sum_{i=1}^{m} v_i (X_{io})_\alpha^L + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,t_0)}}$$

$$\text{s.t.} \; \frac{\sum_{r=1}^{s} u_r (Y_{ro})_\alpha^U + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,p)}}{\sum_{i=1}^{m} v_i (X_{io})_\alpha^L + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,t_0)}} \leq 1,$$

$$\frac{\sum_{r=1}^{s} u_r (Y_{rj})_\alpha^L + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{dj}^{(k,p)}}{\sum_{i=1}^{m} v_i (X_{ij})_\alpha^U + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,t_0)}} \leq 1; \quad \forall j \neq o,$$

$$\frac{\sum_{r \in r^1} u_r (Y_{ro}^{(1,t)})_\alpha^U + \sum_{l \in l^1} \hat{c}_{lo}^{(1,t)} + \sum_{d \in d^1} \hat{z}_{do}^{(1,t)}}{\sum_{i \in i^1} v_i (X_{io}^{(1,t)})_\alpha^L + \sum_{d \in d^{(1)}} \hat{Z}_{do}^{(1,t-1)}} \leq 1; \quad k = 1,$$

$$\frac{\sum_{r \in r^1} u_r (Y_{rj}^{(1,t)})_\alpha^L + \sum_{l \in l^1} \hat{c}_{lj}^{(1,t)} + \sum_{d \in d^1} \hat{z}_{dj}^{(1,t)}}{\sum_{i \in i^1} v_i (X_{ij}^{(1,t)})_\alpha^U + \sum_{d \in d^{(1)}} \hat{Z}_{dj}^{(1,t-1)}} \leq 1; \; \forall j \neq o; \; k = 1,$$

$$\frac{\sum_{r \in r^k} u_r (Y_{ro}^{(k,t)})_\alpha^U + \sum_{l \in l^k} \hat{c}_{lo}^{(k,t)} + \sum_{d \in d^k} \hat{z}_{do}^{(k,t)}}{\sum_{i \in i^k} v_i (X_{io}^{(k,t)})_\alpha^L + \sum_{l \in l^k} \hat{c}_{lo}^{(k-1,t)} + \sum_{d \in d^{(k)}} \hat{Z}_{do}^{(k,t-1)}} \leq 1; \; \forall k \neq 1, K,$$

$$\frac{\sum_{r \in r^k} u_r (Y_{rj}^{(k,t)})_\alpha^L + \sum_{l \in l^k} \hat{c}_{lj}^{(k,t)} + \sum_{d \in d^k} \hat{z}_{dj}^{(k,t)}}{\sum_{i \in i^k} v_i (X_{ij}^{(k,t)})_\alpha^U + \sum_{l \in l^k} \hat{c}_{lj}^{(k-1,t)} + \sum_{d \in d^{(k)}} \hat{Z}_{dj}^{(k,t-1)}} \leq 1; \; \forall j \neq o; \; \forall k \neq 1, K,$$

$$\frac{\sum_{r \in r^K} u_r (Y_{ro}^{(K,t)})_\alpha^U + \sum_{d \in d^k} \hat{z}_{do}^{(K,t)}}{\sum_{i \in i^K} v_i (X_{io}^{(K,t)})_\alpha^L + \sum_{l \in l^K} \hat{c}_{lo}^{(K-1,t)} + \sum_{d \in d^{(K)}} \hat{Z}_{do}^{K(t-1)}} \leq 1; \; k = K,$$

$$\frac{\sum_{r\in r^K} u_r (Y_{ro}^{(K,t)})_\alpha^L + \sum_{d\in d^k} \hat{z}_{do}^{(K,t)}}{\sum_{i\in i^K} v_i (X_{io}^{(K,t)})_\alpha^U + \sum_{l\in l^K} \hat{c}_{lo}^{(K-1,t)} + \sum_{d\in d^{(K)}} \hat{Z}_{do}^{K(t-1)}} \le 1\ \forall j \ne o;\ k = K,$$

$$w_l (C_{lj}^{(k,t)})_\alpha^L \le \hat{c}_{lj}^{(k,t)} \le w_l (C_{lj}^{(k,t)})_\alpha^U;\ \forall k;\ \forall t$$

$$f_d (Z_{dj}^{(k,t)})_\alpha^L \le \hat{z}_{dj}^{(k,t)} \le f_d (Z_{dj}^{(k,t)})_\alpha^U;\ \forall k;\ \forall t$$

$$v_i \ge \epsilon\ \forall i;\ u_r \ge \epsilon\ \forall r;\ f_d \ge \epsilon\ \forall d;\ w_l \ge \epsilon\ \forall l.$$

Upper bounds of system and process efficiencies in each α-cut by using optimal weights derived from Model-4(a) are defined as follows:

$$(E_o^S)_\alpha^U = \frac{\sum_{r=1}^{s} u_r^* (Y_{ro})_\alpha^U + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,p)}}{\sum_{i=1}^{m} v_i^* (X_{io})_\alpha^L + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,t_0)}}$$

$$(E_o^{S(t)})_\alpha^U = \frac{\sum_{k=1}^{K} \sum_{r=1}^{s} u_r^* (Y_{ro}^{(k,t)})_\alpha^U + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,t)}}{\sum_{k=1}^{K} \sum_{i=1}^{m} v_i^* (X_{io}^{(k,t)})_\alpha^L + \sum_{k=1}^{K} \sum_{d=1}^{D} \hat{z}_{do}^{(k,t-1)}}$$

$$(E_o^{(1,t)})_\alpha^U = \frac{\sum_{r\in r^1} u_r^* (Y_{ro}^{(1,t)})_\alpha^U + \sum_{l\in l^1} \hat{c}_{lo}^{(1,t)} + \sum_{d\in d^1} \hat{z}_{do}^{(1,t)}}{\sum_{i\in i^1} v_i^* (X_{io}^{(1,t)})_\alpha^L + \sum_{d\in d^1} \hat{Z}_{do}^{(1,t-1)}}$$

$$(E_o^{(k,t)})_\alpha^U = \frac{\sum_{r\in r^k} u_r^* (Y_{ro}^{(k,t)})_\alpha^U + \sum_{l\in l^k} \hat{c}_{lo}^{(k,t)} + \sum_{d\in d^k} \hat{z}_{do}^{(k,t)}}{\sum_{i\in i^k} v_i^* (X_{io}^{(k,t)})_\alpha^L + \sum_{d\in d^k} \hat{Z}_{do}^{(k,t-1)}};\ k = 2, \ldots, K-1$$

$$(E_o^{(K,t)})_\alpha^U = \frac{\sum_{r\in r^K} u_r^* (Y_{ro}^{(K,t)})_\alpha^L + \sum_{d\in d^k} \hat{z}_{do}^{(K,t)}}{\sum_{i\in i^K} v_i^* (X_{io}^{(K,t)})_\alpha^U + \sum_{l\in l^K} \hat{c}_{lo}^{(K-1,t)} + \sum_{d\in d^K} \hat{Z}_{do}^{K(t-1)}}$$

Model-4(b)

$$(E_o^s)_\alpha^L = \min\ \theta - \epsilon \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ + \sum_{l=1}^{L} \sum_{k=1}^{K} s_{lk} + \sum_{d=1}^{D} \sum_{k=1}^{K} s_{dk} \right)$$

$$\text{s.t.} \sum_{t=1}^{p} \sum_{k=1}^{K} \left(\lambda_o^{(k,t)} \cdot (X_{io}^{(k,t)})_\alpha^U + \sum_{j=1, j\ne k}^{n} \lambda_j^{(k,t)} \cdot (X_{ij}^{(k,t)})_\alpha^L \right) + s_i^- = \theta (X_{io})_\alpha^U;\ \ \forall i,$$

$$\sum_{t=1}^{p} \sum_{k=1}^{K} \left(\lambda_o^{(k,t)} \cdot (Y_{ro}^{(k,t)})_\alpha^L + \sum_{j=1, j\ne k}^{n} \lambda_j^{(k,t)} \cdot (Y_{rj}^{(k,t)})_\alpha^U \right) - s_r^+ = (Y_{ro})_\alpha^L;\ \ \forall r,$$

$$\sum_{t=1}^{p} \sum_{j=1}^{n} \lambda_j^{(k,t)} \cdot c_{lj}^{(k,t)} - \sum_{t=1}^{p} \sum_{j=1}^{n} \lambda_j^{k+(1,t)} \cdot c_{lj}^{(k,t)} - s_{lk} = 0;\ \ \forall k;\ l \in l^k,$$

$$\sum_{t=1}^{p}\sum_{j=1}^{n}\lambda_j^{(k,t)}\cdot\left(z_{dj}^{(k,t)}-z_{dj}^{(k,t-1)}\right)+z_{do}^{(1,p)}-s_{dk}=\theta z_{do}^{(1,1)};\quad \forall k;\ d\in d^k,$$
$$(C_{lj})_\alpha^L\le c_{lj}\le (C_{lj})_\alpha^U;\quad l=1,\ldots,L,$$
$$(Z_{dj})_\alpha^L\le z_{dj}\le (Z_{dj})_\alpha^U;\quad d=1,\ldots,D,$$
$$\lambda_j^{(k,t)},s_i^-,s_r^+,s_{lk},s_{dk}\ge 0,\theta \text{ is free};\ \forall j;\ \forall k;\ \forall t.$$

By using optimal weights derived from Model-4(b), lower bounds of system and process efficiencies in each α-cut are defined as follows:

$$(E_o^S)_\alpha^L=\frac{\sum_{r=1}^{s}u_r^*(Y_{ro})_\alpha^L+\sum_{k=1}^{K}\sum_{d=1}^{D}f_d^*\hat{z}_{do}^{(k,p)^*}}{\sum_{i=1}^{m}v_i^*(X_{io})_\alpha^U+\sum_{k=1}^{K}\sum_{d=1}^{D}f_d^*\hat{z}_{do}^{(k,t_0)^*}}$$
$$(E_o^{S(t)})_\alpha^L=\frac{\sum_{k=1}^{K}\sum_{r=1}^{s}u_r^*(Y_{ro}^{(k,t)})_\alpha^L+\sum_{k=1}^{K}\sum_{d=1}^{D}f_d^*\hat{z}_{do}^{(k,t)^*}}{\sum_{k=1}^{K}\sum_{i=1}^{m}v_i^*(X_{io}^{(k,t)})_\alpha^U+\sum_{k=1}^{K}\sum_{d=1}^{D}f_d^*\hat{z}_{do}^{(k,t-1)^*}}$$
$$(E_o^{(1,t)})_\alpha^L=\frac{\sum_{r\in r^1}u_r^*(Y_{ro}^{(1,t)})_\alpha^L+\sum_{l\in l^1}w_l^*c_{lo}^{(1,t)}+\sum_{d\in d^1}f_d^*z_{do}^{(1,t)^*}}{\sum_{i\in i^1}v_i^*(X_{io}^{(1,t)})_\alpha^U+\sum_{d\in d^1}f_d^*z_{do}^{(1,t-1)^*}}$$
$$(E_o^{(k,t)})_\alpha^L=\frac{\sum_{r\in r^k}u_r^*(Y_{ro}^{(k,t)})_\alpha^L+\sum_{l\in l^k}w_l^*c_{lo}^{(k,t)^*}+\sum_{d\in d^k}f_d^*z_{do}^{(k,t)^*}}{\sum_{i\in i^k}v_i^*(X_{io}^{(k,t)})_\alpha^U+\sum_{d\in d^k}f_d^*z_{do}^{(k,t-1)^*}};\ k=2,\ldots,K-1$$
$$(E_o^{(K,t)})_\alpha^L=\frac{\sum_{r\in r^K}u_r^*(Y_{ro}^{(K,t)})_\alpha^L+\sum_{d\in d^k}f_d^*z_{do}^{(K,t)^*}}{\sum_{i\in i^K}v_i^*(X_{io}^{(K,t)})_\alpha^U+\sum_{l\in l^K}w_l^*c_{lo}^{(K-1,t)^*}+\sum_{d\in d^K}z_{do}^{K(t-1)^*}}$$

Olfat et al. [50] presented an interval type-2 fuzzy dynamic DEA model to deal with uncertainties and measure the performance of DMUs in a dynamic environment. Ebrahimi et al. [41] developed a slacks-based DNDEA model in which different weights are assigned to different divisions, and the divisions are linked through four types of links, namely input link, output link, free link, and fixed or non-discretionary link. All the inputs, outputs, and links are in the form of interval numbers.

10.4.3 Applications of FDDEA

Jafarian-Moghaddam and Ghoseiri [21, 51] proposed fuzzy dynamic DEA in a multi-objective framework and evaluated the performance of 49 railways from all over the world with fuzzy data in a dynamic environment. Kordrostami et al. [31] assessed the efficiency of ten bank areas in Iran by proposing a DNDEA model. Each area comprises three bank branches considered as subunits for three (six-month) periods with interval data for inputs and outputs. Keikha-Javan et al. [32] also used the same data as in Kordrostami et al. [31] to present an application of their model and provided better results than Kordrostami et al. [31]. Yaghoubi et al. [37] and Yaghoubi and Amiri [38] applied a dynamic random fuzzy DEA model on Iranian petroleum company and evaluated the efficiency of five gas stations over two financial periods using the DRF-DEA model and multi-objective stochastic fuzzy DEA (MOFS-DEA) model, respectively. The data used for inputs–outputs were in the form of random triangular fuzzy number with normal distribution, and the efficiency results from both the above-mentioned approaches turned out to be better than the hybrid genetic algorithm proposed by Qin and Liu [52] to deal with fuzzy random inputs–outputs.

Considering the importance of sustainable development, Olfat et al. [26] suggested an extension of DNSBM to calculate the sustainable performance of 28 airports in Iran over two periods. The whole structure is divided into three nodes: (i) airport node, (ii) community node, and (iii) passenger node, and the efficiency is evaluated for each node in different periods as well as the system efficiency in the presence of interval type-2 fuzzy data for inputs and outputs when some of the inputs–outputs are undesirable. Olfat and Pishdar [48] investigated the efficiency of same 28 Iranian airports with the same structure as studied in Olfat et al. [26], but by using both the efficient and inefficient production frontiers, i.e., evaluated the efficiencies from both optimistic and pessimistic viewpoints, whereas, in Olfat et al. [26], efficiencies were evaluated using only optimistic viewpoint and revealed that the former approach exhibits more discrimination power.

Soltanzadeh and Omrani [49] presented a DNDEA model to calculate the efficiency of seven Iranian airlines for the period 2010–12. The network structure of airlines consists of two stages, namely production and consumption, and the data for inputs–outputs belong to the set of triangular fuzzy numbers. In the same manner, Olfat et al. [50] introduced a DNDEA model to assess the performance of the 20 most popular passenger airports in Iran from the viewpoint of sustainability while using interval type-2 fuzzy data for inputs–outputs.

Based on the ideas of Tone and Tsutsui [45] and Wang and Chin [53], Zhou et al. [54] developed a double frontier dynamic network DEA approach for performance evaluation of sustainable supply chains (SSCs) with a network structure of three stages: (i) supplier stage, (ii) manufacturer stage, and (iii) distributor stage. Twenty SSCs are considered for efficiency evaluation (system and period efficiencies) over three periods with interval type-2 fuzzy data for customer satisfaction (desirable output) and environmental pollution (undesirable output). Ebrahimi et al. [41] also evaluated the efficiency of supply chains by using a slacks-based DNDEA model. Thirty Iranian printing supply chains with three divisions (production, assembly, and distribution) for three consecutive periods (2015–2017) are chosen for the case study. The interval overall and period divisional efficiencies are evaluated along with projected values for all the divisions of inefficient DMUs. Hasani and Mokhtari [55] proposed a hybrid fuzzy multi-criteria decision-making (DEA-MCDM) model to evaluate the efficiency of 11 Iranian hospitals with three nodes, namely hospital, community, and patient node. Torabandeh et al. [56] presented a dynamic network DEA model to evaluate and compare the performance of Iran with other countries.

Zhou et al. [25] assessed the efficiency of 20 suppliers for three periods and also set the practical goals (targets) for suppliers by using a goal sequence based on a dynamic DEA model in an uncertain environment. Bansal and Mehra [42] investigated the interval efficiency of 11 Indian airlines over three consecutive periods in the presence of integer and negative data by using dynamic interval DEA. Table 10.1 represents the categorization of publications on applications of FDDEA studies which depicts its implementation in sectors like airlines, supply chains, gas stations, banks, and various other sectors, including railways, oil refineries, bus companies, and hospitals.

10.4.4 Integration of FDDEA with Other Techniques

Khodaparasti and Maleki [57] proposed an integrated approach in a dynamic fuzzy environment by combining a dynamic location model and fuzzy simultaneous DEA model for emergency medical services (EMS). Yaghoubi et al. [37] presented a DRF-DEA model with fuzzy data, which is further converted to a multi-objective programming problem and later on to a single-objective programming problem for performance evaluation. Further, an integrated Monte Carlo simulation and genetic algorithm have been designed to solve the single-objective programming.

Yaghoubi and Amiri [38] proposed a multi-objective stochastic fuzzy DEA (MOFS-DEA) model to evaluate performance in a dynamic environment and designed an integrated meta-heuristic algorithm using imperialist competitive algorithm and Monte Carlo simulation to solve the one objective stochastic model obtained from the initial MOFS-DEA model by using infinite norm approach.

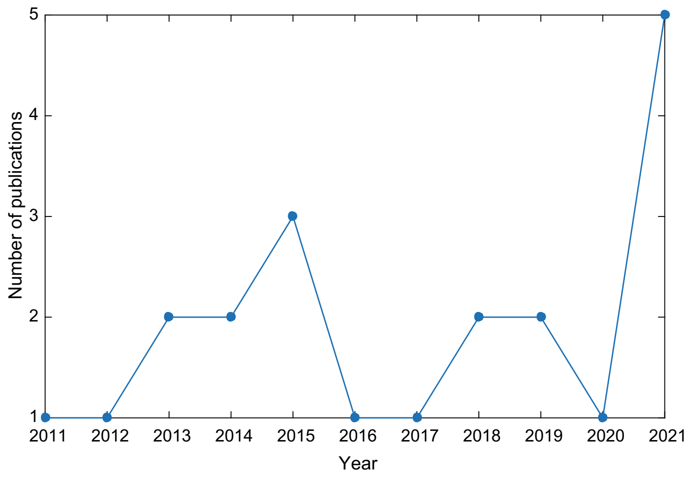

Fig. 10.4 Year-wise publications of fuzzy dynamic data envelopment analysis

Hasani and Mokhtari [55] developed a hybrid fuzzy DEA-MCDM model incorporating fuzzy decision-making trial and evaluation laboratory (DEMATEL) and best-worst method (BWM) to evaluate the system and period efficiencies from interval type-2 fuzzy data. To measure the future as well as past efficiency of suppliers, Nikabadi and Moghaddam [58] developed a hybrid approach by combining an adaptive neuro-fuzzy inference system and dynamic DEA. Figure 10.4 depicts the year-wise publications of FDDEA studies for the periods 2011–2021, from which it can be seen that each year has at least one publication, and the current year 2021 has the highest number of publications on FDDEA in the last decade.

Table 10.1 Publications based on applied study and their characteristics

Area	Study	DMUs	Inputs/outputs/links	Citations
Airports	Olfat et al. [26]	28	Inputs: budget, policy making based on sustainable development concept Outputs: non-aviation income, level of pollution, satisfaction Links: number of aircrafts (takeoff and landing), service quality Carryovers: corporate reputation	63
	Olfat and Pishdar [48]	28	Inputs: policy making based on sustainable development concept, budget Outputs: non-aviation income, pollution levels, satisfaction Links: number of aircrafts (takeoff and landing), service quality, perceived social responsibility Carryovers: corporate reputation	8
	Soltanzadeh and Omrani [49]	7	Inputs: number of employees Outputs: passenger-kilometer performed, passenger ton-kilometer performed Link: number of scheduled flights, available ton-kilometer, available seat-kilometer	25

(continued)

Table 10.1 (continued)

Area	Study	DMUs	Inputs/outputs/links	Citations
	Olfat et al. [50]	20	–	–
	Bansal and Mehra [42]	11	Inputs: operating expenses Outputs: operating revenue, passengers carried per month, pax load factor per month, cargo carried per month Carryovers: losses carried forward after tax, fleet size	–
Supply chains	Zhou et al. [54]	20	Inputs: cost of labor safety, other costs Outputs: degree of environmental pollution Links: value of raw material, value of finished products Carryovers: unrecovered revenue, unpaid cost	24
	Zhou et al. [25]	20	Inputs: technical and financial capability, cost of work safety Outputs: value of raw material, environmental pollution, degree of customer satisfaction Carryovers: accounts receivable, accounts payable	2
	Ebrahimi et al. [41]	30	Inputs: production capacity, planning cost, cardboard and ink cost, electricity cost, machinery cost, labor cost, transportation cost, environmental cost Outputs: label and catalog income, income Links: finished goods, wasted product, recycled waste Carryover: depreciation	–

(continued)

Table 10.1 (continued)

Area	Study	DMUs	Inputs/outputs/links	Citations
Gas stations	Yaghoubi et al. [37]	5	Inputs: employees salaries, operation costs, net profit Outputs: gasoline, net profit Carryover: net profit	3
	Yaghoubi and Amiri [38]	5	Inputs: employees salaries, operation costs, net profit Outputs: gasoline, net profit Carryover: net profit	1
Banks	Kordrostami et al. [31]	30	Inputs: personnel Outputs: usage Carryover: resources	6
	Keikha-Javan et al. [32]	30	Inputs: personnel Outputs: usage Carryover: resources	2
Railways	Jafarian-Moghaddam and Ghoseiri [21]	49	Inputs: length of single, double and electrify track, number of state and private own wagons, fleet size of locomotives, coaches and railcars, and employees Outputs: total train kilometers, gross train tonne kilometers, gross tonne kilometers, gross tonne carried, passengers, passenger kilometers Carryovers: gross tonne kilometers and passenger kilometers	38
Oil refineries	Tavana et al. [59]	9	Inputs: feed, energy consumption, fuel, personal staff, degree of complexity, API Outputs: ratio of light to heavy product, waste (non-permissible CO_2), permissible CO_2	17

(continued)

Table 10.1 (continued)

Area	Study	DMUs	Inputs/outputs/links	Citations
Bus companies	Yen and Chiou [39]	10	Inputs: operating network, number of buses Outputs: operating revenue, number of bus runs, passenger kilometers, passenger satisfaction Carryover: number of buses	1
Hospitals	Hasani and Mokhtari [55]	11	Inputs: policy making based on sustainable concept, budget Outputs: non-healthcare service income, hospital waste, satisfaction Links: social responsibility, population coverage, total bed number, service quality Carryover: hospital reputation	1

10.5 Conclusion

Dynamic DEA with fuzzy set theory is used to measure the inter-temporal efficiency of similar DMUs in an uncertain environment. This study launches a taxonomy and review of recent developments in FDDEA studies in the last decade, and it has been found that FDDEA is still in its initial stage of development. Based on the types of publications used in this paper, FDDEA studies are grouped into four categories, (i) theoretical development of FDDEA models with different fuzzy sets, (ii) FDDEA with network structure, (iii) application of FDDEA, and (iv) integration of FDDEA with other techniques. Figure 10.4 and Table 10.1 clearly depict that FDDEA has been emerging over the years with its concrete applications in various sectors.

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